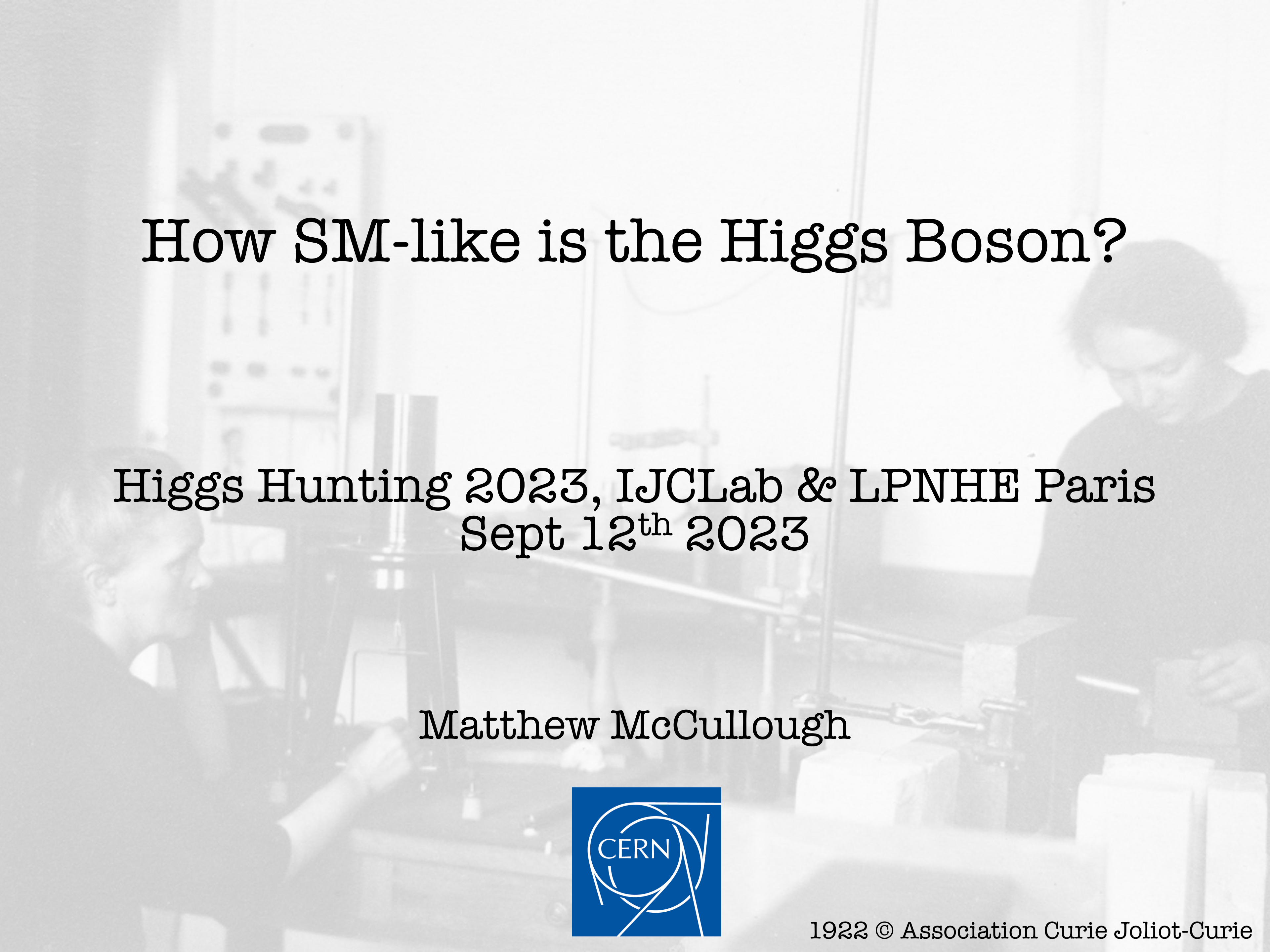
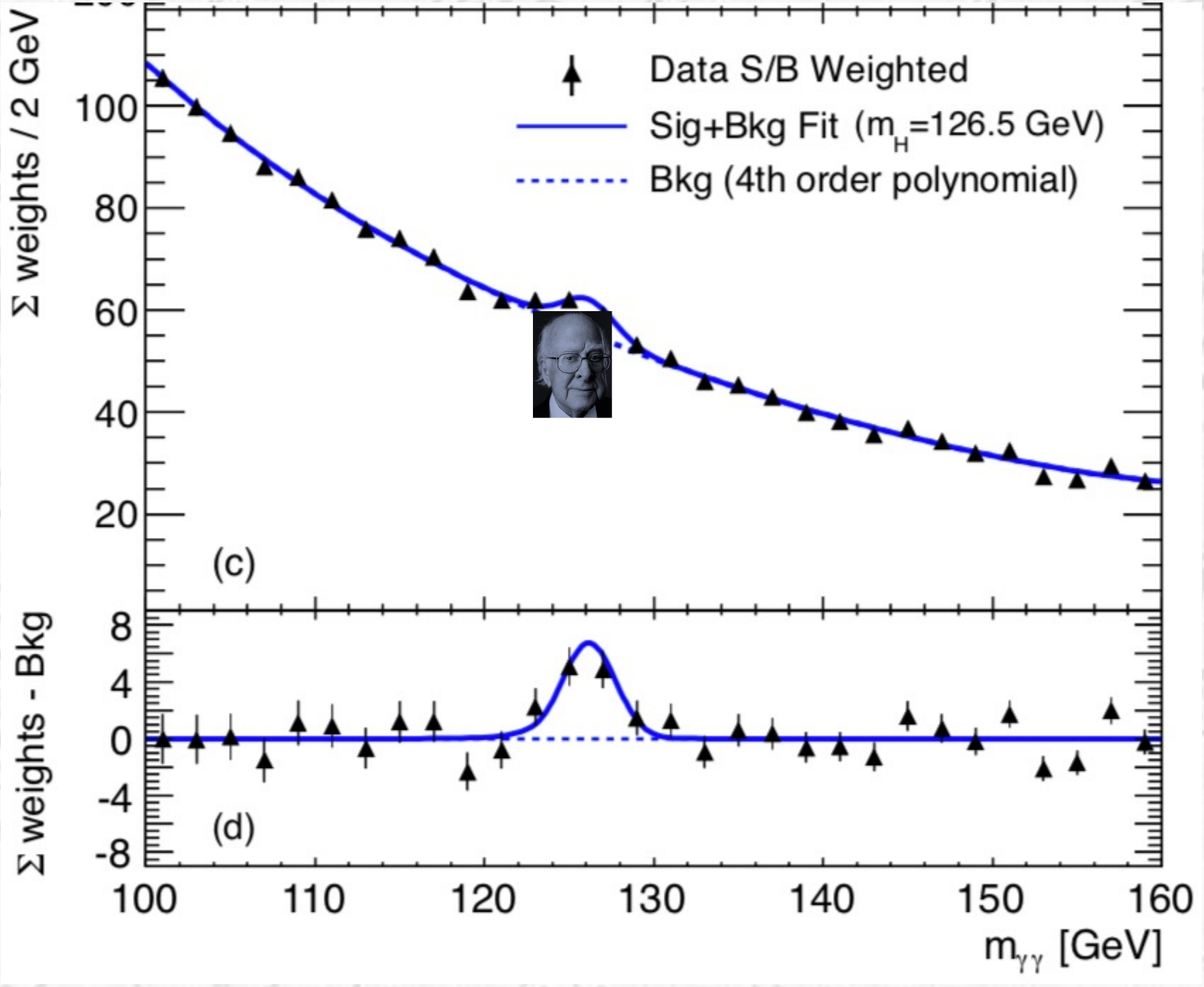


How SM-like is the Higgs Boson?

Higgs Hunting 2023, IJCLab & LPNHE Paris
Sept 12th 2023

Matthew McCullough

How well do we know the Higgs?



Organizing the Unknown

To understand the origin and nature of the Higgs boson, we need to study how it behaves.

$$\frac{c_T}{\ldots}(H^\dagger \overleftrightarrow{D}^\mu H)^2 \qquad \mathcal{O}_W = \frac{ig\, c_W}{2M^2}(H^\dagger \sigma^a \overleftrightarrow{D}^\mu H)D^\nu W^a_{\mu\nu} \qquad \mathcal{O}_{2B} = -\frac{c_{2B}}{4M^2}(\partial_\rho B_{\mu\nu})^2$$

$$c_\Box \ldots |\Box H|^2 \qquad \mathcal{O}_{WW} = \frac{g^2\, c_{WW}}{M^2}|H|^2 W^{a\,\mu\nu} W^a_{\mu\nu}$$

$$\ldots \frac{\ldots^2\, c_{GG}}{\ldots}|H|^2 G^{a,\mu\nu} G^a_{\mu\nu}$$

$$\mathcal{O}_B = \frac{ig'\, c_B}{2M^2}(H^\dagger \ldots H) \ldots$$

$$\mathcal{O}_H = \frac{c_H}{2M^2}\left(\partial^\mu |H|^2\right) \ldots \qquad \mathcal{O}_R = \frac{\ldots}{M^2} \ldots$$

$$\mathcal{O}_{BB} = \frac{g'^2\, c_{BB}}{M^2}|H|^2 B^{\mu\nu} B_{\mu\nu}$$

$$\mathcal{O}_{2W} = -\frac{c_{2W}}{4M^2}(D_\rho W^a_{\mu\nu})^2 \qquad \mathcal{O}_{WB} = \frac{gg'\, c_{WB}}{M^2} H^\dagger \sigma^a H B^{\mu\nu} W^a_{\mu\nu}$$

Let us suppose that any new physics is heavy. Not necessary, just for our purposes.

Operators like those above capture leading effects of heavy physics beyond the standard model. Probing them could reveal origins.

Organizing the Unknown

To understand the origin and nature of the Higgs boson, we need to study how it behaves.

$$\mathcal{O}_T = \frac{c_T}{2M^2}(H^\dagger \overleftrightarrow{D}^\mu H)^2 \qquad \mathcal{O}_W = \frac{ig\, c_W}{2M^2}(H^\dagger \sigma^a \overleftrightarrow{D}^\mu H) D^\nu W^a_{\mu\nu} \qquad \mathcal{O}_{2B} = -\frac{c_{2B}}{4M^2}(\partial_\rho B_{\mu\nu})^2$$

$$\mathcal{O}_{2G} = -\frac{c_{2G}}{4M^2}(D_\rho G^a_{\mu\nu})^2 \qquad \mathcal{O}_\Box = \frac{c_\Box}{M^2}|\Box H|^2 \qquad \mathcal{O}_{WW} = \frac{g^2\, c_{WW}}{M^2}|H|^2 W^{a\,\mu\nu} W^a_{\mu\nu}$$

$$\mathcal{O}_B = \frac{ig'\, c_B}{2M^2}(H^\dagger \overleftrightarrow{D}^\mu H)\partial^\nu B_{\mu\nu} \qquad \mathcal{O}_6 = \frac{c_6}{M^2}|H|^6 \qquad \mathcal{O}_{GG} = \frac{g_s^2\, c_{GG}}{M^2}|H|^2 G^{a,\mu\nu} G^a_{\mu\nu}$$

$$\mathcal{O}_H = \frac{c_H}{2M^2}\left(\partial^\mu |H|^2\right)^2 \qquad \mathcal{O}_R = \frac{c_R}{M^2}|H|^2 |D^\mu H|^2$$

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Operators like those above capture leading effects of heavy physics beyond the standard model. Probing them could reveal origins.

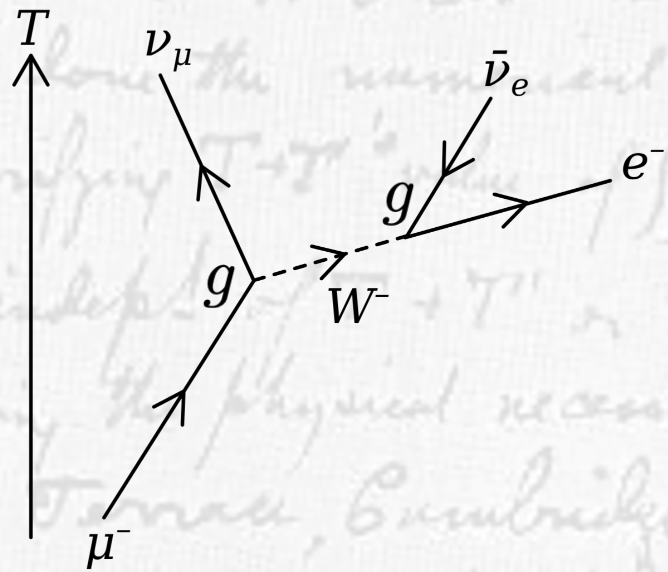
Organizing the Unknown

Naïve dimensional analysis:

$$[H] = [A_\mu] = \frac{1}{LC} \quad , \quad [\psi] = \frac{1}{L^{3/2}C}$$

Fields carry not only dimension of inverse length, but also inverse coupling.

Example: Muon Decay



Fermi Scale

Interaction: $\mathcal{L} \sim \frac{\psi^4}{\Lambda^2}$

Dimension: $[\Lambda] = [G_F^{-1/2}] = \frac{[M_W]}{[g]}$

UV-completion

Coupling

Organizing the Unknown

Higgs Only

$[g_*^0]$

$\mathcal{O}_\Box = \frac{c_\Box}{M^2}|\Box H|^2$

$[g_*^2]$

$\mathcal{O}_H = \frac{c_H}{2M^2}\left(\partial^\mu |H|^2\right)^2$

$\mathcal{O}_T = \frac{c_T}{2M^2}(H^\dagger \overleftrightarrow{D}^\mu H)^2$

$\frac{c_R}{\ldots}|H|^2|D^\mu H|^2$

$[g_*^4]$

$\mathcal{O}_6 = \frac{c_6}{M^2}|H|^6$

Any new physics concerning primarily with Higgs and gauge sectors matches, at leading order, to these operators.

