Constraining ΛCDM and Extensions with the Effective Field Theory of Large-Scale Structures

Théo SIMON

Based on arXiv:2210.14931
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GDR CoPhy - 18/01/2023

The Effective Field Theory of Large-Scale Structures (EFTofLSS)

To constrain ΛCDM and extensions, there are two main observables:

1. **CMB** → CMB power spectra: C_l^{TT}, C_l^{EE}, C_l^{TE}, etc.
2. **LSS** → the galaxy power spectrum: $P_g(z, k, \mu)$

In **linear perturbation theory**, there are two popular ways to use LSS data:

1. Extract information from the full galaxy power spectrum:

$$P_g(z, k, \mu) \simeq \left[b_1(z) + f\mu^2\right]^2 P_m(z, k)$$

[Kaiser '87]

   b_1: bias parameter, f: growth factor and $\mu = \hat{z} \cdot \hat{k}$

2. Redshift Space Distortion (RSD) information: $f\sigma_8$

LSS collaborations conventionally use the second method (+BAO)

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The effective field theory of large-scale structures (EFTofLSS)

The galaxy power spectrum in the framework of the EFTofLSS:

See Pierre Zhang's presentation

$$P_g(k,\mu) \simeq \left[b_1 + f\mu^2\right]^2 P_m(k) = Z_1(\mu)^2 P_m(k)$$

At $\mathcal{O}(k^2/k_M^2)$, we go from 1 to 10 free parameters

$$\begin{aligned}
P_g(k,\mu) &= Z_1(\mu)^2 P_{11}(k) + 2Z_1(\mu)P_{11}(k)\left(c_{\mathrm{ct}}\frac{k^2}{k_{\mathrm{M}}^2} + c_{r,1}\mu^2\frac{k^2}{k_{\mathrm{M}}^2} + c_{r,2}\mu^4\frac{k^2}{k_{\mathrm{M}}^2}\right) \\
&+ 2\int \frac{d^3q}{(2\pi)^3}\, Z_2(\boldsymbol{q},\boldsymbol{k}-\boldsymbol{q},\mu)^2 P_{11}(|\boldsymbol{k}-\boldsymbol{q}|)P_{11}(q) + 6Z_1(\mu)P_{11}(k)\int \frac{d^3q}{(2\pi)^3}\, Z_3(\boldsymbol{q},-\boldsymbol{q},\boldsymbol{k},\mu)P_{11}(q) \\
&+ \frac{1}{\bar{n}_g}\left(c_{\epsilon,0} + c_{\epsilon,1}\frac{k^2}{k_{\mathrm{M}}^2} + c_{\epsilon,2} f\mu^2\frac{k^2}{k_{\mathrm{M}}^2}\right),
\end{aligned}$$

[Perko et al. '16]

The effective field theory of large-scale structures (EFTofLSS)

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[Perko et al. '16]

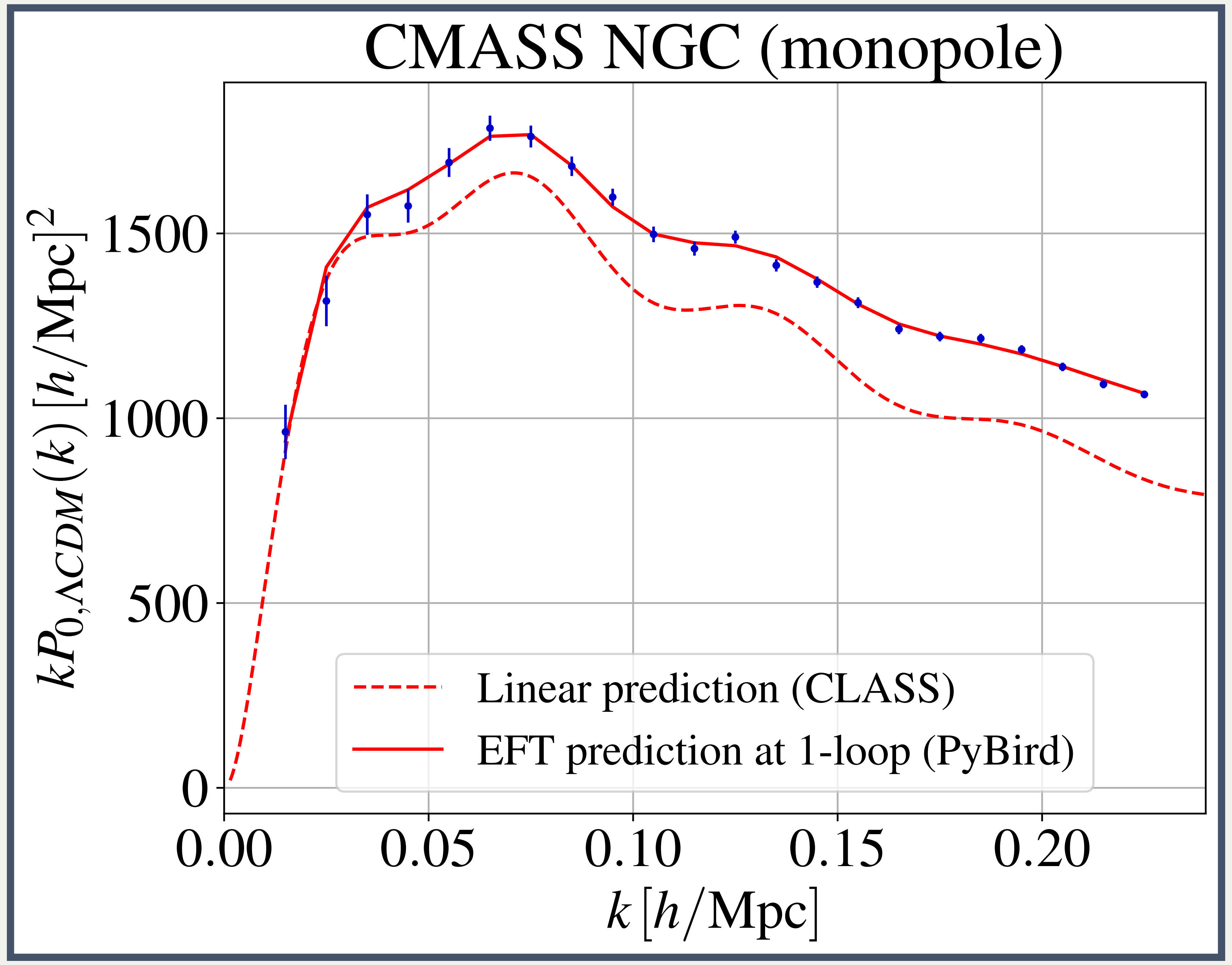
$P_g(k,\mu)$ can be determined directly from $P_{11}(k) = P_m^{\rm lin}(k)$

EFTofLSS applied to BOSS data

Multipoles of the galaxy power spectrum, obtained through a **Legendre** polynomials ($\mathscr{L}_\ell$) decomposition:

