QCD effects in $b \to s$ decays

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Mostly based on:

- Gubernari, MR, van Dyk, Virto 2206.03797
- Amhis, Bordone, MR 2208.08937

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IP3

I. Introduction

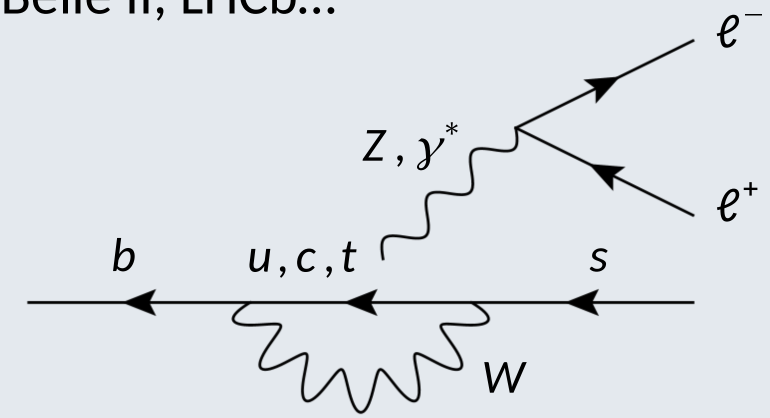
Why b → s (μμ)?

- FCNC → highly suppressed in the SM (GIM, CKM, loop)

  → **Very rare** processes: BR ~ 10^{-7} or smaller

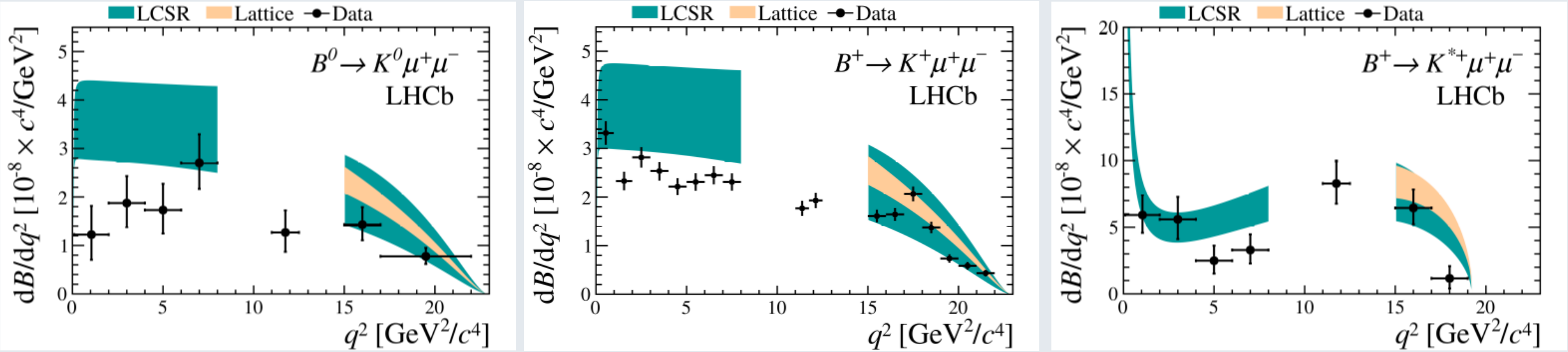
- **Rich BSM pattern** due to the flavour structure

- **Experimentally accessible** at BaBar, Belle, Belle II, LHCb…



Experimental results (I)

[LHCb 2014]

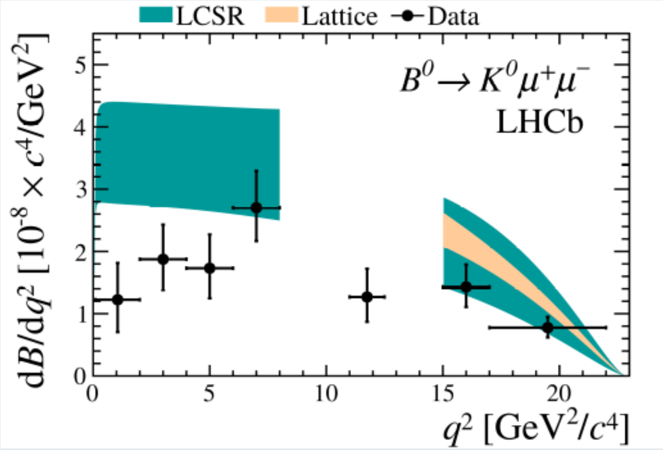


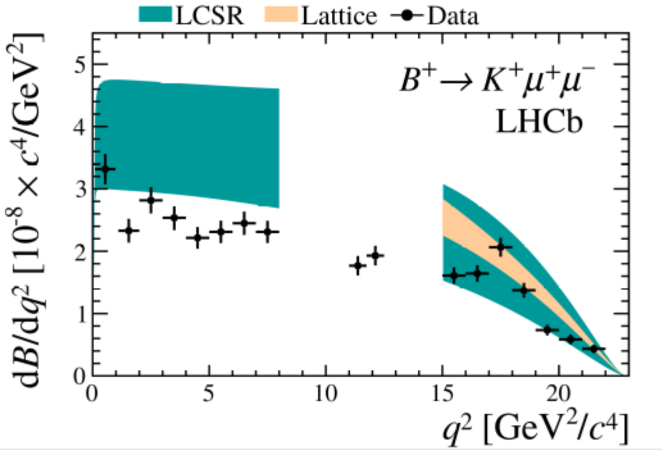
- Empty bins correspond to J/ψ and ψ(2S) resonances
- Branching ratio are **normalized using B → J/ψ K$^{(*)}$** (known to a few percent)
- Systematic uncertainties are partially correlated (which makes the combination harder)
- **Theory uncertainties mostly come from QCD** and are the aim of this talk

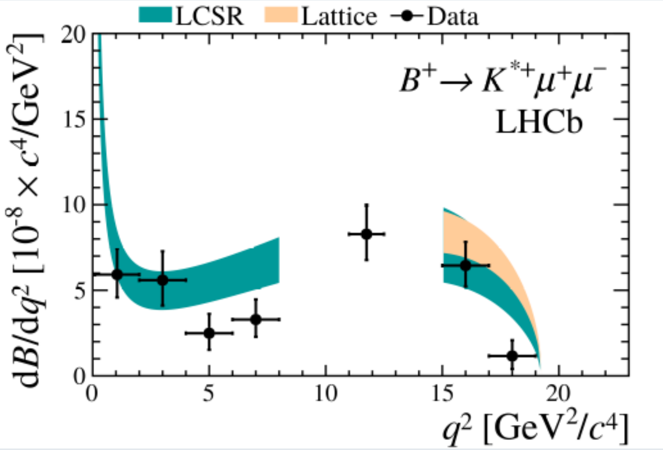
Other measurements exist, see below

Experimental results (II)

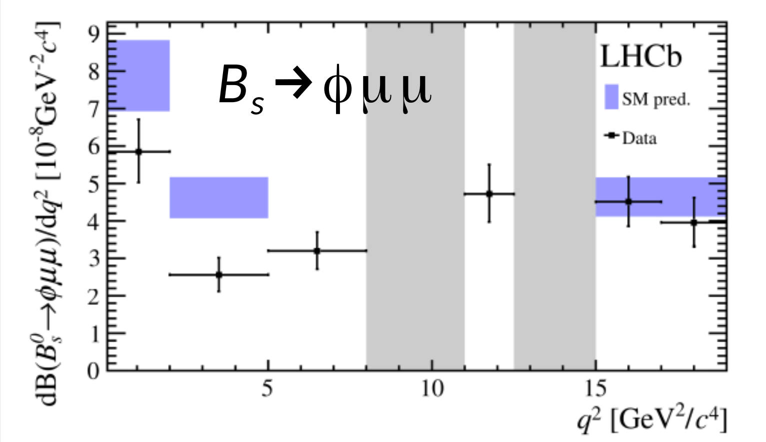
[LHCb 2014]



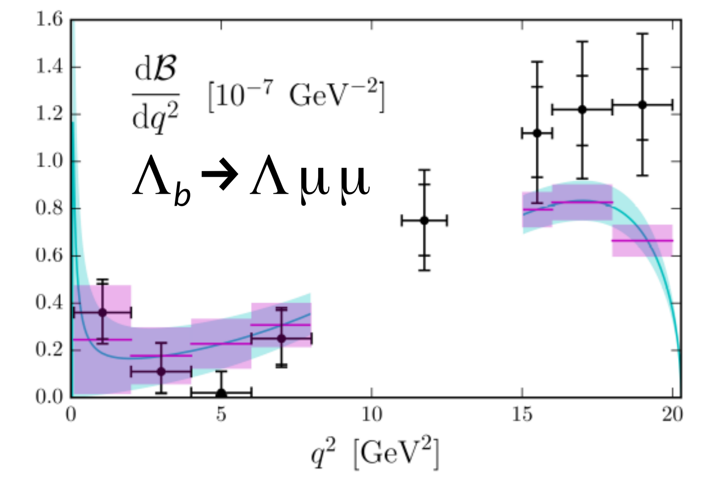




[LHCb 2015]



[LHCb 2015; Detmold&Meinel 2016]



