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Anomalous Z' and the $g-2$ of the muon

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Orsay, 9 February 2023

Intro/motivation

- Standard Model (SM) is the most accurate theory even made.
- However, we know that it is not the final theory. It is effective.
- In this talk, we will extend the SM with an additional gauge boson (often called Z').
- However, we are going to explore an "exotic" version of the Z': an anomalous Z'.

Intro/motivation

- After a small introduction in anomalies, we will discuss anomalies in effective theories.
- Such effective theories can come from string theory, GUTs, theories with a Higgs mech at a higher scale.
- We will show that anomalies in such models can be "superficial" and therefore not forbidden.
- We will argue that in effective theories, the additional Z' is more likely to be anomalous than not.
- We will analyse such models and we will discuss phenomenological implications ($g-2$ of the muon).

Anomalies

Anomalies

- Anomalies are violations of symmetries via quantum corrections (loop-effects).
- They split into: violations of
  - global symmetries: acceptable (for example the $\pi \to \gamma\gamma$).
  - gauge symmetries: unacceptable. Have to be omitted!
- In this talk, we will give to gauge anomalies another chance!

Anomalies

- Consider a single chiral fermion ψ_L, charged under a gauge field A^μ, and the action

$$\mathcal{L} = -\frac{1}{4}F_A^2 + \bar{\psi}_L\gamma_\mu(i\partial^\mu - g_A q A^\mu)\psi_L$$

invariant under

$$\begin{aligned} A^\mu &\to A^\mu + \partial^\mu \epsilon \\ \psi_L &\to e^{i g_A q \epsilon}\psi_L \end{aligned} \quad \longrightarrow \quad \delta\mathcal{L} = 0$$

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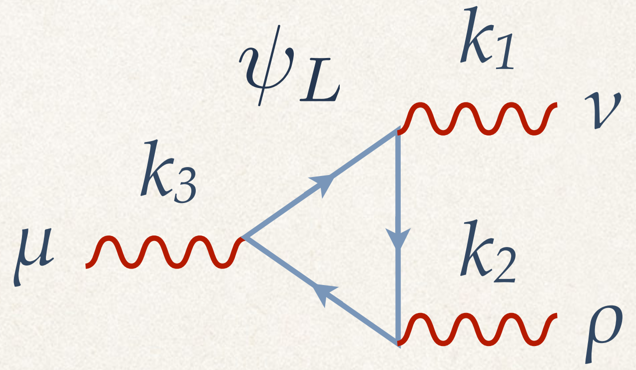
- However, the Ward ID is violated at 1-loop (the gauge symmetry is broken)

"the anomaly"

$k_3^\mu \quad \times$ 

$\neq 0$

$$\delta\mathcal{L}_{\text{1-loop}} \sim \epsilon\; qqq \neq 0$$

- This theory is for the trash bin. It has an "anomaly".

Anomalies

- Consider a ~~single~~ many chiral fermions ψ_L^f, charged under a gauge field A^μ, and the action

$$\mathcal{L} = -\frac{1}{4}F_A^2 + \bar{\psi}_L^f \gamma_\mu (i\partial^\mu - g_A q^f A^\mu)\psi_L^f$$

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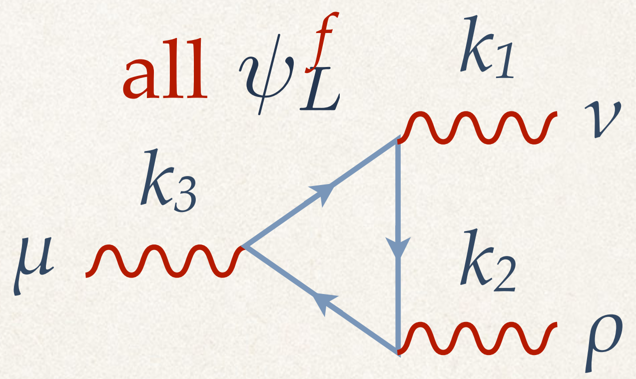

$\cancel{\neq 0} \quad = 0$

$$\delta\mathcal{L}_{\text{1-loop}} \sim \epsilon\, \cancel{qqq} \neq 0$$

$$\epsilon \sum_f q^f q^f q^f \to 0$$

- This theory is ~~for the trash bin~~ fine. It has ~~an~~ no "anomaly".

Anomalies

- Consider a ~~single~~ many chiral fermions ψ_L^f, charged under a gauge field A^μ, and the action

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invariant under

that happens in the Standard Model

$$\begin{aligned} A^\mu &\to A^\mu + \partial^\mu \epsilon \\ \psi_L^f &\to e^{i g_A q^f \epsilon}\psi_L^f \end{aligned} \quad \longrightarrow \quad \delta\mathcal{L} = 0$$

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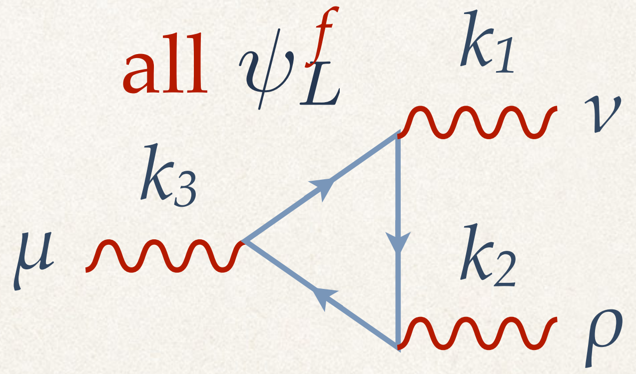
all ψ_L^f

$k_3^\mu \quad \times$ 

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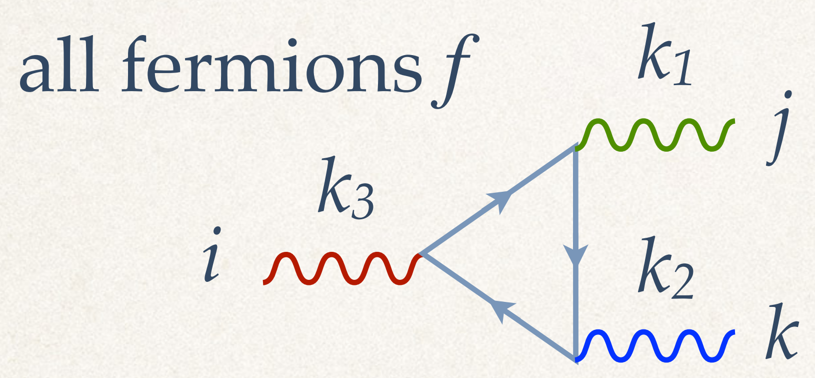
Anomalies

Therefore,

- in models with chiral fermions charged under gauge symmetries the 1-loop diagram is present.



$$\overbrace{\sum_f q_i^f\, q_j^f\, q_k^f}^{t_{ijk}} \neq 0 \quad \longrightarrow \quad \text{anomalous} \quad \longrightarrow \quad \text{inconsistent}$$

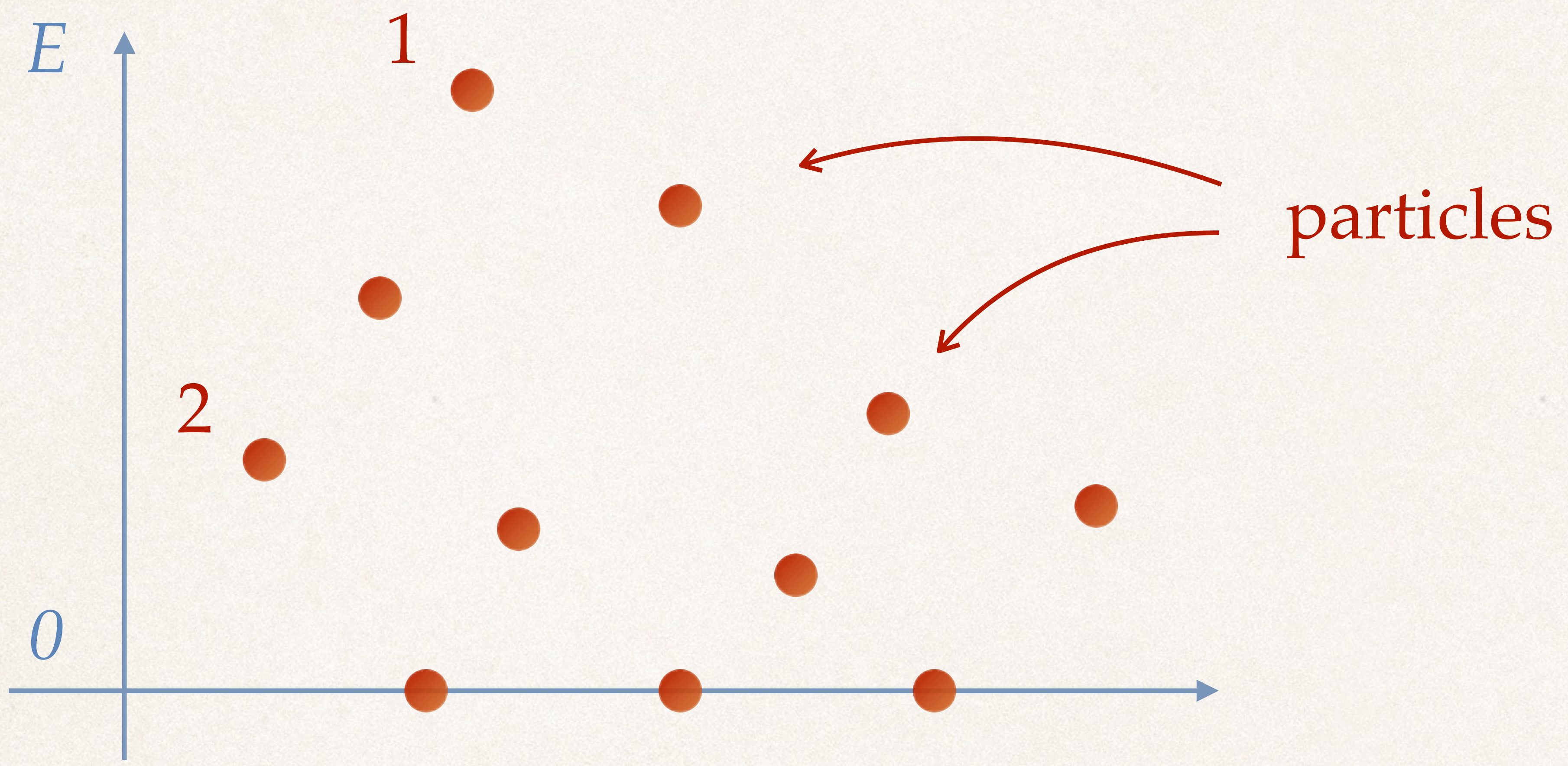
$$\sum_f q_i^f\, q_j^f\, q_k^f = 0 \quad \longrightarrow \quad \text{anomaly free} \quad \longrightarrow \quad \text{fine}$$

- This is the case for fundamental theories: gauge anomalies should cancel.

- What about effective theories?

Anomalies and effective theories

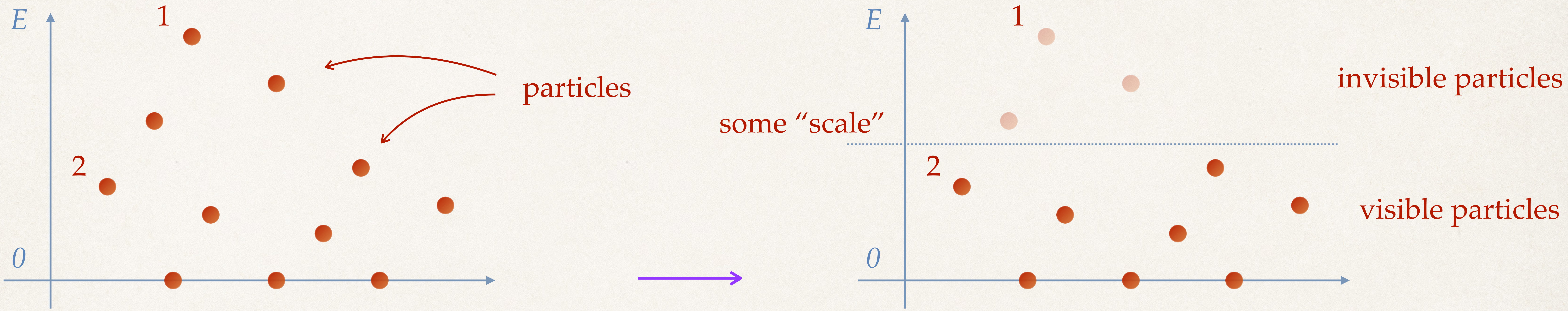
Fundamental vs Effective theories



$$\mathcal{L}_{\text{fundamental}} = \mathcal{L}_1 + \mathcal{L}_2 + \ldots + \mathcal{L}_{\text{1-2}}^{\text{interactions}} + \ldots.$$

all particles

Fundamental vs Effective theories



$$\mathcal{L}_{\text{fundamental}} = \mathcal{L}_1 + \mathcal{L}_2 + \ldots + \mathcal{L}_{\text{1-2}}^{\text{interactions}} + \ldots.$$

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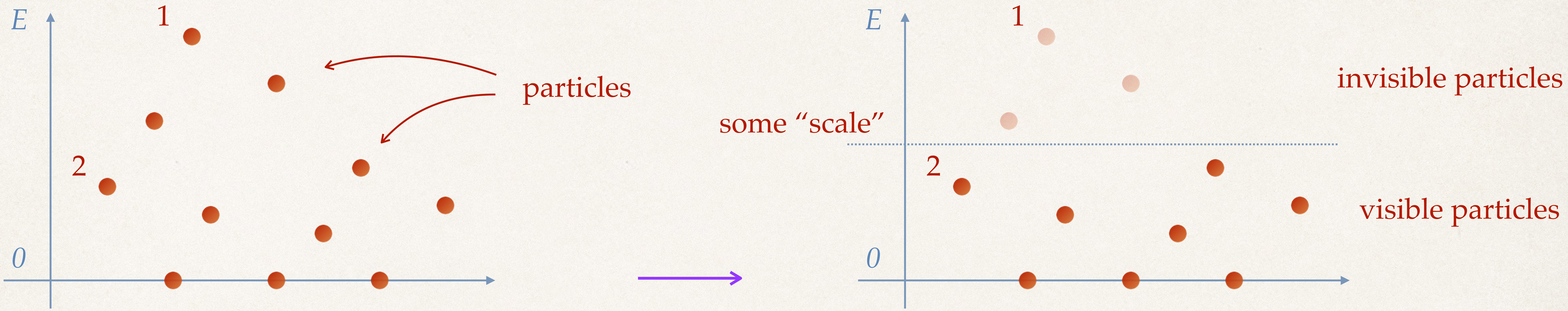
$$\mathcal{L}_{\text{effective}} = \cancel{\mathcal{L}_1} + \mathcal{L}_2 + \ldots + \cancel{\mathcal{L}_{\text{1-2}}^{\text{interactions}}} + \ldots.$$
$$+\mathcal{L}_{\text{due to 1}}^{\text{effective}} + \ldots.$$

only *visible* particles

terms from diagrams with *visible* external and virtual *invisible* particles

Fundamental vs Effective theories



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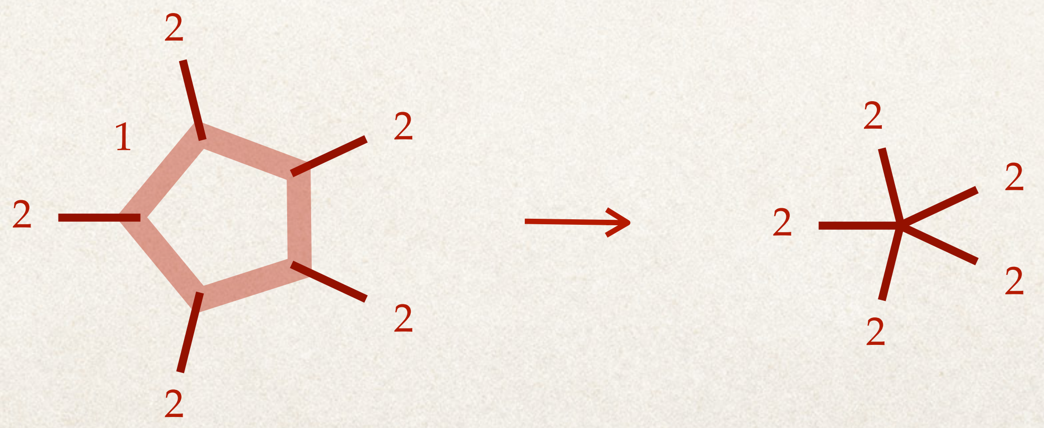
An example:



$\sim \phi_2\phi_2\phi_2\phi_2\phi_2$

to be added

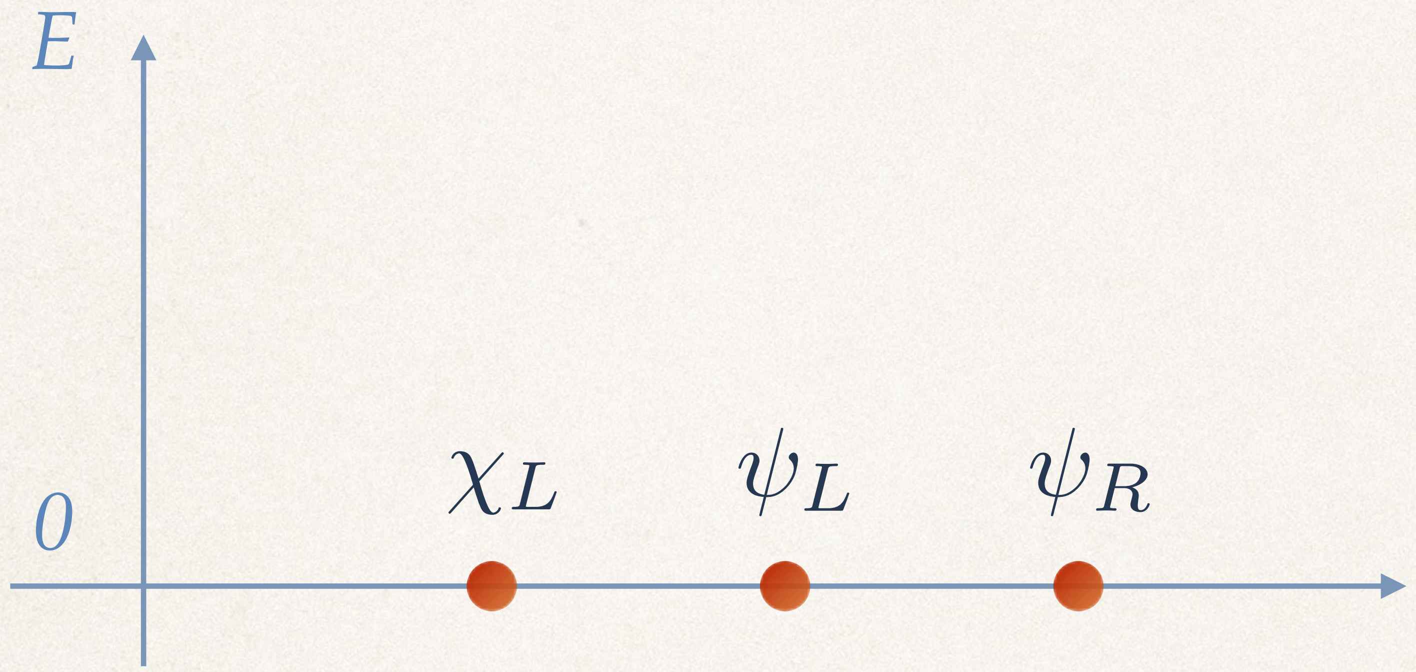
Fundamental vs Effective theories

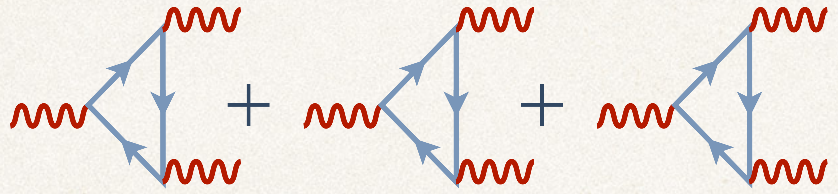
Next,

- we discuss a case of an effective theory after some Higgs mechanism of a fundamental theory.
- We will focus also on the anomalies.

Anomalies in effective theories

- Consider three chiral fermions (χ_L, ψ_L, ψ_R), one Higgs Φ, and one gauge field A^μ





$= 0$ anomaly free

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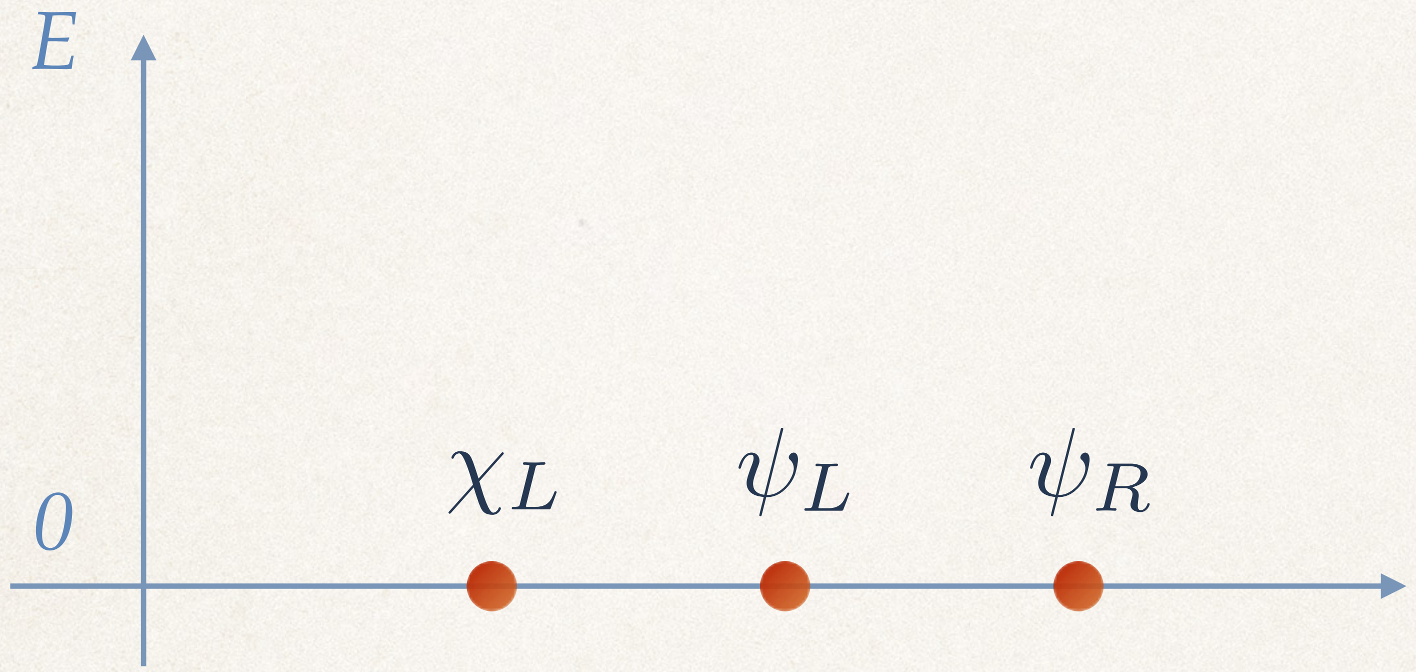

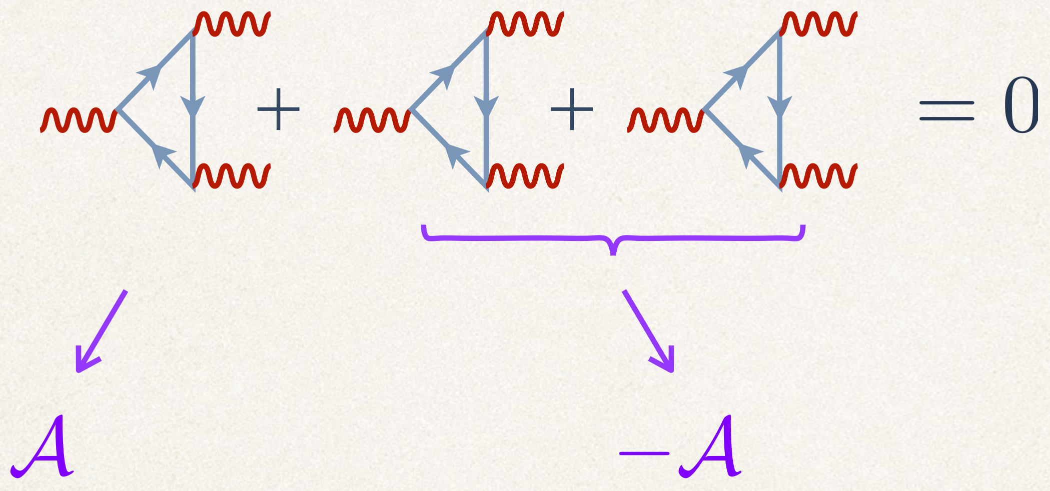

anomaly free

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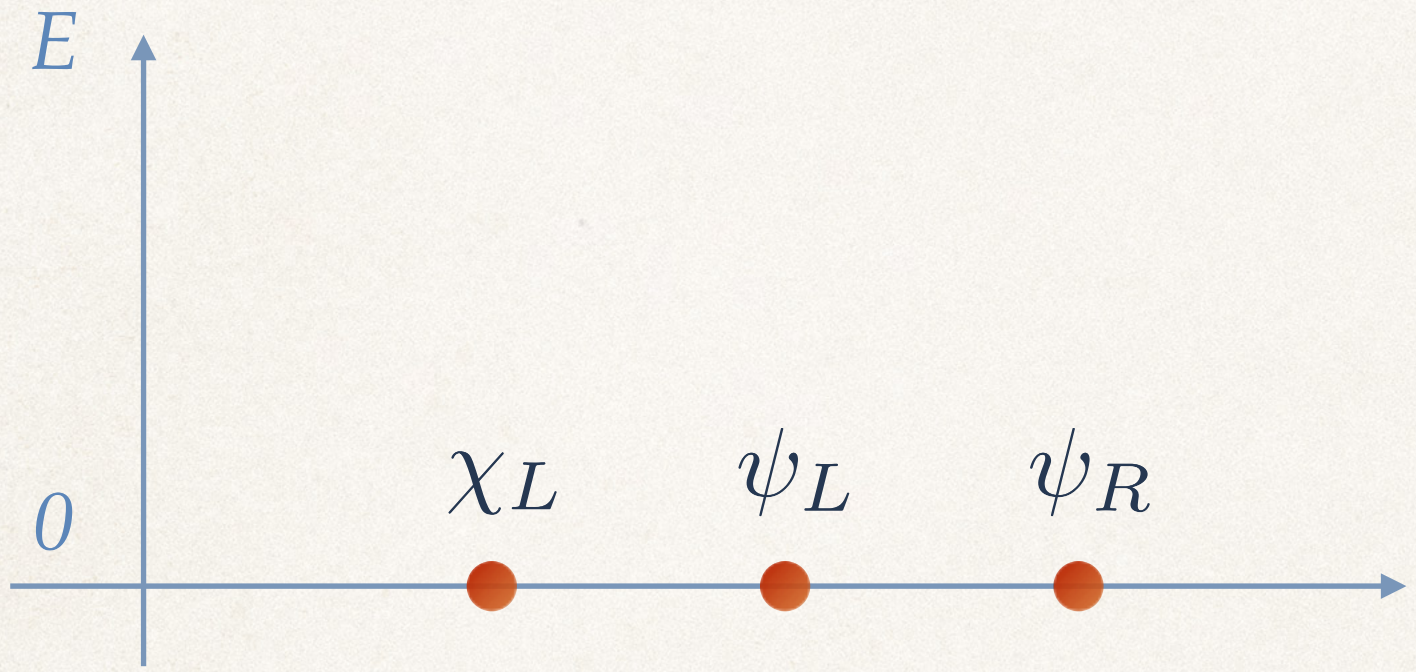
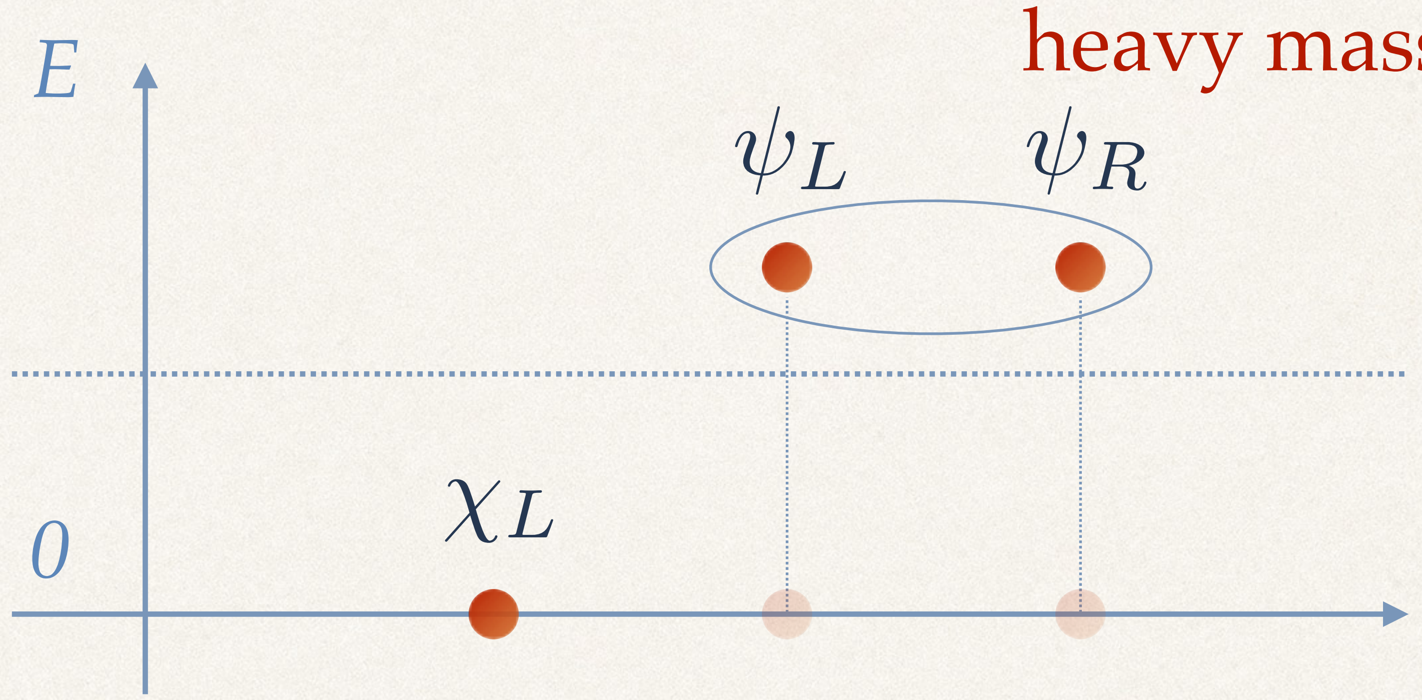
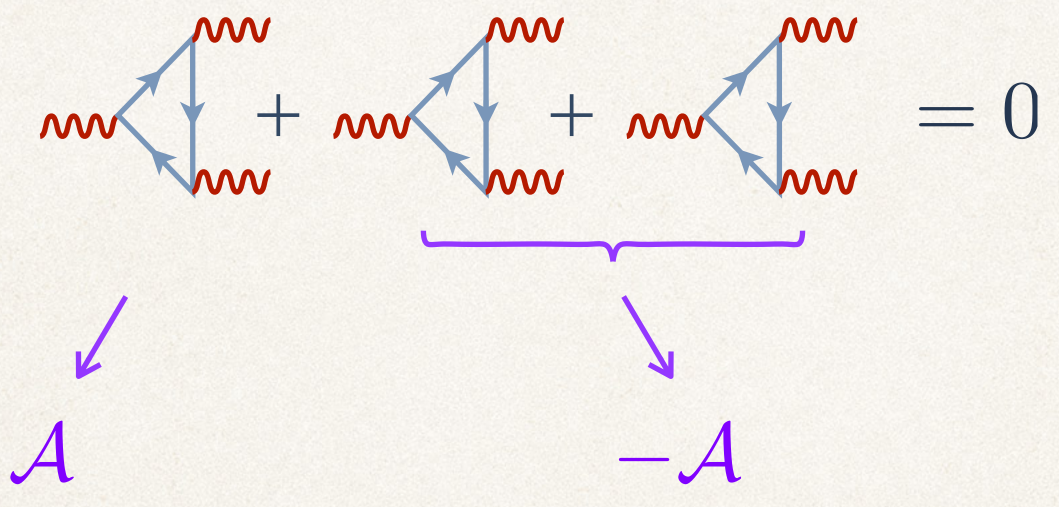


