Magrathea-Pathfinder: a 3D adaptive-mesh code for geodesic ray tracing in N-body simulations

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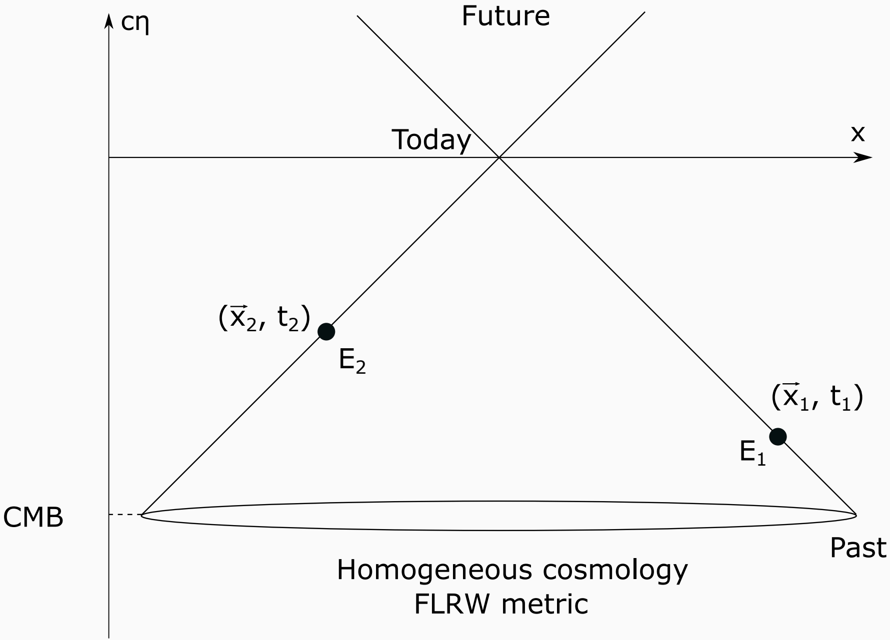
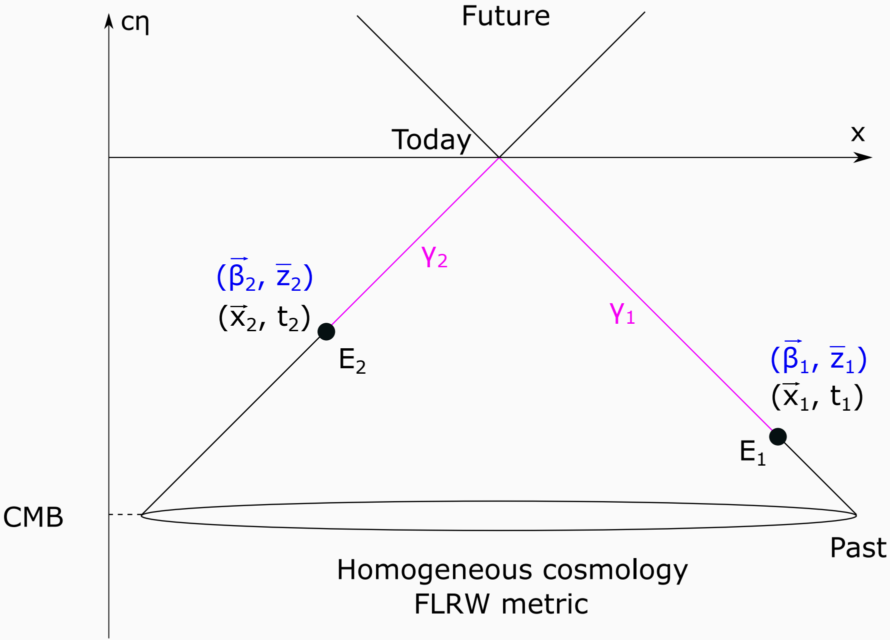
17/01/2023

Light propagation



cη
Future
Today
x
$(\vec{x}_2, t_2)$
E_2
$(\vec{x}_1, t_1)$
E_1
CMB
Past
Homogeneous cosmology
FLRW metric



$c\eta$
Future
Today
x
γ_2
$(\vec{\beta}_2, \bar{z}_2)$
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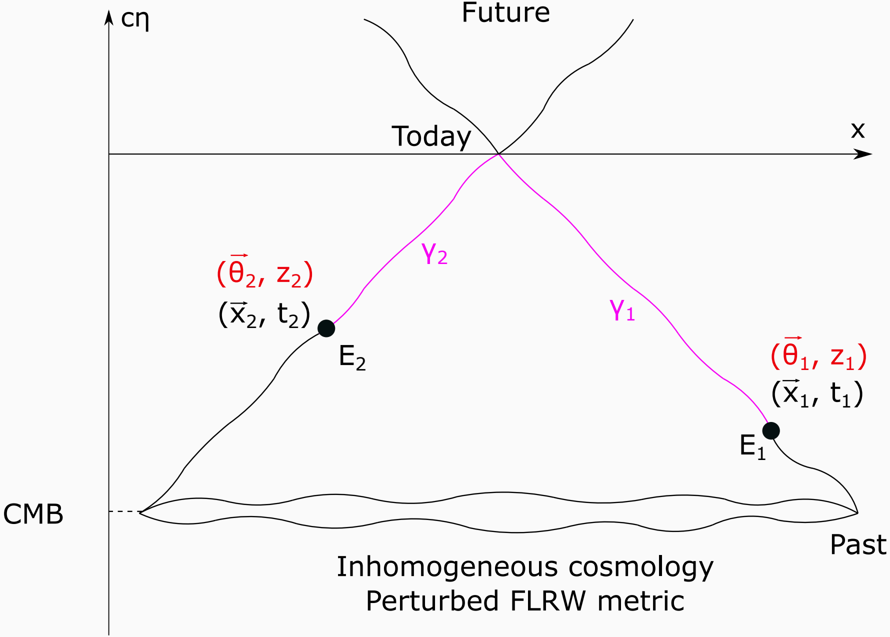