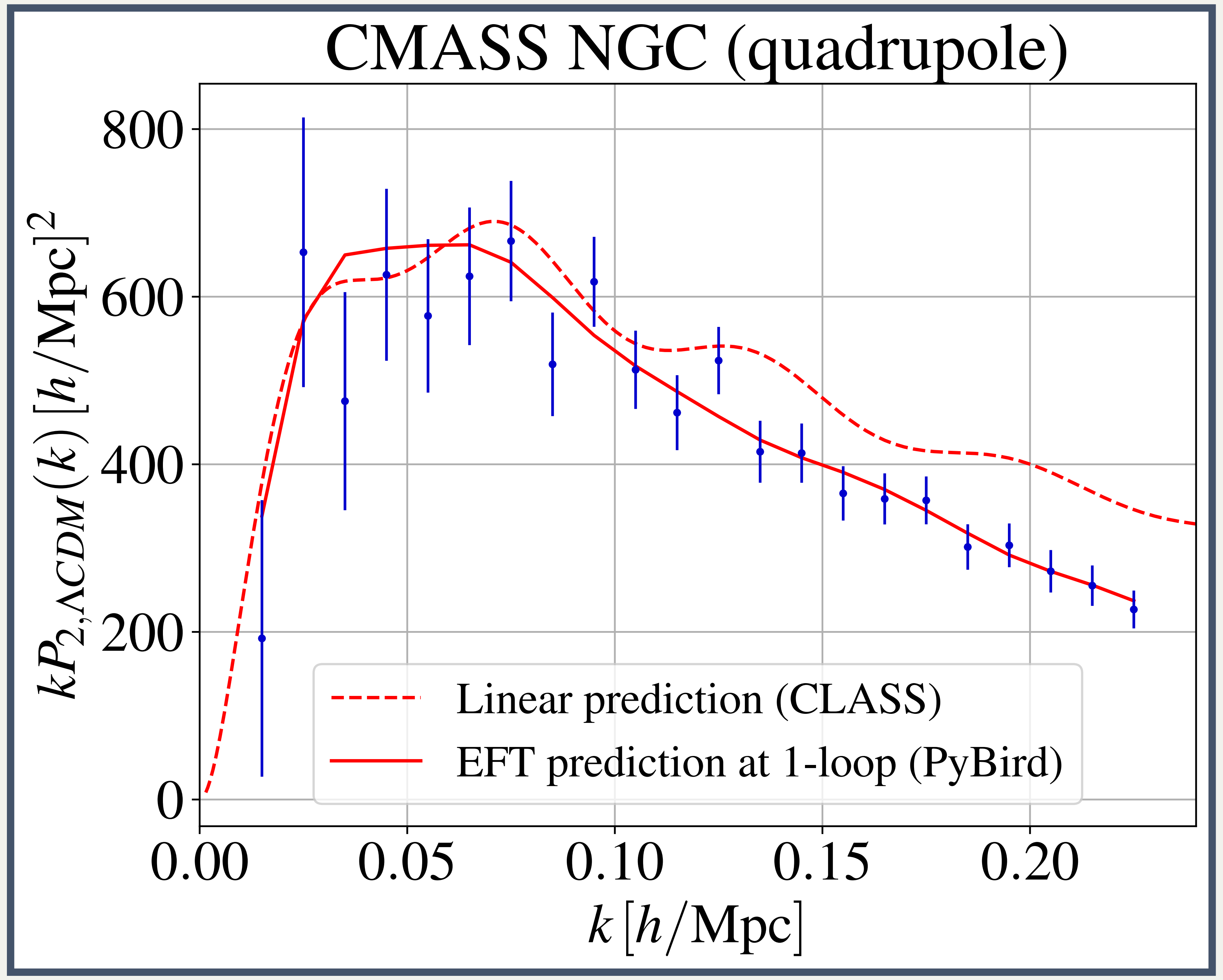
$$P_g(z,k,\mu) = \sum_{\ell \text{ even}} \mathscr{L}_\ell(\mu) P_\ell(z,k)$$

→ the two main contributions to $P_g(z,k,\mu)$ are the **monopole** ($\ell = 0$) and the **quadrupole** ($\ell = 2$).



[Colas et al. '19, Zhang et al. '21]

Made with PyBird: github.com/pierrexyz/pybird



See also [TS et al. '22, arXiv:2208.05929]

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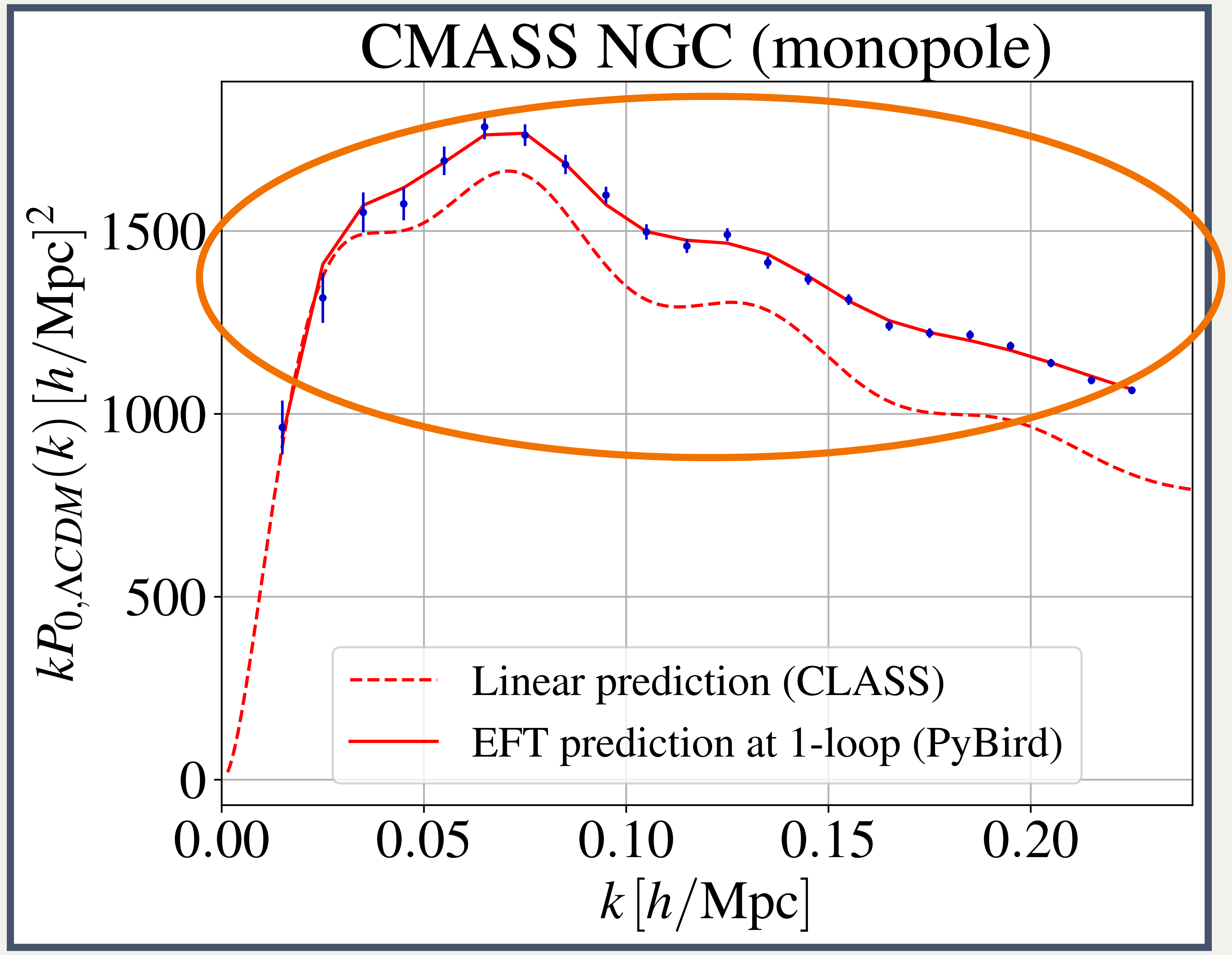
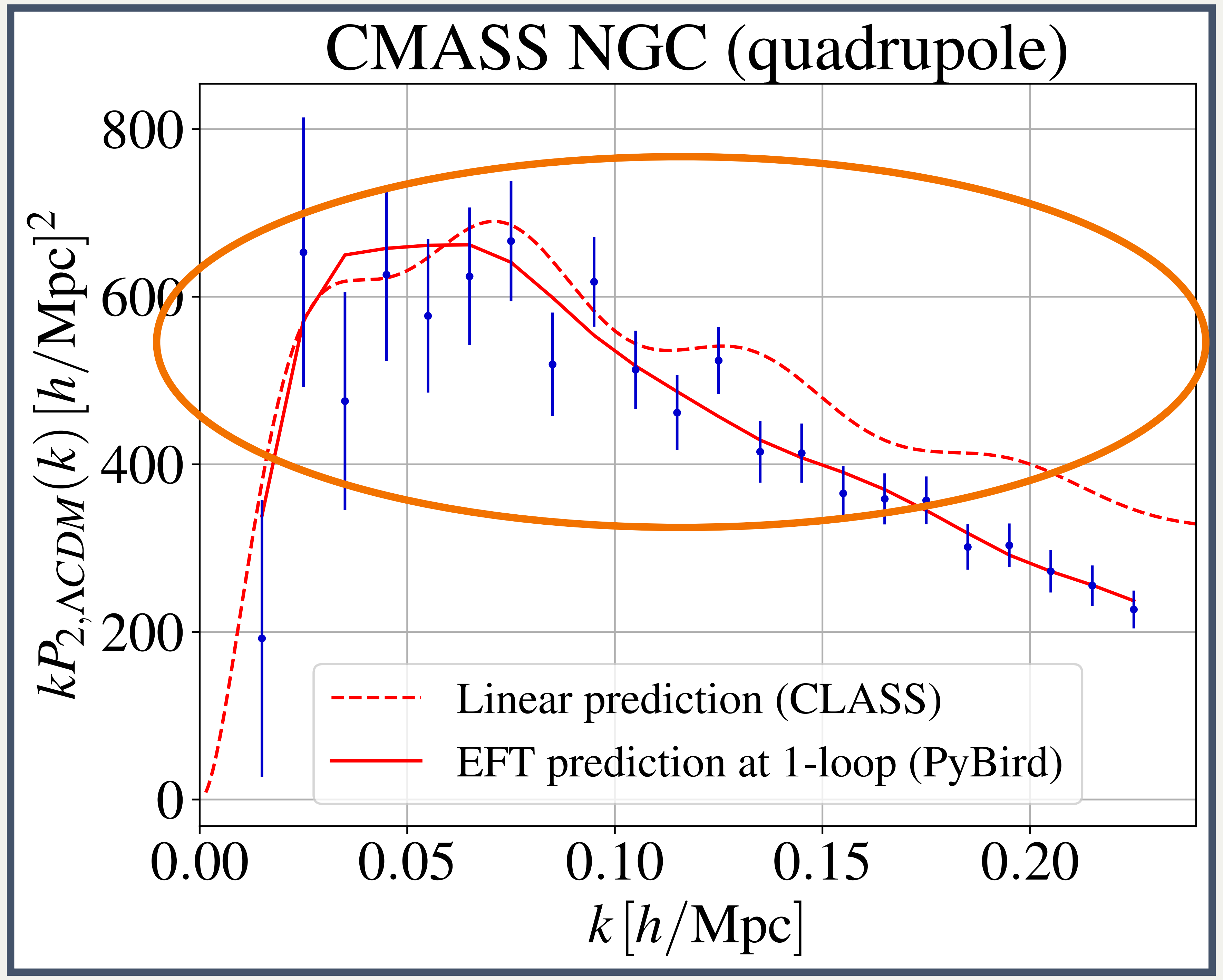


