

Georgi-Machacek and Beyond

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Outline

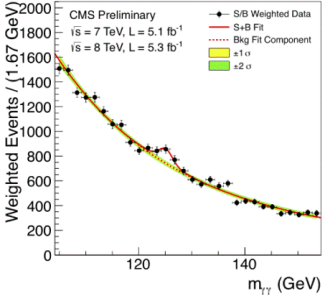
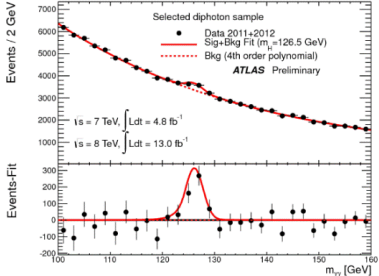
- 1 Introductory motivations
- 2 The Model
 - pre-custodial
 - Georgi-Machacek
 - The ρ parameter
 - The custodial multiplets
 - $\delta\rho$ in Georgi-Machacek
- 3 'conclusion'

Introductory motivations

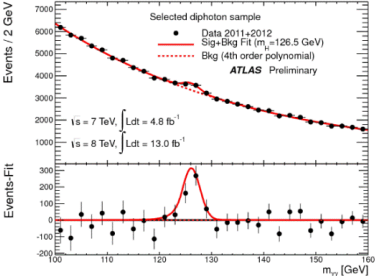
Many motivations to go beyond the SM, that we all know!

Introductory motivations

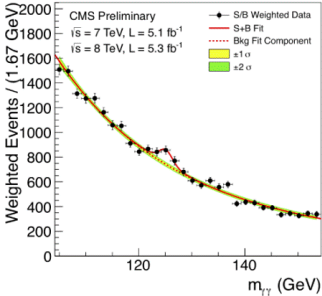
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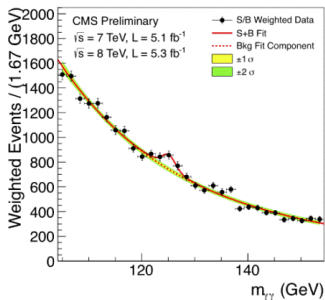
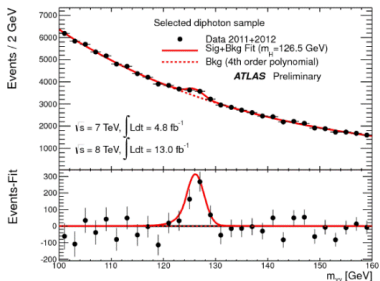
Introductory motivations



a theorem by Nelson Mandela
(not to be confused with the Coleman-Mandula theorem)



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→ Some SM-extensions with scalar triplets can relate this to Majorana neutrinos masses; e.g. type-II seesaw,

$$\mathcal{L}_{Yukawa} \supset Y_\nu L^T C \otimes i\sigma_2 \begin{matrix} A & L \\ \downarrow & \\ SU(2)_L \text{ triplet} & \end{matrix}$$

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$H \sim (1, 2, 1), A \sim (1, 3, 2), B \sim (1, 3, 0)$ under $SU(3)_c \times SU(2)_L \times U(1)_Y$.

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$$H = \begin{pmatrix} \phi^+ \\ \phi^0 \end{pmatrix}, A = \frac{1}{\sqrt{2}} \begin{pmatrix} \chi^+/\sqrt{2} & -\chi^{++} \\ \chi^0 & -\chi^+/\sqrt{2} \end{pmatrix}, B = \frac{1}{\sqrt{2}} \begin{pmatrix} \xi^0/\sqrt{2} & -\xi^+ \\ -\xi^{+*} & -\xi^0/\sqrt{2} \end{pmatrix}$$

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The most general, **renormalizable**, $SU(2)_L \times U(1)_Y$ -invariant Lagrangian:

$$H \rightarrow e^{i\alpha} \mathcal{U}_L H, \quad A \rightarrow e^{i2\alpha} \mathcal{U}_L A \mathcal{U}_L^\dagger, \quad B \rightarrow \mathcal{U}_L B \mathcal{U}_L^\dagger,$$

$$\mathcal{L} = (D_\mu H)^\dagger (D^\mu H) + \text{Tr}[(D_\mu A)^\dagger (D^\mu A)] + \frac{1}{2} \text{Tr}[(D_\mu B)^\dagger (D^\mu B)] - V(H, A, B) \\ + \mathcal{L}_{\text{Yukawa}} + \dots$$