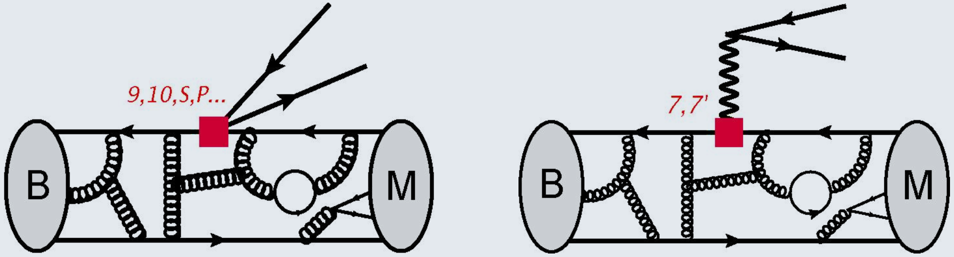
Other measurements exist, see below

II. Predictions

$$\mathcal{H}(b \to s\ell\ell) = -\frac{4G_F}{\sqrt{2}} V_{tb}V_{ts}^* \sum_{i=1}^{10} C_i(\mu)\mathcal{O}_i(\mu)$$

$$\mathcal{O}_{9(10)} = \frac{e^2}{16\pi^2}\left(\bar{s}_L\gamma_\mu b_L\right)\left(\bar{\ell}\gamma^\mu(\gamma_5)\ell\right)$$

$$\mathcal{O}_7 = \frac{e}{16\pi^2}\left(\bar{s}_L\sigma_{\mu\nu}b_R\right)F^{\mu\nu}$$



$$\mathcal{A}_\lambda^{L,R}(B \to M_\lambda\ell\ell) = \mathcal{N}_\lambda\left\{(C_9 \mp C_{10})\mathcal{F}_\lambda(q^2) + \frac{2m_bM_B}{q^2}\left[C_7\mathcal{F}_\lambda^T(q^2) - 16\pi^2\frac{M_B}{m_b}\mathcal{H}_\lambda(q^2)\right]\right\}$$

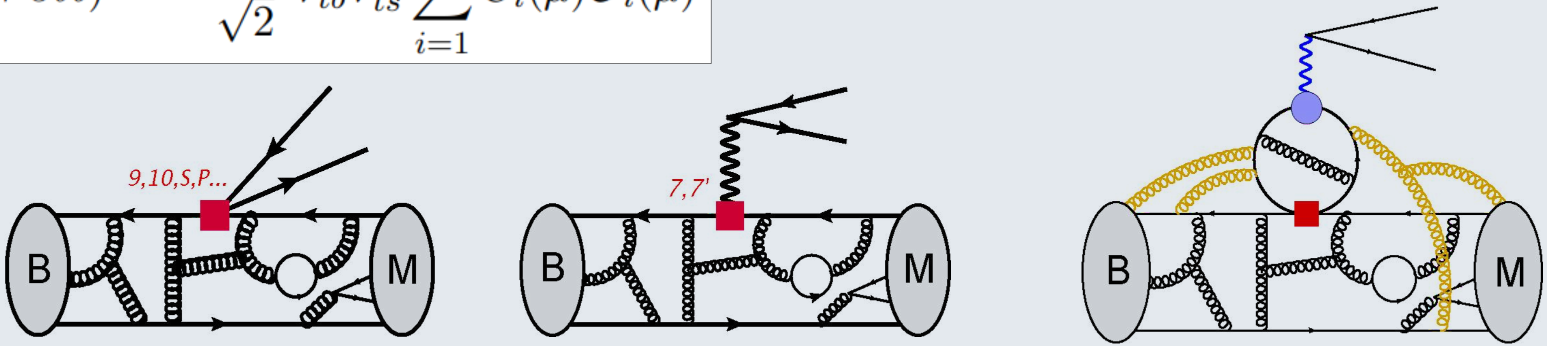
- $B \to K^{(*)}\mu\mu$
- $B_s \to \varphi\mu\mu$
- $\Lambda_b \to \Lambda^{(*)}\mu\mu$

Local form-factors, involves e.g.

$$\mathcal{F}_\mu(k,q) = \langle\bar{M}(k)|\bar{s}\gamma_\mu b_L|\bar{B}(q+k)\rangle$$

Form factors in b → sℓℓ

$$\mathcal{H}(b \to s\ell\ell) = -\frac{4G_F}{\sqrt{2}} V_{tb}V_{ts}^* \sum_{i=1}^{10} C_i(\mu)\mathcal{O}_i(\mu)$$



$$\mathcal{A}_\lambda^{L,R}(B \to M_\lambda \ell\ell) = \mathcal{N}_\lambda \left\{ (C_9 \mp C_{10})\mathcal{F}_\lambda(q^2) + \frac{2m_b M_B}{q^2}\left[C_7 \mathcal{F}_\lambda^T(q^2) - 16\pi^2 \frac{M_B}{m_b}\mathcal{H}_\lambda(q^2) \right] \right\}$$

$$\mathcal{H}_\mu(k,q) = i \int d^4x\, e^{iq\cdot x} \langle \bar{M}(k) | T\{\mathcal{J}_\mu^{\text{em}}(x), \mathcal{C}_i\mathcal{O}_i\} | \bar{B}(q+k)\rangle$$

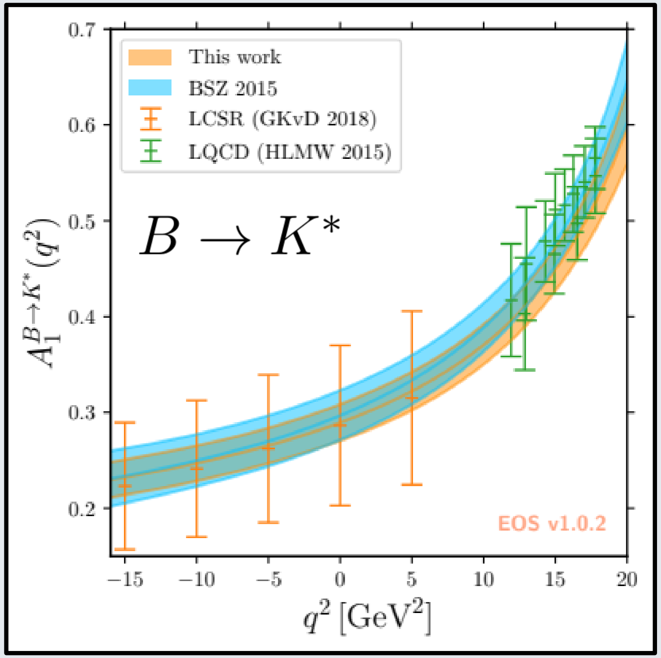
Non-local form-factors

→ Main contributions: the **"charm-loops"** $\mathcal{O}_{2(1)}^c = (\bar{s}_L\gamma_\mu(T^a)c_L)(\bar{c}_L\gamma^\mu(T^a)b_L)$

- **2 main approaches**
  - Lattice QCD → most feasible at **large q^2**
  - Light-cone sum rules → most feasible at **small q^2**

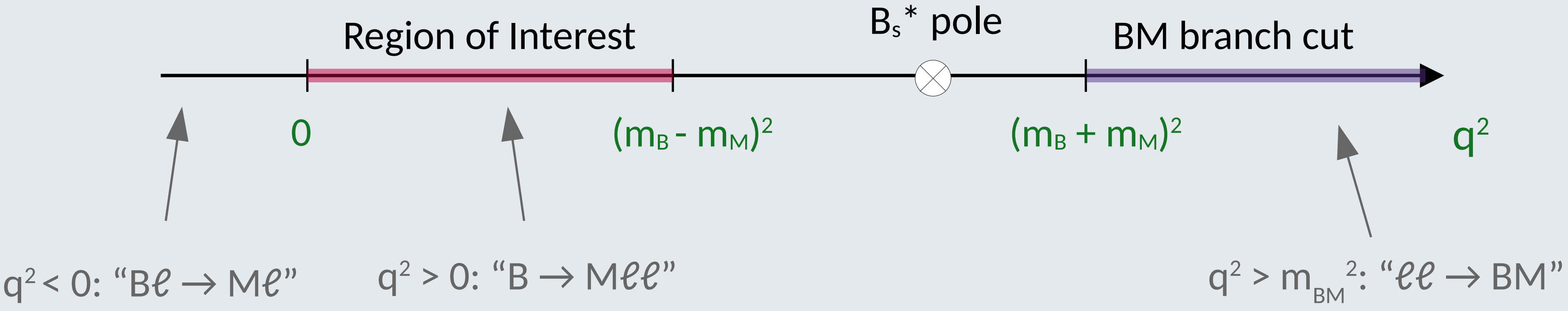
- 2 possible LCSRs:
  - Light meson LCDA [recent works: Bharrucha, Straub, Zwicky '15; Khodjamirian, Rusov '17]
  - B meson LCDA [recent works: Khodjamirian, Mannel, Pivovarov, Wang '10; Gubernari, Kokulu, van Dyk '18]

→ **Interpolation** in the physical range



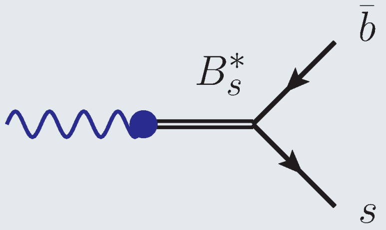
Form Factor Properties

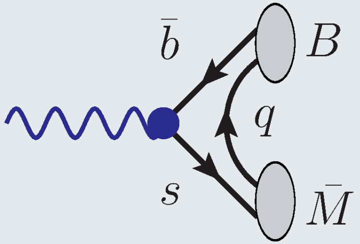
$$\mathcal{F}_\mu(k,q) = \langle \bar{M}(k) | \bar{s}\gamma_\mu b_L | \bar{B}(q+k) \rangle$$