$\mathcal{O}_{2G} = -\frac{c_{2G}}{4M^2}(D_\rho G^a_{\mu\nu})^2$

Mixed

$\mathcal{O}_B = \frac{ig' c_B}{2M^2}(H^\dagger \overleftrightarrow{D}^\mu H)\partial^\nu B_{\mu\nu}$

$\mathcal{O}_W = \frac{ig\, c_W}{2M^2}(H^\dagger \sigma^a \overleftrightarrow{D}^\mu H)D^\nu W^a_{\mu\nu}$

$\mathcal{O}_{GG} = \frac{g_s^2 c_{GG}}{M^2}|H|^2 G^{a,\mu\nu}G^a_{\mu\nu}$

$\mathcal{O}_{WB} = \frac{gg' c_{WB}}{M^2}H^\dagger \sigma^a H B^{\mu\nu}W^a_{\mu\nu}$

$\mathcal{O}_{WW} = \frac{g^2 c_{WW}}{M^2}|H|^2 W^{a\,\mu\nu}W^a_{\mu\nu}$

$\mathcal{O}_{BB} = \frac{g'^2 c_{BB}}{M^2}|H|^2 B^{\mu\nu}B_{\mu\nu}$

Organizing the Unknown

Higgs Only

$[g_*^0]$

$$\mathcal{O}_\Box = \frac{c_\Box}{M^2}|\Box H|^2$$

$[g_*^2]$

$$\mathcal{O}_H = \frac{c_H}{2M^2}\left(\partial^\mu |H|^2\right)^2$$

$$\mathcal{O}_T = \frac{c_T}{2M^2}(H^\dagger \overleftrightarrow{D}^\mu H)^2$$

$$\mathcal{O}_R = \frac{c_R}{M^2}|H|^2|D^\mu H|^2$$

$[g_*^4]$

$$\mathcal{O}_6 = \frac{c_6}{M^2}|H|^6$$

Gauge Only

$$\mathcal{O}_{2G} = -\frac{c_{2G}}{4M^2}(D_\rho G^a_{\mu\nu})^2$$

$$\mathcal{O}_{2W} = -\frac{c_{2W}}{4M^2}(D_\rho W^a_{\mu\nu})^2$$

$$\mathcal{O}_{2B} = -\frac{c_{2B}}{4M^2}(\partial_\rho B_{\mu\nu})^2$$

Mixed

$$\mathcal{O}_B = \frac{ig'\, c_B}{2M^2}(H^\dagger \overleftrightarrow{D}^\mu H)\partial^\nu B_{\mu\nu}$$

$$\mathcal{O}_W = \frac{ig\, c_W}{2M^2}(H^\dagger \sigma^a \overleftrightarrow{D}^\mu H)D^\nu W^a_{\mu\nu}$$

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$$\mathcal{O}_{WW} = \frac{g^2\, c_{WW}}{M^2}|H|^2 W^{a\,\mu\nu}W^a_{\mu\nu}$$

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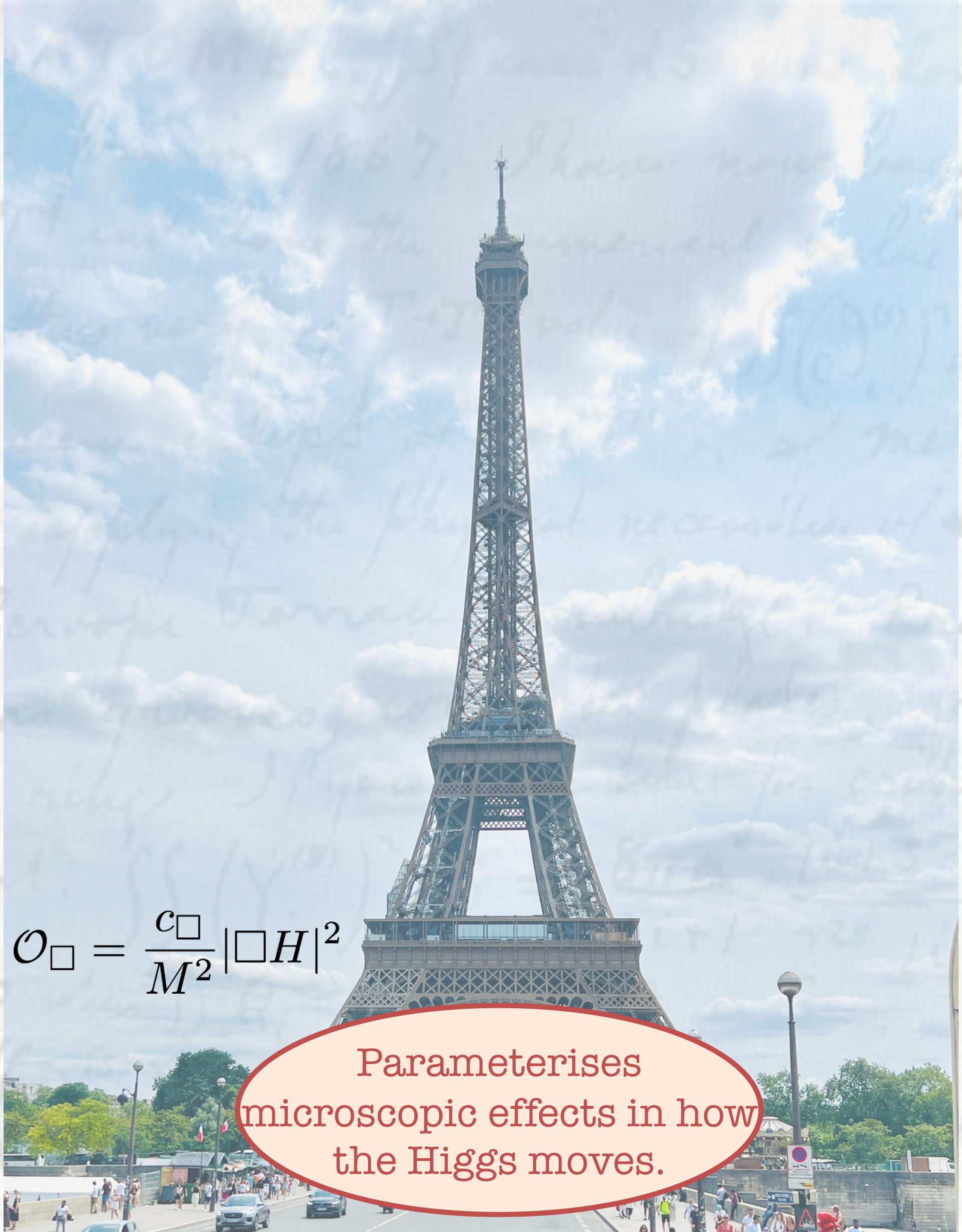
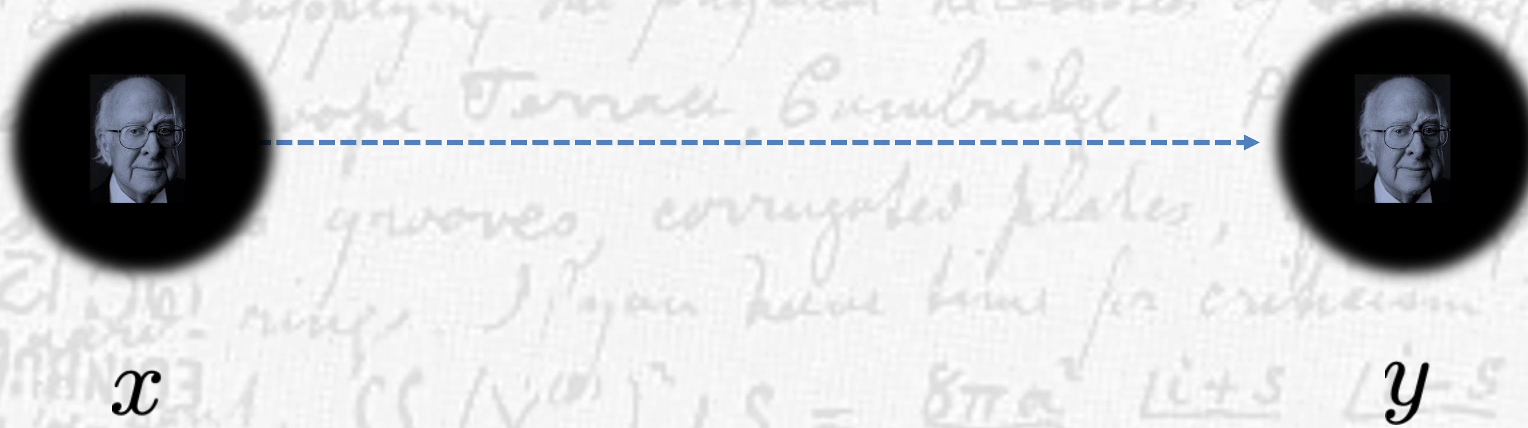


$$\mathcal{O}_\Box = \frac{c_\Box}{M^2}|\Box H|^2$$

Parameterises microscopic effects in how the Higgs moves.

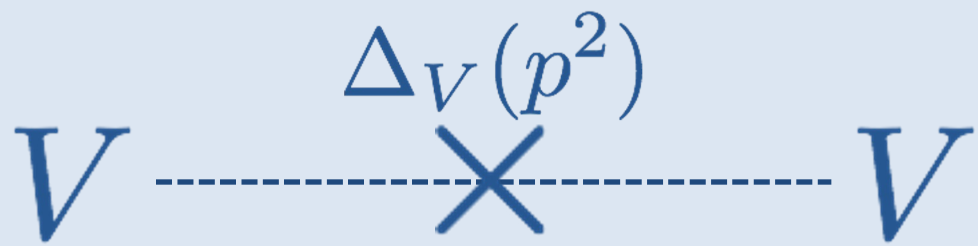
How does the Higgs move?



"Oblique" Corrections

Oblique corrections have formerly been a formidable toolkit in the effort to explore propagation in the electroweak sector.

- S-parameter
- T-parameter
- W-parameter
- Y-parameter



The latter two contribute to processes in an "energy-growing" manner:

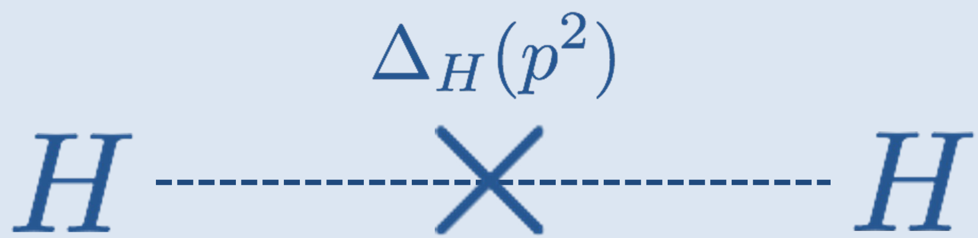
$$\Delta_W(p^2) \approx \frac{1}{p^2 - M_W^2} - \frac{\hat{W}}{M_W^2}$$

Making these oblique parameters an excellent target for high energy colliders...

“Oblique” Corrections

Makes sense to extend to the Higgs sector. Especially since the Higgs can easily interact with new states…

- H-parameter:



1903.07725

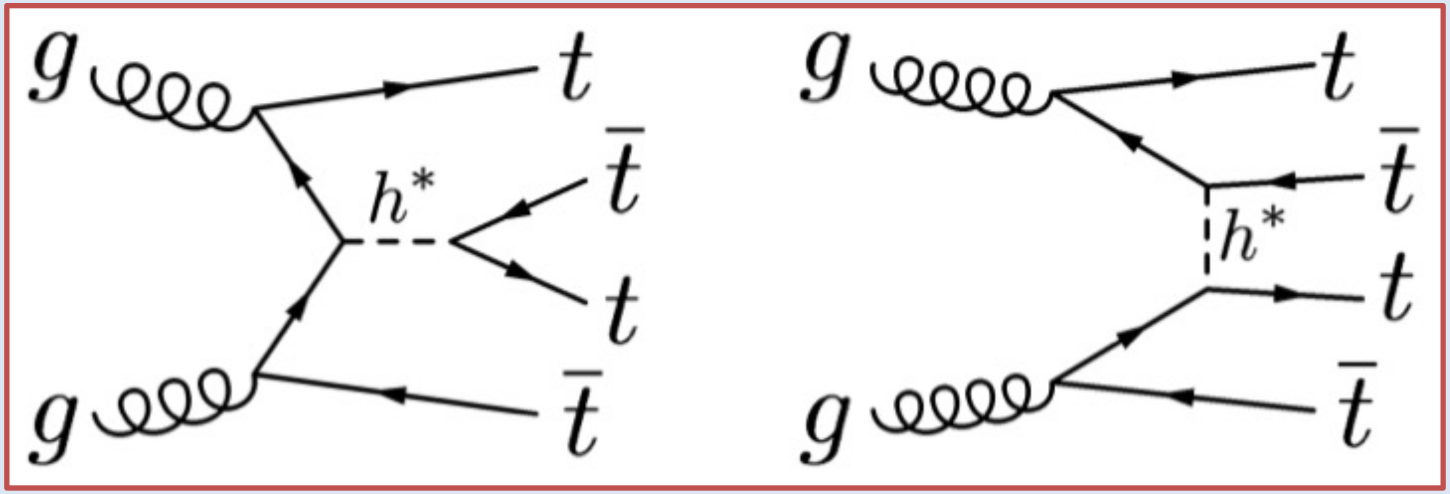
This also contributes to processes in an “energy-growing” manner:

$$\Delta_H(p^2) \approx \frac{1}{p^2 - m_h^2} - \frac{\hat{H}}{m_h^2} + \ldots$$

However, one needs to take the Higgs momentum far from mass-scale, which isn’t easy…

Oblique Corrections

Most promising avenue to take the Higgs momentum high is through four-top production:

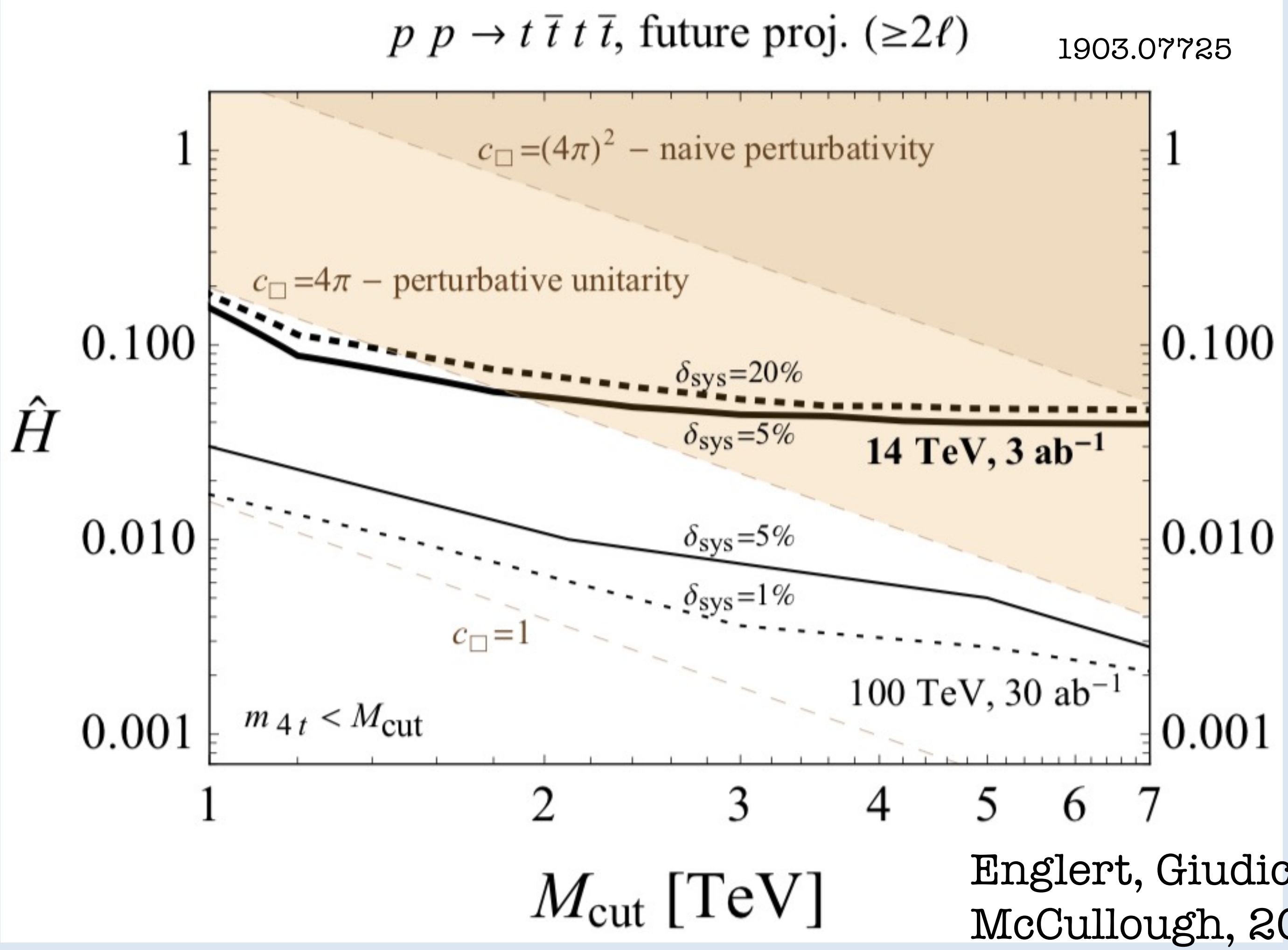


We may relate the effective field theory coefficient to the scale of new physics as:

$$\frac{\hat{H}}{m_h^2} = \frac{c_\Box}{M^2}$$

Oblique Corrections

Our estimate suggests the practical way to probe this special operator is with future colliders:

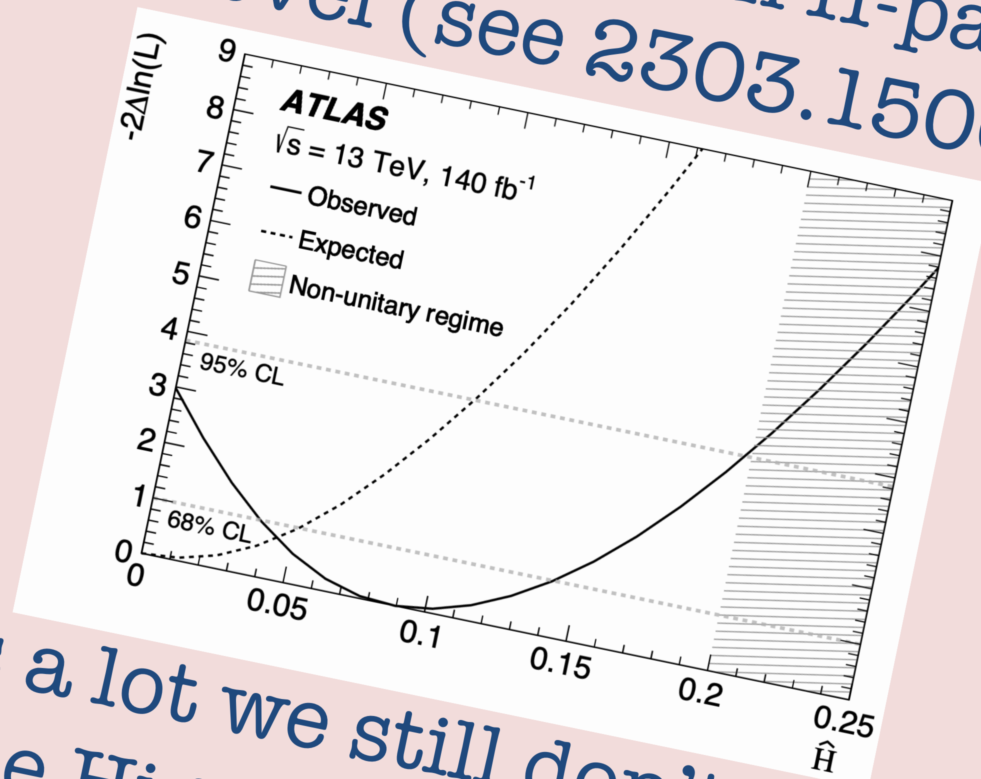


Englert, Giudice, Greljo, McCullough, 2019.

Oblique Corrections

Ou… …etical way to probe
th… …ns:

Currently only constrain H-parameter at 5% level (see 2303.15061).

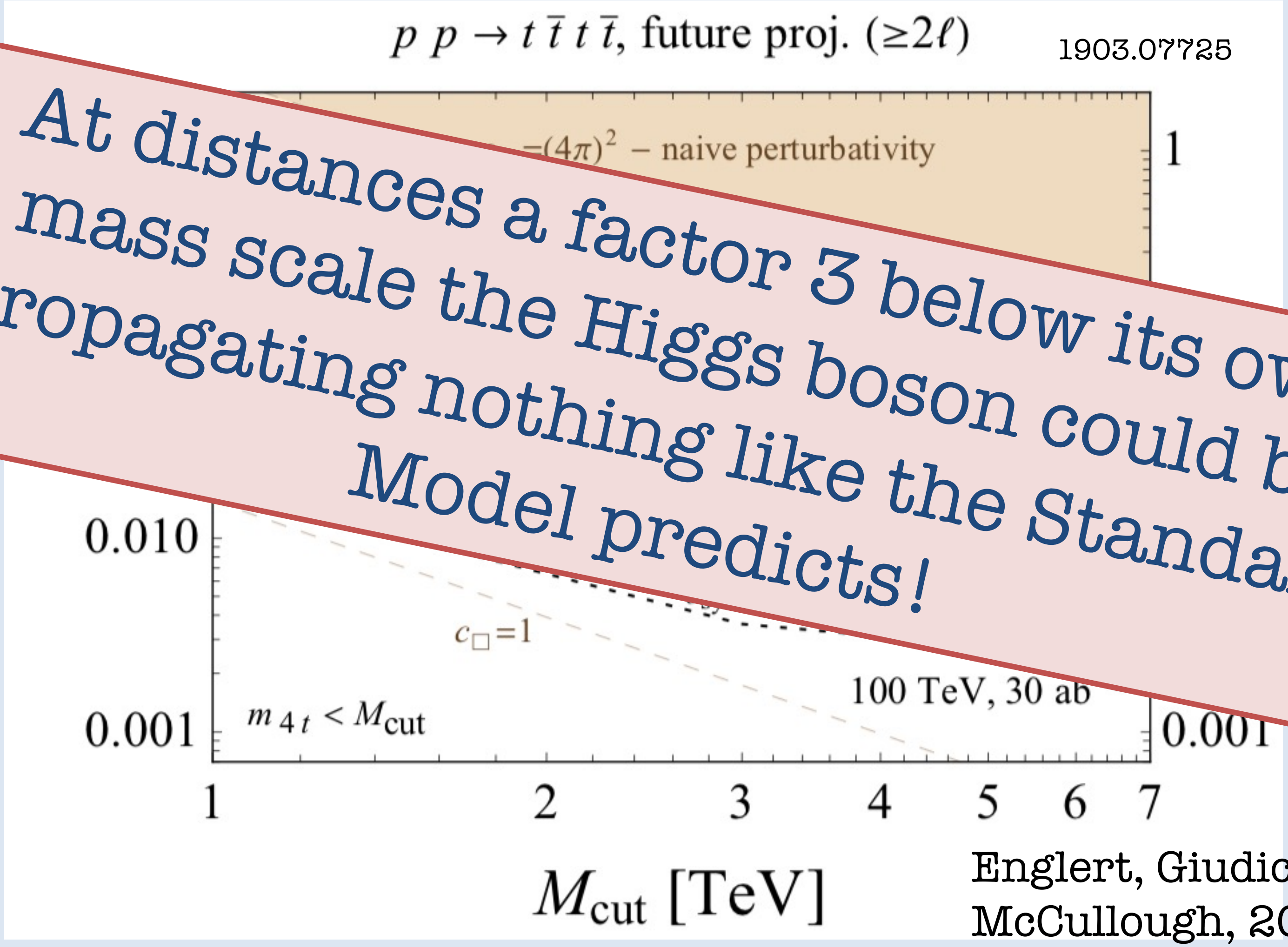


There is a lot we still don't know about how the Higgs moves from X to Y…

…o,
M… 2019.

Oblique Corrections

Our estimate suggests the practical way to probe this special operator is with future colliders:



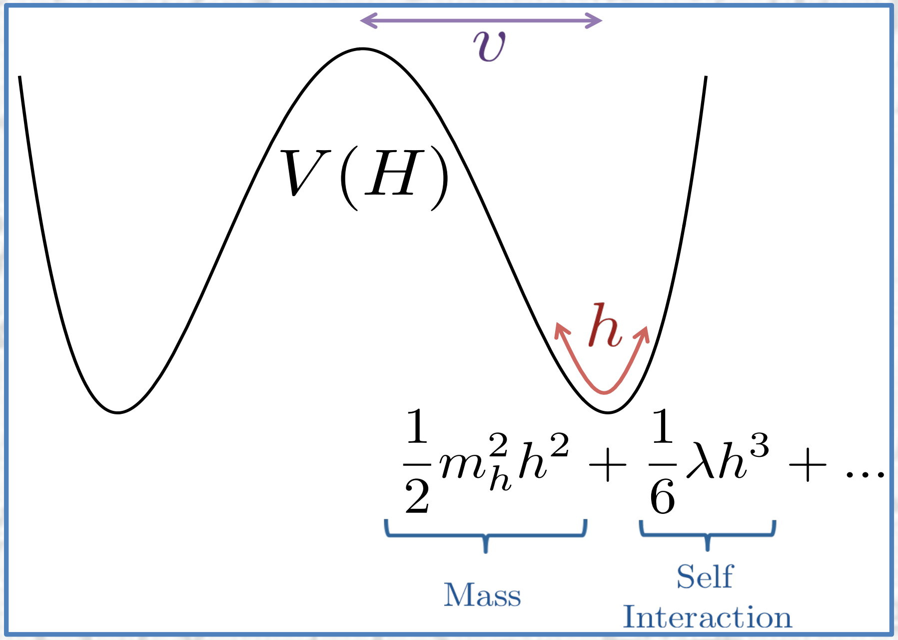
At distances a factor 3 below its own mass scale the Higgs boson could be propagating nothing like the Standard Model predicts!

Englert, Giudice, Greljo, McCullough, 2019.

$$\mathcal{O}_6 = \frac{c_6}{M^2}|H|^6$$

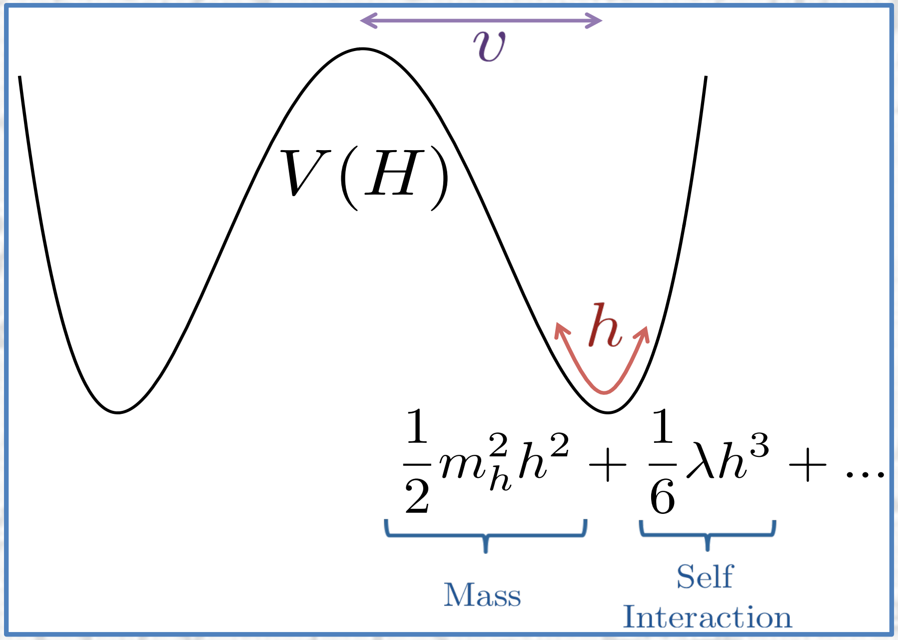
Parameterises
BSM deviations in sole
self-interaction of SM.

What is the Higgs Field Potential?



Important because it determines how the Universe froze in the EW sector, giving mass to gauge bosons, fermions, the Higgs...

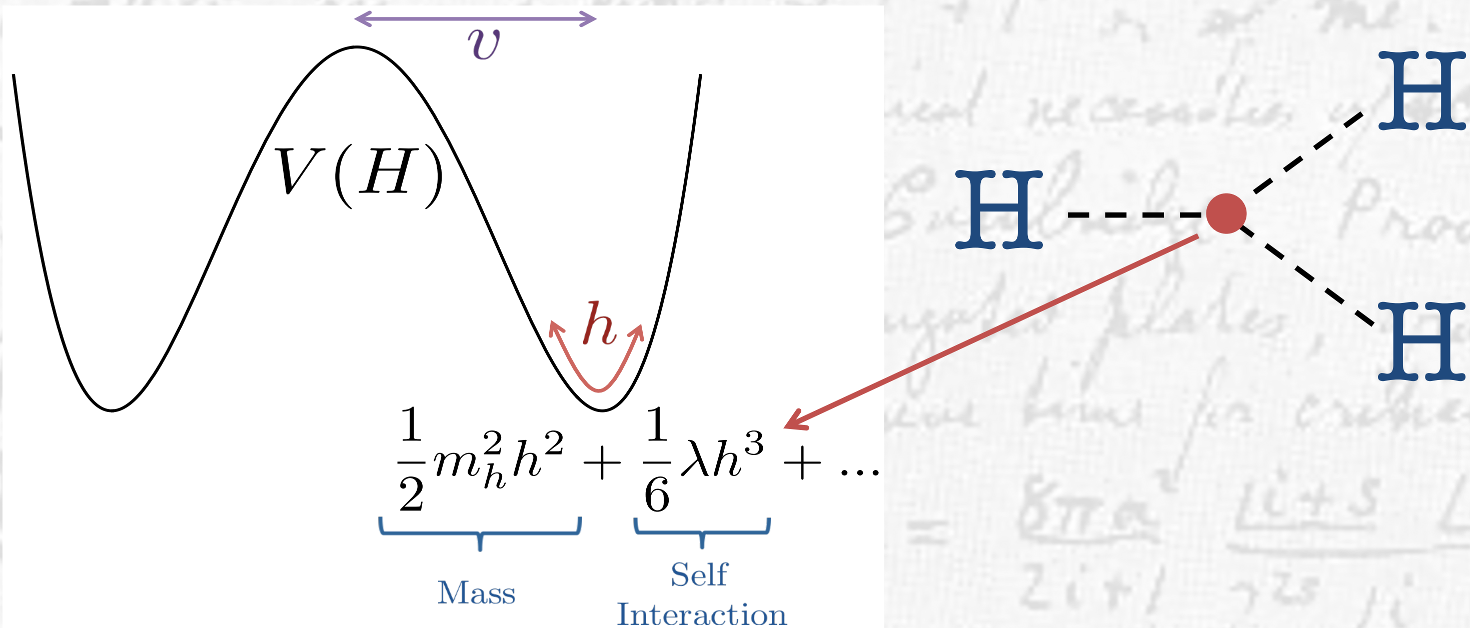
What is the Higgs Field Potential?



...because it determines how the Universe will end...

Naïve Dimensional Analysis

It's known that O_6 contributes to Higgs self-interaction, etc.



But less-well appreciated are the NDA aspects underlying it…

Naïve Dimensional Analysis

The fact that

$$[c_6] = [g^4]$$

and all other operator coefficients have

$$[c_j] \leq [g^2]$$

makes the self-coupling special, with one important implication I'll highlight today.