[Colas et al. '19, Zhang et al. '21]

Improvement in precision

Made with PyBird: github.com/pierrexyz/pybird



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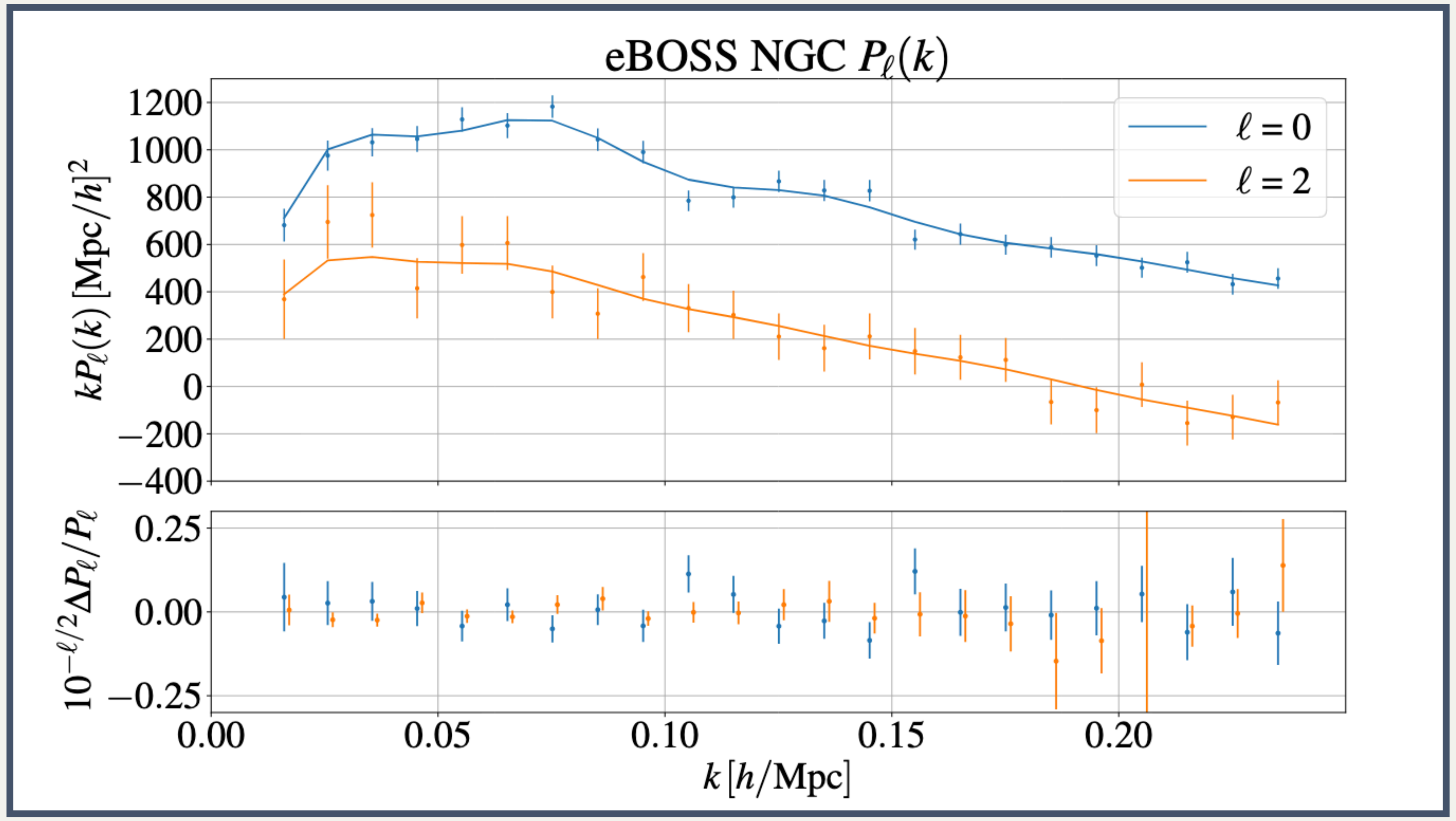
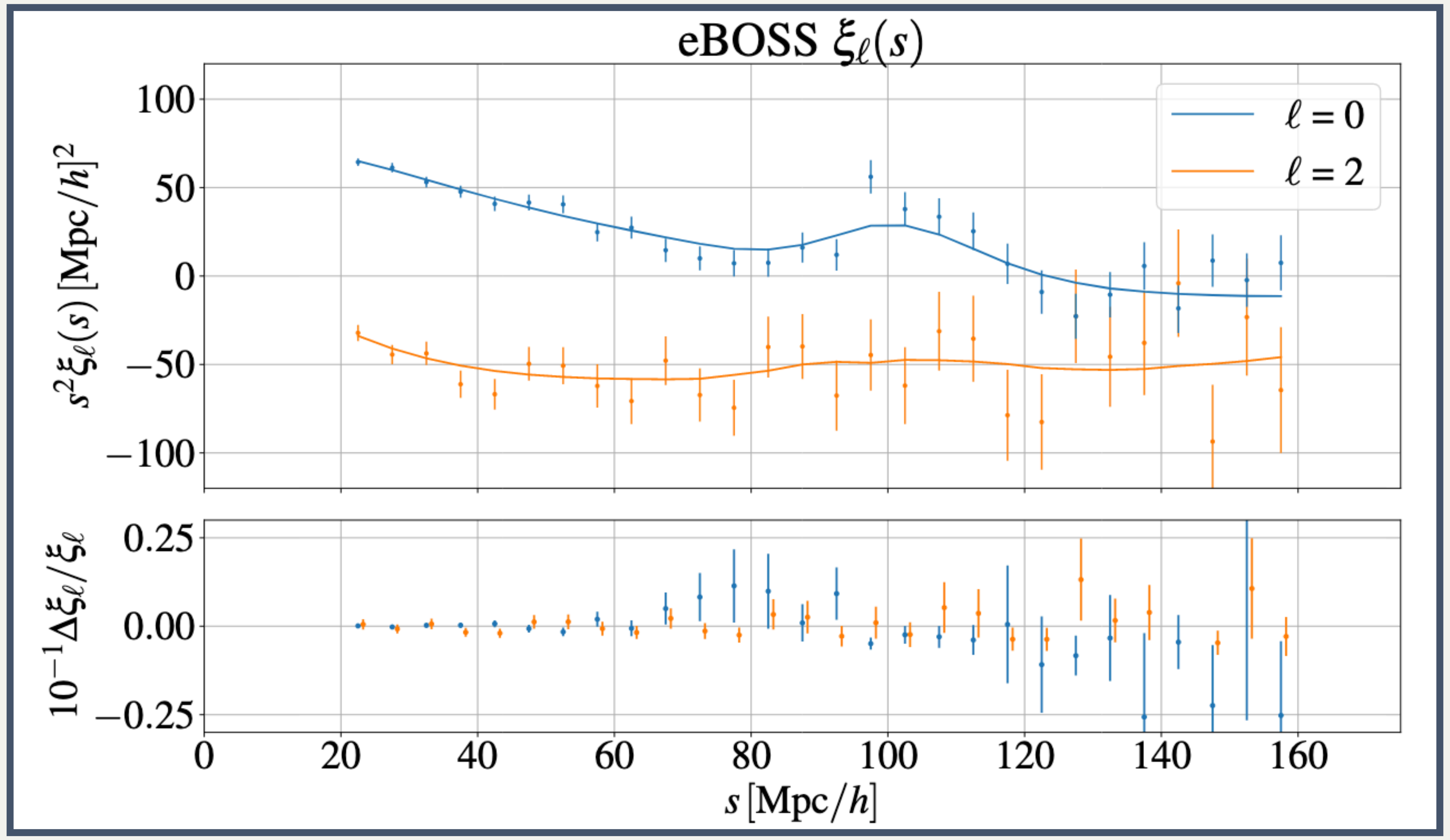
EFTofLSS applied to eBOSS QSO data