after some Higgs mechanism



heavy massive fermion (invisible)



anomaly free

Anomalies in effective theories

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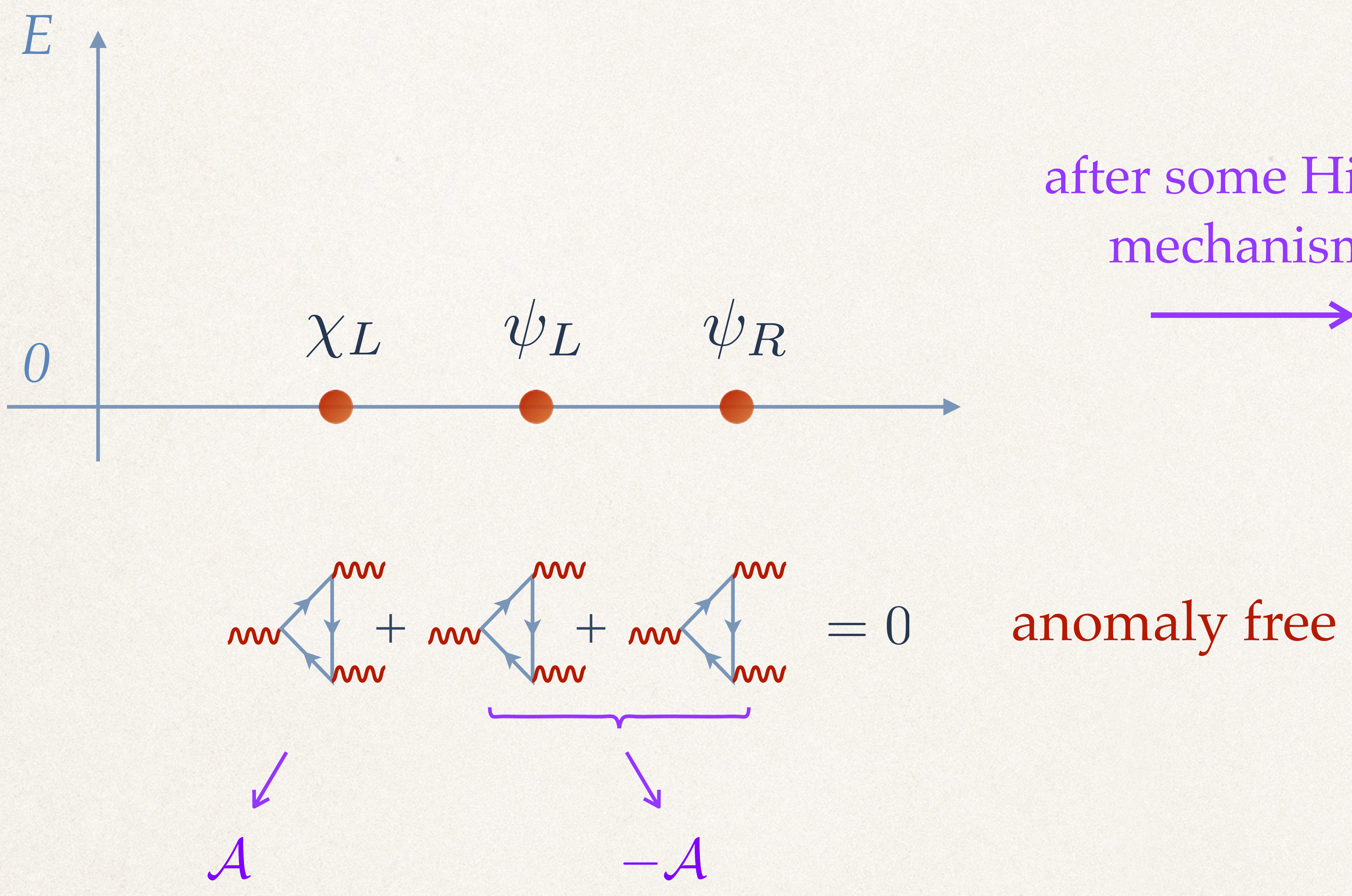

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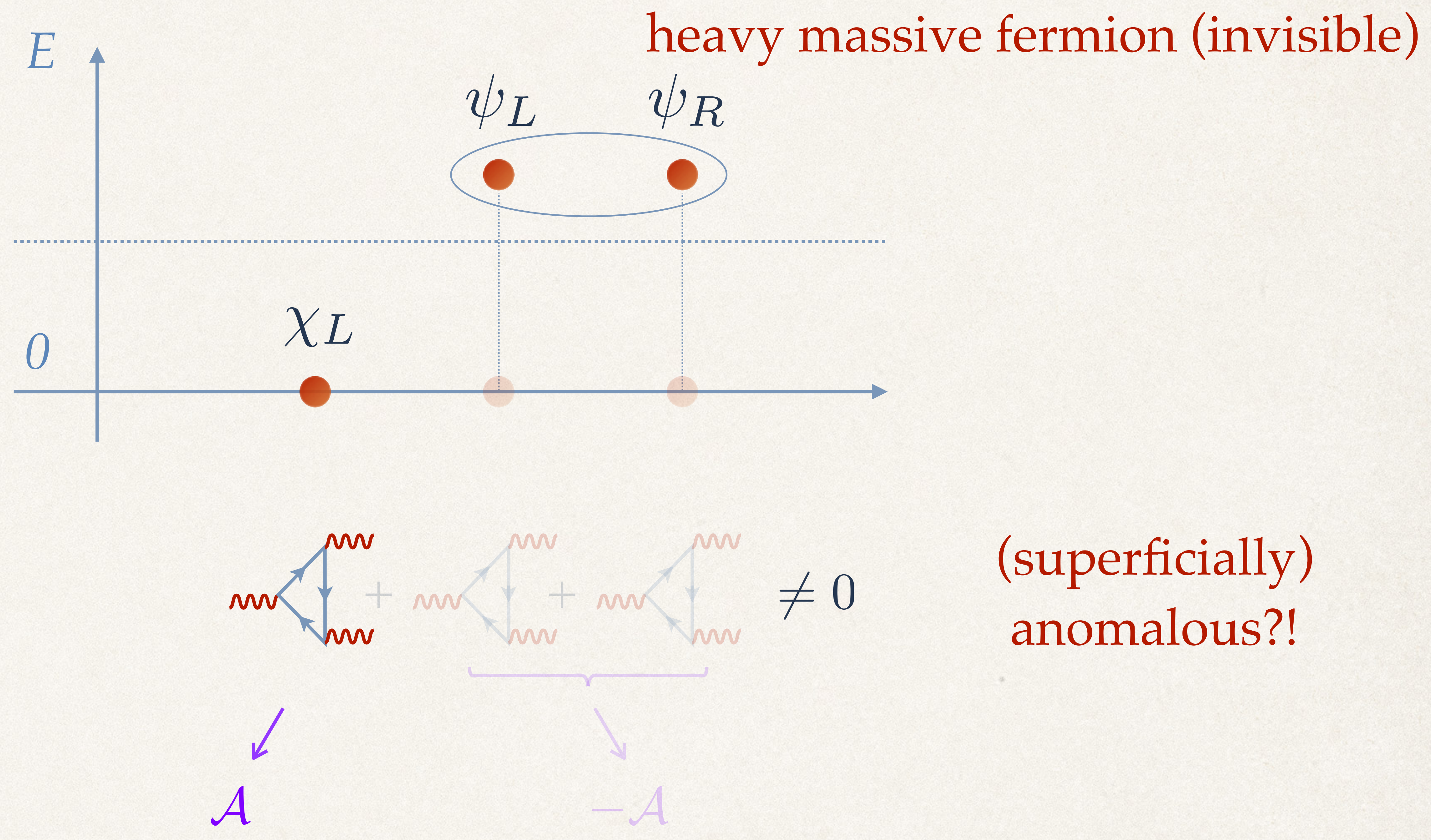

Anomalies in effective theories

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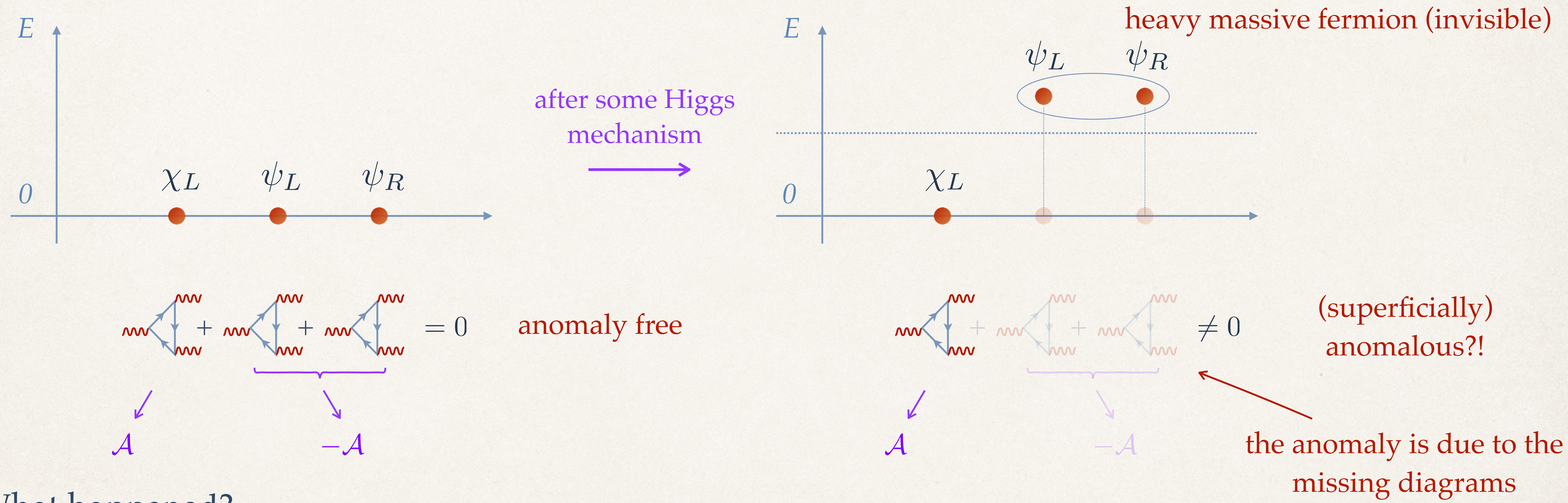

- What happened?

- A consistent (anomaly free) theory became inconsistent (anomalous) after a Higgs mechanism?!

Anomalies in effective theories

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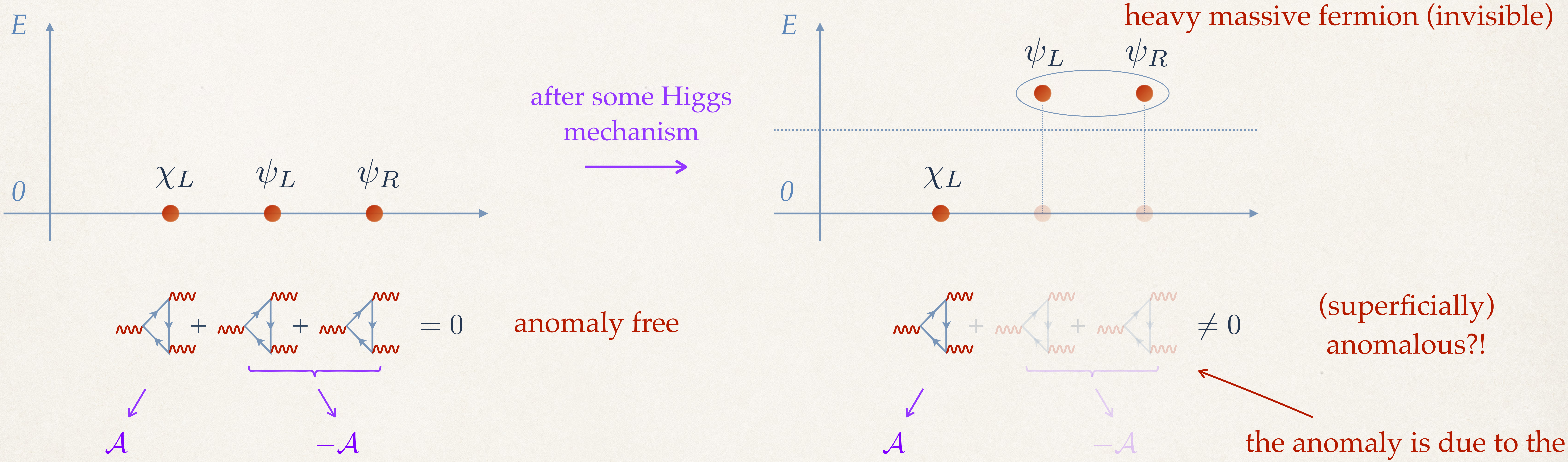


- What happened?
- A consistent (anomaly free) theory became inconsistent (anomalous) after a Higgs mechanism?!

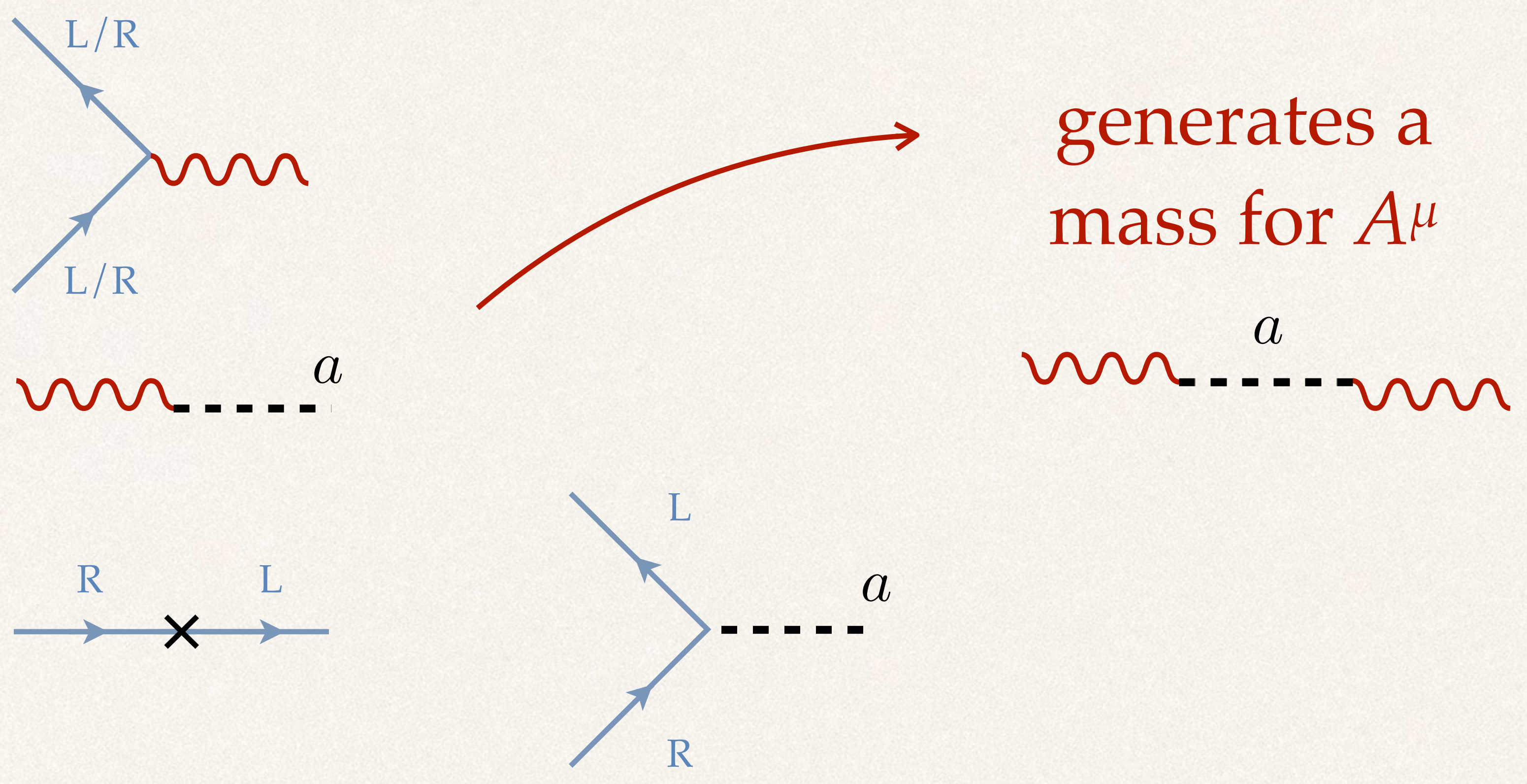
Anomalies in effective theories

- After the Higgs mechanism (in a higher scale), the Higgs field $\Phi = (v+r)e^{ia/v}$ gives

$$\bar{\psi}_{L/R}\gamma^{\mu}D_{\mu}\psi_{L/R} \longrightarrow$$

$$D_{\mu}\Phi D^{\mu}\Phi^{*} \longrightarrow$$

$$\Phi\bar{\psi}_{L}\psi_{R} + h.c. \longrightarrow$$



standard Higgs mechanism

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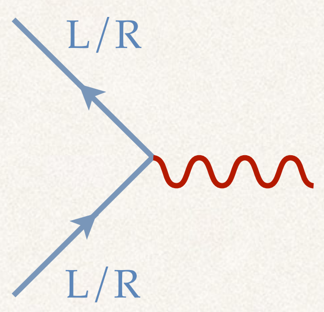
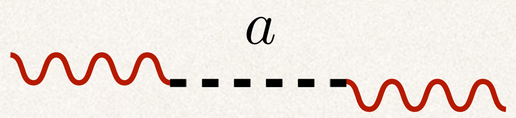
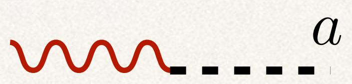
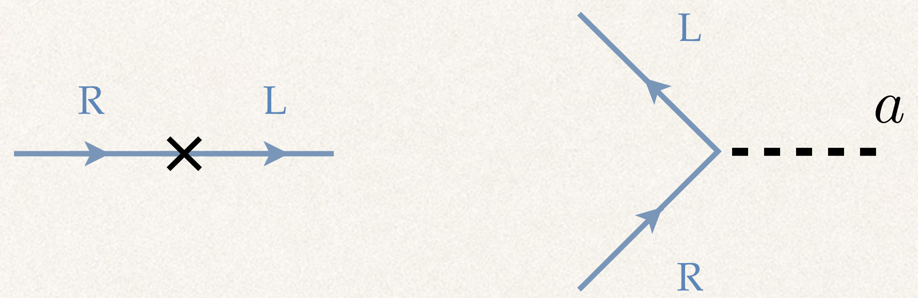
$\bar{\psi}_{L/R}\gamma^{\mu}D_{\mu}\psi_{L/R} \longrightarrow$ 

generates a mass for A^{μ}



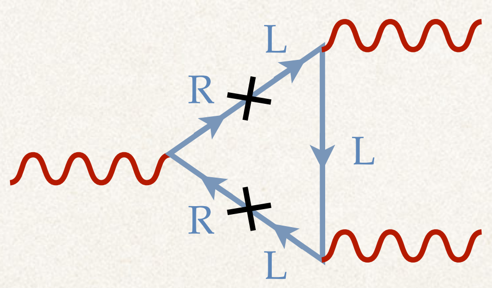
$D_{\mu}\Phi D^{\mu}\Phi^{*} \longrightarrow$ 

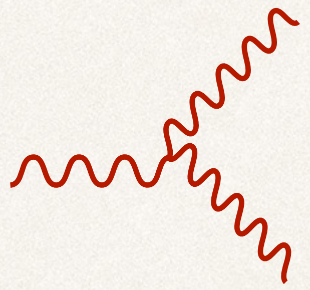
$\Phi\bar{\psi}_{L}\psi_{R} + h.c. \longrightarrow$ 

standard Higgs mechanism

- Out of which we have diagrams that generate effective terms

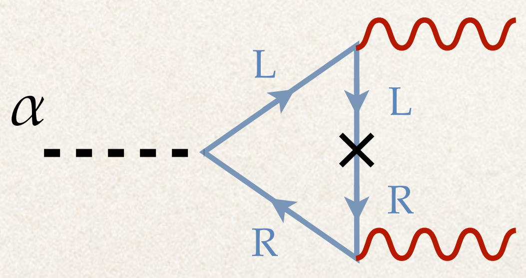
zero for a single A, but present for more

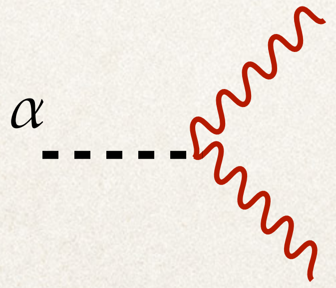


$\longrightarrow$  $\sim A \wedge A \wedge F_A$

generalised Chern-Simons (GCS)

heavy fermion



$\longrightarrow$ 

$\sim a\ F_A \wedge F_A$

"axionic" terms

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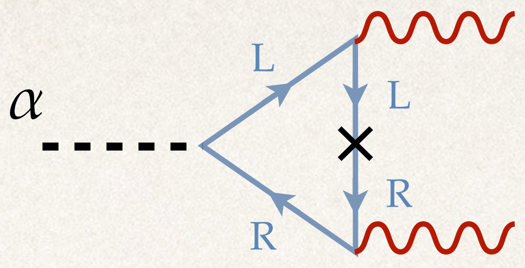
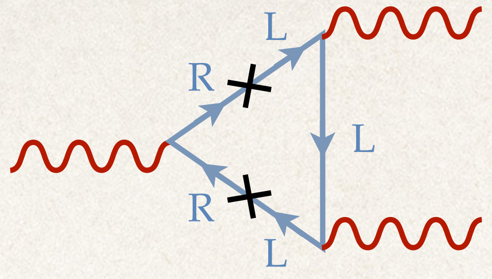
- The effective action (after integrating out heavy states)





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zero in this case

massless fermion

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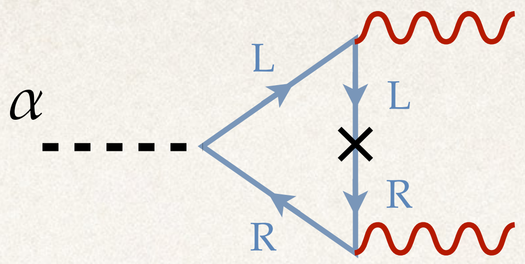

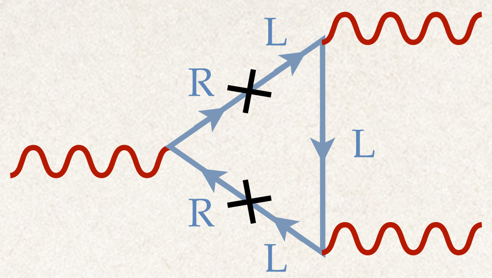

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- Field transformations

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axion

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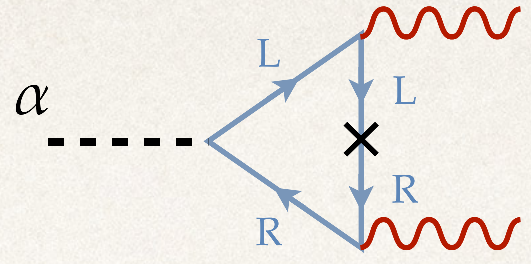

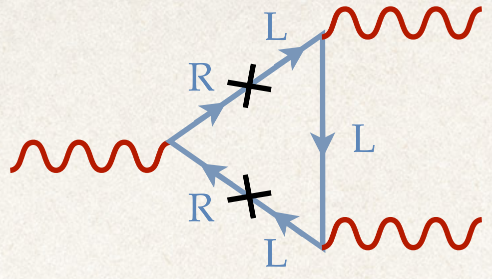

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$$\longrightarrow \qquad \delta\mathcal{L}_{effective} = -\frac{cM}{24\pi^2}\epsilon F_A \wedge F_A$$

- Is this action gauge invariant? **No!**

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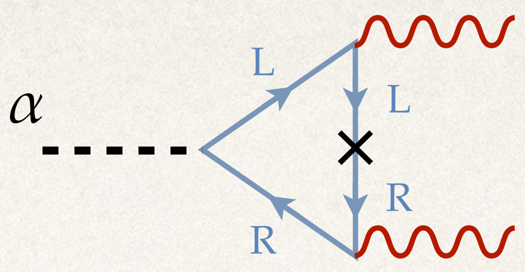
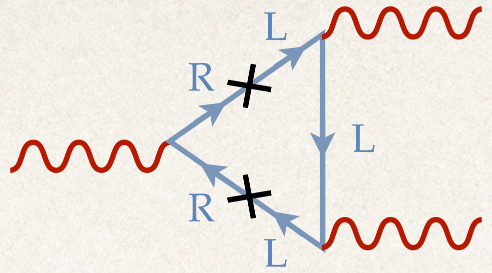
- The effective action (after integrating out heavy states)





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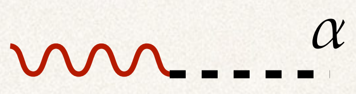

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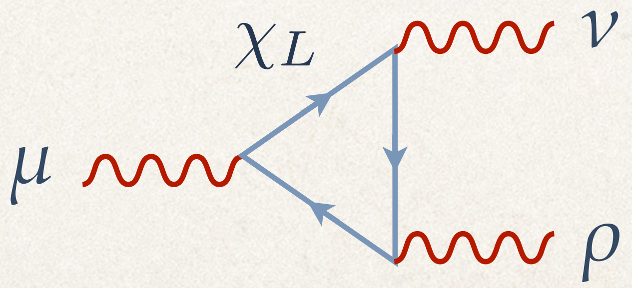
axion

$$\longrightarrow \qquad \delta\mathcal{L}_{effective} = -\frac{cM}{24\pi^2}\epsilon F_A\wedge F_A$$

- Is this action gauge invariant? **No!**

- Remember the anomaly, from the diagram:



$$\longrightarrow \qquad \delta\mathcal{L}_{1-loop} = \frac{qqq}{24\pi^2}\epsilon F_A\wedge F_A$$

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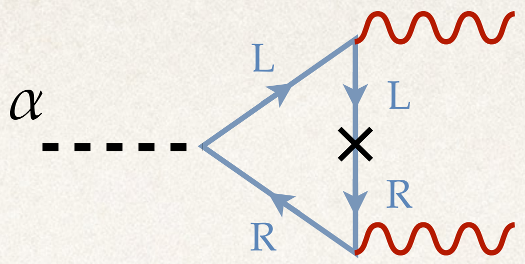
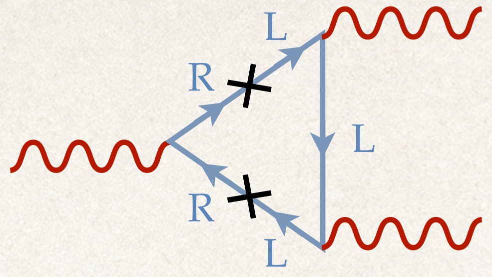
- The effective action (after integrating out heavy states)





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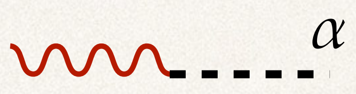

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$$A^\mu \to A^\mu + \partial^\mu\epsilon$$

$$\chi_L \to e^{ig_A q\epsilon}\chi_L$$

$$\Phi \to e^{-ig_A q\epsilon}\Phi \Longrightarrow a \to a - \overbrace{g_A q v}^{M}\epsilon$$

$$\Phi = (v+r)e^{ia/v}$$

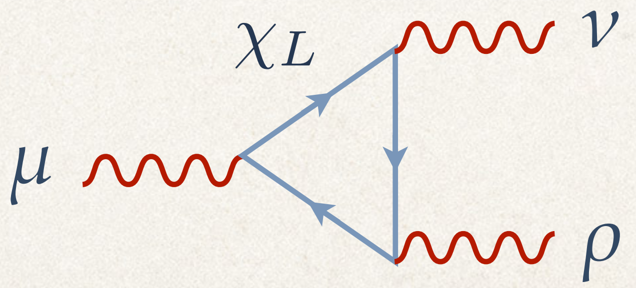
axion

$$\delta\mathcal{L}_{effective} = \cancel{-\frac{cM}{24\pi^2}\epsilon F_A\wedge F_A}$$

fixing the coefficient $c = q^3/M$

- Is this action gauge invariant? **No!**

- Remember the anomaly, from the diagram:



$$\delta\mathcal{L}_{1-loop} = \cancel{\frac{qqq}{24\pi^2}\epsilon F_A\wedge F_A}$$

- Anomaly cancellation fixes all coefficients.

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Generalization

- In a generic theory, we can have several gauge fields A_i^μ, and several axions a^I.
- The generic action is

$$\mathcal{L}_{effective} = -\frac{1}{4}F_i^2 + \frac{1}{2}(\partial a^I + M_i^I A_i)^2 + \frac{C_{ij}^I}{24\pi^2} a^I F_i \wedge F_j + \frac{E_{ij,k}}{24\pi^2} A_i \wedge A_j \wedge F_k$$

- Anomaly conditions

$$t_{ijk} + E_{ij,k} + E_{ik,j} + M_i^I C_{jk}^I = 0$$

fix all coefficients, as functions of the anomalies $t_{ijk} = Tr[q_i q_j q_k] = \sum_f q_i^f q_j^f q_k^f$.