Self-Coupling Dominance

No obstruction to having Higgs self-coupling modifications a “loop factor” greater than **all** other couplings. Could have

$$\left|\frac{\delta_{h^3}}{\delta_{VV}}\right| \lesssim \min\left[\left(\frac{4\pi v}{m_h}\right)^2, \left(\frac{M}{m_h}\right)^2\right]$$

without fine-tuning any parameters, as big as,

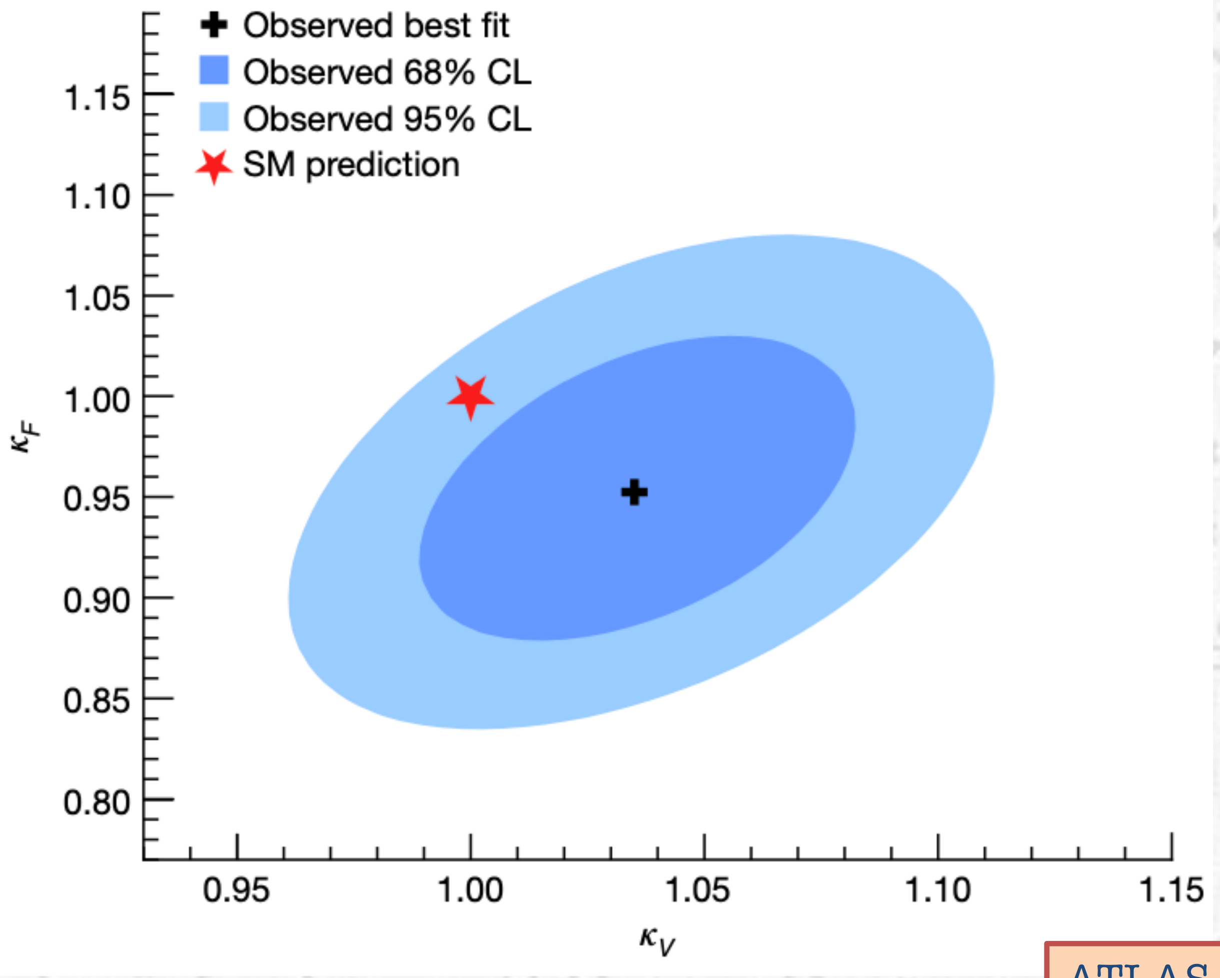
$$(4\pi v/m_h)^2 \approx 600$$

which is significant!

Durieux, MM, Salvioni. 2022

Status of Higgs Couplings

What are experimental limits on modifications of couplings relative to Standard Model prediction?



ATLAS, Nature, 2022

Status of Higgs Couplings

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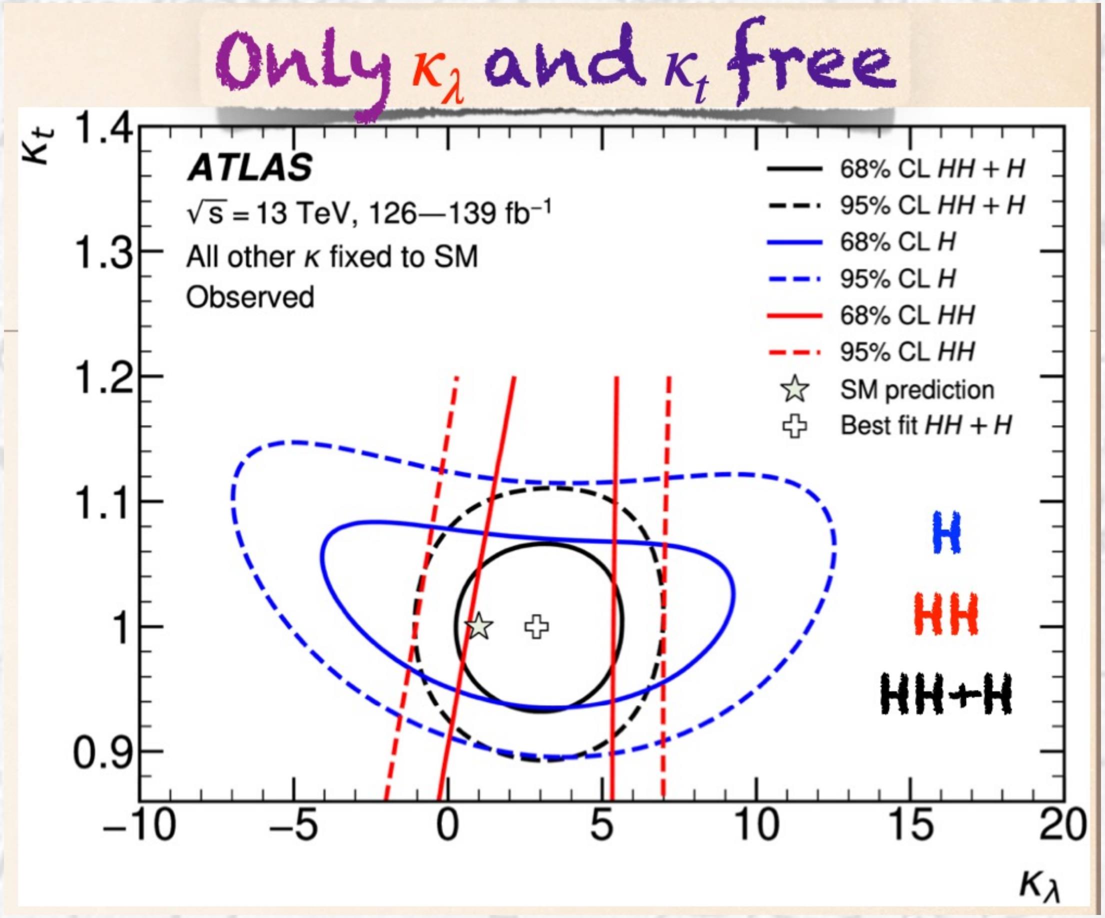

Lidija Živković, Yesterday

Status of Higgs Couplings

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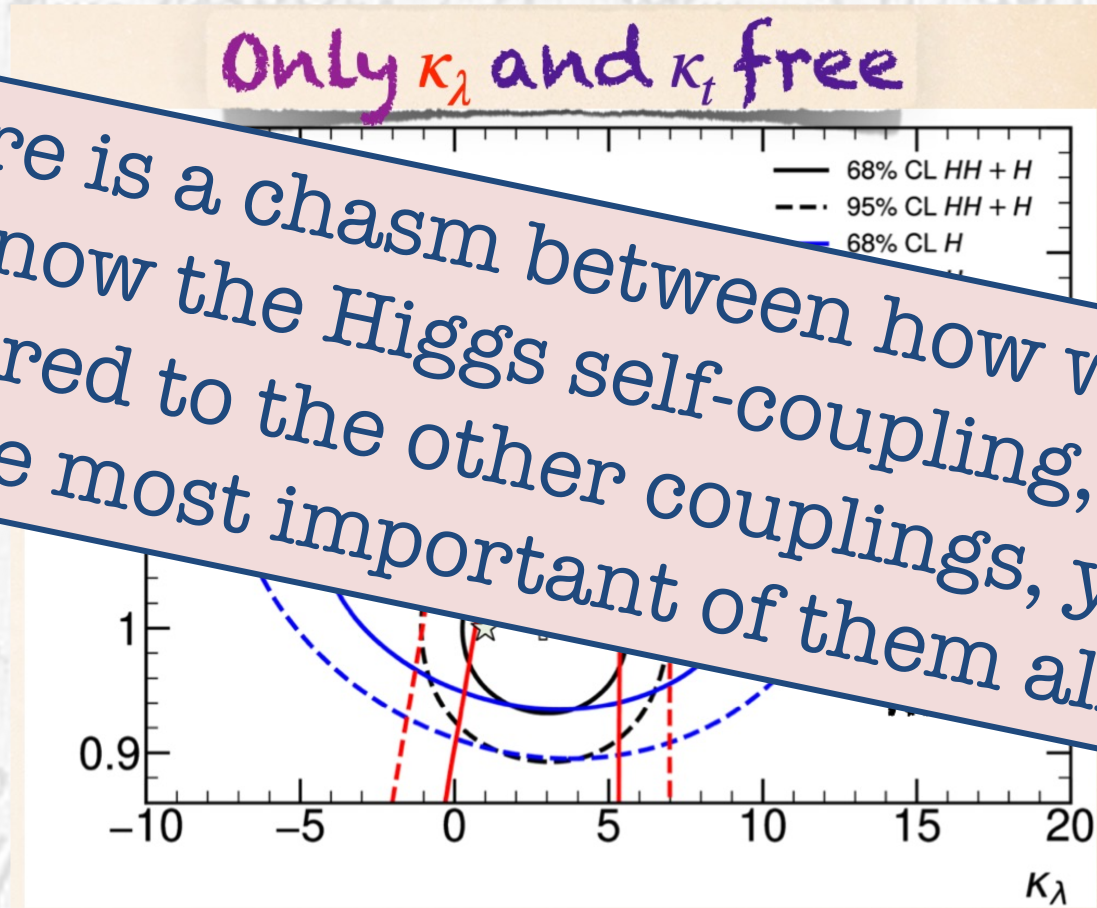


There is a chasm between how well we know the Higgs self-coupling, as compared to the other couplings, yet it is the most important of them all!

Lidija Živković, Yesterday

Self-Coupling Dominance

No obstruction from to having Higgs self-coupling modifications a loop factor greater than **all** other couplings. Could have

$$\left| \delta_{VV} \right| \quad \left[\left(\frac{4\pi v}{} \right)^2, \left(\frac{M}{m_h} \right)^2 \right]$$

But can such a theory exist in practise?

without fine-tuning any parameters, as

$$(4\pi v/m_h)^2 \approx 600$$

which is significant!

Durieux, MM, Salvioni. 2022

Custodial Quadruplet

This is all well and good, but does such a theory exist? Yes: The custodial quadruplet scalar. Projecting the $(\mathbf{4},\mathbf{4})$ of $\mathrm{SU}(2)_L \times \mathrm{SU}(2)_R$ onto EW group we have

$$(\mathbf{4},\mathbf{4}) \quad \rightarrow \quad \mathbf{4}_{1/2} + \mathbf{4}_{3/2}$$

and including all couplings to the Higgs we have for scalar quadruplet

$$\mathcal{L}_{\mathrm{SO}(4)} = -\lambda\Big(H^* H^* (\epsilon H)\Phi + \frac{1}{\sqrt{3}} H^* H^* H^* \widetilde{\Phi}\Big) + \mathrm{h.c.}$$

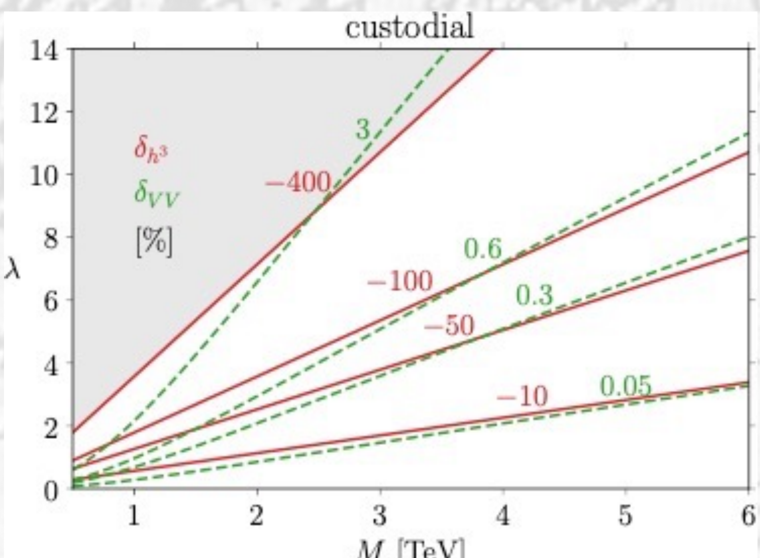
which has exactly the pattern described.

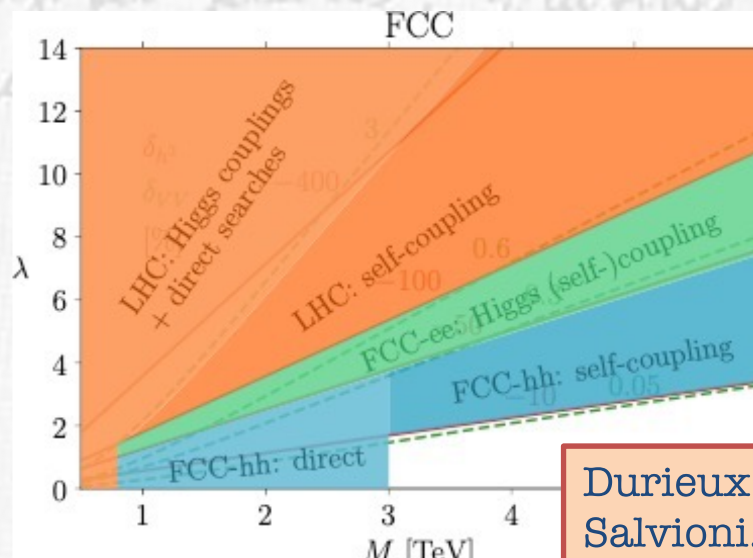
Custodial Quadruplet

Higgs self-coupling is modified at dim-6 at tree-level, all other couplings modified at dim-6 one-loop, or dim-8. All calculable, giving

$$-\frac{\delta_{VV}}{\delta_{h^3}} = 3\left(\frac{m_h}{4\pi v}\right)^2 + \left(\frac{m_h}{M}\right)^2 \approx \frac{1}{200} + \frac{1}{580}\left(\frac{3\ \mathrm{TeV}}{M}\right)^2$$

Remarkably close to NDA estimate!