The model: pre-custodial

$$V_{\text{p-c}} = V_{\text{p-c}}^{(2,3)} + V_{\text{p-c}}^{(4)}$$

dim-2, -3 operators

$$\begin{aligned} V_{\text{p-c}}^{(2,3)} = & -m_H^2 H^\dagger H + M_A^2 \text{Tr}(AA^\dagger) + \frac{1}{2} M_B^2 \text{Tr}(B^2) \\ & + [\mu_A (H^\top i \sigma^2 A^\dagger H) + \text{h.c.}] + \mu_B H^\dagger B H + \mu_{AB} \text{Tr}(AA^\dagger B), \end{aligned}$$

dim-4 operators

$$\begin{aligned} V_{\text{p-c}}^{(4)} = & \frac{\lambda_H}{4} (H^\dagger H)^2 + \frac{\lambda_A^{(1)}}{4} (\text{Tr} AA^\dagger)^2 + \frac{\lambda_A^{(2)}}{4} \text{Tr}(AA^\dagger)^2 + \frac{\lambda_B}{4!} (\text{Tr} B^2)^2 \\ & + \lambda_{AH}^{(1)} H^\dagger H \text{Tr} AA^\dagger + \lambda_{AH}^{(2)} H^\dagger AA^\dagger H + \frac{\lambda_{BH}}{2} H^\dagger H \text{Tr} B^2 \\ & + \frac{\lambda_{AB}^{(1)}}{2} \text{Tr} AA^\dagger \text{Tr} B^2 + \frac{\lambda_{AB}^{(2)}}{2} \text{Tr} AB \text{Tr} A^\dagger B \\ & + \frac{i}{2} \lambda_{ABH} (H^\top \sigma^2 A^\dagger B H - H^\dagger B A \sigma^2 H^*). \end{aligned}$$

The model: pre-custodial \rightarrow Georgi-Machacek

To increase the symmetry of the **purely** scalar sector to $SU(2)_L \times SU(2)_R^{\text{global}}$, correlations (fine-tuning) are needed among (tree-level) mass parameters and couplings \rightarrow GM model:

$$M_A^2 = M_B^2, \quad \mu_B = \sqrt{2}\mu_A,$$

$$\lambda_A^{(1)} = 2\lambda_{AB}^{(1)} + 3\lambda_{AB}^{(2)}, \quad \lambda_A^{(2)} = -2\lambda_{AB}^{(2)}, \quad \lambda_{ABH} = \sqrt{2}\lambda_{AH}^{(2)},$$

$$\lambda_B = 3(\lambda_{AB}^{(1)} + \lambda_{AB}^{(2)}), \quad \lambda_{BH} = \lambda_{AH}^{(1)} + \frac{1}{2}\lambda_{AH}^{(2)}.$$

The model: Georgi-Machacek

$$\mathcal{L} = \frac{1}{2} \text{Tr}[(D_\mu \Phi)^\dagger (D^\mu \Phi)] + \frac{1}{2} \text{Tr}[(D_\mu X)^\dagger (D^\mu X)] - V_{\text{G-M}} + \mathcal{L}_{\text{Yukawa}} + \dots$$

$$V_{\text{G-M}} = V_{\text{G-M}}^{(2,3)} + V_{\text{G-M}}^{(4)}$$

$$V_{\text{G-M}}^{(2,3)} = \frac{\mu_2^2}{2} \text{Tr} \Phi^\dagger \Phi + \frac{\mu_3^2}{2} \text{Tr} X^\dagger X \\ - \left(M_1 \text{Tr} \Phi^\dagger \tau^a \Phi \tau^b + M_2 \text{Tr} X^\dagger t^a X t^b \right) (UXU^\dagger)_{ab}$$

$$V_{\text{G-M}}^{(4)} = \hat{\lambda}_1 (\text{Tr} \Phi^\dagger \Phi)^2 + \hat{\lambda}_2 \text{Tr}(\Phi^\dagger \Phi) \text{Tr}(X^\dagger X) + \hat{\lambda}_3 \text{Tr}(X^\dagger X X^\dagger X) \\ + \hat{\lambda}_4 (\text{Tr} X^\dagger X)^2 - \hat{\lambda}_5 \text{Tr}(\Phi^\dagger \tau^a \Phi \tau^b) \text{Tr}(X^\dagger t^a X t^b)$$

$$\Phi \equiv \begin{pmatrix} H^c & H \\ \phi^{0*} & \phi^+ \\ -\phi^{+*} & \phi^0 \end{pmatrix}, X \equiv \begin{pmatrix} A^c & B & A \\ \chi^{0*} & \xi^+ & \chi^{++} \\ -\chi^{+*} & \xi^0 & \chi^+ \\ \chi^{+++} & -\xi^{+*} & \chi^0 \end{pmatrix}$$