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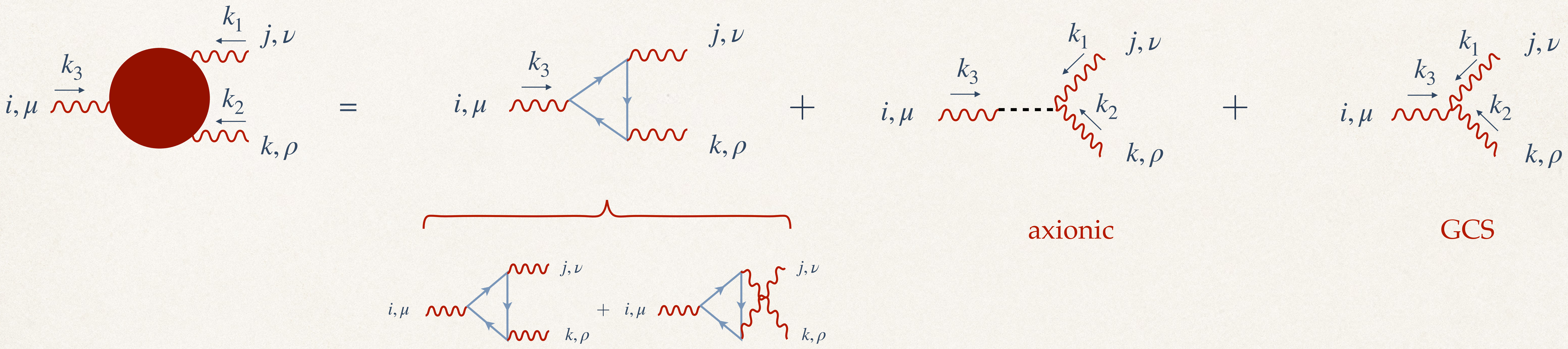
Anomaly cancellation in diagrams

- The 3-point coupling



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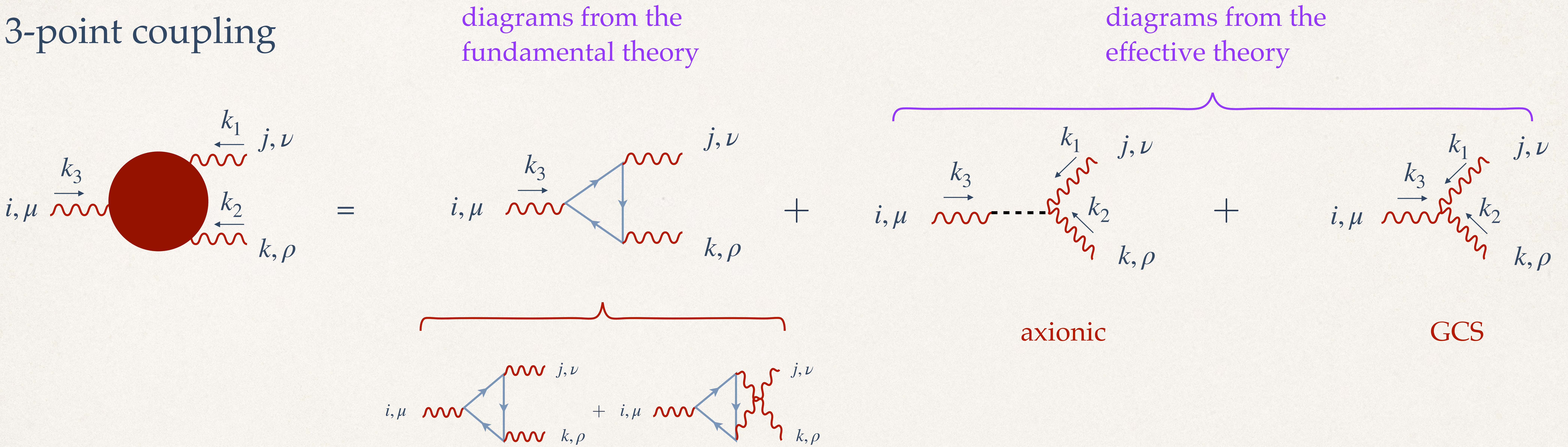
Anomaly cancellation in diagrams

- The 3-point coupling



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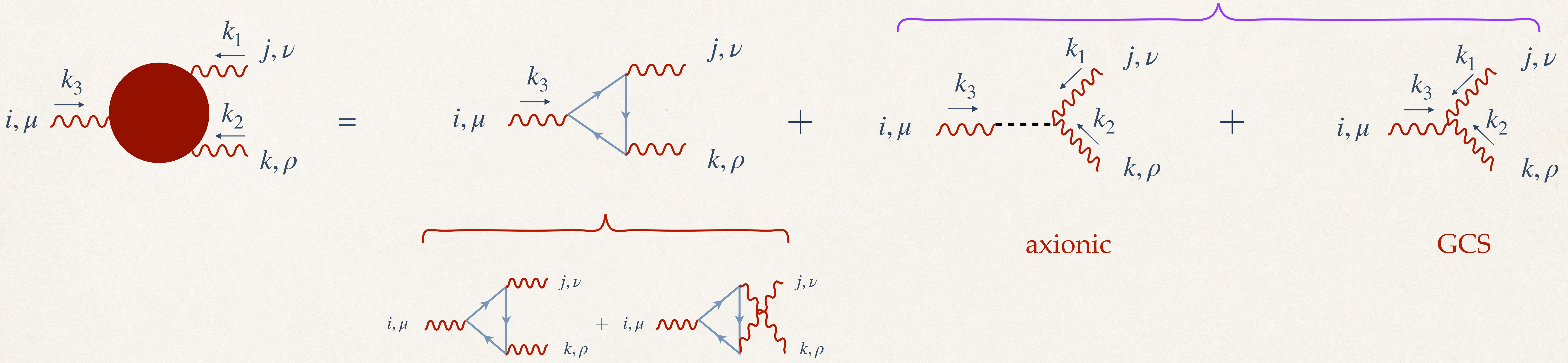
Anomaly cancellation in diagrams

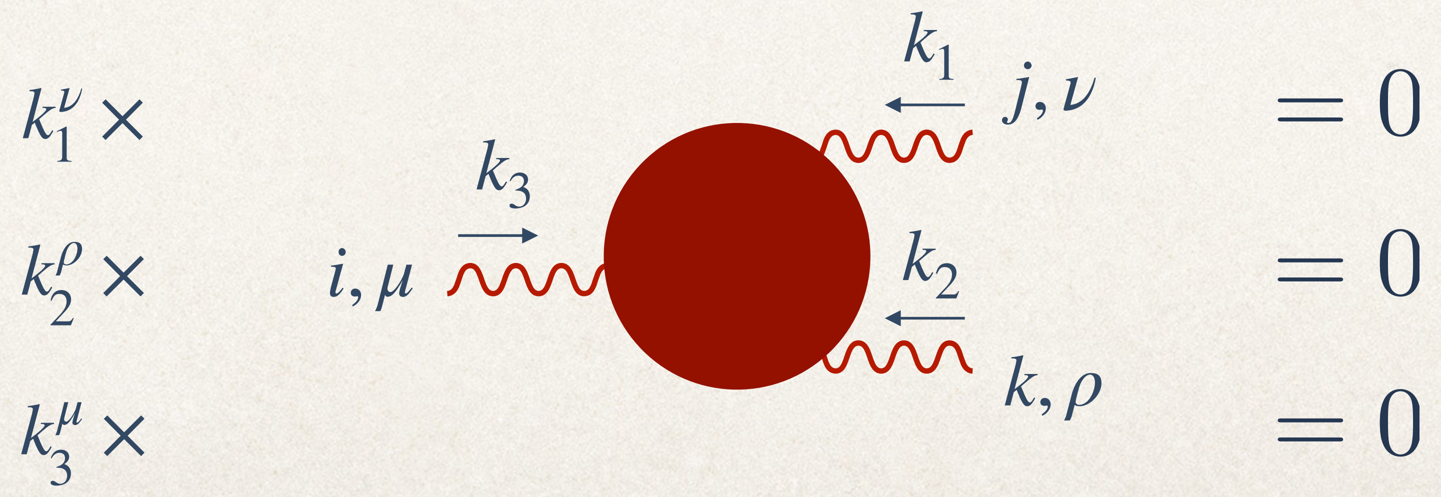
- The 3-point coupling

diagrams from the fundamental theory

diagrams from the effective theory



- The Ward IDs (anomaly cancellation condition)



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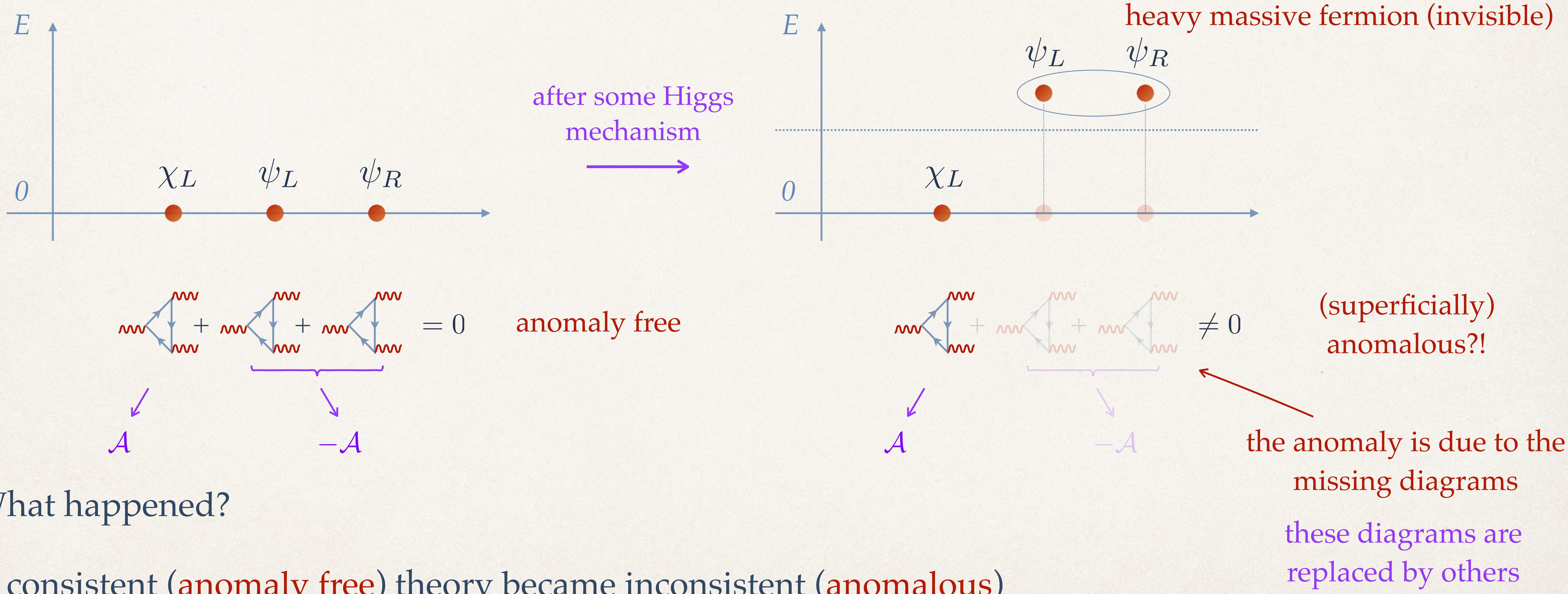
Anomalies in effective theories (back to that question)

- Consider three chiral fermions (χ_L, ψ_L, ψ_R), one Higgs Φ, and one gauge field A^μ

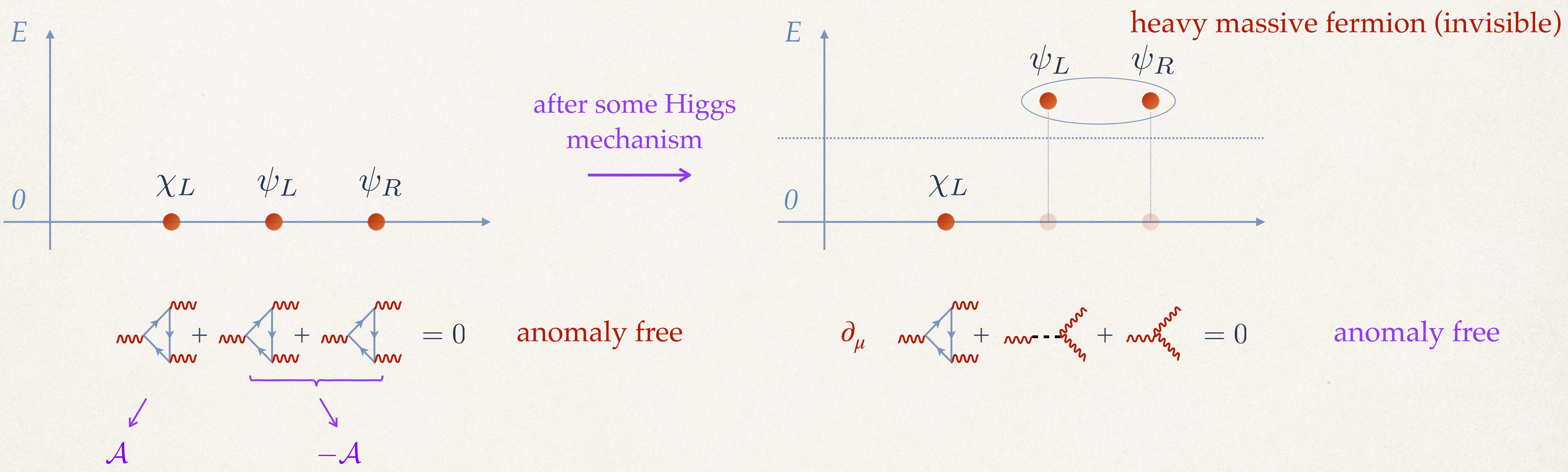


- What happened?

- A consistent (anomaly free) theory became inconsistent (anomalous) after a Higgs mechanism?!

Anomalies in effective theories (back to that question)

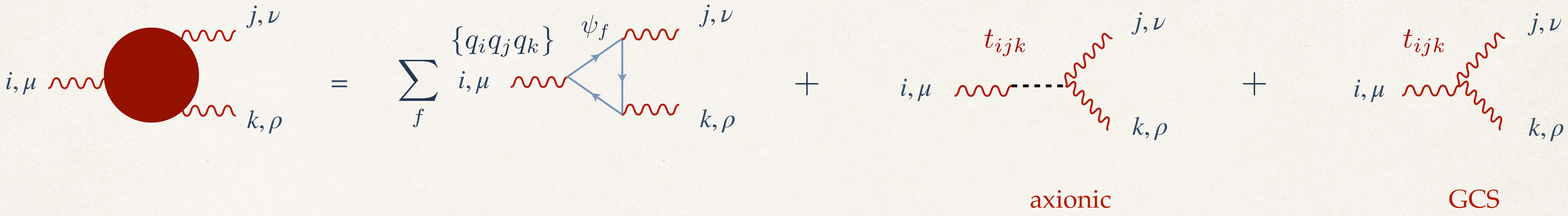
- Consider three chiral fermions (χ_L, ψ_L, ψ_R), one Higgs Φ, and one gauge field A^μ



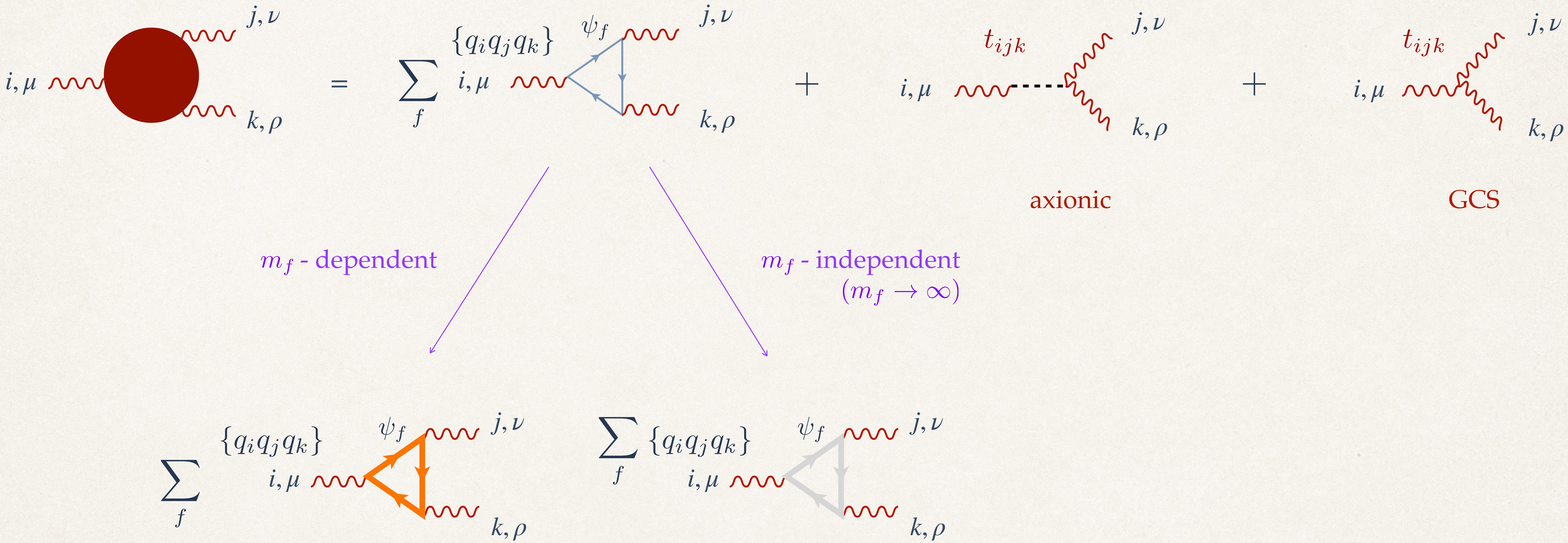
- Axionic and GCS terms "replace" the missing triangle diagrams and cancel the anomalies.
- Notice: the Ward ID is zero (current is conserved), *not* the diagrams.

Analysis of the anomaly related terms

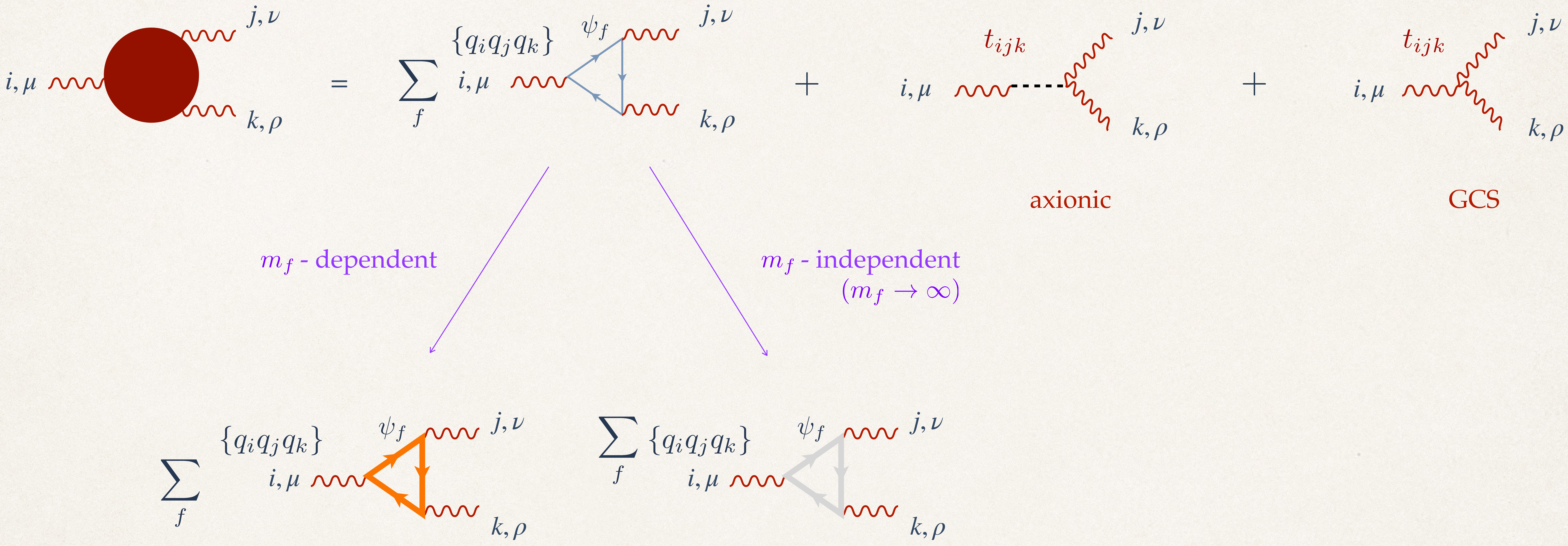
The diagrams again



The diagrams again



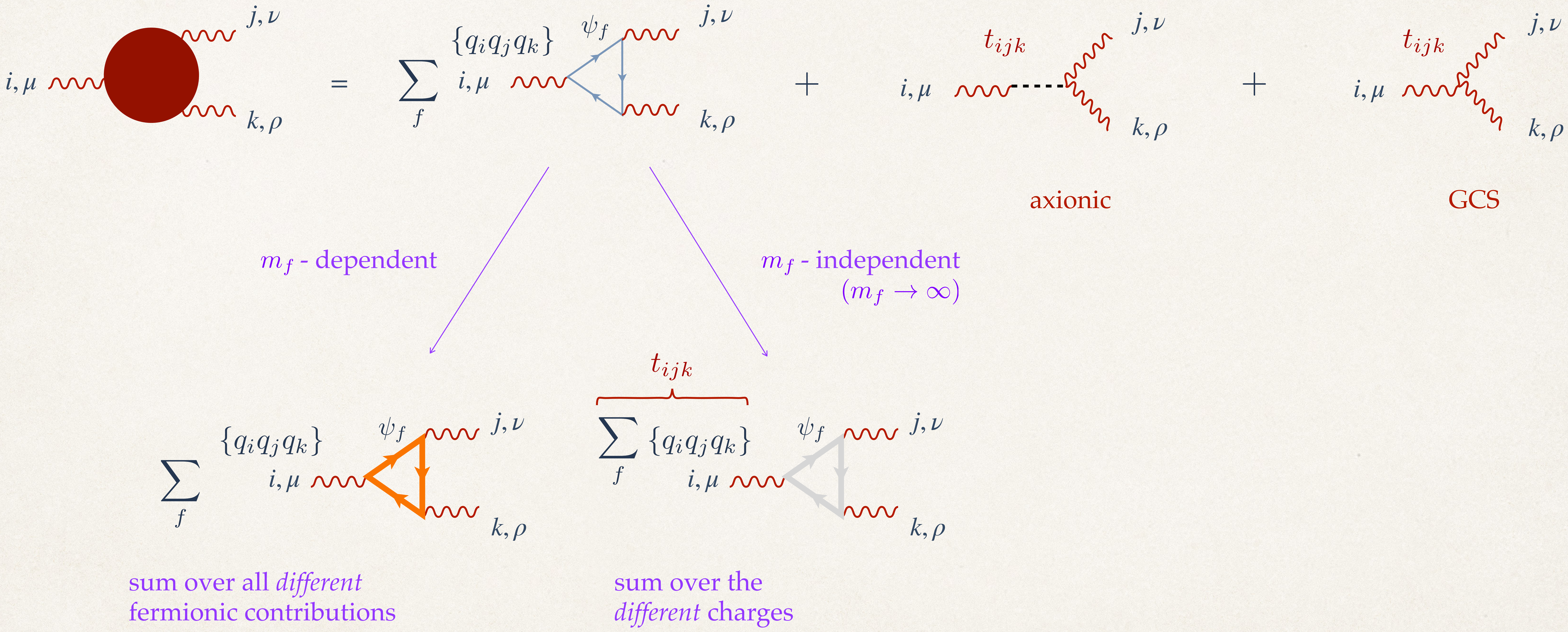
The diagrams again



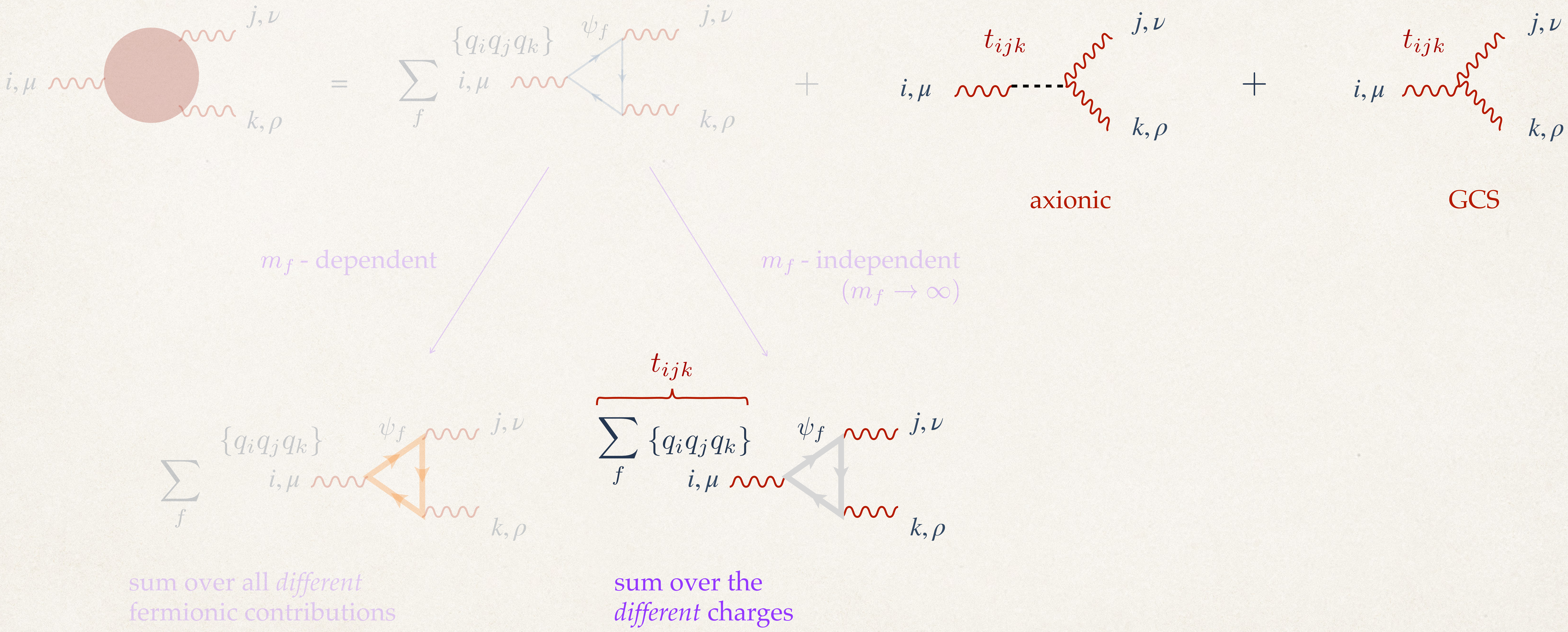
sum over all *different* fermionic contributions

sum over the *different* charges

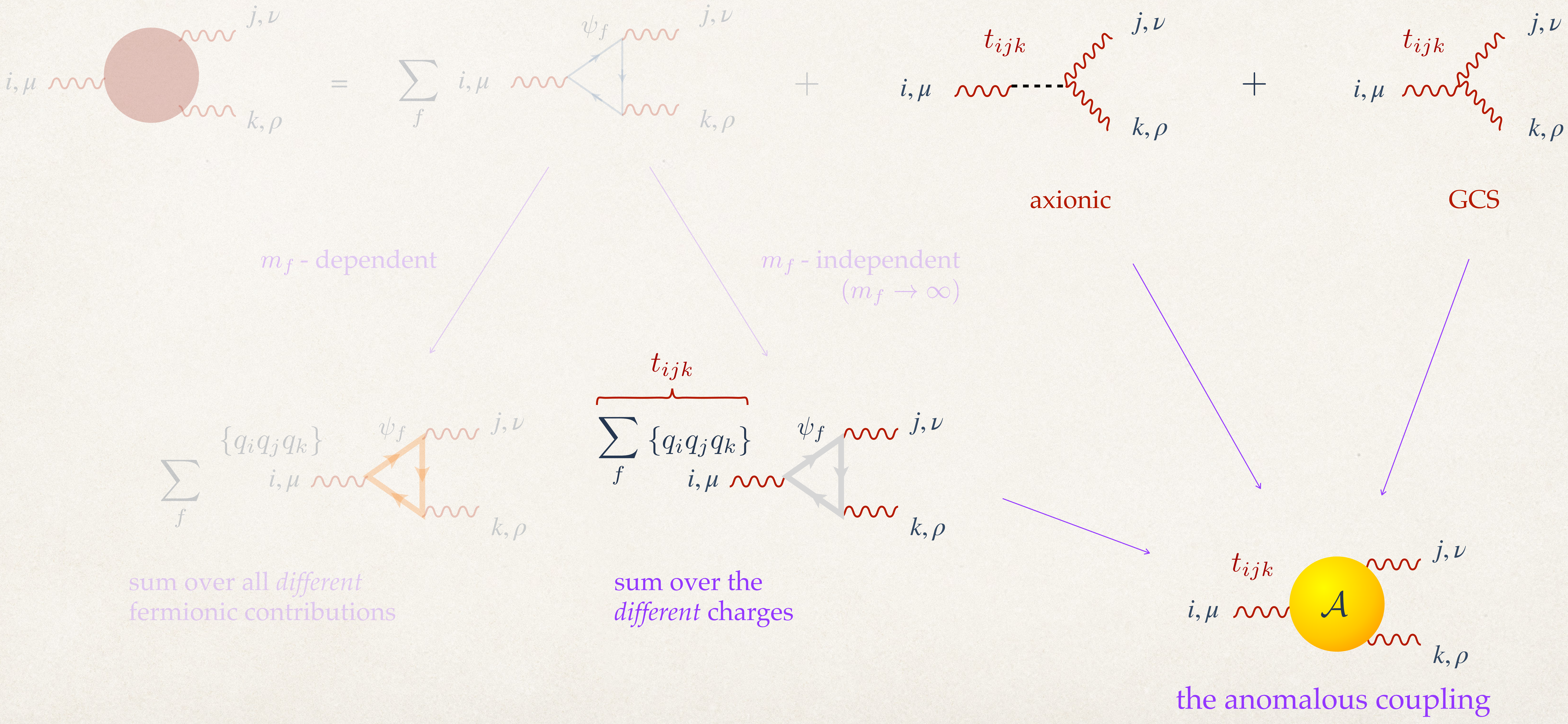
The diagrams again



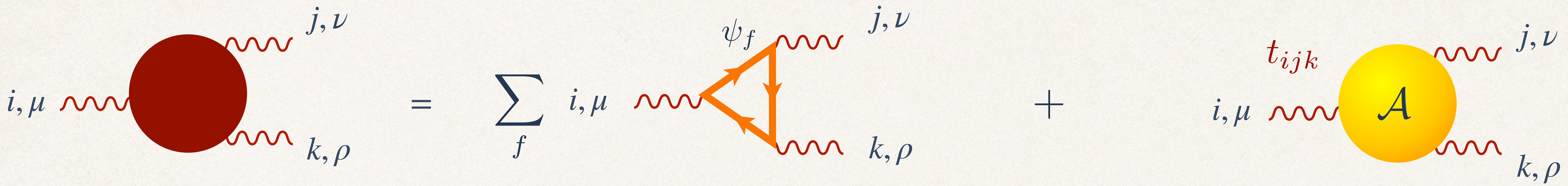
The diagrams again



The diagrams again



The diagrams again



m_f - dependent

the anomalous coupling

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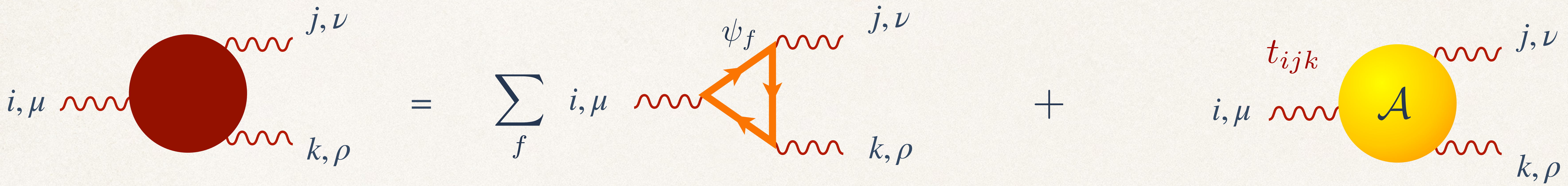
The diagrams again



m_f - dependent

the anomalous coupling

- always present
(anomalous/non-anomalous models)
- drops in anomaly free models
- depends on the mass m_f
The heavier the fermion the smaller the contribution
- does not depend on any mass
- depends on individual charges of the fermion in the loop
- depends on the full anomaly

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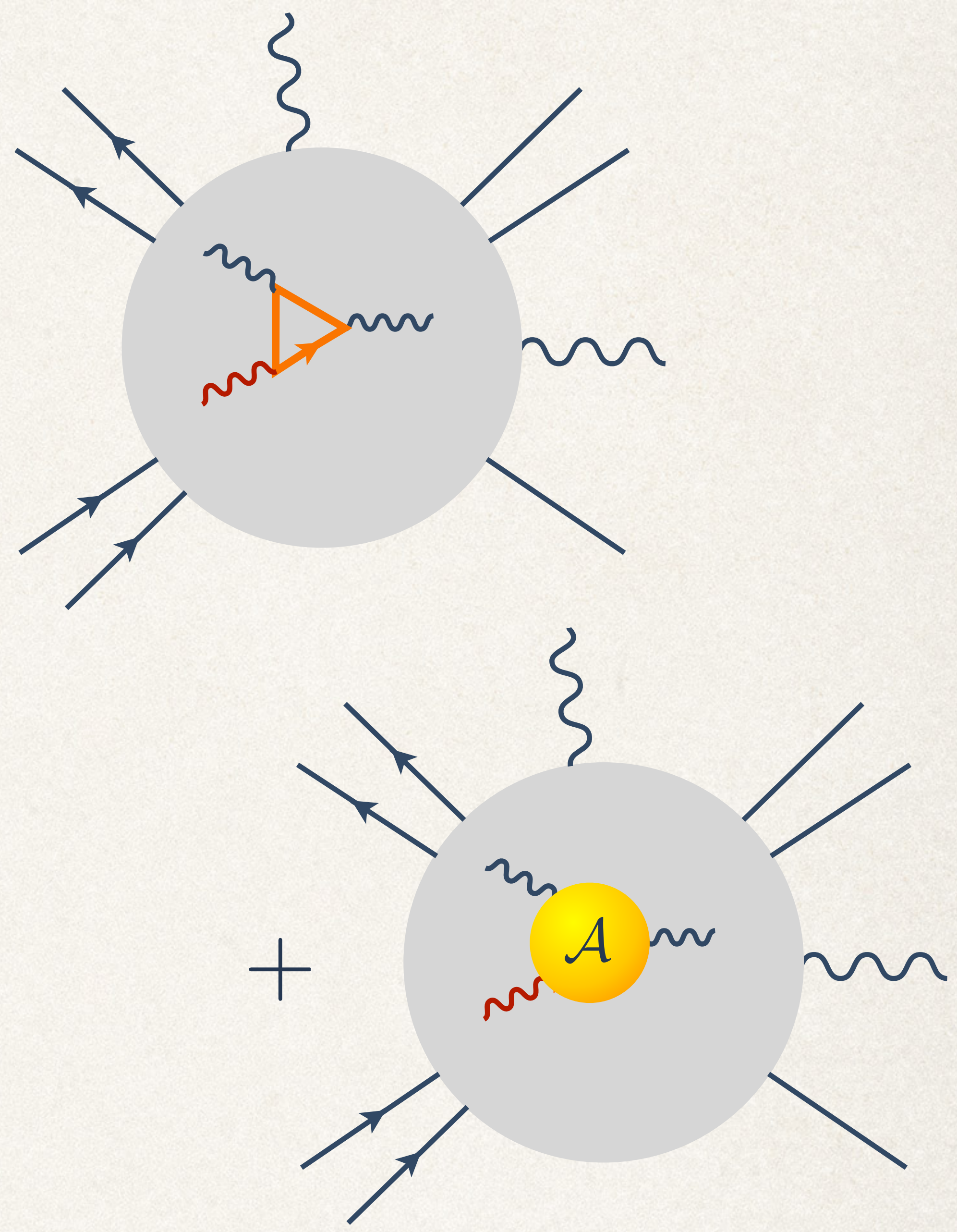
Phenomenology

- I argued that
  - fundamental theories (anomaly free in the UV) can appear to be anomalous in the IR.
  - Effective 3-point couplings (axionic & GCS) cure the “anomalies”.
  - These new couplings depend on the anomaly.
  - They have the same “structure” like the standard fermionic triangle diagram.
  - New diagrams appear where the triangle sub-diagram is replaced by the anomaly.