Durieux, MM, Salvioni. 2022

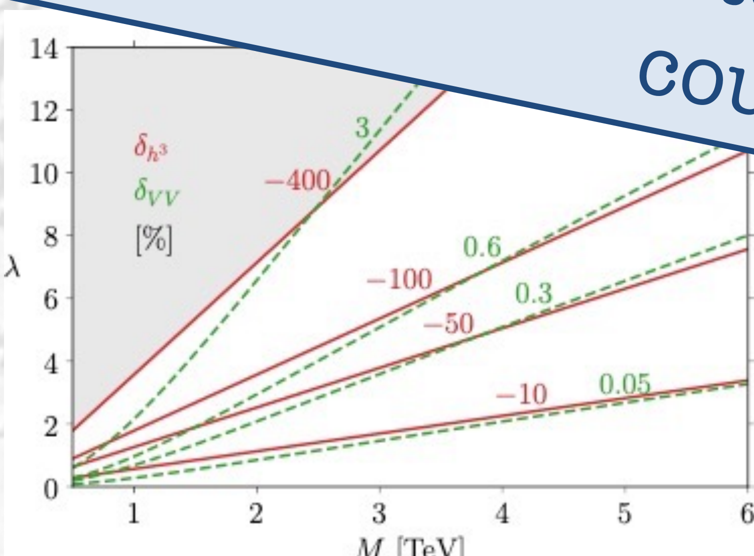
Custodial Quadruplet

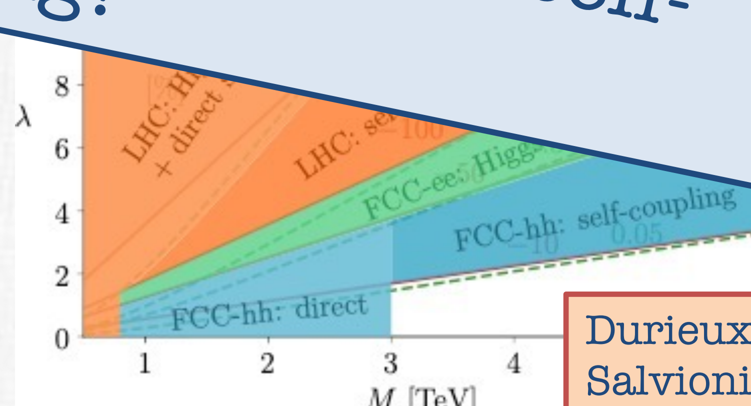
Higgs self-coupling is modified at dim-6 at tree-

lev… other couplings modified at dim-6 one-

loo… calculable, giving

Punchline: Currently only know the self-interaction at the level of 100's %. There is plenty of room for enormous new physics effects to show up in the self-coupling!

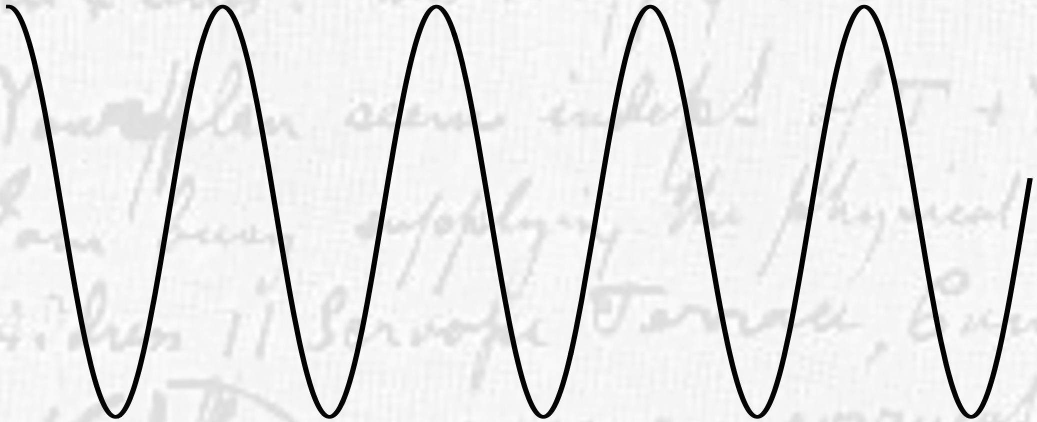




Durieux, MM, Salvioni. 2022

Is the Higgs Fundamental?

The Higgs boson has a size/wavelength. What's inside?



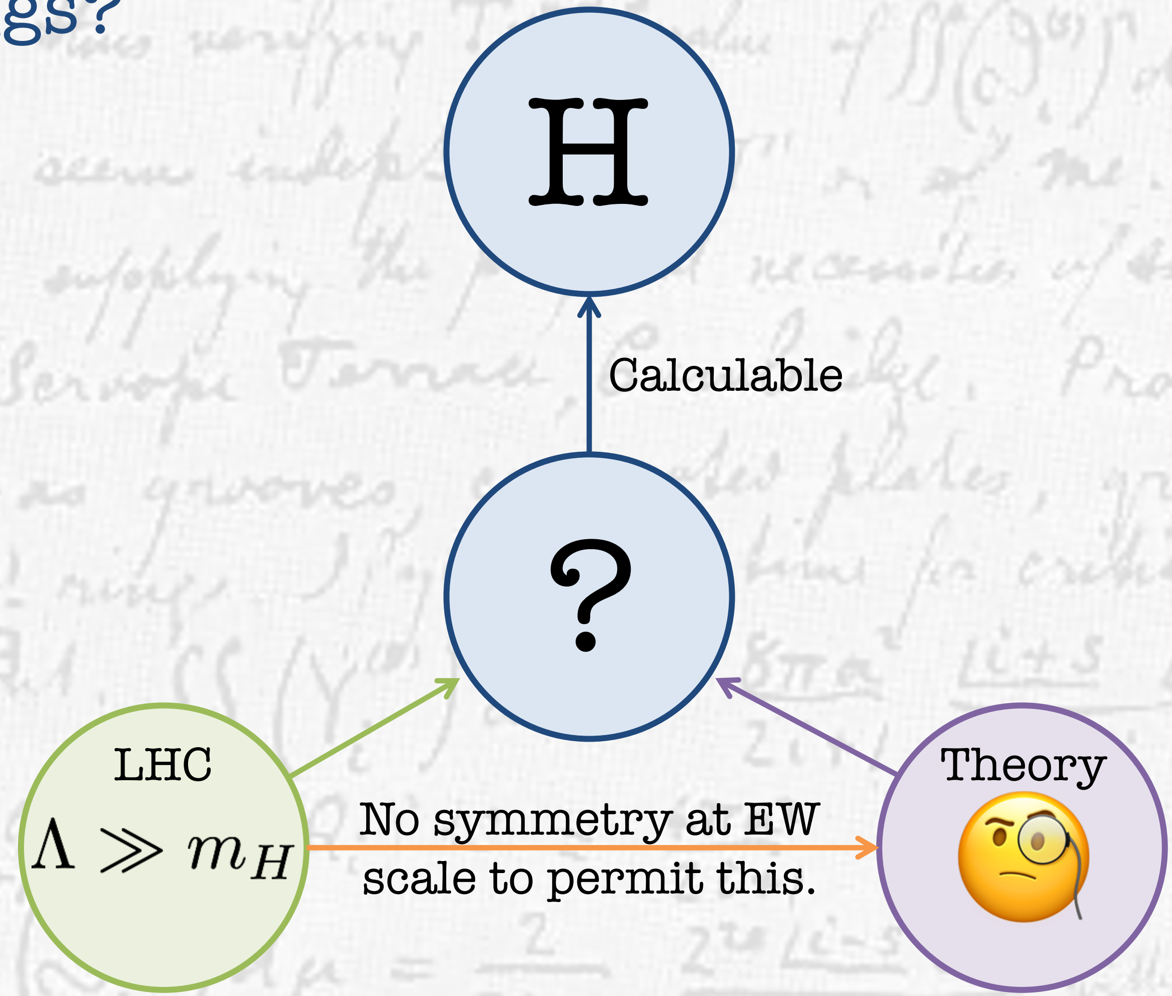
Precision measurements are different ways of probing the "compositeness of the Higgs".

$\lambda_h \approx 10^{-17}$ m

$\lambda_{10 \text{ TeV}} \approx 10^{-19}$ m

Backdrop

We know what happens with pions… what about the Higgs?

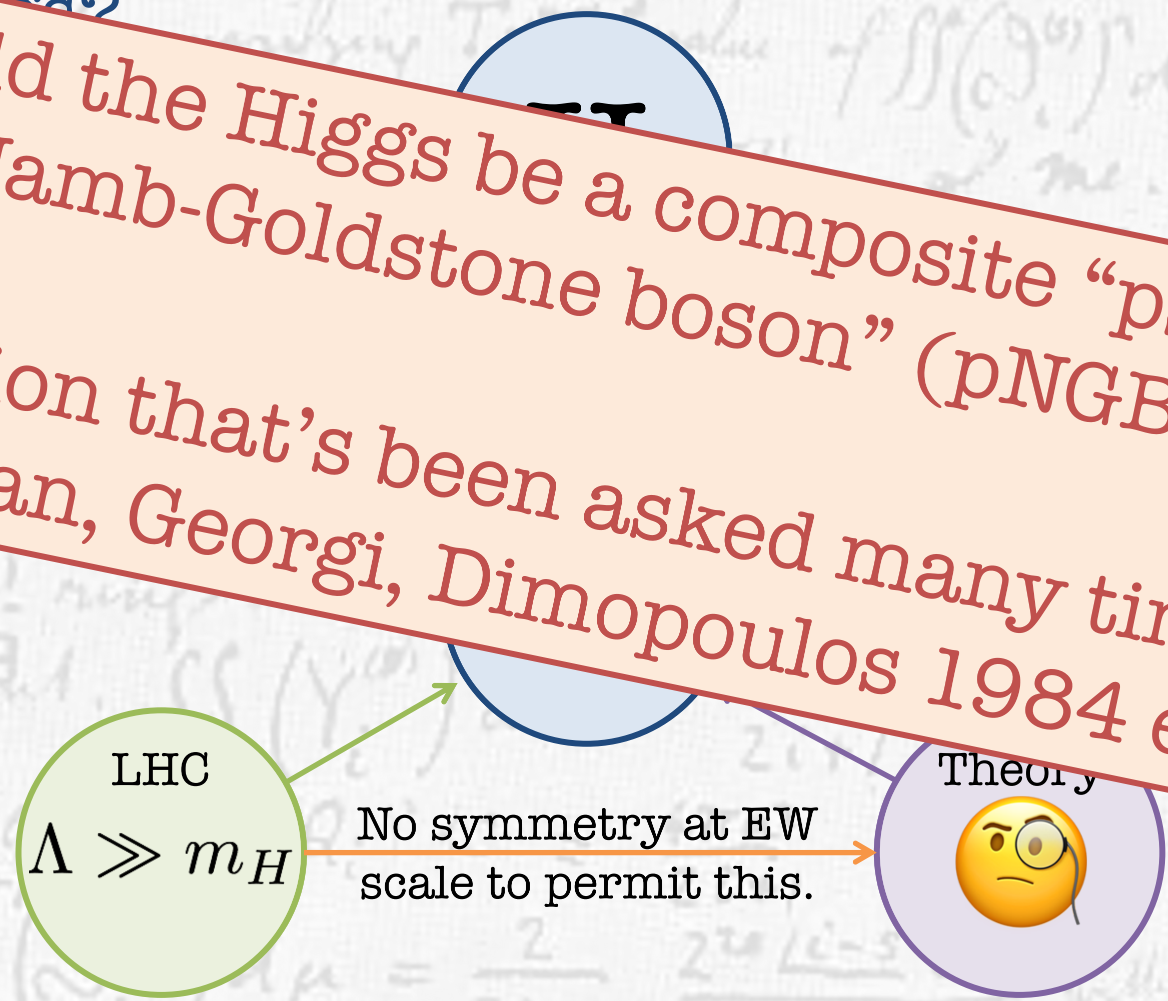


Backdrop

We know what happens with pions… what about t… ?

Could the Higgs be a composite "pseudo-Namb-Goldstone boson" (pNGB)?

Question that's been asked many times… Kaplan, Georgi, Dimopoulos 1984 etc.



Naturalness – Composite Higgs

Vanilla composite Higgs scenarios have a potential which looks like

"Compositeness" Scale

$$V(h) = \epsilon f^2 \Lambda^2 F(h/f)$$

Where F is a generic function. Not so difficult to have a light Higgs

$$m_h^2 \sim \epsilon \Lambda^2$$

If one has $\epsilon \ll 1$. This is not fully possible in concrete models, since this is controlled by a symmetry which is already broken in SM.

However...

Naturalness – Composite Higgs

Vanilla composite Higgs scenarios have a potential which looks like

"Compositeness" Scale

$$V(h) = \epsilon f^2 \Lambda^2 F(h/f)$$

Where F is a generic function. The position of the minimum of the potential doesn't care about this parameter:

$$V'(h) = 0 \Leftrightarrow F'(h/f) = 0$$

So, if this is to occur at $h = v \ll f$ then one has to fine-tune the contributions to the potential from the composite physics.

Naturalness – Composite Higgs

Vanilla composite Higgs scenarios have a potential which looks like

Compositeness Scale

$$V(h) = \epsilon f^2 \Lambda^2 F(h/f)$$

Where F is a generic function. However, it is generic that the operator

$$\mathcal{O}_H \sim \frac{1}{f^2}\left(\partial^\mu |H|^2\right)^2$$

is generated. This modifies all Higgs couplings by an amount

$$\delta_\kappa \sim \frac{v^2}{f^2}$$

Naturalness – Composite Higgs

Vanilla composite Higgs scenarios have a p… which looks like

Compositeness Scale

(h/f)

So, in vanilla scenarios, Higgs coupling measurements suggest that if the Higgs is composite then there must be some fine-tuning of parameters at least at the 10% or so level!

is generated. This modifies all Higgs … by an amount

$$\delta_\kappa \sim \frac{v^2}{f^2}$$

Naturalness – Composite Higgs

Var… composite Higgs scenarios have a po… …ks like

Compositeness Scale

The Composite Nambu-Goldstone Higgs

Giuliano Panico, Andrea Wulzer

Download PDF

The composite Higgs scenario, in which the Higgs emerges as a composite pseudo-Nambu-Goldstone boson, is extensively reviewed in these Notes. The material is presented in a pedagogical fashion, with great emphasis on the conceptual and technical foundations of the construction. A comprehensive summary of the flavor, collider and electroweak precision phenomenology is also presented.

is generated. This modifies a… …gs by an amount

$$\delta_\kappa \sim \frac{v^2}{f^2}$$

Naturalness – Composite Higgs

Let's scrutinize the assumptions...

$$V(h) = \epsilon f^2 \Lambda^2 F\left(h/f\right)$$

How much symmetry breaking → ϵ

How the symmetry is broken... → F

Assumption until now has been that the symmetry is broken in the most minimal ways.

Technically: Breaking "spurion" is in a low-index irrep of the global symmetry.

Beyond Minimality

Consider a simple scenario that could apply to the Higgs boson.

Example SO(N+1):

$$\mathcal{L} = \frac{1}{2}\partial_\mu \phi \cdot \partial^\mu \phi - \frac{\lambda}{4}\left(\phi \cdot \phi - \frac{f^2}{2}\right)^2$$

We get N massless pNGBs with decay constant "f" and unbroken SO(N).