cη
Future
Today
x
γ2
γ1
(θ⃗2, z2)
(x⃗2, t2)
E2
(θ⃗1, z1)
(x⃗1, t1)
E1
CMB
Past
Inhomogeneous cosmology
Perturbed FLRW metric

Problematic

How to properly map the *'real'* universe to the *observed* universe in order to extract relevant cosmological information ?

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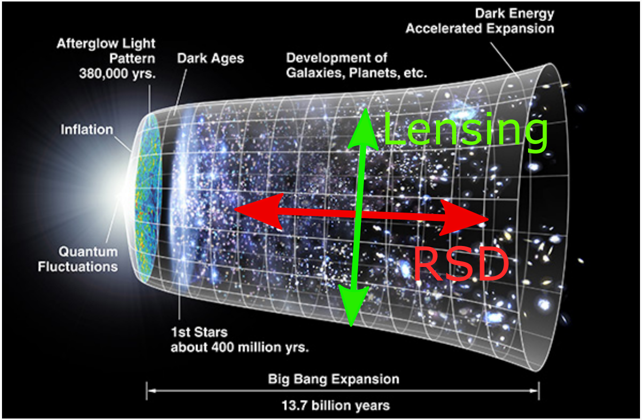
Our methodology

$\longrightarrow$ Full forward modelling with N-body simulations and ray-tracing !

The inhomogeneous universe

Light propagation in an inhomogeneous FLRW universe :

- $ds^2 = -(1+2\Psi)dt^2 + a(t)^2(1-2\Phi)d\mathbf{x}^2$
- $1+z = \frac{(g_{\mu\nu}k^\mu k^\nu)_s}{(g_{\mu\nu}k^\mu k^\nu)_o} \neq$ background FLRW (Redshift-space distortions)
- apparent angles $\neq$ true angles (Lensing)



Light-cone perturbations (terms at the observer are set to zero)

$$1 + z = \frac{a_0}{a}\left\{1 + \frac{\boldsymbol{v}\cdot\boldsymbol{n}}{c} - \frac{\Psi}{c^2} + \frac{1}{2}\left(\frac{v}{c}\right)^2 - \frac{1}{c^2}\int_{\eta}^{\eta_0}\frac{\partial(\Phi+\Psi)}{\partial\eta}d\eta'\right\} \quad (1)$$

$$\boldsymbol{\theta} = \boldsymbol{\beta} + \boldsymbol{\alpha} \quad (2)$$

- Hubble flow (background redshift)
- Doppler (standard RSD)
- Gravitational redshift
- Transverse Doppler (2^{nd} order)
- ISW/RS
- Weak Lensing

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- Hubble flow (background redshift)
- Doppler (standard RSD) (Kaiser 1987)
- Gravitational redshift (McDonald 2009)
- Transverse Doppler (2^{nd} order) (Zhao+13)
- ISW/RS
- Weak Lensing (Matsubara 2000)

etc... modify the observed number counts

(Yoo et al. 2009 ; Bonvin & Durrer 2011 ; Challinor & Lewis 2011)

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+ NON-LINEARITIES → need simulations !

Numerical methods

The RayGal simulation suite

Breton et al. (2019) [MNRAS, 483, 2671]
Rasera et al. (2022) [A&A, 661, A90]

Newtonian N-body simulations of dark matter particles only

Code : RAMSES [Teyssier 2002]

PM - AMR method

Gravity lightcone

RayGal simulations :

4096^3 particles, 2.625 Gpc/h box size

$M_{part} = 1.9 \times 10^{10} M_{\odot}$

Lightcone halo detection : pFoF [Roy+14]

Initial conditions : MPGRAFIC [Prunet+08]

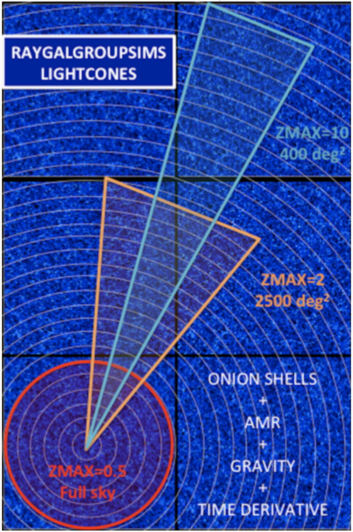
Calibrated on WMAP7 [Komatsu+11]

2 cosmologies :

$\Lambda CDM - wCDM$

Availability :

https://cosmo.obspm.fr/public-datasets/



GENCI TGCC CINES

Magrathea-Pathfinder : an HPC code for 3D ray tracing

- Reverdy (2014) [Thesis] : Magrathea library : management of refined hyperoctrees
- Breton (2018) [Thesis] : Magrathea-Pathfinder : application to ray tracing
- Breton & Reverdy (2022) [A&A, 662, A114] : Public release
  https://github.com/vreverdy/magrathea-pathfinder

