Quantum signatures in cosmological perturbations?

J. Martin, **A. Micheli**, and V. Vennin, arXiv:2211.10114
J. Martin, **A. Micheli**, and V. Vennin, JCAP. **2022**, 051
A. Micheli and P. Peter, arXiv:2211.00182 in Handbook of Quantum Gravity

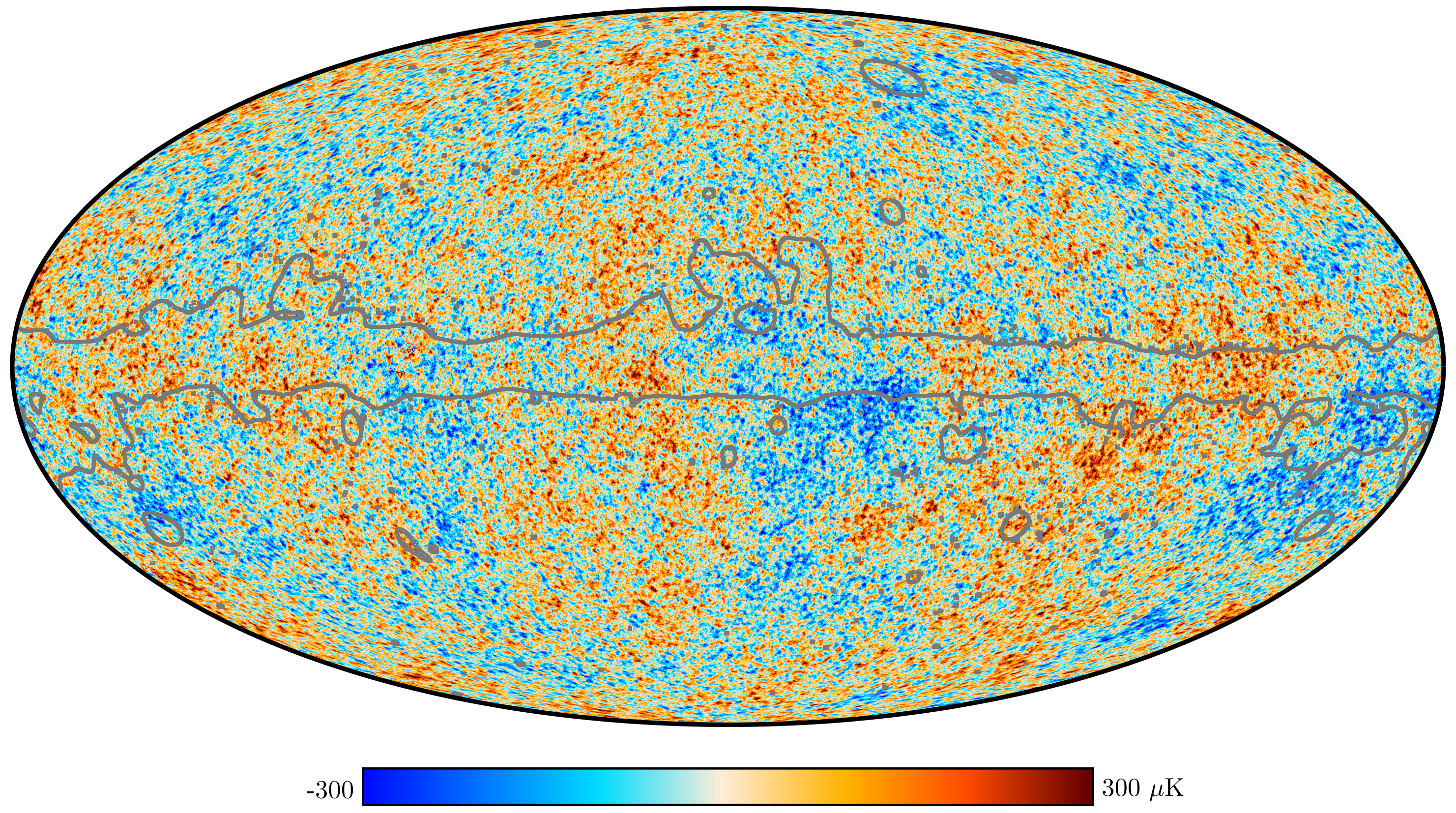
Amaury Micheli

IJCLab, Orsay
IAP, Paris

Kick-Off meeting GDR CoPhy - 19th January 2023

Inhomogeneities in the early Universe - I

- Primordial inhomogeneities come from (vacuum) quantum fluctuations at the beginning of inflation stretched to cosmological scales by expansion.



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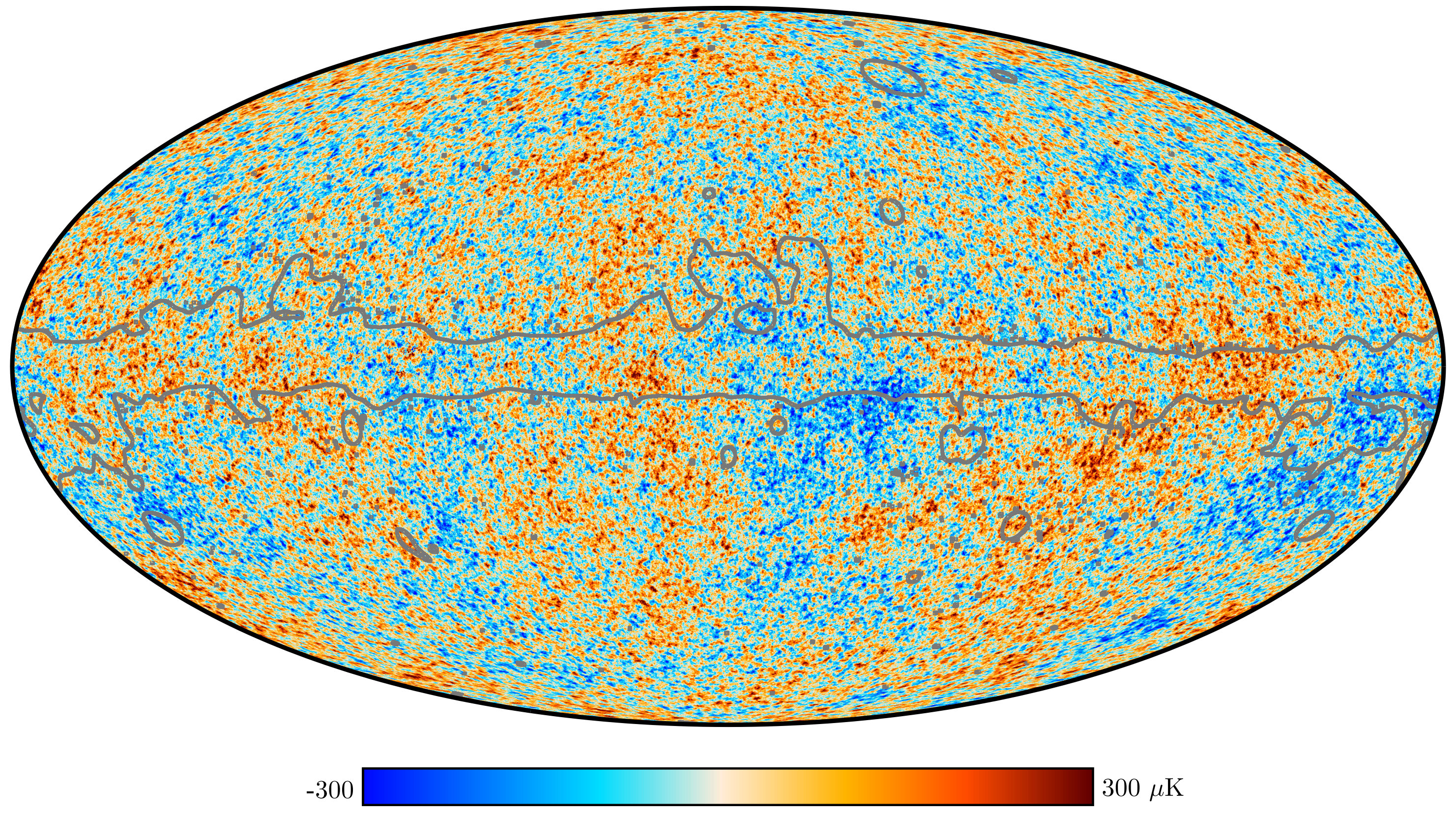

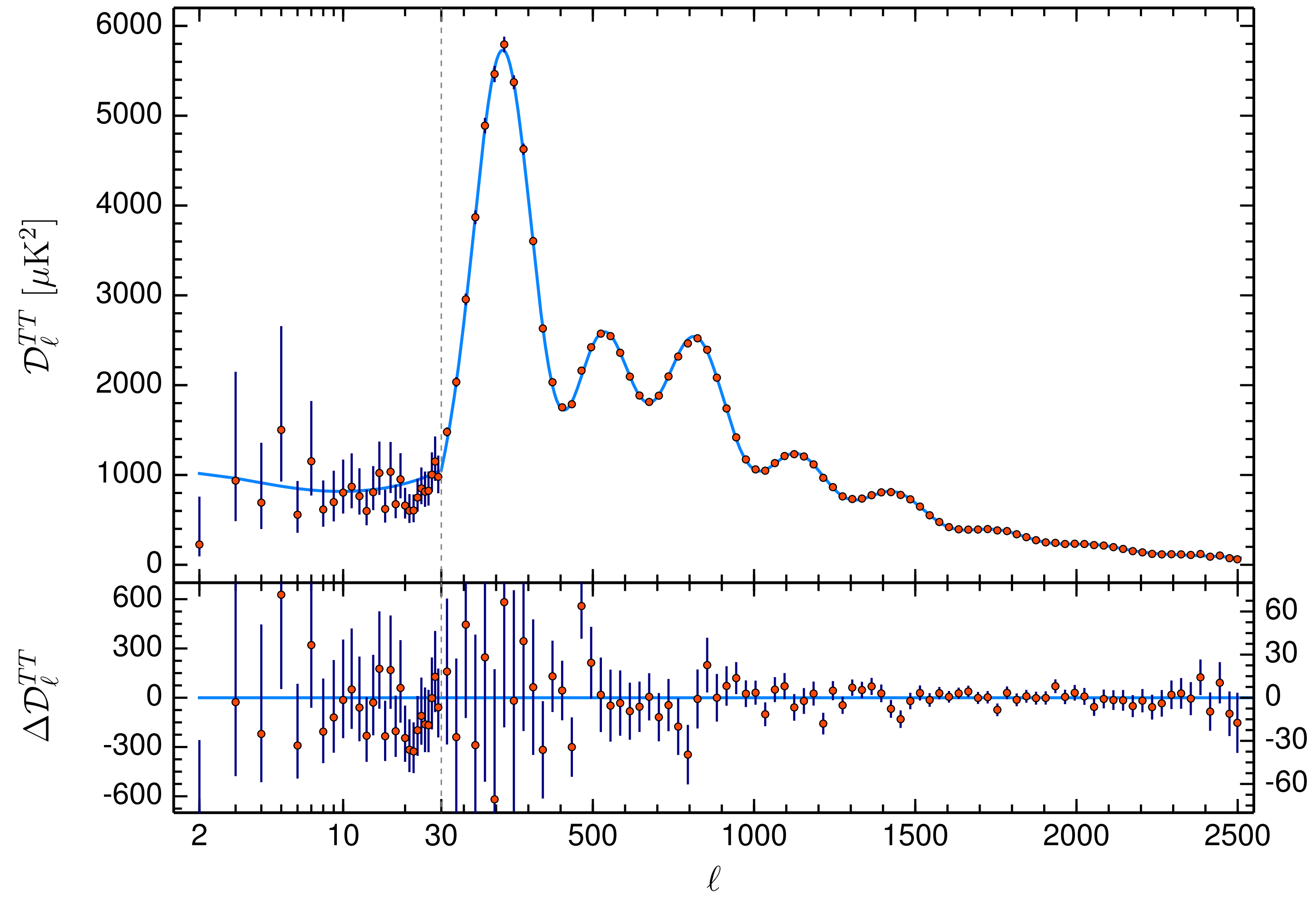

- Indirect proof : very good agreement with observational data[1].

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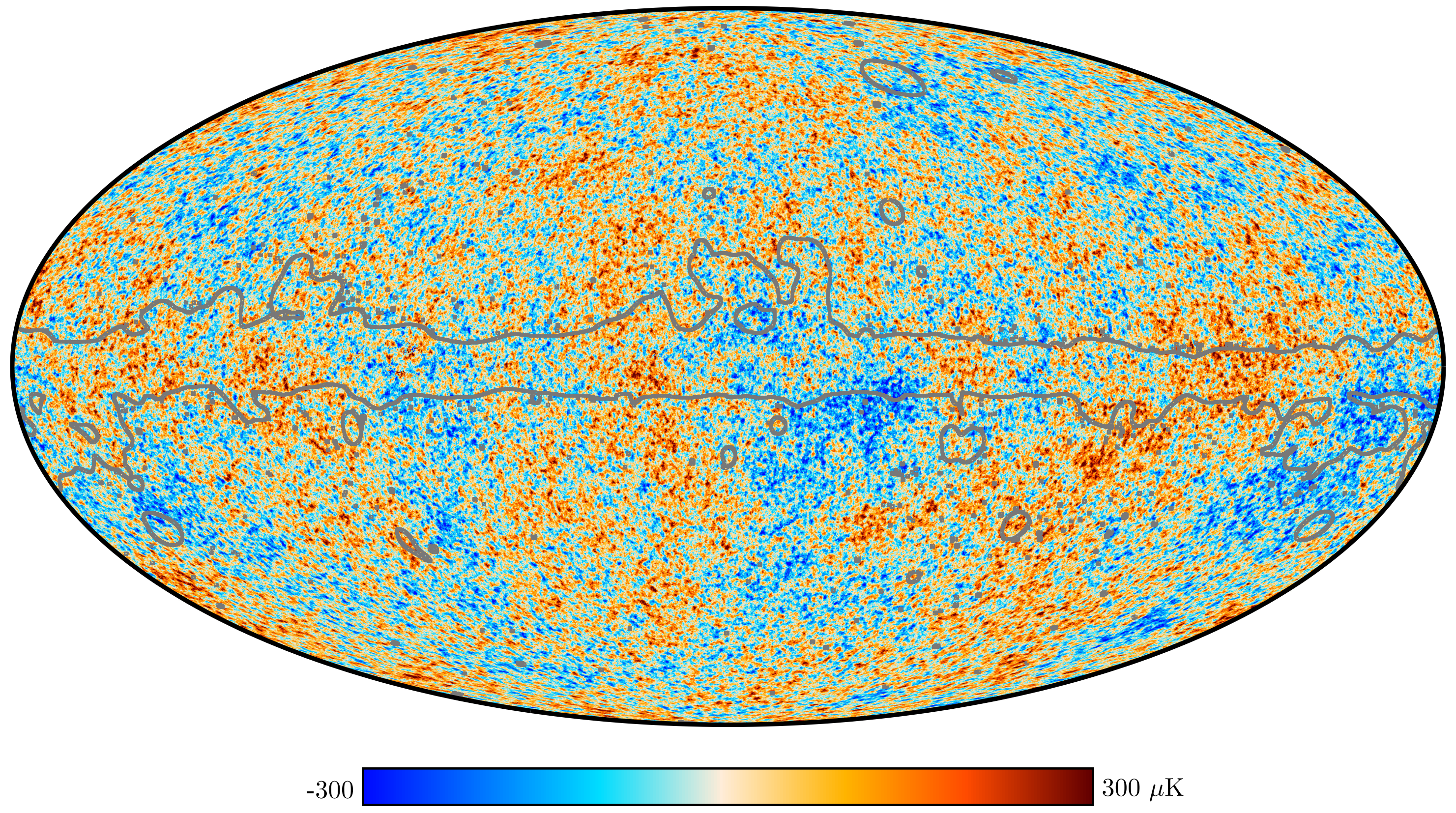

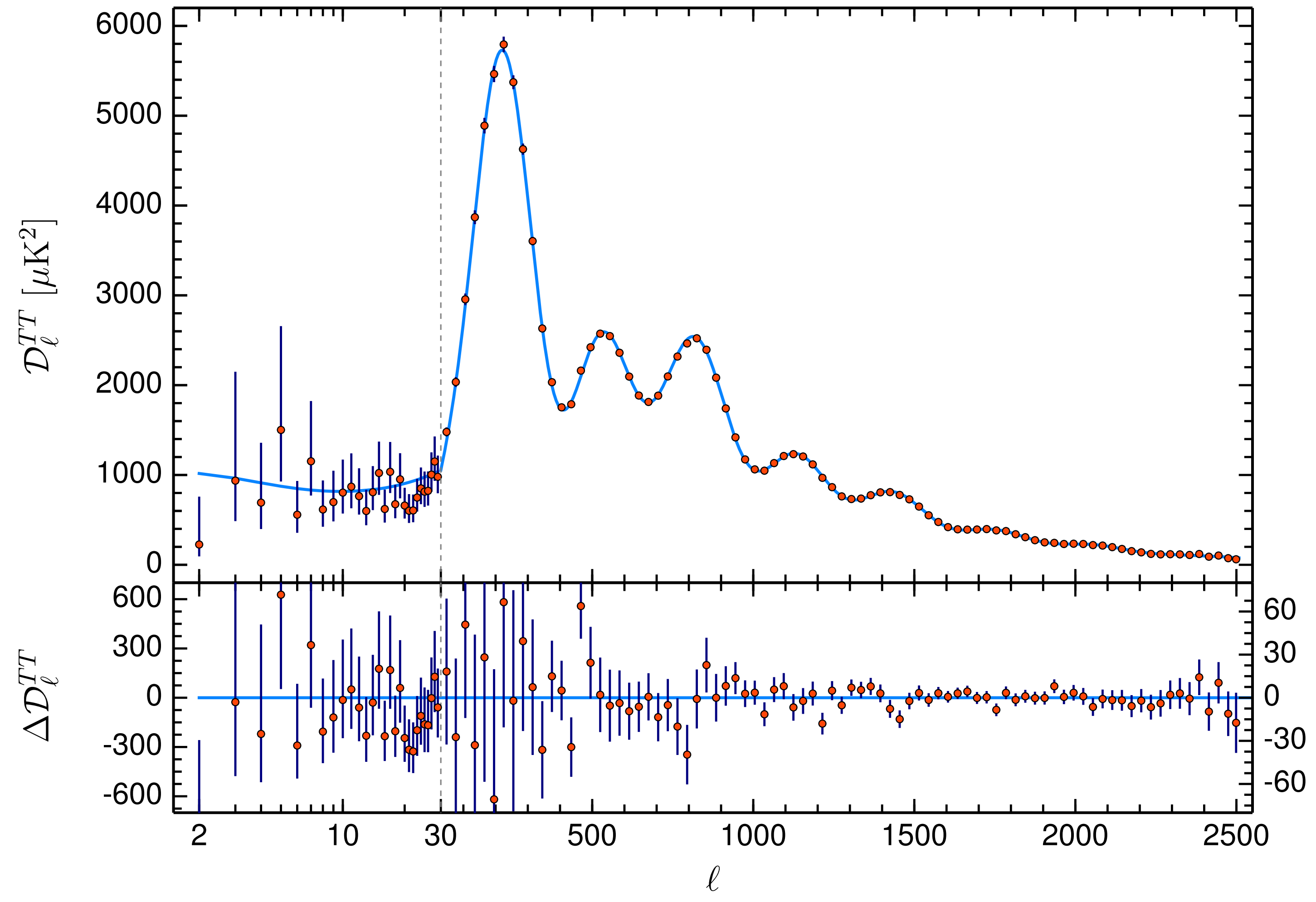

Questions

- Direct proof that initial fluctuations cannot be classical? Would show quantisation of gravity.
- If quantum then and classical now, how did the transition happen? Quantum-to-classical transition problem.

