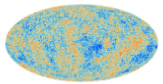
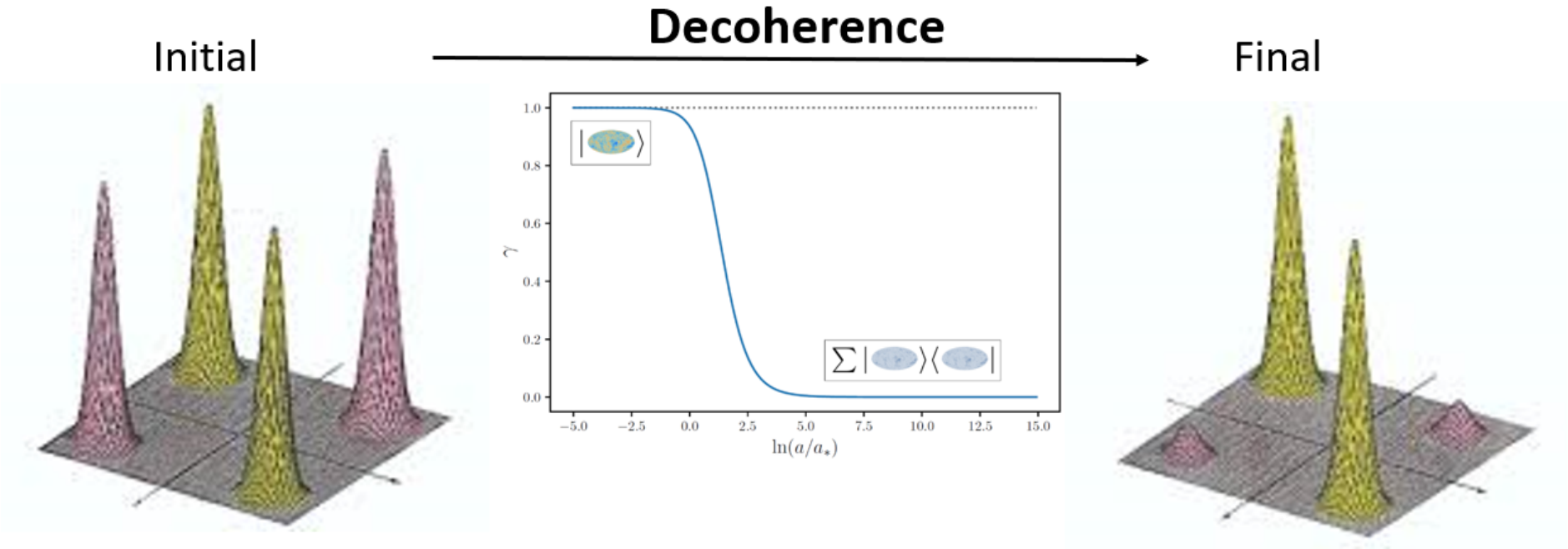
The Universe (not quite) as a Quantum Lab

$|$  $\rangle$?

Thomas Colas

GdR CoPhy

Quantum decoherence in the lab



Pure state

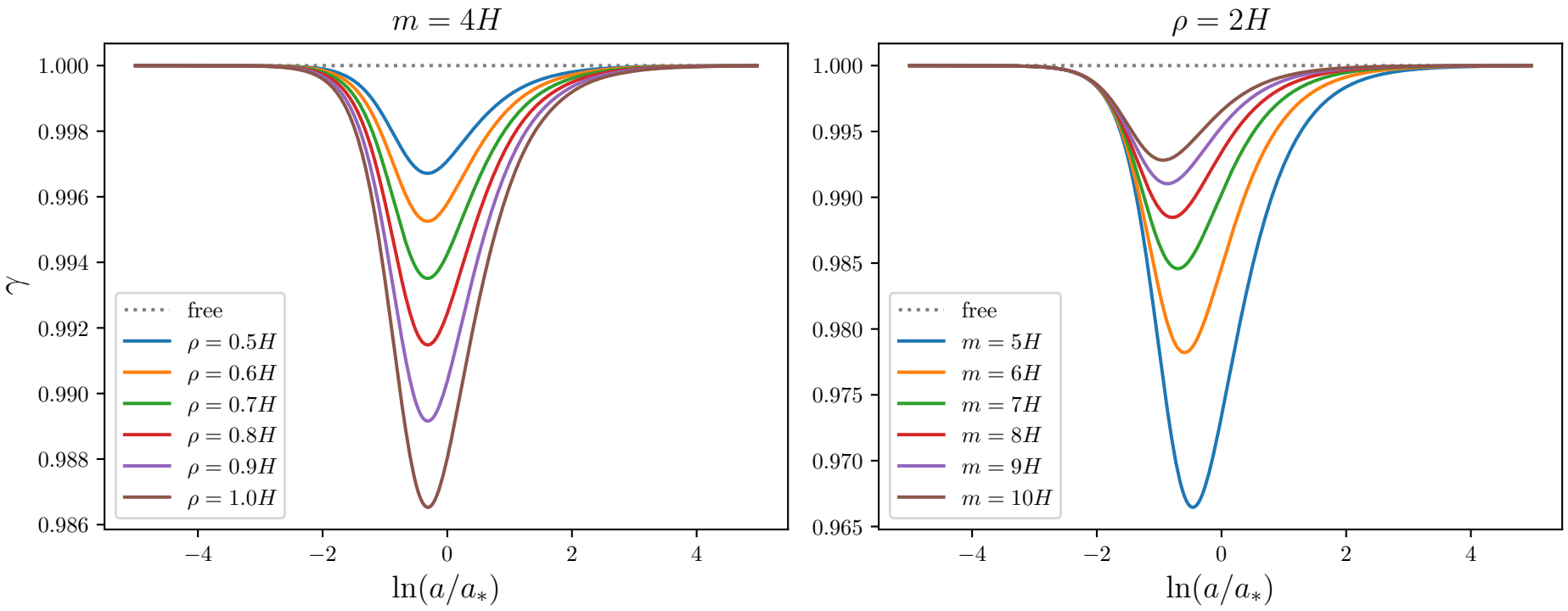
- $|\text{cat}_+\rangle = \frac{1}{\sqrt{2}}\left(|\uparrow\rangle + |\downarrow\rangle\right)$

Mixed state

- $\widehat{\rho}_{\text{red}} \to \frac{1}{2}\left(|\uparrow\rangle\langle\uparrow| + |\downarrow\rangle\langle\downarrow|\right)$

Quantum recoherence in the early universe 2212.09486

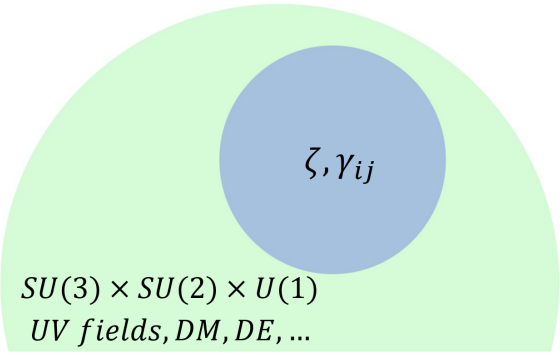
$$\mathcal{L} = \overbrace{a^2\epsilon M_{\rm Pl}^2\zeta'^2 - a^2\epsilon M_{\rm Pl}^2\left(\partial_i\zeta\right)^2}^{\text{System}} + \overbrace{\frac{1}{2}a^2\mathcal{F}'^2 - \frac{1}{2}a^2\left(\partial_i\mathcal{F}\right)^2 - \frac{1}{2}m^2a^4\mathcal{F}^2}^{\text{Environment}} - \overbrace{\rho a^3\sqrt{2\epsilon}M_{\rm Pl}\zeta'\mathcal{F}}^{\text{Interaction}}$$



Outline

How do we do QFT with hidden sectors?

More data = ***More sensible*** *to the interplay* ***visible-hidden*** *sectors.*



Limitations to *model building:*

- Some dof are **unkownn**;

Limitations to *Wilsonian EFTs:*

- Extensions are **less symmetric**;

Open Effective Field Theories:

- Dissipation: *BH mergers, warm inflation, thermal corrections*;
- Decoherence: *quantum-to-classical transition, BH information*;
- Non-unitary extensions to Wilsonian EFTs: *bootstrap validity*;
- Late-time resummation: *secular divergences and DRG*;
- Backreaction and UV/IR mixing: *stochastic inflation*.

Classical and Quantum Open Systems

What happens to particles immersed into an environment? [Brown, 1827]

Classical Brownian motion

Langevin equation

$$\mathrm{d}\mathcal{O} = \{H, O\}\mathrm{d}t + \mathrm{d}\xi$$

Wiener path integral [Wiener, 1923]

$$P = \int \mathcal{D}q e^{-S_0(q)}$$

Fokker-Planck equation

$$\frac{\mathrm{d}P}{\mathrm{d}t} = \mathcal{L}_{\mathrm{FP}}[P]$$

Quantum Brownian motion

Stochastic Schrödinger equation

$$|\mathrm{d}\psi\rangle = -i\left[\widehat{H}, \widehat{\mathcal{O}}\right]\mathrm{d}t + \mathrm{d}\widehat{\xi}$$

Influence functional [Vernon, 1959]

$$\widehat{\rho}_{\mathrm{red}} = \int \mathcal{D}\phi_0^{\pm}\mathcal{I}\left[\phi_0^{\pm}\right]\widehat{\rho}_{\mathrm{red},0}$$

Master equation

$$\frac{\mathrm{d}\widehat{\rho}_{\mathrm{red}}}{\mathrm{d}t} = \mathcal{V}[\widehat{\rho}_{\mathrm{red}}]$$

- Need to specify the **coupling** and the **nature of the environment**
  - See 2209.01929 for the importance of out-of-equilibrium and non-Markovian environments in cosmology.
- Access **observables** and **quantum information properties**.

Scalar-field extension

Add a **massive scalar field** to EFTI [Assassi *et al.*, 2013]: Open EFT treatment of the adiabatic and entropic decomposition of multifield inflation.

- System: *massless scalar*

$$S_\zeta^{(2)} = \int \mathrm{d}^4x \epsilon a^2 M_{\mathrm{Pl}}^2 \left[\zeta'^2 - (\partial_i \zeta)^2\right]$$

- Environment: *massive scalar*

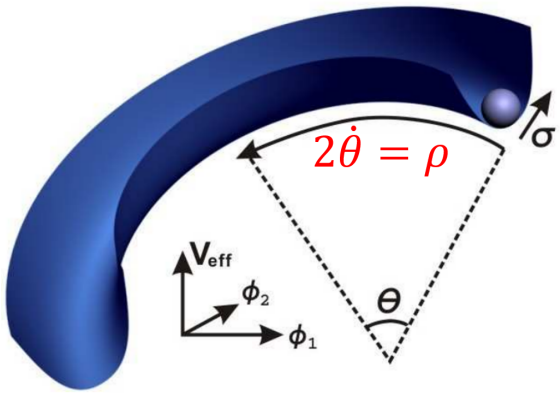
$$S_{\mathcal{F}}^{(2)} = \int \mathrm{d}^4x a^2 \left[\frac{1}{2}\mathcal{F}'^2 - \frac{1}{2}(\partial_i \mathcal{F})^2 - \frac{1}{2}m^2 a^2 \mathcal{F}^2\right]$$

- Interaction: *momentum coupling*

$$S_{\zeta\mathcal{F}}^{(2)} = -\int \mathrm{d}^4x \rho a^3 \sqrt{2\epsilon} M_{\mathrm{Pl}} \zeta' \mathcal{F}$$

Use **numerical exact solution** and **effective single-field open dynamics**.

