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Higgs Vacuum Metastability

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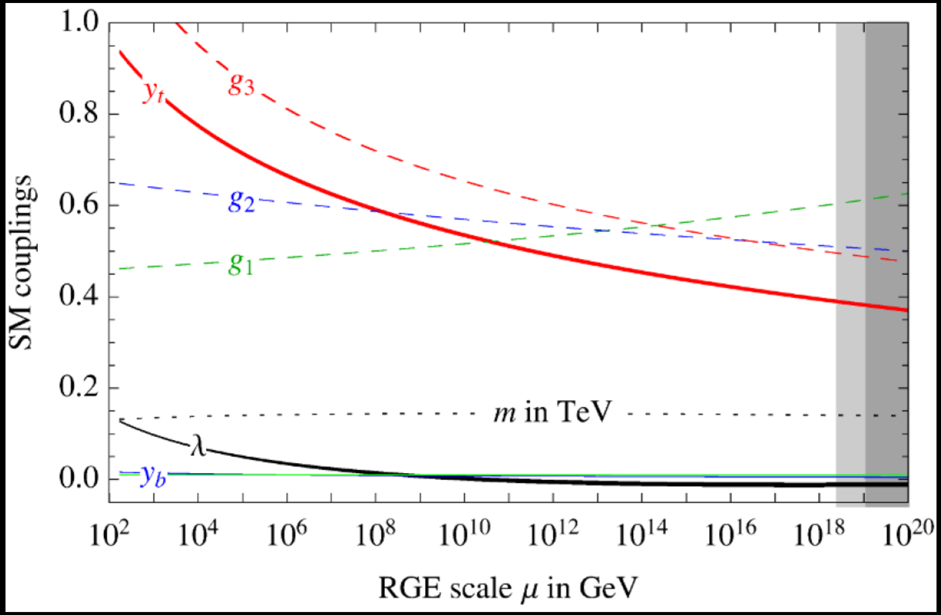
Standard Model of Particle Physics

- All renormalisable terms allowed by symmetries in Minkowski space
- 19 parameters – all have been measured
- Can be extrapolated all the way to Planck scale
- For central experimental values $M_{\rm H} = 125.18$ GeV and $M_{\rm t} = 173.1$ GeV
  - λ becomes negative at $\mu_\Lambda \approx 9.9 \times 10^9$ GeV
  - Minimum value $\lambda_{\rm min} \approx -0.015$ at $\mu_{\rm min} \approx 2.8 \times 10^{17}$ GeV



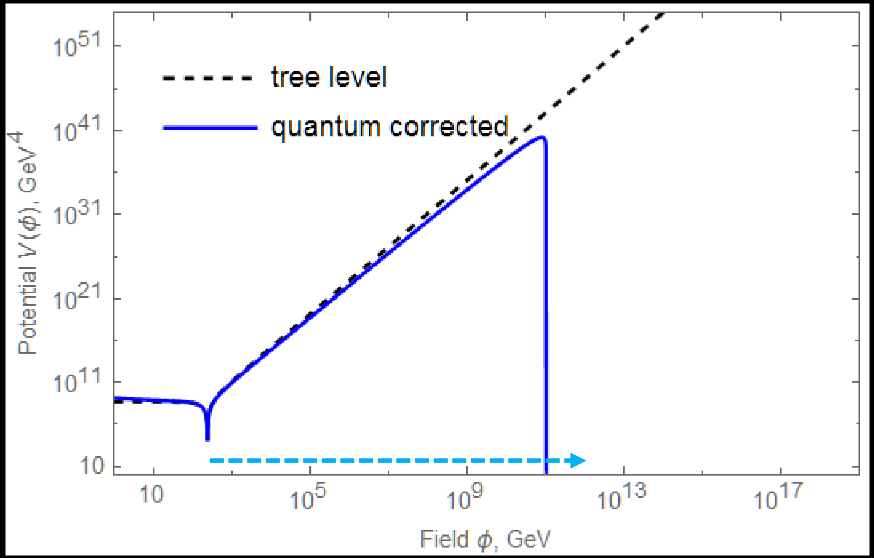
(Buttazzo et al 2013)

Vacuum Instability

- Renormalisation group improved Higgs effective potential

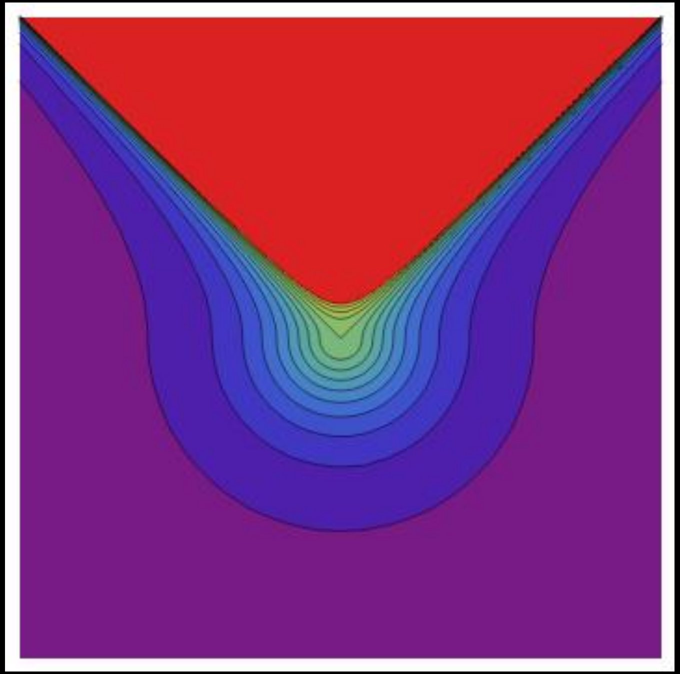
$$V(\phi) \approx \lambda(g\phi)\phi^4$$