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Plan

I. Quantum description of the state of the perturbations

II. Quantum signatures

III. Decoherence: Destruction of quantum signatures

I - Quantum state of perturbations

Classical cosmological perturbations in inflation

- GR $g_{\mu\nu}$ with a single inflaton field ϕ: $S = \int \mathrm{d}^4x \sqrt{-g} \left[\frac{R}{16\pi \mathrm{G}} - \frac{1}{2} g^{\mu\nu} \partial_\mu \phi \partial_\nu \phi - V(\phi) \right]$

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- In slow-roll, amplification of perturbations for super-Hubble modes $k(aH)^{-1} \gg 1$

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- Quantisation: Conjugated field $\hat{\pi}_{\pm\mathbf{k}} = \hat{v}'_{\pm\mathbf{k}} - \frac{z'}{z}\hat{v}_{\pm\mathbf{k}}$ and $\left[\hat{v}_{\mathbf{k}}, \hat{\pi}_{\mathbf{k}'}\right] = \hbar\delta\left(\mathbf{k} + \mathbf{k}'\right)$

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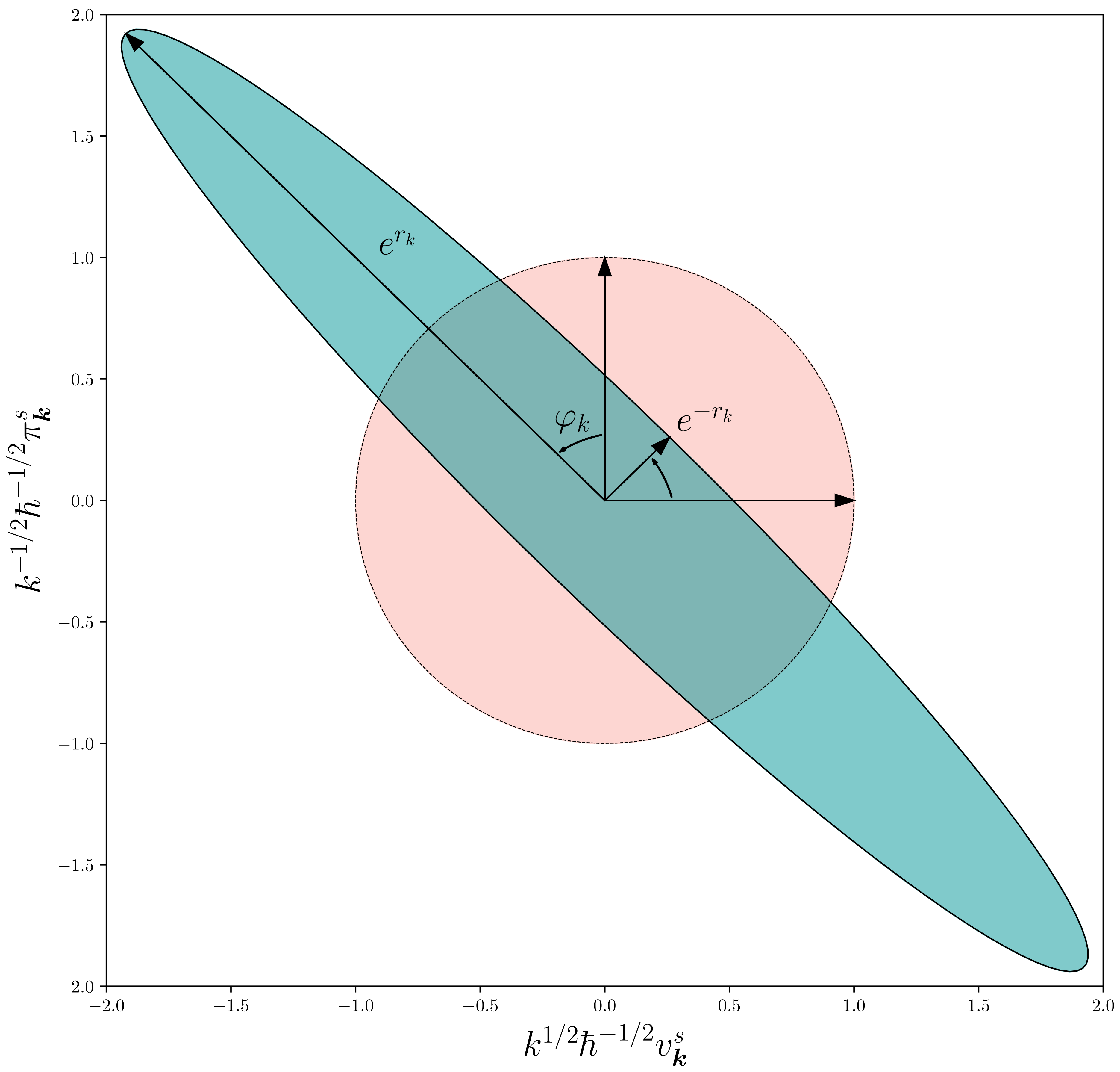
- Area S_k controlled by the purity p_k of the state $S_k = \pi\hbar/\sqrt{p_k}$, here $p_k = 1$

Quantum state of perturbations - II



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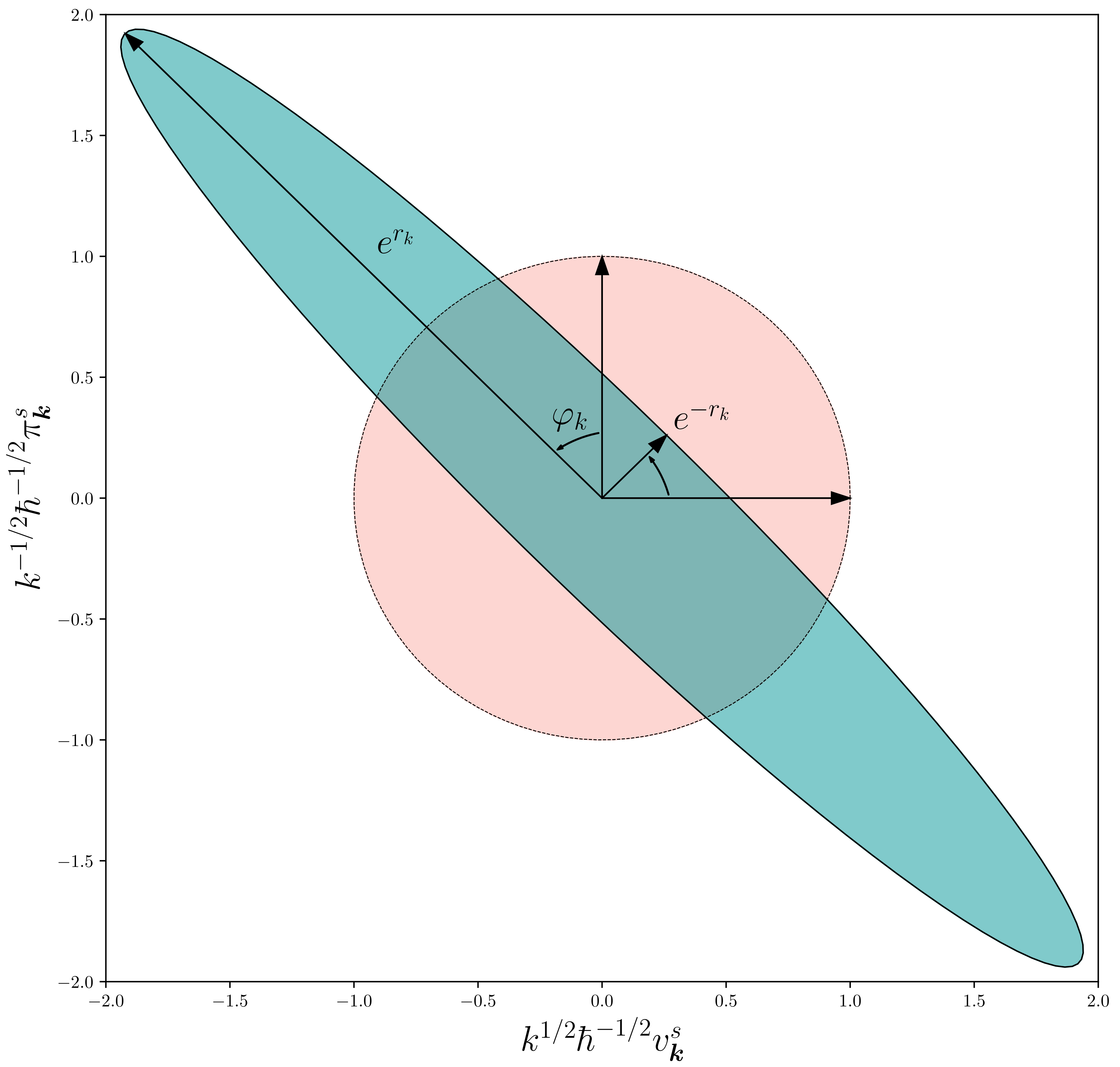

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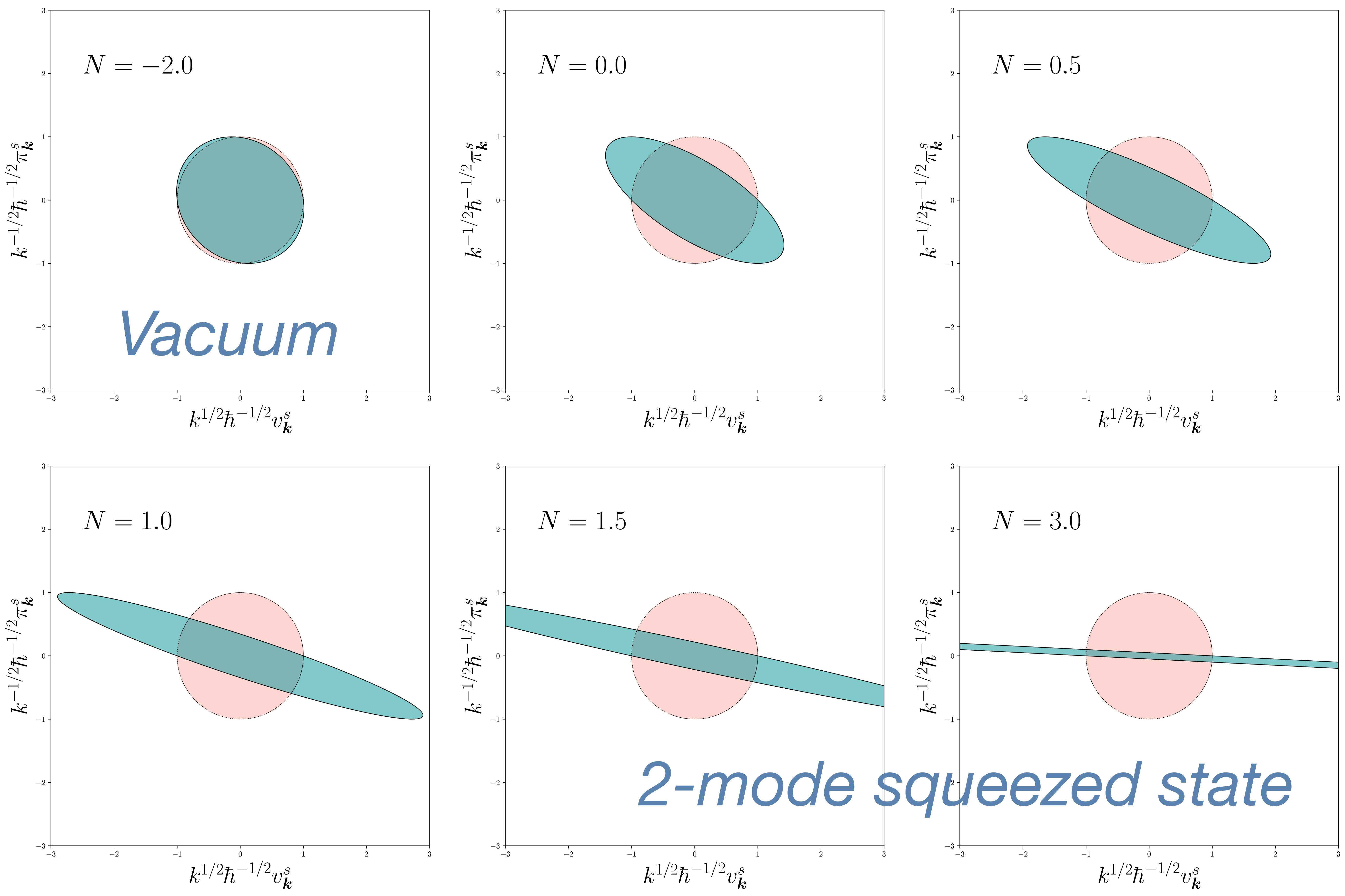

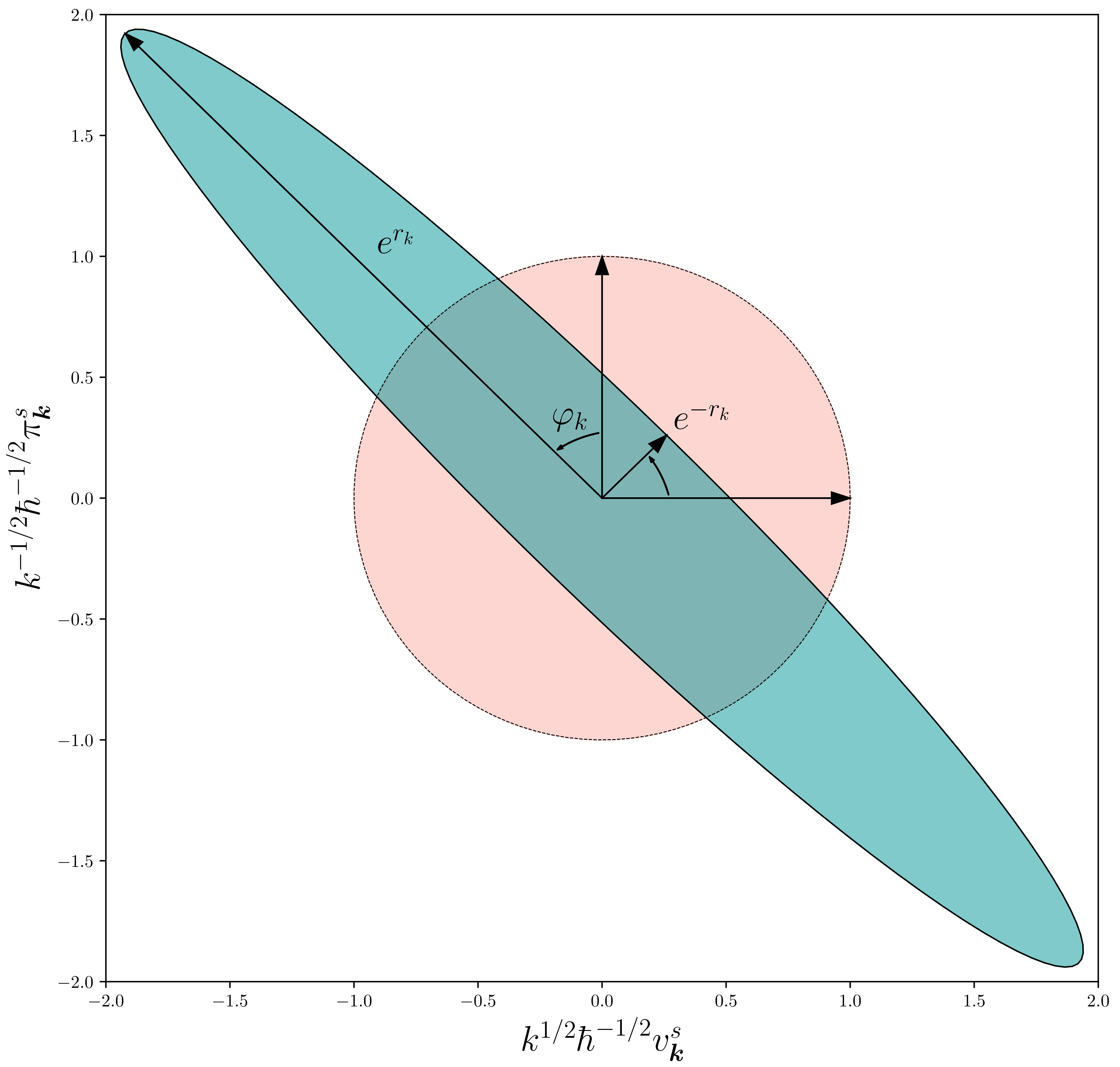

- Inflationary dynamics: very strong squeezing $r_k \approx N$ in the direction of $v_{\mathbf{k}}$: amplified fluctuation of $v_{\mathbf{k}}$