Standard observables



[Credits: Chen *et al.*, 2010]

ME exactly **recovers WEFT result**:

- Leading physics captured by effective speed of sound

$$c_s^2 = 1 - \frac{\rho^2}{m^2}$$

- Scale correction to the power spectrum

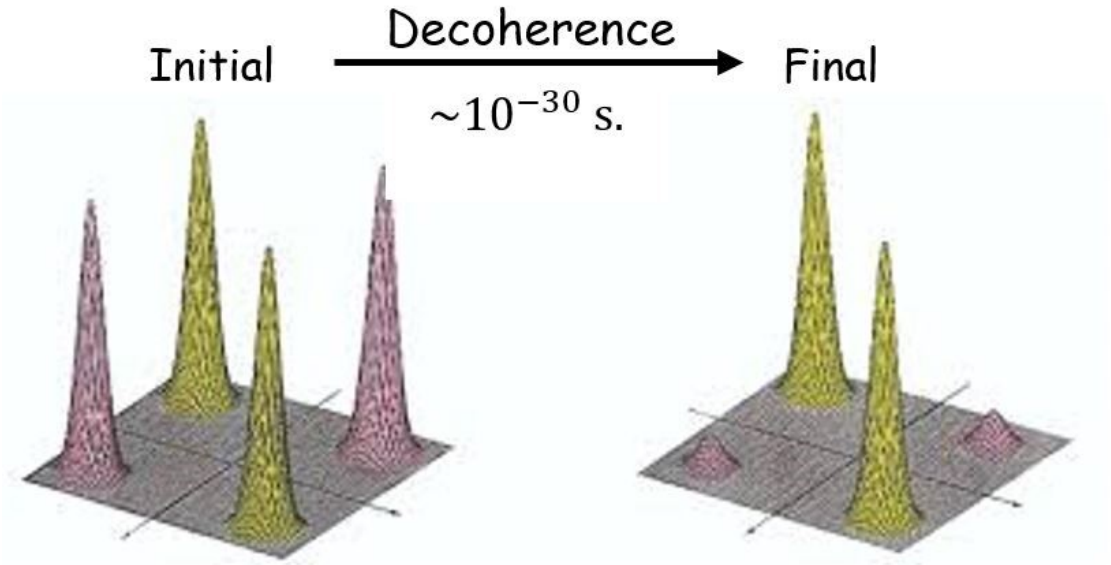
$$\mathcal{P}_\zeta = \mathcal{P}_\zeta^{(0)} \left[1 + \frac{1}{2} \frac{\rho^2}{m^2} \right]$$

No dramatic resummation in this model $\Rightarrow$ ***strengthens in-in result***.

(Explanation: no secular divergences $\Rightarrow$ no late-time resummation).

Purity as a measure of decoherence

$$\gamma \equiv \mathrm{Tr}_{\mathcal{S}}\, \widehat{\rho}_{\mathrm{red}}^2 \qquad \text{with} \qquad \widehat{\rho}_{\mathrm{red}} \equiv \mathrm{Tr}_{\mathcal{E}}\, \widehat{\rho}$$



Pure state

- $|\mathrm{cat}_+\rangle = \frac{1}{\sqrt{2}}\left(|\uparrow\rangle + |\downarrow\rangle\right)$
- $\gamma = 1$

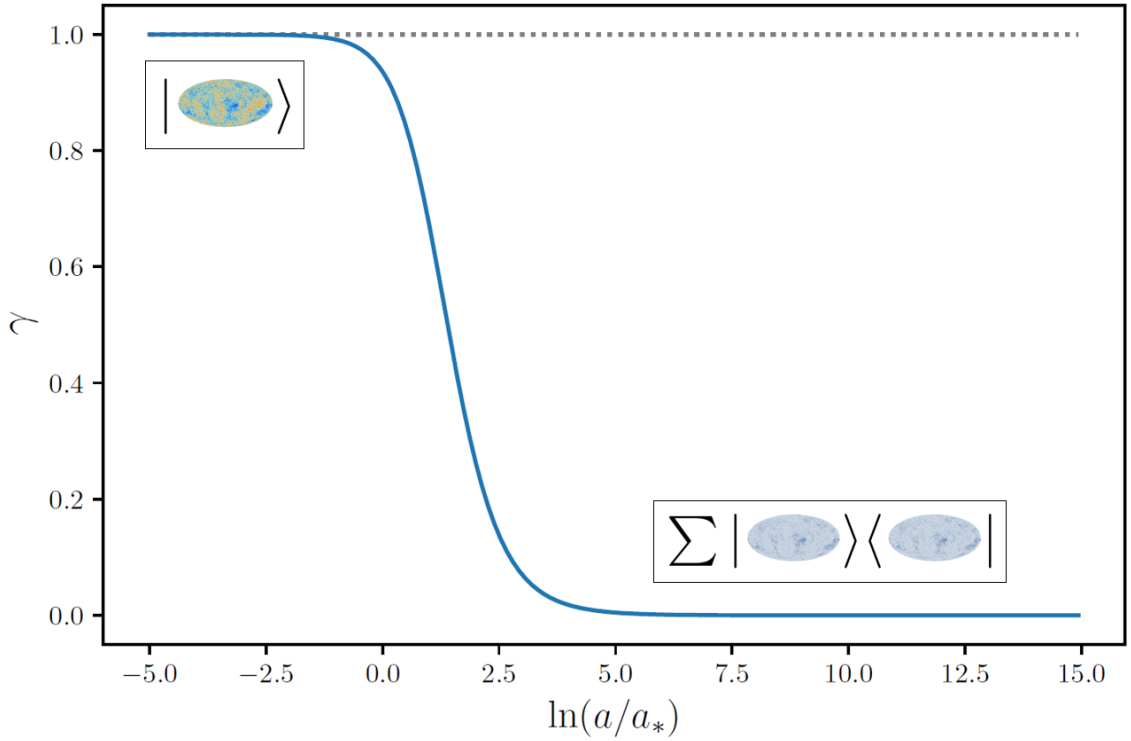
Mixed state

- $\widehat{\rho}_{\mathrm{red}} \to \frac{1}{2}\left(|\uparrow\rangle\langle\uparrow| + |\downarrow\rangle\langle\downarrow|\right)$
- $\gamma \to 1/\dim \mathcal{H}_N$

Obstructions to Bell CMB Experiments [Martin & Vennin, 2017]

Pair creation process $\hat{a}^\dagger_{\boldsymbol{k}}\hat{a}^\dagger_{-\boldsymbol{k}}$ = Large amount of **non-classical correlations**.

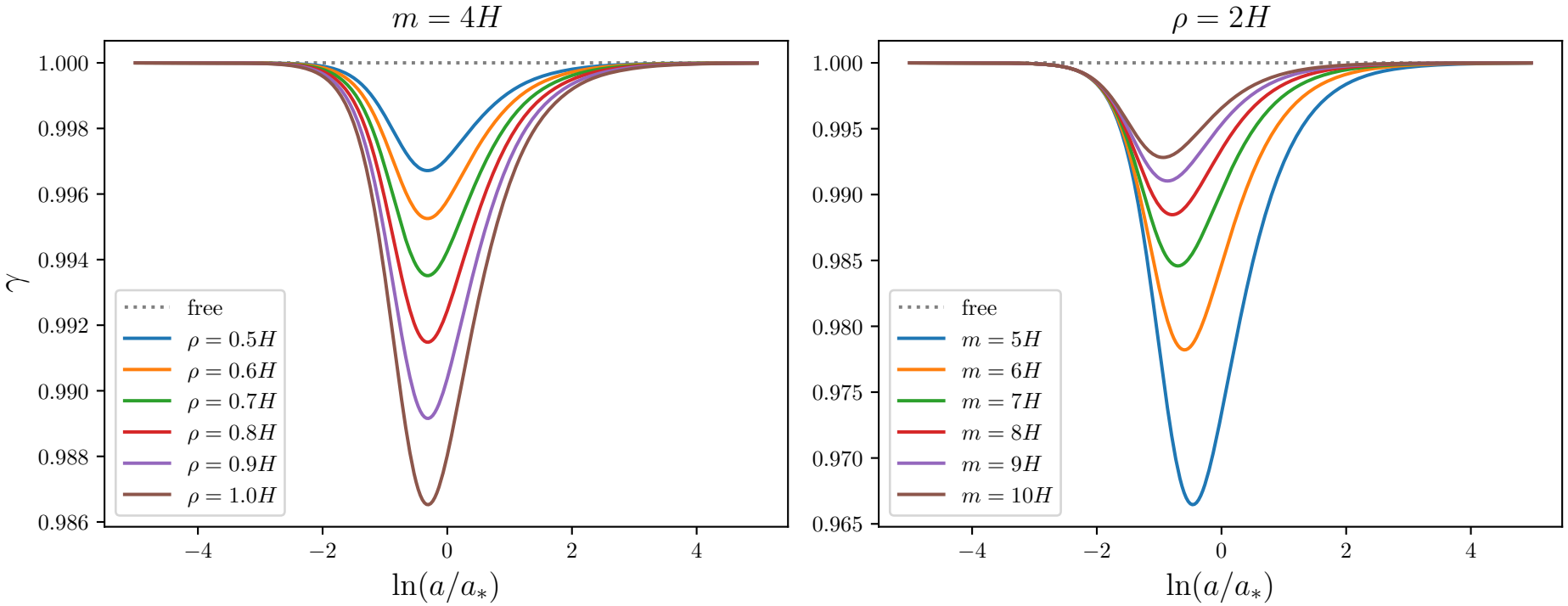
Can we exhibit quantum correlations in the CMB?



Decoherence of ζ may erase any quantum signature.

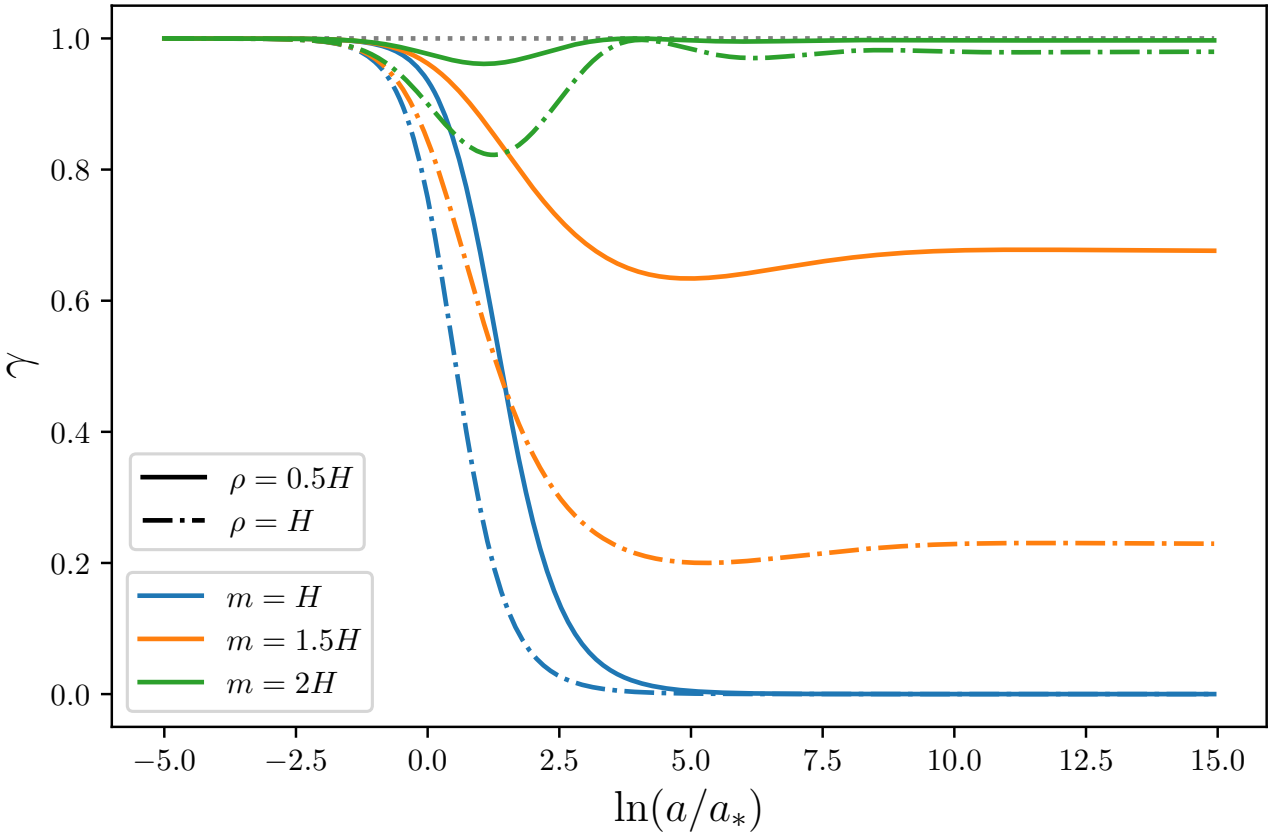
Do isocurvature modes generate quantum decoherence?