Analytic properties of the form factors:

- Pole due to **$b\bar{s}$ bound state**
- **Branch cut** due to on-shell BM production





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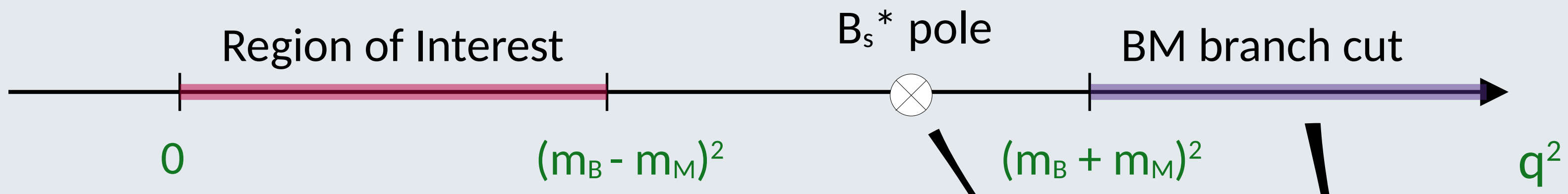
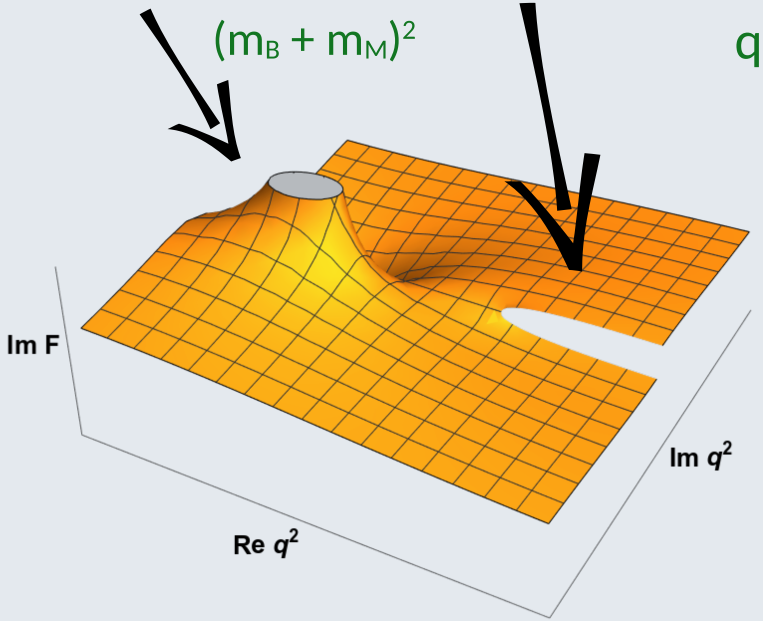
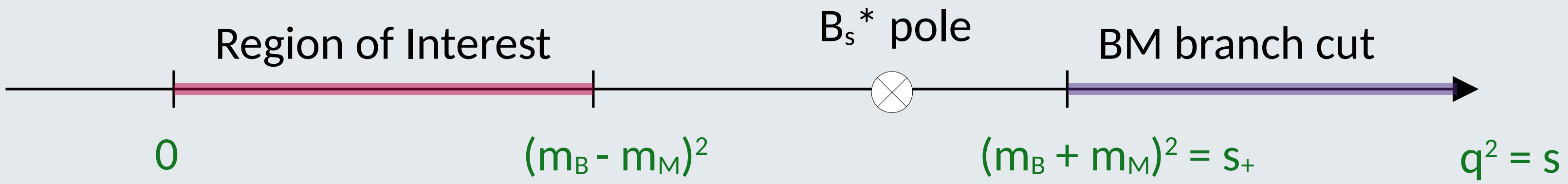




Analytic properties of the form factors:

- Pole due to $\bar{b}$s **bound state**
- **Branch cut** due to on-shell pair production

Form Factor Parametrization



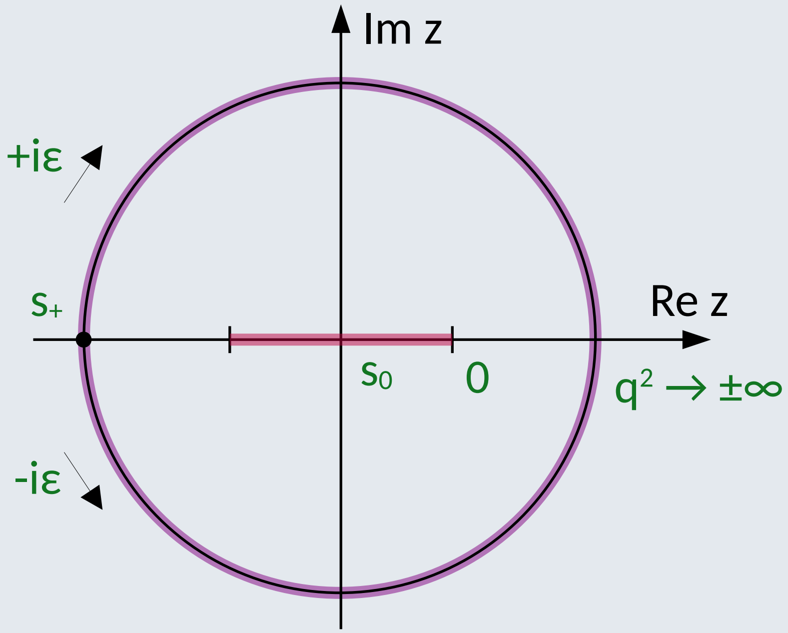
Conformal mapping [Boyd, Grinstein, Lebed '97]

$$z(s) \equiv \frac{\sqrt{s_+ - s} - \sqrt{s_+ - s_0}}{\sqrt{s_+ - s} + \sqrt{s_+ - s_0}}$$

Simplified Series expansion [Bourrely, Caprini, Lellouch, '08; Bharucha, Feldmann, Wick '10]

$$\mathcal{F}_\lambda^{(T)}(q^2) = \frac{1}{q^2 - m_{B_s^*}^2} \sum_{k=0}^{N} \alpha_{\lambda,k} z^k$$

N = 2 is enough to provide an **excellent description of the data** (p-values > 70%)

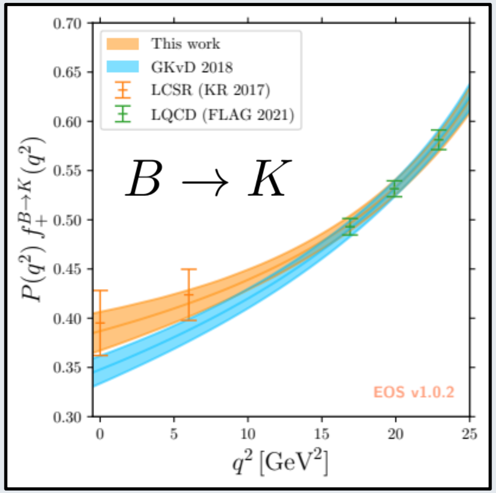


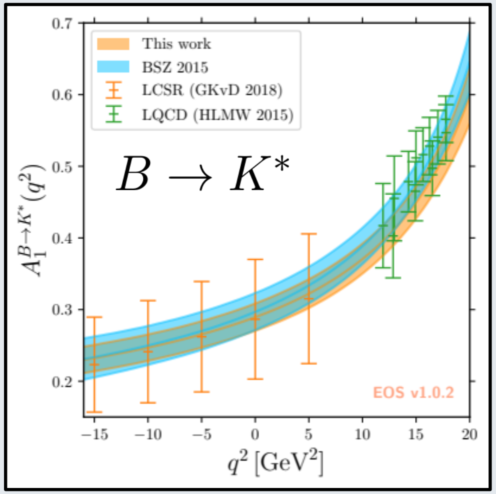
Local form factors

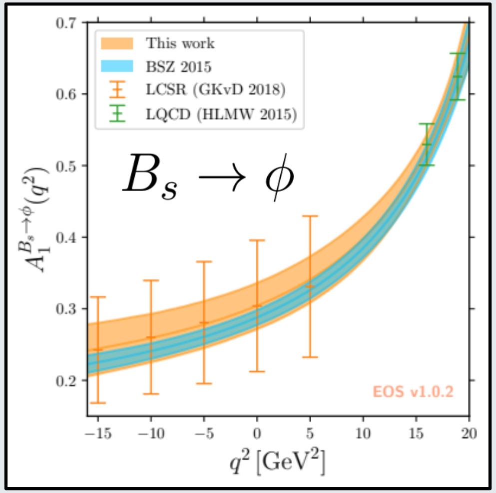
[Gubernari, MR, van Dyk, Virto '22]

Combined fit to LCSR and lattice QCD
Inputs:

- B → K:
  - [HPQCD'17; FNAL/MILC '17]
  - [Khodjamiriam, Rusov '17]
- B → K*:
  - [Horgan, Liu, Meinel, Wingate '15]
  - [Gubernari, Kokulu, van Dyk '18]
- B_s → φ:
  - [Horgan, Liu, Meinel, Wingate '15]
  - [Gubernari, van Dyk, Virto '20]







What about the **model uncertainties**? What if we only have LQCD?