II - Quantum signatures of the state

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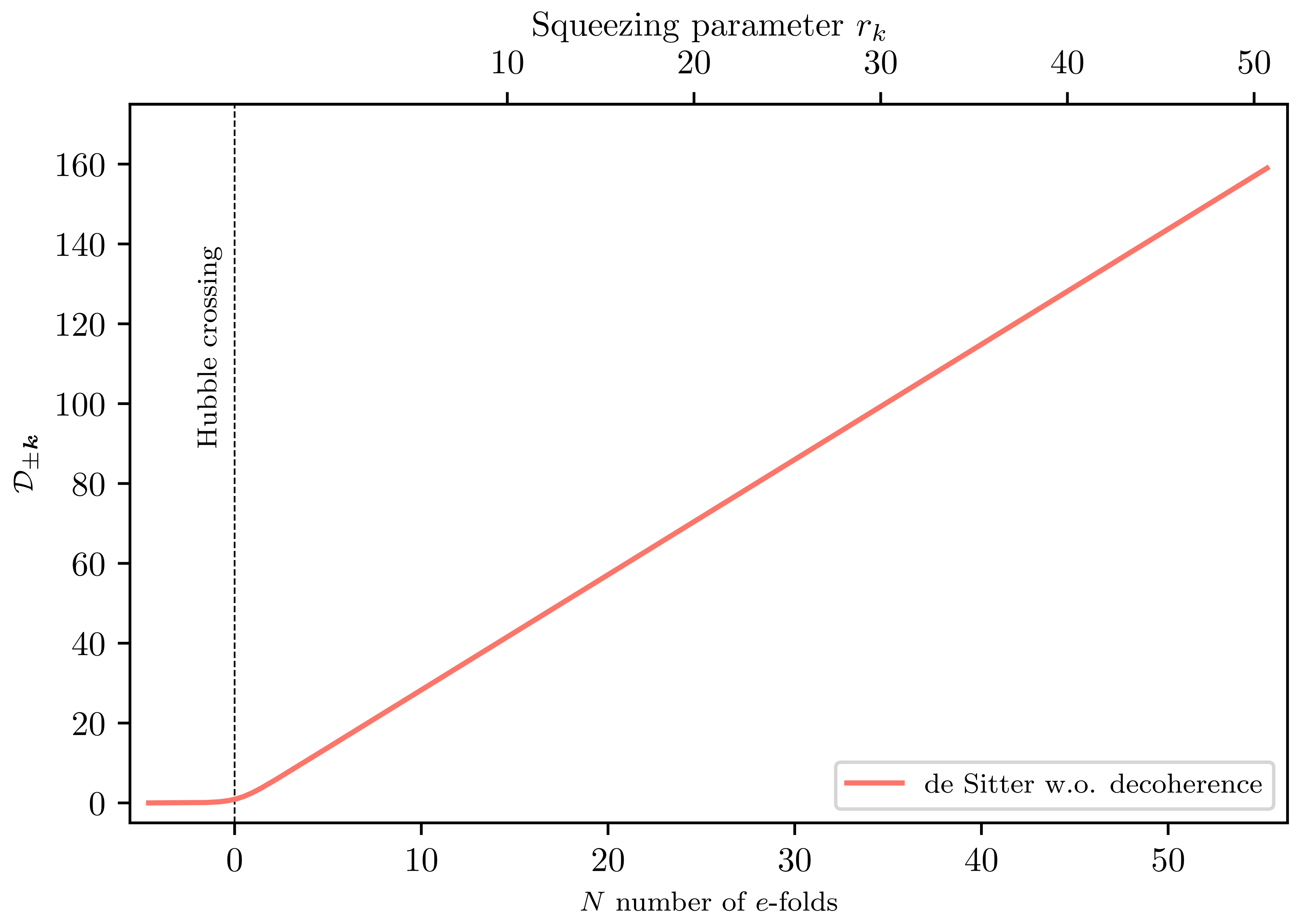

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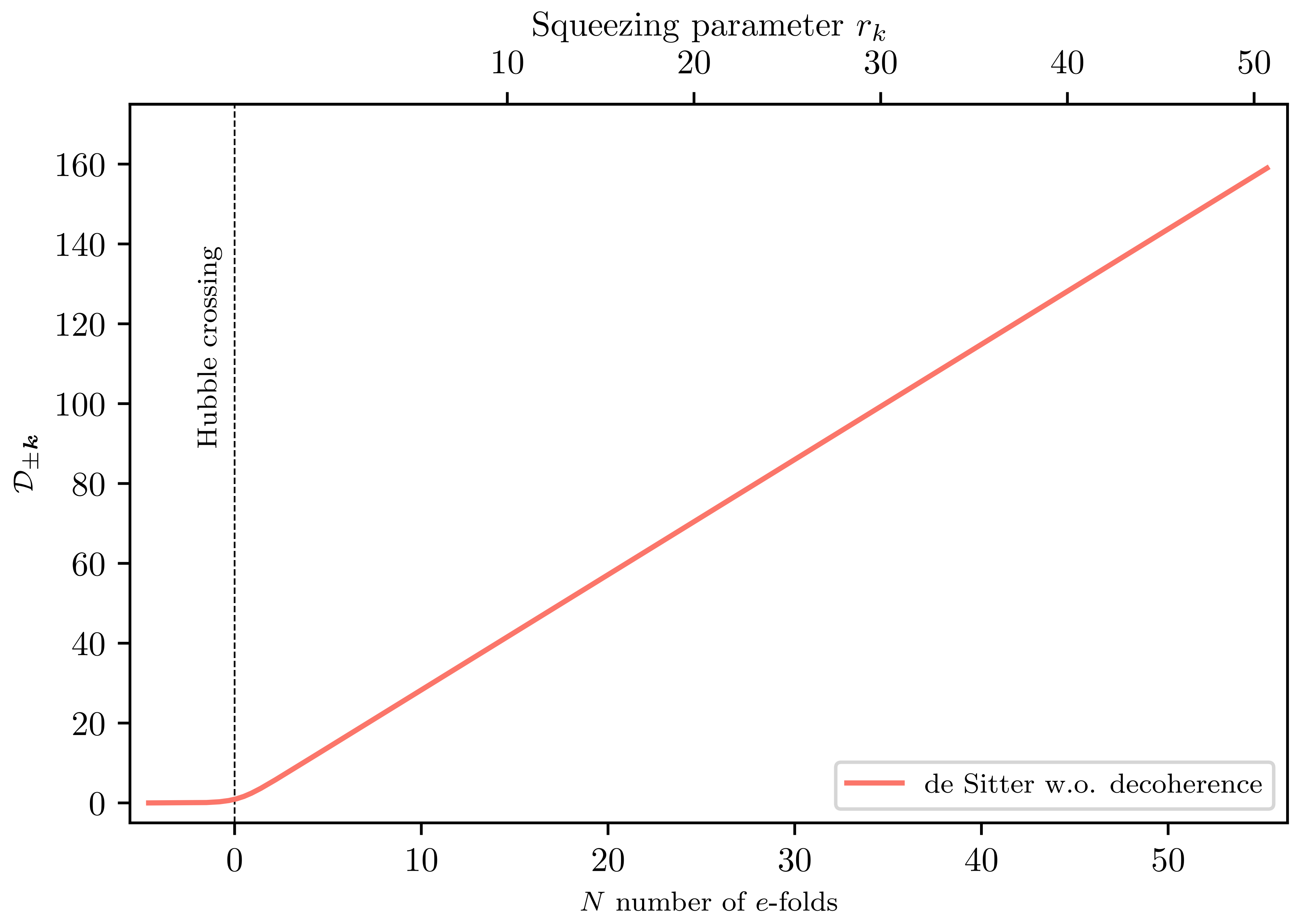

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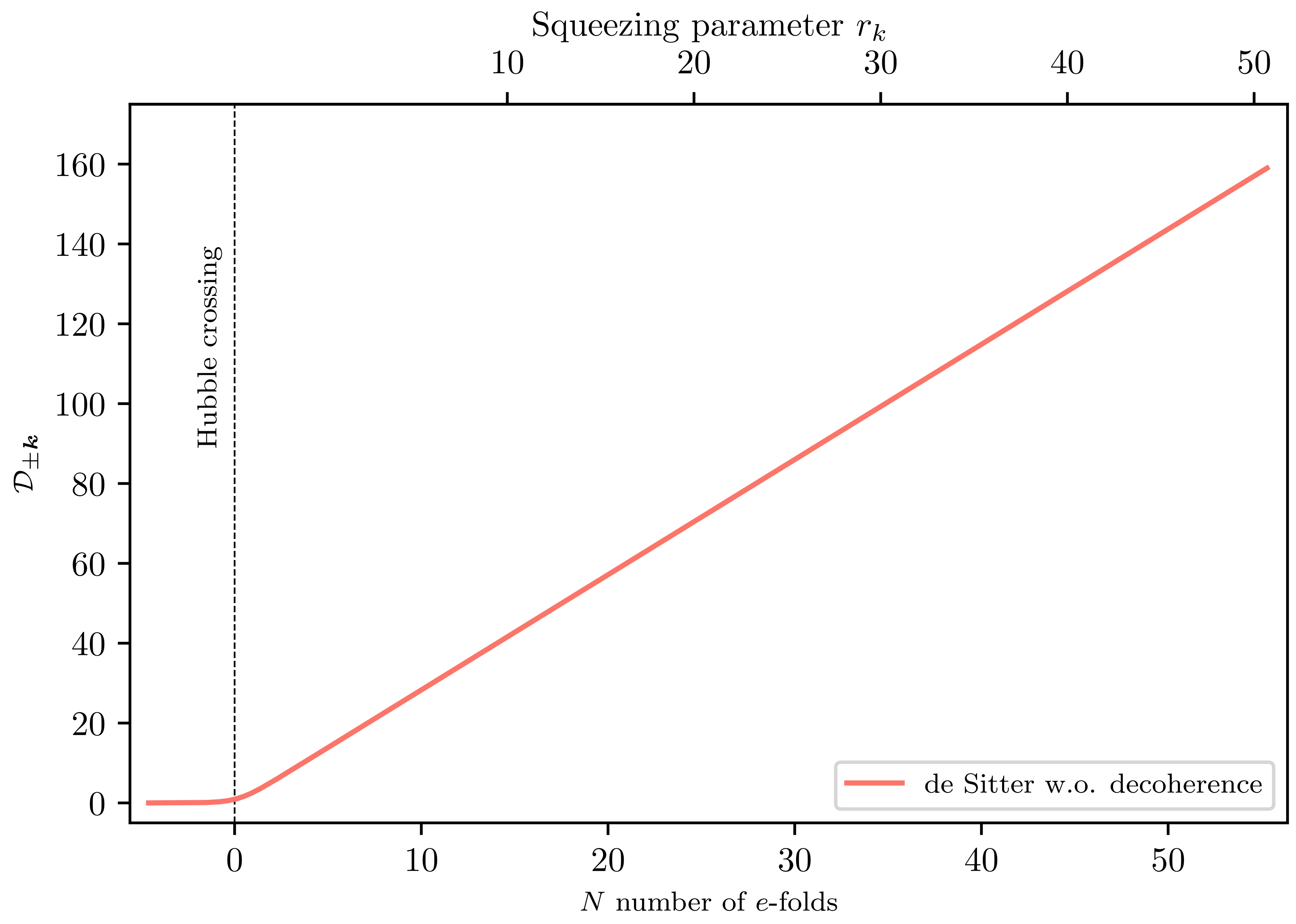

Take Home Message 1

Squeezing generates strong quantum correlations between $\pm k$ modes in the sense of several non-classicality criteria.

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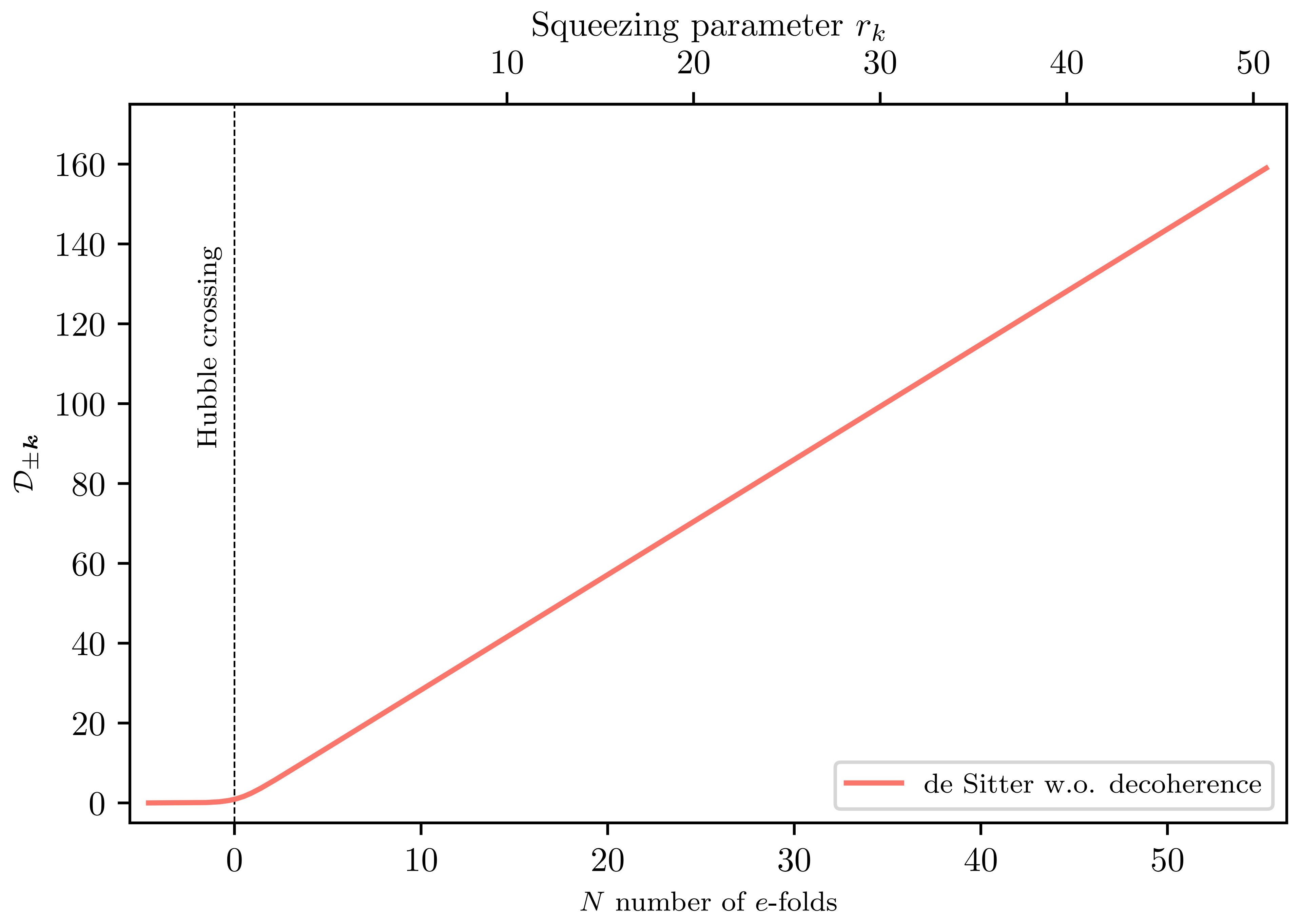
Is this due to oversimplified models?



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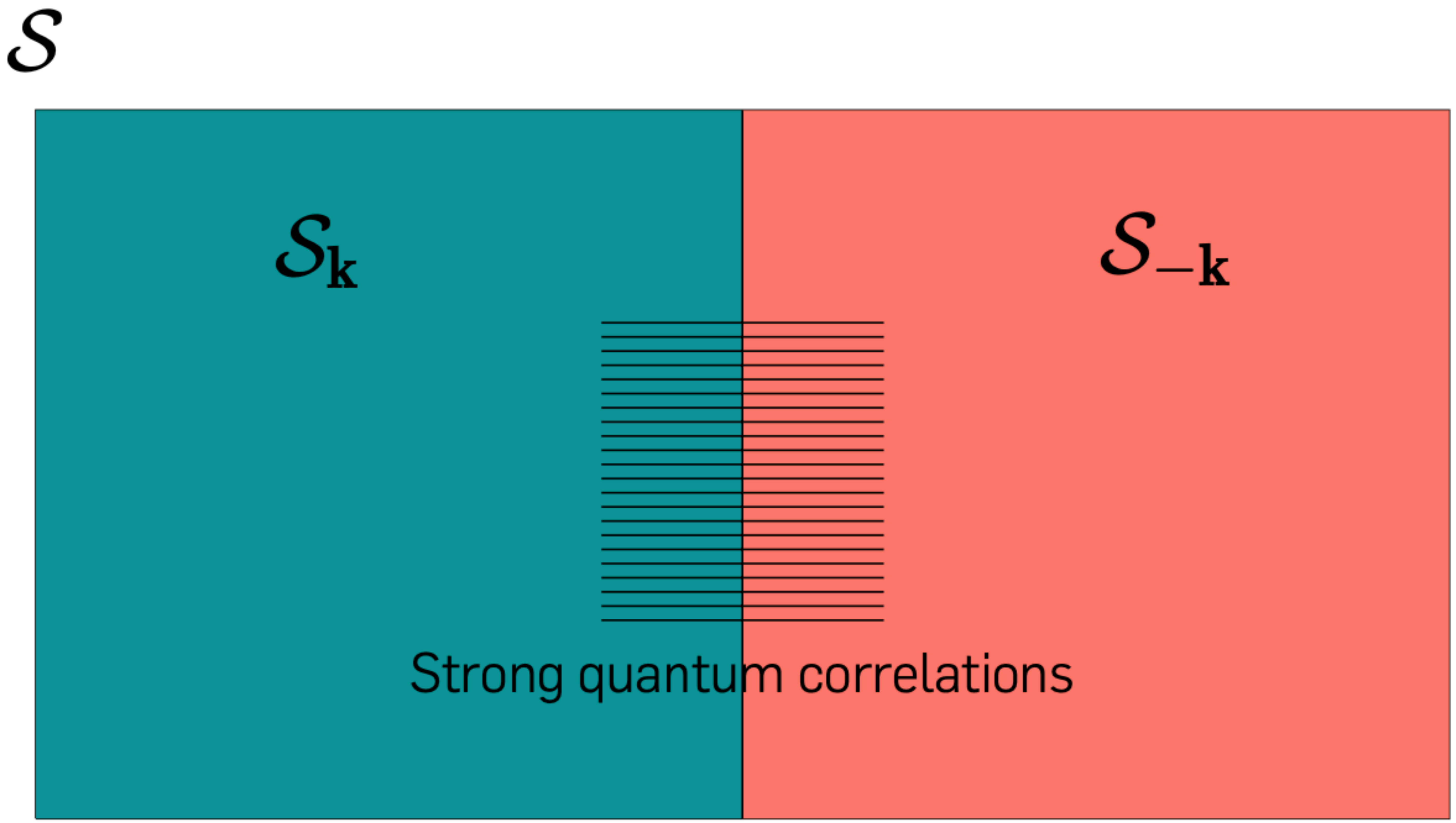
III - Decoherence: Destruction of quantum correlations