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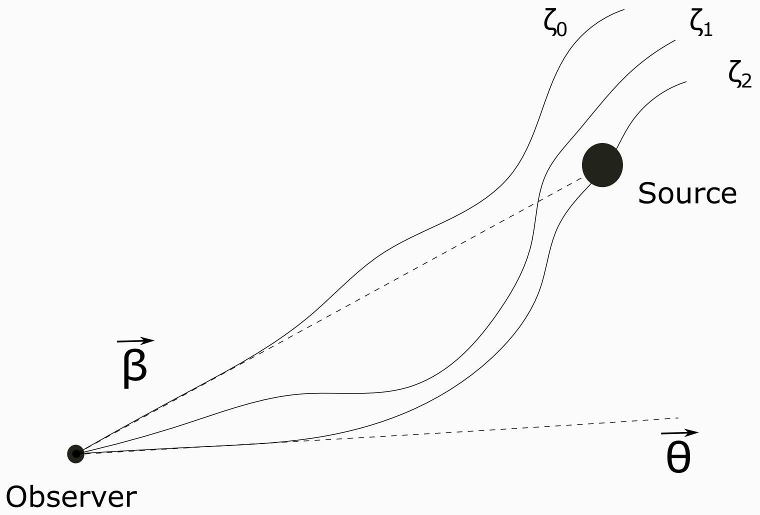
Main features

- C++11 template metaprogramming
- MPI + multi-threading
  Tested with ∼ 10,000 CPUs and ∼ 25 TB of RAM
- Solves directly the geodesic equations in the **3D adaptive mesh** of N-body simulations
- *New* : Unified high-resolution approach to all the general relativistic effects at first order

Pathfinder

New : Find the geodesics which connect the sources to the observer

→ Production of realistic observables ! Breton et al. (2019) [MNRAS, 483, 2671]

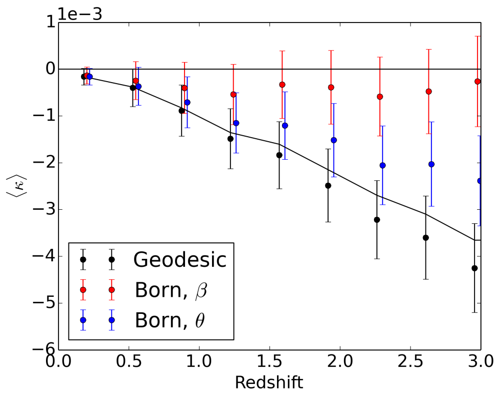


Weak lensing analysis (two examples of applications)

Question 1

For a source sample, what is its mean convergence/magnification ?

Source-averaged convergence Breton & Reverdy (2022) [A&A, 662, A114]



- Born approximation

  (straight line propagation)

  $\langle \kappa \rangle = 0 \rightarrow \langle \boldsymbol{\mu} \rangle > \mathbf{1}$
- Null geodesics

  $\langle \boldsymbol{\mu} \rangle = \mathbf{1} \rightarrow \langle \kappa \rangle < 0$

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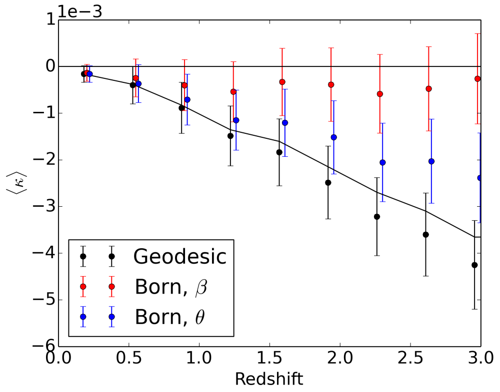

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Implications

- *Flux-limited simulated samples **overestimate** the number of observed sources if the Born approximation is used*
- Biases the distance-redshift relation depending on the averaging procedure

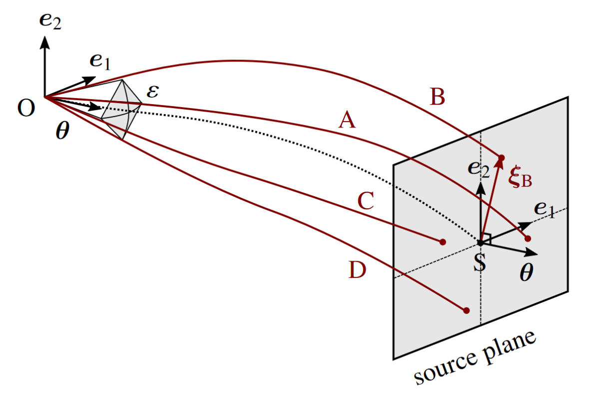
  Breton & Fleury (2021) [A&A, 655, A54]

