Quantum-Darwinism-encoding Transitions

Xiangyu Cao
joint work with Benoît Ferté

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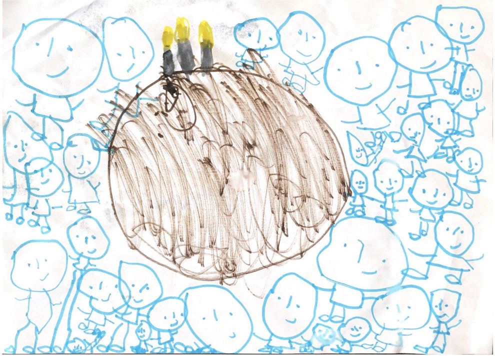
Classical objectivity from quantum mechanics

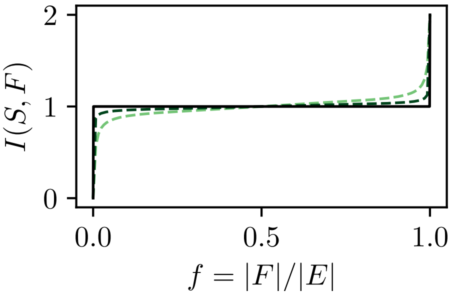
Consider a system S and its environment E. E has **objective knowledge** of S only if many small fractions $F \subset E$ are correlated with S. [Zurek *et al*]



[illustration by a 4 y.o.]

The Quantum Darwinism (QD) plateau

A strong indication of the emergence of objectivity is the "QD plateau" of the **mutual information** $I(S, F)$ between the system and a fraction of the environment. When S is a single qubit, we have:



A perfect QD plateau (black) and some approximate ones (dashed green).

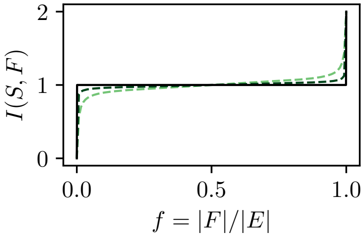
Reminder: mutual information

The mutual information between A and B is defined as

$$I(A,B) := H(A) + H(B) - H(AB)$$

where $H(X) := -\mathrm{Tr}[\rho_X \log_2 \rho_X]$ is the von Neumann entropy of the reduced density matrix to X (in unit of bits).

- Without correlation ($\rho_{AB} = \rho_A \otimes \rho_B$), $I(A,B) = 0$.
- If A and B are classically correlated bits, $I(A,B) = 1$.
- If A and B are maximally entangled qubits, $I(A,B) = 2$.



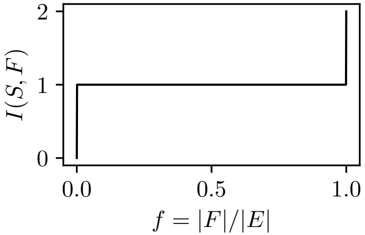
Basic example

The GHZ state

$$|\mathsf{GHZ}\rangle = \left(|0\rangle_S|00\ldots0\rangle_E + |1\rangle_S|11\ldots1\rangle_E\right)/\sqrt{2}$$

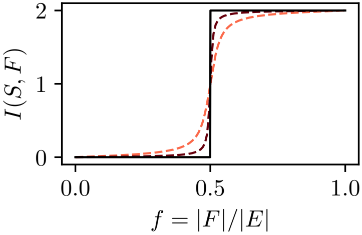
has a perfect Darwinism plateau:

$$I(S, F\subset E) = \begin{cases} 0 & f=0 \\ 1 & 0<f<1 \\ 2 & f=1 \end{cases} \qquad (f=|F|/|E|)$$



Opposite of objectivity: encoding

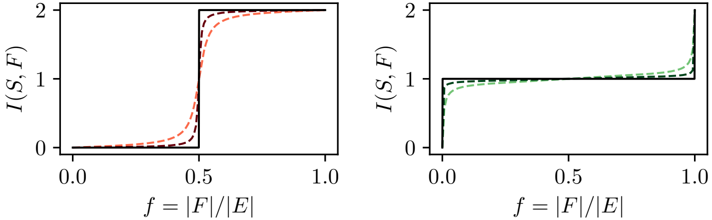
A random state in the Hilbert space $\mathcal{H}_{SE}$ does *not* have a plateau. Instead, it exhibits **encoding**:



A small subsystem ($f < 1/2$) knows nothing of S! (This is a consequence of the Page curve.)