- **343 708 quasars** selected in the redshift range $0.8 < z < 2.2$
- $z_{\text{eff}} = 1.5$
- 2 skycuts : NGC and SGC

[eBOSS Collaboration '20, arXiv:2007.08991]





Determination of the cut-off scale $k_{\max}$ of the one-loop prediction

At **one-loop order**, the galaxy power spectrum reads:

$$P_g(k,\mu) = Z_1(\mu)^2 P_{11}(k) + 2Z_1(\mu)P_{11}(k)\left(c_{\rm ct}\frac{k^2}{k_{\rm M}^2} + c_{r,1}\mu^2\frac{k^2}{k_{\rm M}^2} + c_{r,2}\mu^4\frac{k^2}{k_{\rm M}^2}\right)$$
$$+2\int\frac{d^3q}{(2\pi)^3}\, Z_2(\boldsymbol{q},\boldsymbol{k}-\boldsymbol{q},\mu)^2 P_{11}(|\boldsymbol{k}-\boldsymbol{q}|)P_{11}(q) + 6Z_1(\mu)P_{11}(k)\int\frac{d^3q}{(2\pi)^3}\, Z_3(\boldsymbol{q},-\boldsymbol{q},\boldsymbol{k},\mu)P_{11}(q)$$
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One can add the **NNLO terms** (*i.e.*, the dominant two-loop terms):

$$P_{\rm NNLO}(k,\mu) = \frac{1}{4}b_1\left(c_{r,4}b_1 + c_{r,6}\mu^2\right)\mu^4\frac{k^4}{k_{\rm M}^4}P_{11}(k)$$

If the contribution of $P_{\rm NNLO}(k,\mu)$ becomes **too large,** the one-loop prediction is **not accurate enough** → this determines the **cut-off scale** $k_{\max}$ of the prediction

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2 new EFT parameters

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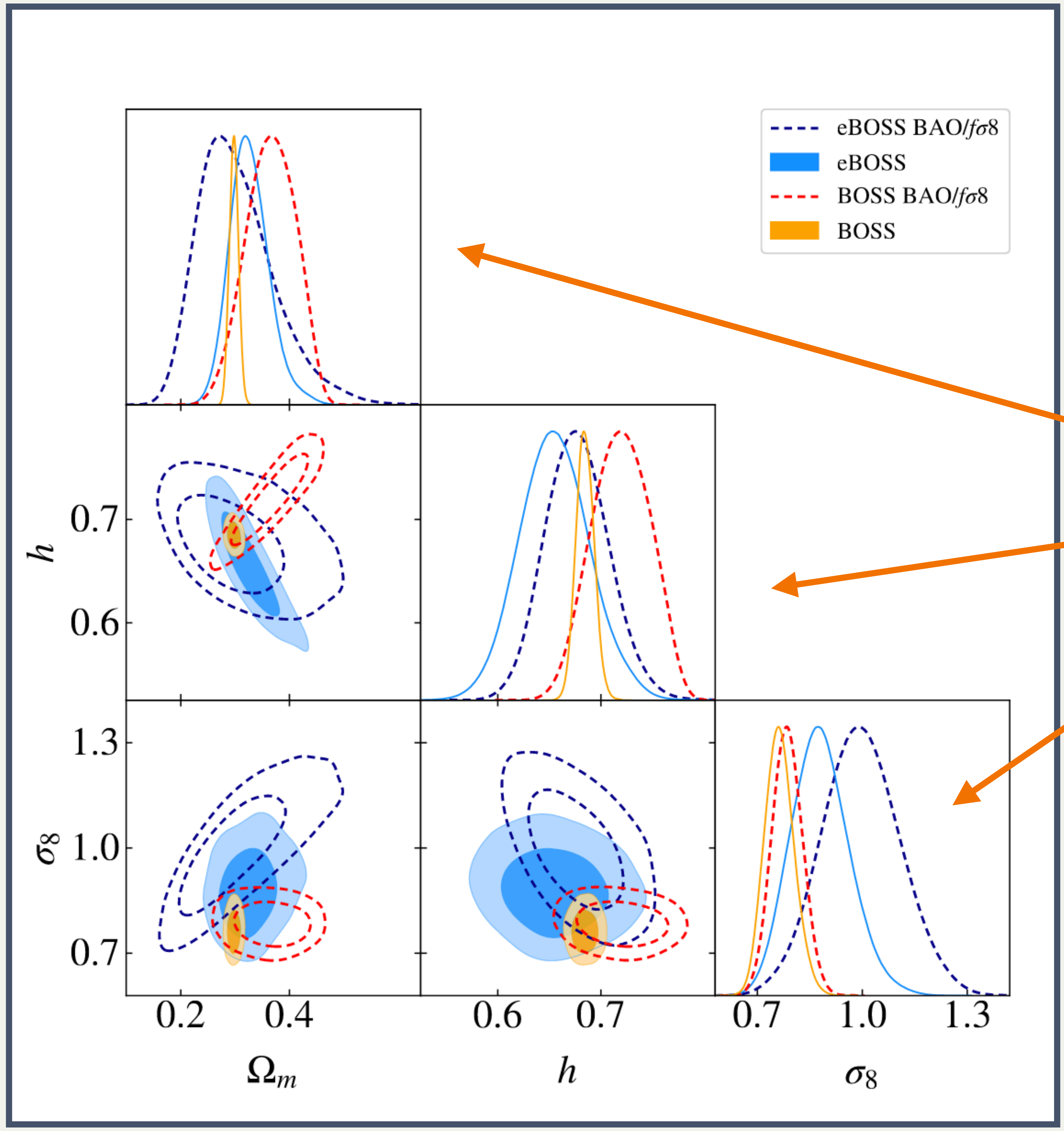
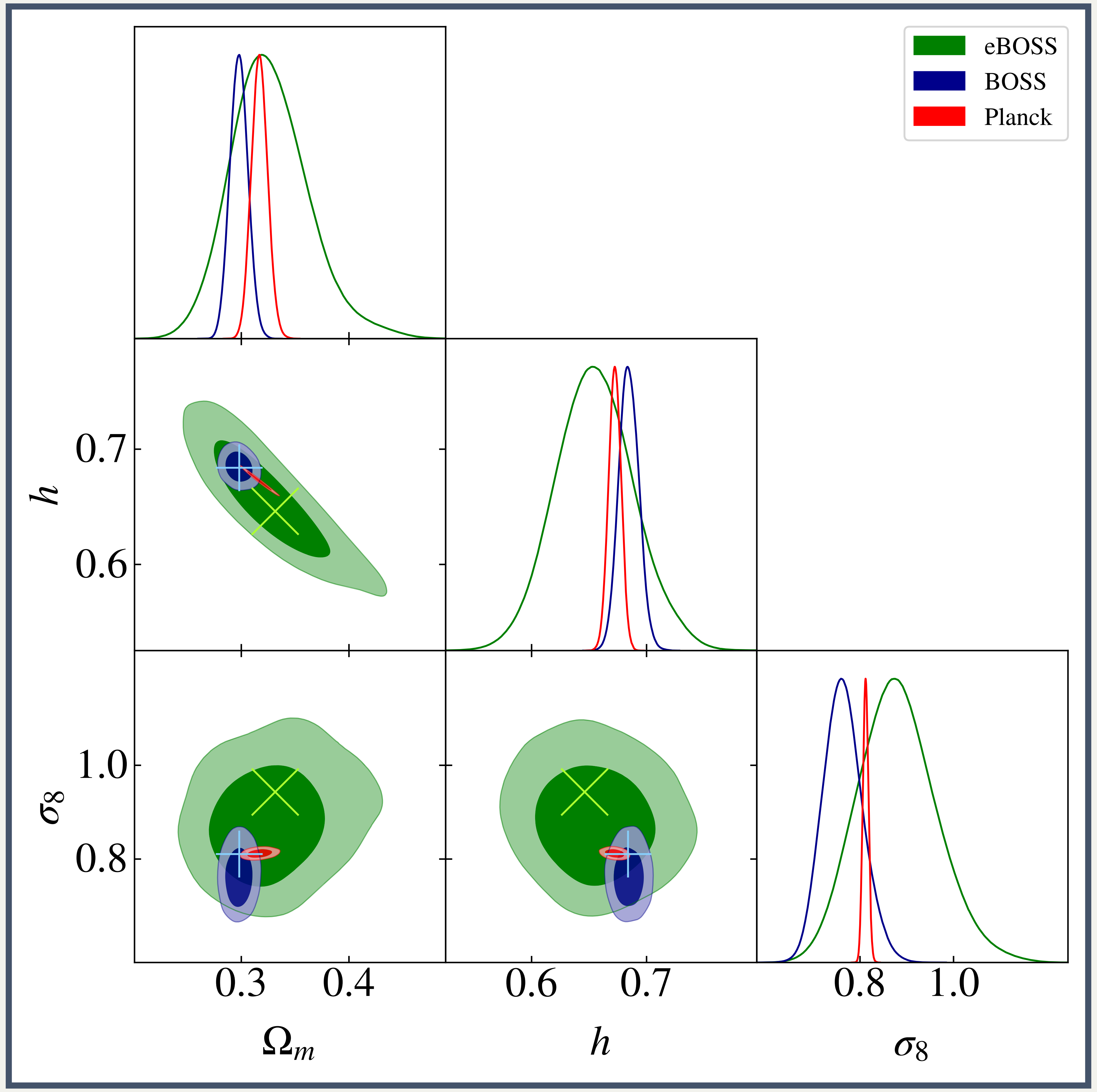
BAO/$f\sigma_8$ vs EFTofLSS