Quantum recoherence in the early universe

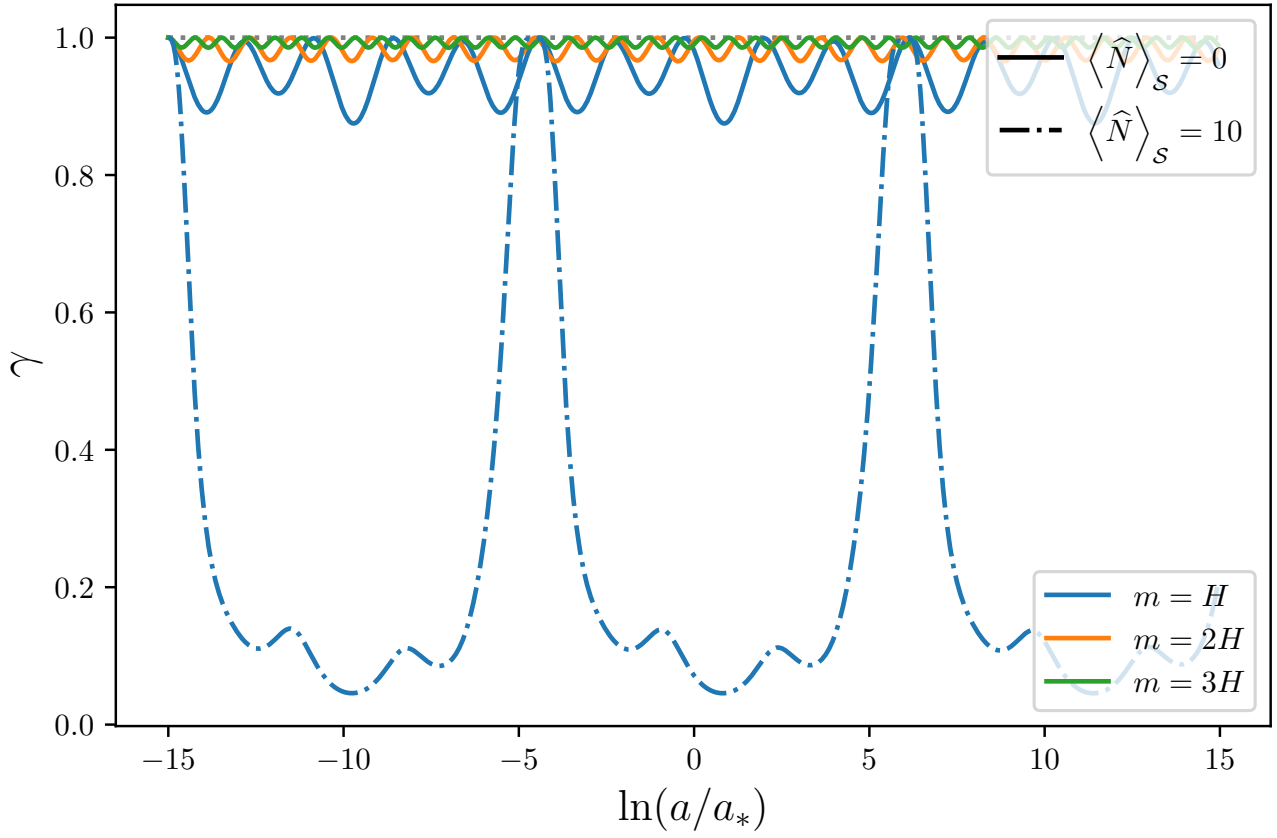


$\mathcal{F}$ **preserves late-time coherence** of ζ.

Heavy *versus* light environments

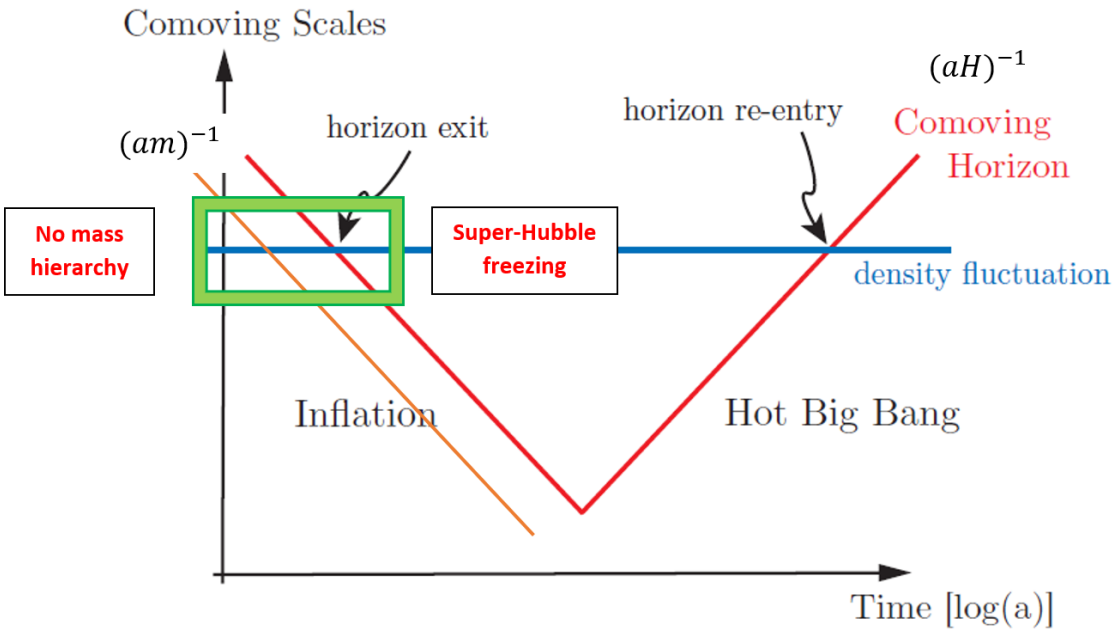


Flat space limit



Poincaré recurrence is unavoidable in Minkowski.

Scale hierarchies



1. $k > am$: no mass hierarchy $\Rightarrow$ no oscillations;
2. $am > k > aH$: degeneracy breaks down $\Rightarrow$ starts to oscillate;
3. $k < aH$: super-Hubble freezing $\Rightarrow$ $\dot{\zeta}\mathcal{F}$ quenched off.

<u>*Exception*</u>: $\mathcal{F}$ light $\Rightarrow$ growing mode in $\dot{\zeta}\mathcal{F}$ drives $\widehat{\rho}_{\rm red}$ into a mixed state.

What did we learn?

1. Cosmology breaks common sense of lab-based experiments;
2. **Heavy decoupling** even at level of quantum information properties.

Need for quantitative investigation of decoherence channels and non-unitary extensions in the early universe.

Two ongoing projects:

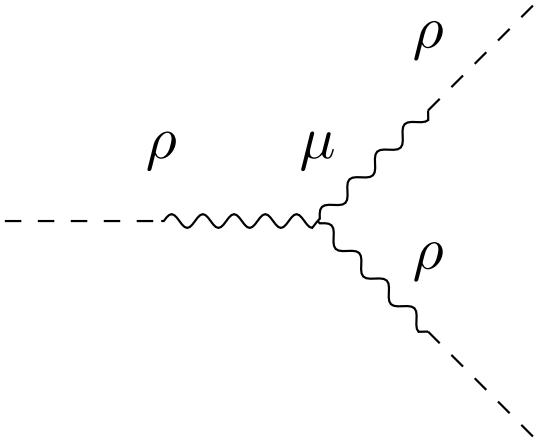
- Connecting **Primordial Non-Gaussianities** and QI properties.
- What is the relation between **Open** and **Wilsonian** EFTs?

Primordial Non-Gaussianities and QIP with S. Brahma and J. Calderón-Figueroa

Consider quasi-single field inflation [Chen & Wang, 2010]:

$$\mathcal{L} = \overbrace{-\frac{1}{2}(\partial\varphi)^2}^{\text{System}} \overbrace{- \frac{1}{2}(\partial\sigma)^2 - \frac{1}{2}m^2\sigma^2 - \mu\sigma^3}^{\text{Environment}} \overbrace{+ \rho\dot{\varphi}\sigma}^{\text{Interaction}}$$

Goal: Open EFT treatment for **bispectrum computation**. In-in results:



$$\lim_{k_1 \ll k_2 \approx k_3} \langle \widehat{\varphi}_{\boldsymbol{k}_1} \widehat{\varphi}_{\boldsymbol{k}_2} \widehat{\varphi}_{\boldsymbol{k}_3} \rangle \propto P(k_2) P(k_1) \left(\frac{k_1}{k_2}\right)^{\frac{3}{2}-\nu(m)}$$

Link between decoherence timescales and observational signatures?
What does the Open EFT resum in this case?

WEFT and their non-unitary embedding with C. de Rham and G. Kaplanek

Partial UV-completion

$$S_{\rm UV}[\varphi,\chi] = \int d^4x \left[-\frac{1}{2}(\partial\varphi)^2 - \frac{1}{2}(\partial\chi)^2 - \frac{1}{2}M^2\chi^2 + \frac{g}{\Lambda_*}(\partial\varphi)^2\,\chi\right]$$

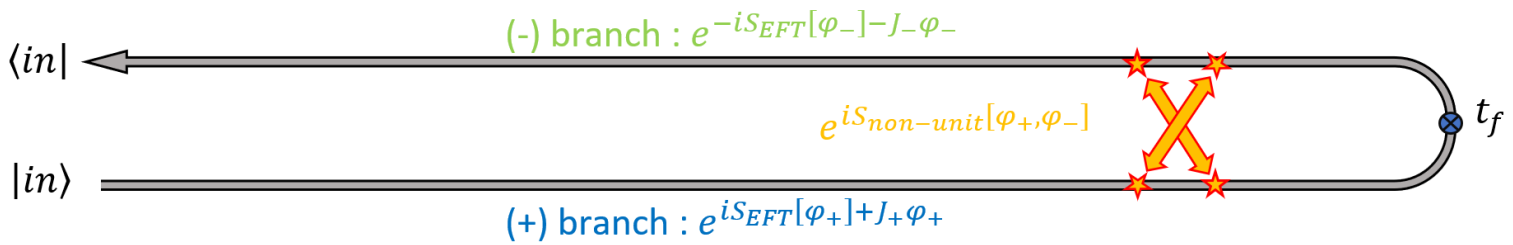
1. **WEFT** after integrating χ: local and unitary

$$S_{\rm EFT}[\varphi] \simeq \int d^4x \left[-\frac{1}{2}(\partial\varphi)^2 + \frac{g^2}{2M^2\Lambda_*^2}(\partial\varphi)^4 + \mathcal{O}\left((\partial\varphi)^6/M^4\right)\right]$$

2. **IF** after integrating χ: non-local and non- unitary

$$Z_{\rm IF}[J_+,J_-] = \int \mathcal{D}\varphi_+\mathcal{D}\varphi_-\, e^{iS_{\rm EFT}[\varphi_+]+J_+\varphi_+-iS_{\rm EFT}[\varphi_-]-J_-\varphi_-+iS_{\rm non-unit}[\varphi_+,\varphi_-]}$$

$$\langle\varphi_+|\rho_{\rm ref}(t_0)|\varphi_-\rangle\delta\left(\varphi_+(\boldsymbol{x},t_f)-\varphi_-(\boldsymbol{x},t_f)\right)$$



Summary and outlook

Open EFTs may be useful to:

1. Access **QI properties** of inflationary models [2212.XXXXX];
2. Perform **late-time resummations** [2209.01929];

Possible applications:

- Scalar-induced Gravitational Waves;
- Black Hole information.