Question

Are QD and encoding “phases of information”, separated by some sharp transition(s)?

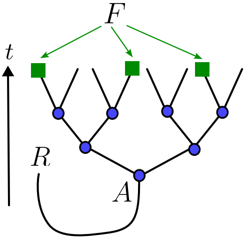


Since generic chaotic unitary dynamics leads to encoding, such a transition would have a similar flavor to MBL and measurement-induced transitions.

In this talk

We consider two models with Quantum Darwinism-encoding transitions (QDETs).

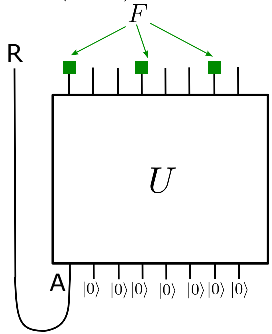
- Both are “mean-field” models, defined on an expanding tree (equivalent to all-to-all interaction).
- Model 1 is a simple random Clifford model [2305.03694].
- Model 2 is deterministic and non-Clifford [to appear].



Models for QDET: general setup

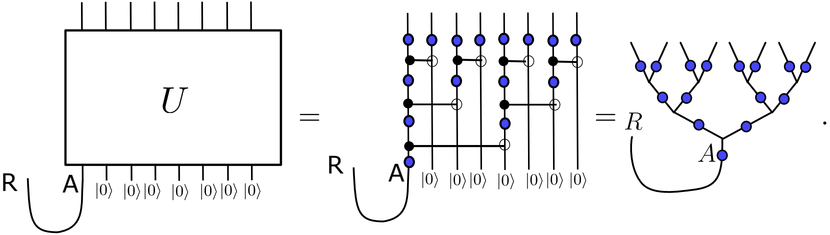
- Start with an entangled pair RA and $N-1$ environment qubits ("recruits") in a factorized state.
- A and the recruits undergo a unitary evolution U depending on a parameter p.
- Pick a subset F of the output qubits; $f = |F|/N$.

Figure of merit: $I(F, R)$ as a function of f and p

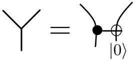


Models on a expanding tree

We now further specify the geometry:



Here, a branching duplicates the in-going bit (in the Z basis)

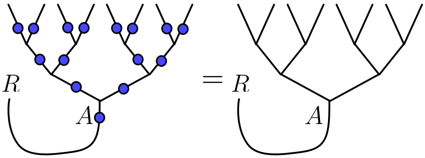


$$= \sum_{s=\pm 1} |s\rangle|s\rangle\langle s| \,,$$

and ● is a one-site unitary depending on the parameter p.

The QD limit ($p = 0$)

In both models, ● $= I$ when $p = 0$. So



$(p = 0)\,,$

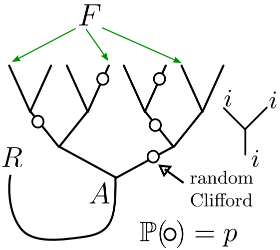
generates a GHZ state on R and the N output bits. Then

$$I(F, R) = 1\,,\ (0 < |F| < N)$$

The information on R is perfectly broadcast.
When $p > 0$, the ●'s will tend to "scramble " the information, and drive transitions towards encoding.

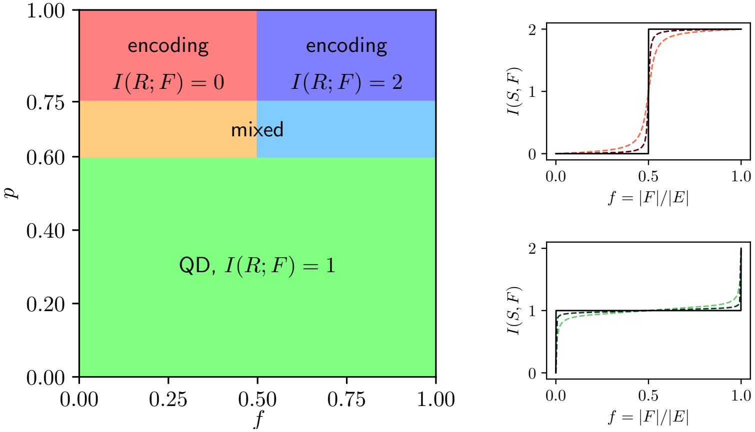
Model 1: definition

- Each $u = \bullet$ is a random one-site Clifford[1] with probability p, and identity otherwise.
- The fraction F is chosen randomly: every leaf $\in F$ with probability f.