Decoherence : how to destroy quantum features

Current situation



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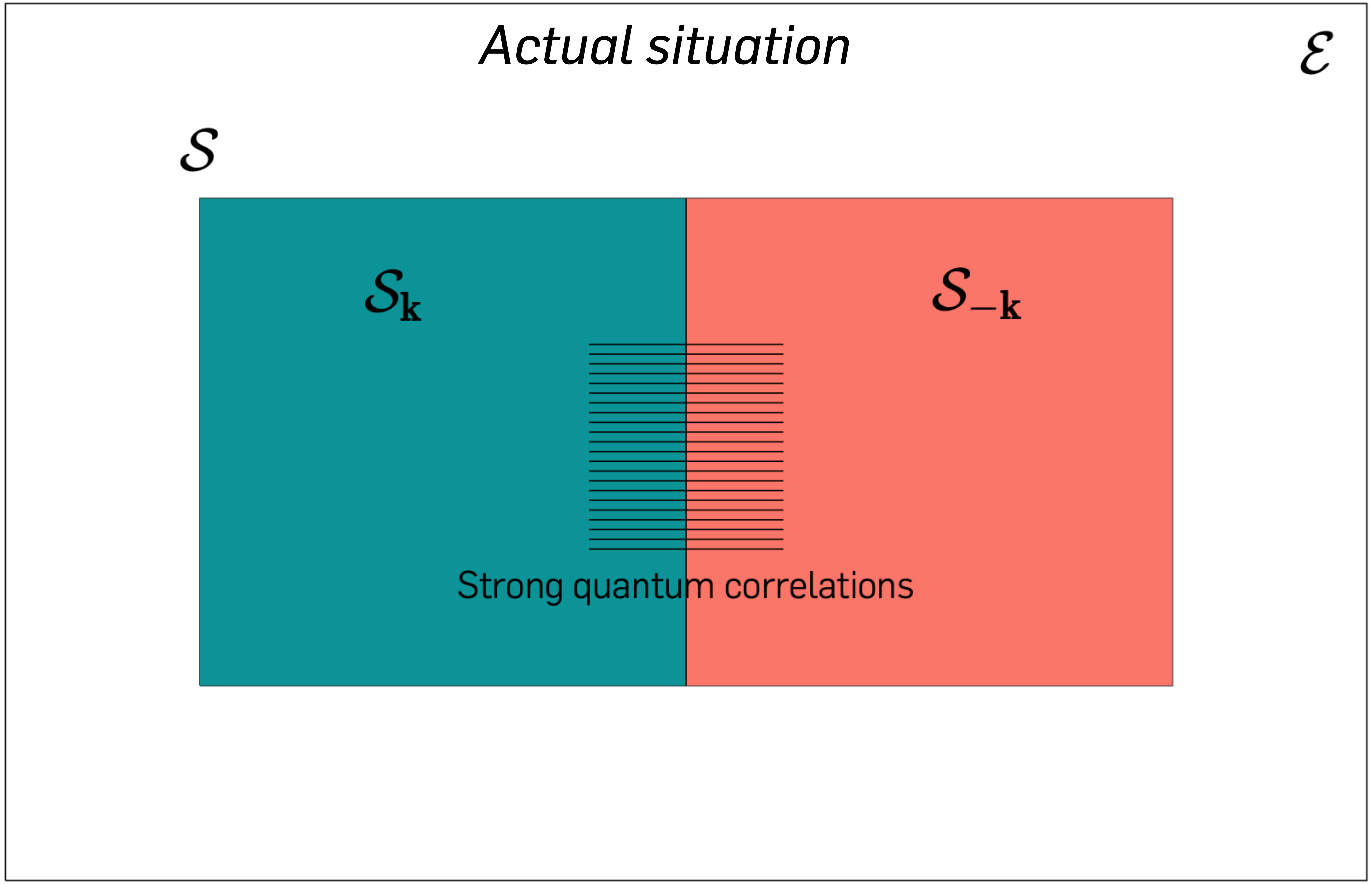

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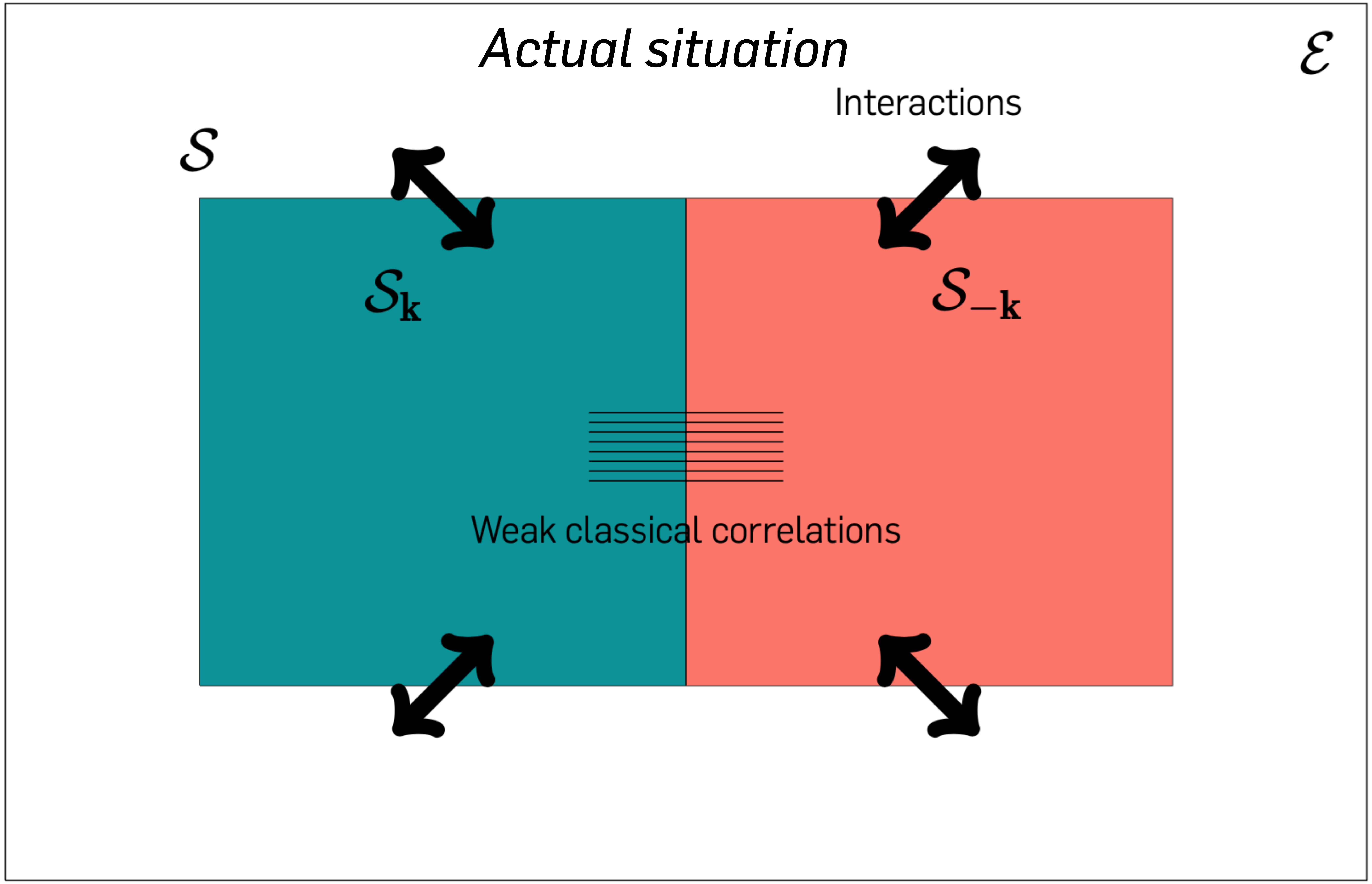

Interactions with extra d.o.f lead to decoherence of quantum systems.

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- Under a few generic assumptions (perturbative coupling, $\mathscr{E}$ large w.r.t $\mathscr{S}$ and stationnary[3]) can derive Lindblad equation (non-unitary) and show that state becomes mixed 2-mode squeezed state[4] parametrized by r_k, φ_k and the purity $0 \le p_k \le 1$.

3. [arXiv:2209.01929 Colas, Grain and Vennin]
4. [arXiv:2112.05037 Martin, Micheli and Vennin]

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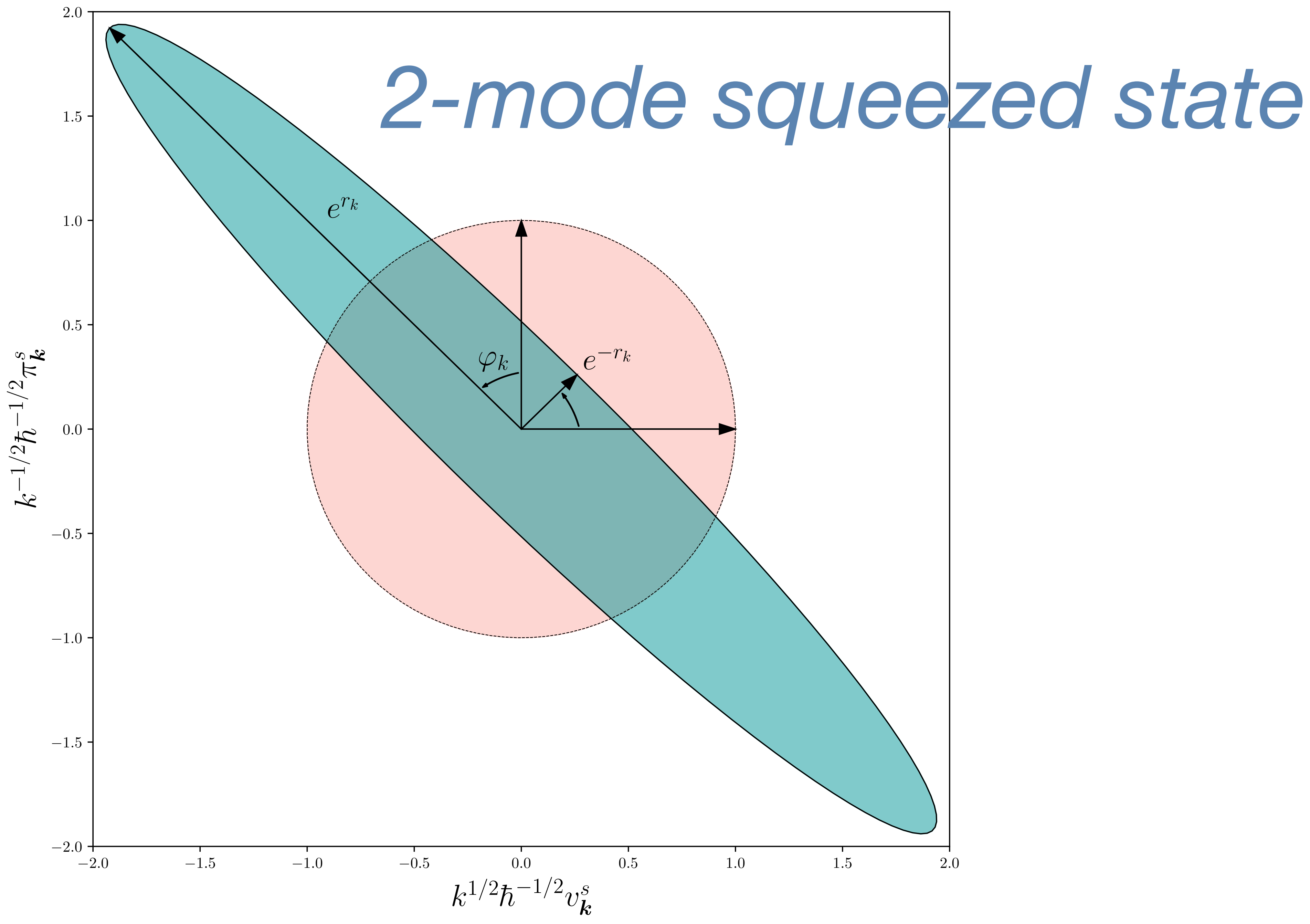

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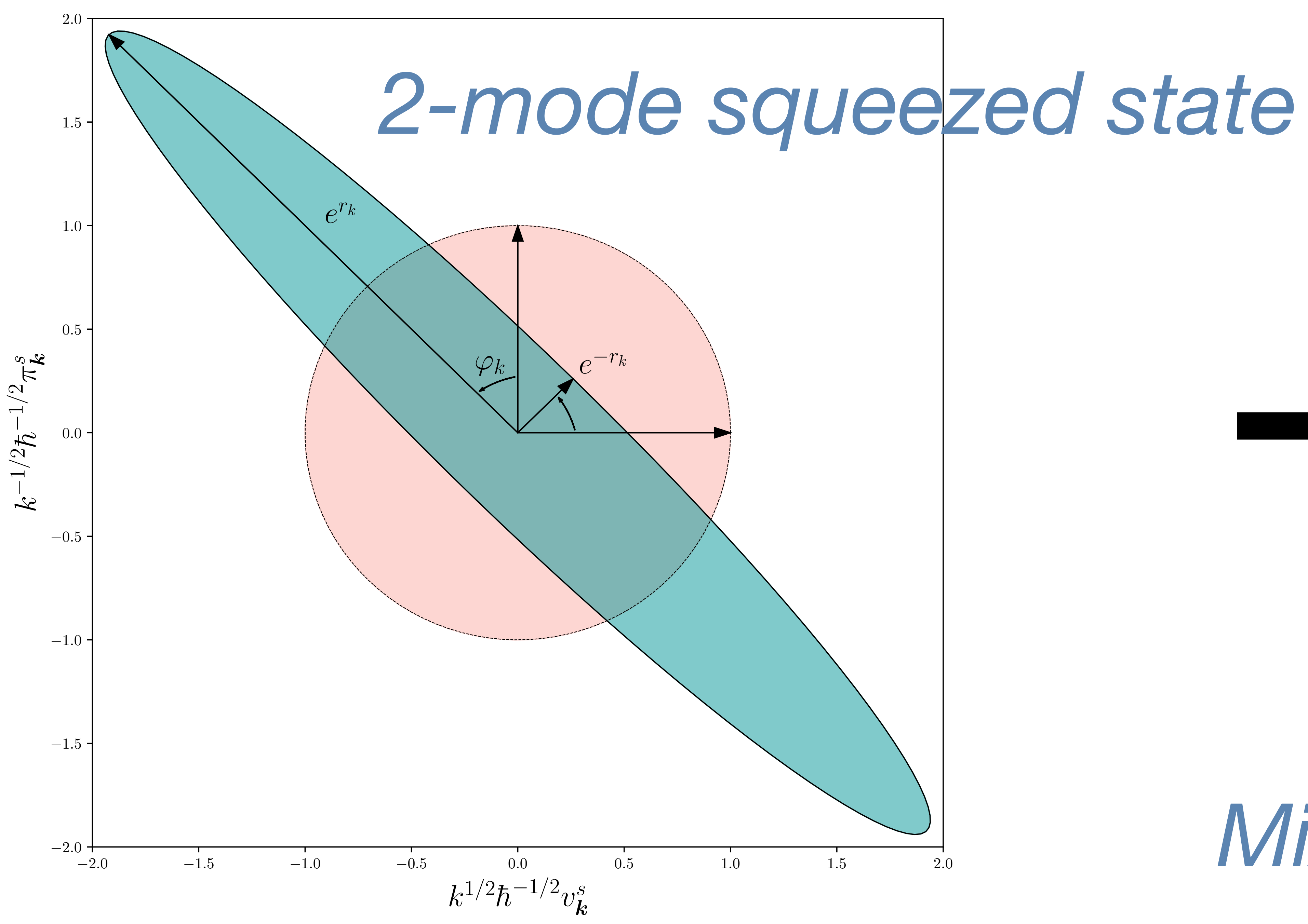
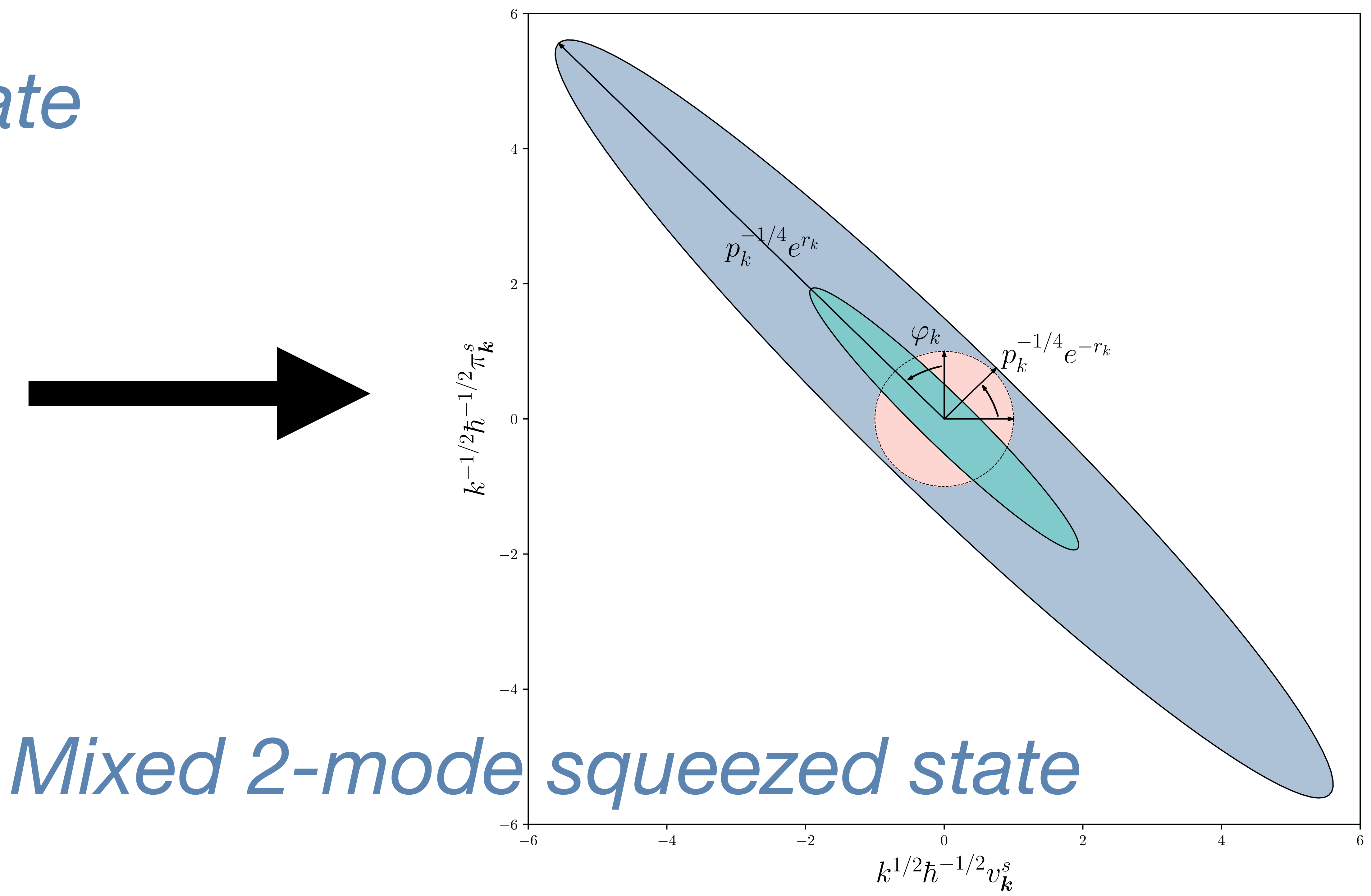
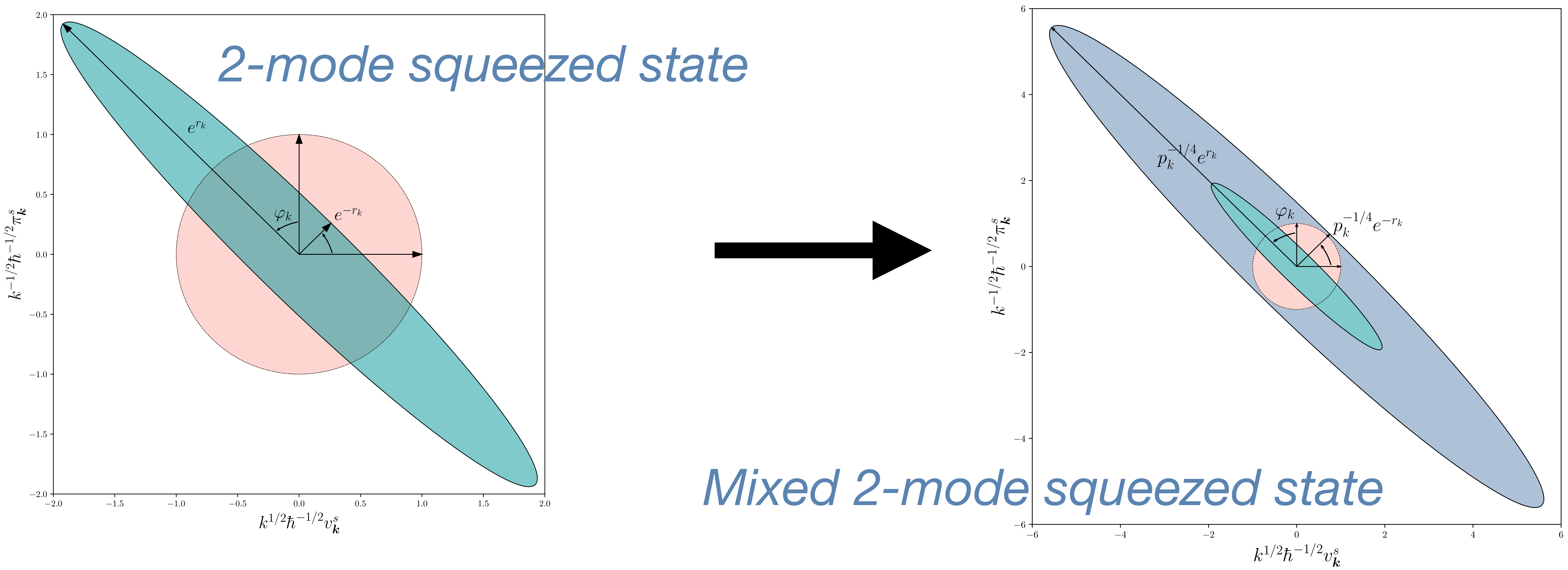




Environment destroys quantum correlations

- Geometrically: growth of the ellipse area $S_k = \pi\hbar/\sqrt{p_k}$



How does decoherence affect quantumness of correlations?