- Natural questions
  - What if SM is an anomalous theory after very massive states (via some mechanism) are integrated out?
  - What are the phenomenological consequences?

Standard Model and effective theory

- Consider
  - SM + some other fermions charged under $SU(3) \times SU(2) \times U(1)_Y \times U(1)_A$.
  - an *extra* Higgs field Φ, coupled only to the extra fermions and charged only under $U(1)_A$.

Standard Model and effective theory

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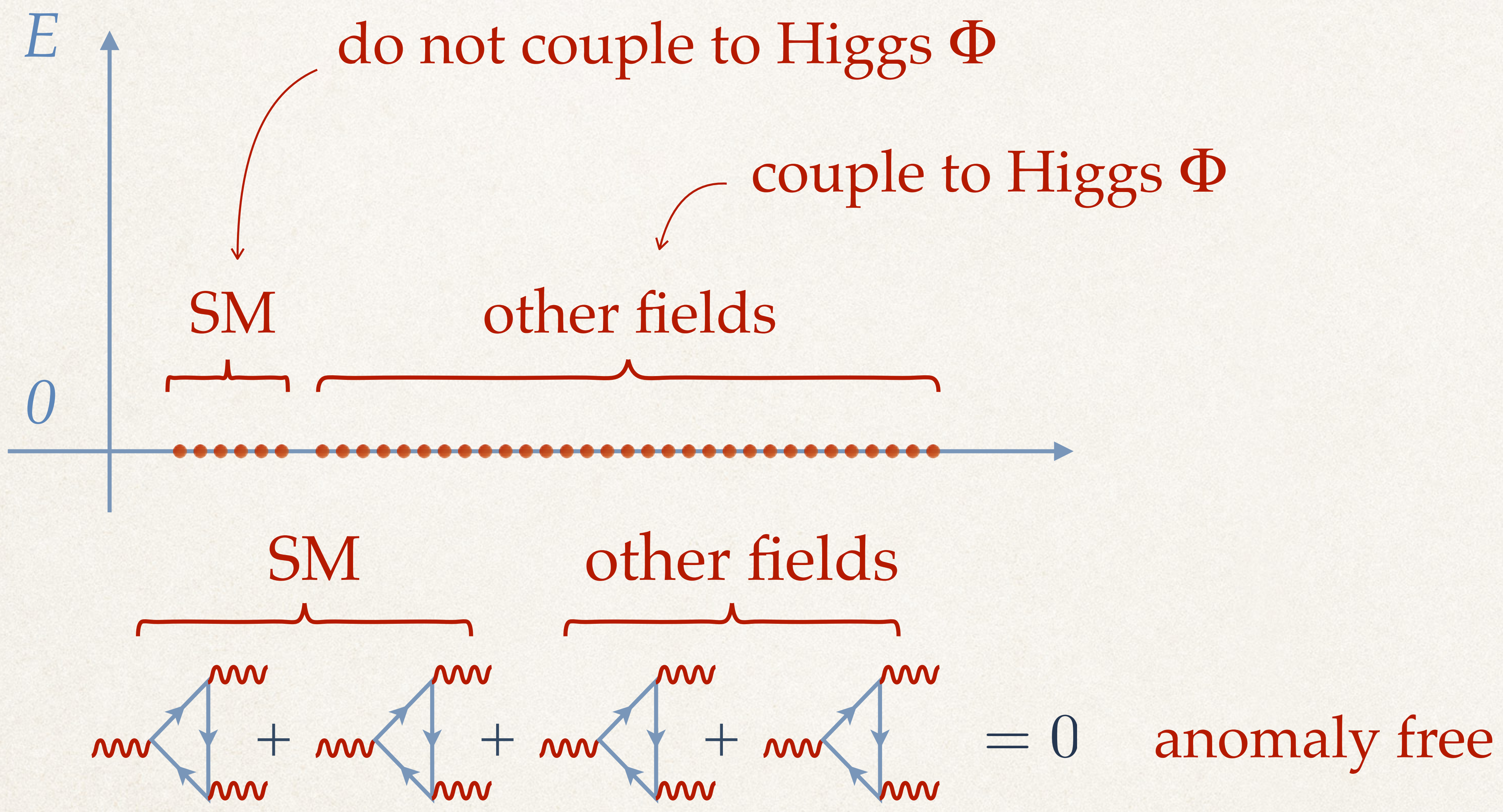

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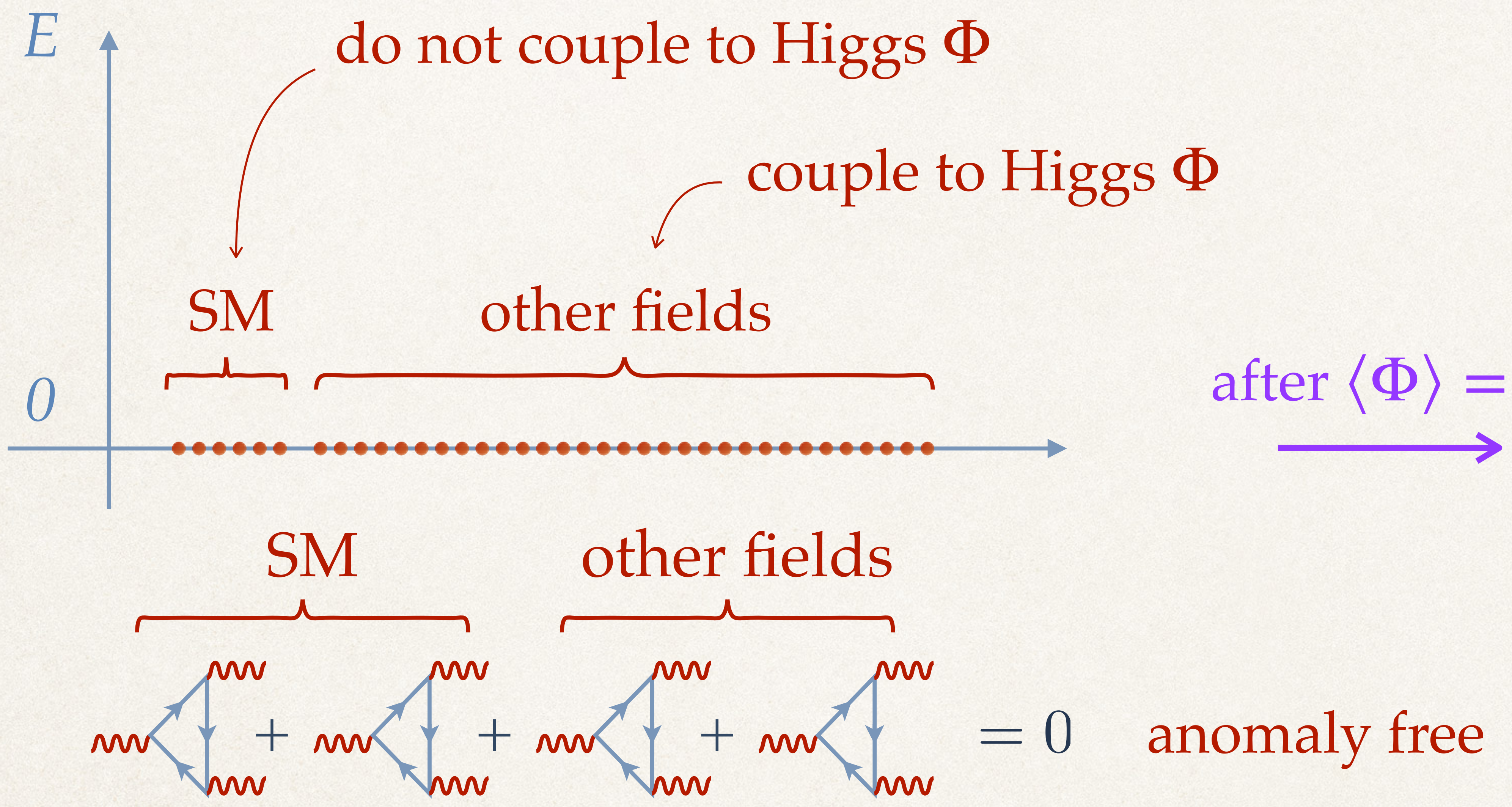
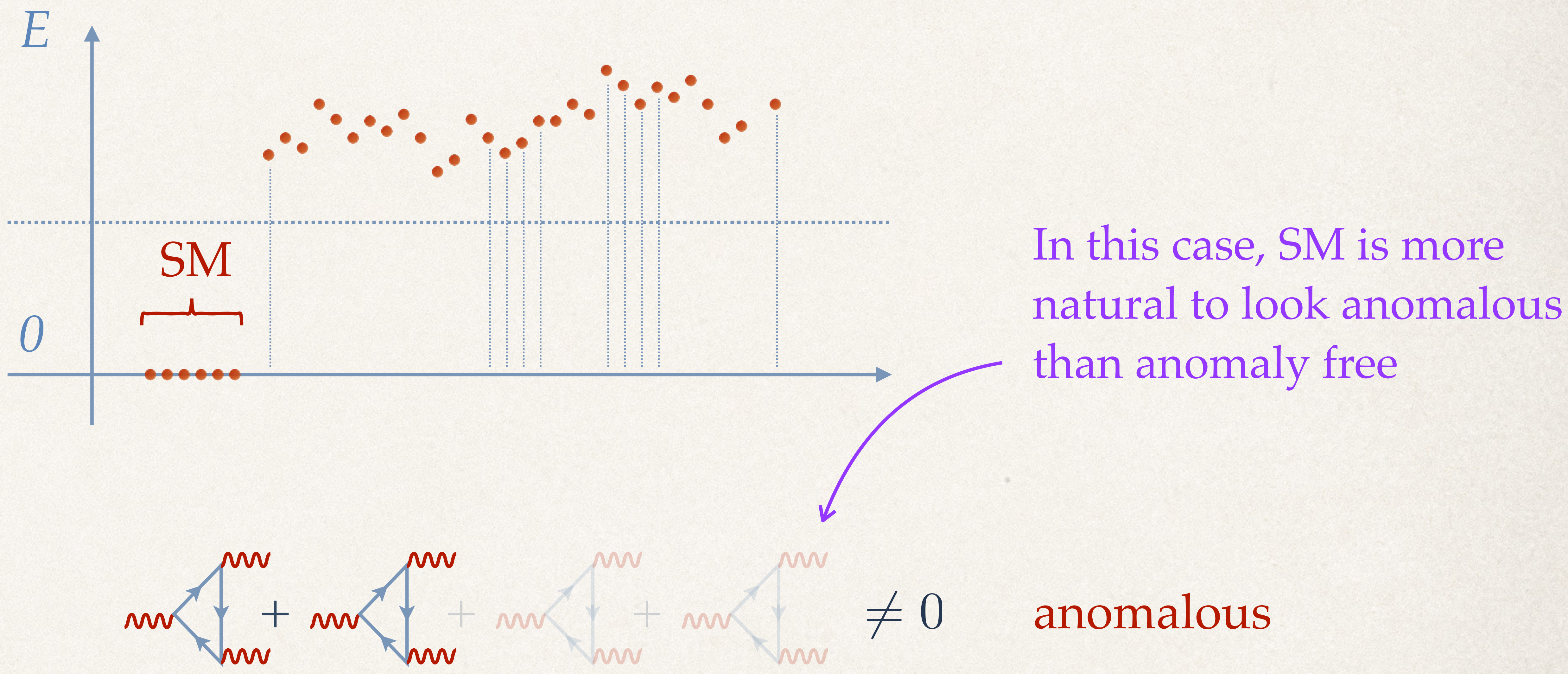


after $\langle\Phi\rangle = V$



- The effective model would be anomalous.

Standard Model and effective theory

- Consider
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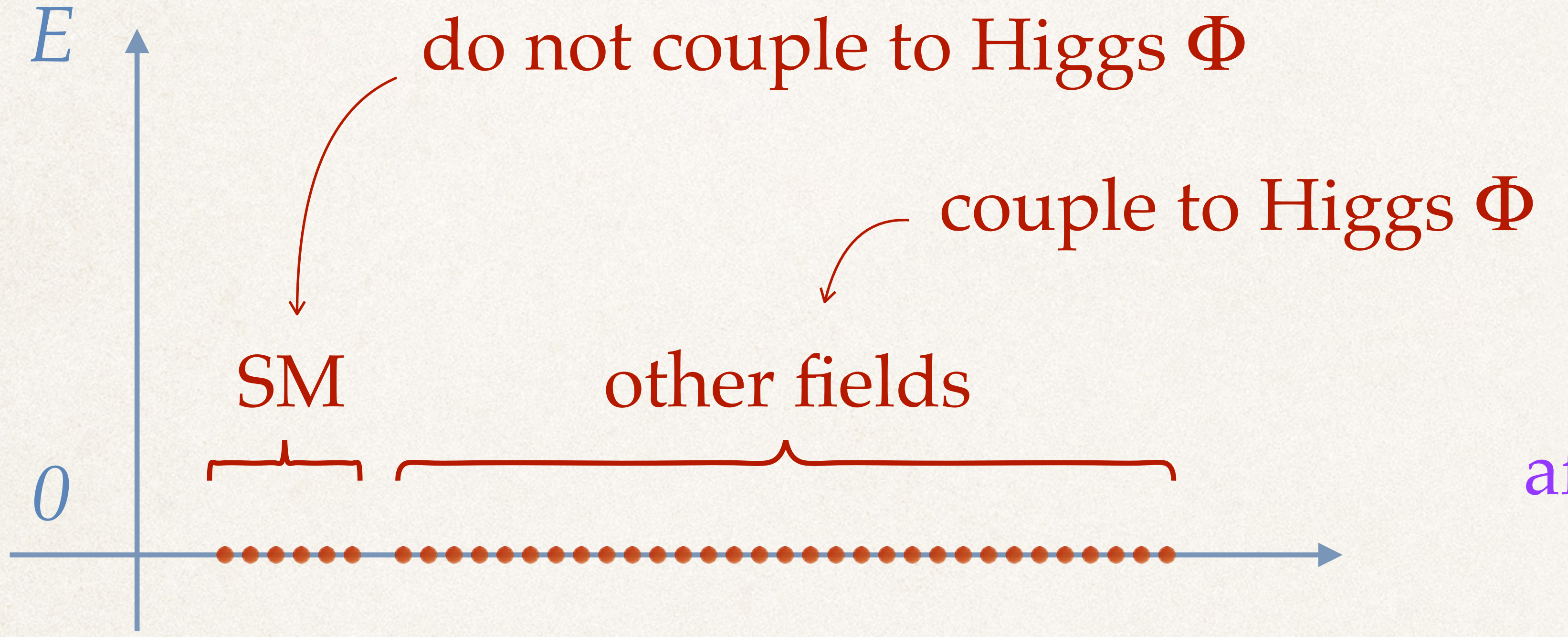
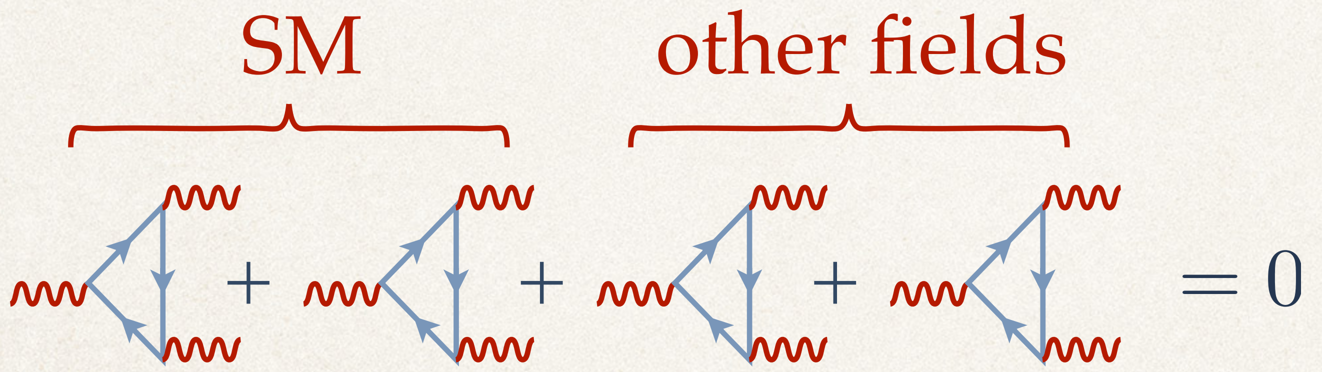
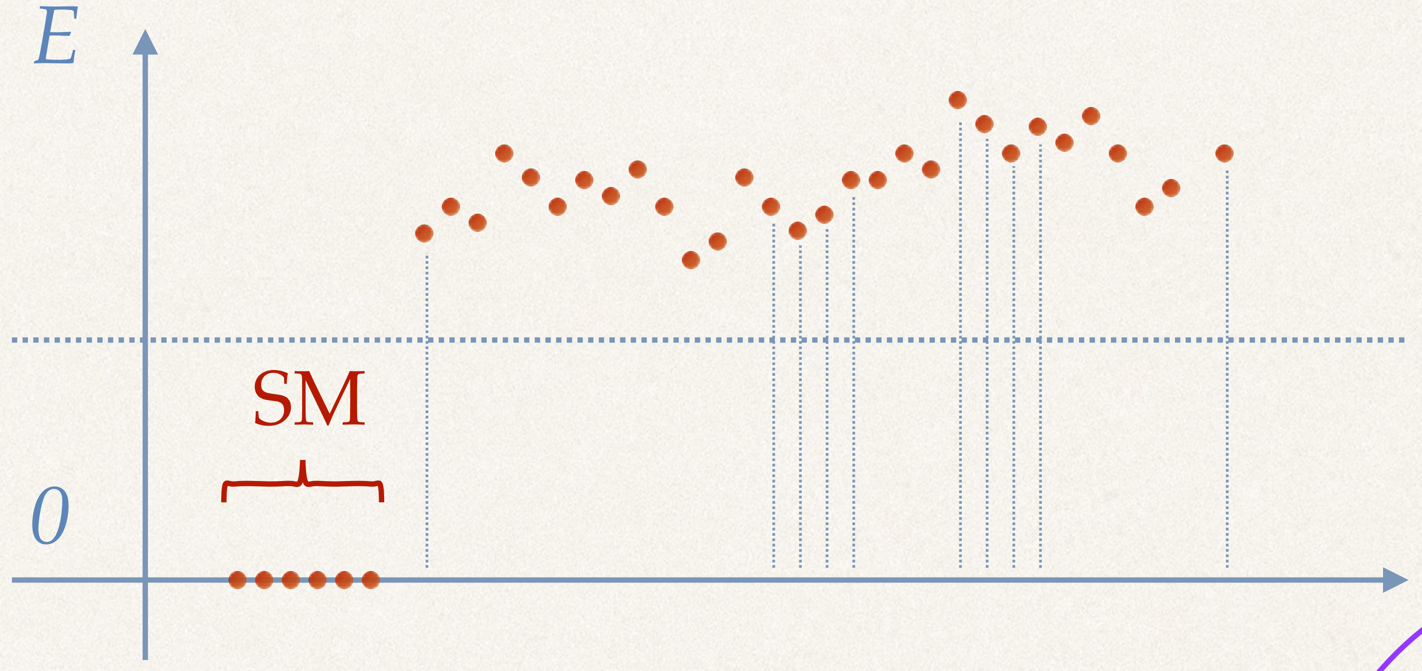
  - an *extra* Higgs field Φ, coupled only to the extra fermions and charged only under $U(1)_A$.



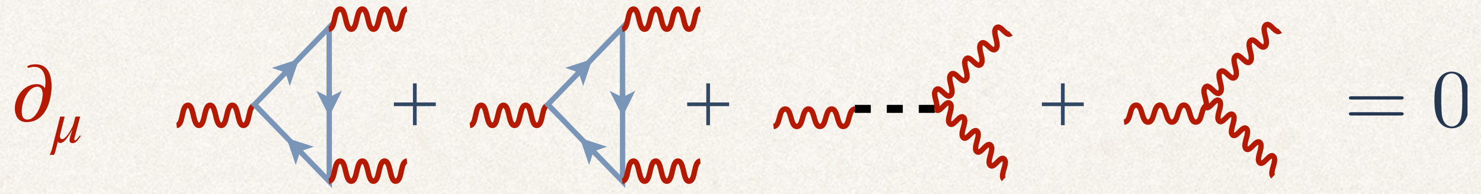


anomaly free

after $\langle \Phi \rangle = V$



In this case, SM is more natural to look anomalous than anomaly free



anomaly free

- The effective model would be superficially anomalous.

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Towards Phenomenology

Anomalies and the Standard Model

- In the next, we study a model where the Standard Model is a part of a Fundamental theory.
- We discuss the effective theory, at the scale below the breaking of the extra Higgs field Φ.
- All heavy fields are not included in our model (their effects are included via effective terms).
- Therefore, we expand the Standard Model with
  - and additional gauge boson A^μ
  - an axion a (coming from a theory in the UV).

Standard Model + an anomalous U(1)

- Standard Model fields are charged under the additional A^μ.

	$SU(3)$	$SU(2)$	Y	A
Q_L	$\mathbf{3}$	$\mathbf{2}$	$1/6$	q^{Q-A}
u_R^c	$\bar{\mathbf{3}}$	$\mathbf{1}$	$-2/3$	q^{u-A}
d_R^c	$\bar{\mathbf{3}}$	$\mathbf{1}$	$1/3$	q^{d-A}
L_L	$\mathbf{1}$	$\mathbf{2}$	$-1/2$	q^{L-A}
l_R^c	$\mathbf{1}$	$\mathbf{1}$	1	q^{l-A}
H	$\mathbf{1}$	$\mathbf{2}$	$1/2$	q^{H-A}

generic charges under the additional A^μ

- For these generic charges, we have the non-vanishing traces (leading to anomalies)

standard SM conditions

$$Tr[q_Y] = Tr[q_Y q_Y q_Y] = Tr[q_Y T_i^a T_i^a] = 0$$

$$\left.\begin{aligned}
Tr[q_A q_Y q_Y] &= \frac{q^{Q-A}}{6} + \frac{4q^{u-A}}{3} + \frac{q^{d-A}}{3} + \frac{q^{L-A}}{2} + q^{l-A} &&= t_{AYY} \\
Tr[q_Y q_A q_A] &= (q^{Q-A})^2 - 2(q^{u-A})^2 + (q^{d-A})^2 - (q^{L-A})^2 + (q^{l-A})^2 &&= t_{YAA} \\
Tr[q_A q_A q_A] &= 6(q^{Q-A})^3 + 3(q^{u-A})^3 + 3(q^{d-A})^3 + 2(q^{L-A})^3 + (q^{l-A})^3 &&= t_{AAA} \\
Tr[q_A T^a_{SU(2)} T^a_{SU(2)}] &= 3q^{Q-A} + q^{L-A} &&= T_{A,2} \\
Tr[q_A T^a_{SU(3)} T^a_{SU(3)}] &= 2q^{Q-A} + q^{u-A} + q^{d-A} &&= T_{A,3}
\end{aligned}\right\}$$

zero in anomaly free extensions of the SM

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Standard Model + an anomalous U(1)

- The action for the SM + the (anomalous) A^μ + the axion a becomes

$$\mathcal{L} = \mathcal{L}_{\rm SM} + \mathcal{L}_{\rm extra\ fields}$$

- The extra terms

$$\begin{aligned}
\mathcal{L}_{\rm extra\ fields} \quad &= \quad -\frac{1}{4}F^A_{\mu\nu}F_A^{\mu\nu} + \frac{1}{2}(\partial_\mu a + MA_\mu)^2 + g_A q^{\psi-A} A_\mu \bar{\psi}\gamma^\mu\psi \\
&+2ig_A q^{Q-A} H^\dagger A^\mu \Big(\partial_\mu - ig_2 T^\alpha W^\alpha_\mu - ig_Y q^{H-Y} Y_\mu - \frac{i}{2} g_A q^{H-A} A_\mu\Big) H \\
&+\frac{1}{24\pi^2} a\Big(C_{YY} F_Y \wedge F_Y + C_{YA}\ F_Y \wedge F_A + C_{AA}\ F_A \wedge F_A + \sum_{i=2,3} D_i\ Tr_i[G \wedge G]\Big) \\
&+\frac{1}{24\pi^2} A \wedge Y \wedge \Big(E_{AY,A} F_A + E_{AY,Y} F_Y\Big) \\
&+\frac{1}{24\pi^2} \sum_{i=2,3} Z_i\ A \wedge Tr_i\Big[A \wedge (dA - \frac{2}{3} A \wedge A)\Big]
\end{aligned}$$

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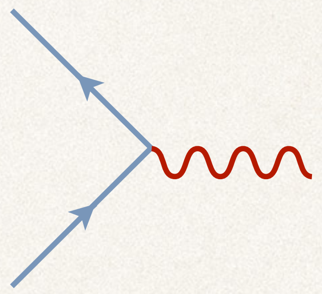
Standard Model + an anomalous U(1)

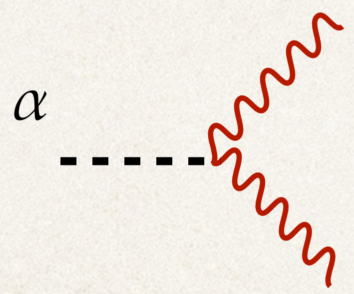
- The action for the SM + the (anomalous) A^μ + the axion a becomes

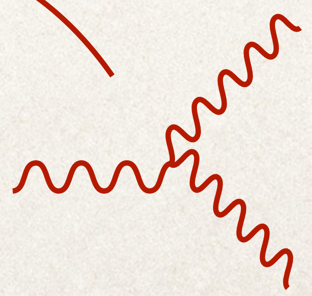
$$\mathcal{L} = \mathcal{L}_{\rm SM} + \mathcal{L}_{\text{extra fields}}$$

- The extra terms



$$\begin{aligned}\mathcal{L}_{\text{extra fields}} \quad = \quad & -\frac{1}{4}F^A_{\mu\nu}F^{\mu\nu}_A + \frac{1}{2}(\partial_\mu a + MA_\mu)^2 + g_A q^{\psi-A} A_\mu \bar{\psi}\gamma^\mu\psi \\ & +2ig_A q^{Q-A} H^\dagger A^\mu \Big(\partial_\mu - ig_2 T^\alpha W^\alpha_\mu - ig_Y q^{H-Y} Y_\mu - \frac{i}{2} g_A q^{H-A} A_\mu\Big) H \\ & +\frac{1}{24\pi^2} a\Big(C_{YY} F_Y \wedge F_Y + C_{YA}\, F_Y \wedge F_A + C_{AA}\, F_A \wedge F_A + \sum_{i=2,3} D_i\, Tr_i[G\wedge G]\Big) \\ & +\frac{1}{24\pi^2} A\wedge Y \wedge \Big(E_{AY,A} F_A + E_{AY,Y} F_Y\Big) \\ & +\frac{1}{24\pi^2}\sum_{i=2,3} Z_i\, A \wedge Tr_i\Big[A\wedge(dA - \frac{2}{3}A\wedge A)\Big] \end{aligned}$$