[1] By definition, $O \mapsto u^\dagger O u$ permutes randomly the Pauli operators.

Model 1: exact phase diagram



Two transitions and a mixed phase (stochastic mixture).

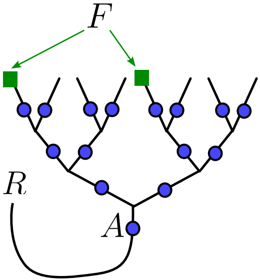
Model 2: non-Clifford and deterministic

- The scramblers are deterministic and identical

$$u = \bullet = \exp(i\theta Y) = \begin{pmatrix} \cos\theta & -\sin\theta \\ \sin\theta & \cos\theta \end{pmatrix}, \theta = \frac{p\pi}{4}$$

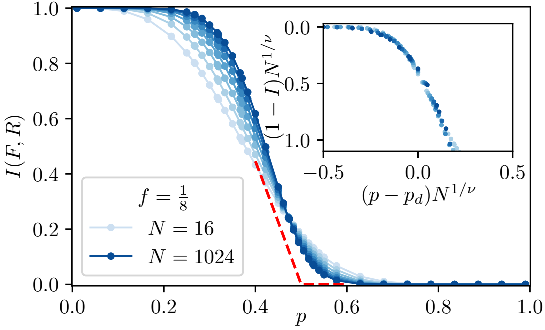
$u_{p=0} = I$, $u_{p=1}$ swaps Z and X.

- F is deterministic, with $f = 1/2^k$ $(k > 1)$.



(Example of $f = 1/4$)

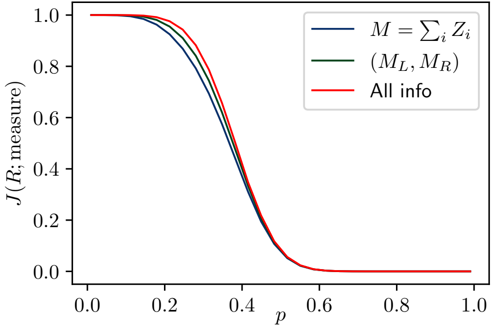
Model 2: phase diagram



- An encoding phase at $p > p_c = \frac{1}{2}$ (exact prediction).
- A mixed phase at $p_d < p < p_c$ and a QD phase at $p < p_d$ for some $p_d \approx 0.33$ (inset).

Coarse-grained measure

What can we learn about R by measuring only the total spin $M = \sum_{j \in F} Z_j$? Or the total spin in the two half trees (M_L, M_R), etc? Almost everything!

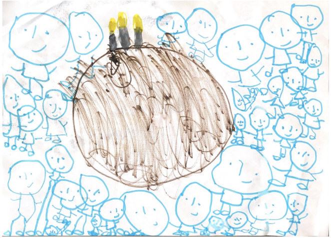


Useful for observing the transitions in practice.

Summary

Unitary dynamics can exhibit both Quantum Darwinism and encoding phases, separated by transitions. The phases are characterized by how information dissipates. Questions for future work include:

- Universal critical properties in finite dimensions (in an expanding spacetime?).
- Relevance of statistical physics for foundational questions, e.g., Wigner's friend paradox.

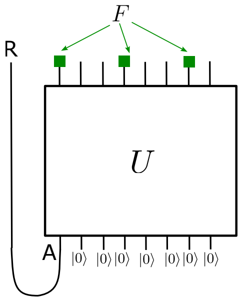


Quantities of interest

- Mutual information $I(F, R)$.
- Conditional entropy: does F know about R? Or, can measuring F (in a basis of your choice) disentangle R?

$$J(R; F) = H(R) - \min_{\text{basis}} \langle H(R) \rangle_{\text{measure}}$$

In general $J(R; F) \leq I(F, R)$. $I(F, R) - J(R; F)$ is called the "quantum discord".