- Becomes negative at $\phi > \phi_c \approx 10^{10}\mathrm{GeV}$
- True vacuum at Planck scale?
- Current vacuum metastable against quantum tunnelling
- Barrier at $\phi_{\mathrm{bar}} \approx 4.6 \times 10^{10}\ \mathrm{GeV}$, height $V(\phi_{\mathrm{bar}}) \approx \left(4.3 \times 10^{9}\ \mathrm{GeV}\right)^4$ (Based on a 3-loop calculation by Bednyakov et al. 2015)



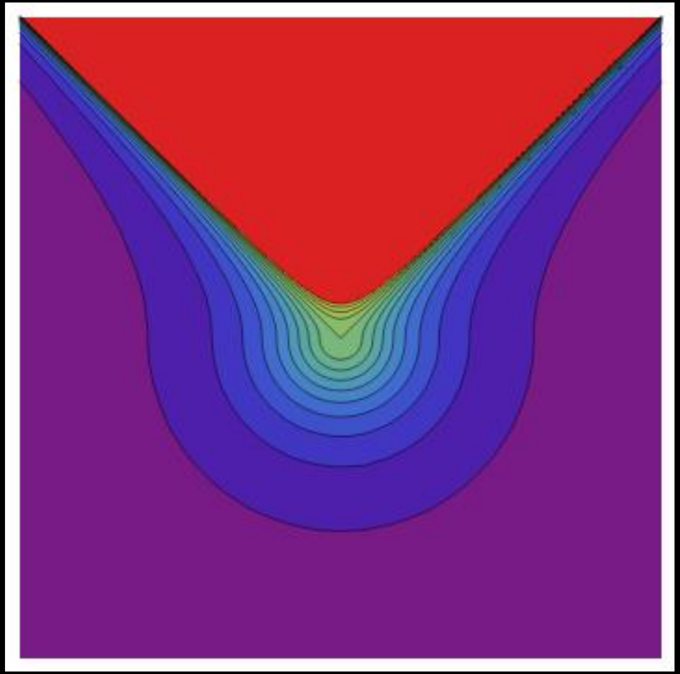
Bubble Nucleation

- Bubble nucleation rate:
  - $\Gamma \sim e^{-B}$, where
  - $B =$ "bounce" instanton action (Coleman 1977)
  - Solution of Euclidean equation of motion
- Spherical symmetric bounce
  ⇒ Lorentz invariant
- Bubble expands at the speed of light
  (see, however, De Luca, Kehagias & Riotto JCAP 2022)
- Bubble interior:
  Gravitational collapse in a microscopic time
  (Unless trapped behind an event horizon)



Bubble Nucleation

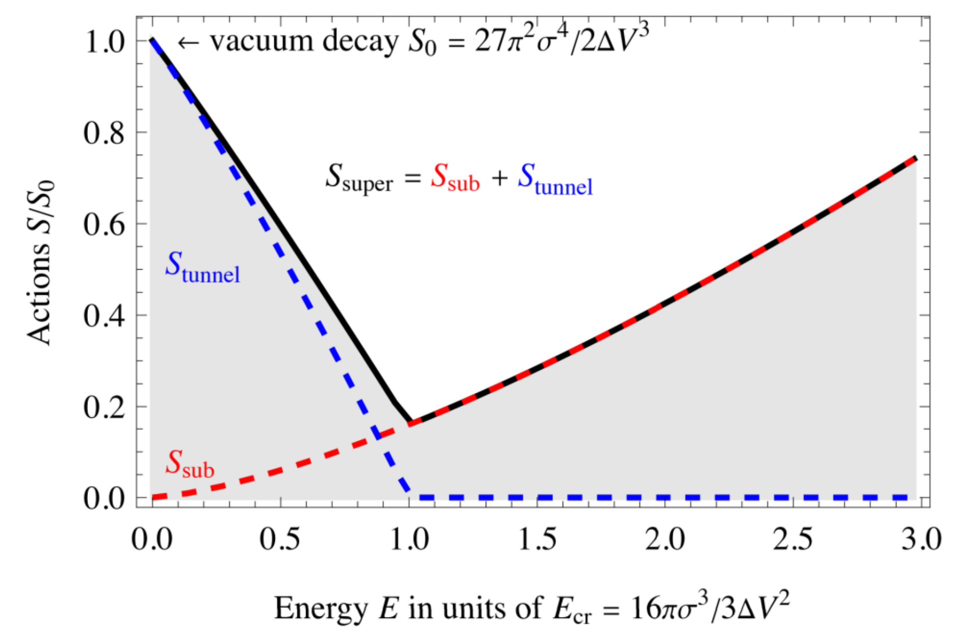
- Simple toy model: (Fubini 1976)
  $V(\phi) = \frac{1}{4}\lambda\phi^4$ with constant $\lambda < 0$
- Exact solution
  $$\phi(r) = \sqrt{\frac{2}{|\lambda|}}\frac{2R}{r^2+R^2}$$
- Action $B = \frac{8\pi^2}{3|\lambda|}$
- When λ runs, $B \approx \frac{8\pi^2}{3|\lambda_{\min}|} \approx 1800$
  (depending on Higgs and top masses)
  $\Rightarrow$ extremely slow rate $\Gamma \sim \mu_{\min}^4 e^{-B}$



- Can be enhanced by higher-dimensional operators, e.g., ϕ^6 (Branchina et al 2014)