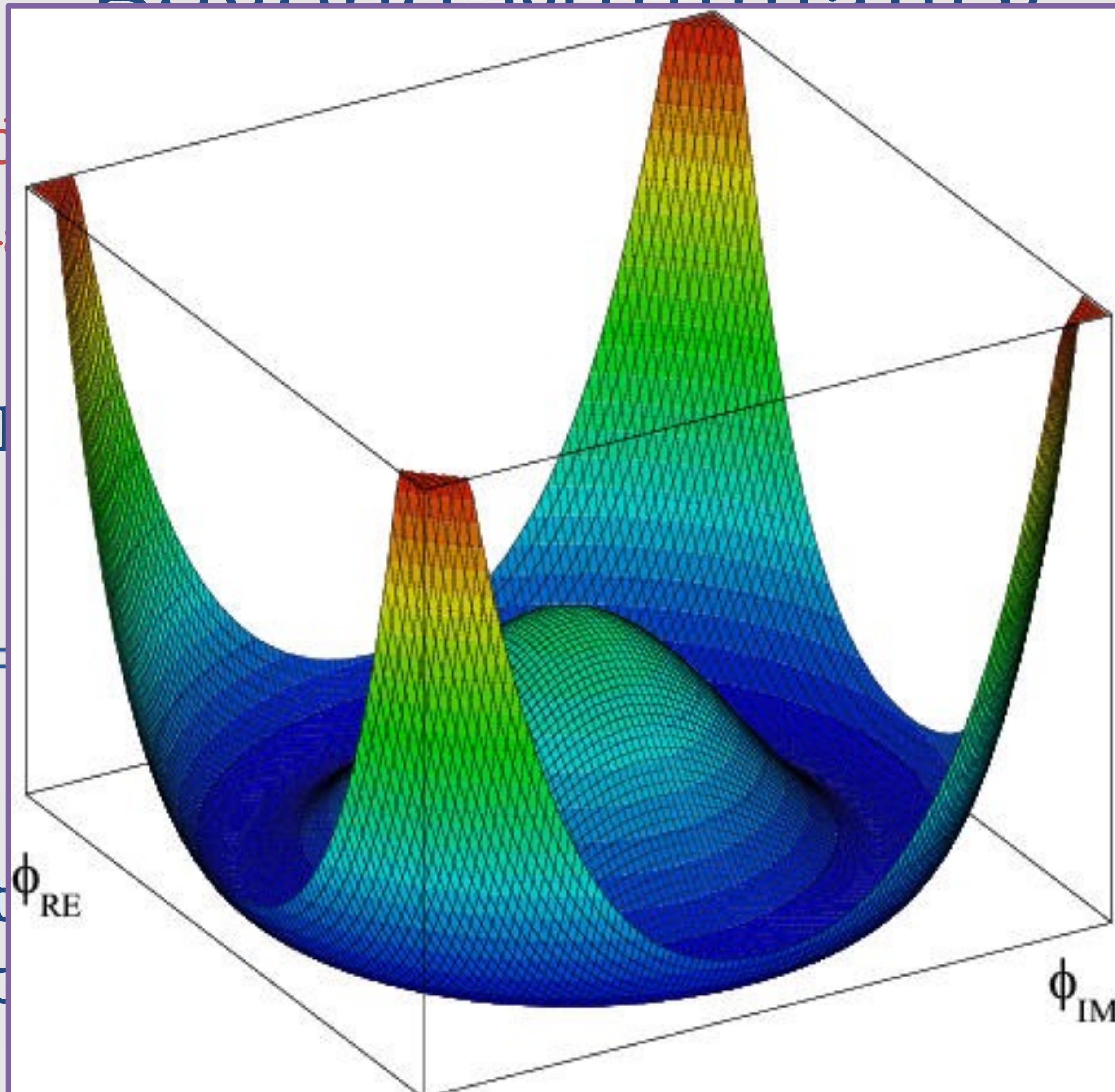
Beyond Minimality

Consid… …ply to
the Hi…

Examp…

$\mathcal{L} = \ldots \Big)^2$

We get… …stant
"f" and…



Beyond Minimality

Now assume some small explicit breaking "spurion" in a symmetric irrep with "n" indices:

$$V_\epsilon = \frac{\lambda}{f^{n-4}} \epsilon_{a_1, a_2, \ldots, a_n} \phi^{a_1} \phi^{a_2} \ldots \phi^{a_n}$$

How the symmetry is broken...

For the pNGB fields this generates a potential:

$$V = \epsilon m_\rho^2 f^2 \; G_n^{(N-1)/2}(\cos \Pi / f)$$

Gegenbauer function!

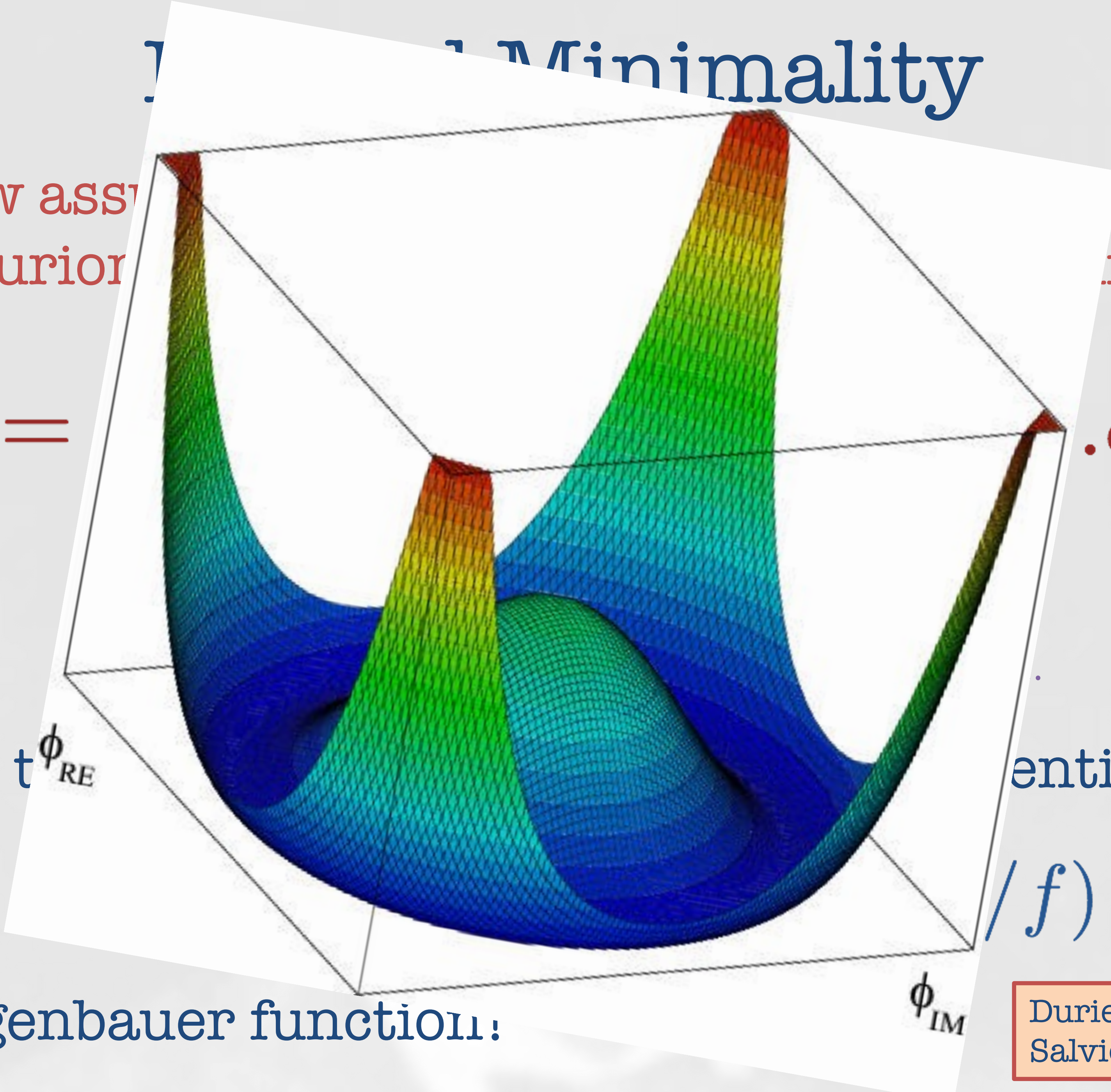
Durieux, MM, Salvioni. 2021

Minimality

Now ass

"spurion ... ices:

$$V_\epsilon = \quad .\phi^{a_n}$$



For t ... ential:

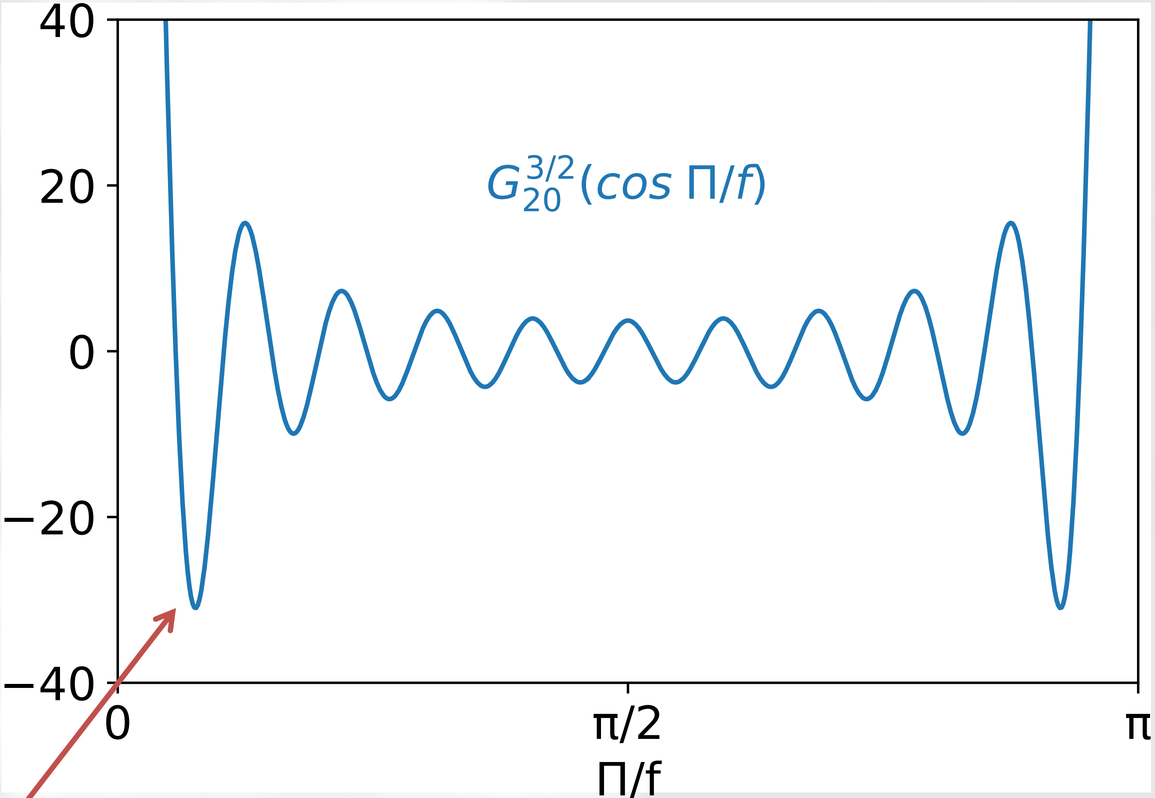
$$/f)$$

Gegenbauer functio

Durieux, MM, Salvioni. 2021

Getting to know Gegenbauer

The Gegenbauer potential looks like:



Global minimum at naturally small field values:

$$\frac{\langle \Pi \rangle}{f} \approx \frac{j_{\lambda+1/2,1}}{n+\lambda} \approx \frac{5.1}{n}$$

Gegenbauer's Twin

Durieux, MM, Salvioni. 2022

Gegenbauer contribution allows to naturally realise v<<f. On the other hand, for a standard composite Higgs model the top sector doesn't allow ϵ to be arbitrarily small...

Twin Higgs models, however, address that particular aspect. Could "Gegenbauer's Twin" allow both $\epsilon \ll 1$ and $v \ll f$?

Application

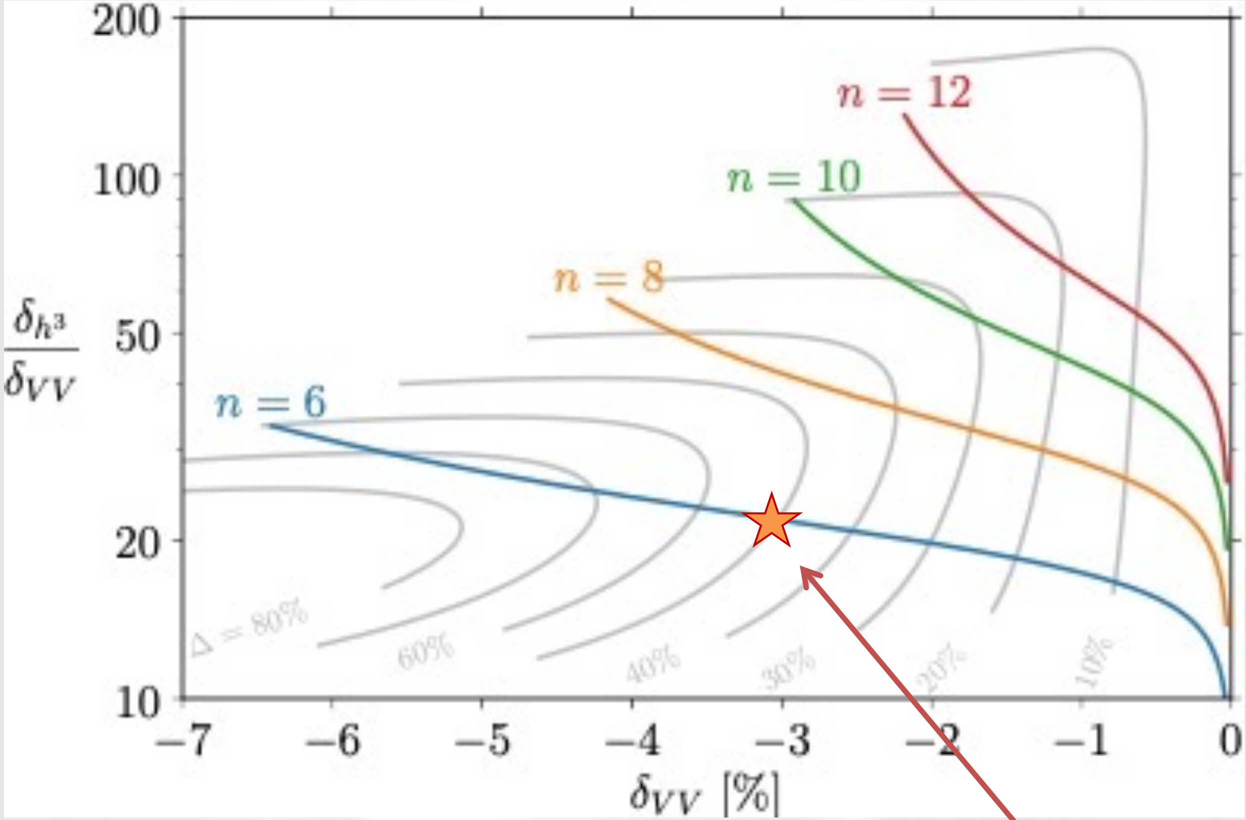
Consider some standard pNGB Higgs construction and, inspired by pions, allow for an additional source of explicit symmetry breaking, in n-index irrep of global symmetry.

$$\mathcal{L} = \mathcal{L}_{\mathrm{Old}} + \epsilon \mathcal{L}_{S_n \neq 0}$$

What happens?