Environment destroys quantum correlations

- Discord in presence of decoherence[2,3]

$$\mathscr{D}_{\pm\mathbf{k}} = f\left[p_k^{-1/2}\cosh\left(2r_k\right)\right] - 2f\left(p_k^{-1/2}\right) + f\left[\frac{p_k^{-1/2}\cosh\left(2r_k\right) + p_k^{-1}}{p_k^{-1/2}\cosh\left(2r_k\right) + 1}\right]$$

2. [arXiv:2211.10114 Martin, Micheli, and Vennin]
3. [arXiv:2112.05037 Martin, Micheli and Vennin]

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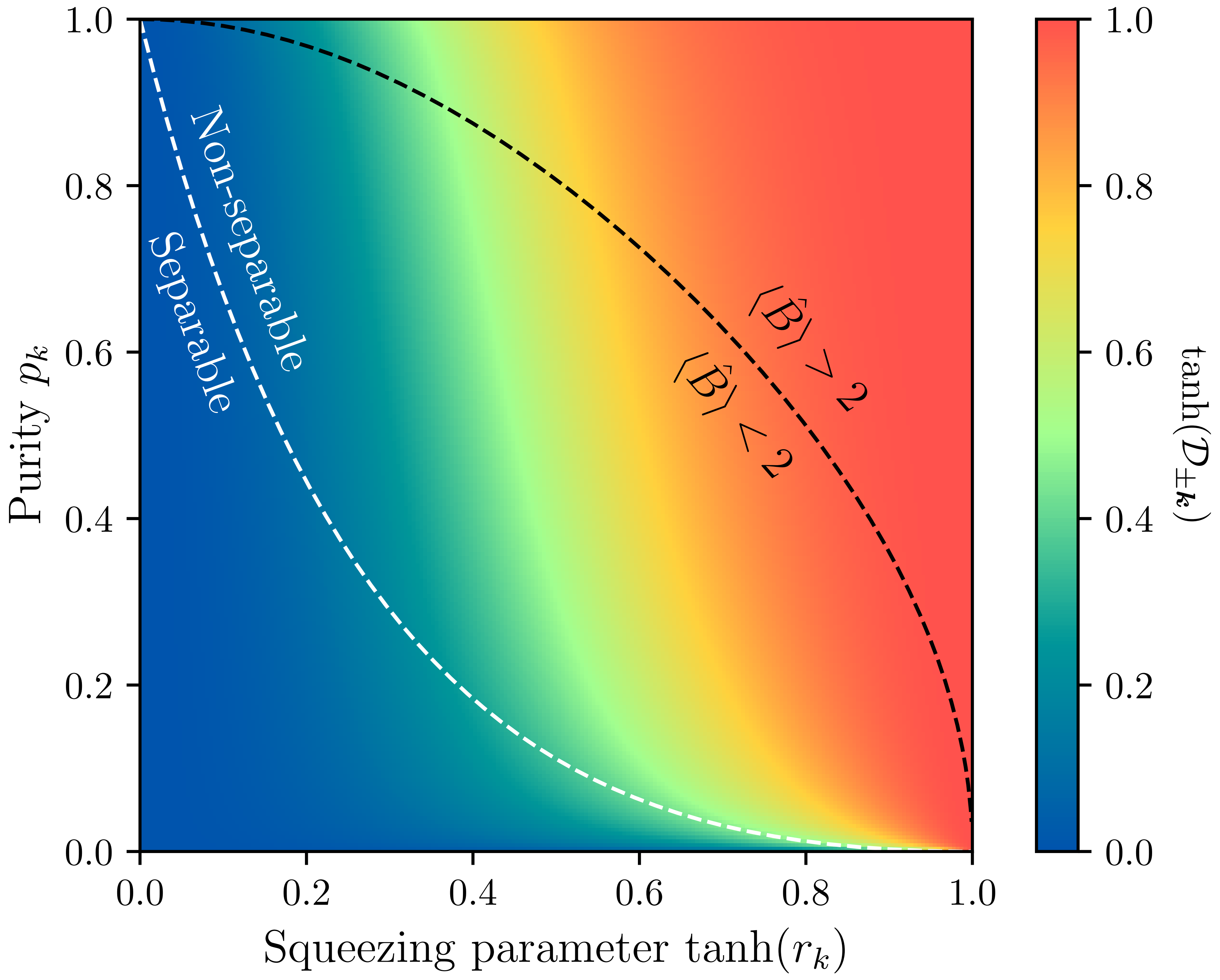

Take Home Message 2

Presence or absence of quantum correlations is the result of a competition between correlation build up and interaction erasing quantum features.[3]

2. [arXiv:2211.10114 Martin, Micheli, and Vennin]
3. [arXiv:2112.05037 Martin, Micheli and Vennin]

Future directions

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- Is there some quantum correlations left? Get/use realistic estimations of the level of decoherence to see where we are in the previous plot[4].

4. [arXiv:2211.11046 Burgess et al.]

Future directions

- Is there some quantum correlations left? Get/use realistic estimations of the level of decoherence to see where we are in the previous plot[4].

- Observability: So far no proposed protocol to measure these criteria. To evade that problem would require, either to have several times, or go beyond Gaussian level[5] or consider more complicated models[6].

4. [arXiv:2211.11046 Burgess et al.]
5. [arXiv:2001.09149 Green and Porto]
6. [arXiv:1508.01082 Maldacena]

Thank you for your attention!

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Based on:

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and:

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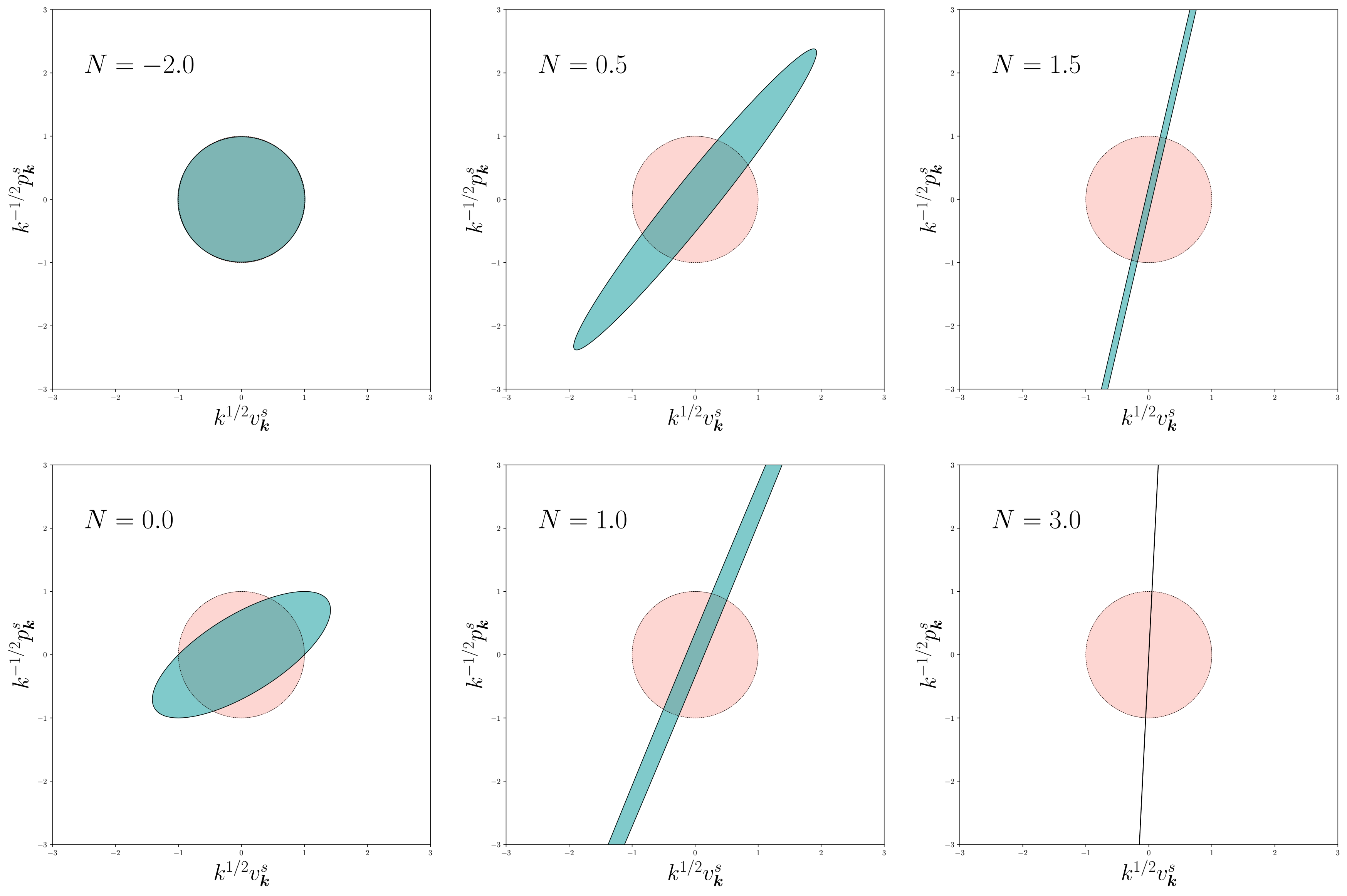
Extra

Limit Wigner to a Dirac delta

Apparent classicality - II



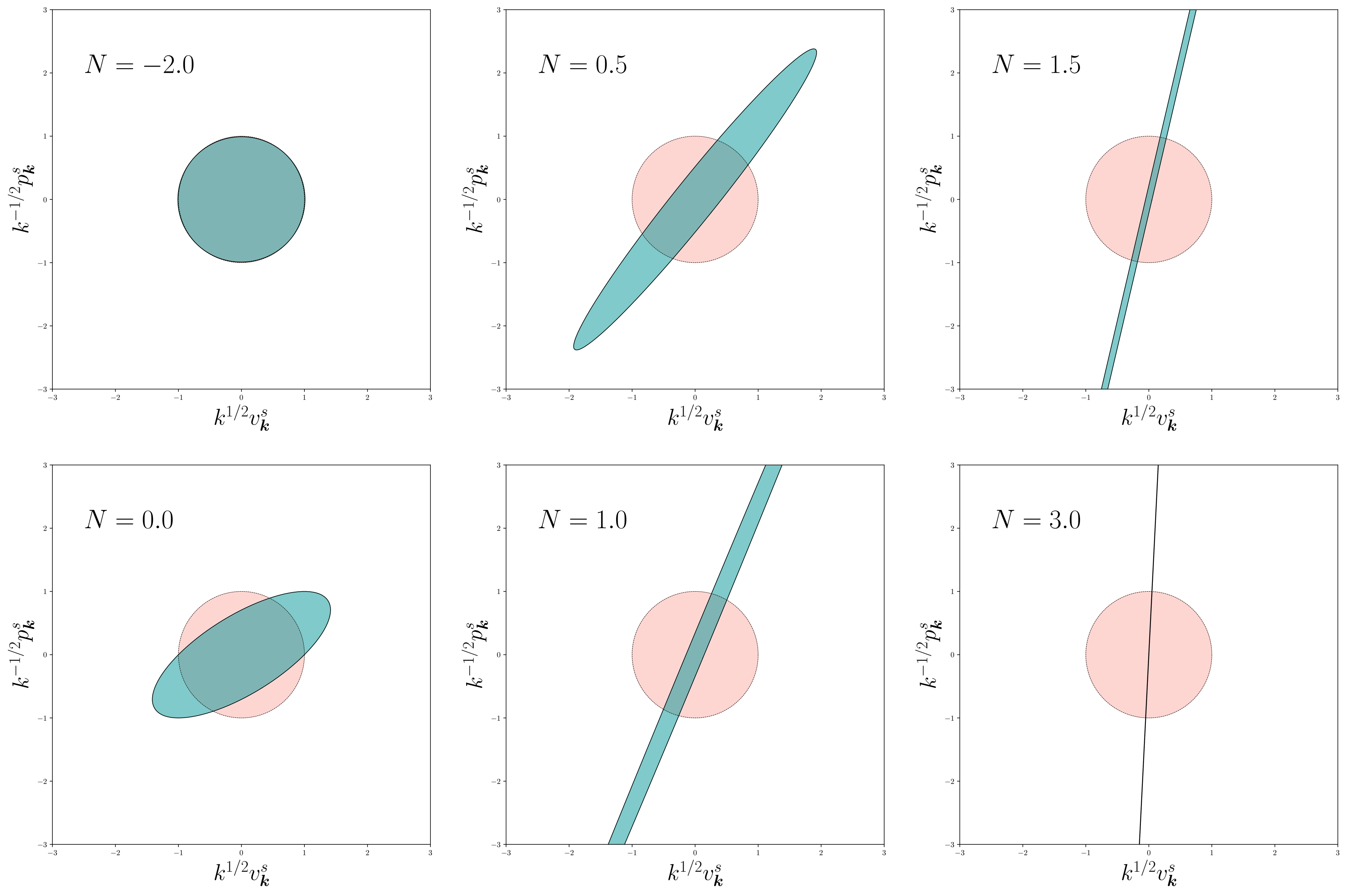
- <u>Cosmological perturbations:</u> Inflation leads to very strong squeezing $r_k \sim 60$.

- Can decompose the Wigner function as $W^s(v,\pi) = P(v)\sqrt{\frac{\gamma_{11}}{k\pi\hbar}} e^{-\frac{\gamma_{11}}{\hbar k}\left(\pi - \frac{\gamma_{12}}{\gamma_{11}} k v\right)^2}$

  where $P(v) = \left(\frac{k}{\pi\hbar\gamma_{11}}\right)^{1/2} e^{-\frac{kv^2}{\hbar\gamma_{11}}}$
  probability distribution for v

$$\delta\left(\pi - \frac{\gamma_{12}}{\gamma_{11}} k v\right)$$

Apparent classicality - II



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  probability distribution for v