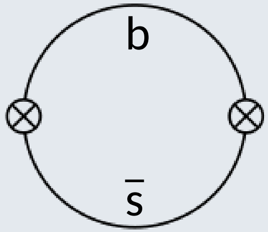
[BSZ = Bharucha, Straub, Zwicky '15]

II. Dispersive bound

Dispersive bound

- **Main idea:** Compute the inclusive $e^+e^- \to \bar{b}s$ cross-section and relate it to the form factors [Bharucha, Feldmann, Wick '10]

Insertion of a scalar, vector or tensor current



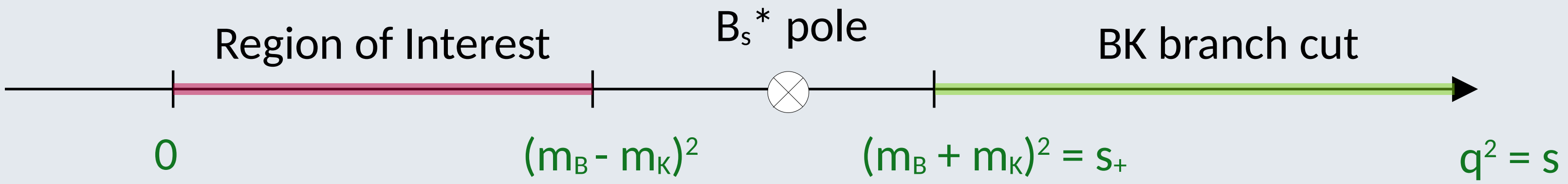
+ other diagrams: loops, quark and gluon condensates…

- Unitarity gives **shared bounds** for **all the b → s processes**: (schematically)

$$1 > 2\int_{(m_B+m_K)^2}^{\infty} \left|\hat{\mathcal{F}}_X^{B\to K}(t)\right|^2 dt \;+\; 2\int_{(m_B+m_{K^*})^2}^{\infty} \left|\hat{\mathcal{F}}_X^{B\to K^*}(t)\right|^2 dt \;+\; \ldots$$

$$\text{known functions} \times \mathcal{F}_X^{B\to K}(t)$$

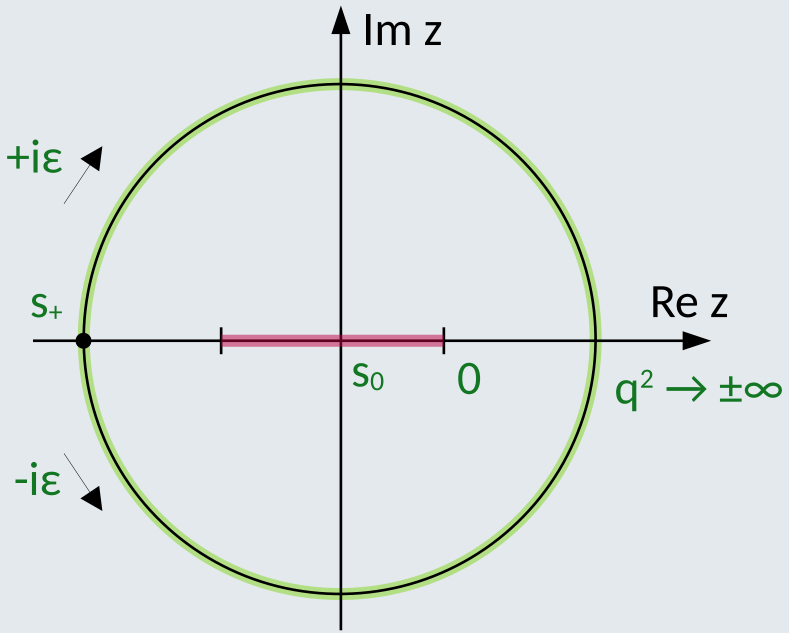
Simple case: B → K



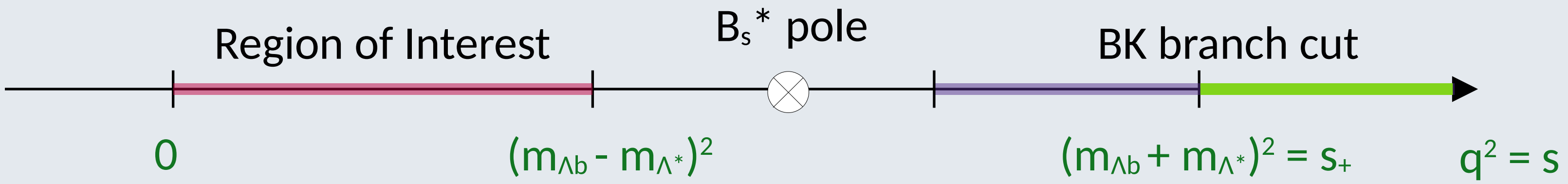
- The branch cut starts **at** the pair production threshold
- The monomial z^k are **orthogonal** on the unit circle

$$\hat{\mathcal{F}}_X^{B\to K} = \sum_{k=0}^{N} a_{X,k}\, z^k$$

$$\int_{(m_B+m_K)^2}^{\infty} \left|\hat{\mathcal{F}}_X^{B\to K}(t)\right|^2 dt = \sum_{k=0}^{N} |a_{X,k}|^2$$



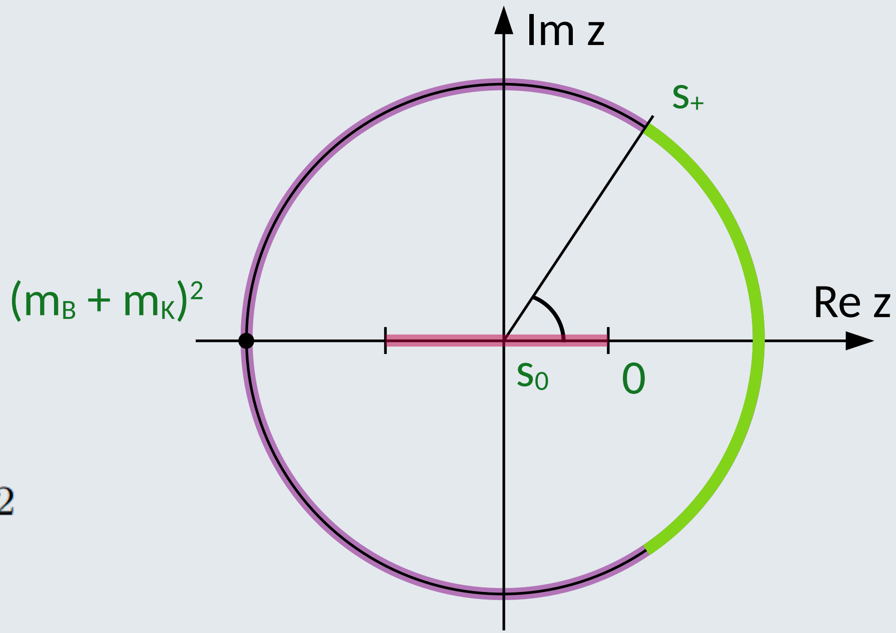
Less simple case, e.g. $\Lambda_b \to \Lambda^*$



- The first branch cut (BK) starts **before** the pair production threshold
- Introduce orthonormal polynomials of the **arc of the unit circle**

$$\hat{\mathcal{F}}_X^{\Lambda_b \to \Lambda^*} = \sum_{k=0}^{N} a_{X,k}\, p_k(z)$$

$$\int_{(m_{\Lambda_b}+m_{\Lambda^*})^2}^{\infty} \left|\hat{\mathcal{F}}_X^{\Lambda_b \to \Lambda^*}(t)\right|^2 dt = \sum_{k=0}^{N} |a_{X,k}|^2$$



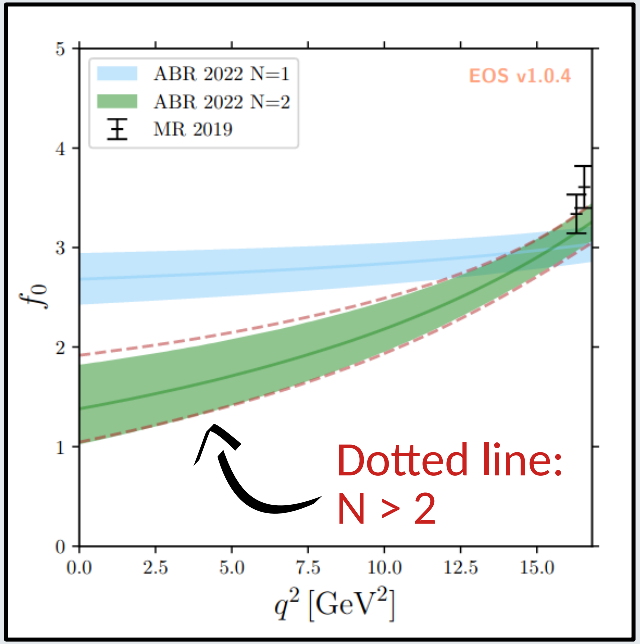
III. Numerical results for $\Lambda_b \to \Lambda^*$