No gauge invariant terms

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Standard Model + an anomalous U(1)

- Variation of the action under $A^\mu \to A^\mu + \partial^\mu \epsilon\,,\; Y^\mu \to Y^\mu + \partial^\mu \zeta$ gives (not gauge invariant!)

$$\delta S_{\rm extrafields} = -\frac{1}{24\pi^2}\int \Big\{\epsilon\Big((E_{AY,A}+MC_{AYY})F_Y\wedge F_Y + (E_{AY,Y}+MC_{YAA})F_A\wedge F_Y + MC_{AAA}F_A\wedge F_A + MD_{A,i}Tr[G_i\wedge G_i]\Big) - \zeta\Big(E_{AY,A}F_A\wedge F_A + E_{AY,Y}F_A\wedge F_Y\Big)\Big\}$$

- At 1-loop, we get the non-vanishing terms (due to the anomalies t_{ijk})

$$\delta S_{1-\rm loop} = -\frac{1}{24\pi^2}\int \Big\{\epsilon\Big(t_{AYY}F_Y\wedge F_Y + t_{YAA}F_A\wedge F_Y + t_{AAA}F_A\wedge F_A + T_{A,i}Tr[G_i\wedge G_i]\Big) + \zeta\Big(t_{YAA}F_A\wedge F_A + t_{AYY}F_A\wedge F_Y\Big)\Big\}$$

- Cancellation of anomalies requires

$$\delta S_{\text{1-loop}} + \delta S_{\text{extra fields}} = 0 \;\to\; \left\{\begin{array}{ll} MC_{YY} = -2t_{AYY} & MC_{YA} = -2t_{YAA} \\ MC_{AA} = -t_{AAA} & 2MD_i = -3Z_i = -6T_{A,i} \\ E_{AY,A} = t_{YAA} & E_{AY,Y} = t_{AYY} \end{array}\right.$$

all coefficients are fixed by the anomaly cancellation

Standard Model + an anomalous U(1)

- Next, we have the standard EW breaking.
- Assuming that the SM Higgs H is charged under the anomalous A^μ the Y^μ, W_3^μ, A^μ mix

photon remains massless

$$\begin{pmatrix} Y \\ W_3 \\ A \end{pmatrix} = \begin{pmatrix} \cos\theta_W^0 & -\cos\zeta_+ \sin\theta_W^0 & +\cos\zeta_- \sin\theta_W^0 \\ \sin\theta_W^0 & -\cos\zeta_+ \cos\theta_W^0 & -\cos\zeta_- \cos\theta_W^0 \\ 0 & \sin\zeta_+ & \sin\zeta_- \end{pmatrix} \begin{pmatrix} \gamma \\ Z \\ Z' \end{pmatrix}$$

$$m_\gamma^2 = 0$$
$$8m_Z^2 = m_-^2 + 2g^2v^2$$
$$8m_{Z'}^2 = m_+^2 + 2g^2v^2$$

with the coefficients

$$g^2 \equiv g_Y^2 + g_2^2, \quad \sin\theta_W^0 \equiv g_Y/g, \quad \cos\theta_W^0 \equiv g_2/g,$$
$$m_\pm^2 \equiv 2M^2 + (g_A q^{H-A} v)^2 - g^2v^2 \pm \sqrt{(2M^2 + (g_A q^{H-A} v)^2 - g^2v^2)^2 + 4g^2v^2(g_A q^{H-A} v)^2},$$
$$\sin\zeta_\pm \equiv \frac{2g g_A q^{H-A} v^2}{\sqrt{m_\pm^4 + 4g^2v^2(g_A q^{H-A} v)^2}}, \quad \cos\zeta_\pm \equiv \frac{m_\pm^2}{\sqrt{m_\pm^4 + 4g^2v^2(g_A q^{H-A} v)^2}}.$$

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New couplings

- Anomalous terms generate new couplings (all are absent in anomaly-free models)

$$Y \wedge A \wedge F_A, \quad Y \wedge A \wedge F_Y \qquad \longrightarrow \qquad \gamma\gamma Z, \quad \gamma\gamma Z', \quad \gamma ZZ, \quad \gamma ZZ', \quad \gamma Z'Z', \quad ZZZ', \quad ZZ'Z'$$

$$A \wedge Tr\Big[W \wedge (dW + \frac{2}{3}W \wedge W)\Big] \qquad \longrightarrow \qquad ZW^+W^-, \quad Z'W^+W^-, \quad \gamma ZW^+W^-, \quad \gamma Z'W^+W^-$$

$$A \wedge Tr\Big[G \wedge (dG + \frac{2}{3}G \wedge G)\Big] \qquad \longrightarrow \qquad Z \text{ gluons gluons}, \quad Z' \text{ gluons gluons}, \quad \ldots$$

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$$Y \wedge A \wedge F_A, \quad Y \wedge A \wedge F_Y \qquad \longrightarrow \qquad \gamma\gamma Z, \quad \gamma\gamma Z', \quad \gamma Z Z, \quad \gamma Z Z', \quad \gamma Z' Z', \quad Z Z Z', \quad Z Z' Z'$$

$$A \wedge Tr\Big[W \wedge (dW + \frac{2}{3} W \wedge W\Big)\Big] \qquad \longrightarrow \qquad Z W^+ W^-, \quad Z' W^+ W^-, \quad \gamma Z W^+ W^-, \quad \gamma Z' W^+ W^-$$

$$A \wedge Tr\Big[G \wedge (dG + \frac{2}{3} G \wedge G\Big)\Big] \qquad \longrightarrow \qquad Z \text{ gluons gluons}, \quad Z' \text{ gluons gluons}, \quad \ldots$$

- Notice (anomaly generated) terms with *only* SM fields.
- Such terms are new (anomaly related) and they modify the standard SM predictions.

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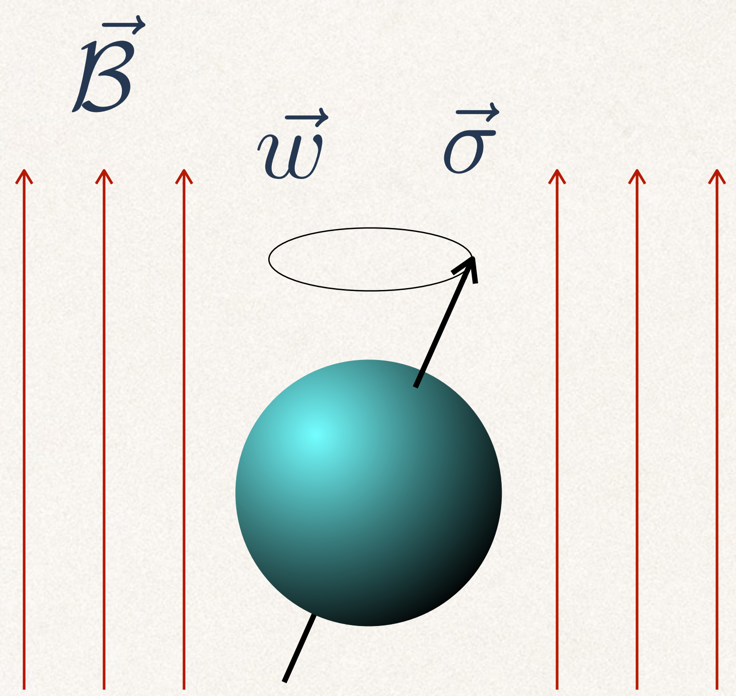
Let’s apply all these.

The $g - 2$ of the muon

The $g - 2$ of a fermion



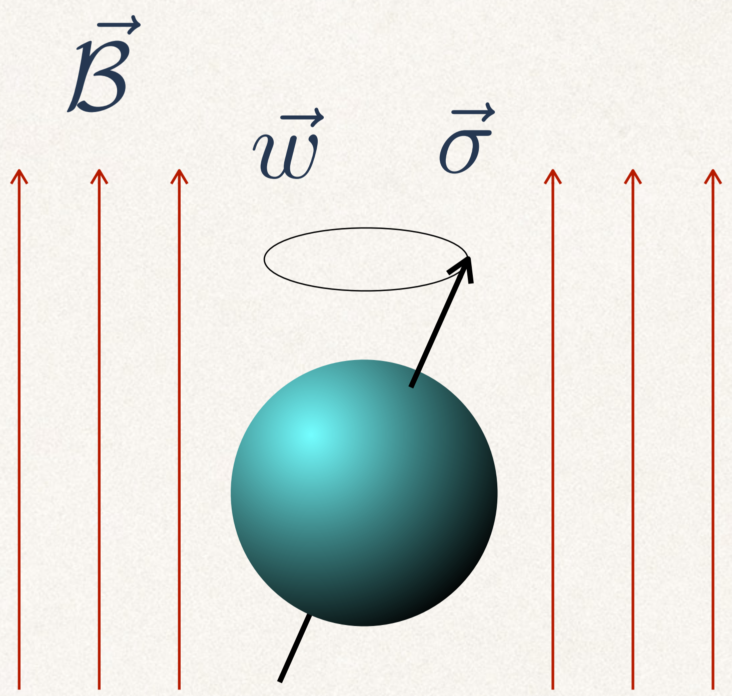
$$\mu = \frac{qg}{2m}$$

Landé g factor

how much is it?

$$H = -\mu \, \frac{\vec{\sigma}}{2} \cdot \vec{\mathcal{B}}$$

The $g-2$ of a fermion



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Landé g factor

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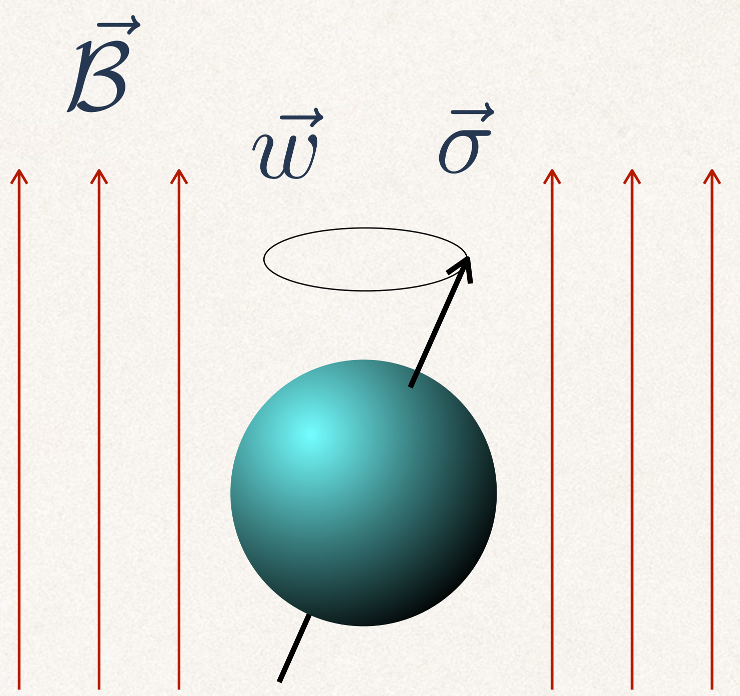
$$H = -\mu\,\frac{\vec{\sigma}}{2}\cdot\vec{\mathcal{B}}$$

- Predictions / Results
  - Classically: $g = 1$
  - Quantum (Dirac): $g = 2$

The $g-2$ of a fermion

subatomic world
obeys quantum laws!



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Landé g factor

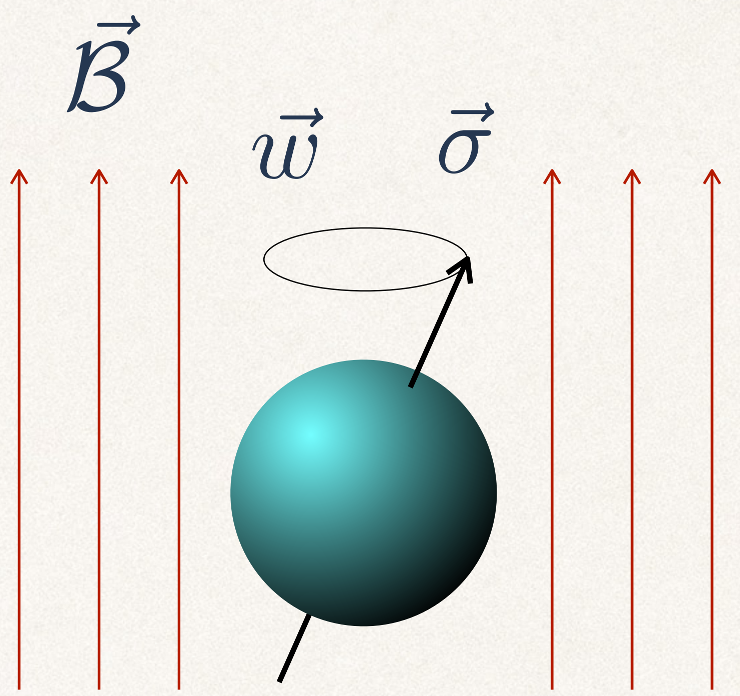
how much is it?

$$H = -\mu \, \frac{\vec{\sigma}}{2} \cdot \vec{\mathcal{B}}$$

- Predictions / Results
  - ~~Classically~~: $g = 1$
  - Quantum (Dirac): $g = 2$
  - Experiment: $g \simeq 2$

Over the years, the value of the $g-2$ is one of the greatest *challenges* of theory vs experiment

The $g-2$ of the muon



subatomic world
obeys quantum laws!

$$\mu = \frac{qg}{2m}$$

Landé g factor

how much is it?

$$H = -\mu\,\frac{\vec{\sigma}}{2}\cdot\vec{B}$$

- The $g-2$ of the muon
  - Experimental value : $g-2|^{\rm FNAL}_{\rm exp} = 2\times 116592061(41)\times 10^{-11}$
  - Theoretical (SM) prediction : $g-2|_{\rm SM} = 2\times 116591810(43)\times 10^{-11}$

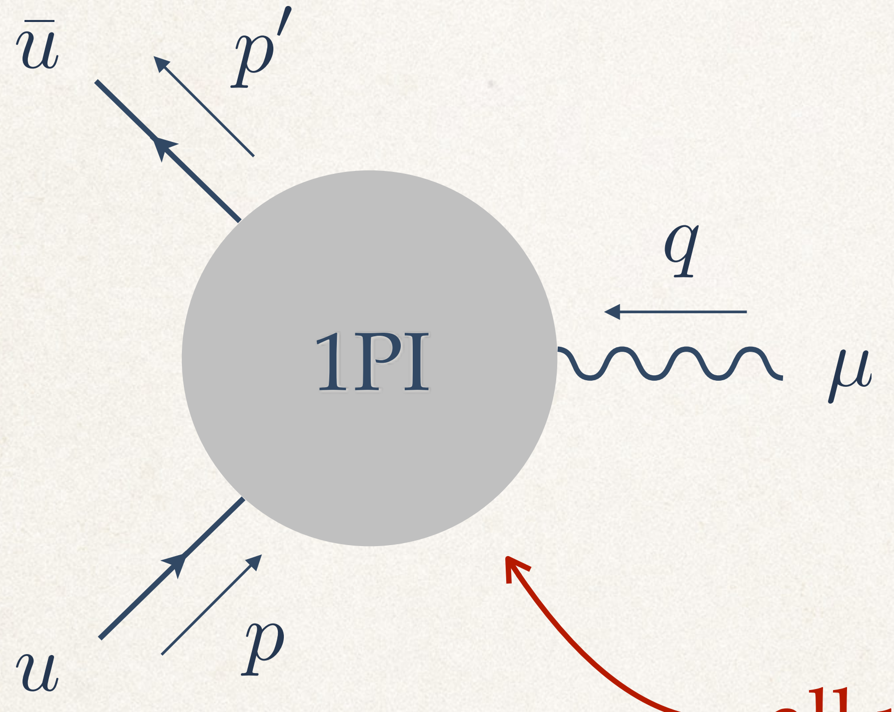
$$\Delta g/2 = 251(59)\times 10^{-11}$$

Standard Model is the *most accurate* theory ever made!

however.. the difference approaches the 4.2σ..

The $g-2$ of the muon

- Theoretical evaluation of the $g-2$ of a fermion.



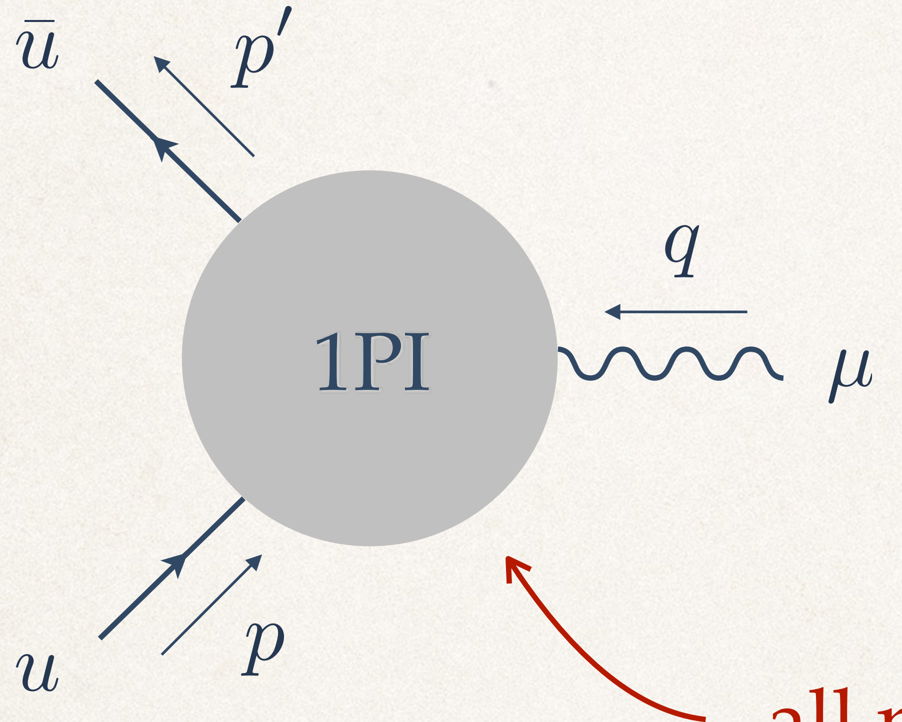
$$= (-ie)\bar{u}(p')\Big(\gamma^\mu F_1(q) + i\frac{\sigma^{\mu\nu}q_\nu}{2m_\ell}F_2(q) + \gamma^5\frac{\sigma^{\mu\nu}q_\nu}{2m_\ell}F_3(q) + \gamma^5(q^2\gamma^\mu - \gamma_\nu q^\nu q^\mu)F_4(q)\Big)u(p)$$

all possible diagrams

Schwinger

The $g - 2$ of the muon

- Theoretical evaluation of the $g - 2$ of a fermion.



quantum corrections to the coupling

$(g - 2)/2$ of the fermion

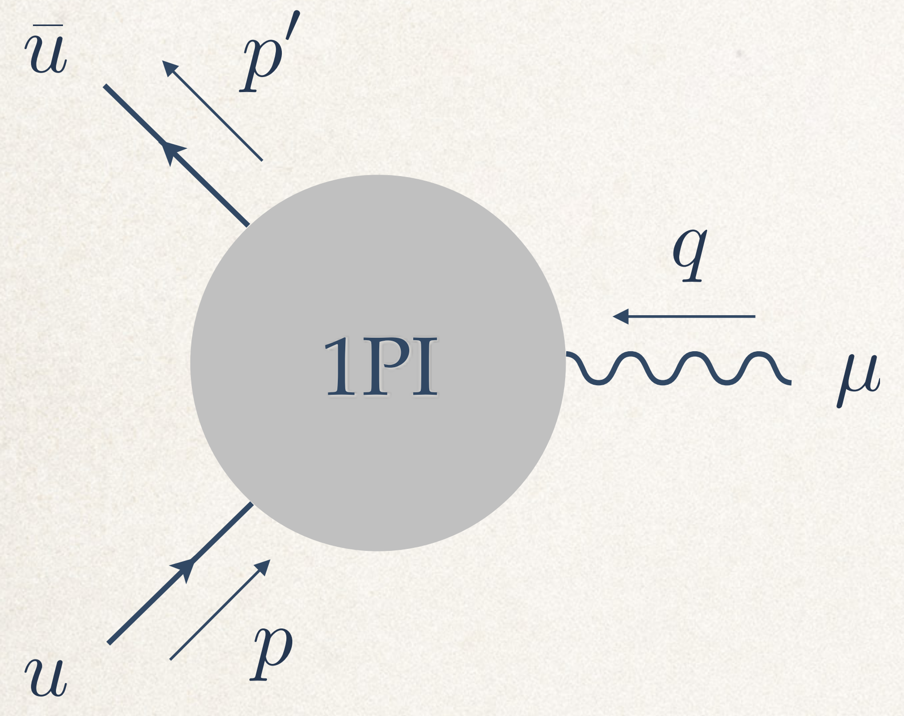
$$= (-ie)\bar{u}(p')\Big(\gamma^\mu F_1(q) + i\frac{\sigma^{\mu\nu}q_\nu}{2m_\ell}F_2(q) + \gamma^5\frac{\sigma^{\mu\nu}q_\nu}{2m_\ell}F_3(q) + \gamma^5(q^2\gamma^\mu - \gamma_\nu q^\nu q^\mu)F_4(q)\Big)u(p)$$

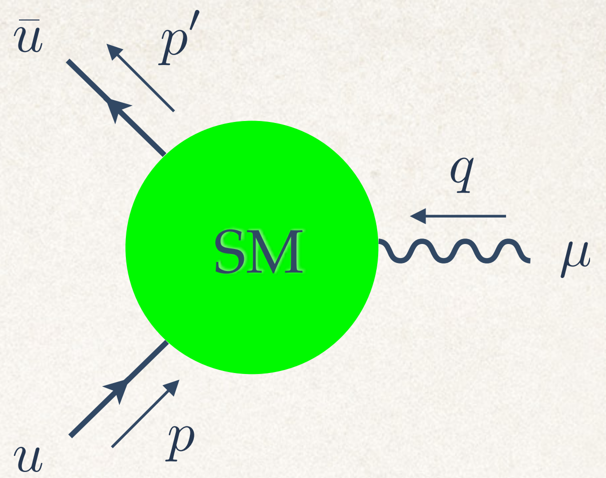
electric dipole moment (EDM)

all possible diagrams

Schwinger

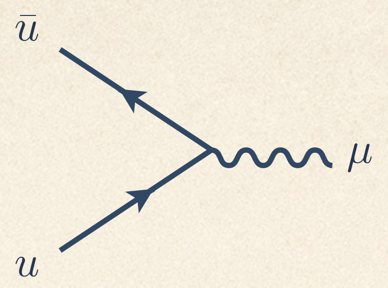
- In "all possible diagrams", propagate *all* possible fields in a model.
- Several models have been built, to explain the discrepancy (extra Higgs, Z', …).
- Our goal is to evaluate the contribution of an anomalous Z' to the $g - 2$ of the muon.



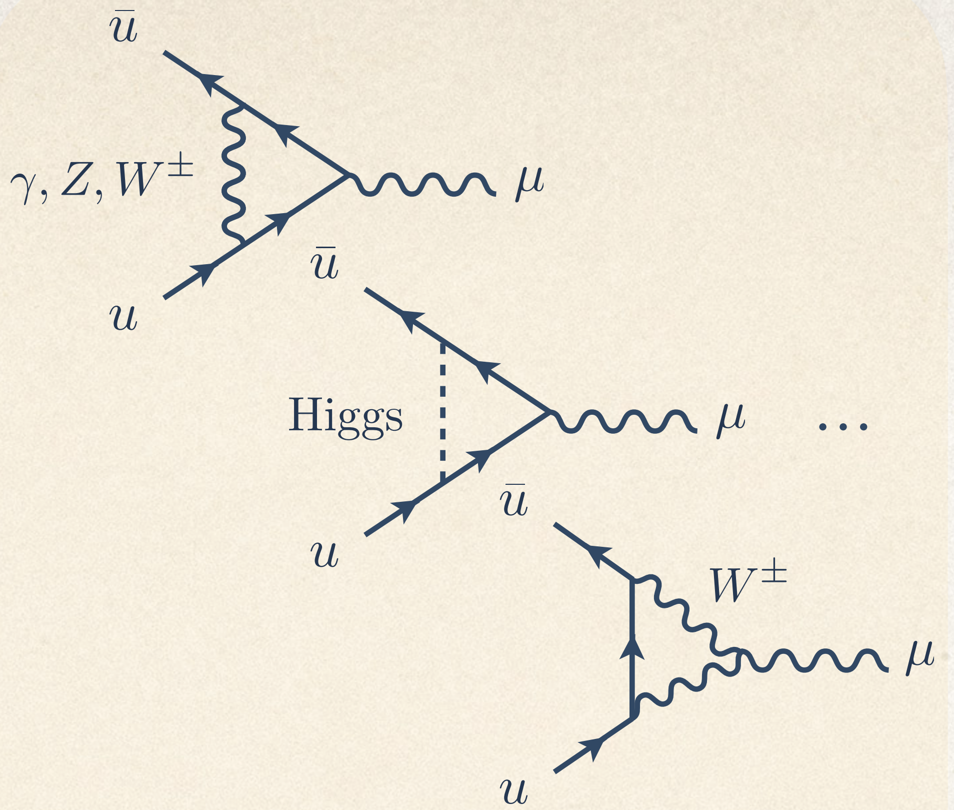


diagrams that contain *only*
SM particles and couplings

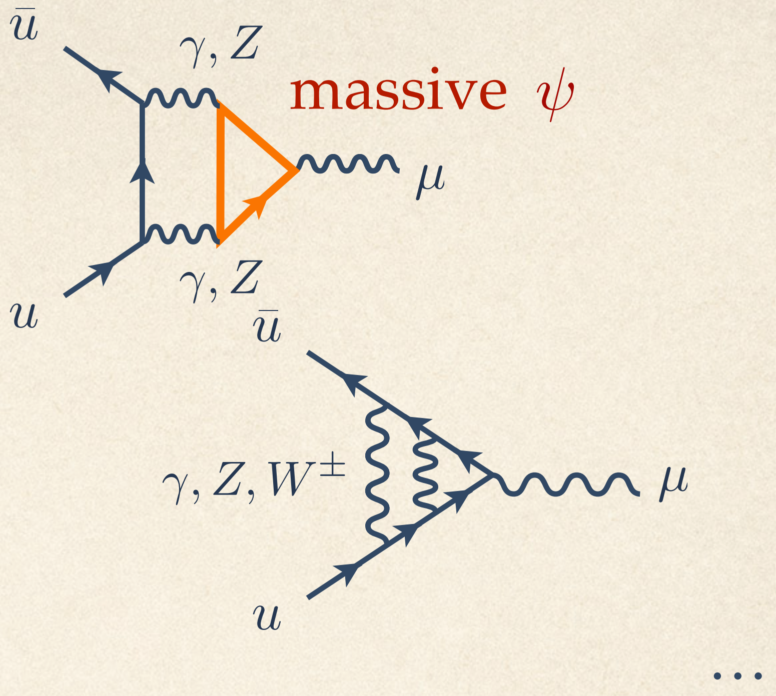
$g - 2 = 0$: the triumph of
quantum mechanics