Question 2

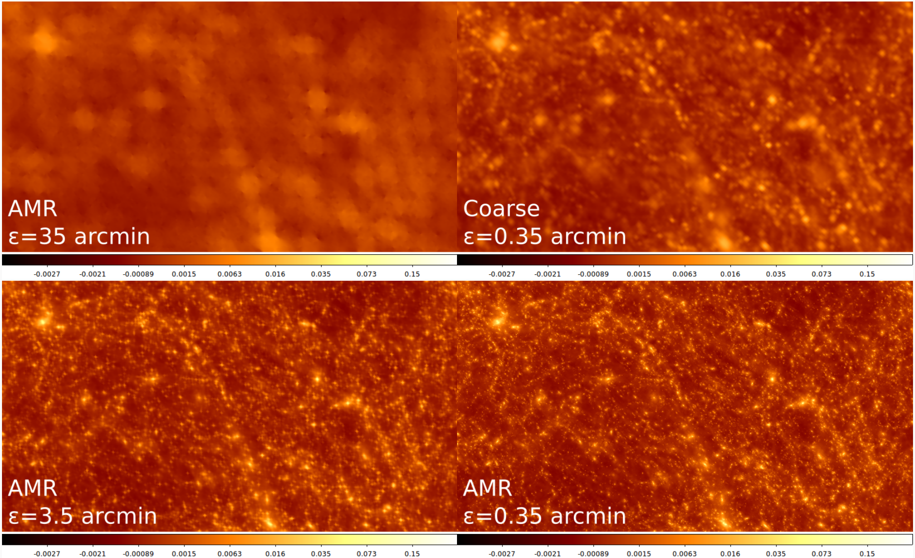
If sources have a finite extension, then so do the light beams (they are not infinitesimal). What is the impact of light beam finiteness on weak lensing statistics ?

Geometry of the light beam Breton & Fleury (2021) [A&A, 655, A54]

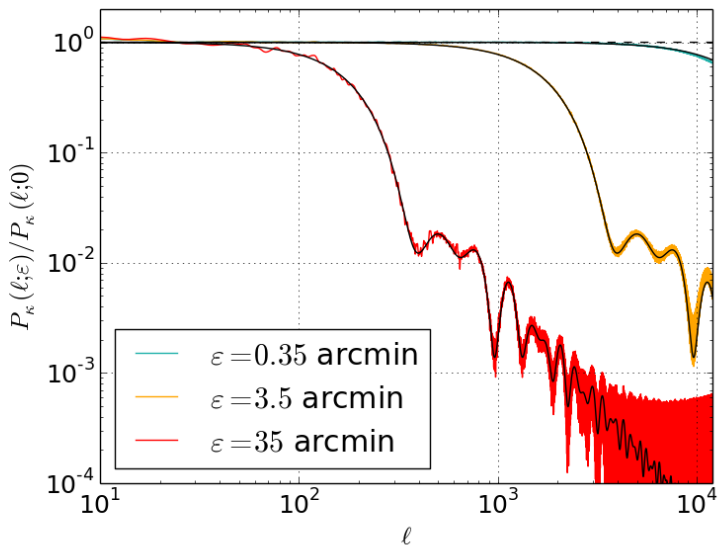
- Ray-bundle method
- Every photon follows $ds^2 = 0$
- Relevant for extended sources
- Compute $A_{ij} = \partial\beta_i/\partial\theta_j$



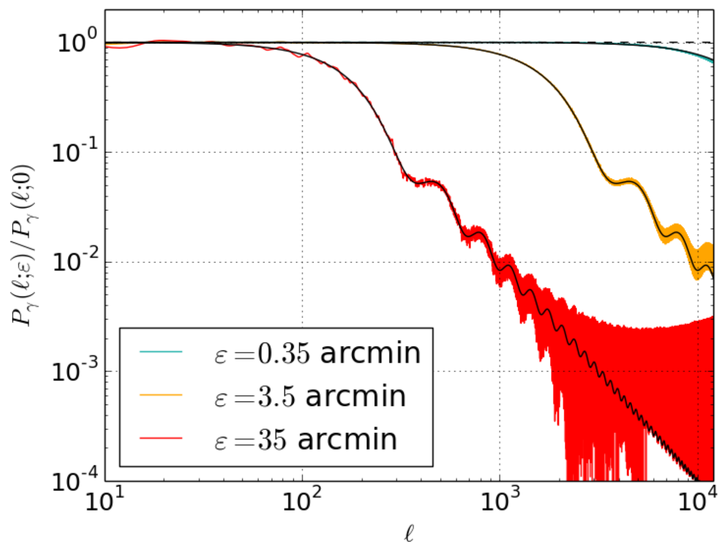
Convergence maps [extended beams] Breton & Fleury (2021) [A&A, 655, A54]



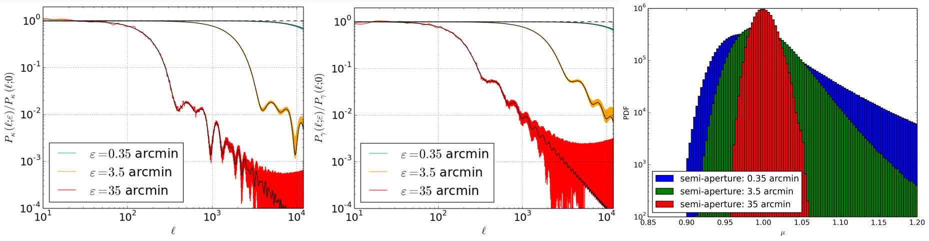
Convergence power specrum Breton & Fleury (2021) [A&A, 655, A54]



Shear power specrum Breton & Fleury (2021) [A&A, 655, A54]