Gegenbauer's Twin

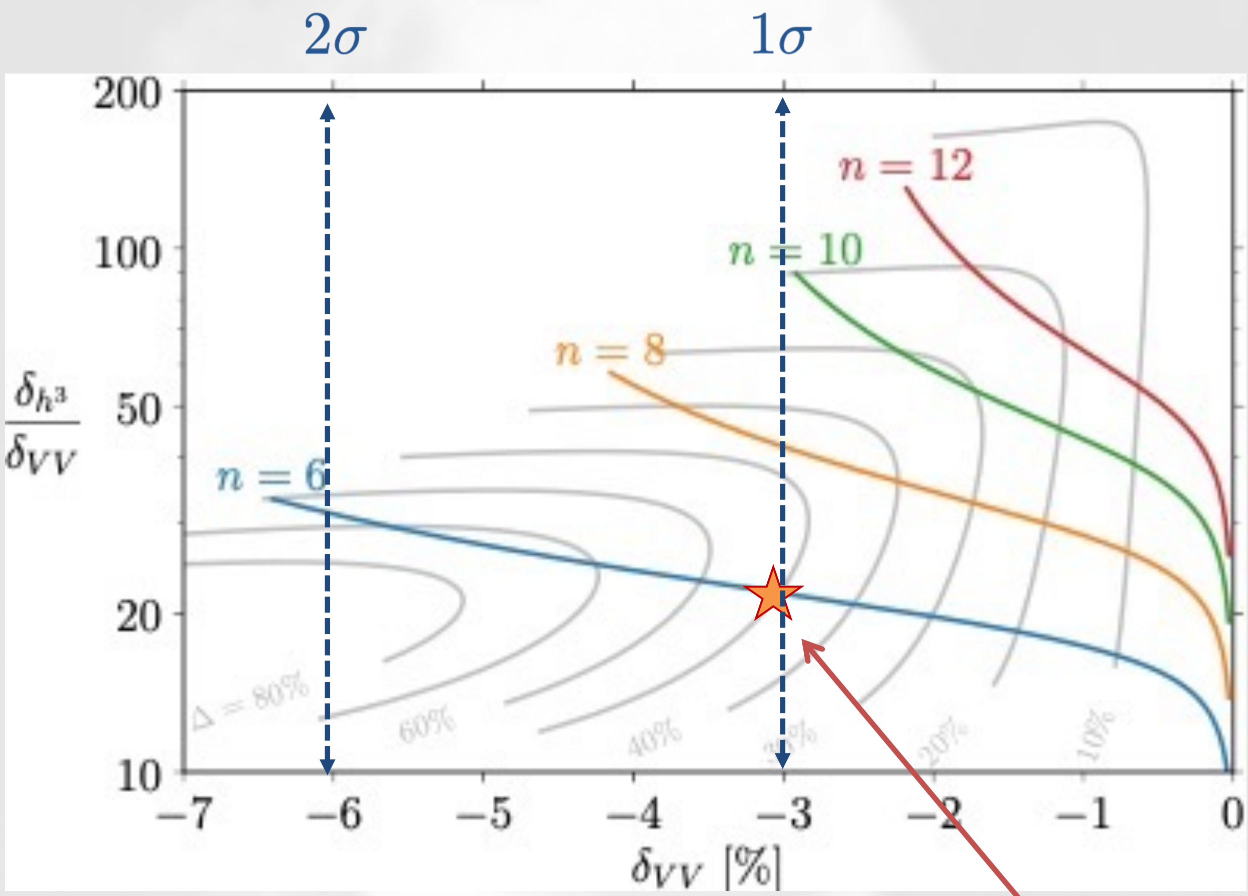
Predictions, in absolute terms:



Example point. Low tuning (low cutoff), 3% single-coupling correction, 70% self-coupling correction.

Gegenbauer's Twin

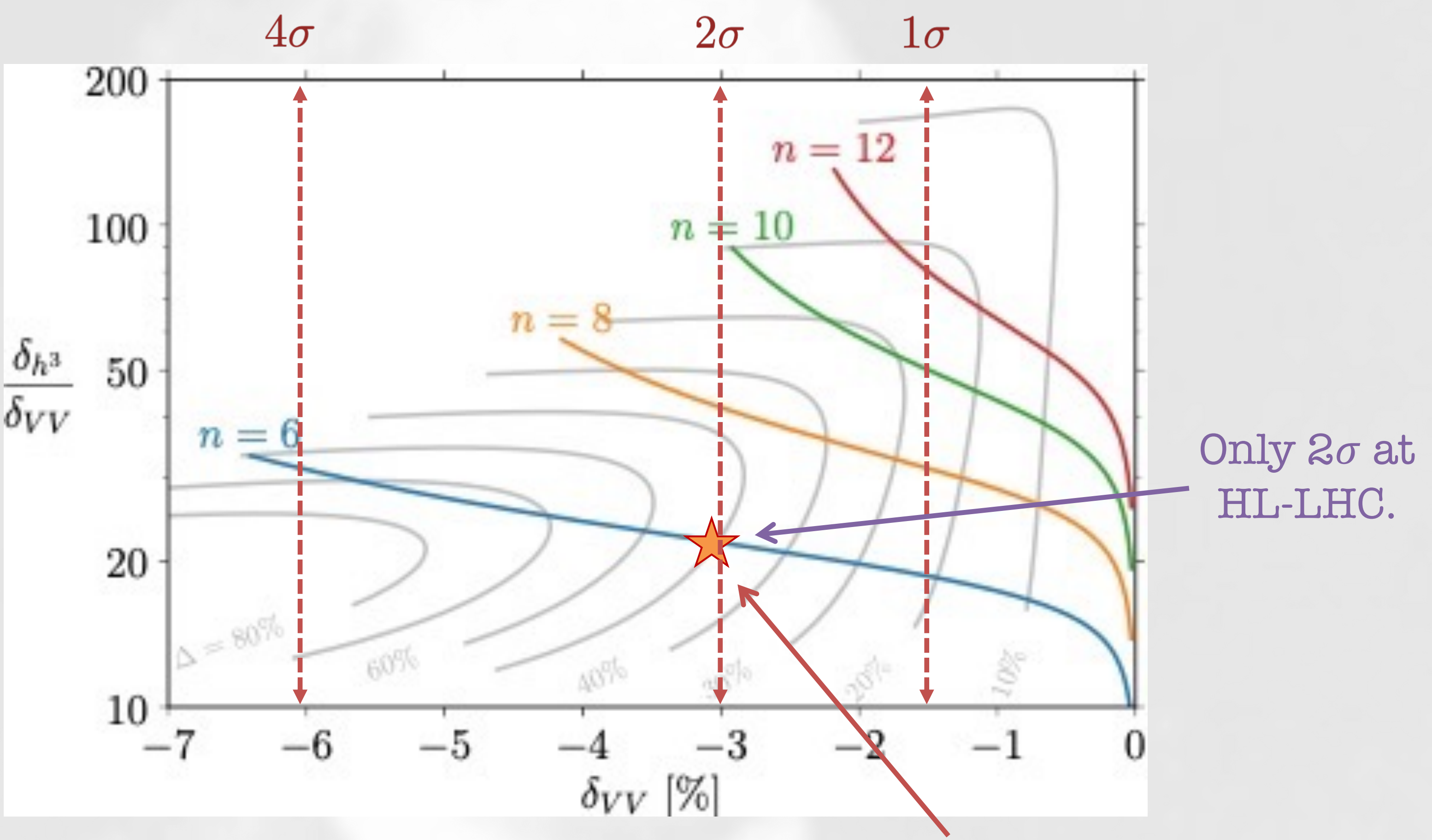
Present Limits



Example point. Low tuning (low cutoff), 3% single-coupling correction, 70% self-coupling correction.

Gegenbauer's Twin

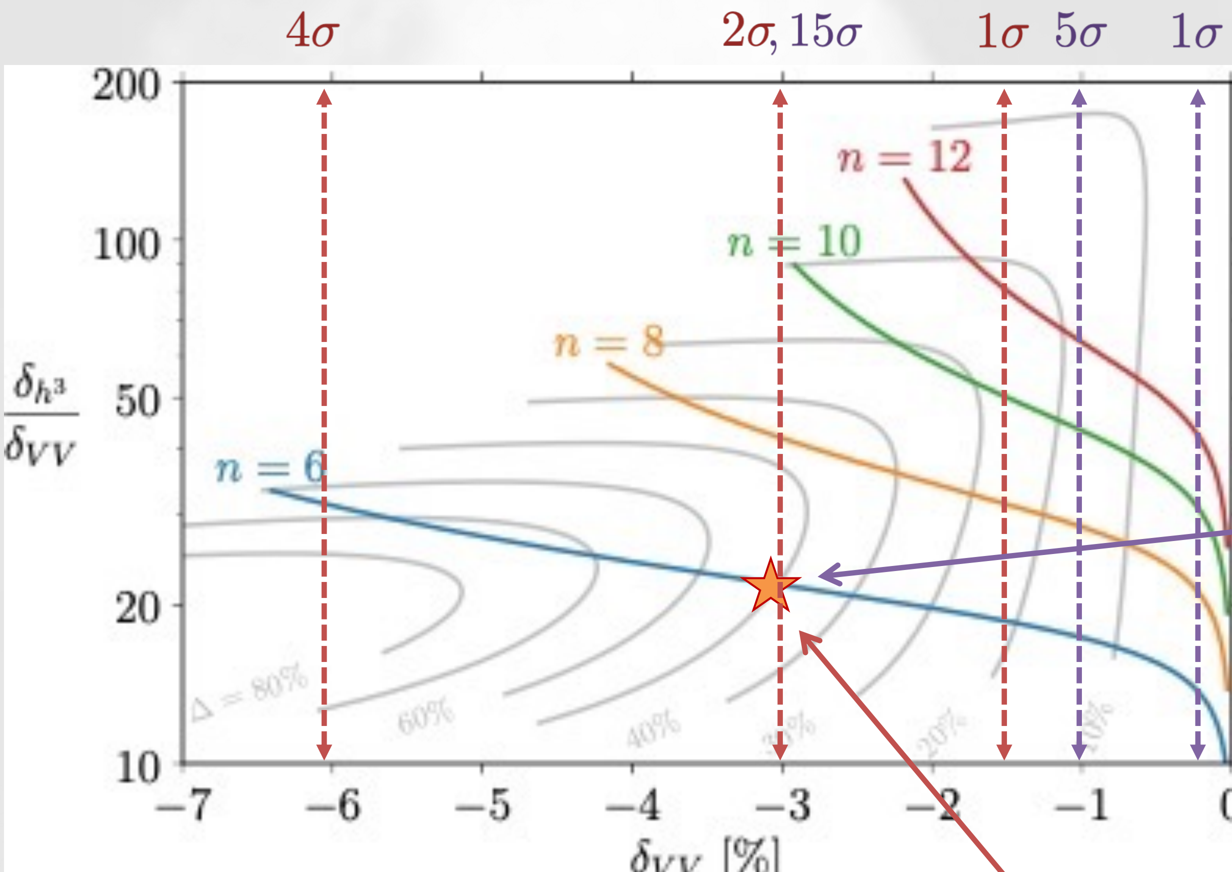
HL-LHC Expectations



Example point. Low tuning (low cutoff), 3% single-coupling correction, 70% self-coupling correction.

Gegenbauer's Twin

HL-LHC Expectations & FCC-ee

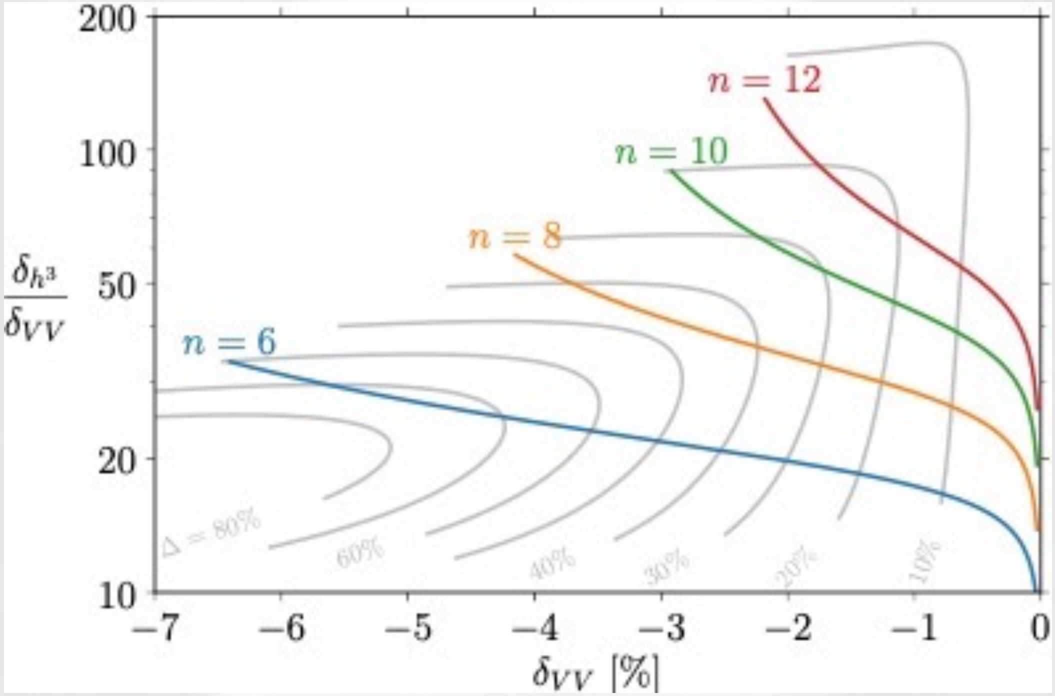


Only 2σ at HL-LHC but 15σ at FCC-ee!

Example point. Low tuning (low cutoff), 3% single-coupling correction, 70% self-coupling correction.

Gegenbauer's Twin

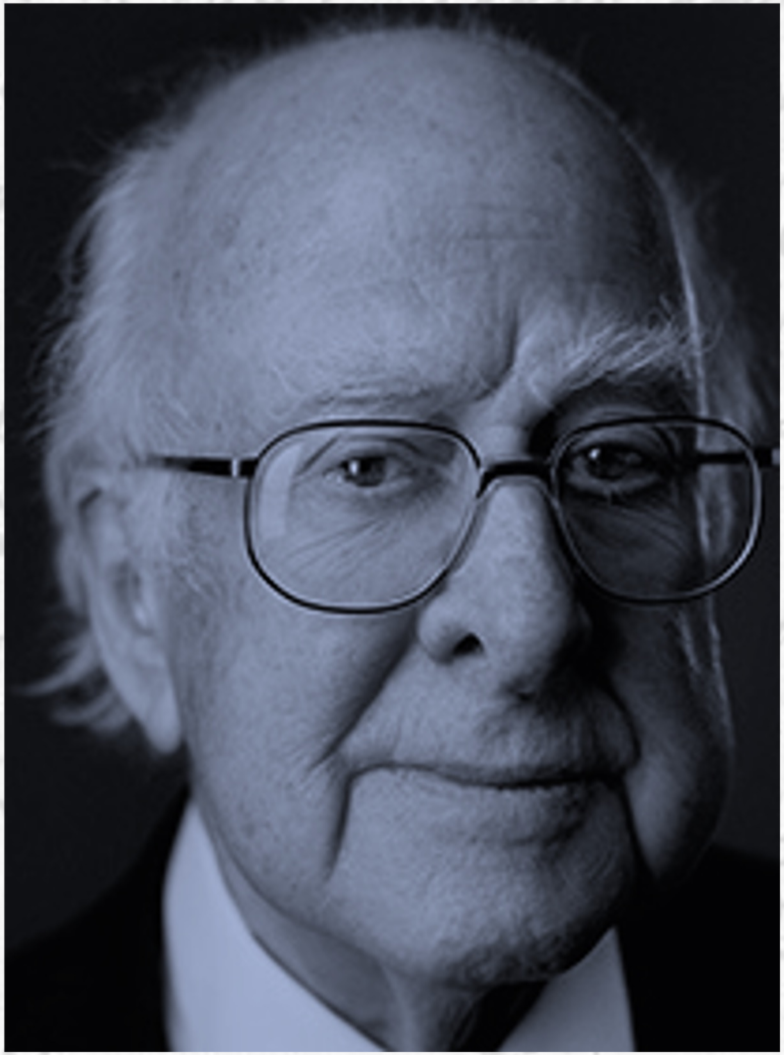
Modifications to self-interaction relative to other couplings are huge:



Naturalness could show up in self-interaction!