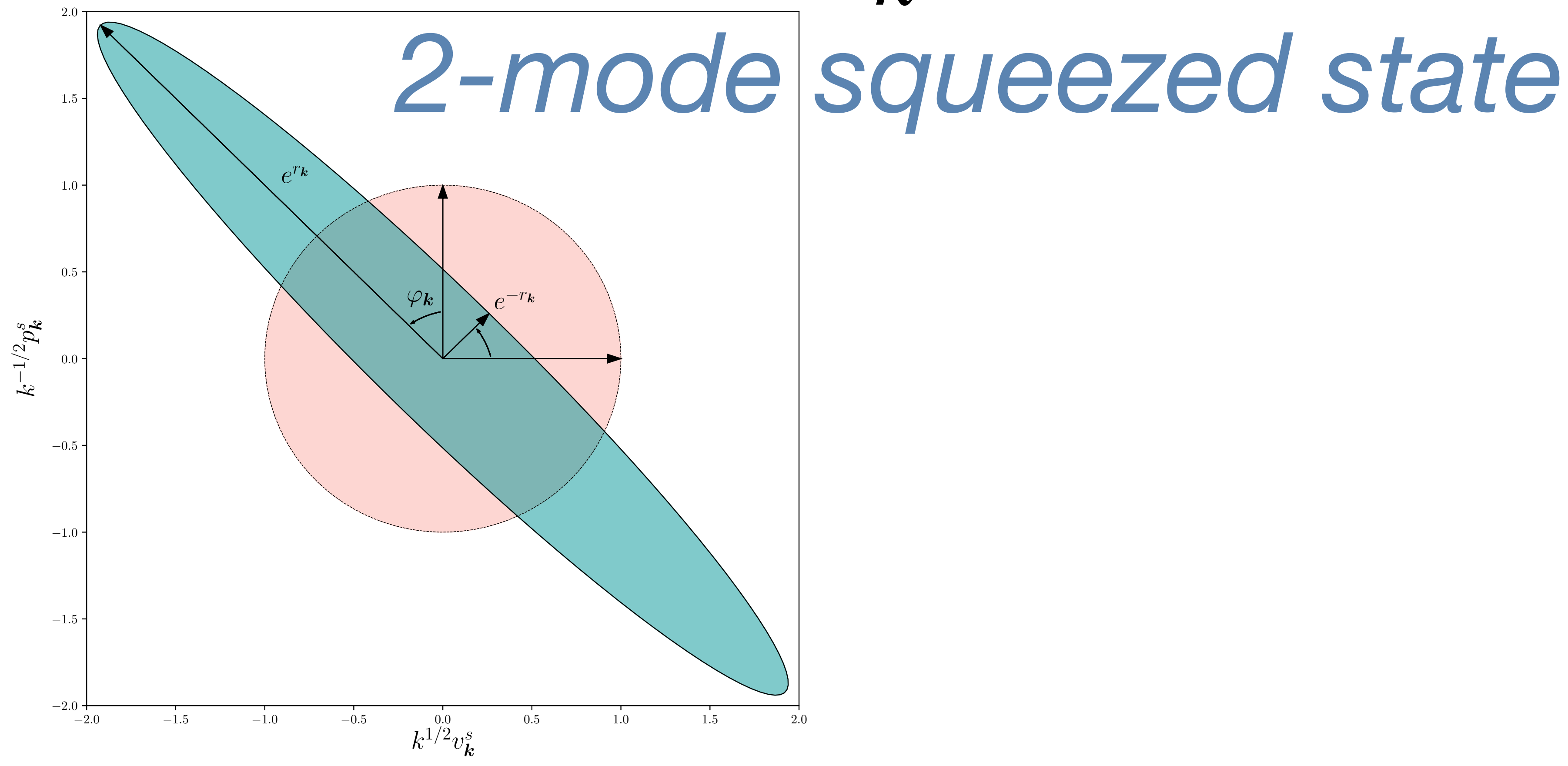
$$\downarrow$$

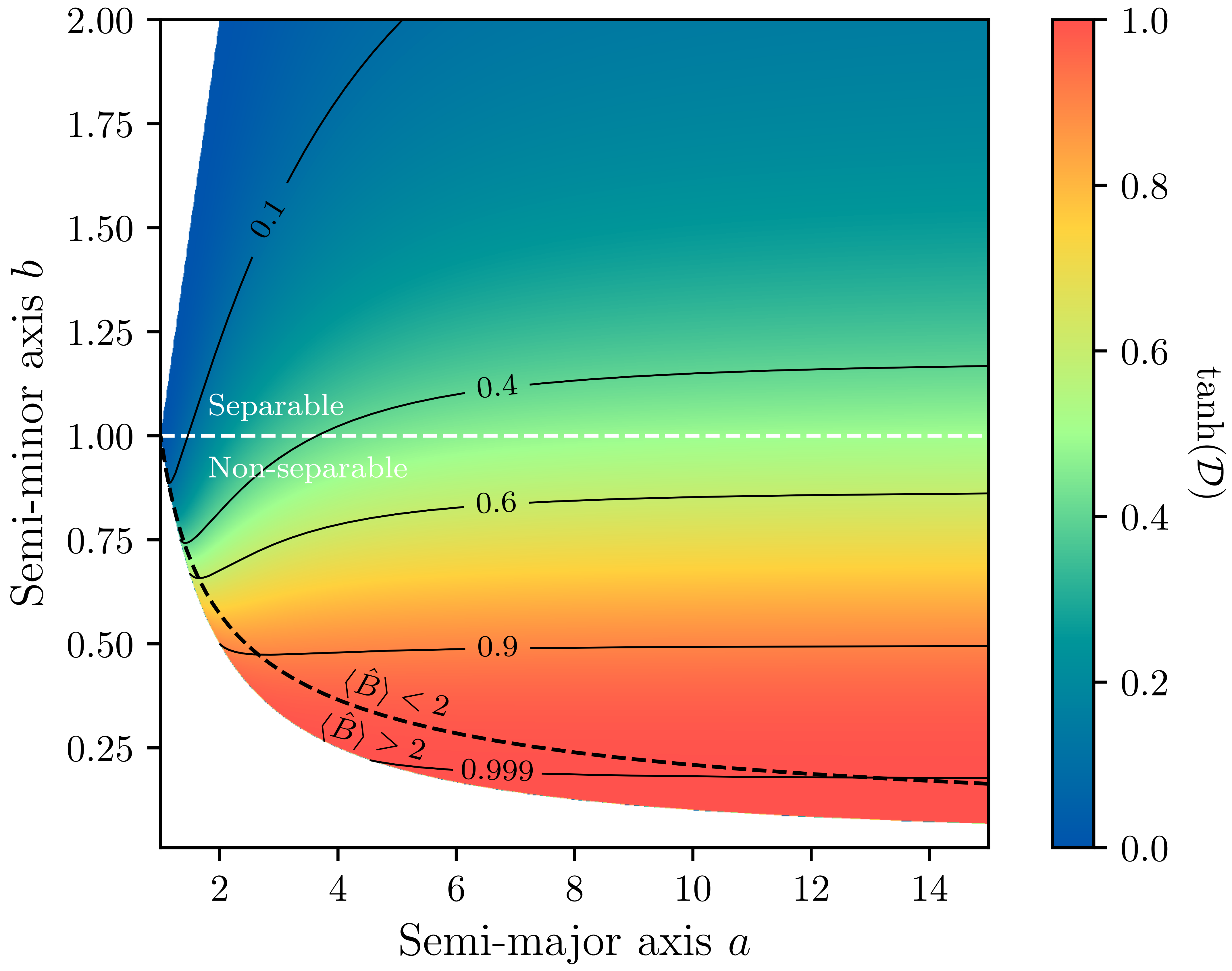
$$\delta\left(\pi - \frac{\gamma_{12}}{\gamma_{11}}kv\right)$$

Sub-fluctuant mode

Environment destroys quantum correlations

- Semi-minor axis $p_k^{-1/4}e^{-r_k}$ can remove sub-fluctuant direction.





Take Home Message 5

Large quantum Discord and non-separability for cosmological perturbations require existence of a sub-fluctuant direction.[5]

3. [Martin, Micheli and Vennin, 2022 (to be published)]
5. [arXiv:2112.05037 Martin, Micheli and Vennin, 2021]

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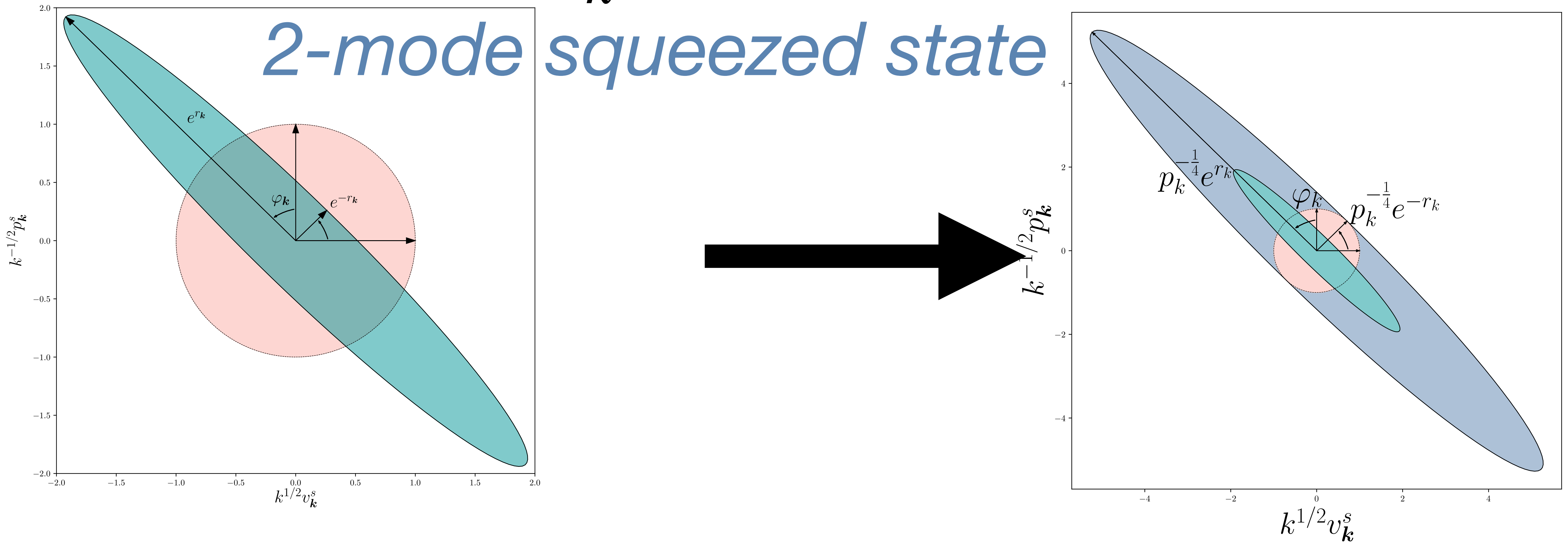

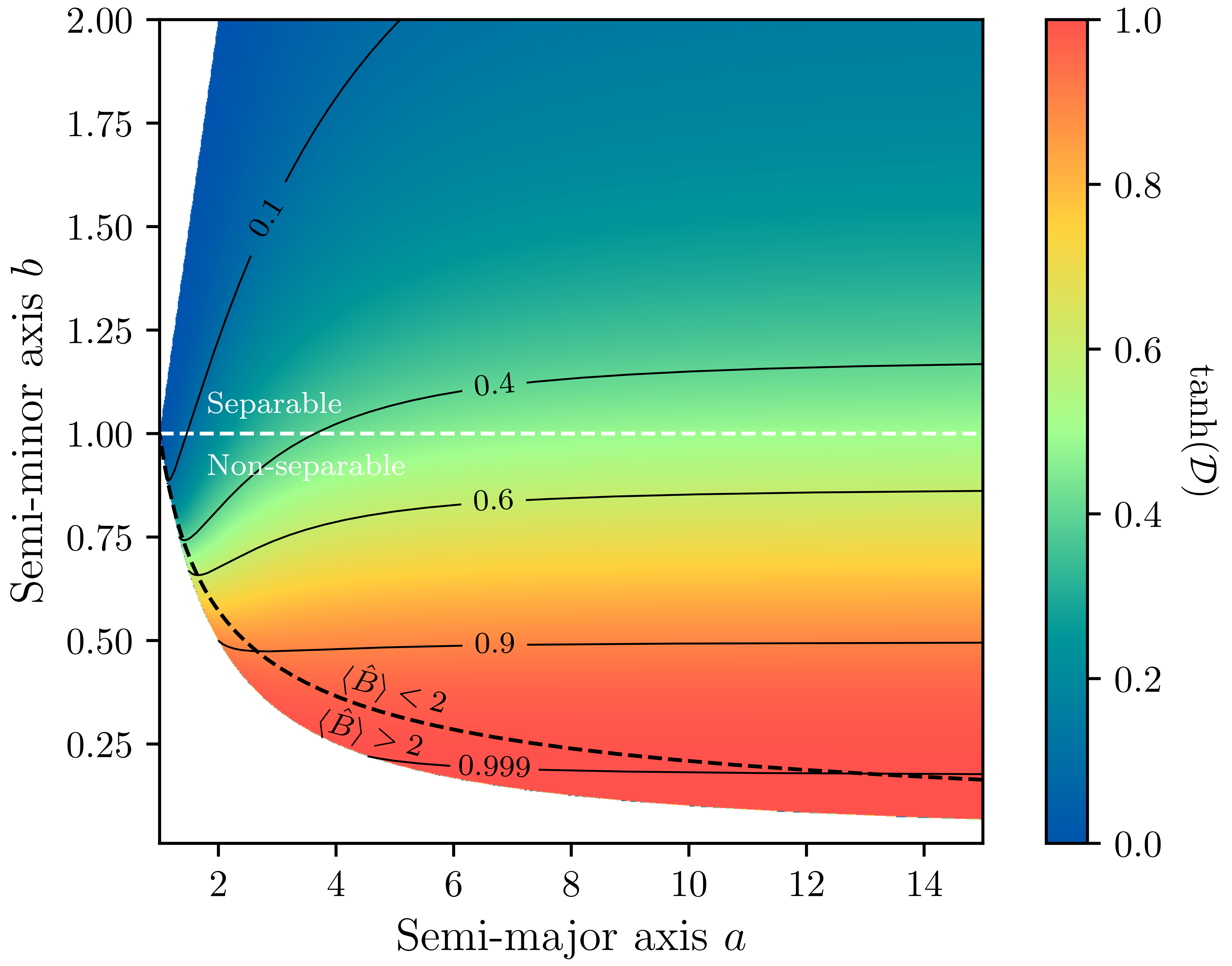

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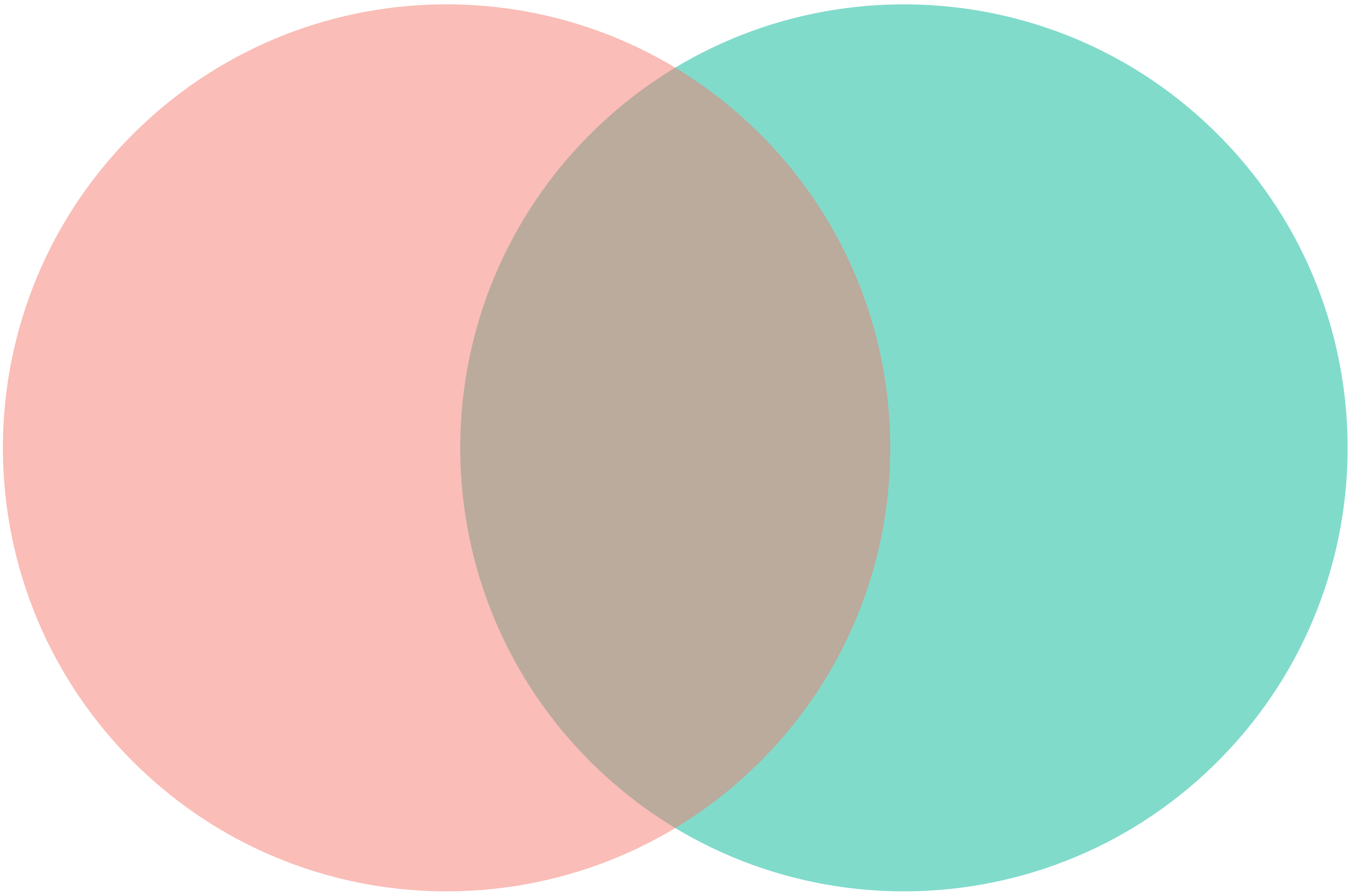
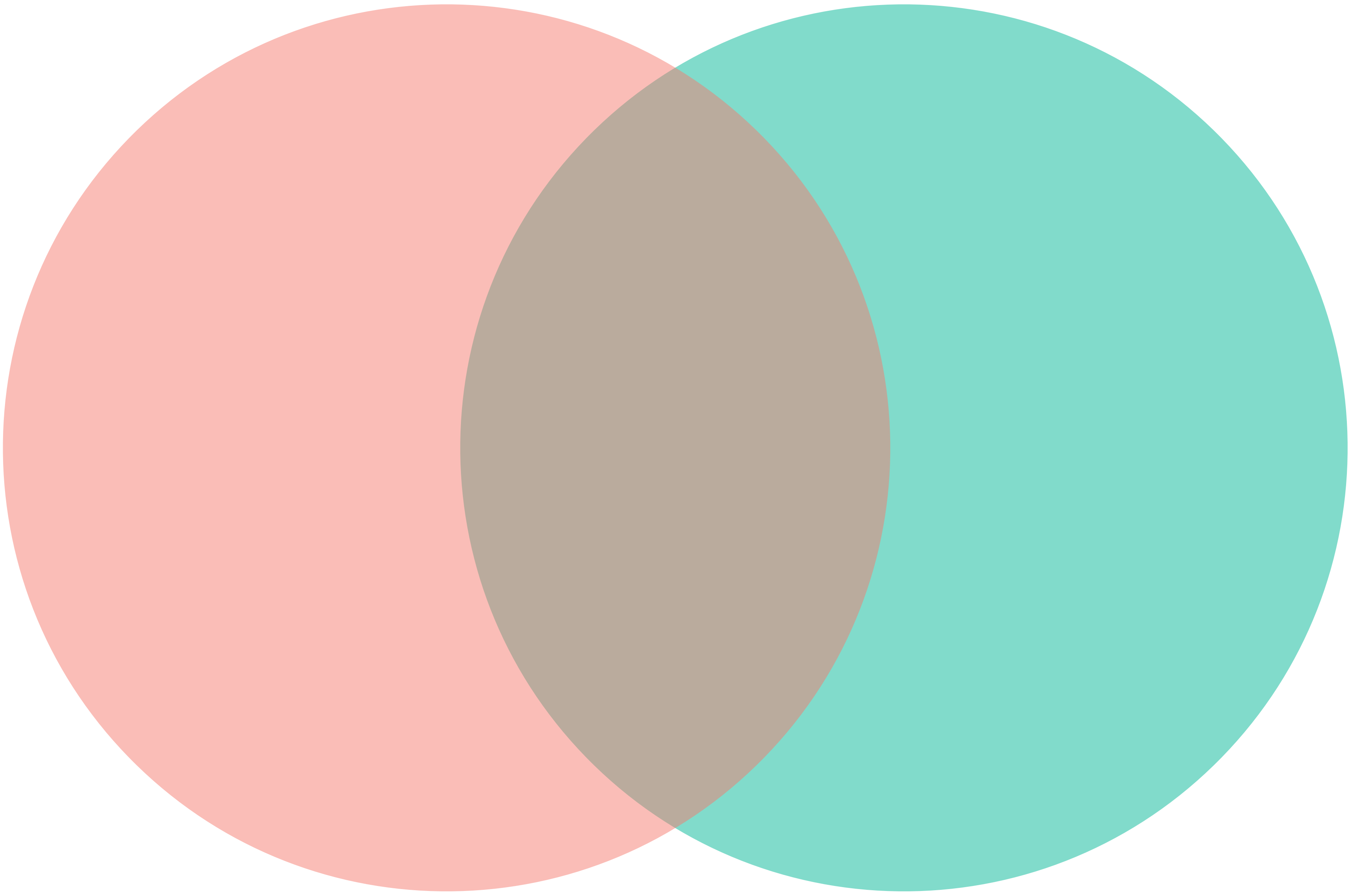
Quantum Discord