Additional information

- For **eBOSS**, the error bars of Ω_m and σ_8 are reduced by a factor ~ 2.0 and ~ 1.3
- For **BOSS**, the error bars of Ω_m and h are reduced by a factor ~ 5.4 and ~ 3.2

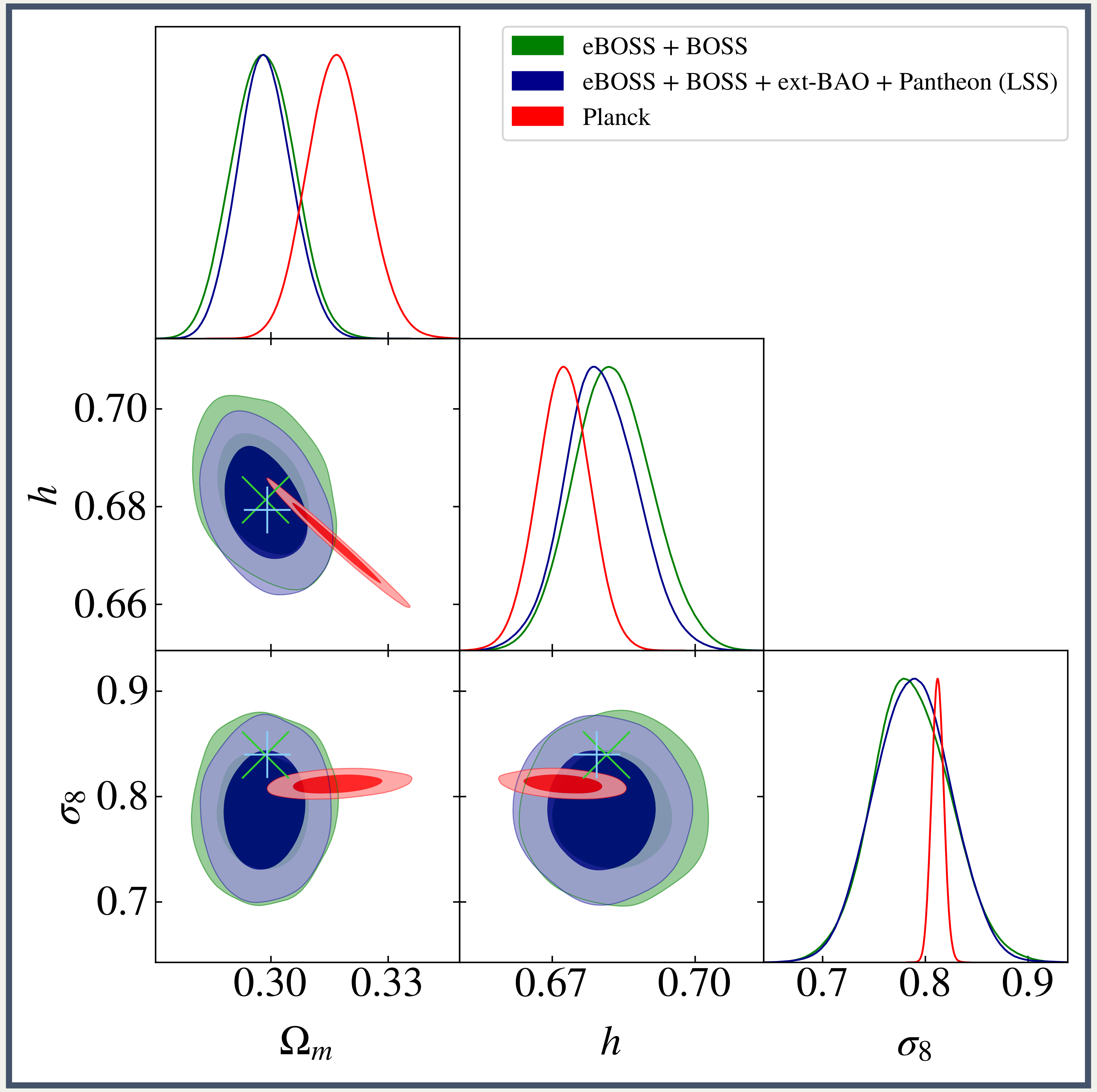
LSS data vs Planck



- **eBOSS, BOSS and Planck are consistent** at $\lesssim 1.8\sigma$ on all cosmological parameters
- h is $\sim 1\sigma$ lower for eBOSS than for BOSS, while σ_8 is $\sim 1.5\sigma$ higher
- The h and σ_8 Planck values are in-between those of BOSS and eBOSS