Future directions:

- Bottom-up constructions of Open EFTs [Glorioso & Liu, 2018];
- **Positivity bounds** for non-unitary constructions;
- **Consistency relations** for entanglement tracers: soft-mode decoherence.

Outline

The master equation zoo

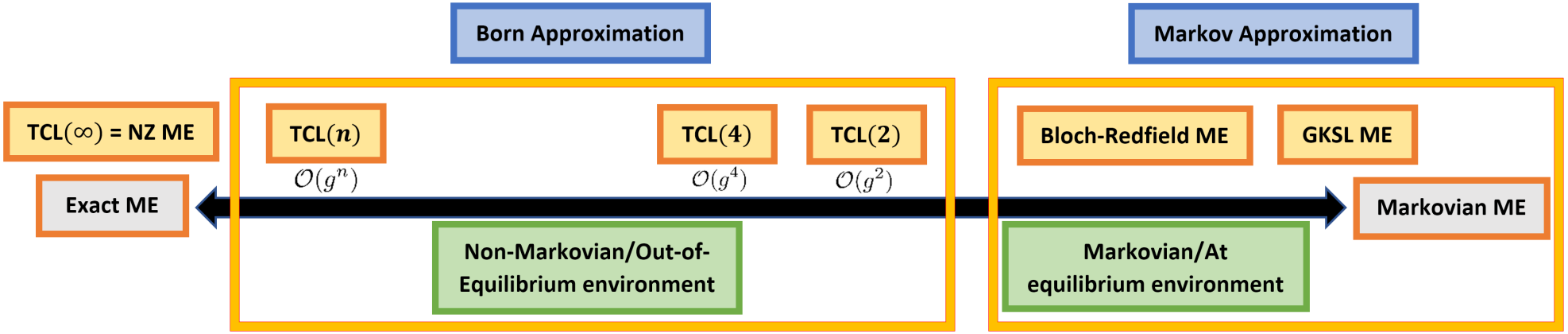
- Fundamental observables are correlators

$$\left\langle \widehat{\mathcal{O}}_1 \widehat{\mathcal{O}}_2 \cdots \widehat{\mathcal{O}}_n \right\rangle (t) \equiv \mathrm{Tr}\left[\widehat{\mathcal{O}}_1 \widehat{\mathcal{O}}_2 \cdots \widehat{\mathcal{O}}_n \widehat{\rho}_{\mathrm{red}}(t)\right]$$

- **Master Equations** (ME) are dynamical equations for $\widehat{\rho}_{\mathrm{red}}(t)$

$$\frac{\mathrm{d}\widehat{\rho}_{\mathrm{red}}}{\mathrm{d}t} = \mathcal{V}\left(\widehat{\rho}_{\mathrm{red}}\right)$$

- There exists a **whole bestiary** of MEs [Breuer & Petruccione, 2002]



At which level should we work in cosmology ?

The curved-space Caldeira-Leggett model

- Action for the field sector:

$$S = -\int \mathrm{d}^4x \sqrt{-\det g}\Bigg(\overbrace{\left[\frac{1}{2}g^{\mu\nu}\partial_\mu\varphi\partial_\nu\varphi + \frac{1}{2}m^2\varphi^2\right]}^{\text{System}} + \underbrace{\left[\frac{1}{2}g^{\mu\nu}\partial_\mu\chi\partial_\nu\chi + \frac{1}{2}M^2\chi^2\right]}_{\text{Environment}} + \underbrace{\lambda^2\varphi\chi}_{\text{Interaction}}\Bigg)$$

- **Field redefinition**: rotation in field space of

$$\theta = -\frac{1}{2}\arctan\left(\frac{2\lambda^2}{M^2 - m^2}\right)$$

  decouple the two sectors: fully integrable model.

- **Gaussian system**:
  - All information contained in the system covariance $\mathbf{\Sigma}_{\varphi\varphi}$;
  - Quantum information properties: $\gamma = \det\left[\mathbf{\Sigma}_{\varphi\varphi}\right]^{-1}/4$.

Deriving a master equation (1)

Master equation: dynamical equation for the quantum state of the system.

1. Start with **Liouville-von Neumann equation**

$$\frac{\mathrm{d}\widetilde{\rho}}{\mathrm{d}\eta} = -i\lambda^2 \left[\widetilde{\mathcal{H}}_{\mathsf{int}}(\eta), \widetilde{\rho}(\eta)\right]$$

2. Introduce **projectors** $\widetilde{\rho} \mapsto \mathcal{P}\widetilde{\rho} = \widetilde{\rho}_{\mathsf{red}} \otimes \rho_{\mathrm{E}}$ and $\mathcal{Q}\widetilde{\rho} = \widetilde{\rho} - \mathcal{P}\widetilde{\rho}$

3. Rewrite dynamics as

$$\frac{\mathrm{d}}{\mathrm{d}\eta}\mathcal{P}\widetilde{\rho}(\eta) = \lambda^4 \int_{\eta_0}^{\eta} \mathrm{d}\eta' \mathcal{K}(\eta, \eta')\mathcal{P}\widetilde{\rho}(\eta)$$

$\mathcal{K}(\eta, \eta')$: memory kernel which depends on **the coupling** and **the environment**.

Deriving a master equation (2)

Expand in powers of **the coupling** constant

$$\frac{\mathrm{d}}{\mathrm{d}\eta}\mathcal{P}\widetilde{\rho}(\eta) = \sum_{n=0}^{\infty} \lambda^{2n}\mathcal{K}_n(\eta)\mathcal{P}\widetilde{\rho}(\eta)$$

Lowest order leads to the **non-Markovian ME**

$$\frac{\mathrm{d}\widetilde{\rho}_{\mathsf{red}}}{\mathrm{d}\eta} = -\lambda^4 \int_{\eta_0}^{\eta} \mathrm{d}\eta' \, \mathsf{Tr}_E \left[\widetilde{\mathcal{H}}_{\mathsf{int}}(\eta), \left[\widetilde{\mathcal{H}}_{\mathsf{int}}(\eta'), \widetilde{\rho}_{\mathsf{red}}(\eta) \otimes \rho_{\mathrm{E}}\right]\right]$$

1. This ME is local in time yet Non-Markovian [Breuer & Petruccione, 2002]: **breaks semi-group evolution**; the dissipator has a **negative eigenvalue**.
2. This ME does not assume a large environment [Breuer *et al.*, 2016]: suited to study **finite dimensional effects** in OQS.
3. CPTP can be checked for this class of **Non-Markovian Gaussian ME** [Diósi & Ferialdi, 2014].

Understanding the master equation

- In the **Fock space**: a master equation, with $\widehat{\boldsymbol{z}} = (\widehat{v}_\varphi, \widehat{p}_\varphi)^{\mathrm{T}}$

$$\frac{\mathrm{d}\widehat{\rho}_{\mathrm{red}}}{\mathrm{d}\eta} = \overbrace{-i\left[\widehat{H}_0(\eta) + \widehat{H}^{(\mathrm{LS})}(\eta), \widehat{\rho}_{\mathrm{red}}(\eta)\right]}^{\text{Unitary evolution}}$$
$$\underbrace{+\sum_{i,j} \boldsymbol{\mathcal{D}}_{ij}(\eta)\left[\widehat{\boldsymbol{z}}_i \widehat{\rho}_{\mathrm{red}}(\eta)\widehat{\boldsymbol{z}}_j - \frac{1}{2}\left\{\widehat{\boldsymbol{z}}_j\widehat{\boldsymbol{z}}_i, \widehat{\rho}_{\mathrm{red}}(\eta)\right\}\right]}_{\text{Non-unitary evolution}}$$

- In the **phase space**: a Fokker-Planck equation, with $\boldsymbol{z} = (v_\varphi, p_\varphi)^{\mathrm{T}}$

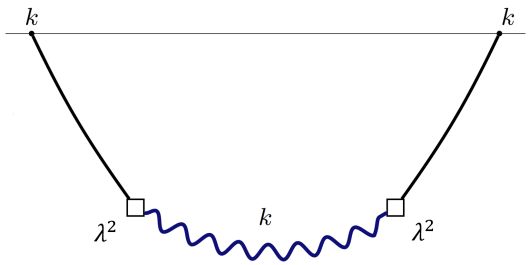
$$\frac{\mathrm{d}W_{\mathrm{red}}}{\mathrm{d}\eta} = \overbrace{\left\{\widetilde{H}_0(\eta) + \widetilde{H}^{(\mathrm{LS})}(\eta), W_{\mathrm{red}}(\eta)\right\}}^{\text{Unitary evolution}}$$
$$\underbrace{+\boldsymbol{\Delta}_{12}(\eta)\sum_i \frac{\partial}{\partial \boldsymbol{z}_i}\left[\boldsymbol{z}_i W_{\mathrm{red}}(\eta)\right] - \frac{1}{2}\sum_{i,j}\left[\boldsymbol{\omega}\boldsymbol{D}(\eta)\boldsymbol{\omega}\right]_{ij}\frac{\partial^2 W_{\mathrm{red}}(\eta)}{\partial \boldsymbol{z}_i \partial \boldsymbol{z}_j}}_{\text{Non-unitary evolution}}$$

Benchmarking cosmological master equations

- For the ME to be an interesting tool, it **needs to do better than standard techniques**.

- Benchmark against standard perturbation theory (SPT) results: **in-in formalism at linear order**:
  1. Write down mode functions equations of motion;
  2. Solve them perturbatively order by order.

  $\Rightarrow$ Observables are computed at a given order in λ^2.



- We compare ME and SPT against exact results on:
  1. Accuracy on the **system covariance** $\mathbf{\Sigma}_{\varphi\varphi}$;
  2. Ability to recover the **purity** $\gamma = \det\left[\mathbf{\Sigma}_{\varphi\varphi}\right]^{-1}/4$.

Perturbative ME is SPT

- ME generates an effective transport equation:

$$\frac{\mathrm{d}\boldsymbol{\Sigma}_{\varphi\varphi}}{\mathrm{d}\eta} = \underbrace{\boldsymbol{\omega}\left(\boldsymbol{H}^{(\varphi)} + \boldsymbol{\Delta}\right)\boldsymbol{\Sigma}_{\varphi\varphi} - \boldsymbol{\Sigma}_{\varphi\varphi}\left(\boldsymbol{H}^{(\varphi)} + \boldsymbol{\Delta}\right)\boldsymbol{\omega}}_{\text{Unitary evolution}} \underbrace{- 2\boldsymbol{\Delta}_{12}\boldsymbol{\Sigma}_{\varphi\varphi} - \boldsymbol{\omega}\boldsymbol{D}\boldsymbol{\omega}}_{\text{Non-unitary evolution}}$$

- If we solve the TCL_2 ME **perturbatively** at LO $\mathcal{O}(\lambda^4)$, we recover exactly SPT results.