Bubbles from Particle Collisions

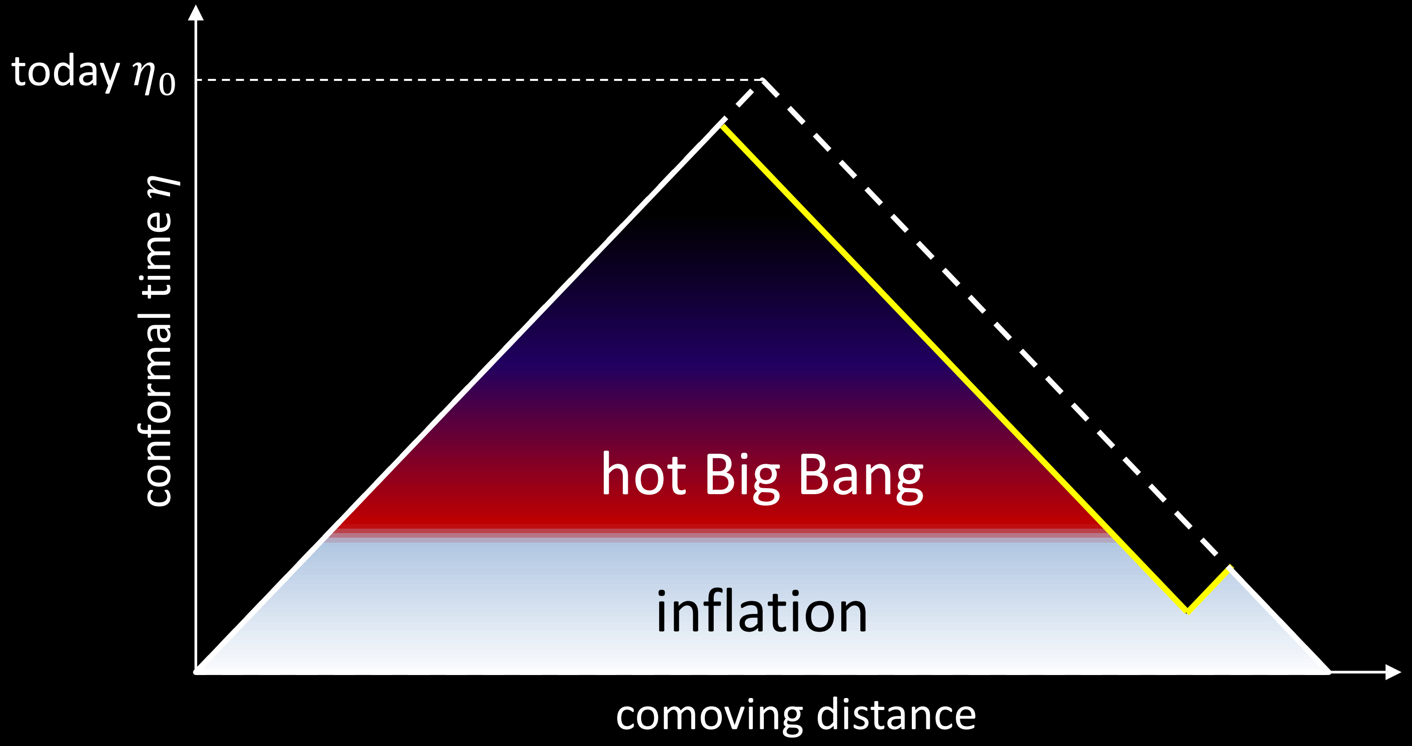
- Non-vacuum initial state makes the calculation more difficult:
  Classical solution on a complex time path
  (Rubakov,Son&Tinyakov 1992)
- Numerical solutions for 1+1D toy model
  (Demidov&Levkov 2015)
  and analytical estimates
  (Kiselev 1992; Gorsky&Voloshin 1993;
  Voloshin 1994; Kuznetson&Tinyakov 1997, …):
  Suppression largely unchanged
- Can be understood as entropy suppression



(Strumia 2023)

Past Light Cone

- Assume: Bubbles grow at the speed of light and destroy everything they hit
  $\Rightarrow$ There cannot have been any bubbles in our past light cone



Past Light Cone

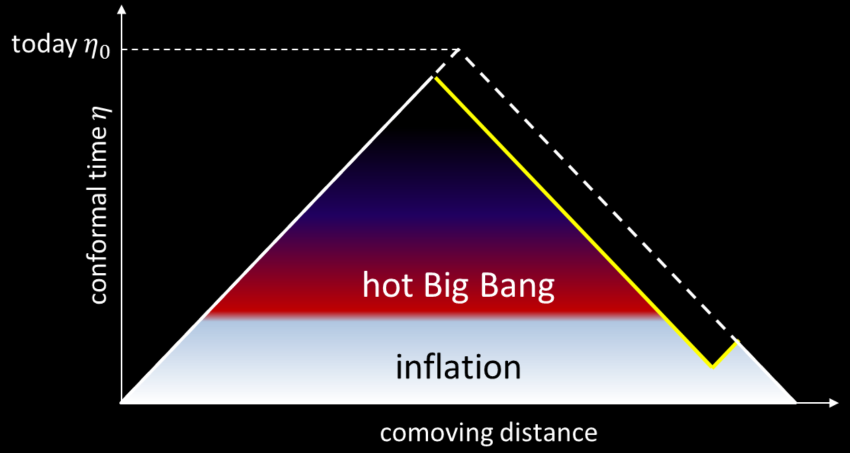
- Probability of no bubble in the past light cone:

$$P(\mathcal{N}=0)=\mathrm{e}^{-\langle\mathcal{N}\rangle},$$

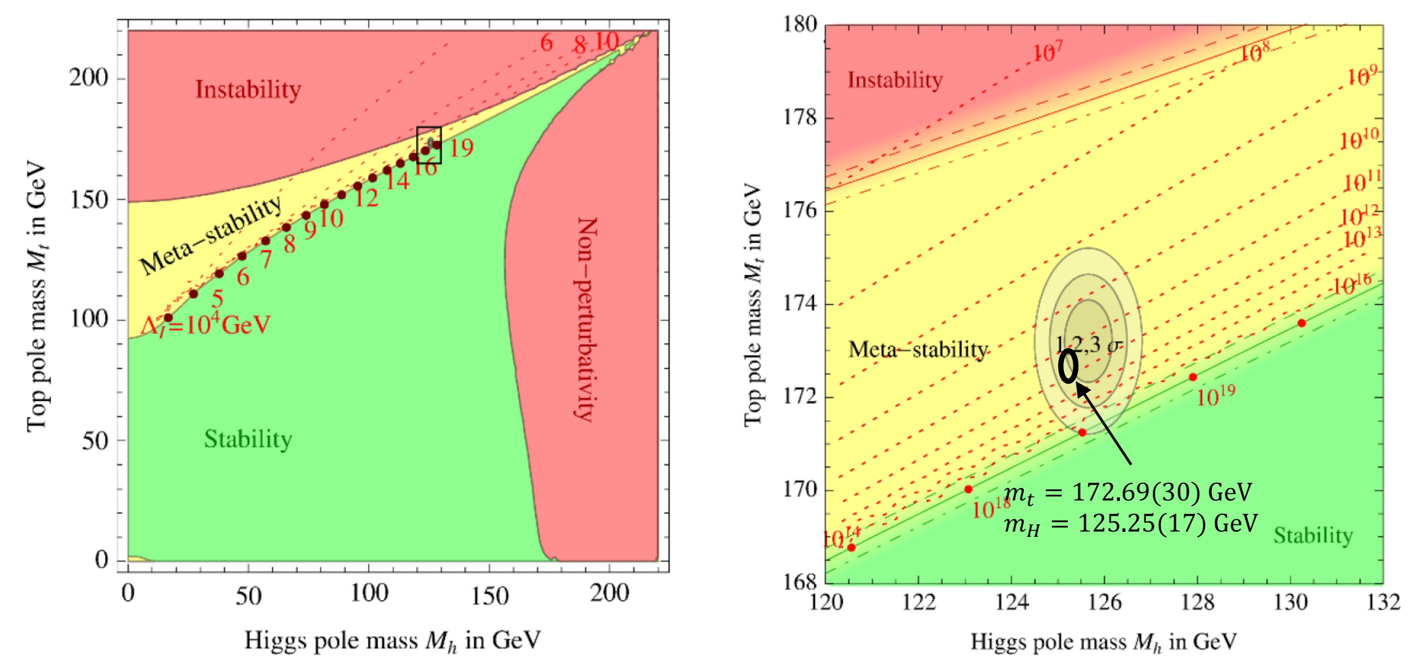
where $\langle\mathcal{N}\rangle$ is the expected number of bubbles ($d\eta = dt/a$),

$$\langle\mathcal{N}\rangle=\frac{4\pi}{3}\int^{\eta_0}d\eta\; a(\eta)^4(\eta_0-\eta)^3\Gamma(\eta)$$

- Therefore, we must have $\langle\mathcal{N}\rangle \lesssim 1$
- Integrate over the whole history of the Universe: inflation, reheating, hot Big Bang, and late Universe
- (For anthropists: $\frac{d\langle\mathcal{N}\rangle}{dt}\Delta t \lesssim 1$)



Late Universe Stability Bounds



(Buttazzo et al. 2013)

- Number of bubbles in past lightcone: $\langle \mathcal{N} \rangle \approx 0.125\Gamma/H_0^4$
- Metastability: $0 < \langle \mathcal{N} \rangle \ll 1$, no contradiction with observations

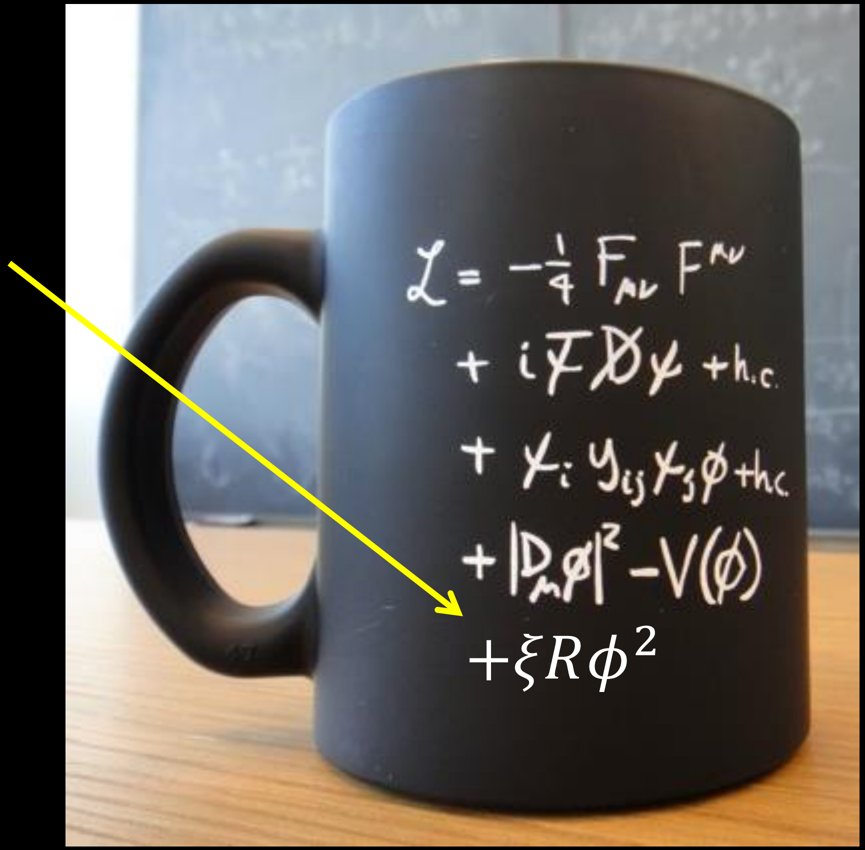
Effect of Gravity

- Bubble interior:
  Gravity important
- Standard Model symmetries allow one more renormalisable term, the Higgs-curvature coupling ξ:

  $$\mathcal{L} = \mathcal{L}_{\mathrm{SM}} + \xi R \phi^\dagger \phi$$

  (Chernikov&Tagirov 1968)

  where R is the spacetime curvature
- Required for renormalisability, runs with energy –
  Cannot be set to zero!
- Last unknown parameter in the Standard Model