Fine-tuning is small. The Higgs could still, naturally, be composite!

How well do we know the Higgs?



Barely.

Conclusions

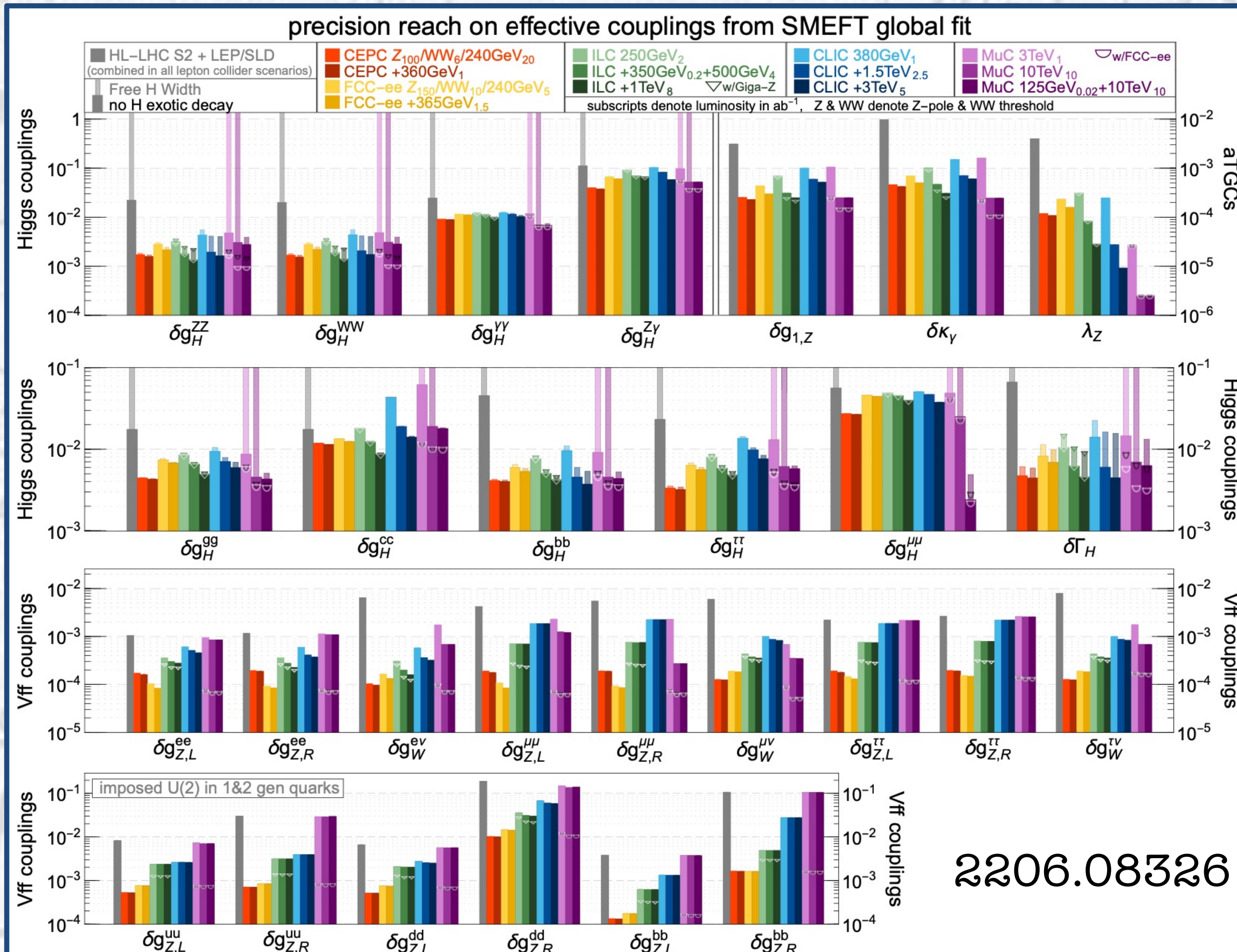
Higgs physics is still in its nascence. Pions were discovered in the early 1940's. Their fundamental origin, QCD, was developed theoretically in the early 1970's and only experimentally established in the late 1970's.

It has been eleven years since the discovery of the Higgs boson.

We must be patient and determined to uncover its origins.

Future of Higgs/EW Couplings

Future facilities can give us valuable new insights into the nature of the Higgs boson.



2206.08326

Conclusions

As it stands, we don't know how the Higgs behaves if we displace it by distances smaller than its Compton wavelength.

As it stands, we don't know how it interacts with itself; a property with far-reaching implications.

As it stands, we don't know if the Higgs boson is composite. However, some clues may already be pointing in a specific theory direction.

O.T.' R.U. ATOME? $\iint$ Sphere dS was done in the most general form in 1867. I have now bagged ξ & η from T & T' and done the numerical value of $\iint (Y_i^{(s)})^2 dS$ in 4 lines, thus verifying T+T's value of $\iint (\vartheta_i^{(s)})^2 dS$

Your plan seems indept of T + T' or of me. Publish!

I am busy supplying the physical necessities of scientific life at 11 Scroope Terrace, Cambridge. Prooves have gone as far as grooves, corrugated plates, gratings and rings. If you have time for criticism then

$$\iint (Y_i^{(s)})^2 dS = \frac{8\pi a^2}{2i+1} \frac{\lfloor i+s \lfloor i-s}{2^{2s} \lfloor i \lfloor i}$$

except when $s=0$ when $\iint (Q_i)^2 dS = \frac{4\pi a^2}{2i+1}$

Hence

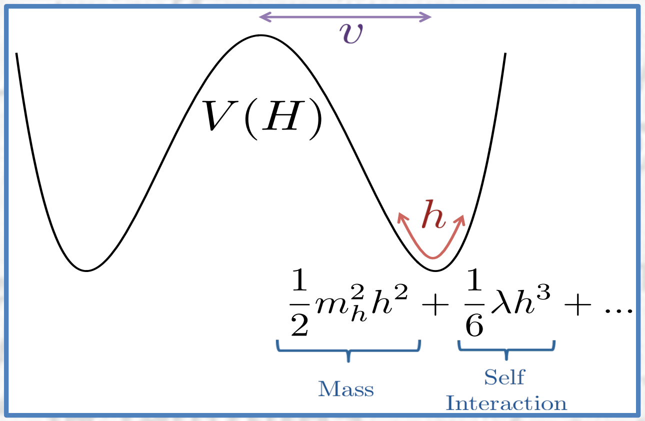
$$\int_{-1}^{+1} (\vartheta_i^{(s)})^2 d\mu = \frac{2}{2i+1} \frac{2^{2s} \lfloor i-s \lfloor s \lfloor s}{\lfloor i+s}$$

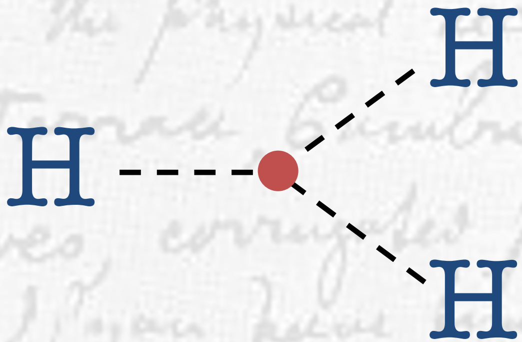
without exception

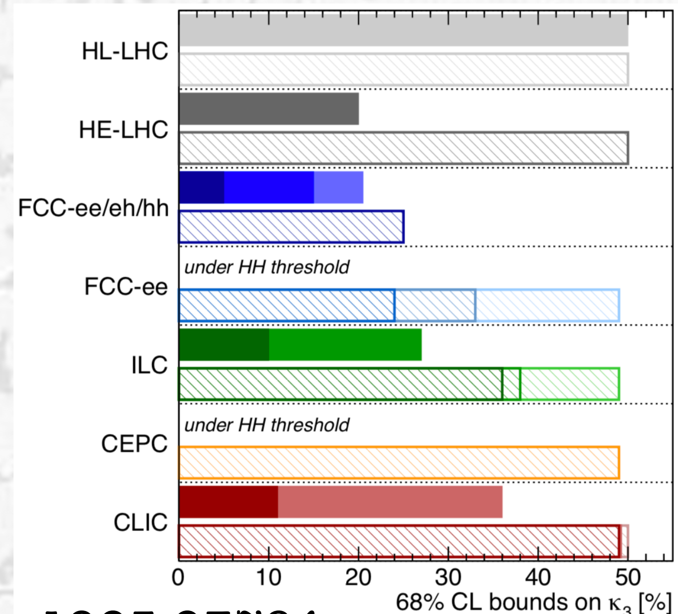
yrs $\frac{dp}{dt}$

Future of Higgs Self-Coupling

Future facilities can give us valuable new insights into the nature of the Higgs potential.







1905.03764

Rich interplay between direct/indirect, HL-LHC, Higgs factory, future High energy machines.

Gegenbauer's Twin

If tuning calculations interest you (I understand if not…), we followed conservative approach

$$\delta = \begin{pmatrix} \frac{\partial \log v^2}{\partial \log \epsilon} & \frac{\partial \log v^2}{\partial \log a} \\ \frac{\partial \log m_h^2}{\partial \log \epsilon} & \frac{\partial \log m_h^2}{\partial \log a} \end{pmatrix} \qquad \Delta = \left(\sum \text{eigenvalues}\,(\delta^T \delta)\right)^{-1/2}$$

As Gegenbauer gives vev, tuning dominated by

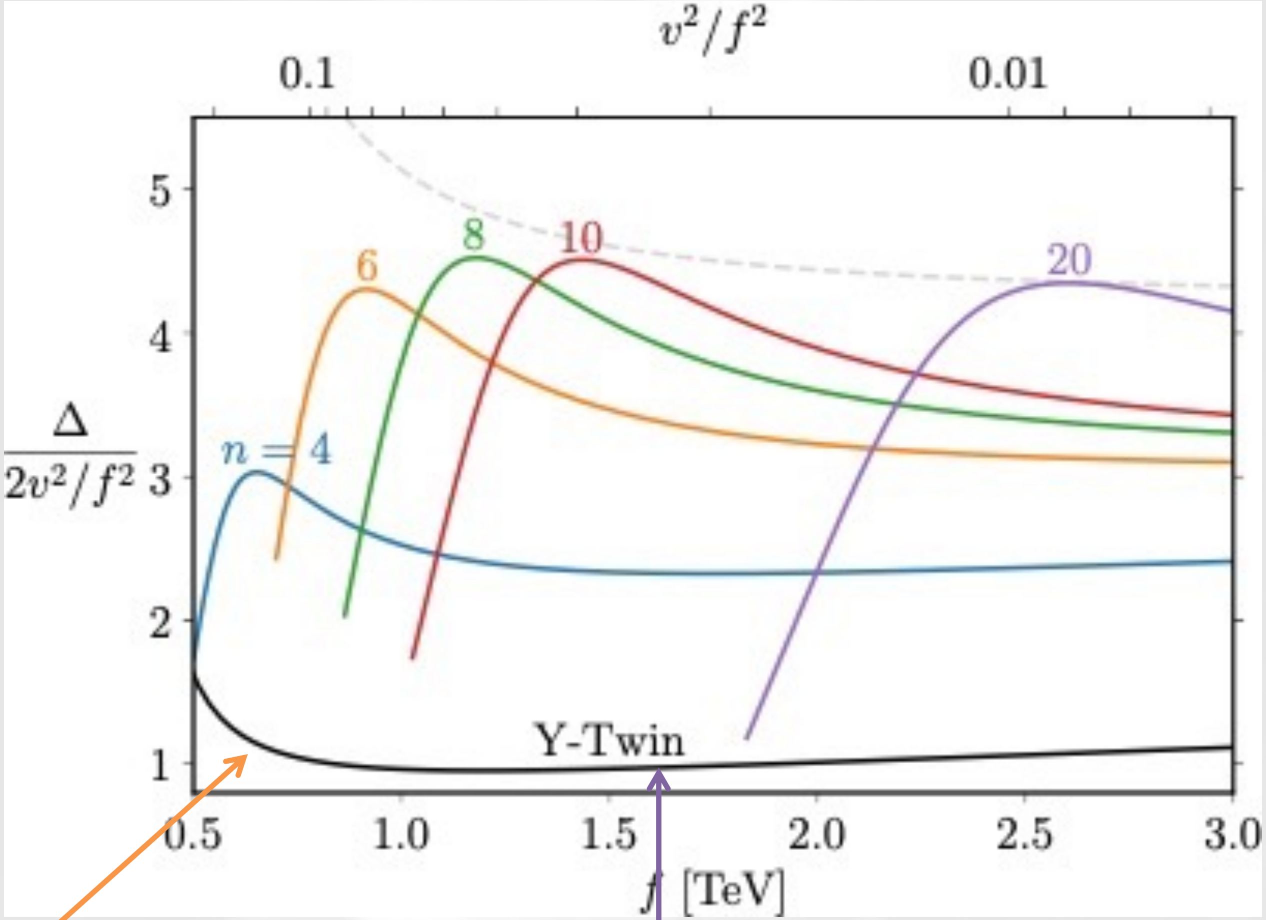
$$\left(\frac{\partial \log v^2}{\partial \log a}\right)^{-1} = \frac{8\pi^2 m_h^2}{3 y_t^4 f^2 \left(1 - \frac{3v^2}{f^2} + \frac{2v^4}{f^4}\right)}$$

So, compared to standard Twin expect improvement of

$$\frac{\Delta}{2v^2/f^2} \approx \frac{4\pi^2 m_h^2}{3 y_t^4 v^2} \approx 4$$

Gegenbauer's Twin

Quantitatively:



Estimate of Craig & Howe seems robust.

Twin model of Barbieri, Greco, Rattazzi, Wulzer.

Gegenbauer's Twin

Generalising Gegenbauer story to pNGB Twin Higgs for SO(8) → SO(7) and going to Unitary gauge the top-sector contributions to the Higgs potential are

$$V_t \approx \frac{3y_t^4 f^4}{64\pi^2}\left[\sin^4\frac{h}{f}\,\log\frac{a}{\sin^2 h/f} + \cos^4\frac{h}{f}\,\log\frac{a}{\cos^2 h/f}\right]$$

Whereas the symmetric n-index irrep gives

$$V_G^{(n)} = \epsilon\, m_\rho^2 f^2 G_n^{3/2}\left(\cos 2h/f\right)$$

Note: This is radiatively stable at all scales.