tree-level

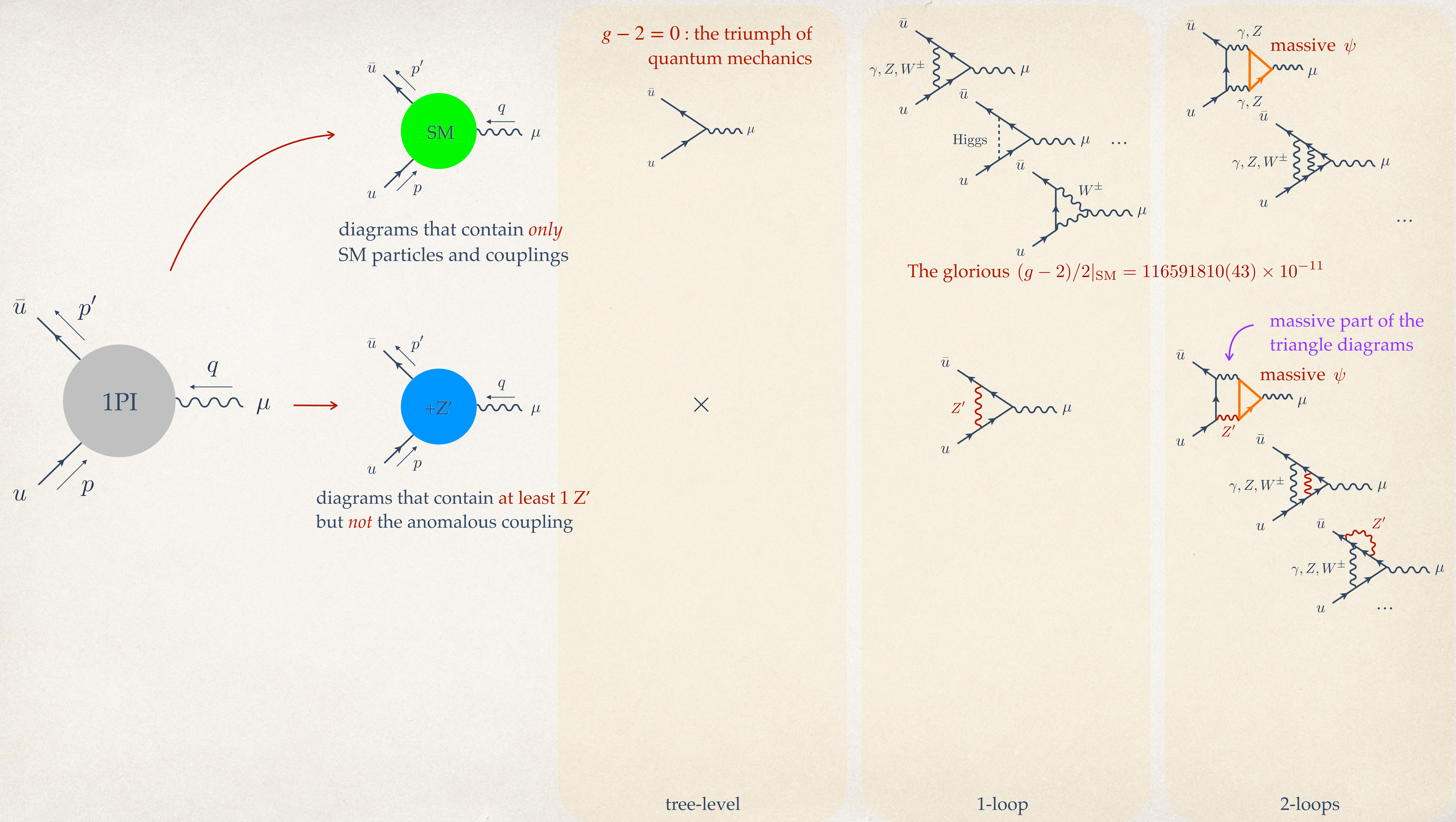


1-loop

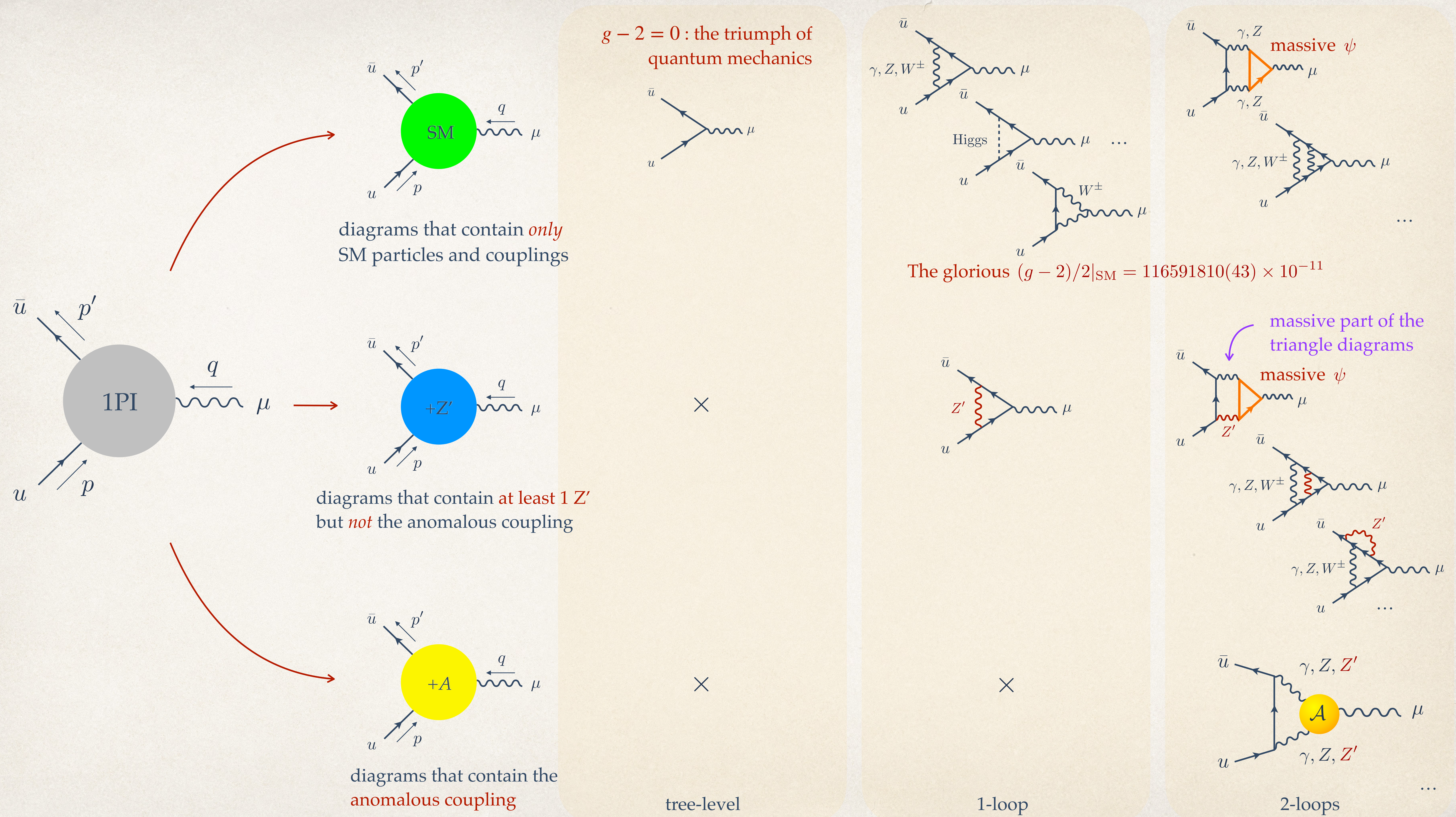


2-loops

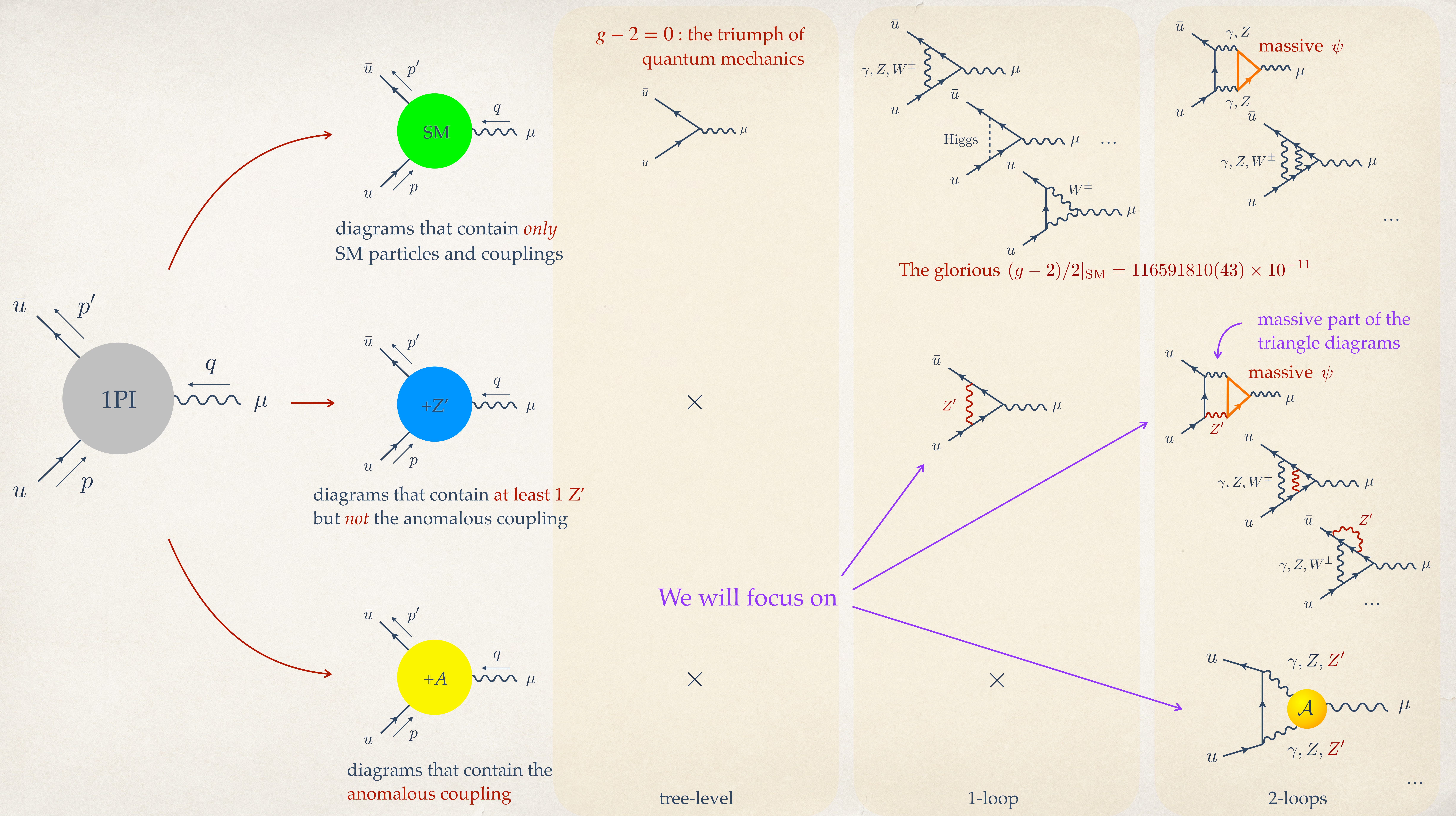
The glorious $(g-2)/2|_{\mathrm{SM}} = 116591810(43) \times 10^{-11}$



1PI
$\bar{u}$ p' q μ u p
SM
diagrams that contain *only* SM particles and couplings
$+Z'$
diagrams that contain at least 1 Z' but *not* the anomalous coupling
$g-2=0$: the triumph of quantum mechanics
$\times$
tree-level
$\gamma, Z, W^{\pm}$
Higgs
$W^{\pm}$
…
The glorious $(g-2)/2|_{\mathrm{SM}} = 116591810(43) \times 10^{-11}$
Z'
1-loop
γ, Z
massive ψ
massive part of the triangle diagrams
2-loops

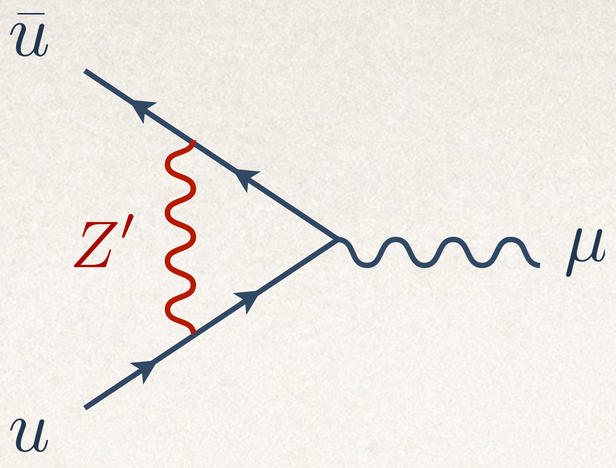


1PI
SM
+Z′
+A
diagrams that contain only SM particles and couplings
diagrams that contain at least 1 Z′ but not the anomalous coupling
diagrams that contain the anomalous coupling
g − 2 = 0 : the triumph of quantum mechanics
Higgs
massive ψ
The glorious (g − 2)/2|SM = 116591810(43) × 10^−11
massive part of the triangle diagrams
massive ψ
tree-level
1-loop
2-loops

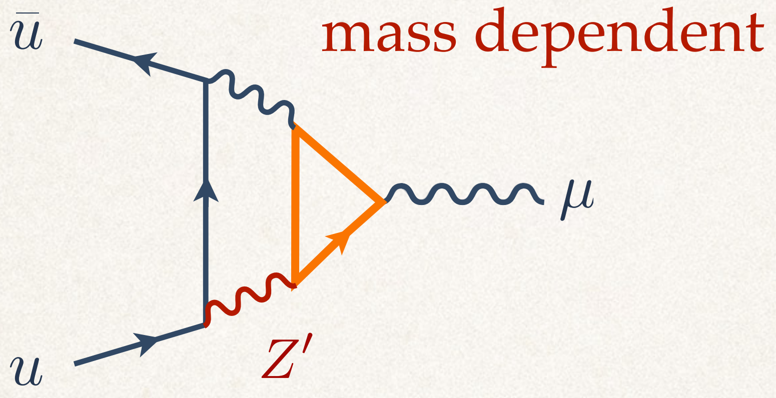


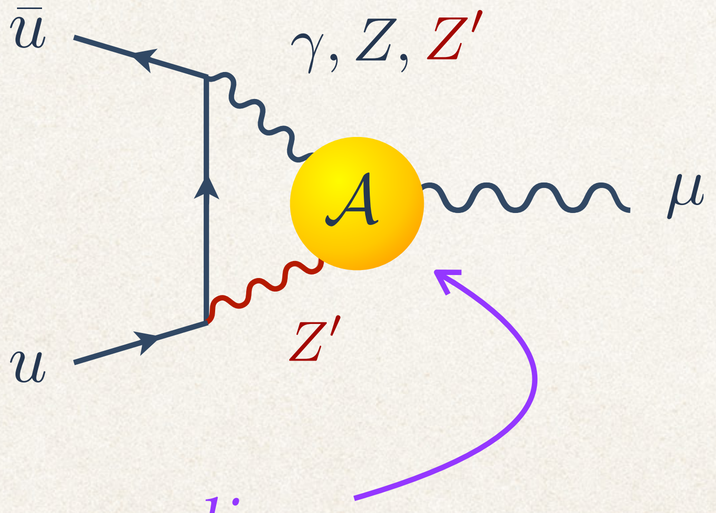
1PI
SM
diagrams that contain only SM particles and couplings
+Z′
diagrams that contain at least 1 Z′ but not the anomalous coupling
+A
diagrams that contain the anomalous coupling
g − 2 = 0 : the triumph of quantum mechanics
γ, Z, W±
Higgs
W±
The glorious (g − 2)/2|SM = 116591810(43) × 10^−11
γ, Z
massive ψ
massive part of the triangle diagrams
massive ψ
Z′
We will focus on
γ, Z, Z′
tree-level
1-loop
2-loops

1-loop

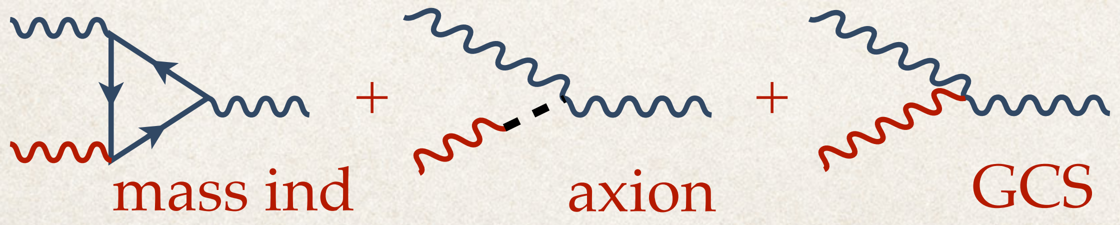


2-loop

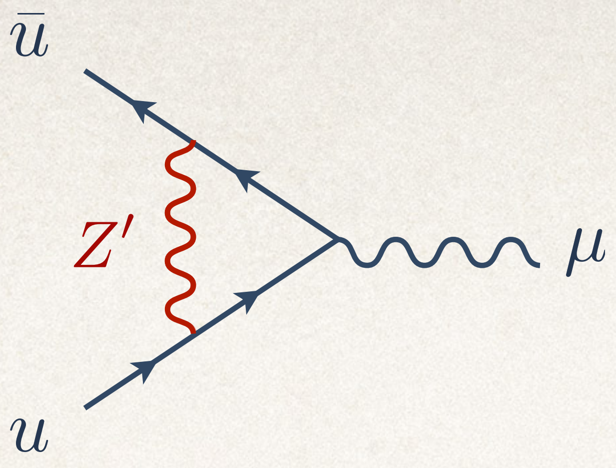




the *anomalous coupling*



1-loop

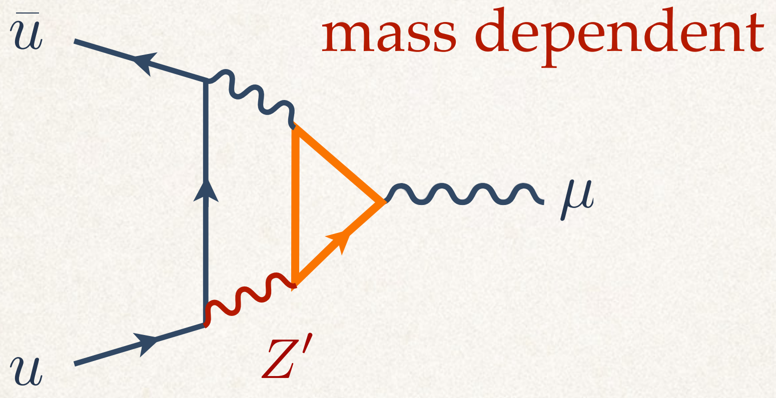


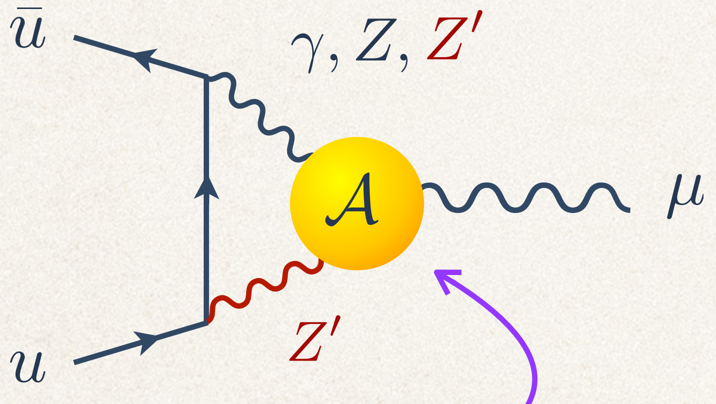
$$\Delta g_\mu/2 = \frac{1}{4\pi^2}\frac{m_\mu^2}{m_{Z'}^2} g_{Z'}^2 \left((q_V^{\mu-Z'})^2 \mathcal{F}_V\left[\frac{m_\mu^2}{m_{Z'}^2}\right] - (q_A^{\mu-Z'})^2 \mathcal{F}_A\left[\frac{m_\mu^2}{m_{Z'}^2}\right] \right)$$

$$\mathcal{F}_V\left[\frac{m_\mu^2}{m_{Z'}^2}\right] = \int_0^1 dx \frac{x^2(1-x)}{1-x+x^2 m_\mu^2/m_{Z'}^2}$$

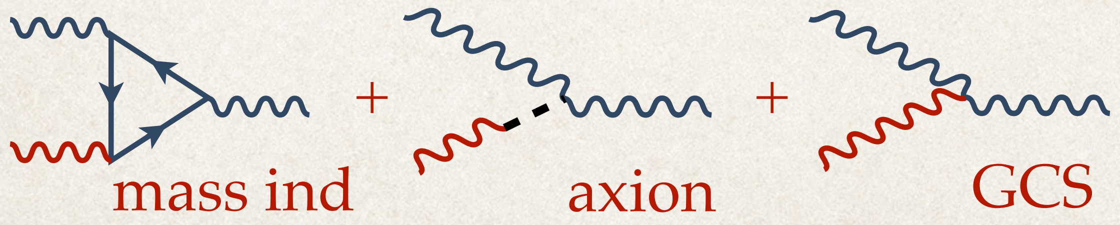
$$\mathcal{F}_A\left[\frac{m_\mu^2}{m_{Z'}^2}\right] = \int_0^1 dx \frac{x(1-x)(4-x)+2x^3 m_\mu^2/m_{Z'}^2}{1-x+x^2 m_\mu^2/m_{Z'}^2}$$

2-loop

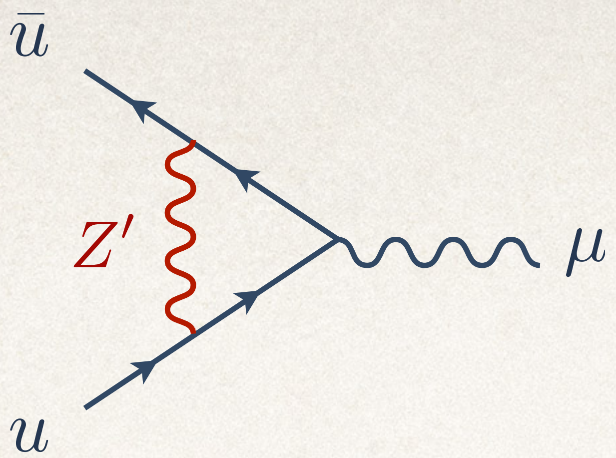




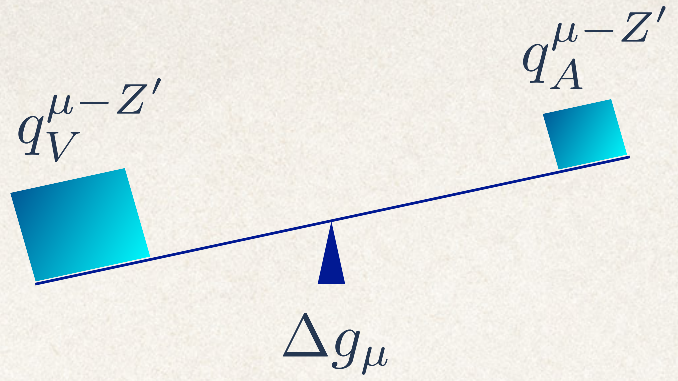
the *anomalous coupling*



1-loop



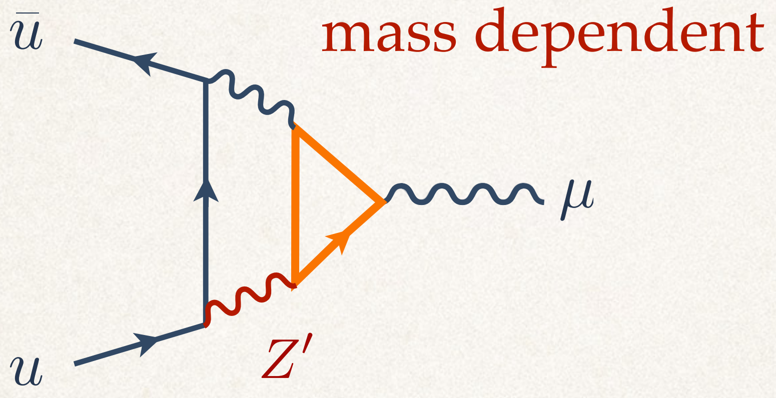
$$\Delta g_\mu/2 \;=\; \frac{1}{4\pi^2}\frac{m_\mu^2}{m_{Z'}^2} g_{Z'}^2 \left((q_V^{\mu-Z'})^2 \mathcal{F}_V\left[\frac{m_\mu^2}{m_{Z'}^2}\right] - (q_A^{\mu-Z'})^2 \mathcal{F}_A\left[\frac{m_\mu^2}{m_{Z'}^2}\right] \right)$$

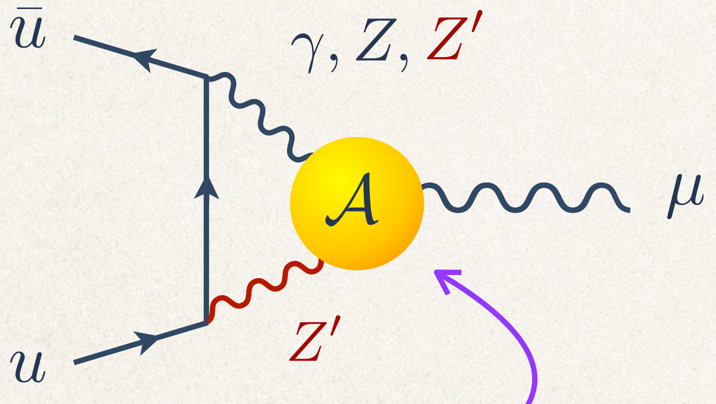


$$\mathcal{F}_V\left[\frac{m_\mu^2}{m_{Z'}^2}\right] = \int_0^1 dx \frac{x^2(1-x)}{1-x+x^2 m_\mu^2/m_{Z'}^2}$$

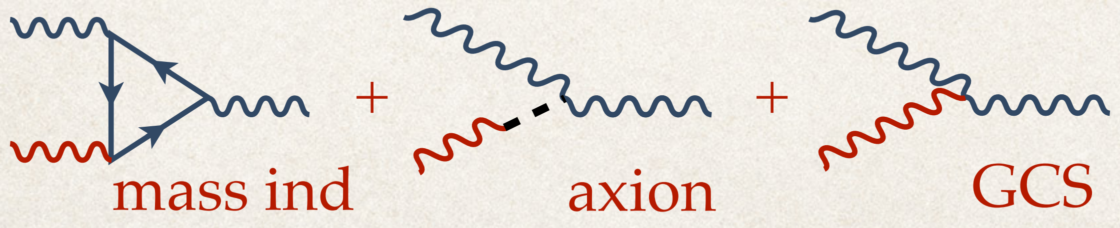
$$\mathcal{F}_A\left[\frac{m_\mu^2}{m_{Z'}^2}\right] = \int_0^1 dx \frac{x(1-x)(4-x)+2x^3 m_\mu^2/m_{Z'}^2}{1-x+x^2 m_\mu^2/m_{Z'}^2}$$

2-loop

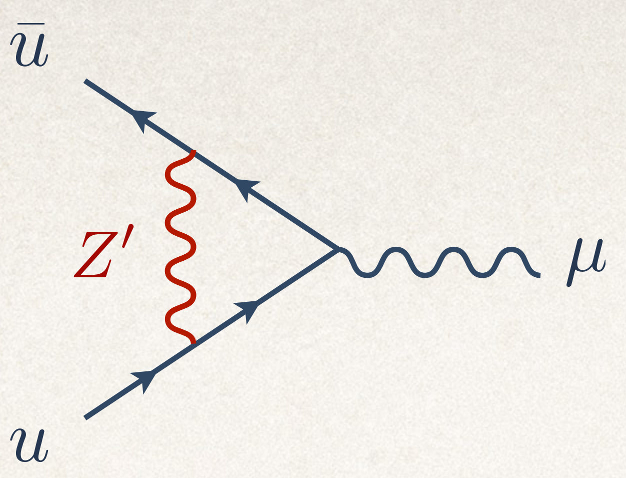




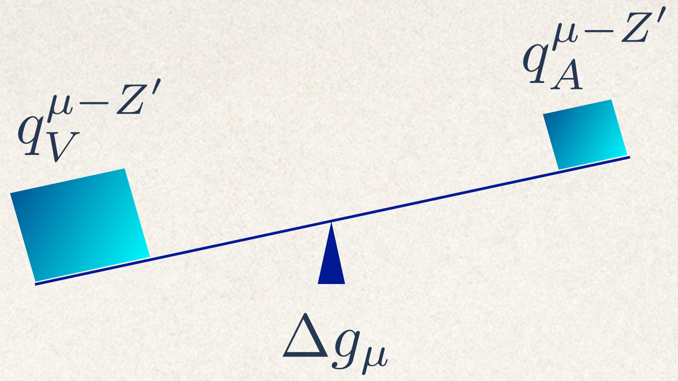
the *anomalous coupling*



1-loop



$$\Delta g_\mu/2 = \frac{1}{4\pi^2}\frac{m_\mu^2}{m_{Z'}^2} g_{Z'}^2 \left((q_V^{\mu-Z'})^2 \mathcal{F}_V\left[\frac{m_\mu^2}{m_{Z'}^2}\right] - (q_A^{\mu-Z'})^2 \mathcal{F}_A\left[\frac{m_\mu^2}{m_{Z'}^2}\right] \right)$$

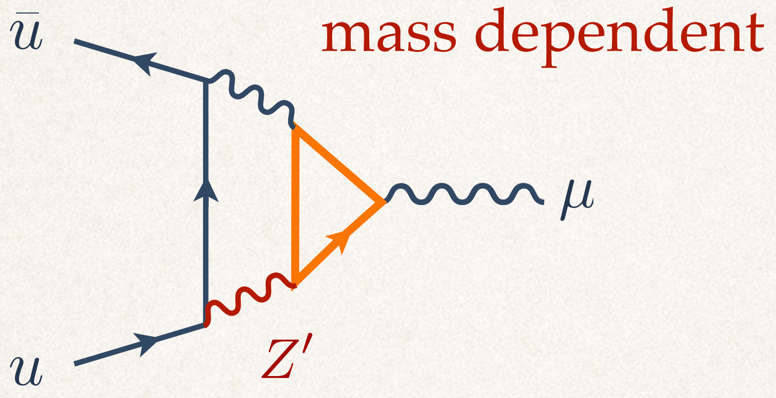


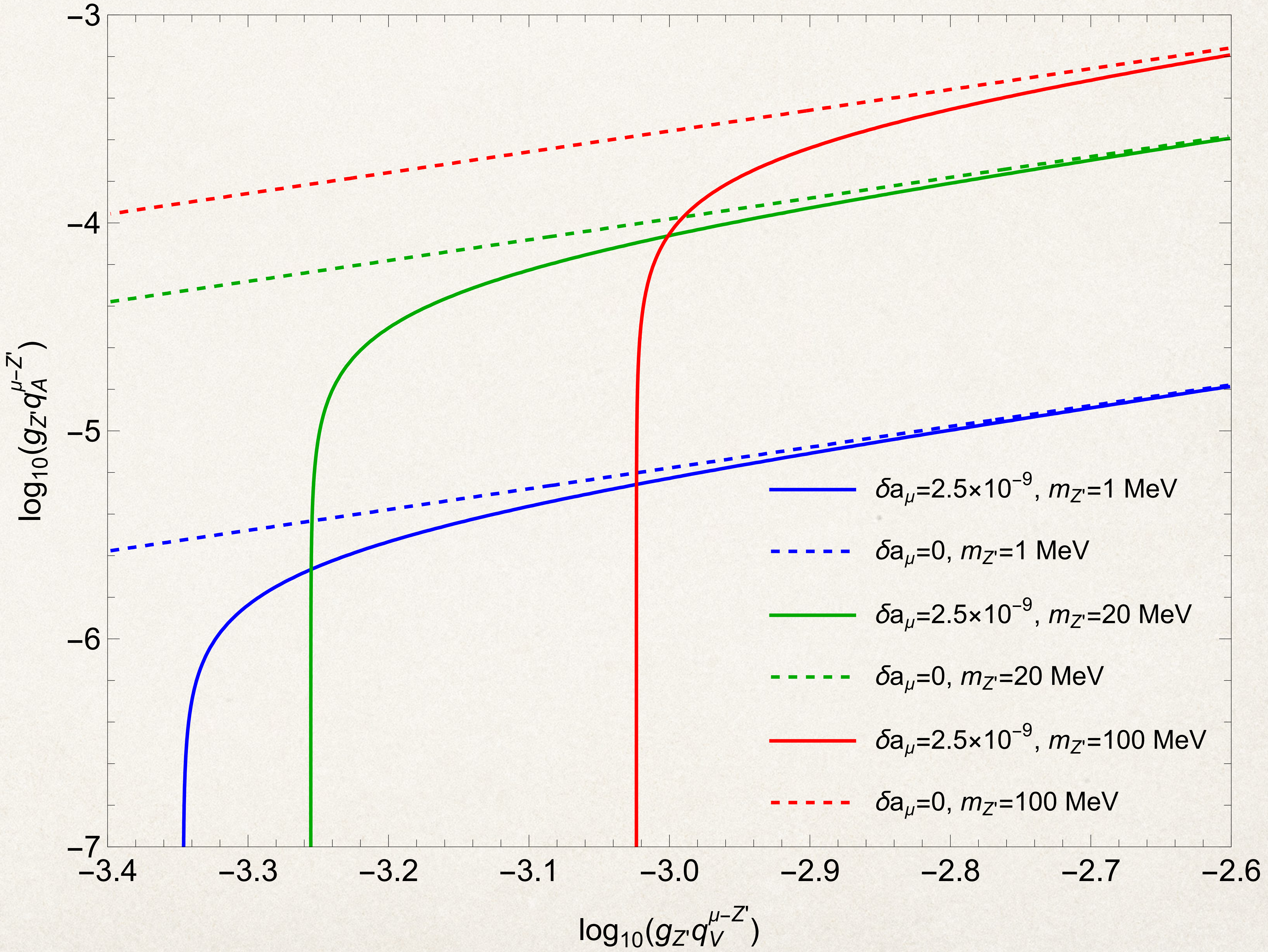
$$\mathcal{F}_V\left[\frac{m_\mu^2}{m_{Z'}^2}\right] = \int_0^1 dx \frac{x^2(1-x)}{1-x+x^2 m_\mu^2/m_{Z'}^2}$$