$\rightarrow$ **there is no tension between Planck and BOSS/eBOSS**

LSS data vs Planck

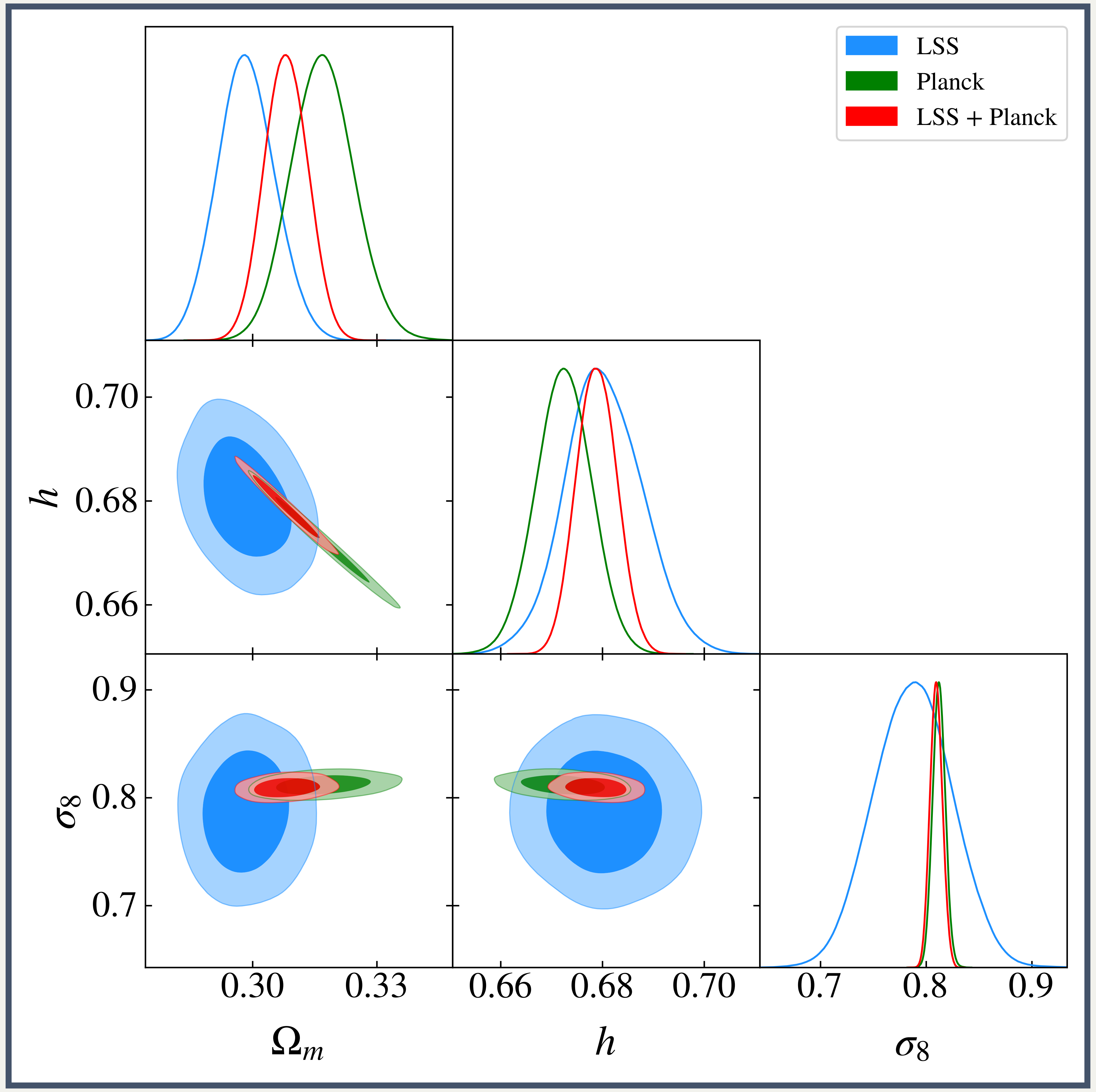


ext-BAO: 6dF & MGS (SDSS) data

- The combination of eBOSS + BOSS allows to determine Ω_m and h at a **precision similar to Planck**
- The combination of LSS data remains consistent with Planck → **we can combine them!**

LSS data combined with Planck

LSS: eBOSS + BOSS + ext-BAO + Pantheon



- **Compared to Planck alone**, the constraints on Ω_m and h are **improved by** $\sim 30\,\%$
- σ_8 and A_s are not significantly impacted

Extensions to ΛCDM: curvature density fraction Ω_k

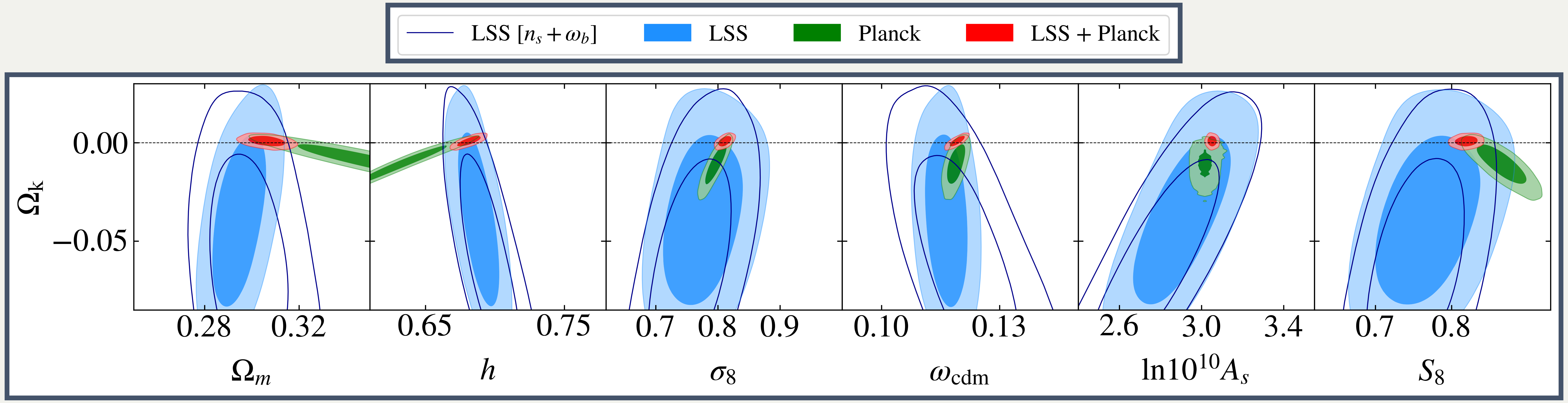
- The EFT analysis **significantly improves the constraints** on Ω_k by $\sim 50\,\%$ compared to the conventional BAO/$f\sigma_8$ analysis
- With LSS data only, we find Ω_k **compatible with zero curvature at** 1.3σ
- The combination of LSS and Planck leads to a **strong constraint** and excludes the (slightly favored) negative values of Ω_k

LSS:

$$\Omega_k = -0.039^{+0.028}_{-0.029}$$

LSS+Planck:

$$\Omega_k = 0.0008^{+0.0018}_{-0.0017}$$



Extensions to ΛCDM: dark energy equation of state w_0

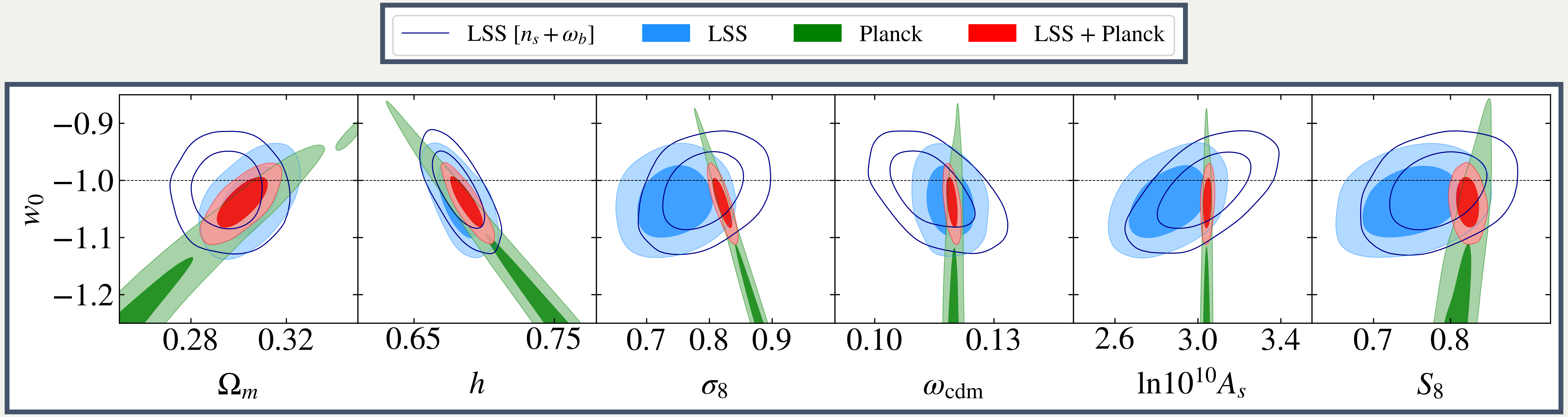
- The EFT analysis **improves the constraints** on w_0 by $\sim 20\,\%$ compared to the conventional BAO/$f\sigma_8$ analysis
- With the LSS data only, we find **no evidence for a universe with** $w_0 \neq -1$
- The addition of LSS data select values of w_0 close to -1, located in the 2σ region reconstructed from Planck data

LSS:

$w_0 = -1.038 \pm 0.041$

LSS+Planck:

$w_0 = -1.039 \pm 0.029$

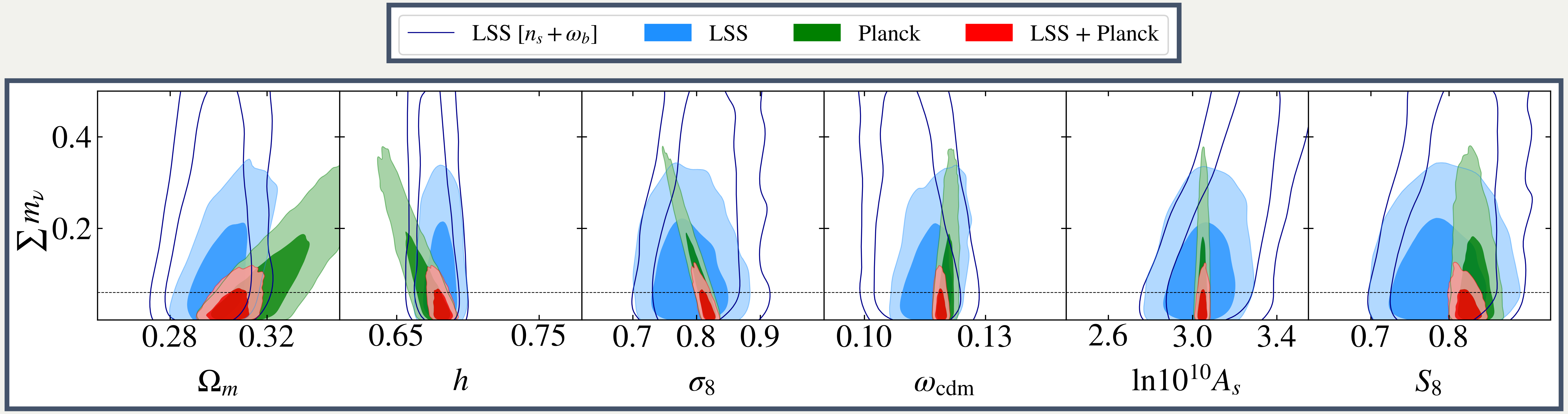


Extensions to ΛCDM: total neutrino mass $\sum m_\nu$

- The EFT analysis **significantly improves the constraints** on $\sum m_\nu$ (by a factor of ~ 18) over the conventional BAO/$f\sigma_8$ analysis ($\sum m_\nu = 4.84 eV$)
- The LSS constraint derived in this work is **only** $\sim 10\,\%$ **weaker than the Planck constraint** ($\sum m_\nu = 0.241 eV$)
- This analysis **disfavors the inverse hierarchy** at $\sim 2.2\sigma$