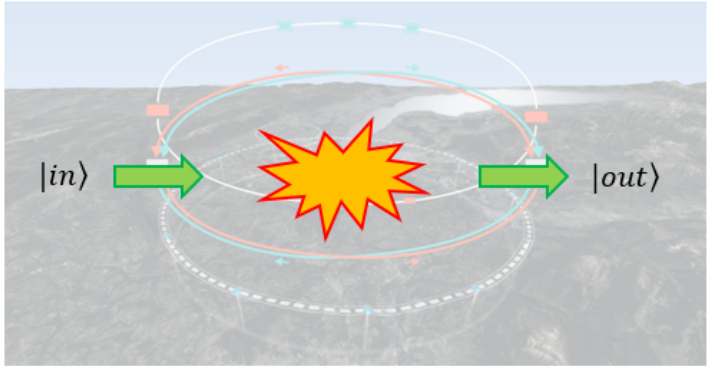
- We explicitly checked at **NLO** $\mathcal{O}(\lambda^8)$ **that SPT and TCL_4 match**. The proof is generalisable at any order:

$$\widetilde{\rho}_{\mathrm{red}}^{(n)}(\eta) = (-i)^n \lambda^{2n} \mathrm{Tr}_E \int_{\eta_0}^{\eta} \mathrm{d}\eta_1 \int_{\eta_0}^{\eta_1} \mathrm{d}\eta_2 \cdots \int_{\eta_0}^{\eta_{n-1}} \mathrm{d}\eta_n \left[\widetilde{\mathcal{H}}_{\mathrm{int}}(\eta_1), \left[\cdots \left[\widetilde{\mathcal{H}}_{\mathrm{int}}(\eta_n), \widehat{\rho}_{\mathrm{i}}\right]\right]\right]$$

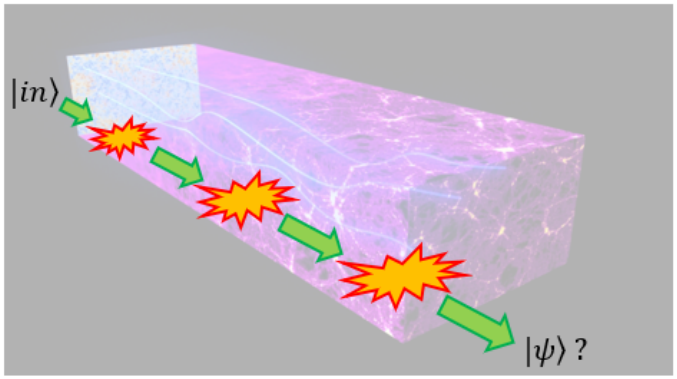
$$\left\langle \widehat{O}(\eta) \right\rangle^{(n)} = i^n \lambda^{2n} \left\langle \int_{\eta_0}^{\eta} \mathrm{d}\eta_1 \int_{\eta_0}^{\eta_1} \mathrm{d}\eta_2 \cdots \int_{\eta_0}^{\eta_{n-1}} \mathrm{d}\eta_n \left[\widetilde{\mathcal{H}}_{\mathrm{int}}(\eta_n), \left[\cdots \left[\widetilde{\mathcal{H}}_{\mathrm{int}}(\eta_1), \widetilde{O}(\eta)\right]\right]\right] \right\rangle_{\widehat{\rho}_{\mathrm{i}}}$$

Non-perturbative resummation

Difference between flat and curved spacetime QFT [Burgess *et al.*, 2015]: **cumulative effects** lead to secular divergences



Credits : FCC, CERN



Credits : Planck, ESA

- ME studied in cosmology for its ability to perform late-time resummations [Boyanovsky, 2015], [Burgess, Holman & Tasinato, 2015], [Brahma, Berera & Calderón-Figueroa, 2021]
- Resumation obtained when solving the effective transport equation **non-perturbatively**, considering ME as a *bona fide* dynamical map.
- **Partial resummation**: all the terms at leading order and some of the higher order terms [Colas, Grain & Vennin, 2022].

Spurious terms (1)

Effect of the environment encoded in

$$\boldsymbol{D}(\eta) = \lambda^4 a^2(\eta) \int_{\eta_0}^{\eta} \mathrm{d}\eta' a^2(\eta') \boldsymbol{G}_\varphi\left(\eta, \eta'\right) \mathrm{Re}\left\{\mathcal{K}^{>}(\eta, \eta')\right\} = \boldsymbol{F}_{\boldsymbol{D}}(\eta) - \boldsymbol{F}_{\boldsymbol{D}}(\eta_0)$$

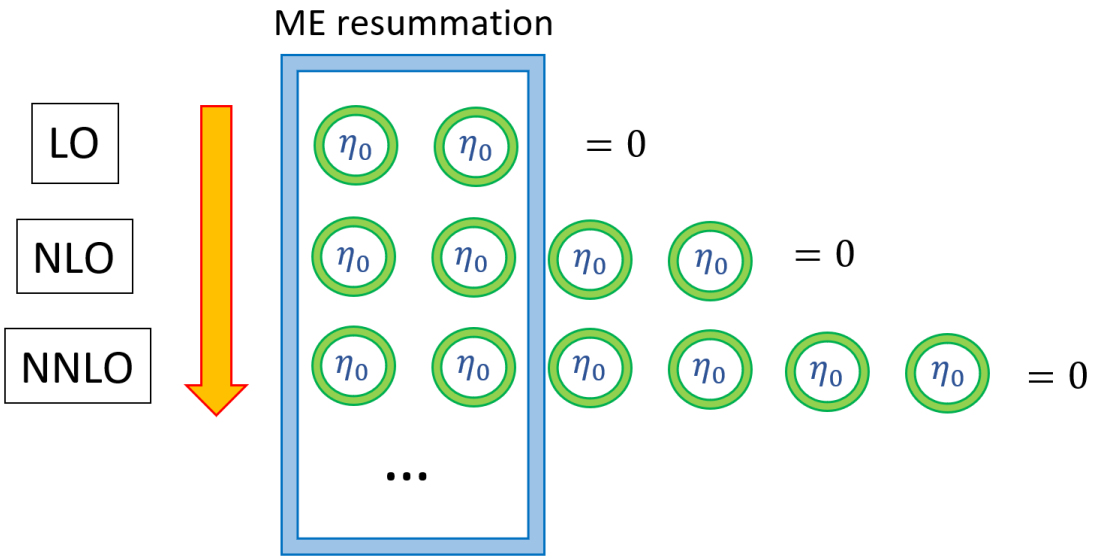
$$\boldsymbol{\Delta}(\eta) = \lambda^4 a^2(\eta) \int_{\eta_0}^{\eta} \mathrm{d}\eta' a^2(\eta') \boldsymbol{G}_\varphi\left(\eta, \eta'\right) \mathrm{Im}\left\{\mathcal{K}^{>}(\eta, \eta')\right\} = \boldsymbol{F}_{\boldsymbol{\Delta}}(\eta) - \boldsymbol{F}_{\boldsymbol{\Delta}}(\eta_0)$$

with the **memory kernel** $\mathcal{K}^{>}(\eta, \eta') = \langle \widehat{v}_\chi(\eta) \widehat{v}_\chi(\eta') \rangle_{\mathrm{free}}$.

- The η_0-dependent terms cancel each other when computing the observables **perturbatively** at any order.
- If we **include them in the non-perturbative resummation**, ME results diverge from exact ones at late time.

Spurious terms (2)

Explanation: partial resummation breaks order-by-order relations:



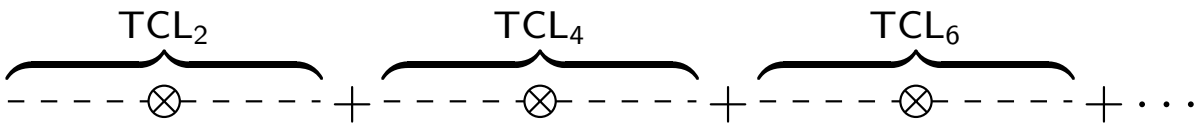
- TCL_n has **all terms** of order g^n and **some terms** of order $g^{m>n}$;
- The **cancellation requires all η_0-dependent terms** at a given order.

$\Rightarrow$ Need to impose the broken relation by hand before resumming.

What is being resummed ?

- In the **exact theory**, there is only one 1PI:

- In the **effective theory**, there is an infinite tower of 1PI:



one for each of the TCL cumulant.

- Moreover, there are **non-unitary contributions** from diffusion and dissipation which do not have diagrammatic representation.
- Hence, the question of knowing **which diagram has been resumed** is **ill-posed**. This feature is shared with WEFT and the DRG.

Late-time resummation technique

Following [Boyanovsky, 2015], [Brahma *et al.*, 2021],

$$\langle \widetilde{v}_\varphi(\eta)\widetilde{v}_\varphi(\eta)\rangle = v_-(\eta)v_-(\eta)\left\langle \widehat{P}_\varphi^2\right\rangle + v_-(\eta)v_+(\eta)\left\langle \widehat{Q}_\varphi\widehat{P}_\varphi + \widehat{P}_\varphi\widehat{Q}_\varphi\right\rangle$$
$$+v_+(\eta)v_+(\eta)\left\langle \widehat{Q}_\varphi^2\right\rangle \to v_+(\eta)v_+(\eta)\left\langle \widehat{Q}_\varphi^2\right\rangle$$

with

$$\frac{\mathrm{d}\left\langle \widehat{Q}_\varphi^2\right\rangle}{\mathrm{d}\eta} = \Gamma(\eta)\left\langle \widehat{Q}_\varphi^2\right\rangle$$

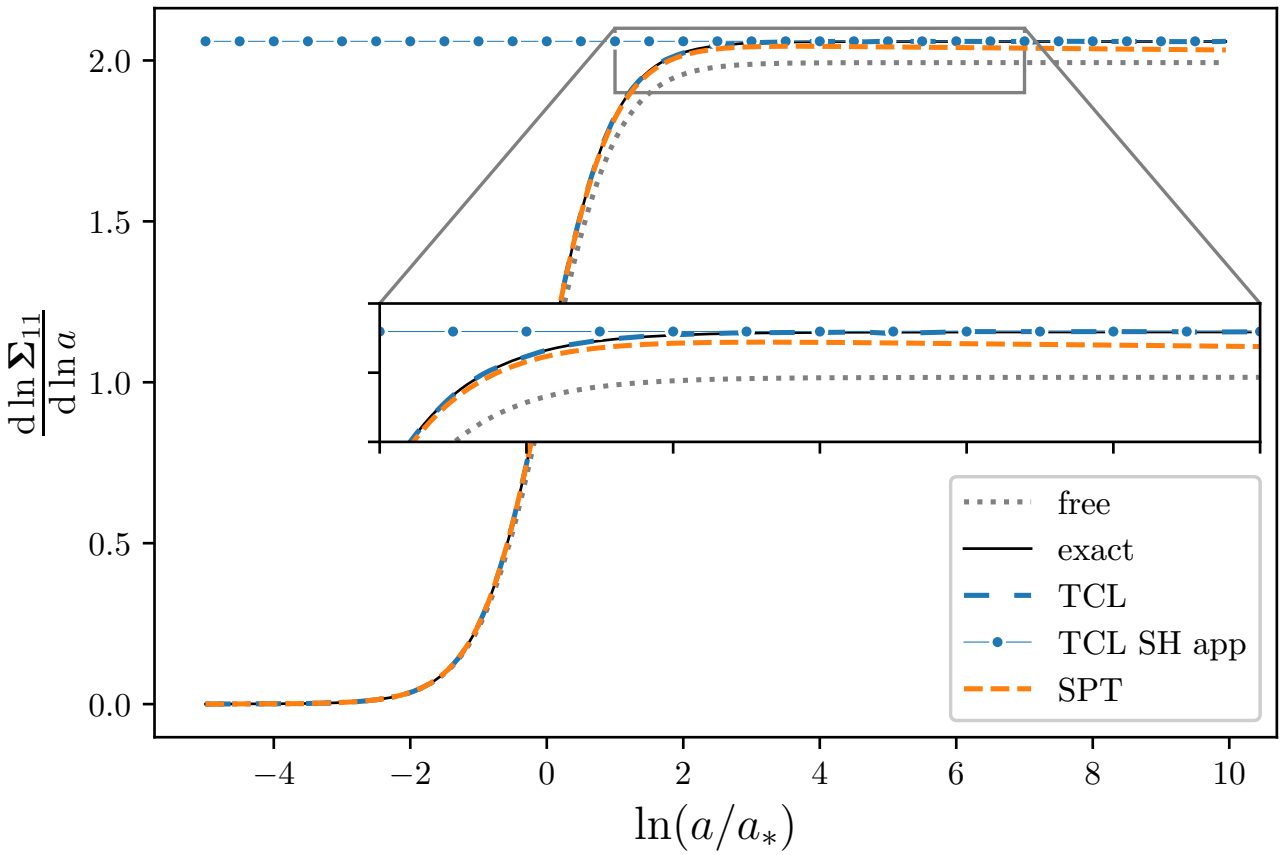
obtained from the **TCL$_2$ ME**.