Conclusion on finite-beam effects



Implications

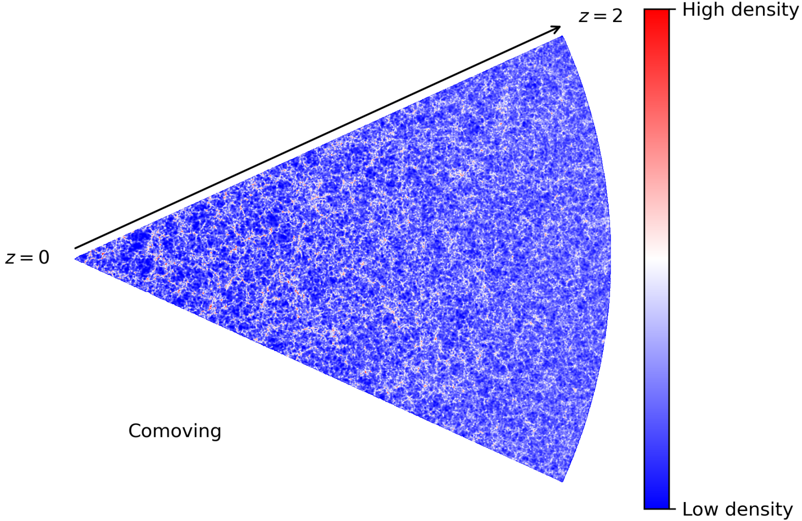
- *For realistic sizes, the effect becomes important at $\ell \sim 10^5$. This is* ***probably not relevant for two-point statistics***
- *However, it also impacts lensing PDFs which* ***can be important for microlensing experiments*** Boscá et al. (2022)

Galaxy clustering analysis (example of application)

Question

What is the impact of first-order light-cone perturbations on the observed distribution of matter ?

RayGal simulation light-cone Rasera, Breton et al. (2022) [A&A, 661, A90]



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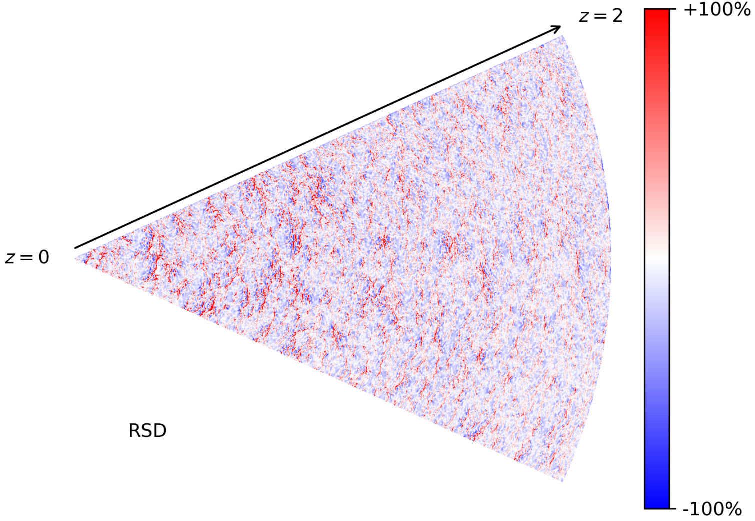

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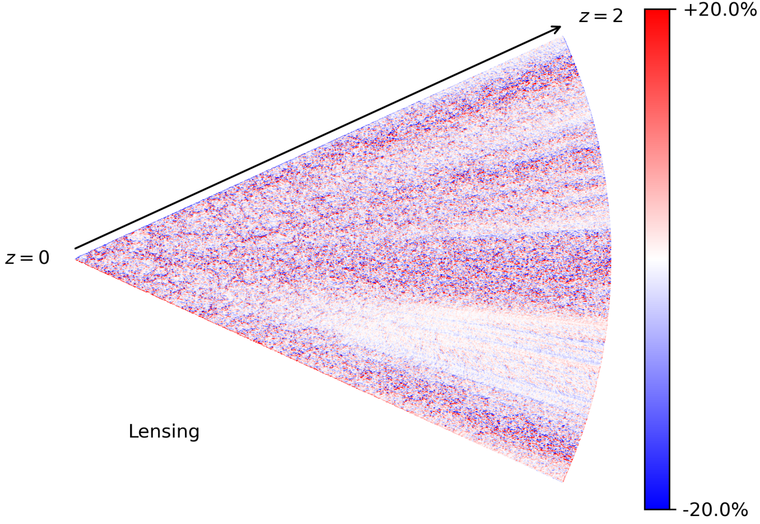
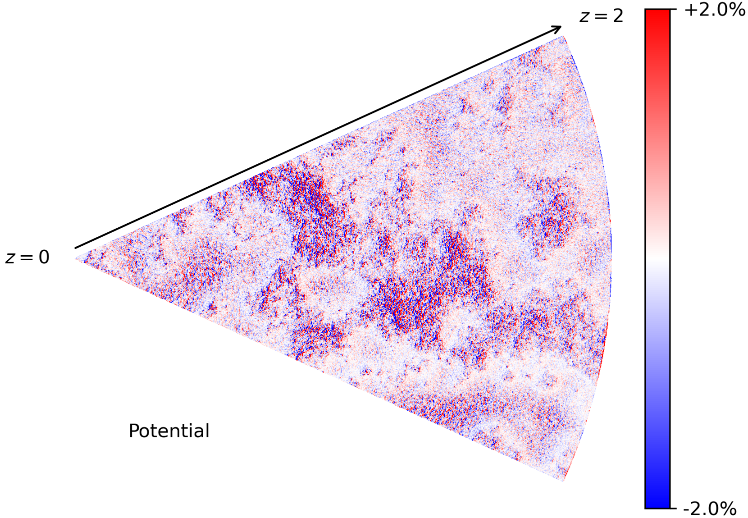