Quantum Discord - I

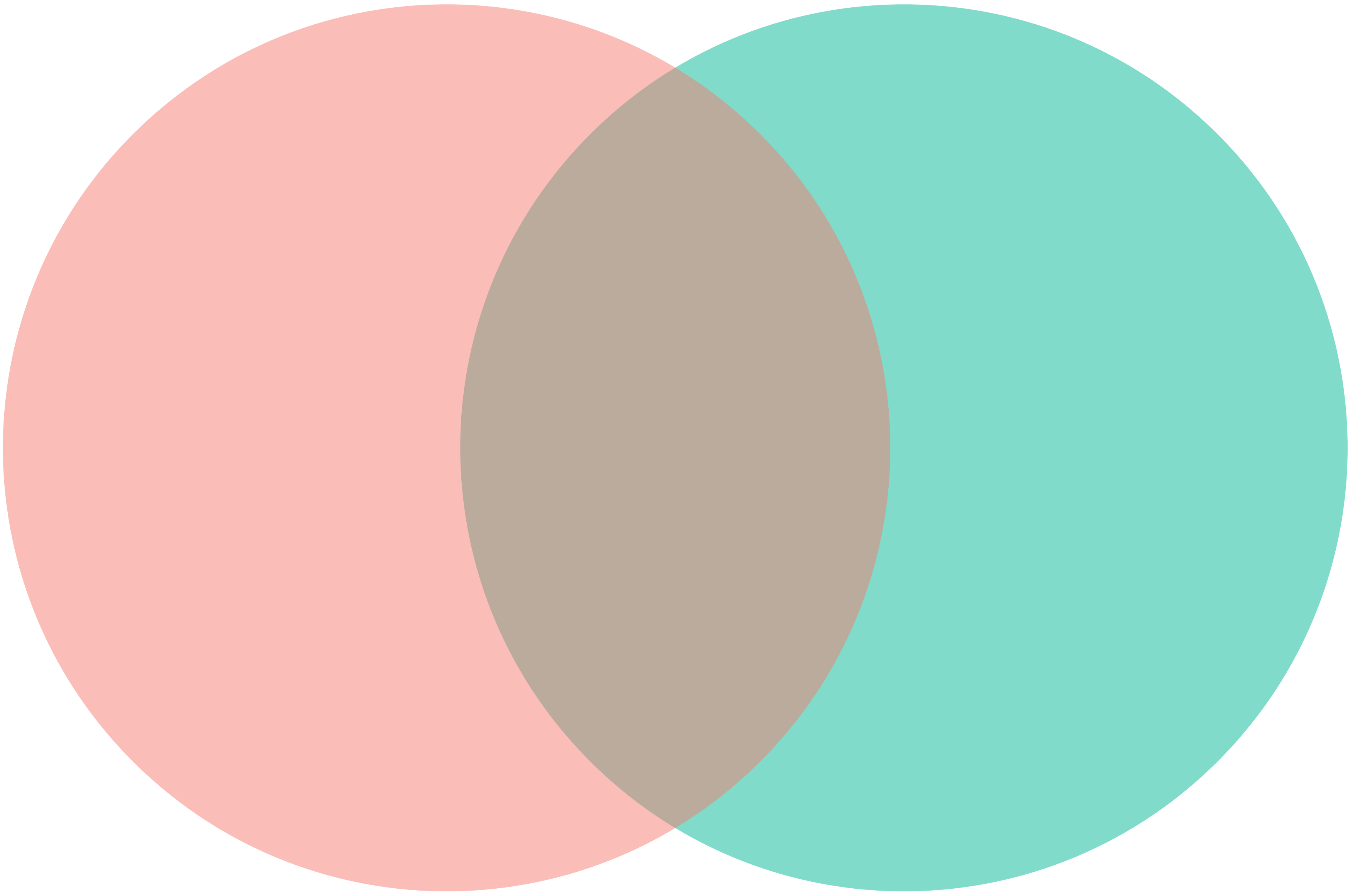


Quantum Discord - I



$$I(\mathscr{S}_1, \mathscr{S}_2) = S\left(\mathscr{S}_1\right) + S\left(\mathscr{S}_2\right) - S\left(\mathscr{S}_1, \mathscr{S}_2\right)$$

Quantum Discord - I



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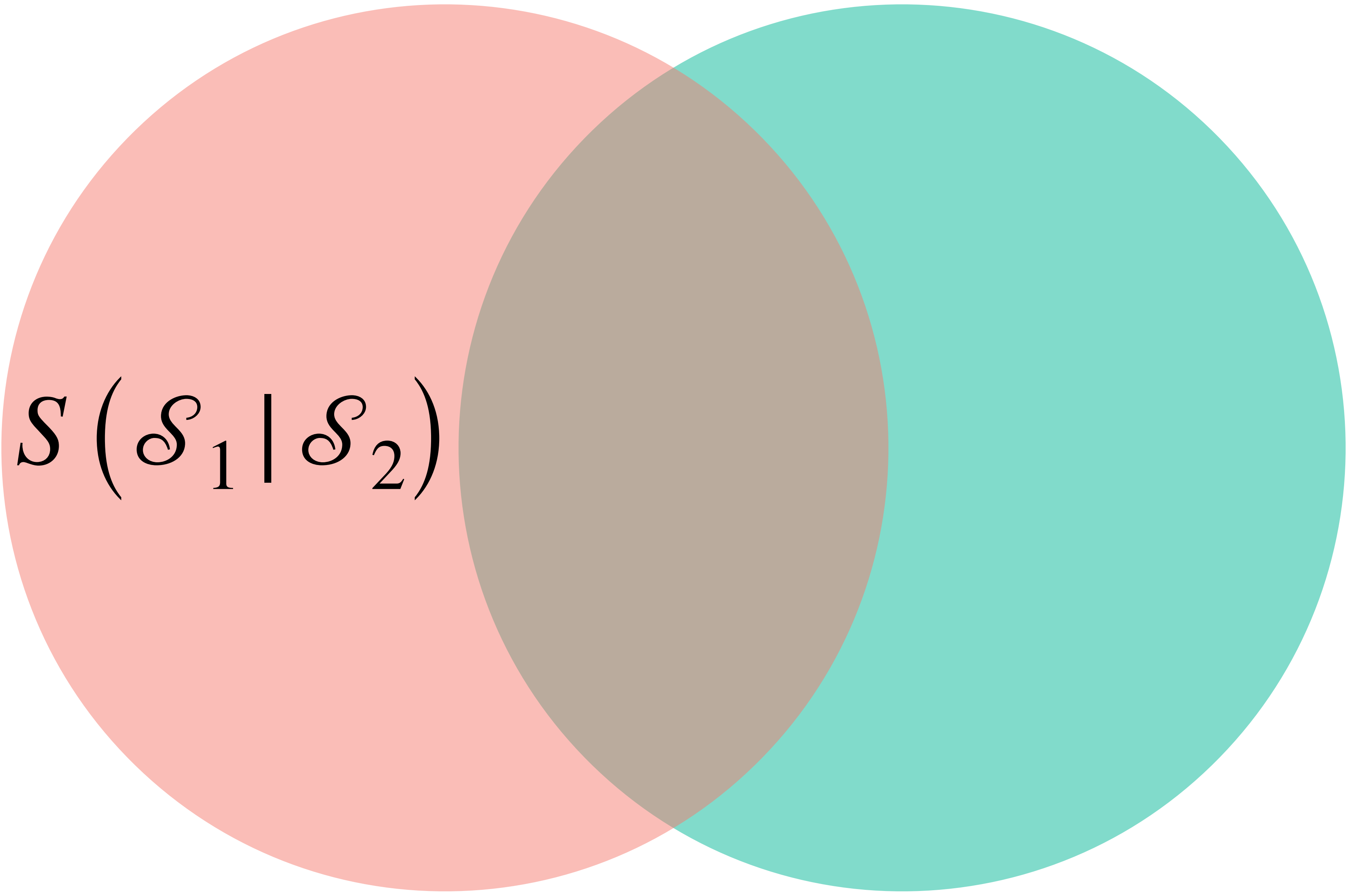
Quantum Discord - I



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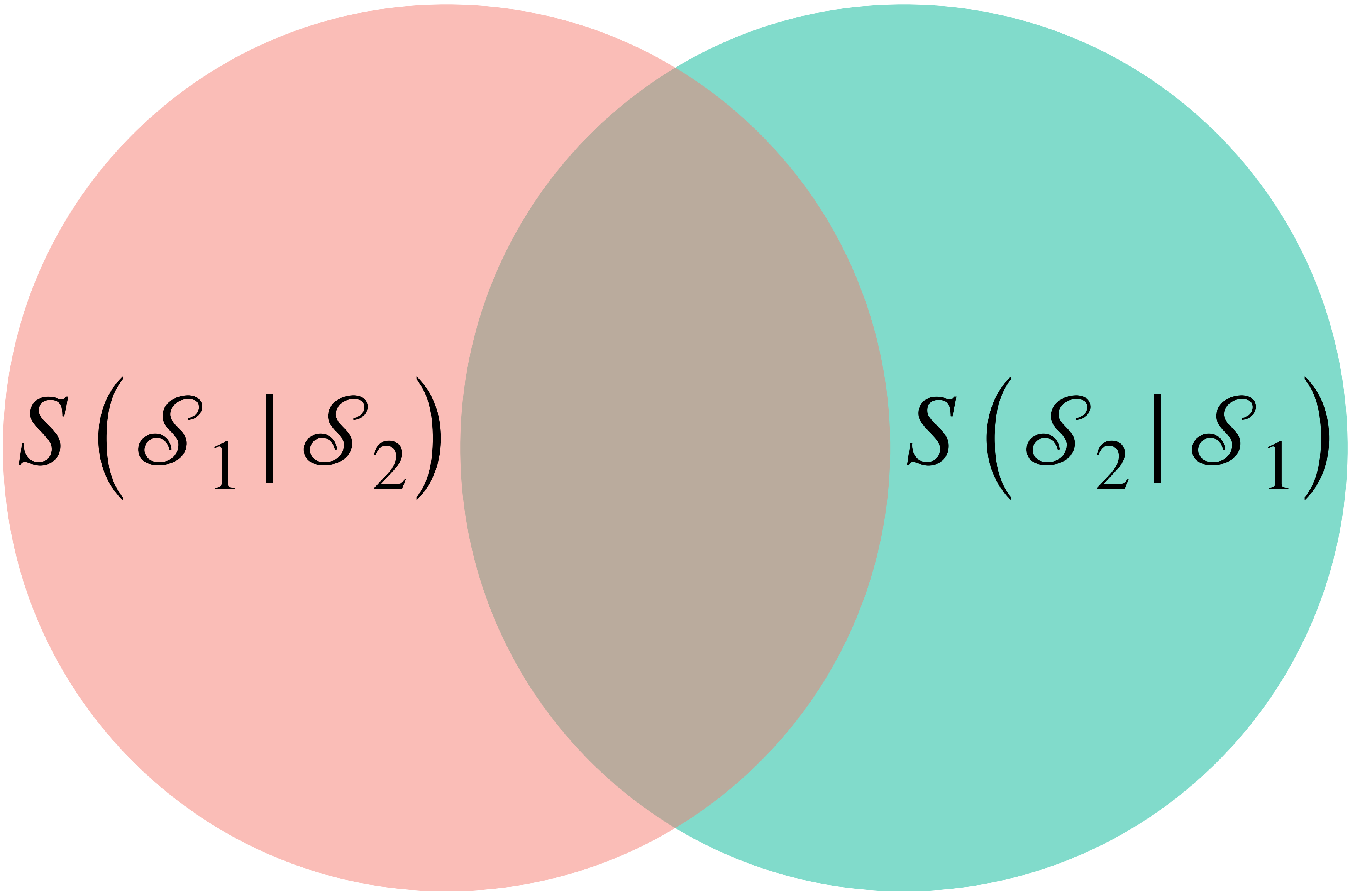

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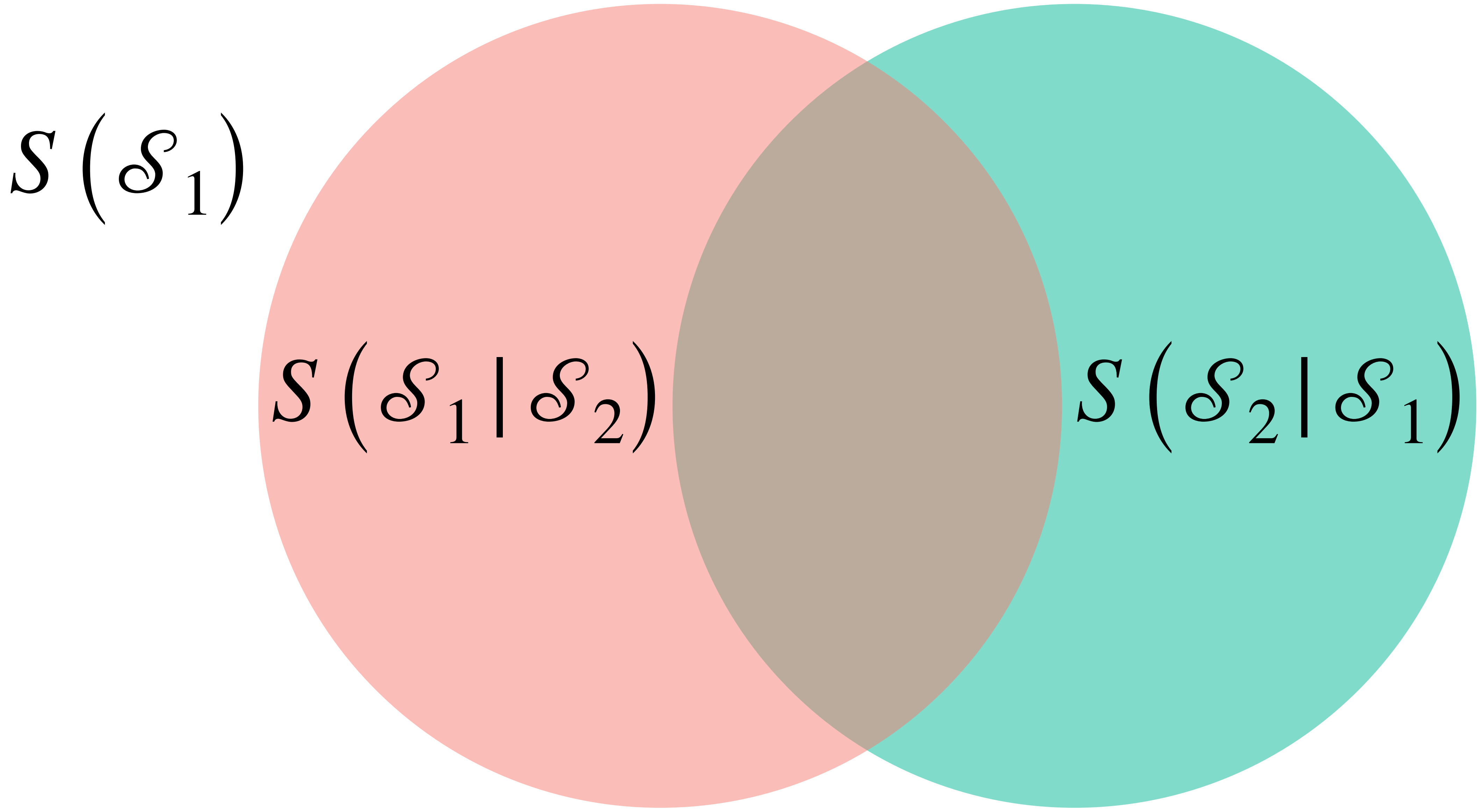

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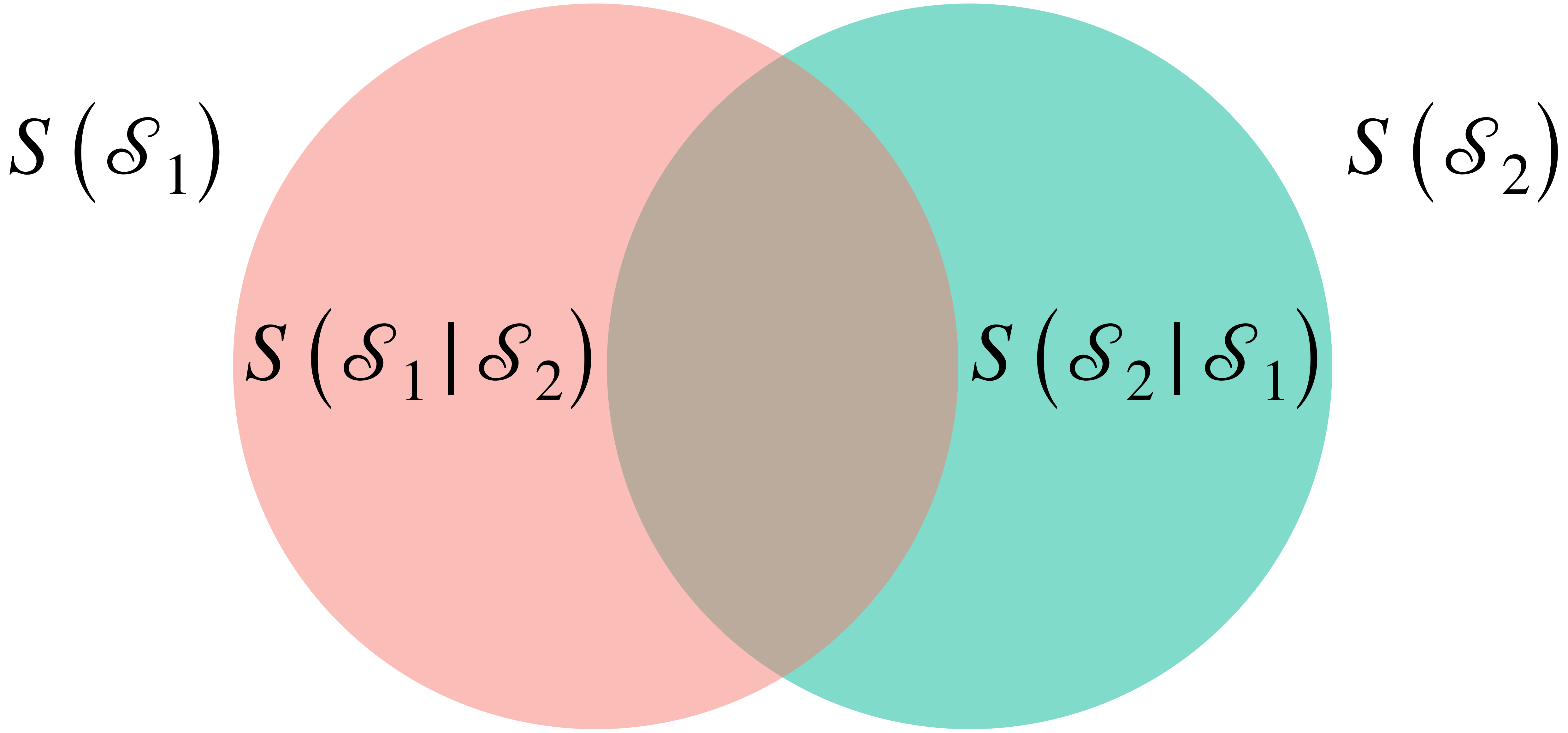