Defining the order parameter (Model 2)

Consider measuring Z's on F. To each outcome $\vec{m} = (m_j)_{j \in F}$ is associated an operator on A:

$$Q_{\vec{m}} = V^\dagger \left[\prod_{j \in F} \frac{1 + m_j Z_j}{2} \right] V = p_{\vec{m}} (I + u_{\vec{m}} Z + v_{\vec{m}} X) \,.$$

where $V|\psi\rangle = U|\psi\rangle|0^{N-1}\rangle$. $p_{\vec{m}}$ is the probability of the outcome. $(u_{\vec{m}}, v_{\vec{m}})$ is the information on R revealed by the measure. The order parameter is a distribution of (u, v) (in the unit disk)

$$P(u, v) = \sum_{\vec{m}} p_{\vec{m}} \delta(u_{\vec{m}} - u) \delta(v_{\vec{m}} - v)$$

Order parameter: properties

$$P(u,v) = \sum_{\vec{m}} p_{\vec{m}} \delta(u_{\vec{m}} - u)\delta(v_{\vec{m}} - v)$$

"Distribution of knowledge of F on R" (no longer discrete).

- It determines both conditional entropy and mutual information ($f \leq 1/2$)

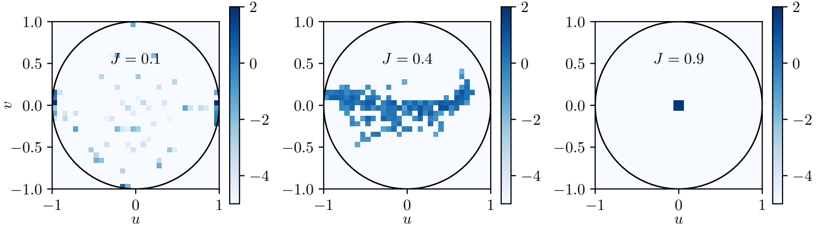
  $$J(F;R) = I(F,R) = 1 + \int (\lambda_+ \log_2 \lambda_+ + \lambda_- \log_2 \lambda_-) P(u,v)$$

  where $\lambda_\pm = (1 \pm \sqrt{u^2+v^2})/2$ (no discord).
- It satisfies a backward recursion relation (harder to solve).

  $$P'(u,v) = \int (1+u_1u_2)\delta(u - \frac{u_1+u_2}{1+u_1u_2})\delta(v - \frac{v_1v_2}{1+u_1u_2})P(u_1,v_1)P(u_2,v_2)$$

Order parameter distribution

Plots of $\log_{10} P(u, v)$ ($f = 1/8$, $N = 64$):



- In the QD phase, $P(u, v)$ is concentrated on the unit circle.
- In the encoding phase, $P(u, v)$ is concentrated on $(0, 0)$.
- Nontrivial in the mixed phase.

The order parameter (Model 1)

Consider a product of Pauli's (*Pauli string*) P on F. $U^\dagger PU$ then a acting on A and $N-1$ recruits. Contracting the recruits gives an operator on A:

$$Q_A[P] := \langle 0^{N-1} | U^\dagger PU | 0^{N-1} \rangle \in \{I, X, Y, Z, 0\}$$

It is not hard to show that

$$\mathbf{s} := \{Q_A[P] : P \text{ Pauli string on } F\} \cap \{I, X, Y, Z\}$$

is a subgroup of $\{I, X, Y, Z\} \simeq \mathbb{Z}_2^2$, i.e. $\mathbf{s} \in \{\mathbf{n}, \mathbf{z}, \mathbf{x}, \mathbf{y}, \mathbf{a}\}$ where

$$\mathbf{n} = \{I\}, \mathbf{z} = \{I, Z\}, \mathbf{x} = \{I, X\}, \mathbf{y} = \{I, Y\}, \mathbf{a} = \{I, Z, X, Y\}.$$

The order parameter

The subgroup

$$\mathbf{s} := \{Q_A[P] : P \text{ Pauli string on } F\} \cap \{I, X, Y, Z\}$$

tells us what F knows about R (since AR form an EPR pair):

- $\mathbf{s} = \mathbf{n}$: F and R are uncorrelated, $I(F, R) = 0$.
- $\mathbf{s} = \mathbf{z}$: some operator on F is perfectly correlated with Z_R: $I(F, R) = J(R; F) = 1$.
- $\mathbf{s} = \mathbf{a}$: we can distill from F a qubit that is maximally entangled with R: $I(F, R) = 2$ $(J = 1)$.