RayGal simulation light-cone Rasera, Breton et al. (2022) [A&A, 661, A90]



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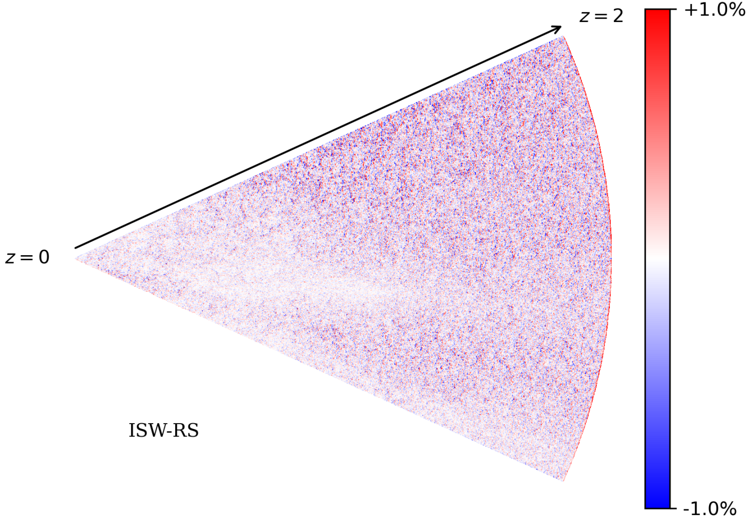


Multipoles of the two-point correlation function

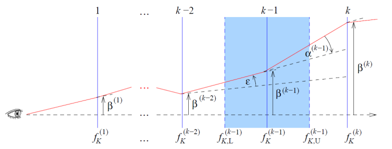
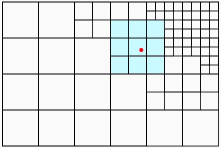
RayGal simulation, redshift bins : $[\mathbf{0.8} - \mathbf{1.0}]$ and $[\mathbf{1.6} - \mathbf{1.9}]$

ξ_ℓ	Doppler	v_o	Grav. redshift	Lensing	T. Doppler	ISW
ξ_0	$> 20\%$	3%	$< 1\%$	$1 - 10\%$	$< 1\%$	$< 1\%$
ξ_1	$> 20\%$	$< 1\%$	$> 20\%$	$< 1\%$	$< 1\%$	$< 1\%$
ξ_2	$> 20\%$	2%	$< 1\%$	5%	$< 1\%$	$< 1\%$
ξ_4	$> 20\%$	-	$< 1\%$	$1 - 10\%$	$< 1\%$	$< 1\%$

Implications

- Parameter estimation from **RSD analysis can be very sensitive to lensing** (bias on $f\sigma_8$ of a few %) Breton et al. (2022) [A&A, 661, A154]
- **The dipole** of the 2PCF **probes the gravitational potential** of galaxies Breton et al. (2019) [MNRAS, 483, 2671]

Summary : ray-tracing algorithms comparison

	Standard approach	**Magrathea-Pathfinder**
		
Dimensions	2D	3D
Spacing	Fixed	Adaptive
Approximations	Weak field Flat sky (Born approximation : undeflected photons)	Weak field
Applications	**Unique** : Gravitational lensing	**Various** : Gravitational lensing Galaxy clustering General relativity Distance measures Halo/void profiles

Conclusion

Summary

- Introducing Magrathea-Pathfinder : an HPC code for ray-tracing in N-body simulations
- Unified treatment of light propagation to produce realistic observables
- Very flexible tool : wide range of applications

Short-term perspectives

- New simulation available [220 Mpc/h, 2048^3 particles] : development of modules for strong lensing
- Your idea !

Data availability

- Magrathea-Pathfinder (+ small test simulation dataset) https://github.com/vreverdy/magrathea-pathfinder
- RayGal simulations https://cosmo.obspm.fr/public-datasets/

Appendix

Raytracing integration

$$ds^2 = a(t)^2[-(1+2\Phi/c^2)d\eta^2 + (1-2\Phi/c^2)\delta_{ij}dx^i dx^j]$$

Geodesic equation

$$\frac{d^2\eta}{d\lambda^2} = -\frac{2a'}{a}\frac{d\eta}{d\lambda}\frac{d\eta}{d\lambda} - \frac{2}{c^2}\frac{d\phi}{d\lambda}\frac{d\eta}{d\lambda} + 2\frac{\partial\phi}{\partial\eta}(\frac{d\eta}{d\lambda})^2$$

$$\frac{d^2x}{d\lambda^2} = -\frac{2a'}{a}\frac{d\eta}{d\lambda}\frac{dx}{d\lambda} + \frac{2}{c^2}\frac{d\phi}{d\lambda}\frac{dx}{d\lambda} - 2\frac{\partial\phi}{\partial x}(\frac{d\eta}{d\lambda})^2$$

$$\frac{d^2y}{d\lambda^2} = -\frac{2a'}{a}\frac{d\eta}{d\lambda}\frac{dy}{d\lambda} + \frac{2}{c^2}\frac{d\phi}{d\lambda}\frac{dy}{d\lambda} - 2\frac{\partial\phi}{\partial y}(\frac{d\eta}{d\lambda})^2$$