Fit results

- Inputs:
  - **LQCD** [Meinel, Rendon '21]
  - no LCSR → use **SCET relations** [Descotes-Genon, Novoa-Brunet '19]

$$f_{\perp'}(0) = 0 \pm 0.2\,, \quad g_{\perp'}(0) = 0 \pm 0.2\,, \quad h_{\perp'}(0) = 0 \pm 0.2\,,$$
$$\tilde{h}_{\perp'}(0) = 0 \pm 0.2\,, \quad f_+(0)/f_\perp(0) = 1 \pm 0.2\,, \quad f_\perp(0)/g_0(0) = 1 \pm 0.2\,,$$
$$g_\perp(0)/g_+(0) = 1 \pm 0.2\,, \quad h_+(0)/h_\perp(0) = 1 \pm 0.2\,, \quad f_+(0)/h_+(0) = 1 \pm 0.2\,,$$

$O(\alpha_s/\pi, \Lambda_{QCD}/m_b)$

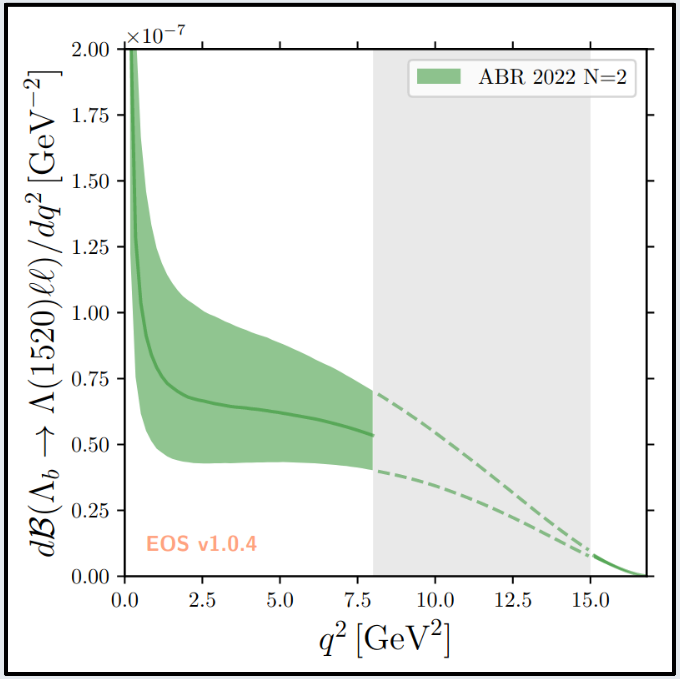


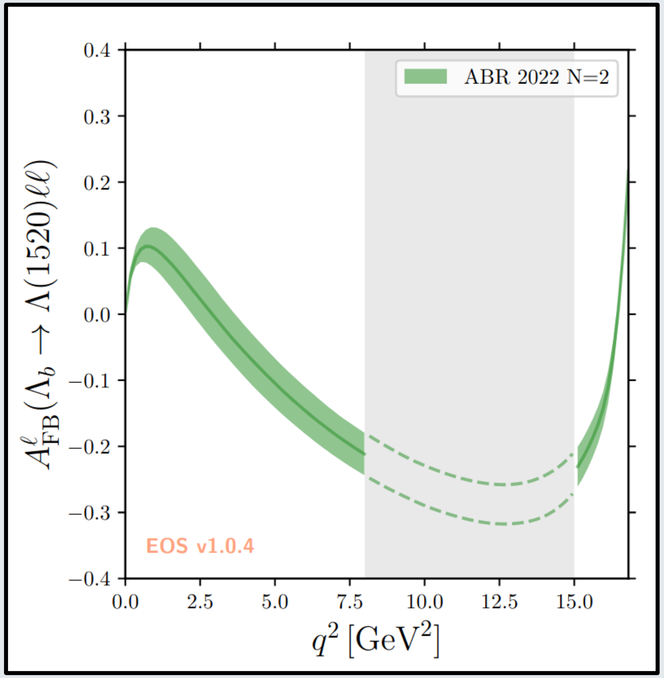
- Use an **under-constrained fit** (N>1) and allows for saturation of the dispersive bound

  → The uncertainties are model-independent, increasing the expansion order does not change their size

Phenomenology

- Uncertainties are large but **under control** and **systematically improvable**
- LHCb analysis is **ongoing**



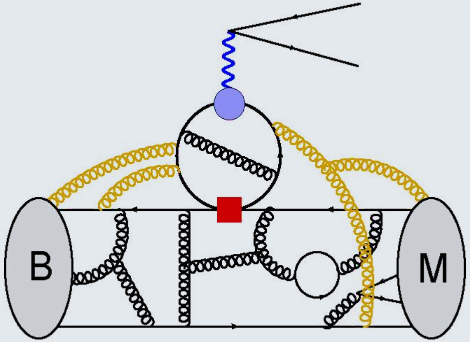


IV. Non-local contributions

Non-local form factors

$$\mathcal{A}_\lambda^{L,R}(B \to M_\lambda \ell\ell) = \mathcal{N}_\lambda \left\{ (C_9 \mp C_{10}) \mathcal{F}_\lambda(q^2) + \frac{2 m_b M_B}{q^2} \left[C_7 \mathcal{F}_\lambda^T(q^2) - 16\pi^2 \frac{M_B}{m_b} \mathcal{H}_\lambda(q^2) \right] \right\}$$

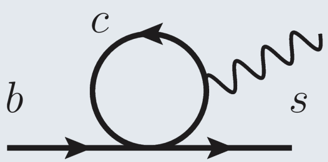
$$\mathcal{H}_\mu(k,q) = i \int d^4x\, e^{iq\cdot x} \langle \bar{M}(k) | T\{ \mathcal{J}_\mu^{\text{em}}(x), \mathcal{C}_i \mathcal{O}_i \} | \bar{B}(q+k) \rangle$$



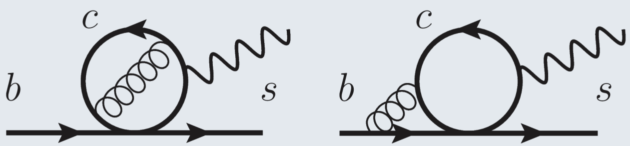
- Problematic because **they can mimic a BSM signal!**
  - $\mathcal{H}_\lambda$ can be interpreted as a shift to C_9 and C_7
  - This shift is lepton-flavour universal (as now seen in the data)
- Notably **harder to estimate**, no lattice computation so far
- **Different parametrizations** are suggested

$\mathcal{H}_\lambda$ can still be calculated in **two kinematics regions**:

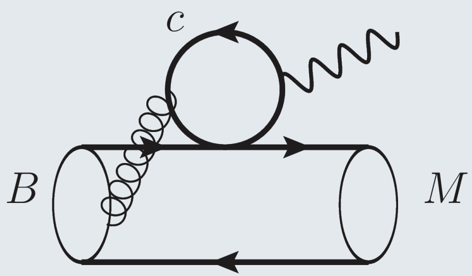
- **Local** OPE $|q|^2 \gtrsim m_b^2$ [Grinstein, Piryol ‘04; Beylich, Buchalla, Feldmann ‘11]
- **Light Cone** OPE $q^2 \ll 4m_c^2$ [Khodjamirian, Mannel, Pivovarov, Wang ‘10]



LO and α_s corrections

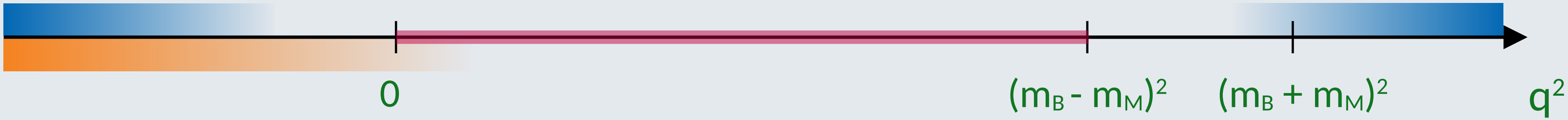


[Asatarian, Greub, Virto ‘19]



Non-perturbative soft gluon corrections

[Gubernari, van Dyk, Virto ‘20]

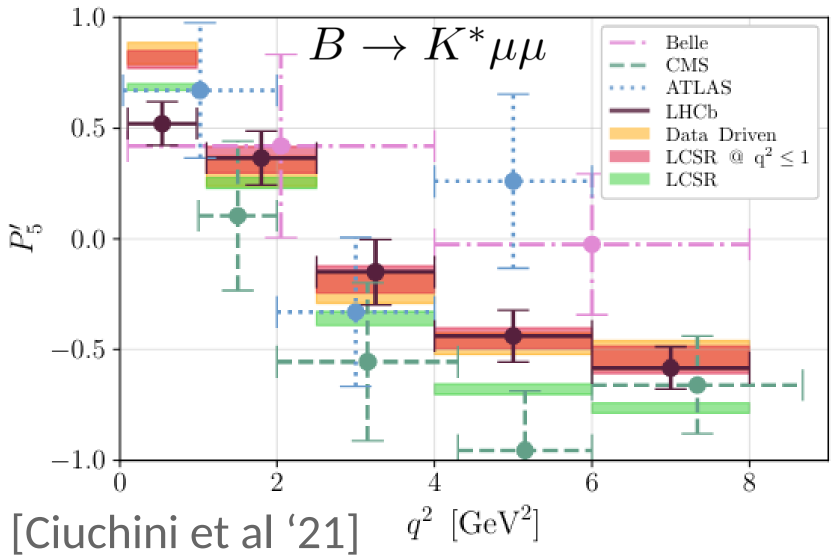


Parametrization #1

- **Simple q² expansion** [Jäger, Camalich ‘12; Ciuchini et al. ‘15]

$$\mathcal{H}_\lambda(q^2) = \mathcal{H}_\lambda^{\text{QCDF}}(q^2) + h_\lambda(0) + \frac{q^2}{m_B^2} h'_\lambda(0) + \ldots$$