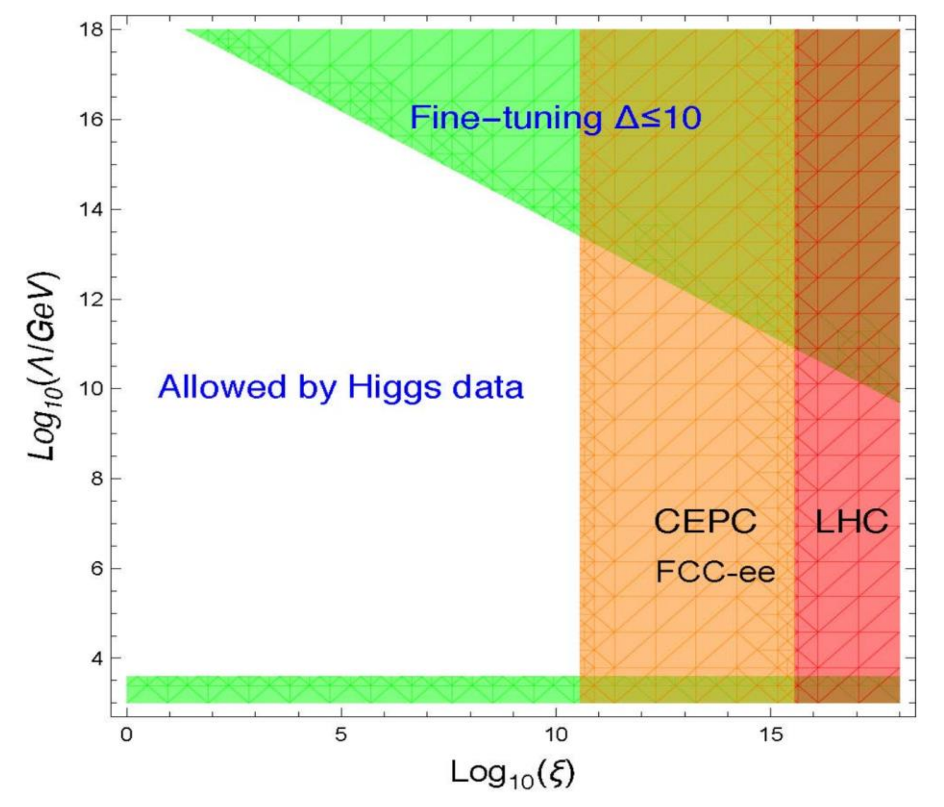


Measuring ξ

- Curved spacetime:

$$\mathcal{L} = \mathcal{L}_{\mathrm{SM}} + \xi R \phi^\dagger \phi$$

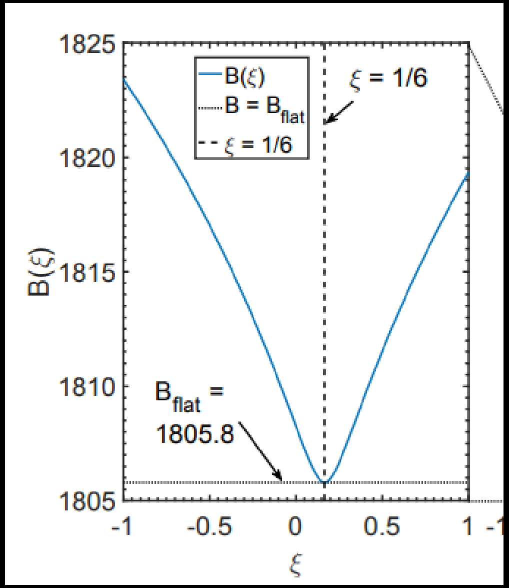
- Ricci scalar R very small today
  $\Rightarrow$ Difficult to measure ξ
- Colliders: Suppresses Higgs couplings
  - LHC Bound $|\xi| \lesssim 2.6 \times 10^{15}$ (Atkins&Calmet 2012)
  - Future colliders: $|\xi| \lesssim 10^{11}$ (Wu et al. 2019)