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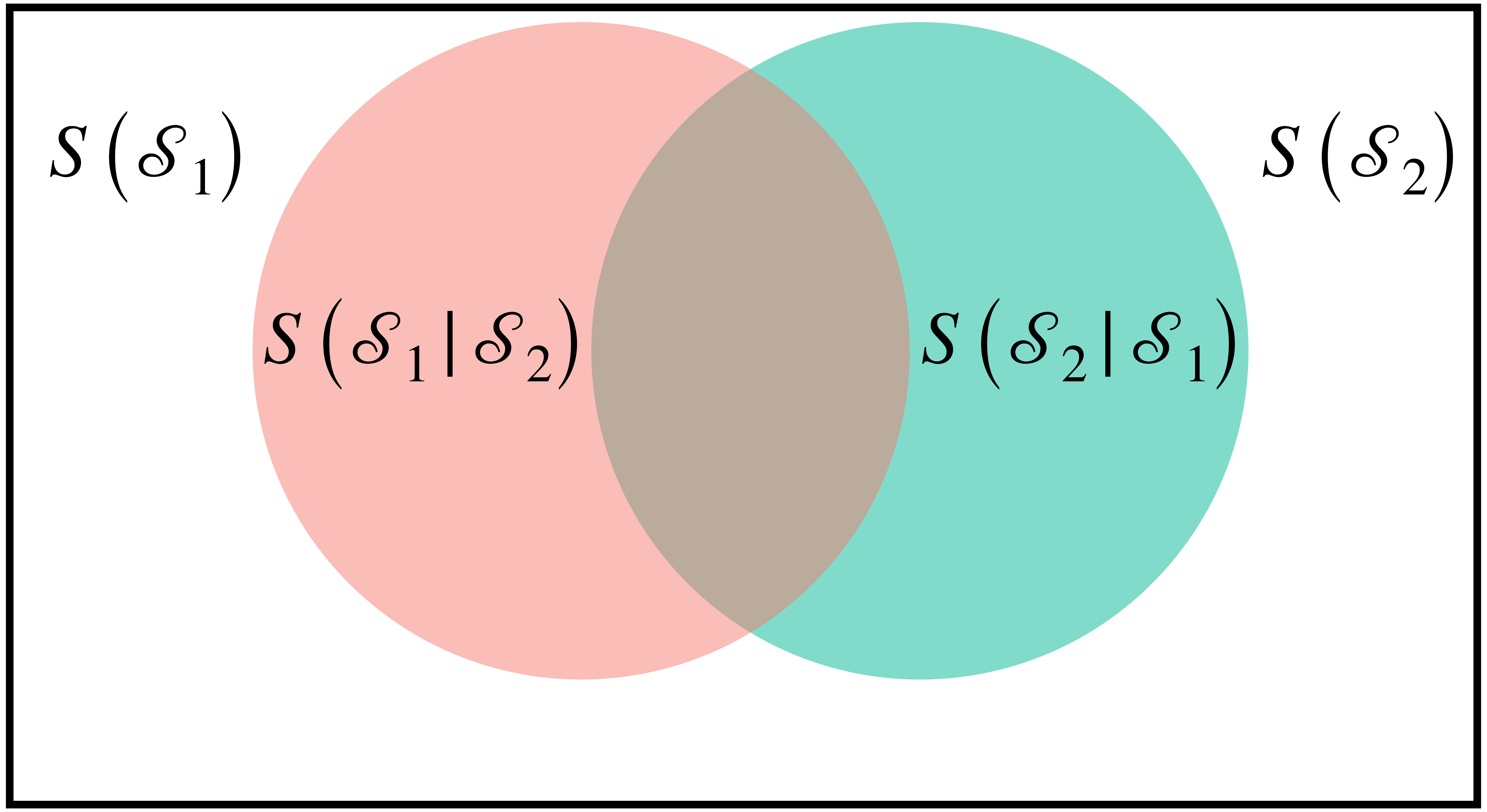

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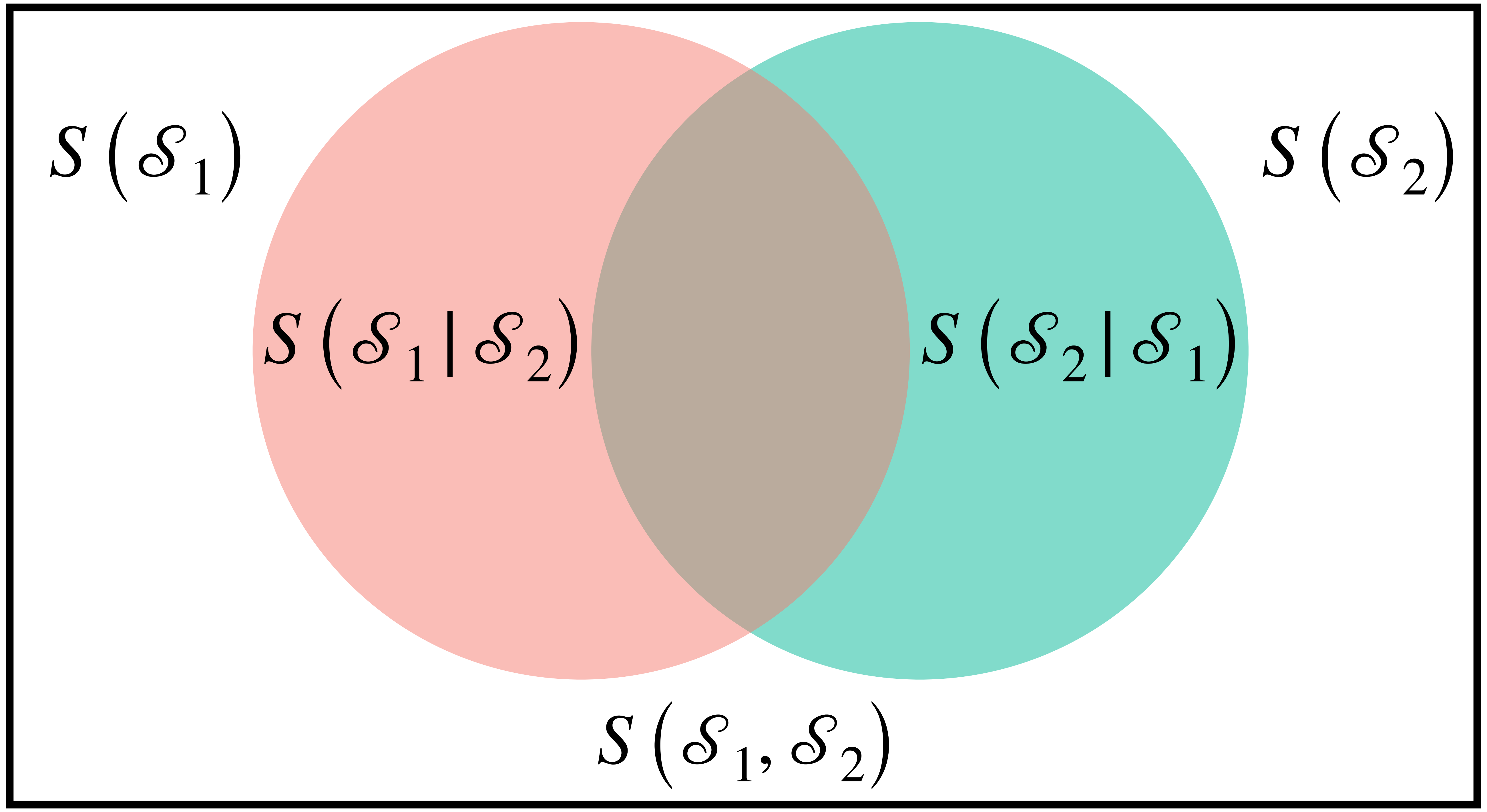

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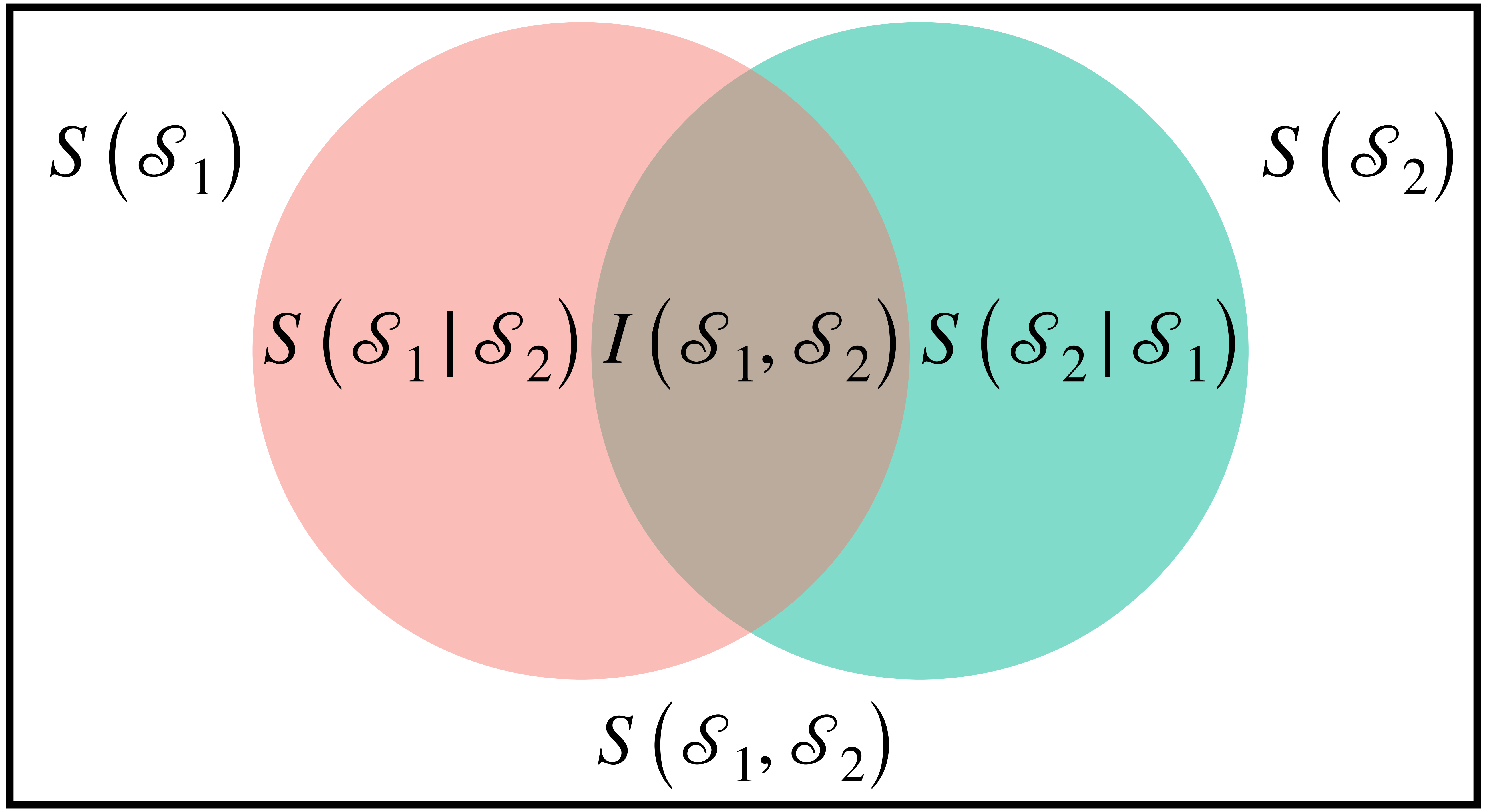

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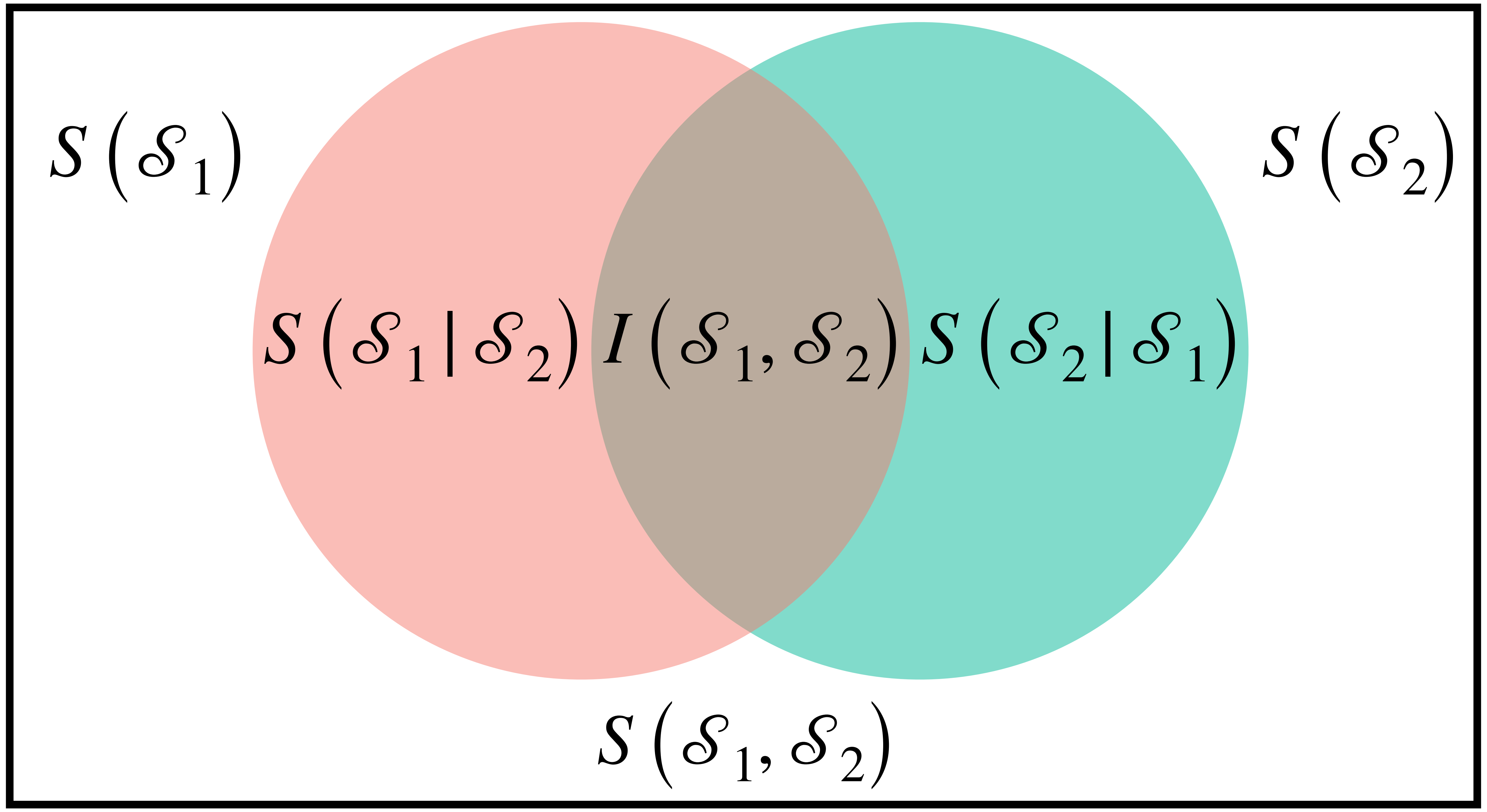
Classically, Bayes' theorem $p(\mathcal{S}_1,\mathcal{S}_2) = p\left(\mathcal{S}_1 \cap \mathcal{S}_2\right)/p\left(\mathcal{S}_2\right)$ implies

$$J(\mathcal{S}_1,\mathcal{S}_2) = I(\mathcal{S}_1,\mathcal{S}_2)$$

Quantum Discord - I

- Idea: Two different expressions I, J of mutual information between $\mathcal{S}_{1/2}$.



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Quantum Discord - II

Quantum Discord - II

$$J\left(\mathscr{S}_1, \mathscr{S}_2; \{\hat{\Pi}_j^{\mathscr{S}_2}\}\right) = S\left(\mathscr{S}_1\right) - S\left(\mathscr{S}_1 | \mathscr{S}_2; \{\hat{\Pi}_j^{\mathscr{S}_2}\}\right)$$

Quantum Discord - II

$$S\left(\mathscr{S}_1 | \mathscr{S}_2; \{\hat{\Pi}_j^{\mathscr{S}_2}\}\right) \equiv \Sigma p_j S\left(\hat{\rho}_{\mathscr{S}_1|\{\hat{\Pi}_j^{\mathscr{S}_2}\}}\right)$$

Define $$J\left(\mathscr{S}_1, \mathscr{S}_2; \{\hat{\Pi}_j^{\mathscr{S}_2}\}\right) = S\left(\mathscr{S}_1\right) - S\left(\mathscr{S}_1 | \mathscr{S}_2; \{\hat{\Pi}_j^{\mathscr{S}_2}\}\right)$$

and

Quantum Discord - II

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and $$\delta\left(\mathscr{S}_1, \mathscr{S}_2\right) \equiv I(\mathscr{S}_1, \mathscr{S}_2) - \max_{\{\hat{\Pi}_j^{\mathscr{S}_2}\}} J\left(\mathscr{S}_1, \mathscr{S}_2, \{\hat{\Pi}_j^{\mathscr{S}_2}\}\right) \geq 0$$

Quantum Discord - II

- Quantum counterpart of $S\left(\mathcal{S}_1 | \mathcal{S}_2\right)$ requires to specify set of measurements $\left\{\hat{\Pi}_j^{\mathcal{S}_2}\right\}_j$ applied to 'learn' the state of $\mathcal{S}_2$

$$S\left(\mathcal{S}_1 | \mathcal{S}_2; \left\{\hat{\Pi}_j^{\mathcal{S}_2}\right\}\right) \equiv \Sigma p_j S\left(\hat{\rho}_{\mathcal{S}_1|\left\{\hat{\Pi}_j^{\mathcal{S}_2}\right\}}\right)$$

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Quantum Discord - II

- Von Neumann entropy $S\left(\mathcal{S}_i\right) = \hat{\rho}_i \log\left(\hat{\rho}_i\right)$
- Quantum counterpart of $S\left(\mathcal{S}_1 \,|\, \mathcal{S}_2\right)$ requires to specify set of measurements $\left\{\hat{\Pi}_j^{\mathcal{S}_2}\right\}_j$ applied to 'learn' the state of $\mathcal{S}_2$

$$S\left(\mathcal{S}_1 \,|\, \mathcal{S}_2; \left\{\hat{\Pi}_j^{\mathcal{S}_2}\right\}\right) \equiv \Sigma p_j S\left(\hat{\rho}_{\mathcal{S}_1|\{\hat{\Pi}_j^{\mathcal{S}_2}\}}\right)$$

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