Computed in [Beneke, Feldman, Seidel ‘01]

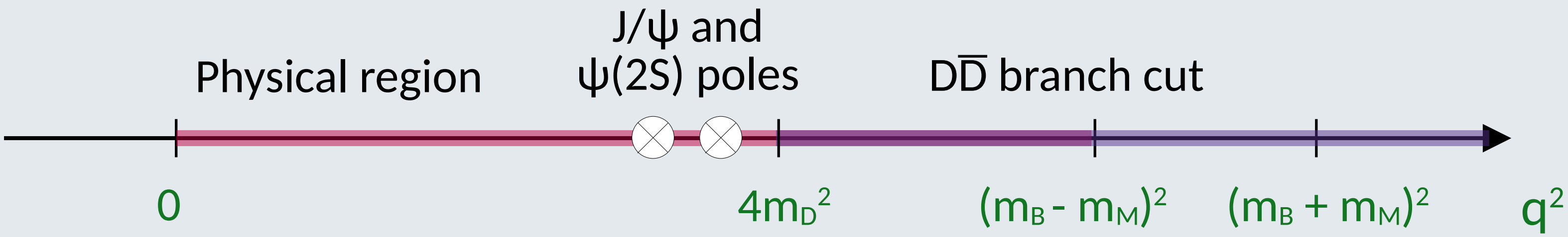


[Ciuchini et al ‘21]

- The h_λ terms can be fitted or varied
- Fitting the h_λ terms on data gives a satisfactory but uninformative result
- This parametrization **cannot account** for the analyticity properties of $\mathcal{H}_\lambda$

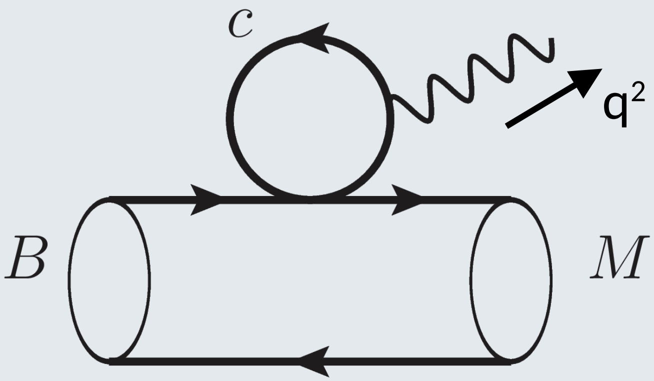
Analyticity properties

$$\mathcal{H}_\mu(k,q) = i\int d^4x\, e^{iq\cdot x}\langle \bar{M}(k)|T\{\mathcal{J}^{\text{em}}_\mu(x), \mathcal{C}_i\mathcal{O}_i\}|\bar{B}(q+k)\rangle$$

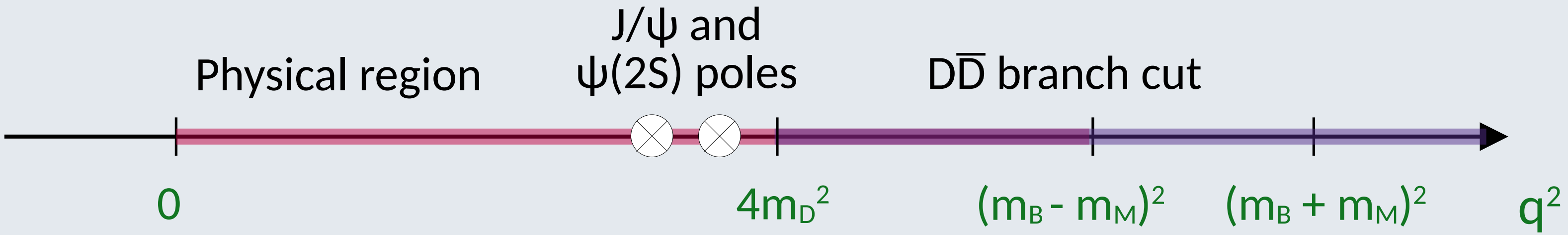


Analyticity properties of the non-local form factors:

- Poles due to **charmonium state**
- **Branch cut** in the physical range due to on-shell D meson production: $B \to MD\bar{D}$

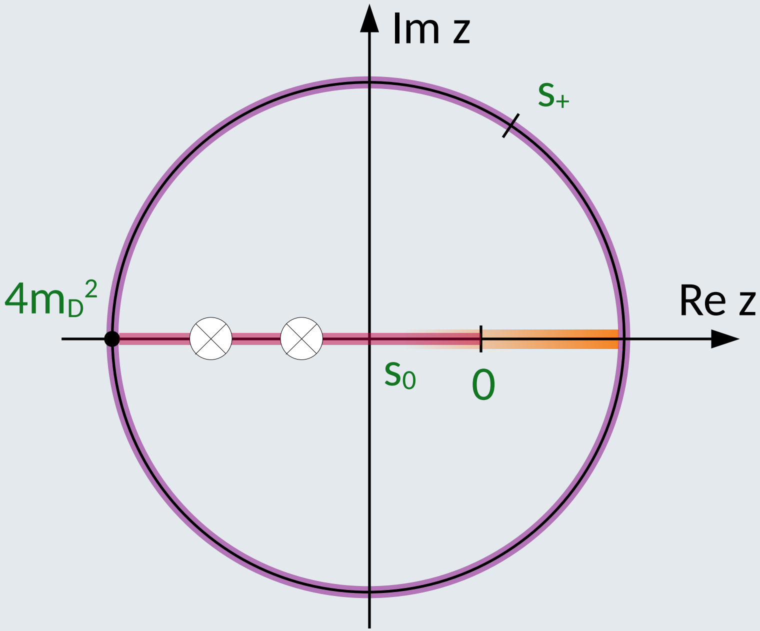


Parametrization #2



- z-expansion: [Bobeth, Chrzaszcz, van Dyk, Virto '17]

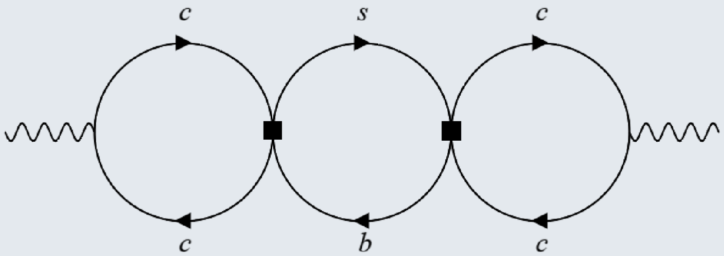
$$z(s) = \frac{\sqrt{4m_D^2 - s} - \sqrt{4m_D^2 - s_0}}{\sqrt{4m_D^2 - s} + \sqrt{4m_D^2 - s_0}} \qquad \mathcal{H}_\lambda(z) = \frac{\mathcal{F}_\lambda(z)}{\mathcal{P}(z)} \sum_{k=0}^{N} a_{\lambda,k}\, z^k$$



- Coefficients can be fitted on the light cone OPE results and the charmonium poles (⊗).
- Main issue: No control of **truncation uncertainties!**

Non-local contributions

- **Main idea:** Compute the charm-loop induced, inclusive $e^+e^- \to \bar{b}s$ cross-section and relate it to $\mathcal{H}_\lambda$ [Gubernari, van Dyk, Virto '20]



+ other diagrams…

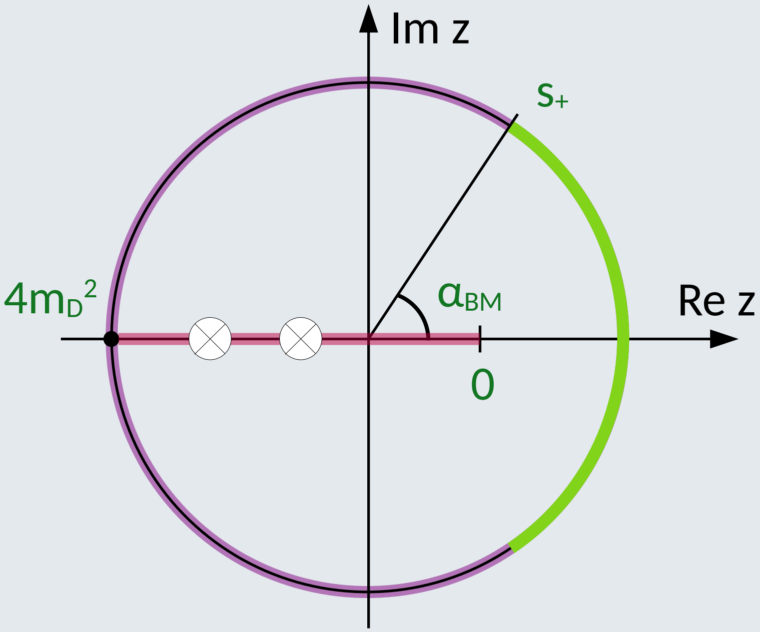
- Unitarity gives a **shared bound** for **all the b → s processes**:

$$1 > 2\int_{-\alpha_{BK}}^{+\alpha_{BK}} d\alpha \left|\hat{\mathcal{H}}_0^{B\to K}(e^{i\alpha})\right|^2 + \sum_\lambda \left[2\int_{-\alpha_{BK^*}}^{+\alpha_{BK^*}} d\alpha \left|\hat{\mathcal{H}}_\lambda^{B\to K^*}(e^{i\alpha})\right|^2 + \int_{-\alpha_{B_s\phi}}^{+\alpha_{B_s\phi}} d\alpha \left|\hat{\mathcal{H}}_\lambda^{B_s\to \phi}(e^{i\alpha})\right|^2\right]$$

$$+\Lambda_b \to \Lambda^{(*)} \ldots$$

Parametrization #3

$$1 > 2 \int_{-\alpha_{BK}}^{+\alpha_{BK}} d\alpha \left| \hat{\mathcal{H}}_0^{B\to K}(e^{i\alpha}) \right|^2 + \sum_\lambda \left[2 \int_{-\alpha_{BK^*}}^{+\alpha_{BK^*}} d\alpha \left| \hat{\mathcal{H}}_\lambda^{B\to K^*}(e^{i\alpha}) \right|^2 + \int_{-\alpha_{B_s\phi}}^{+\alpha_{B_s\phi}} d\alpha \left| \hat{\mathcal{H}}_\lambda^{B_s\to \phi}(e^{i\alpha}) \right|^2 \right]$$