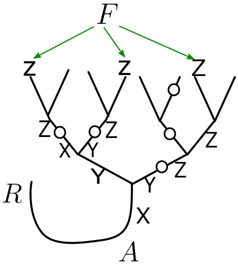
The order parameter is the probability distribution of $\mathbf{s}$,

$$\pi := (\pi_{\mathbf{n}}, \pi_{\mathbf{z}}, \pi_{\mathbf{x}}, \pi_{\mathbf{y}}, \pi_{\mathbf{a}})$$

where $\pi_{\mathbf{n}} = \mathrm{Prob}(\mathbf{s} = \mathbf{n})$ etc.

Calculation on a tree by example

Let's compute $Q_A[P]$ where $P = \prod_{i \in F} Z_i$ in a fixed realization. We contract with the recruit at each branching. The calculation is a "backward recursion" (from top to bottom) and follows the table at each branching:

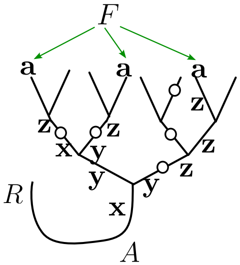


	I	Z	X	Y	0
I	I	Z	0	0	0
Z	Z	I	0	0	0
X	0	0	X	Y	0
Y	0	0	Y	X	0
0	0	0	0	0	0

.

Calculation on a tree by example

We can compute $\mathbf{s}$ in a fixed realization by a similar backward recursion ($\mathbf{n}$ is omitted in the figure):



	$\mathbf{n}$	$\mathbf{z}$	$\mathbf{x}$	$\mathbf{y}$	$\mathbf{a}$
$\mathbf{n}$	$\mathbf{n}$	$\mathbf{z}$	$\mathbf{n}$	$\mathbf{n}$	$\mathbf{z}$
$\mathbf{z}$	$\mathbf{z}$	$\mathbf{z}$	$\mathbf{z}$	$\mathbf{z}$	$\mathbf{z}$
$\mathbf{x}$	$\mathbf{n}$	$\mathbf{z}$	$\mathbf{x}$	$\mathbf{y}$	$\mathbf{a}$
$\mathbf{y}$	$\mathbf{n}$	$\mathbf{z}$	$\mathbf{y}$	$\mathbf{x}$	$\mathbf{a}$
$\mathbf{a}$	$\mathbf{z}$	$\mathbf{z}$	$\mathbf{a}$	$\mathbf{a}$	$\mathbf{a}$

.

Note: a 1-site Clifford permutes $\mathbf{z}, \mathbf{x}, \mathbf{y}$ and leaves $\mathbf{a}$ and $\mathbf{n}$ invariant.

Recursion of order parameter

We can easily turn these rules to a recursion relation between π of a $t+1$-generation tree and that of a t-generation one:

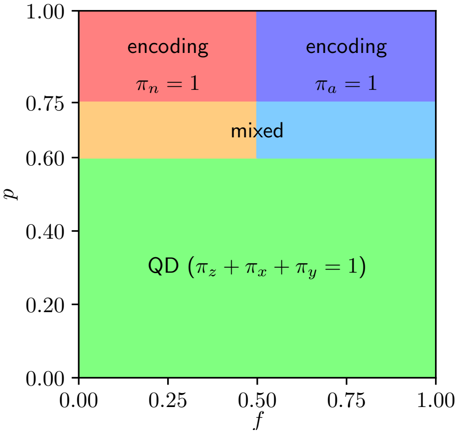
$$\pi(t+1) = M(\pi(t))$$

$M : \mathbb{R}^5 \to \mathbb{R}^5$ is a nonlinear (quadratic) map that depends on p. (See paper for explicit expression.) Initial condition depends on $f = |F|/N$:

$$\pi(t=0) = (1-f, 0, 0, 0, f)\,.$$

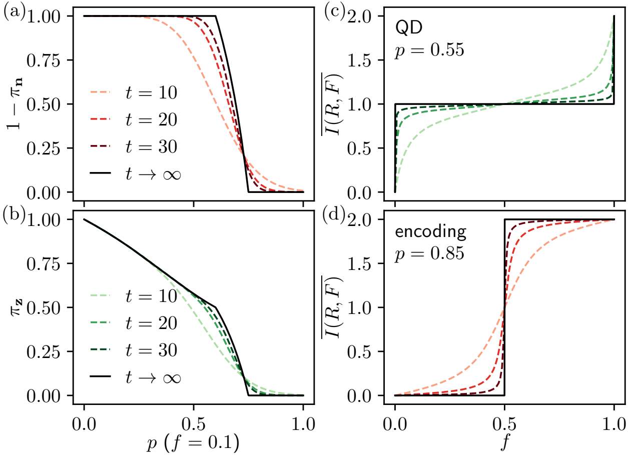
At this point, solving the model amounts to analyzing the asymptotic ($t \to \infty$) behavior of the “flow” generated by M.