In the *curved-space Caldeira-Leggett model*, leads to

$$\boldsymbol{\Sigma}_{\varphi\varphi}^{\mathrm{TCL}} \supset e^{-\frac{1}{\nu_\varphi}\frac{H^2}{M^2-m^2}\frac{\lambda^4}{H^4}\ln(-k\eta)}\boldsymbol{\Sigma}_{\varphi\varphi}^{(0)}$$

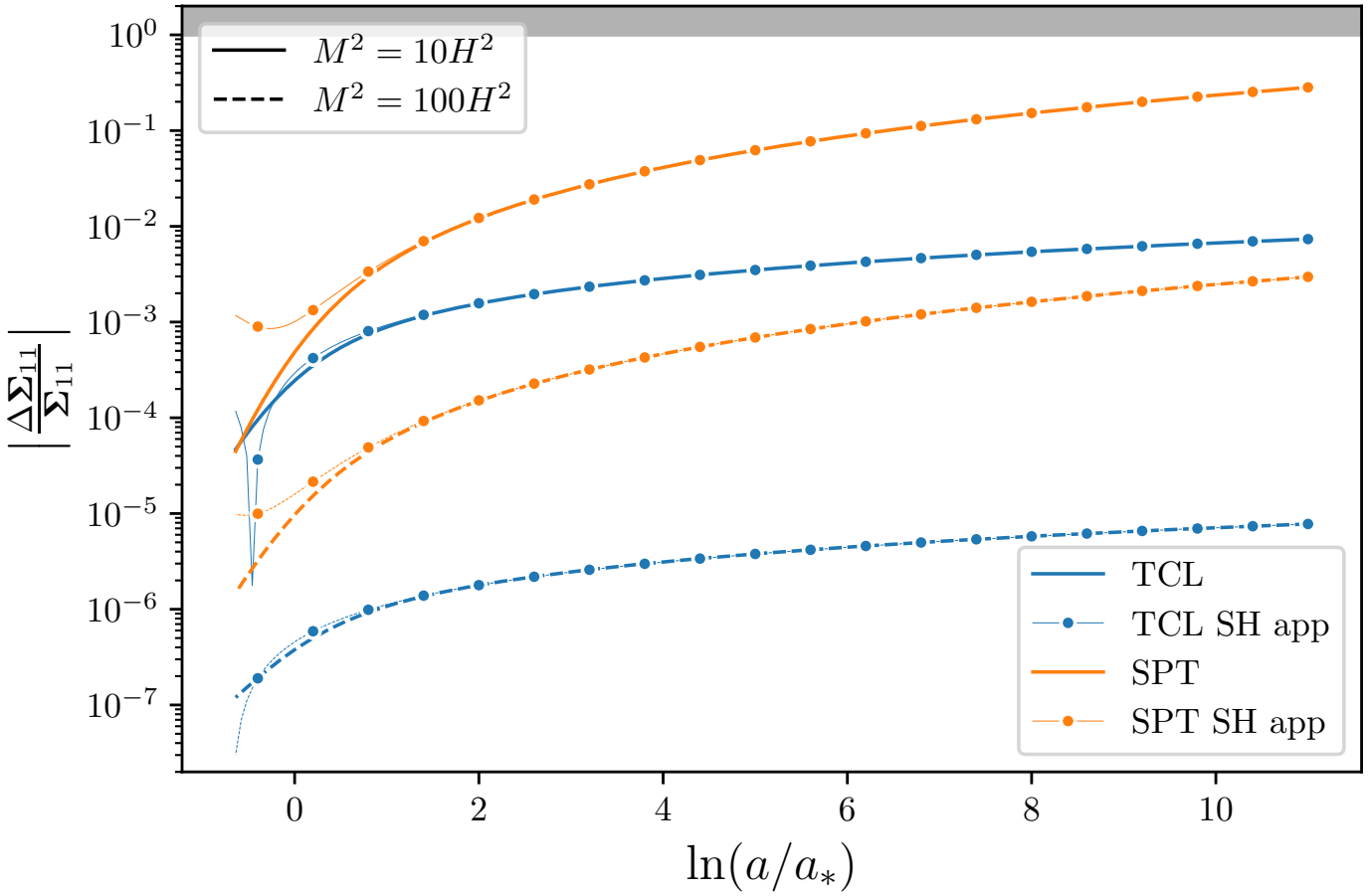
where late-time secular effects have been resummed.

Growth rate and effective mass



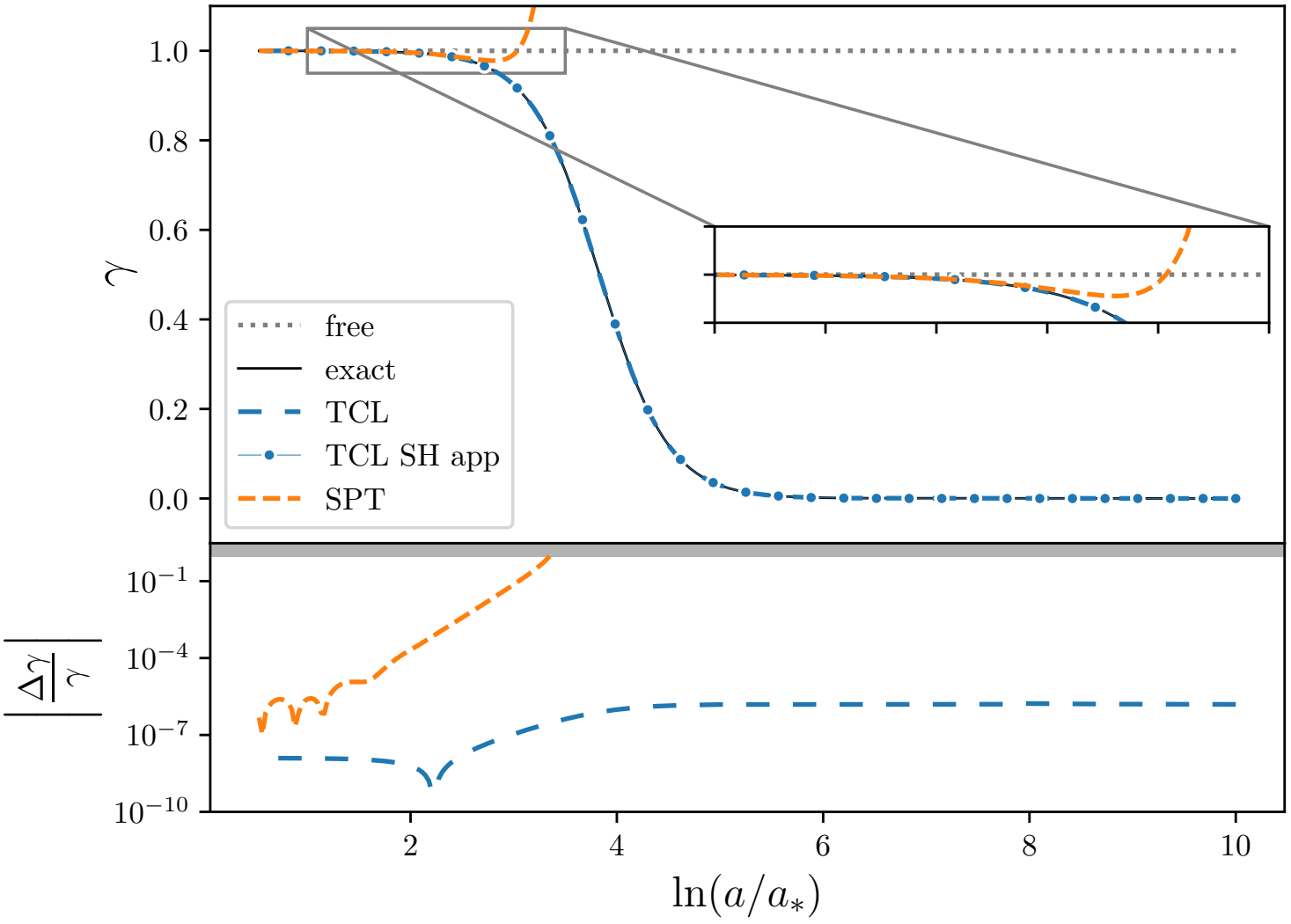
$$m_{\mathrm{eff}}^2 = \underbrace{m^2 - \frac{\lambda^4}{M^2 - m^2}}_{\mathrm{ME}} + \frac{\lambda^8}{(M^2 - m^2)^3} + \cdots$$

Improved late-time results

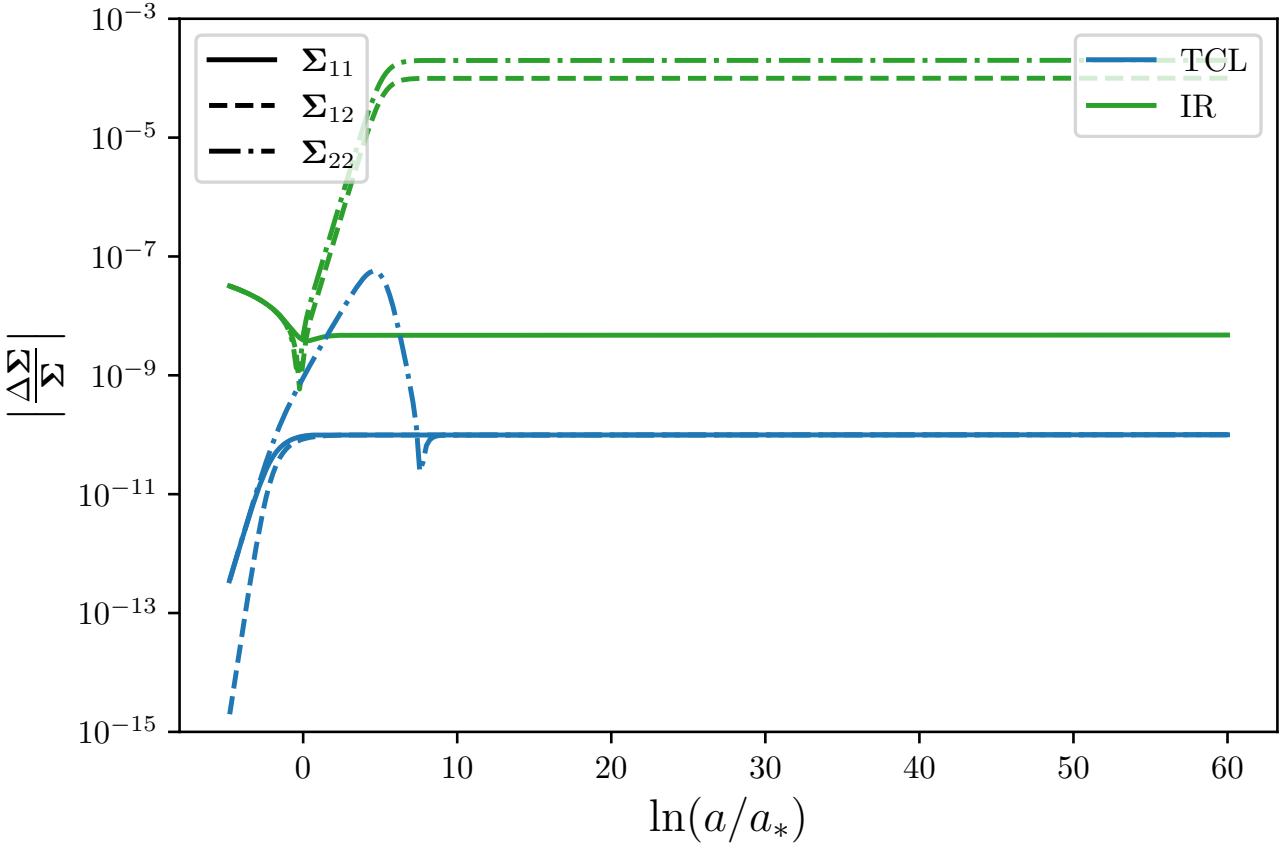


Error: SPT $\sim \frac{\lambda^8}{M^4} \ln^2 a$; ME $\sim \frac{\lambda^8}{M^6} \ln a$