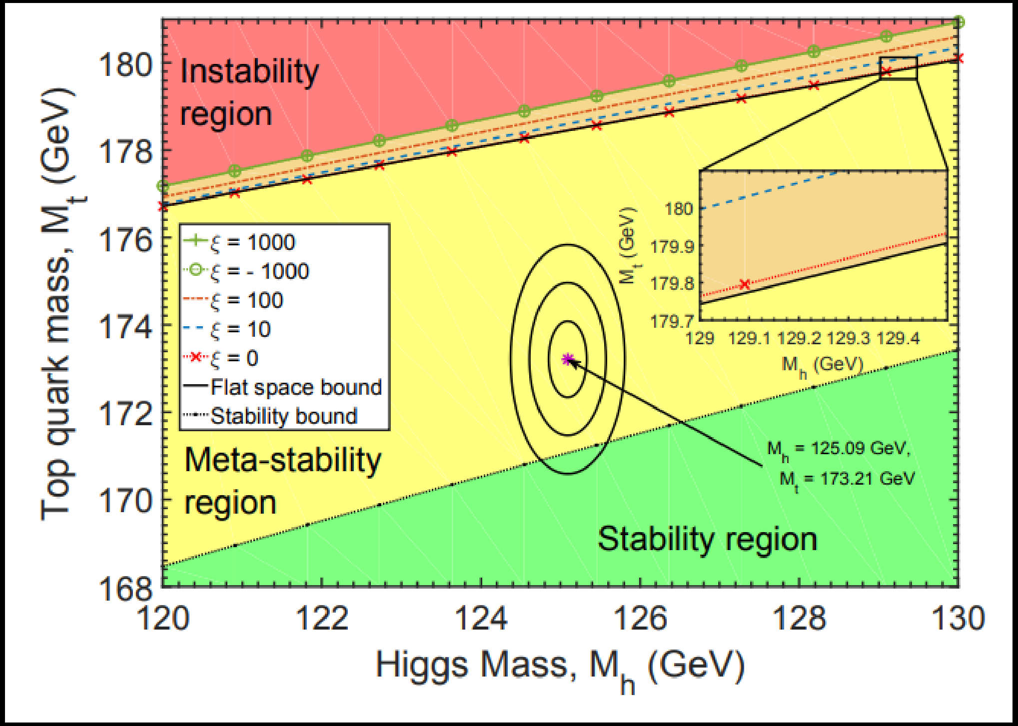


(Wu et al. 2019)

Late Universe Stability Bounds

- Find the gravitational instanton by solving field + Einstein equations numerically (AR&Stopyra 2016)

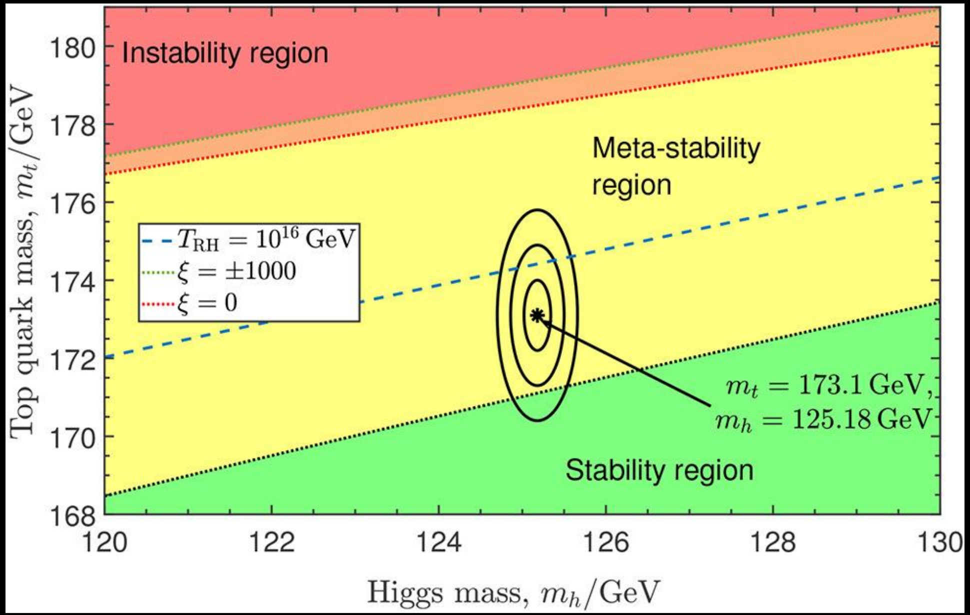




Hot Big Bang

- High temperature:
  Higher bubble nucleation rate
  (Arnold&Vokos 1992; Espinosa et al 2008)
- If reheat temperature T_{RH} is high enough, this dominates over late-time contribution
  ⇒ More unstable
- Top mass bound: (Delle Rose et al 2016)

$$\frac{M_t}{\mathrm{GeV}} < 0.283\left(\frac{\alpha_s - 0.1184}{0.0007}\right) + 0.4612\frac{M_h}{\mathrm{GeV}}$$
$$+1.907\log_{10}\frac{T_{\mathrm{RH}}}{\mathrm{GeV}} + \frac{1.2\times 10^3}{0.323\log_{10}\frac{T_{\mathrm{RH}}}{\mathrm{GeV}} + 8.738}.$$

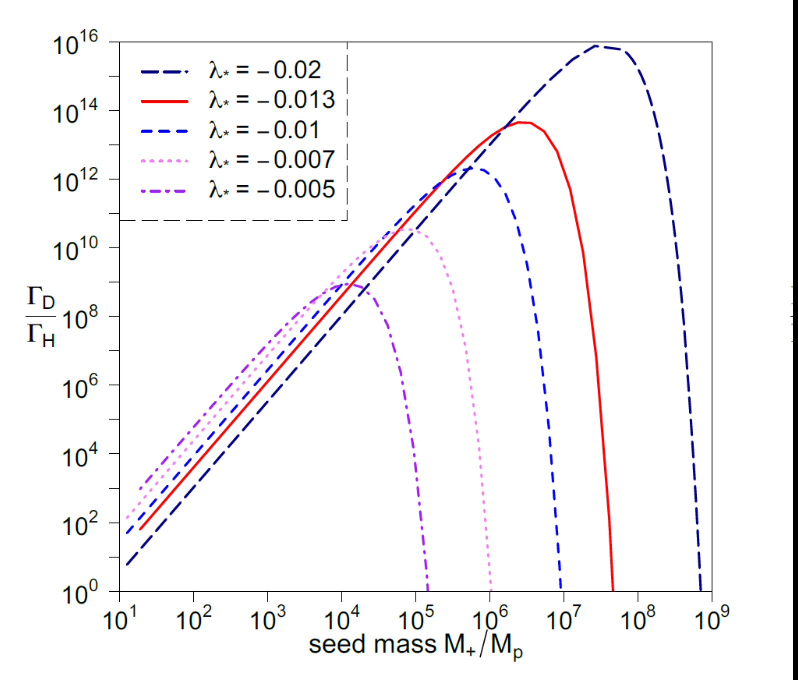


(Markkanen, AR, Stopyra, 2018)

Bubbles from Black Holes

- Black holes can catalyse vacuum decay
  (Hiscock 1987; Berezin et al 1991)
- Burda et al 2016:
  Planck-scale black holes catalyse
  vacuum decay faster than they evaporate
  ⇒ Excludes primordial BHs with mass $M \lesssim 10^{12}\ \mathrm{kg}$
- Shkerin&Sibiryakov 2022:
  Thermal effects due to Hawking radiation stabilise
  (see also Strumia 2022)

⇒ No primordial black hole constraints from vacuum stability?