Phase diagram

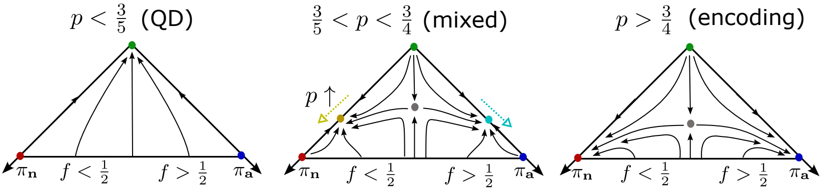


$\pi(t \to \infty)$ is explicitly predicted everywhere (see paper).

More plots with finite size numerics



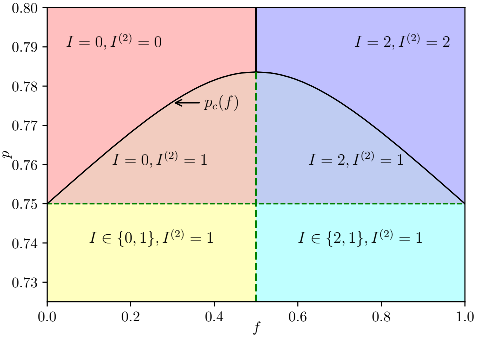
Flow and symmetries



- $\mathbb{Z}_2$: swapping $\mathbf{n} \leftrightarrow \mathbf{a}$, or $f \to 1 - f$. Explicitly broken by the initial condition ($f \neq 1/2$). Spontaneously broken in the encoding and mixed phase, restored in the QD phase.
- $\mathcal{S}_3$: permuting $\mathbf{z}, \mathbf{x}, \mathbf{y}$. Explicitly broken by the branching isometry. Broken in the QD and mixed phase ($\pi_{\mathbf{z}} > \pi_{\mathbf{x}} = \pi_{\mathbf{y}}$), restored in the encoding phase.

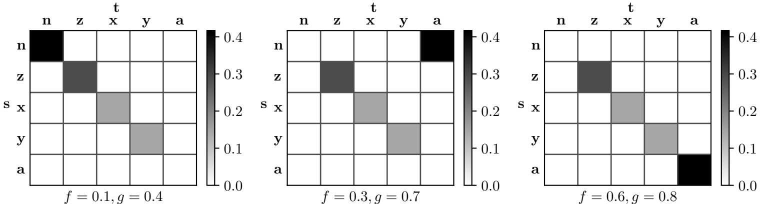
Annealed mutual information (replica trick)

$$I^{(2)}(F,R) := \log_2 \mathrm{Tr}\left[\overline{\rho_{FR}^2}\right] - \log_2 \mathrm{Tr}\left[\overline{\rho_F^2}\right] + 1\,,$$

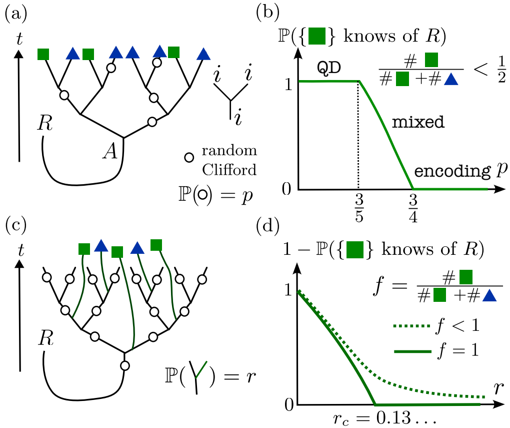


Mixed phase: joint order parameter

Two fractions $F \subset G$, $|F| = fN, |G| = gN$. Joint distribution of order parameter $\Pi_{\mathbf{st}}$:



QDET *vs* measurement-induced phase transitions (MIPT)



(a,b) QDET (Model 1). (c,d) MIPT: a sharp transition requires full access to the environment (measurement ancillas).