- The bound can be "**diagonalized**" with **orthonormal polynomials** of the arc of the unit circle [Gubernari, van Dyk, Virto '20]

$$\mathcal{H}_\lambda(z) = \frac{1}{\phi(z)\mathcal{P}(z)} \sum_{k=0}^{N} a_{\lambda,k}\, p_k(z)$$

- The new coefficients respect the **simple bound:**

$$\sum_{n=0}^{\infty} \left\{ 2\left| a_{0,n}^{B\to K} \right|^2 + \sum_{\lambda=\perp,\|,0} \left[2\left| a_{\lambda,n}^{B\to K^*} \right|^2 + \left| a_{\lambda,n}^{B_s\to \phi} \right|^2 \right] \right\} < 1$$

Numerical analysis

- The new parametrization is fitted to

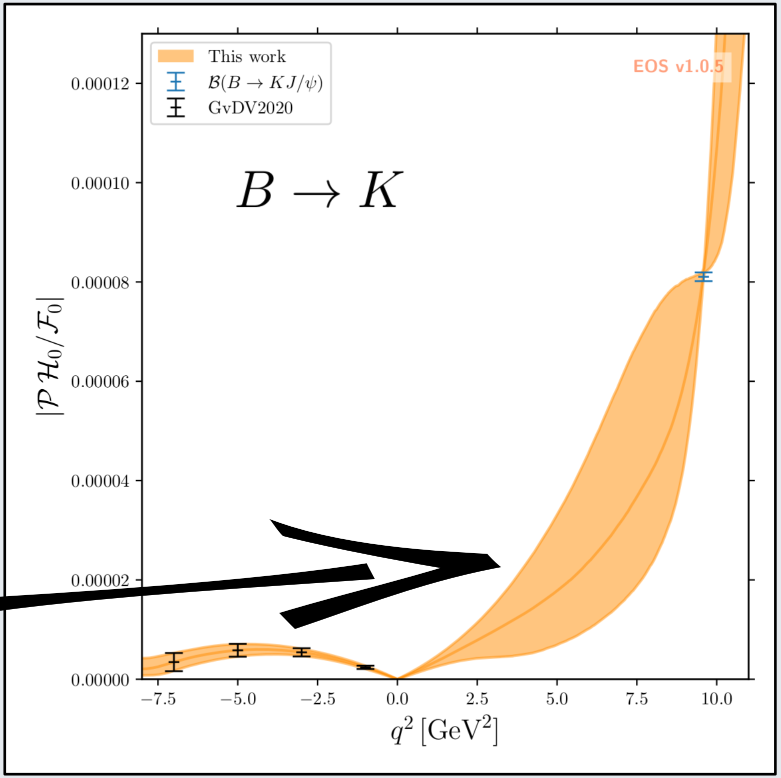
  $B \to K$, $B \to K^*$, $B_s \to \varphi$

  using:
  - 4 theory point at negative q^2 from the light cone OPE
  - Experimental results at the J/ψ
  - Use an **under-constrained fit** and allows for **saturation of the dispersive bound**

→ The uncertainties are **model-independent**, increasing the expansion order does not change their size

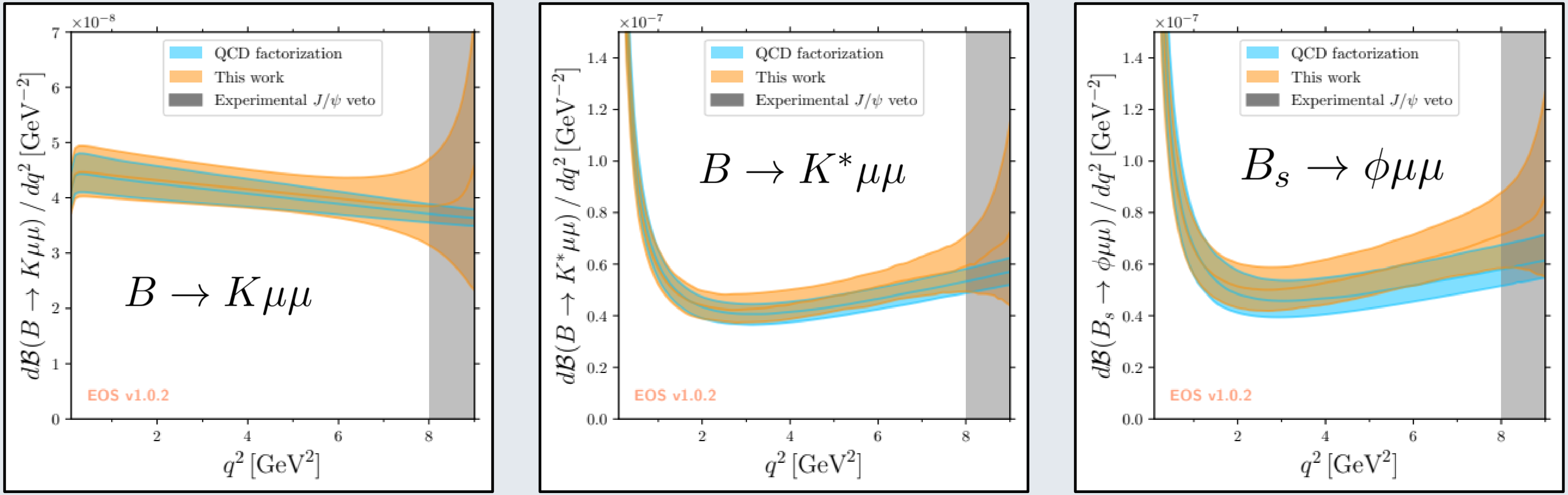
→ All p-values are larger than 11%

[Gubernari, MR, van Dyk, Virto '22]



SM predictions

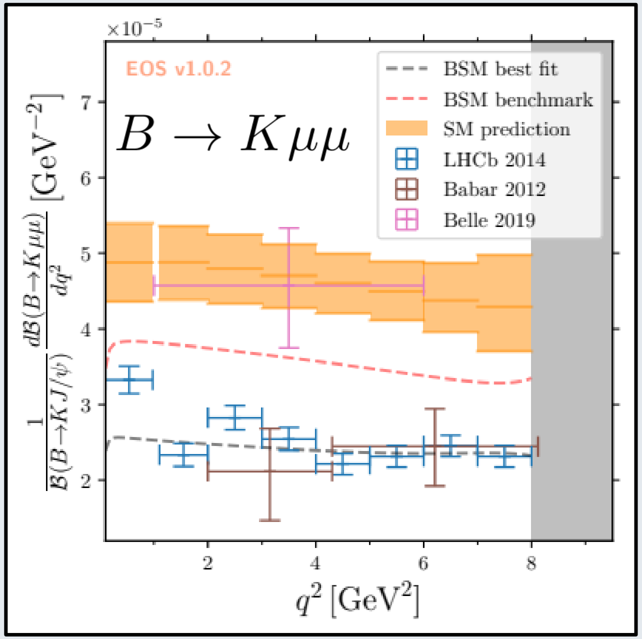
- **Good overall agreement** with previous theoretical approaches
  - Small deviation in the slope of $B_s \to \phi\mu\mu$
- **Larger** but **controlled** uncertainties especially near the J/ψ
  → The approach is **systematically improvable** (new channels, ψ(2S) data…)