LSS:
$\sum m_\nu < 0.274 eV$

LSS+Planck:
$\sum m_\nu < 0.093 eV$



Extensions to ΛCDM: effective number of relativistic species $N_{\rm eff}$

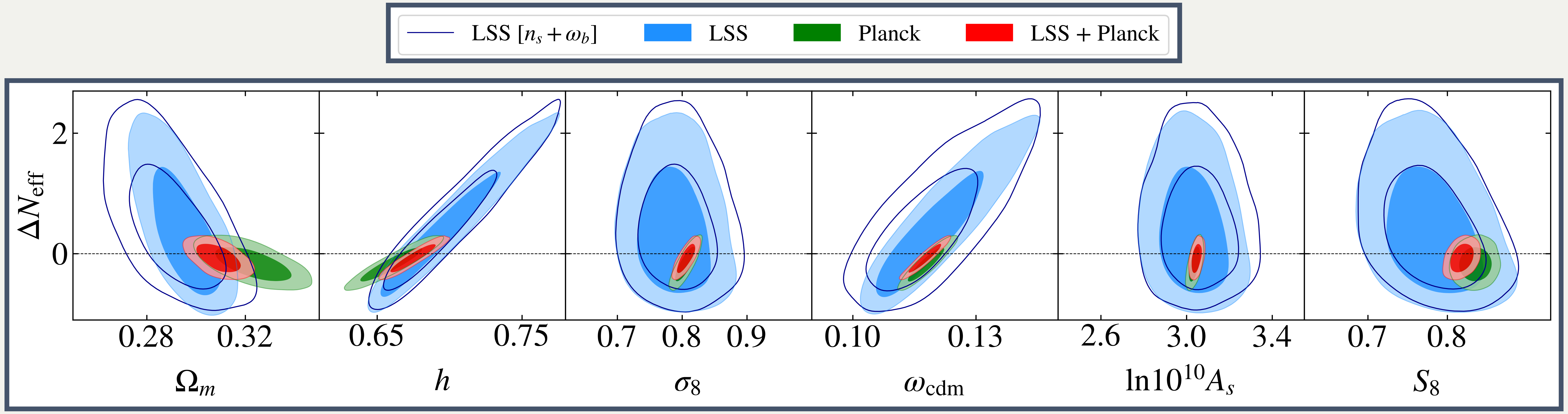
- Unlike EFTofLSS, **the conventional BAO/$f\sigma_8$ analysis is unable to constrain this parameter**
- The value of $\Delta N_{\rm eff}$ is **compatible with the standard model**
- The addition of the LSS data **improves** the results of Planck alone

LSS:

$$\Delta N_{\rm eff} = 0.40^{+0.44}_{-0.91}$$

LSS+Planck:

$$\Delta N_{\rm eff} = -0.07^{+0.15}_{-0.16}$$



Conclusions

- The EFTofLSS is a novel method that provides an **accurate description of LSS data at a controlled precision**
- Constraints from LSS data are **competitive with CMB data** and their combination **improves over Planck alone**
- EFTofLSS allows to highlight that **there is no tension** between current BOSS/eBOSS data and Planck data (but not in tension with weak lensing neither)
- Data are consistent with ΛCDM at $\lesssim 1.3\sigma \rightarrow$ Strong constraints on canonical extensions to ΛCDM
  e.g. LSS+Planck: $\sum m_\nu < 0.093 e$V
- EFTofLSS provides **interesting constraints on non-trivial extensions** of the ΛCDM model:
  $\rightarrow$ see [TS et al. '22, arXiv:2203.07440] for **Decaying Cold Dark Matter**
  $\rightarrow$ see [TS et al. '22, arXiv:2208.05930] for **Early Dark Energy**

Thanks for your attention

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Backup I: EFTofLSS parameters

$$
\begin{aligned}
P(k,\mu) &= Z_1(\mu)^2 P_{11}(k) \\
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&+ 2Z_1(\mu)P_{11}(k)\left(c_{\rm ct}\frac{k^2}{k_{\rm M}^2} + c_{r,1}\mu^2\frac{k^2}{k_{\rm M}^2} + c_{r,2}\mu^4\frac{k^2}{k_{\rm M}^2}\right) + \frac{1}{\bar{n}_g}\left(c_{\epsilon,0} + c_{\epsilon,1}\frac{k^2}{k_{\rm M}^2} + c_{\epsilon,2} f\mu^2\frac{k^2}{k_{\rm M}^2}\right), \qquad (10)
\end{aligned}
$$

with

$$
\begin{aligned}
Z_1(\mathbf{q}_1) &= K_1(\mathbf{q}_1) + f\mu_1^2 G_1(\mathbf{q}_1) = b_1 + f\mu_1^2, \\
Z_2(\mathbf{q}_1,\mathbf{q}_2,\mu) &= K_2(\mathbf{q}_1,\mathbf{q}_2) + f\mu_{12}^2 G_2(\mathbf{q}_1,\mathbf{q}_2) + \frac{1}{2}f\mu q\left(\frac{\mu_2}{q_2}G_1(\mathbf{q}_2)Z_1(\mathbf{q}_1) + \text{perm.}\right), \\
Z_3(\mathbf{q}_1,\mathbf{q}_2,\mathbf{q}_3,\mu) &= K_3(\mathbf{q}_1,\mathbf{q}_2,\mathbf{q}_3) + f\mu_{123}^2 G_3(\mathbf{q}_1,\mathbf{q}_2,\mathbf{q}_3) \\
&\quad + \frac{1}{3}f\mu q\left(\frac{\mu_3}{q_3}G_1(\mathbf{q}_3)Z_2(\mathbf{q}_1,\mathbf{q}_2,\mu_{123}) + \frac{\mu_{23}}{q_{23}}G_2(\mathbf{q}_2,\mathbf{q}_3)Z_1(\mathbf{q}_1) + \text{cyc.}\right),
\end{aligned}
$$

with

$$
\begin{aligned}
K_1 &= b_1, \\
K_2(\mathbf{q}_1,\mathbf{q}_2) &= b_1\frac{\mathbf{q}_1\cdot\mathbf{q}_2}{q_1^2} + b_2\left(F_2(\mathbf{q}_1,\mathbf{q}_2) - \frac{\mathbf{q}_1\cdot\mathbf{q}_2}{q_1^2}\right) + b_4 + \text{perm.}, \\
K_3(k,q) &= \frac{b_1}{504k^3q^3}\left(-38k^5q + 48k^3q^3 - 18kq^5 + 9(k^2-q^2)^3\log\left[\frac{k-q}{k+q}\right]\right) \\
&+ \frac{b_3}{756k^3q^5}\left(2kq(k^2+q^2)(3k^4 - 14k^2q^2 + 3q^4) + 3(k^2-q^2)^4\log\left[\frac{k-q}{k+q}\right]\right)
\end{aligned}
$$