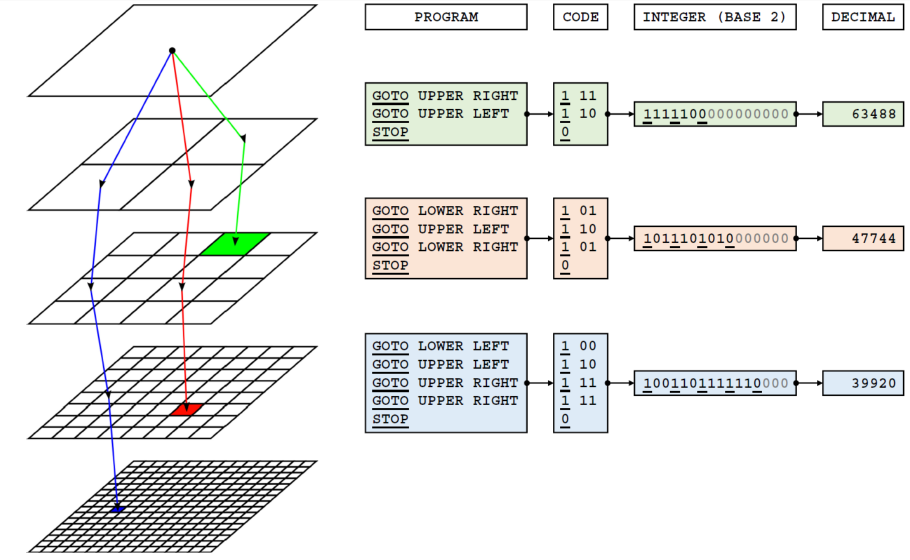
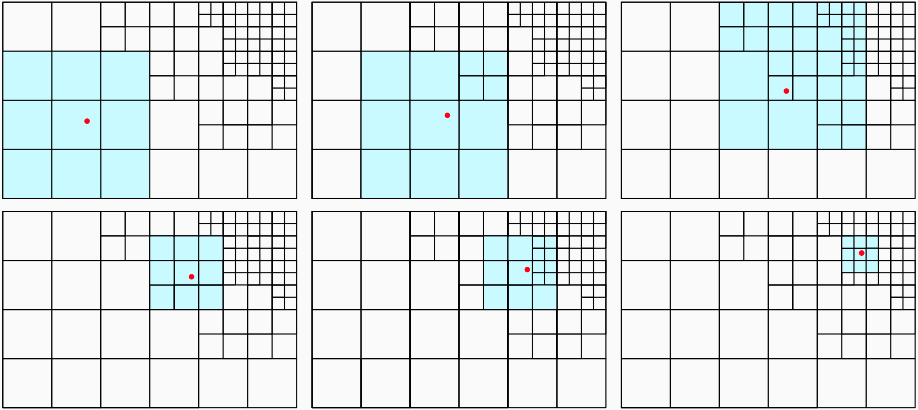


Figure 1 – MAGRATHEA indexing Reverdy (2014)[Thesis]

Inverse TSC interpolation in an adaptive grid



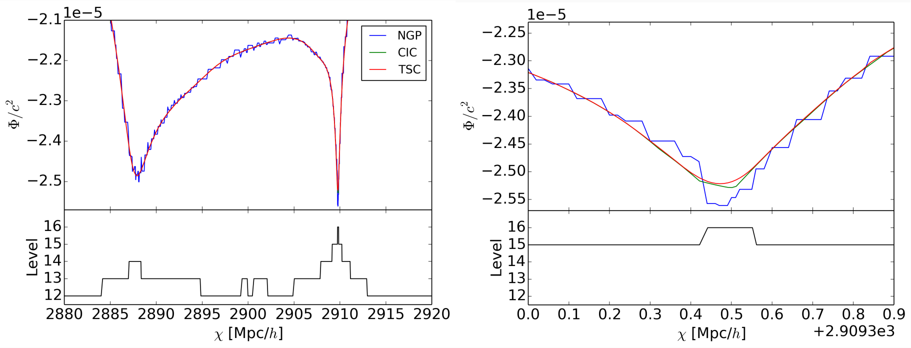
Ray-tracing runtime in RayGal Breton & Reverdy (2022) [A&A, 662, A114]

Performance per time step	
Interpolation	Time (µs)
No	0.55
Order 0 (NGP)	1.10
Order 1 (CIC)	6.00
Order 2 (TSC)	14.6

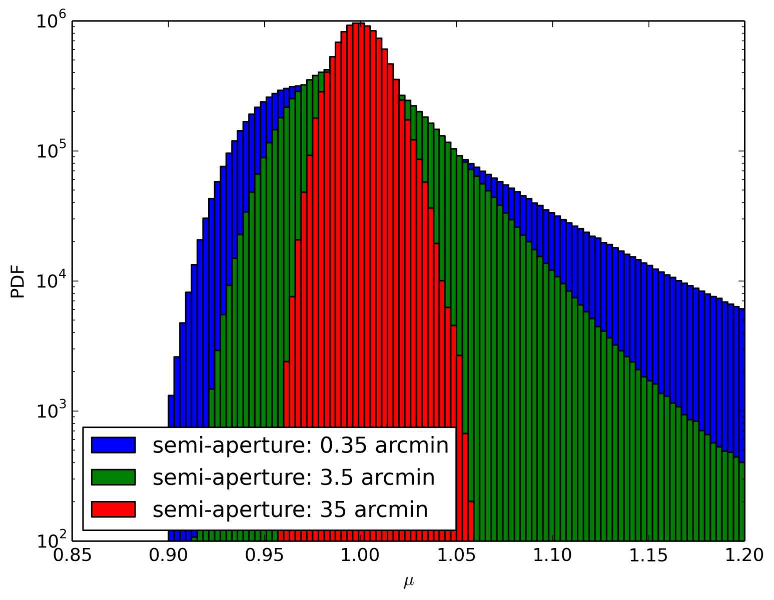
One trajectory $\sim$ 10,000 steps

10^{10} **trajectories : 300.000 hours**

Adaptive-mesh ray tracing Breton & Reverdy (2022) [A&A, 662, A114]