The model: Georgi-Machacek

...the gauge/scalar sector

$$\frac{1}{2} \text{Tr}[(D_\mu \Phi)^\dagger (D^\mu \Phi)] + \frac{1}{2} \text{Tr}[(D_\mu X)^\dagger (D^\mu X)] - V_{\text{G-M}}$$

invariant under: $\Phi \rightarrow U_L \Phi U_R^\dagger$ & $X \rightarrow U_L X U_R^\dagger$ for $g'_{U(1)_Y} \rightarrow 0$

$$\text{SU}(2)_L \times \text{SU}(2)_R \xrightarrow[\text{EWSB}]{??} \text{SU}(2)_{L+R} \equiv \text{SU}(2)_{\text{custodial}}$$

only if $U_{L+R} \langle X \rangle U_{L+R}^\dagger = \langle X \rangle$, i.e. iff $\langle A \rangle = \langle B \rangle$.

The ρ parameter

Recall:
$$\rho := \frac{M_W^2}{M_Z^2 \cos^2 \theta_W},$$

“ ρ_0^{Fit} ” = 1.00038 ± 0.00020 (PDG2020)

$$\rho_{tree}^{theo} = \frac{\sum v_i^2 [I_i(I_i + 1) - \frac{1}{4} Y_i^2]}{\sum \frac{1}{2} v_i^2 Y_i^2} \text{ (@tree - level, general)}$$

→ depends on the group reps. and possibly on the vacuum structure.

→ $\rho_{tree}^{theo} = 1, \forall v_i$ for:

- $I_i = \frac{1}{2}, 0; Y_i = 1, 0$, Higgs doublet + replicas + singlets,
- but also for more exotic replicas, $I_i = 3, \frac{25}{2}, 48, \frac{361}{2}, \dots; Y_i = 4, 15, 56, 209, \dots$

→ or, $\rho_{tree}^{theo} = 1$ for special relations between the v_i → dynamics?
e.g. SM + 1 complex and 1 real n-plets, with $n > 2$

- $I_A = I_B = 1, 8, 49, \dots, Y_A = 2, 12, 70, \dots, Y_B = 0$;
- $I_A = I_B = 2, 24, \dots, Y_A = 3, 30, \dots, Y_B = 0$;
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Let us look under the light of symmetries



The custodial multiplets in Georgi-Machacek

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$H_5^{++}, H_5^+, H_5^0, H_5^-, H_5^{--}$

H_3^+, H_3^0, H_3^- ,

$H_1^0, H_1^{0'} \rightarrow h^0, H^0$

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4 physical masses:

- 5 degenerate states with mass m_5
- 3 degenerate states with mass m_3
- m_{h^0}, m_{H^0}

and some selection rules,

e.g. $H_{3,5}^0 \not\rightarrow h^0 h^0, H^0 H^0, h^0 H^0$, no coupling of H_5 to fermions, ...

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But rad. corr. break $SU(2)_{\text{custodial}}$ and the degeneracy and the selection rules...

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In particular the presence of a doubly-charged state opens up possibilities not allowed by $SU(2)_L$ doublets/singlets:
e.g. unitarity in the WW scattering → sum rules:

$$g_{WWh}^2 \underset{\text{(SM)}}{=} \sum_i g_{W^+W^-\phi_i^0}^2 - \sum_k |g_{W^-W^-\phi_k^{0++}}|^2$$

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→ this and other similar sum-rules allow non-standard neutral scalar states to have larger couplings to WW (ZZ) than in the SM!

$\delta\rho$ in Georgi-Machacek

→ radiative corrections to the ρ parameter: different status of custodial symmetry breaking with triplets as compared to doublets!

- in the SM beyond tree-level, $\rho = 1 + \delta\rho$ with $\delta\rho$ finite and uniquely defined
- in the GM model $\delta\rho$ is divergent and needs to be renormalized. Also, divergent $H_5 - H_3$, $H_5^+ - W^+$ mixings.

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→ many of the pheno features of the GM become renormalization scheme and scale dependent...(including precision observables, $\delta\rho$, M_W , etc.).

'conclusion'

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They might however still need to be supplemented by extra doublets/singlets, if some recent excesses at CMS and/or ATLAS end up satisfying Nelson Mandela's theorem!

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La parole est à François