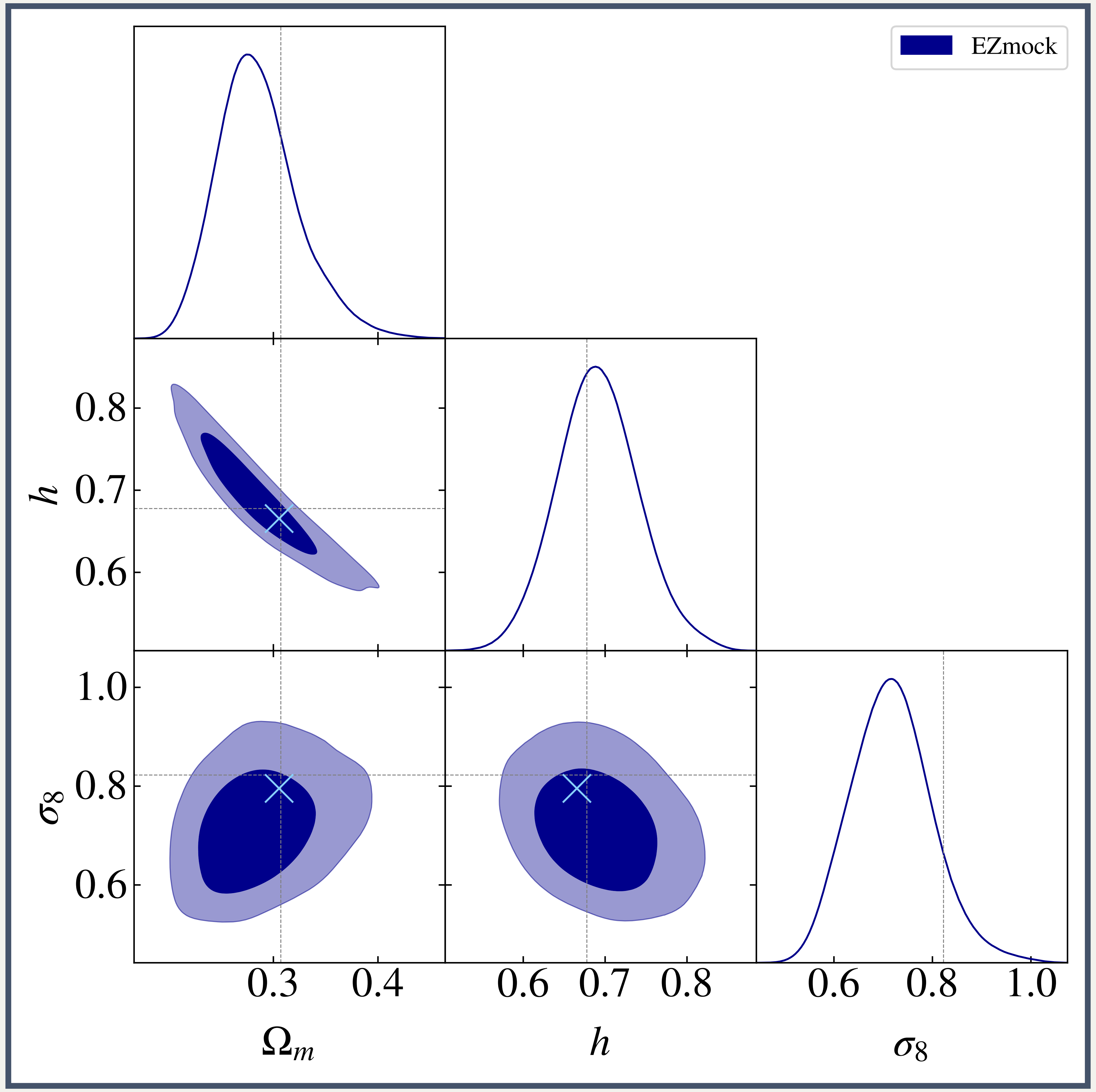
10 parameters

4 parameters b_i $(i = 1,2,3,4)$ to describe the **galaxy bias** which arises from the one-loop contributions.

3 parameters corresponding to **counterterms** (c_{ct} linear combination of a higher derivative bias and the dark matter sound speed, while $c_{r,1}$ and $c_{r,2}$ are the redshift-space counterterms).

3 parameters which describe **stochastic** terms.

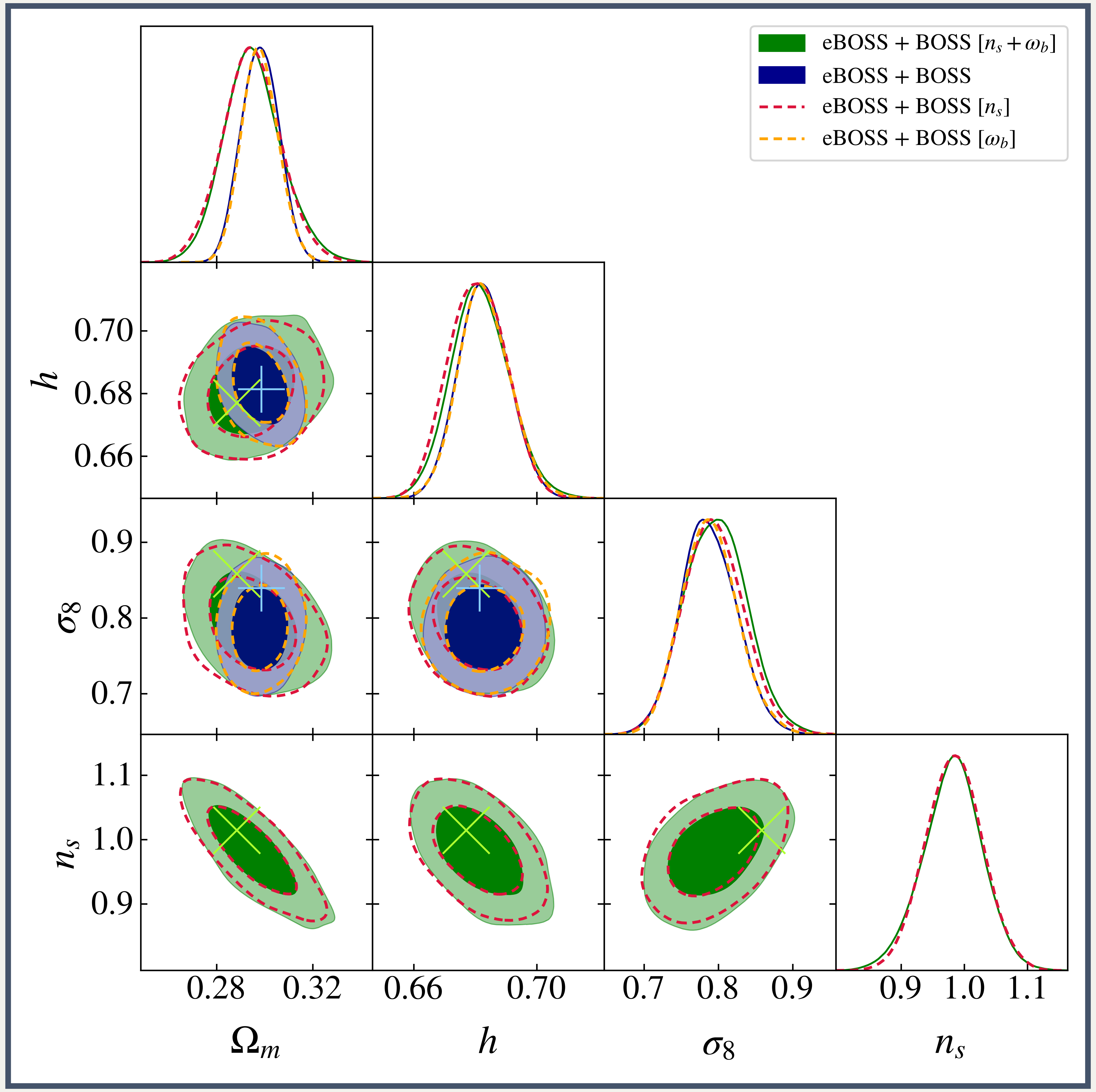
Backup II: test against simulations



- **EZmock**: mocks that are built to simulate eBOSS observational characteristics

- We find that up to $k_{\max} = 0.24h\,\mathrm{Mpc}^{-1}$, the best-fit values of the cosmological parameters are shifted with respect to the truth of the simulations by $\lesssim 1/3 \cdot \sigma$

Backup III: variation of n_s and ω_b



- We impose a uninformative large flat prior on n_s, while we impose a BBN Gaussian prior on ω_b
- The variation of ω_b within the BBN prior has a negligible impact on the cosmological results: we have a relative shift of $\lesssim 0.04\sigma$
- The variation of n_s within a uninformative large flat prior leads to a relative shift $\lesssim 0.4\sigma$

Backup IV: dark energy equation of state $w_0 \geq -1$

- One can see that this new prior shifts the 2D posteriors inferred from the LSS data in a non-negligible way, while it remains globally stable for the LSS + Planck

- For these analyses, $\Delta\chi^2 = 0$ with respect to ΛCDM, since we obtain best-fit values of $w_0 = -1$

LSS:
$w_0 < -0.932$

LSS+Planck:
$w_0 < -0.965$