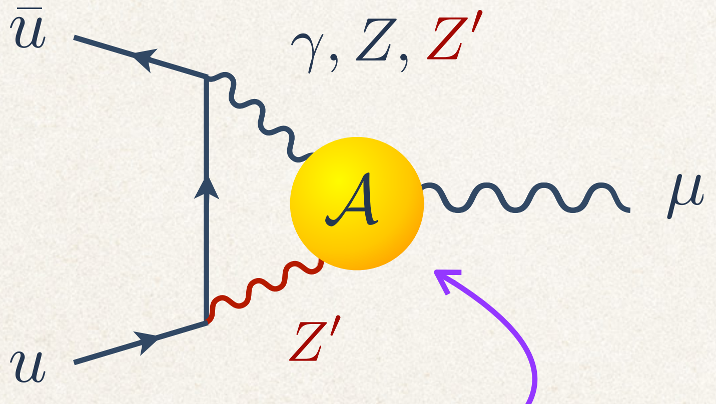
$$\mathcal{F}_A\left[\frac{m_\mu^2}{m_{Z'}^2}\right] = \int_0^1 dx \frac{x(1-x)(4-x)+2x^3 m_\mu^2/m_{Z'}^2}{1-x+x^2 m_\mu^2/m_{Z'}^2}$$

$$q_A^{\mu-Z'} = \mathcal{G}[q_V^{\mu-Z'}, m_{Z'}, \Delta g_\mu]$$

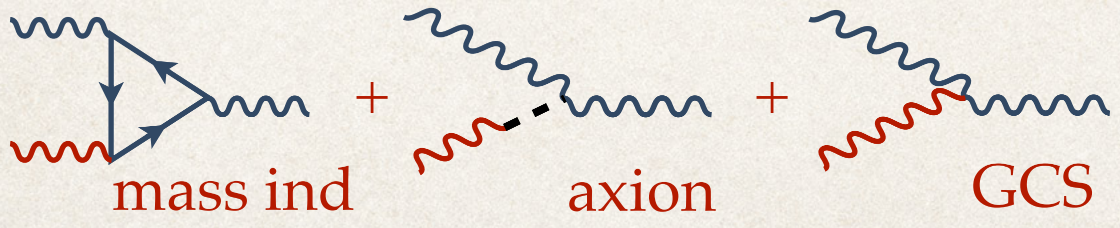
2-loop







the *anomalous coupling*



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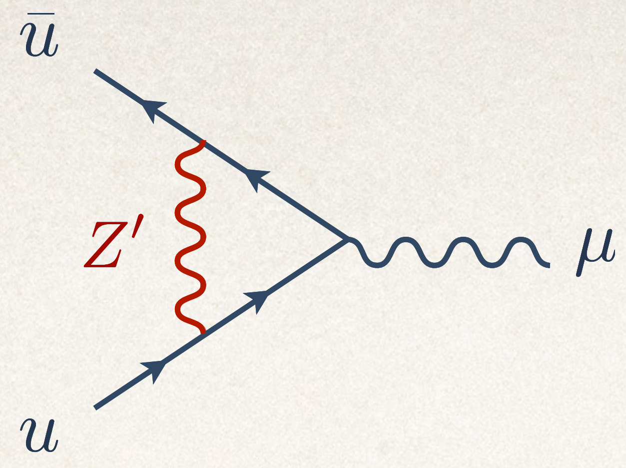
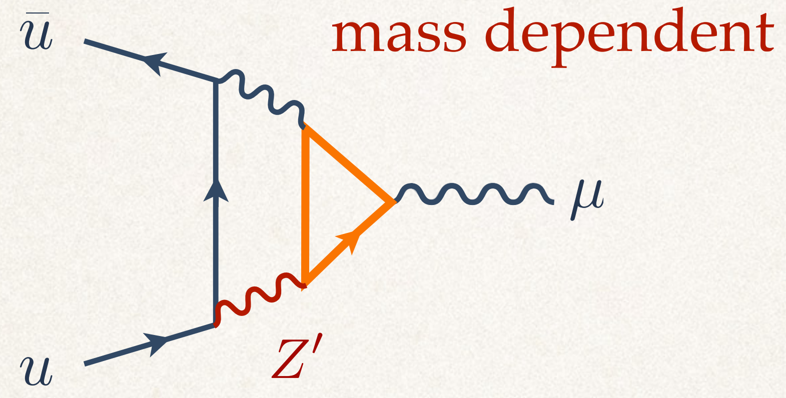
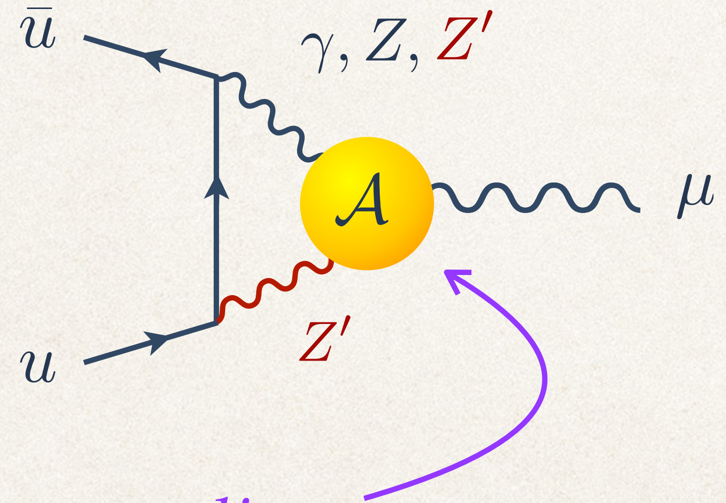
1-loop



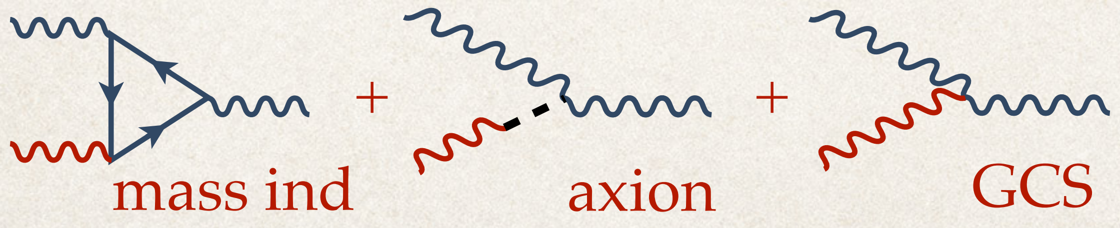
Message: there are areas in the parameter space where the 1-loop contribution is zero / tiny.

2-loop





the *anomalous coupling*



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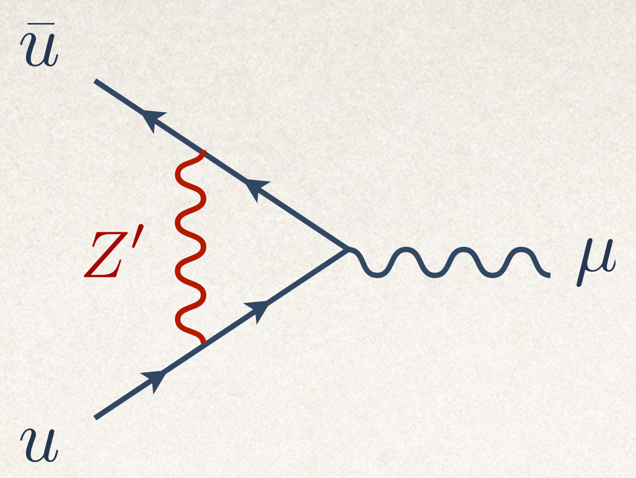
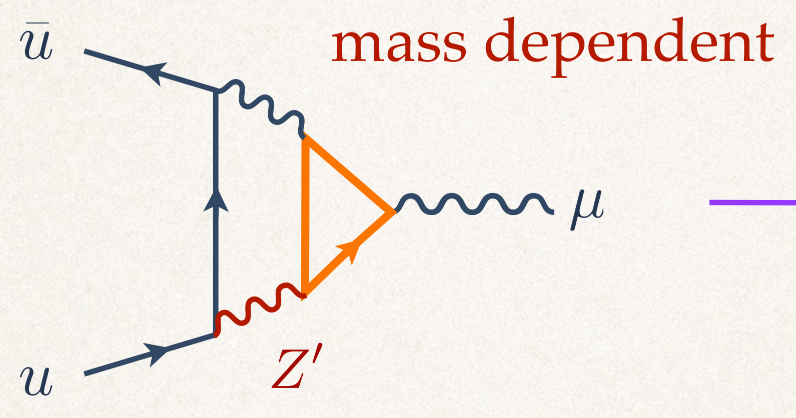
1-loop



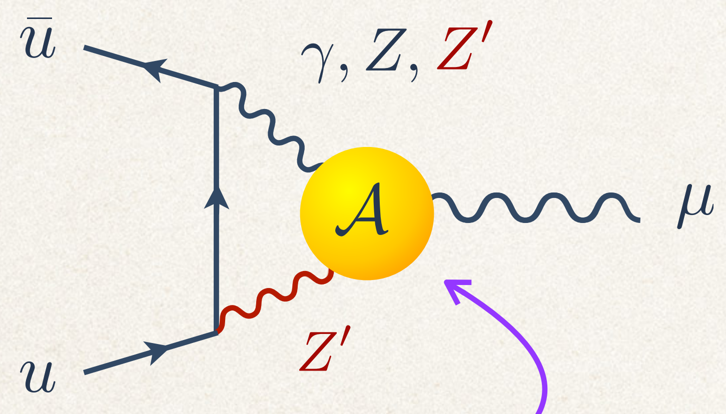
Message: there are areas in the parameter space where the 1-loop contribution is zero / tiny.

2-loop

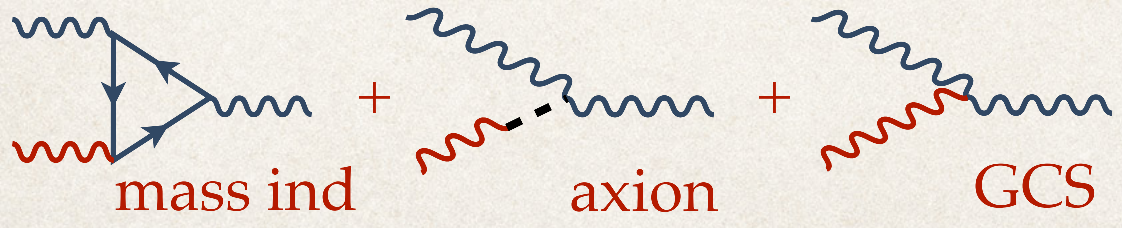


$$\Delta g_\mu^{\triangle(\text{mass dep})}[Z',\gamma]/2 \; = \; q_{V/A}^{\mu-Z'}\, q_{V/A}^{\mu-\gamma}\, q_{V/A}^{\mu-\gamma}\, q_{A}^{\mu-Z'}\; g_{Z'}^2\, g_{EM}^2 \times \ldots \text{ too large to present here}$$

the mass dependent part is proportional to the individual charges

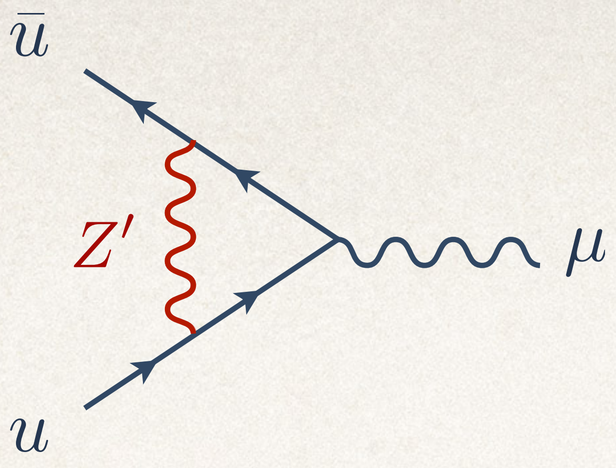


the *anomalous coupling*



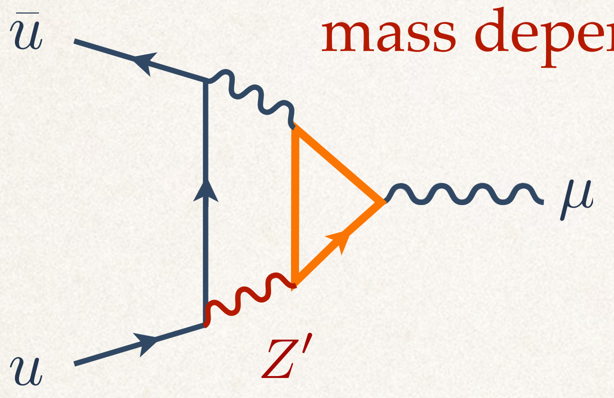
Anastasopoulos Kaneta Kiritsis Mambrini

1-loop



Message: there are areas in the parameter space where the 1-loop contribution is zero / tiny.

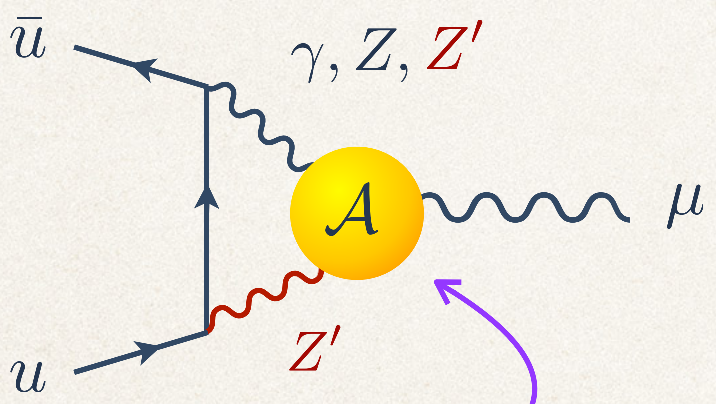
2-loop



the mass dependent part is proportional to the individual charges

$$\Delta g_\mu^{\triangle(\text{mass dep})}[Z',\gamma]/2 \;=\; q_{V/A}^{\mu-Z'}\, q_{V/A}^{\mu-\gamma}\, q_{V/A}^{\mu-\gamma}\, q_A^{\mu-Z'}\; g_{Z'}^2 g_{EM}^2 \times \ldots \text{ too large to present here}$$

these parts depend on the full anomaly



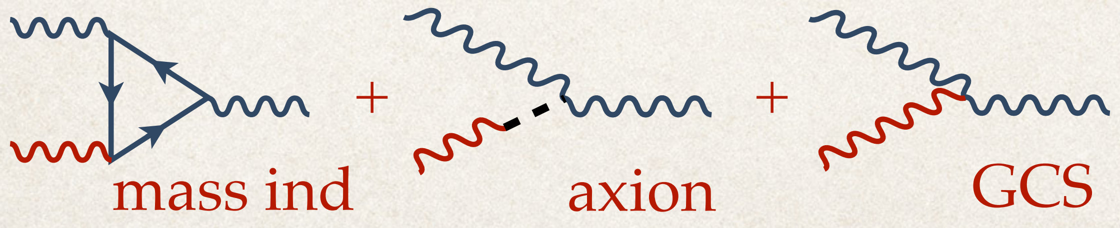
$$\Delta g_\mu^{\triangle(\text{mass ind})}[Z',\gamma]/2 \;=\; t_{Z'\gamma\gamma}\; q_A^{\mu-Z'}\; g_{Z'}^2 g_{EM}^2$$

$$\frac{m_\mu^2}{576\pi^4}\int_0^1 dx \int_0^{1-x} dy \left(\frac{(x+y+1)(2x+y-2)}{m_\mu^2 y^2 + m_{Z'}^2 x} - \frac{(x+y-2)(2x+y)}{m_{Z'}^2(x+y-1) - m_\mu^2 y^2} \right)$$

$$\Delta g_\mu^{\text{axion\&GCS}}[Z',\gamma]/2 \;=\; t_{Z'\gamma\gamma}\; q_A^{\mu-Z'}\; g_{Z'}^2 g_{EM}^2$$

$$\frac{m_\mu^2}{192\pi^4}\int_0^1 dx \int_0^{1-x} dy \left(\frac{(x+y-2)(2x+y)}{m_{Z'}^2(x+y-1) - m_\mu^2 y^2} - \frac{(1+x+y)(2x+y-2)}{m_{Z'}^2 x + m_\mu^2 y^2} \right)$$

the *anomalous coupling*



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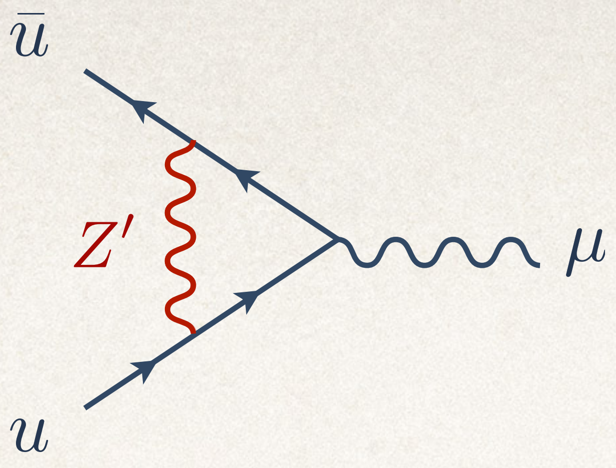
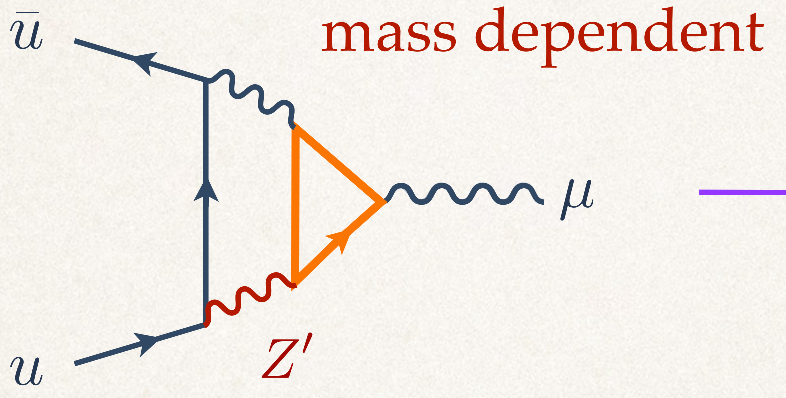
1-loop



Message: there are areas in the parameter space where the 1-loop contribution is zero / tiny.

2-loop

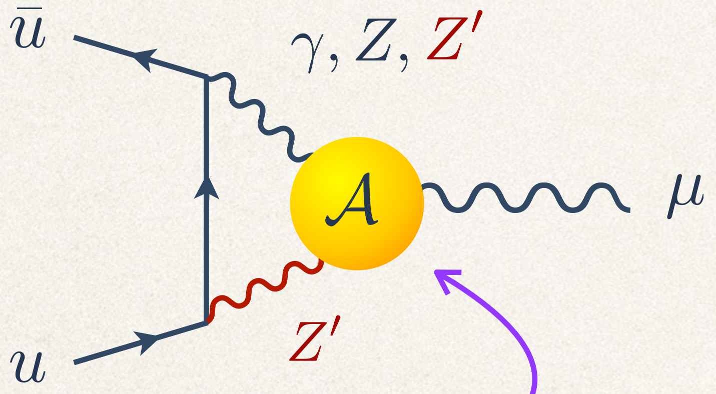


$$\Delta g_\mu^{\triangle(\text{mass dep})}[Z',\gamma]/2 = q_{V/A}^{\mu-Z'} q_{V/A}^{\mu-\gamma} q_{V/A}^{\mu-\gamma} q_A^{\mu-Z'} g_{Z'}^2 g_{EM}^2 \times \ldots \text{ too large to present here}$$

the mass dependent part is proportional to the individual charges

these parts depend on the full anomaly

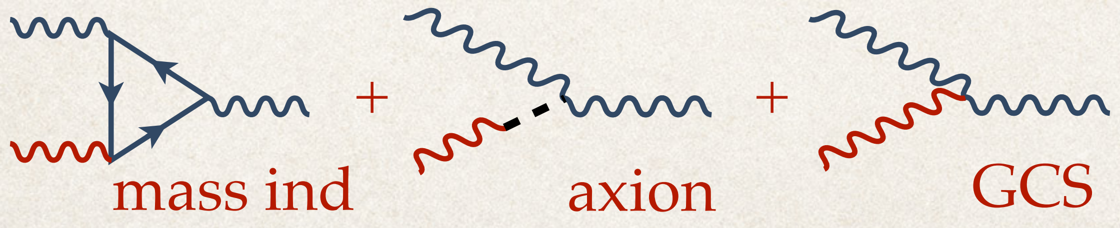
these parts depend only on the masses



$$\Delta g_\mu^{\triangle(\text{mass ind})}[Z',\gamma]/2 = t_{Z'\gamma\gamma}\, q_A^{\mu-Z'} g_{Z'}^2 g_{EM}^2 \; \frac{m_\mu^2}{576\pi^4}\int_0^1 dx \int_0^{1-x} dy \left(\frac{(x+y+1)(2x+y-2)}{m_\mu^2 y^2 + m_{Z'}^2 x} - \frac{(x+y-2)(2x+y)}{m_{Z'}^2(x+y-1) - m_\mu^2 y^2}\right)$$

$$\Delta g_\mu^{\text{axion\&GCS}}[Z',\gamma]/2 = t_{Z'\gamma\gamma}\, q_A^{\mu-Z'} g_{Z'}^2 g_{EM}^2 \; \frac{m_\mu^2}{192\pi^4}\int_0^1 dx \int_0^{1-x} dy \left(\frac{(x+y-2)(2x+y)}{m_{Z'}^2(x+y-1) - m_\mu^2 y^2} - \frac{(1+x+y)(2x+y-2)}{m_{Z'}^2 x + m_\mu^2 y^2}\right)$$

the *anomalous coupling*

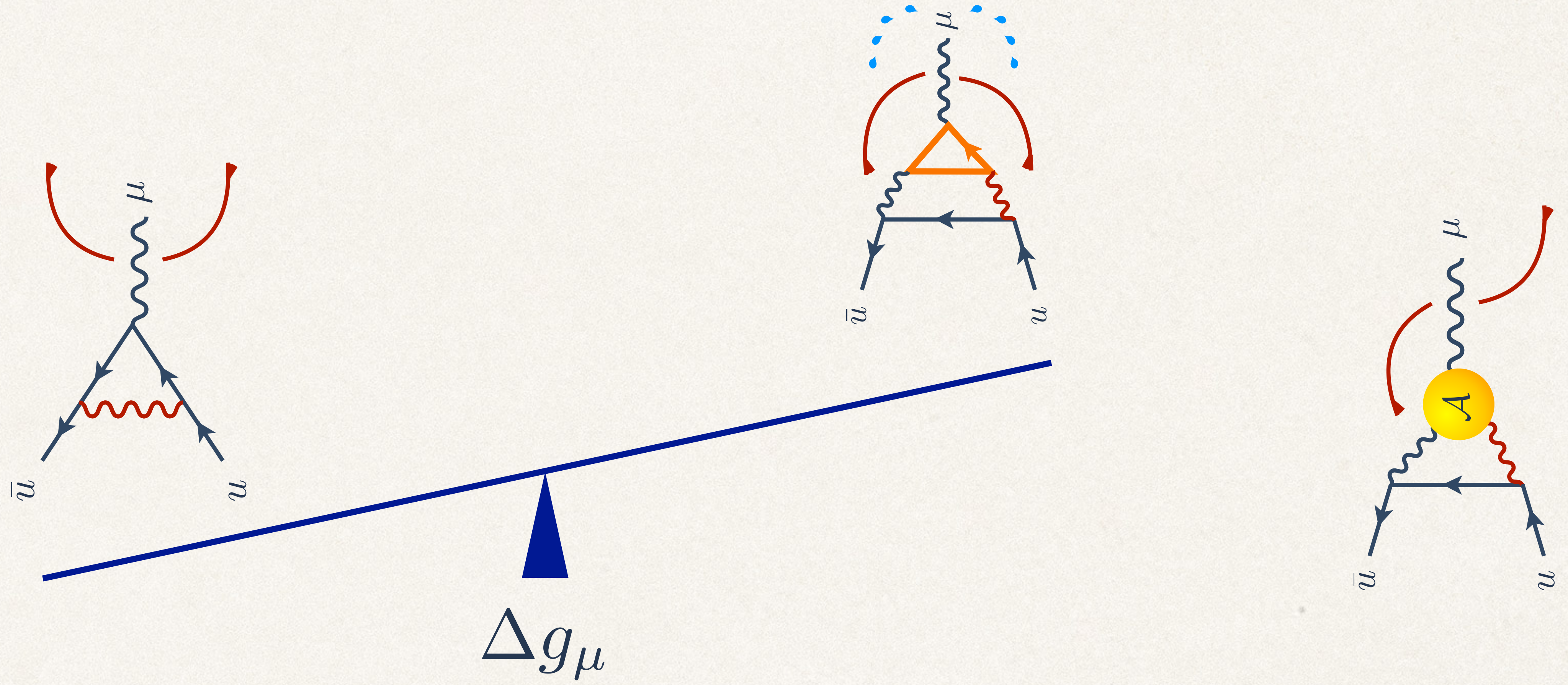


Anastasopoulos Kaneta Kiritsis Mambrini

On the balance

- The 1-loop diagram is leading (in general).
- However, we have seen that there are areas in the parameter space where it becomes tiny or zero.
- In these areas the 2-loop diagrams:
  - mass-dependent triangle part or
  - the anomalous contribution

  take over.
- Which one is the dominant?
- To answer, we have to choose some values for our parameters and compare.



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Free parameters

- The free parameters of the model (in agreement with experimental bounds)
  - mass of Z′ : $m_{Z'} = 10 - 100$ MeV
  - couplings (vectorial and axial)
    - relation between vectorial and axial coupling

$$q_V^{\mu-Z'} / q_A^{\mu-Z'} = 10$$

Remember: $q_V^{\mu-Z'} = q_L^{\mu-Z'} + q_R^{\mu-Z'}$
$q_A^{\mu-Z'} = q_L^{\mu-Z'} - q_R^{\mu-Z'}$
If $q_L^{\mu-Z'} \simeq q_R^{\mu-Z'}$ our assumption is reasonable

    - chosen range for the couplings

$$g_{Z'} q_V^{\mu-Z'} = 10 \; g_{Z'} q_A^{\mu-Z'} = (4 - 10) \times 10^{-4}$$

  - The anomaly

how big is the anomaly?!

$$t_{ijk} = \sum_f \left(q^f_{A,i} q^f_{V,j} q^f_{V,k} + q^f_{V,i} q^f_{A,j} q^f_{V,k} + q^f_{V,i} q^f_{V,j} q^f_{A,k} + q^f_{A,i} q^f_{A,j} q^f_{A,k} \right)$$

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How big is the anomaly?!

- The anomaly is

$$t_{ijk} = \sum_f \left(q^f_{A,i} q^f_{V,j} q^f_{V,k} + q^f_{V,i} q^f_{A,j} q^f_{V,k} + q^f_{V,i} q^f_{V,j} q^f_{A,k} + q^f_{A,i} q^f_{A,j} q^f_{A,k} \right)$$

- and depends on
  - the number of SM fermions : $f = \underset{\text{colors}}{3} \times \underset{\text{quarks}}{6} + \underset{\text{leptons}}{6} = 24$
  - the vectorial and axial charges of these fermions (remember: $q_V^{\mu-Z'} / q_A^{\mu-Z'} = 10$).

- Assuming an (no so) extreme case where all SM fermions have the same charges

$$\begin{array}{l} q_L = 0.6 \\ q_R = 0.5 \end{array} \rightarrow \ t_{ijk} = 10 \qquad \begin{array}{l} q_L = 1.3 \\ q_R = 1.0 \end{array} \rightarrow \ t_{ijk} = 100 \qquad \begin{array}{l} q_L = 2.2 \\ q_R = 1.8 \end{array} \rightarrow \ t_{ijk} = 500$$

- In the next, we will take the anomaly to be at the range: $t_{ijk} = 0 - 500$.

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Where is the cutoff?

- Up to which scale our action is valid, for these values of the parameters?
- The cutoff Λ is given by

$$\Lambda \lesssim 64\pi^3 \frac{M}{g_A(g_A^2 t_{AAA} + 2g_A g_Y t_{YAA} + g_Y^2 t_{AYY} + \sum_{i=2}^{3} g_i^2 T_{A,i})}$$

Preskill

- Using our parameters, and the max considered value for the anomaly ($t_{ijk} = 500$), the cutoff is

$$\Lambda \sim 40 \text{ TeV}$$

- It implies new physics (new massive charged fermions) within the reach of future experiments.