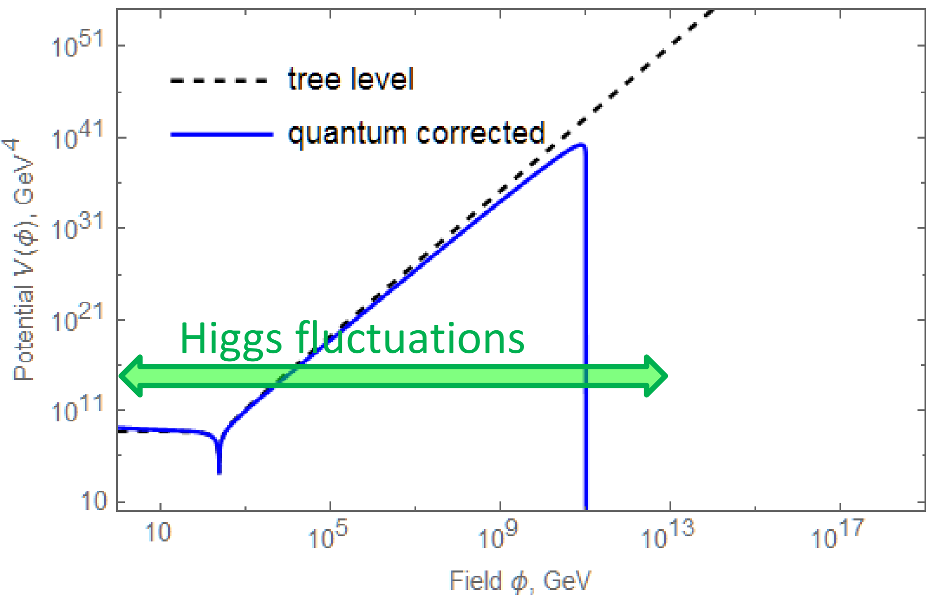
(Burda et al 2016)

Higgs Fluctuations from Inflation

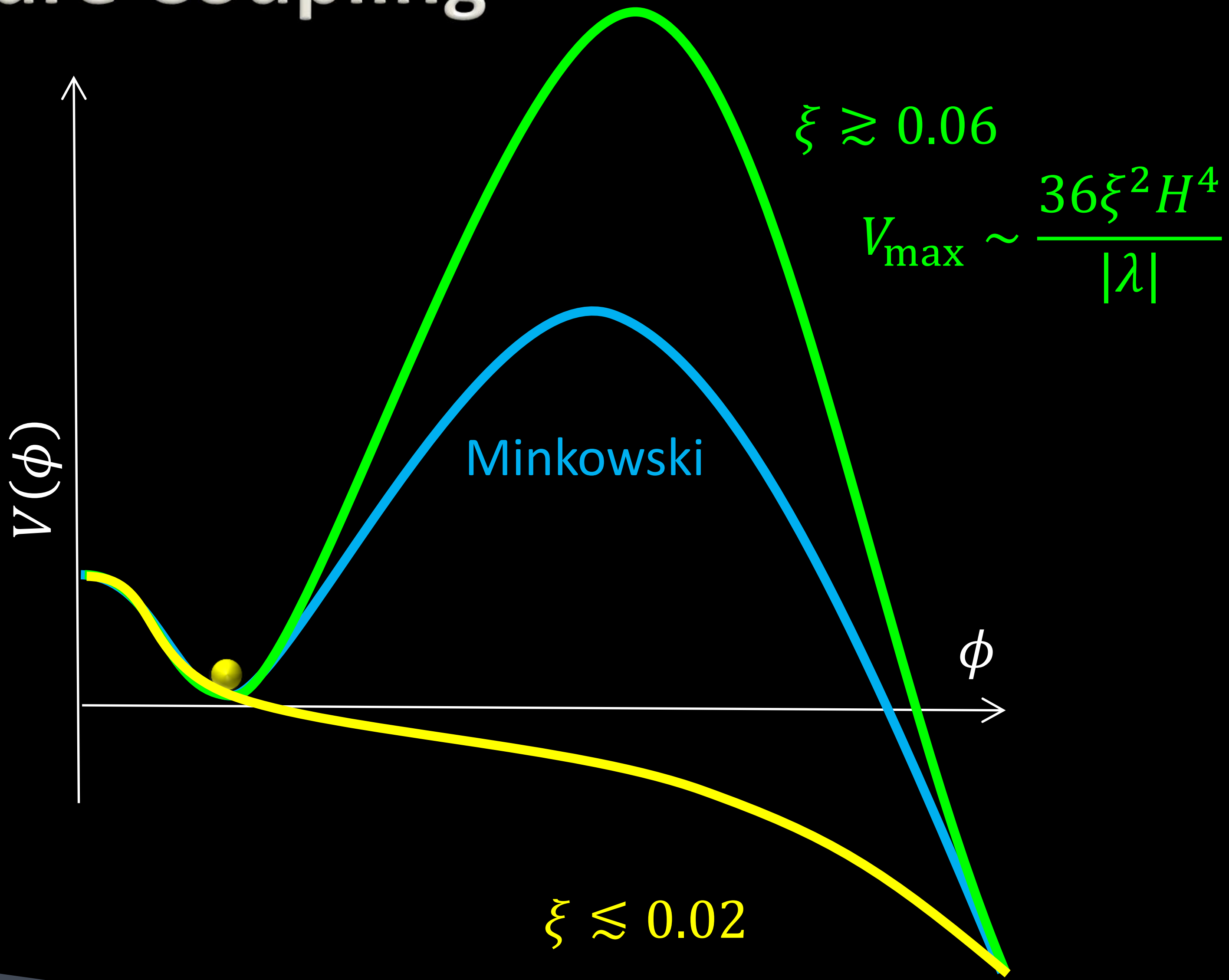
- Quantum fluctuations:

$$P(\phi) \propto \exp\left[-\frac{8\pi^2}{3H^4}V(\phi)\right]$$

- Take the Higgs over the barrier if the Hubble rate is high, $H \gtrsim \phi_{\mathrm{bar}} \approx 10^{10}\mathrm{GeV}$
  (Espinosa et al. 2008;
  Lebedev&Westphal 2013;
  Kobakhidze&Spencer-Smith 2013;
  Fairbairn&Hogan 2014;
  Hook et al. 2014)
- Does this imply an upper limit on the scale of inflation $H \lesssim 10^{10}\mathrm{GeV}$?



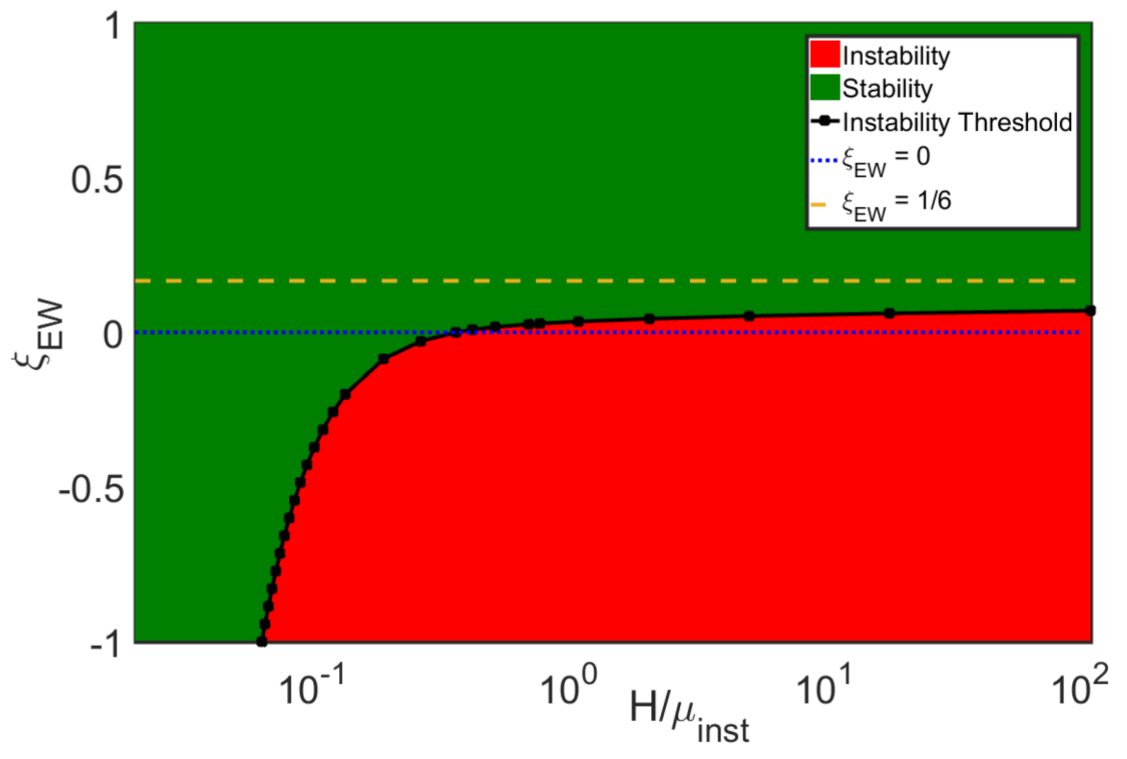
Higgs-Curvature Coupling



$$V_{\max} \sim \frac{36\xi^2 H^4}{|\lambda|}$$

(De)Stabilising the Potential

- Minimal scenario:
  Standard Model + $m^2\chi^2$ chaotic inflation, no direct coupling to inflaton
- If $H \gtrsim \mu_{\text{inst}} = 6.6 \times 10^9 \text{GeV}$ and there is no new physics, vacuum stability during inflation requires $\xi \gtrsim 0.06$
  (Markkanen,Nurmi,AR,Stopyra 2018; Mantziris,Markkanen,AR 2021,2022)
- Stability at the end of inflation requires $\xi \lesssim 9$
  (Herranen,Markkanen,Nurmi,AR 2015; Figueroa,AR&Torrenti 2018)



(MNRS 2018)

Vacuum Stability Constraint on ξ

- Stability both during and through the end of inflation requires

$$0.06 \lesssim \xi \lesssim 9$$

- 15 orders of magnitude stronger than the LHC bound

$$|\xi| \lesssim 2.6 \times 10^{15}$$

- Caveats:
  - Assumes no direct coupling to inflaton (see, e.g., Ema et al. 2016, 2017)
    – Would still need $|\xi| \lesssim O(1)$
  - Assumes no new physics
    – Could stabilise potential altogether, or destabilise further
  - Assumes high scale inflation $H \gtrsim 10^9$ GeV

Summary

- Experimental results
  ⇒ The Standard Model vacuum is metastable unless stabilised by new physics
- No contradiction with observations, but can provide interesting constraints:
  - Reheating temperature
  - Primordial black holes (?)
  - Scale of inflation
  - (Non-minimal) Higgs-curvature coupling
- Particle collisions highly unlikely to trigger vacuum decay

- Depends sensitively on any new physics!