Access quantum information properties



Late-time resummation and the DRG

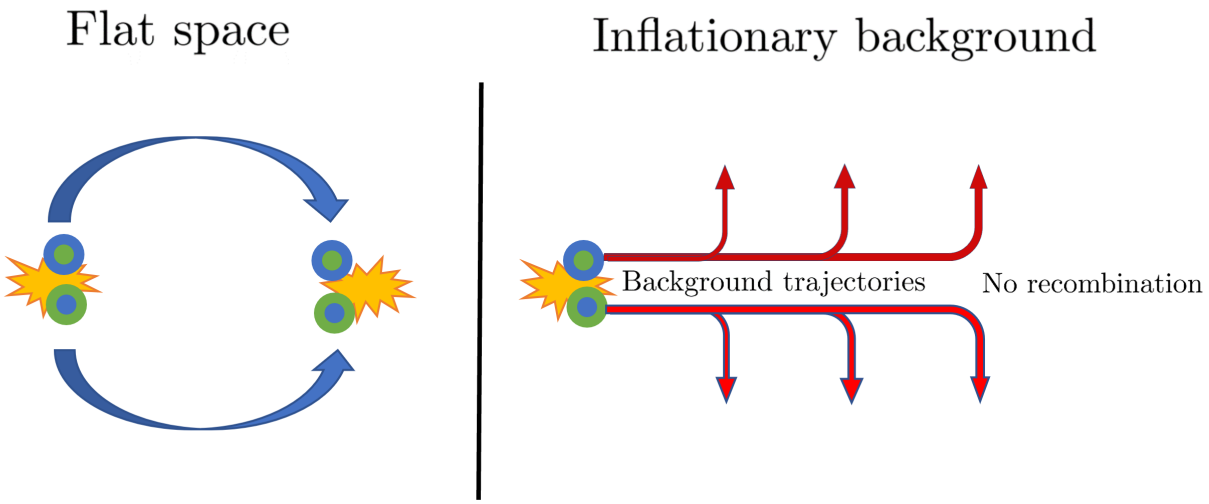


This resummation technique shares many features with the DRG [Burgess *et al.*, 2009].

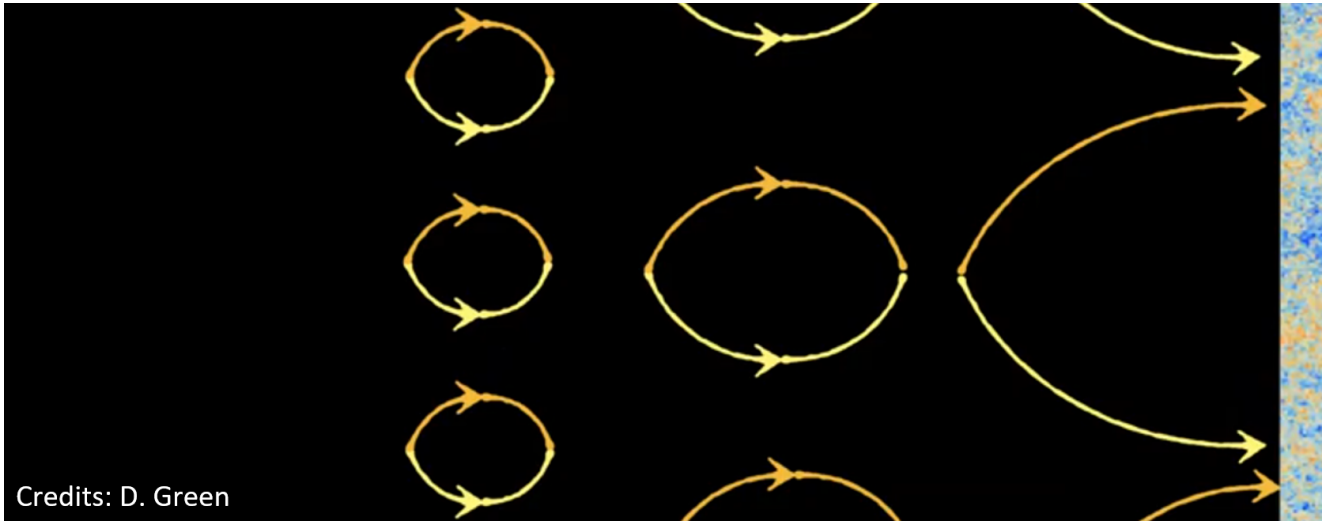
Are they equivalent ?

Outline

Pair creation in the early universe



Universe populated with highly entangled correlations.

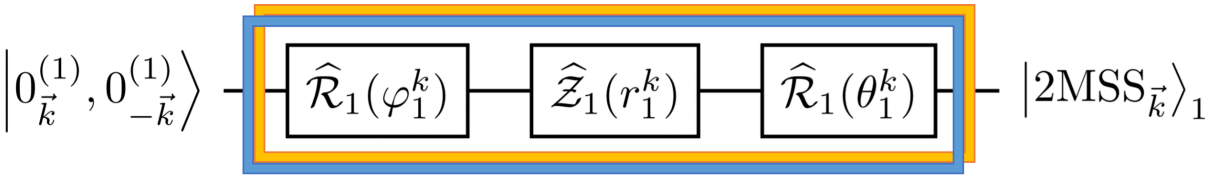


Two-mode squeezed states

Quadratic part of S_π leads to $\widehat{\mathcal{H}}^{(2)}_{\mathcal{F}} = \int_{\mathbb{R}^{3+}} \mathrm{d}^3\vec{k}\, \widehat{\mathcal{H}}_{1,\vec{k}}$ where [Albrecht *et al.*, 1993]

$$\widehat{\mathcal{H}}_{i,\vec{k}} = \overbrace{F_{i,k}\left(\widehat{a}^\dagger_{i,\vec{k}}\widehat{a}_{i,\vec{k}} + \widehat{a}^\dagger_{i,-\vec{k}}\widehat{a}_{i,-\vec{k}} + 1\right)}^{\text{harmonic part}} + \overbrace{R_{i,k}\left(e^{i\Theta_{i,k}}\widehat{a}^\dagger_{i,\vec{k}}\widehat{a}^\dagger_{i,-\vec{k}} + \text{h.c.}\right)}^{\text{parametric amplification}}$$

Starting from a vacuum state, experiences huge squeezing

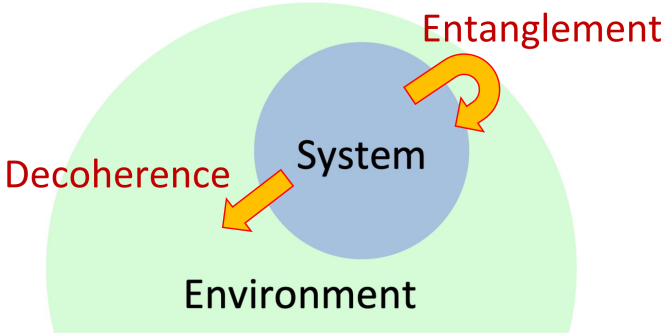


with

$$\overbrace{\widehat{\mathcal{R}}_i(\theta^k) \equiv e^{i\theta^k\left(\widehat{a}^\dagger_{i,\vec{k}}\widehat{a}_{i,\vec{k}} + \widehat{a}^\dagger_{i,-\vec{k}}\widehat{a}_{i,-\vec{k}} + 1\right)}}^{\text{phase shift}}, \qquad \overbrace{\widehat{\mathcal{Z}}_i(r^k) \equiv e^{r^k\left(\widehat{a}^\dagger_{i,\vec{k}}\widehat{a}^\dagger_{i,-\vec{k}} - \widehat{a}_{i,\vec{k}}\widehat{a}_{i,-\vec{k}}\right)}}^{\text{pair creation}}$$

Along the $\vec{k}$ / $-\vec{k}$ bipartition

Entanglement builds up at the same time decoherence proceeds.



$\Rightarrow$ Cannot rely on the flat space intuitions of decoherence.

[Martin, Micheli & Vennin, 2019]

[Martin & Vennin, 2019]

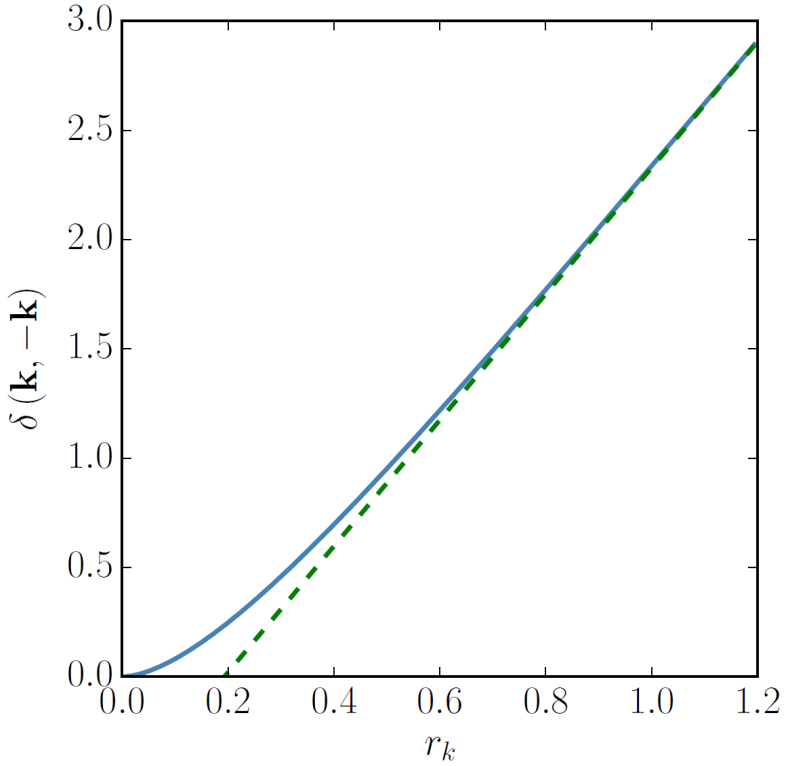


FIG. 1: Quantum discord $\delta(\boldsymbol{k}, -\boldsymbol{k})$ of cosmological scalar perturbations during inflation, as a function of the squeezing parameter r_k. The solid blue line is the result (44) while the dotted green line is the large squeezing expansion (45).