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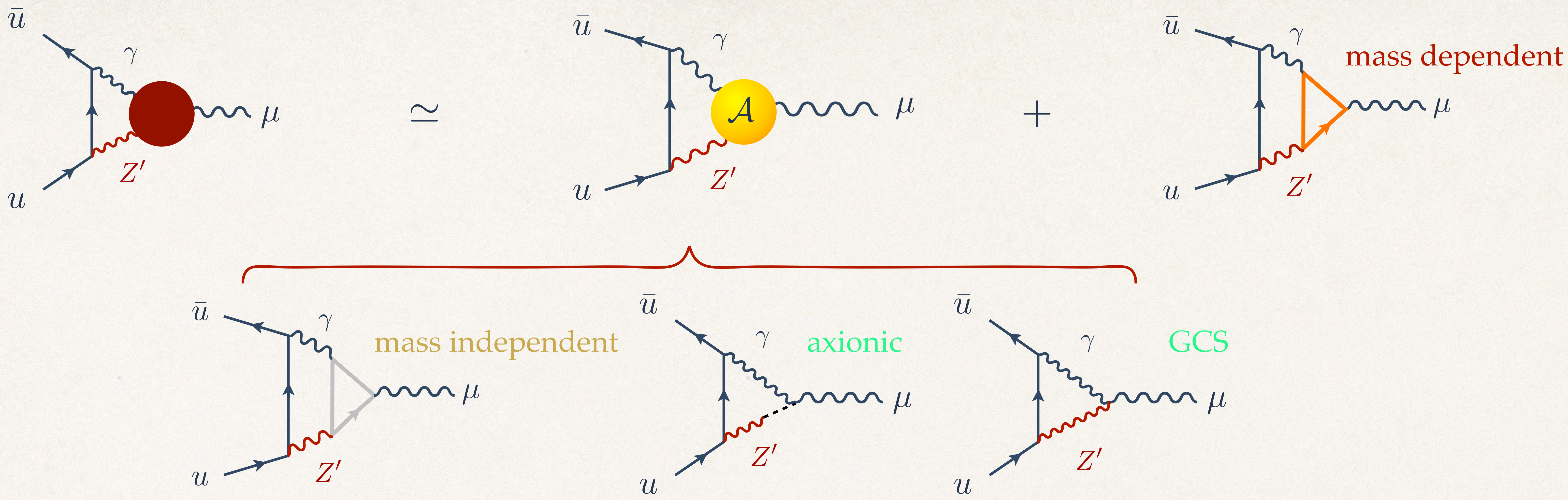

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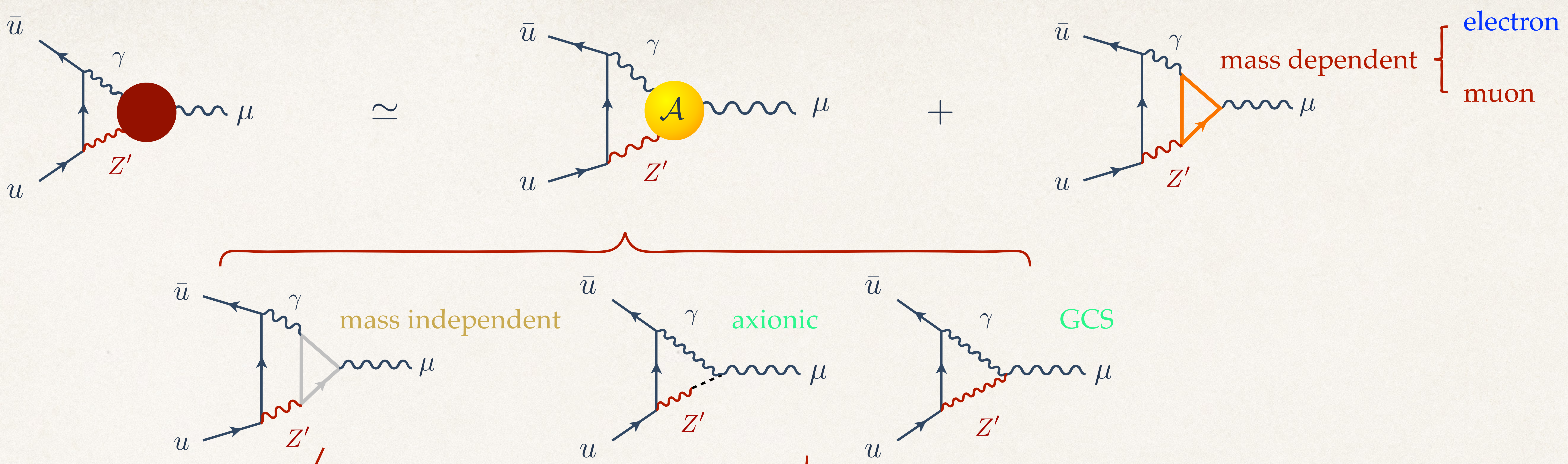
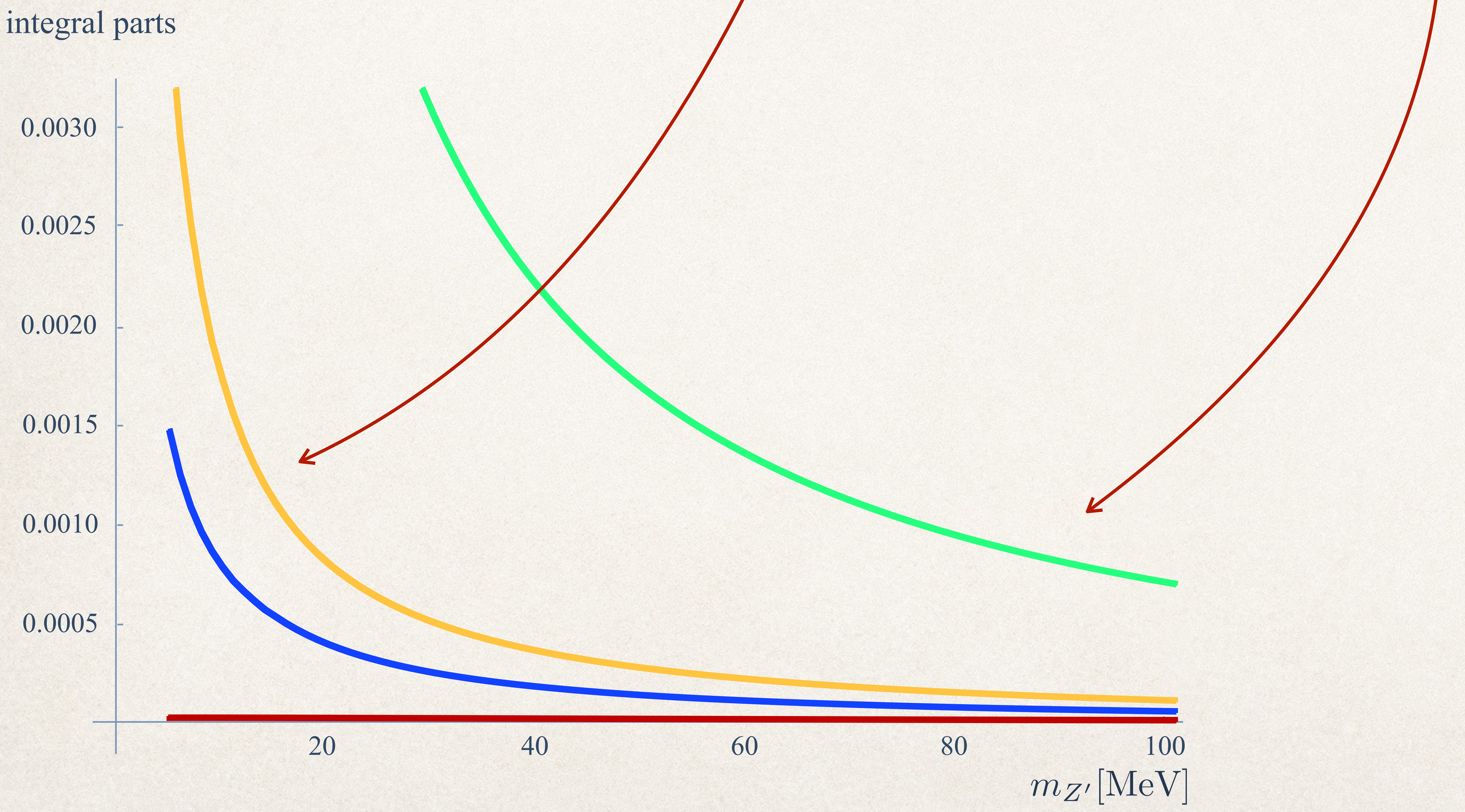
Z′
≃
𝒜
+
mass dependent
mass independent
axionic
GCS





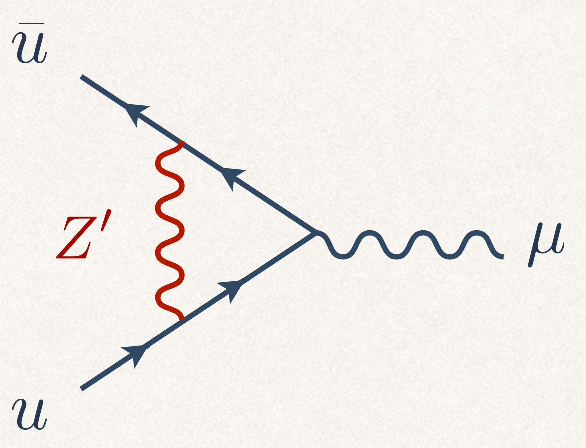
Comparing the integral (Z' mass dependant) part only.

1. The axionic & GCS terms are leading.
2. the mass independent part of the triangle is second.
3. the mass dependent is last.

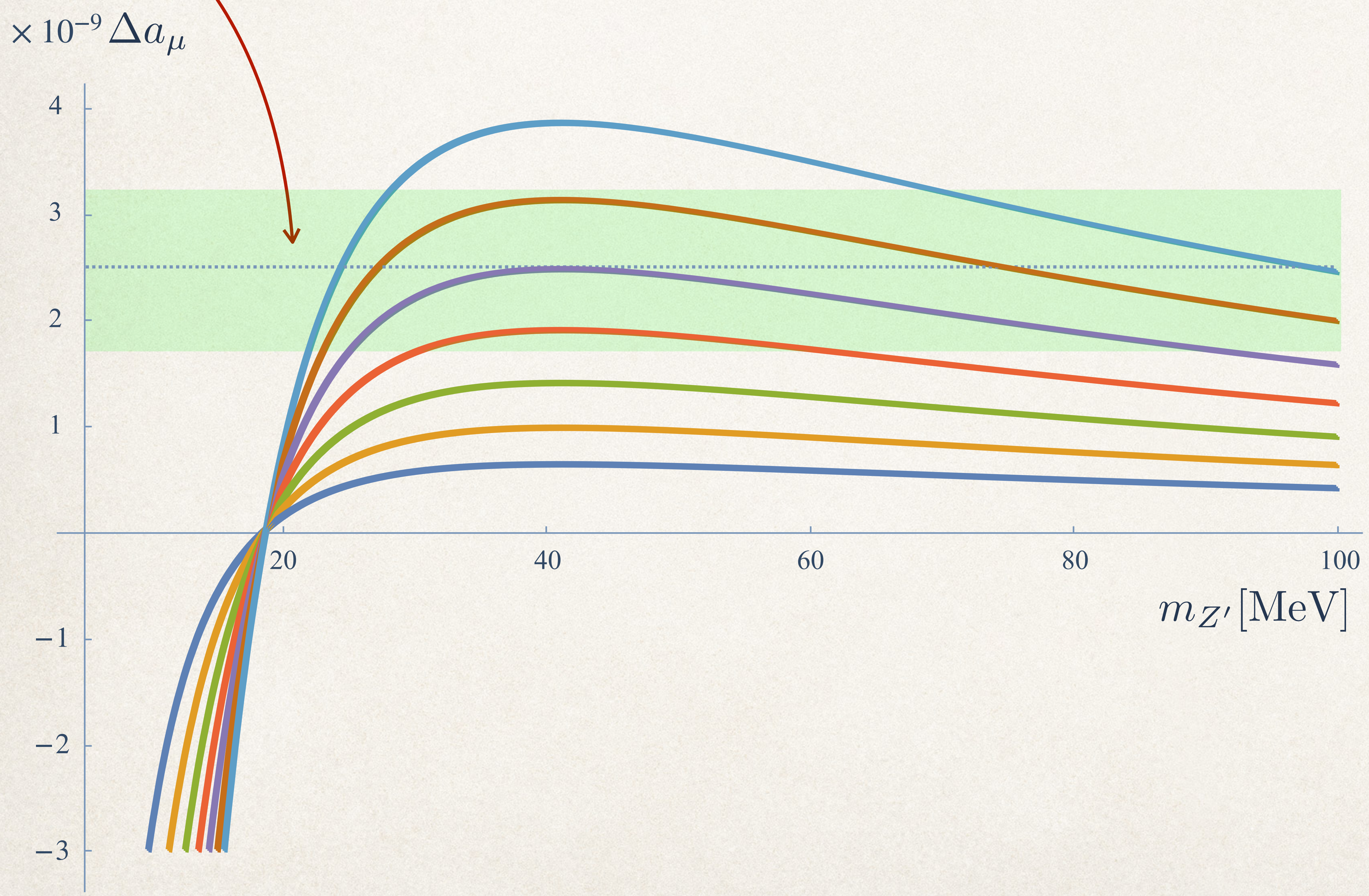
Thus, the anomaly related terms are dominant at 2-loops.

Anomalous Z′ and 1-loop vs 2-loops for the $g-2$ of the muon

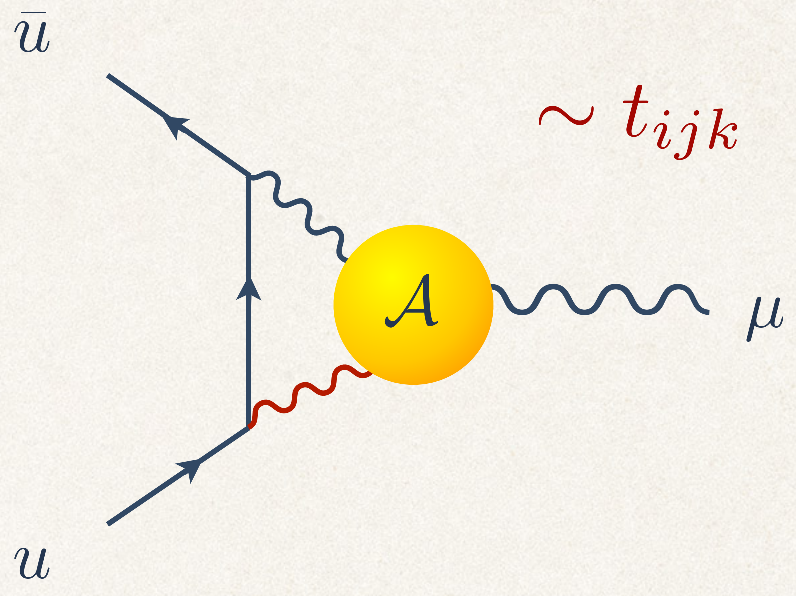
1-loop contribution



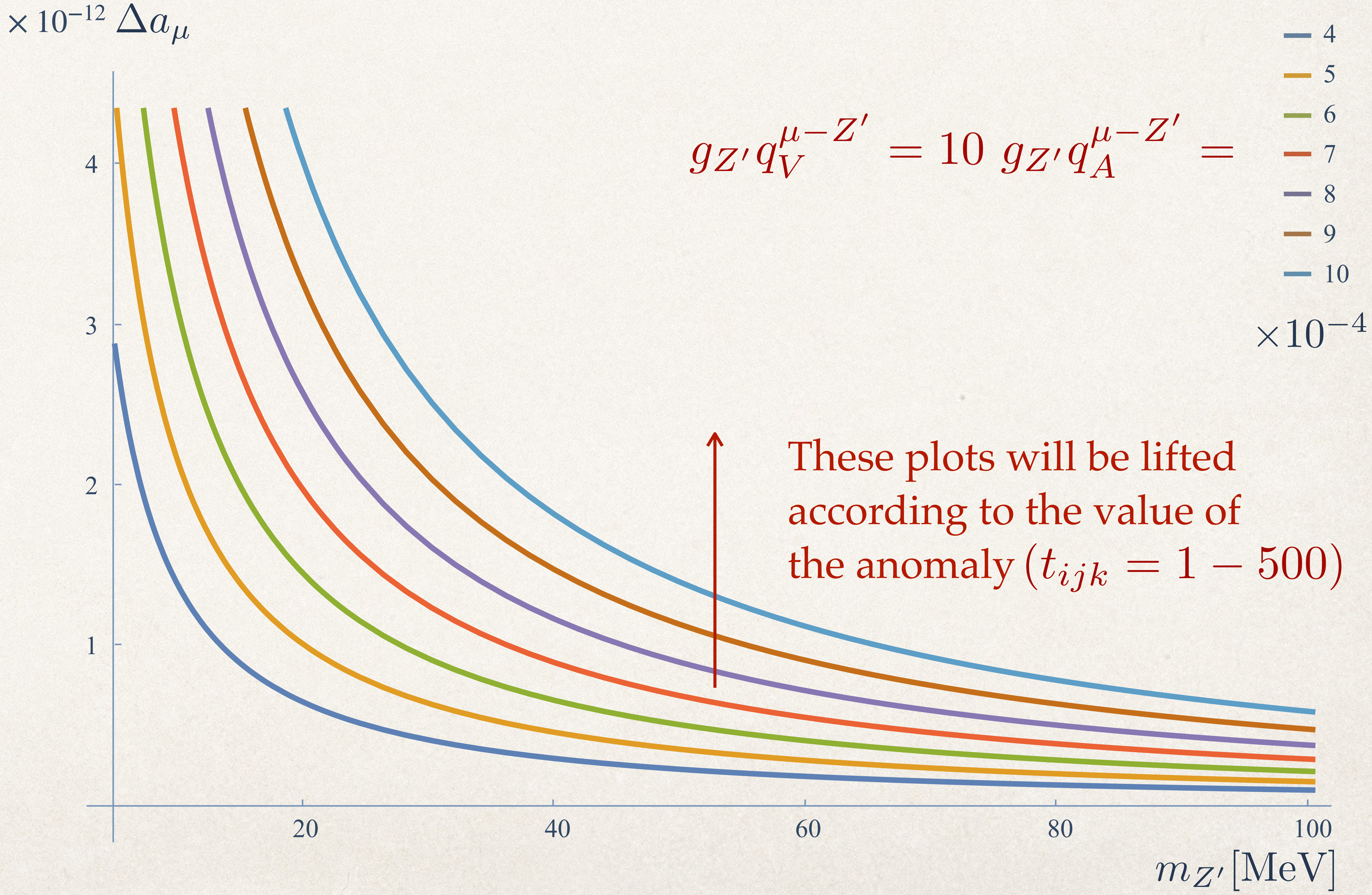
experimental value



Dominant 2-loop contribution



Setting $t_{ijk} = 1$



$$g_{Z'} q_V^{\mu - Z'} = 10\; g_{Z'} q_A^{\mu - Z'} = \times 10^{-4}$$

These plots will be lifted according to the value of the anomaly ($t_{ijk} = 1 - 500$)

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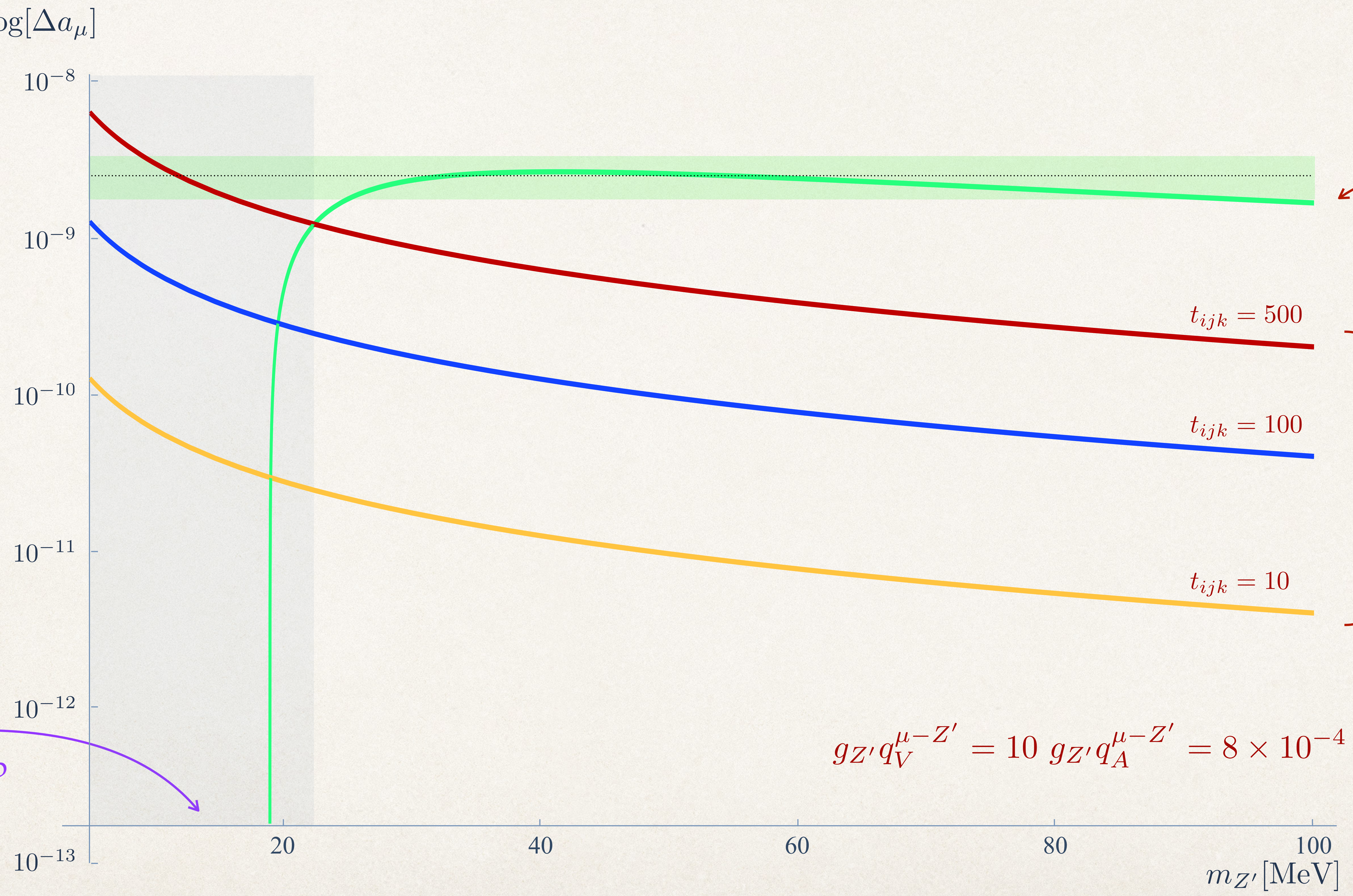
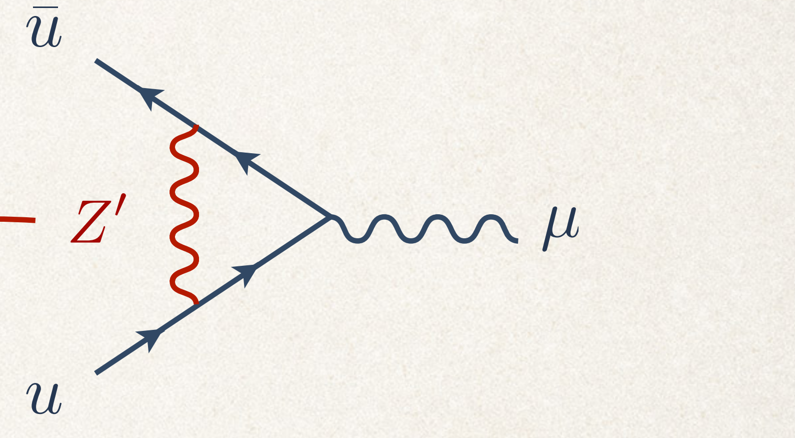
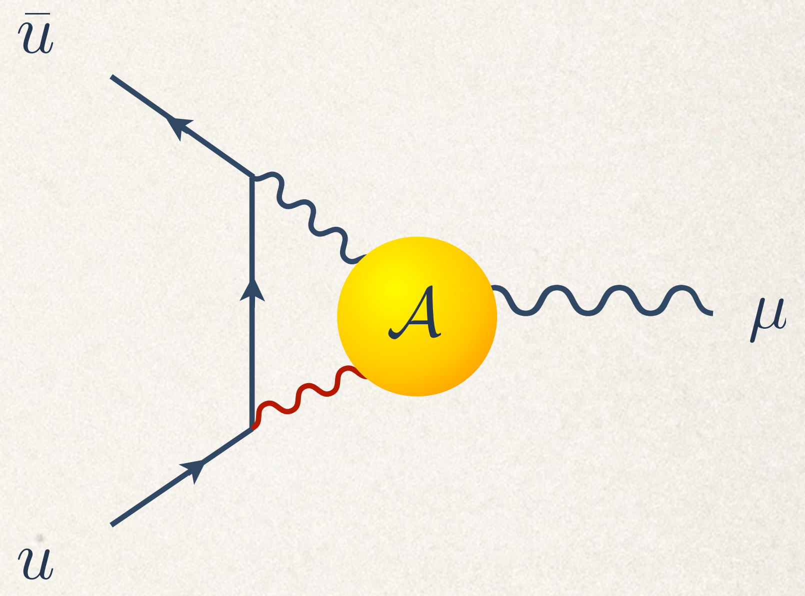
Comparing 1-loop and the anomalous 2-loop contributions



area where the *anomalous* 2-loop dominates the 1-loop

1-loop contribution





Dominant 2-loop contribution

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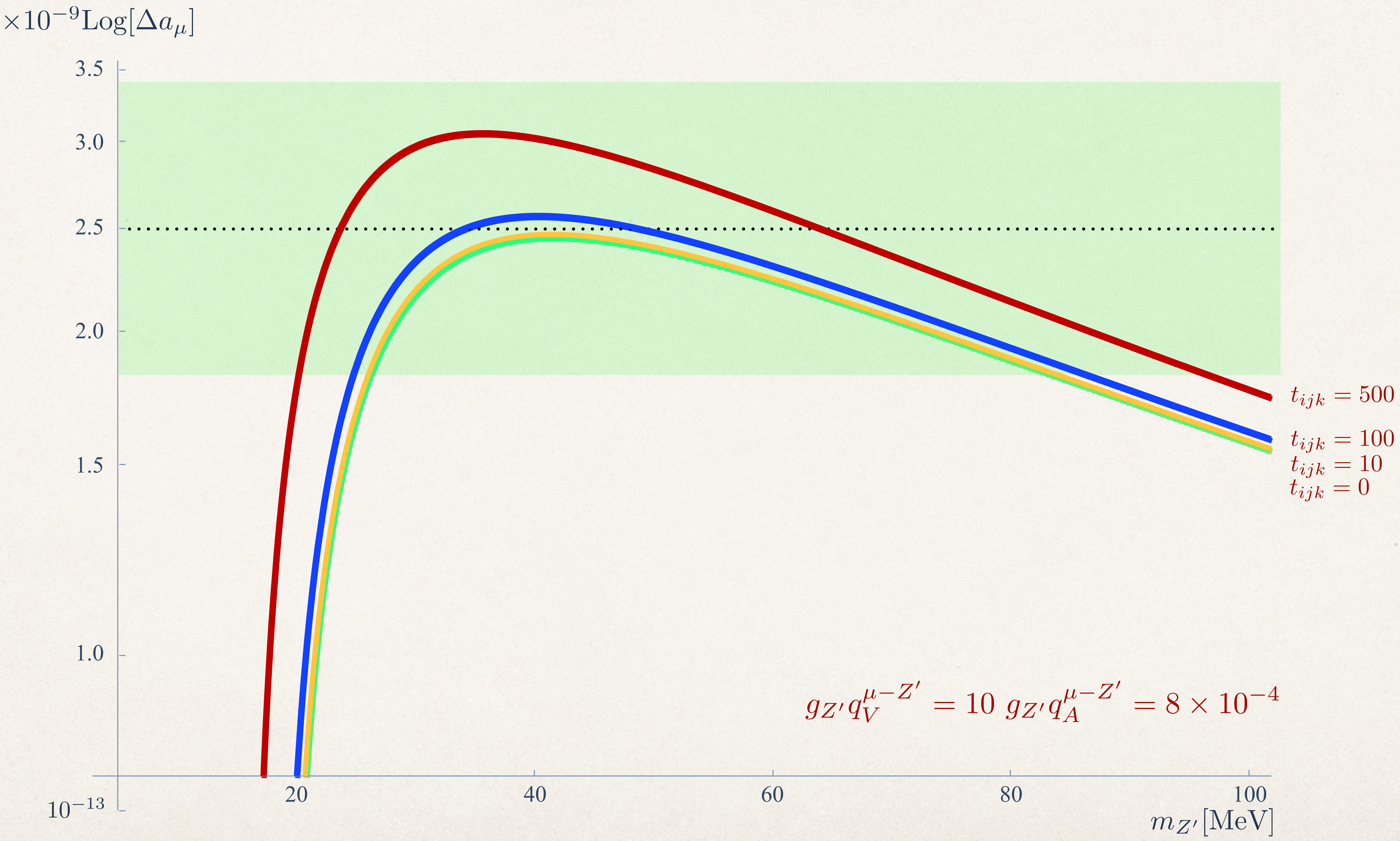
Total contribution to the $g-2$ from an anomalous Z'



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Conclusions and future plans

Conclusions and future plans

- Standard Model is an effective theory.
- If there is an extra gauge boson Z', most likely, will be (superficially) anomalous.
- The anomalies of the spectrum are cancelled by effective terms from the UV part of the theory.
- These new anomaly-generated couplings have interesting phenomenology.

Conclusions and future plans

- We have studied the effects of an anomalous Z' to the $g-2$ of the muon.
- An anomalous Z' can explain the discrepancy between the theoretical and experimental values.
- The 1-loop contribution of an anomalous/non-anomalous Z' is usually dominant.
- At 2-loop, the dominant diagram is the anomalous diagram.
- There are areas of the parameter space where the anomalous diagram dominates the 1-loop contribution.

Conclusions and future plans

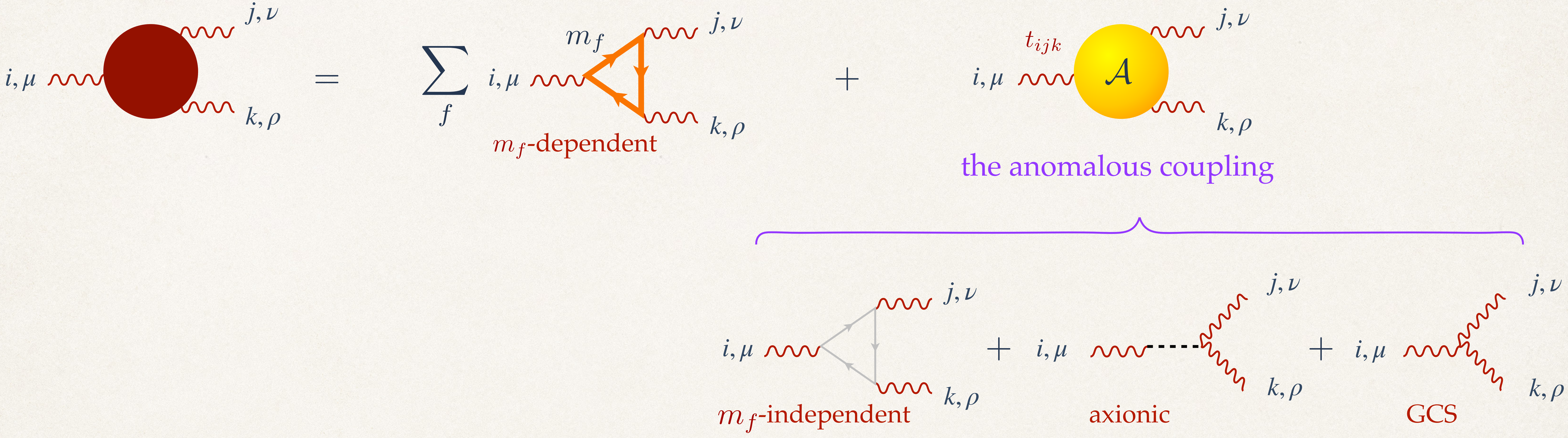
- Anomalies affect also couplings between only SM gauge fields (apart from the Z').
- For example, $\gamma\gamma Z$, $\gamma Z Z$, $Z W^+ W^-$, $\gamma Z W^+ W^-$, Z gluons gluons, Z gluons gluons gluons, …
- Further phenomenological analysis is needed and is under way.

Thank you

Scheme dependance and GCS terms

- It is known that anomalies depend on the scheme.
- We can move the anomaly of the triangle diagram from one leg to another (or spread to all of them).
- That can happened by adding a GCS term (similar to ours).
- How much the scheme changes our analysis here?

Scheme dependance and GCS terms



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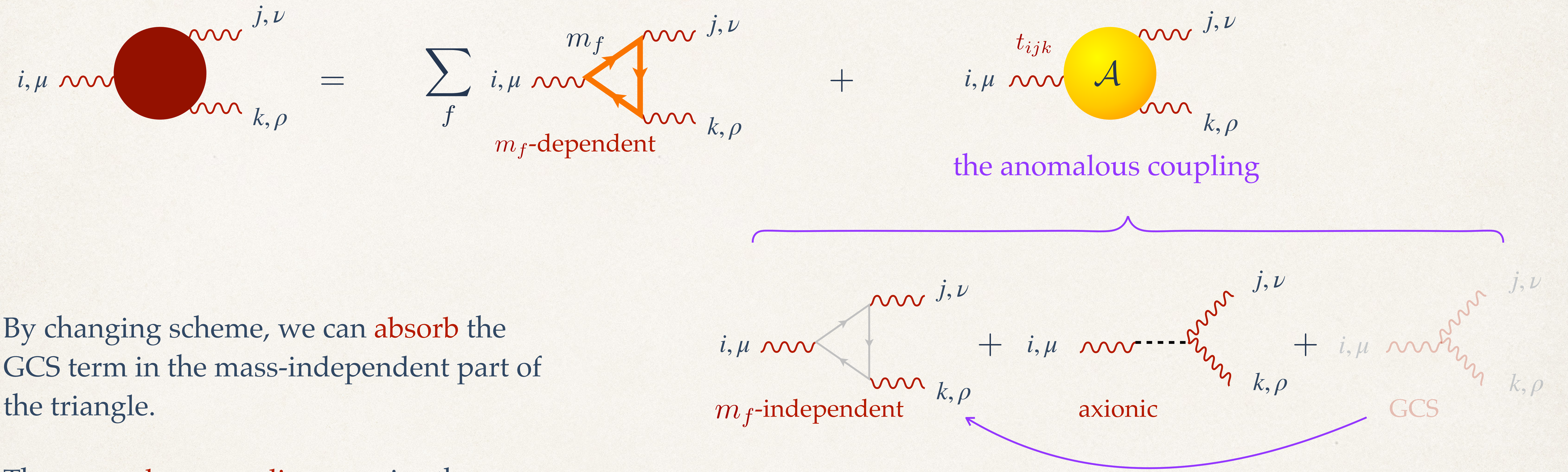
Scheme dependance and GCS terms



- By changing scheme, we can absorb the GCS term in the mass-independent part of the triangle.
- The anomalous coupling remains the same.
- Our analysis is scheme independent.

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