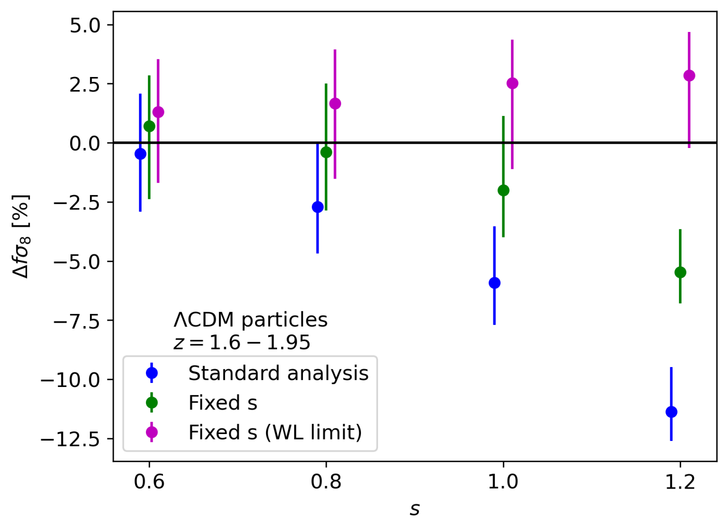


Magnification PDF for extended light beams at $z = 2$



Impact of lensing on $f\sigma_8$ Breton et al. (2022) [A&A, 661, A154]

Power-law luminosity function $n(< m) \propto 10^{ms}$



Redshift-space number count (linear) decomposition

$$\Delta^{acc} = \frac{1}{\mathcal{H}c}\dot{\boldsymbol{v}} \cdot \boldsymbol{n}, \tag{4}$$

$$\Delta^{q} = -\frac{\dot{\mathcal{H}}}{c\mathcal{H}^2}\boldsymbol{v} \cdot \boldsymbol{n}, \tag{5}$$

$$\Delta^{div} = -\frac{2}{\mathcal{H}\chi}\boldsymbol{v} \cdot \boldsymbol{n}, \tag{6}$$

$$\Delta^{pot,(1)} = \frac{1}{\mathcal{H}c}\boldsymbol{\nabla}_r\psi \cdot \boldsymbol{n}, \tag{7}$$

$$\Delta^{pot,(2)} = \left(\frac{\dot{\mathcal{H}}}{\mathcal{H}^2} + \frac{2c}{\mathcal{H}\chi}\right)\psi/c^2 - \frac{1}{\mathcal{H}c^2}\dot{\psi}, \tag{8}$$

$$\Delta^{shapiro} = (\phi + \psi)/c^2, \tag{9}$$

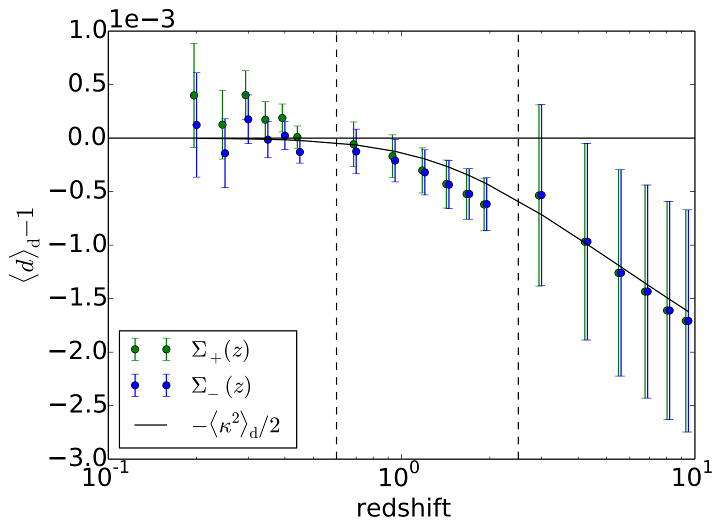
$$\Delta^{lens} = -\frac{1}{c^2}\int_0^{\chi}\frac{(\chi - \chi')\chi'}{\chi}\nabla_{\perp}^2(\phi + \psi)d\chi', \tag{10}$$

$$\Delta^{isw} = \frac{1}{\mathcal{H}c^2}(\dot{\phi} + \dot{\psi}), \tag{11}$$

$$\Delta^{LC} = \boldsymbol{v} \cdot \boldsymbol{n}/c, \tag{12}$$

$$\Delta_{neglect} = \left(\frac{\dot{\mathcal{H}}}{\mathcal{H}^2} + \frac{2c}{\mathcal{H}\chi}\right)\frac{1}{c^2}\int_{\eta}^{\eta_0}\frac{\partial(\phi + \psi)}{\partial\eta}d\eta' + \frac{2}{\chi c^2}\int_0^{\chi}(\phi + \psi)d\chi'. \tag{13}$$

Direction-averaged distance bias Breton & Fleury (2021) [A&A, 655, A54]



Source-averaged distance bias Breton & Fleury (2021) [A&A, 655, A54]