Generalisation

Most generic quadratic two-field quantum system:

$$\widehat{\mathcal{H}} = \int_{\mathbb{R}^{3+}} \mathrm{d}^3\vec{k} \left(\widehat{\mathcal{H}}_{1,\vec{k}} + \widehat{\mathcal{H}}_{2,\vec{k}} + \widehat{\mathcal{H}}_{1\leftrightarrow 2,\vec{k}} \right)$$

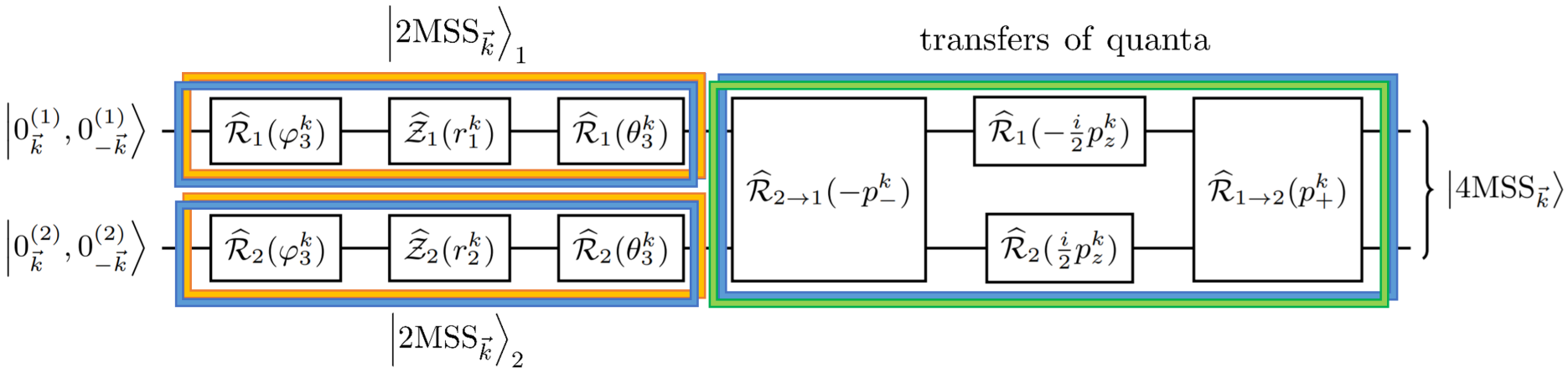
1. Free dynamics:

$$\widehat{\mathcal{H}}_{i,\vec{k}} = \overbrace{F_{i,k}\left(\widehat{a}^\dagger_{i,\vec{k}}\widehat{a}_{i,\vec{k}} + \widehat{a}^\dagger_{i,-\vec{k}}\widehat{a}_{i,-\vec{k}} + 1\right)}^{\text{harmonic part}} + \overbrace{R_{i,k}\left(e^{i\Theta_{i,k}}\widehat{a}^\dagger_{i,\vec{k}}\widehat{a}^\dagger_{i,-\vec{k}} + \text{h.c.}\right)}^{\text{parametric amplification}}$$

2. Interaction Hamiltonian:

$$\widehat{\mathcal{H}}_{1\leftrightarrow 2,\vec{k}} = \overbrace{F_{1\leftrightarrow 2,k}e^{i\varphi_k}\left(\widehat{a}^\dagger_{1,\vec{k}}\widehat{a}_{2,\vec{k}} + \widehat{a}^\dagger_{1,-\vec{k}}\widehat{a}_{2,-\vec{k}}\right)}^{\text{transfers of quanta}}$$
$$\underbrace{+R_{1\leftrightarrow 2,k}e^{i\xi_k}\left(\widehat{a}^\dagger_{1,\vec{k}}\widehat{a}^\dagger_{2,-\vec{k}} + \widehat{a}^\dagger_{2,\vec{k}}\widehat{a}^\dagger_{1,-\vec{k}}\right)}_{\text{entangled pair creation}} + \text{h.c.}$$

Four-mode squeezed states



where we have to introduce a new operation for the transfers of quanta between the two fields

$$\underbrace{\widehat{\mathcal{R}}_{i\to j}(p^k) \equiv e^{ip^k\left(\widehat{a}^\dagger_{j,\vec{k}}\widehat{a}_{i,\vec{k}}+\widehat{a}^\dagger_{j,-\vec{k}}\widehat{a}_{i,-\vec{k}}\right)}}_{\text{transfers of quanta}}$$

Quantum state in Fock basis

- In Fock basis

$$\left|4\mathrm{MSS}_{\vec{k}}\right\rangle = \sum_{n,m=0}^{\infty} \sum_{s,t=-n}^{m} c_k(m,n,s,t) \left|(n+s)^{(1)}_{\vec{k}}, (n+t)^{(1)}_{-\vec{k}}, (m-s)^{(2)}_{\vec{k}}, (m-t)^{(2)}_{-\vec{k}}\right\rangle$$

with

$$c_k(n,m,s,t) = \overbrace{\frac{e^{-2i\left[\theta_3^k(n+m+1)+\varphi_3^k\right]}}{\cosh r_1^k \cosh r_2^k}(-1)^{n+m} e^{p_z^k(m-n)} \tanh^n(r_1^k)\tanh^m(r_2^k)(ip_+^k)^{-s-t}\frac{m!}{n!}}^{\text{parametric amplification}}$$

$$\underbrace{\sqrt{\frac{(m-s)!(m-t)!}{(n+s)!(n+t)!}} \sum_{i=\max(0,s)}^{m} \frac{\left(p_-^k p_+^k e^{-p_z^k}\right)^i (n+i)!}{i!(i-s)!(m-i)!} \sum_{j=\max(0,t)}^{m} \frac{\left(p_-^k p_+^k e^{-p_z^k}\right)^j (n+j)!}{j!(j-t)!(m-j)!}}_{\text{transfers of quanta}}$$

- When we expand in the small coupling limit, we find that

$$\left|4\mathrm{MSS}_{\vec{k}}\right\rangle = \left|2\mathrm{MSS}_{\vec{k}}\right\rangle_1 \otimes \left|2\mathrm{MSS}_{\vec{k}}\right\rangle_2 + \tau_k \left|1 \text{ exchange}\right\rangle$$
$$+ \tau_k^2 \left|2 \text{ exchanges}\right\rangle + \tau_k^3 \left|3 \text{ exchanges}\right\rangle + \cdots$$

Outline

Example 1: an exact ME

Master equation: dynamical equation for the quantum state of the system.

1. Start with Liouville-von Neumann equation

$$\frac{\mathrm{d}\widetilde{\rho}}{\mathrm{d}\eta} = -ig\left[\widetilde{\mathcal{H}}_{\mathsf{int}}(\eta), \widetilde{\rho}(\eta)\right] \equiv g\mathcal{L}(\eta)\widetilde{\rho}(\eta)$$

2. Introduce projectors $\widetilde{\rho} \mapsto \mathcal{P}\widetilde{\rho} = \widetilde{\rho}_{\mathsf{red}} \otimes \rho_{\mathrm{E}}$ and $\mathcal{Q}\widetilde{\rho} = \widetilde{\rho} - \mathcal{P}\widetilde{\rho}$

3. Rewrite dynamics as

$$\frac{\mathrm{d}}{\mathrm{d}\eta}\mathcal{P}\widetilde{\rho}(\eta) = g^2 \int_{\eta_0}^{\eta} \mathrm{d}\eta' \mathcal{K}(\eta, \eta')\mathcal{P}\widetilde{\rho}(\eta)$$

$\mathcal{K}(\eta, \eta')$: **memory kernel** which depends on **the coupling** and **the environment**.

Example 2: an effective ME

1. Expand in powers of **the coupling** constant

$$\frac{\mathrm{d}}{\mathrm{d}\eta}\mathcal{P}\widetilde{\rho}(\eta) = \sum_{n=0}^{\infty} g^n \mathcal{K}_n(\eta)\mathcal{P}\widetilde{\rho}(\eta)$$

2. Lowest order leads to the non-Markovian ME

$$\frac{\mathrm{d}\widetilde{\rho}_{\mathsf{red}}}{\mathrm{d}\eta} = -g^2 \int_{\eta_0}^{\eta} \mathrm{d}\eta' \, \mathsf{Tr}_E \left[\widetilde{\mathcal{H}}_{\mathsf{int}}(\eta), \left[\widetilde{\mathcal{H}}_{\mathsf{int}}(\eta'), \widetilde{\rho}_{\mathsf{red}}(\eta) \otimes \rho_{\mathrm{E}}\right]\right]$$

3. Error function $e_r^{(2)} \sim g^2 ||\mathcal{K}_4(\eta)||/||\mathcal{K}_2(\eta)||$.

Example 3: a Markovian ME

1. When the environment is a **bath** (large number of dof, thermal equilibrium), the dynamics is **Markovian**, the system admits a **semi-group evolution**

$$V(\eta_1)V(\eta_2) = V(\eta_1 + \eta_2)$$

.

2. It implies a specific form for the ME [Lindblad 1976]

$$\frac{\mathrm{d}\widehat{\rho}_{\mathrm{red}}}{\mathrm{d}\eta} = -i\left[\widehat{H}(\eta), \widehat{\rho}_{\mathrm{red}}(\eta)\right] + \sum_k \gamma_k \left[\widehat{\boldsymbol{L}}_k \widehat{\rho}_{\mathrm{red}}(\eta) \widehat{\boldsymbol{L}}_k^\dagger - \frac{1}{2}\left\{\widehat{\boldsymbol{L}}_k^\dagger \widehat{\boldsymbol{L}}_k, \widehat{\rho}_{\mathrm{red}}(\eta)\right\}\right]$$

3. It relies on a fast decay of temporal correlations in the environment.

Question: At which level should we work in cosmology ?

Cosmological environments

In cosmology, we have **background symmetries**: *homogeneity* and *isotropy*

- At *linear order*, there is **no mode coupling**:

$$\left\langle \widehat{\mathcal{O}}_{\boldsymbol{k}} \widehat{\mathcal{O}}_{\boldsymbol{q}} \right\rangle \propto \delta(\boldsymbol{k} + \boldsymbol{q})$$

  $\Rightarrow$ only way to get a large number of dof: infinitely many environmental fields.

- At *non-linear order*, **environmental self-interactions** may relax the constraint on the number of fields.

  $\Rightarrow$ induce primordial non-Gaussianities (PNG): observationally constrained [Assassi *et al.*, 2014].

Conclusion: not so common to have a Markovian ME in cosmology.

The emergence of Markovianity

- Fast decay of environmental correlations

$$\mathcal{K}^{>}(\eta,\eta') \xrightarrow[\text{graining}]{\text{coarse-}} \delta(\eta-\eta')$$

ME reduces to a GKSL equation for which the dynamical map reads

$$\mathcal{L}\left[\widehat{\rho}_{\text{red}}\right] = -i\left[\widehat{\mathcal{H}},\widehat{\rho}_{\text{red}}\right] + \gamma\left(\widehat{L}\widehat{\rho}_{\text{red}}\widehat{L}^{\dagger} - \frac{1}{2}\left\{\widehat{L}^{\dagger}\widehat{L},\widehat{\rho}_{\text{red}}\right\}\right)$$

GKSL equation is CPTP: physical consistency of the solutions ensured.

- Non-Markovian evolution/non-semigroup dynamical map implies dissipator matrix non-positive semi-definite.
- Non-positive semi-definite dissipator matrix is a generic feature of Non-Markovian OQS: not directly related to CPTP properties.
- Curved-space Caldeira-Leggett model ME belongs to the class of Gaussian non-Markovian ME $\Rightarrow$ CPTP ensured by [Diósi & Ferialdi, 2014].