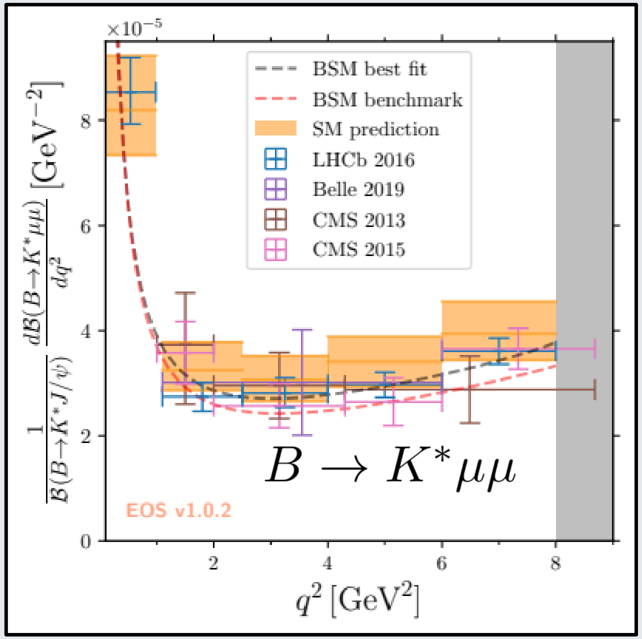


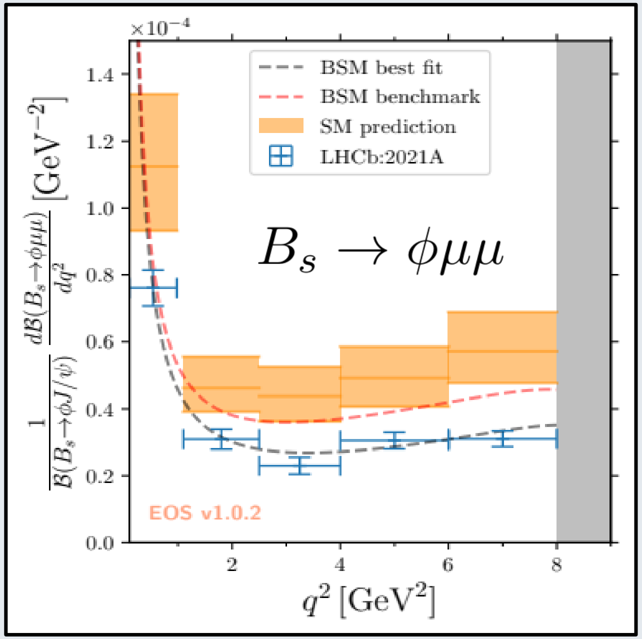
Confrontation with data

- Conservatively accounting for the non-local form factors **does not solve the “B anomalies”**.
- In this approach, the greatest source of theoretical uncertainty now comes from **local form factors**.

Experimental results:
[Babar: 1204.3933; Belle: 1908.01848, 1904.02440; ATLAS: 1805.04000, CMS: 1308.3409, 1507.08126, 2010.13968, LHCb: 1403.8044, 2012.13241, 2003.04831, 1606.04731, 2107.13428]



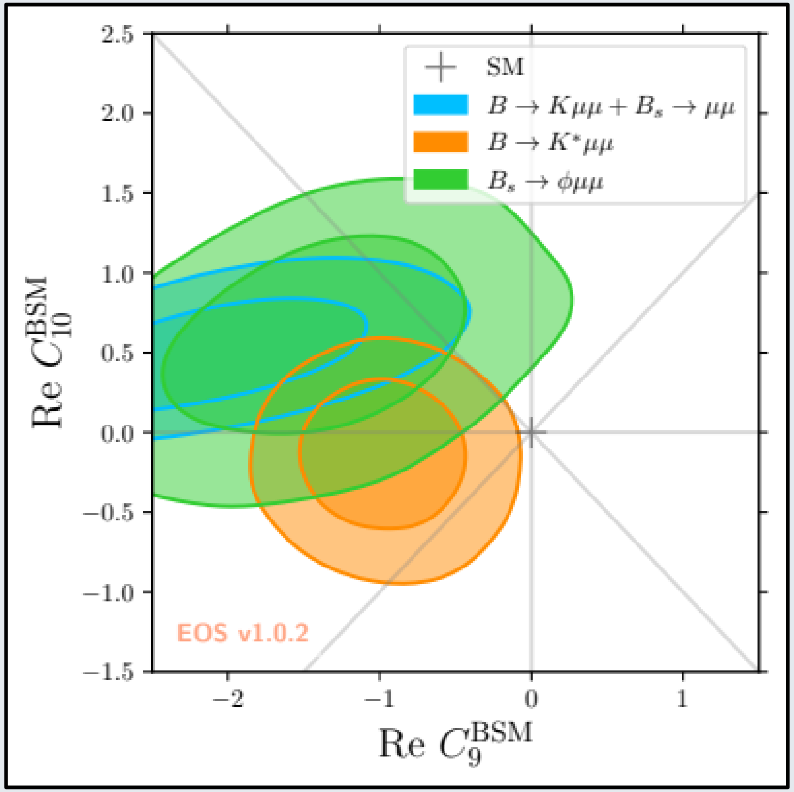




Additional plots can be found in the paper: 2206.03797

BSM analysis

- A combined BSM analysis would be **very CPU expensive** (130 correlated, non-Gaussian, nuisance parameters!)
- Fit **separately** C_9 and C_{10} for the three channels:
  - $B \to K\mu^+\mu^- + B_s \to \mu^+\mu^-$ (*)
  - $B \to K^*\mu^+\mu^-$
  - $B_s \to \varphi\mu^+\mu^-$



(*) CMS recently updated their $B_s \to \mu^+\mu^-$ measurement [2212.10311]

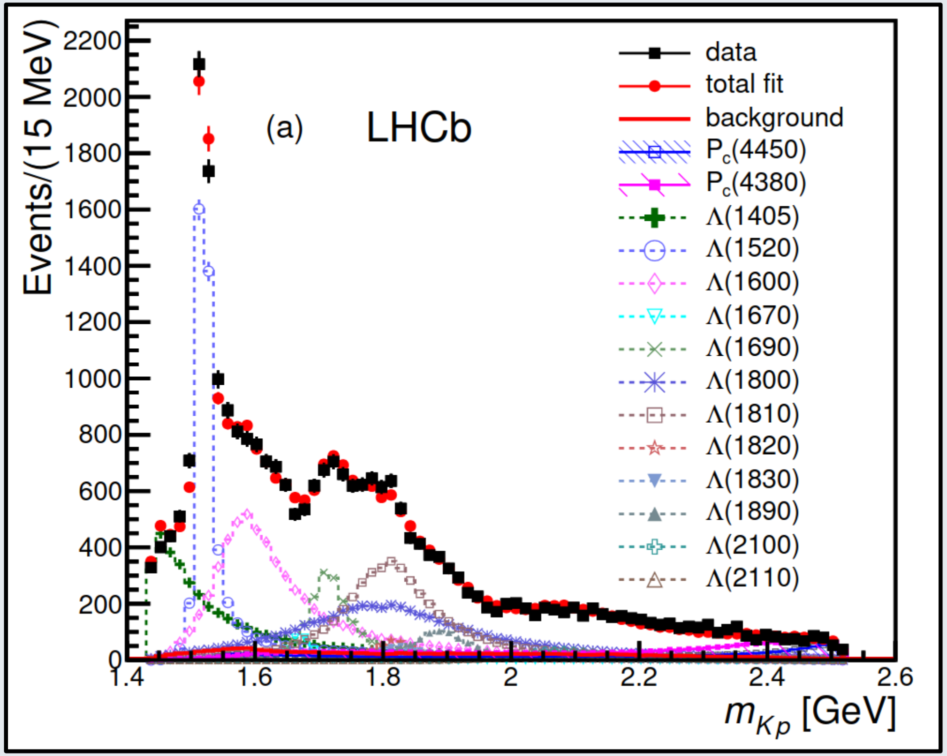
Conclusion

Discussing BSM models requires a solid understanding of the hadronic physics:

- **Local form factors** are obtained by fitting **LQCD results** and **LCSR calculations;**
- **Non-local form factors** can also be constrained by theory calculation and experimental measurements
  - Uncertainties are still large, but controlled by **dispersive bounds**
  - Our approach is **systematically improvable**

Back-up

Resonance structure in $\Lambda_b \to J/\psi p K$

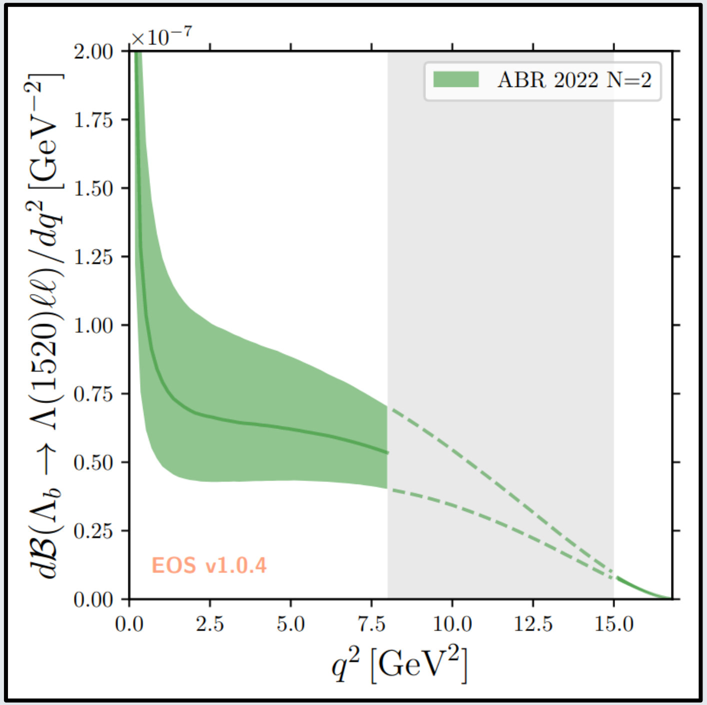


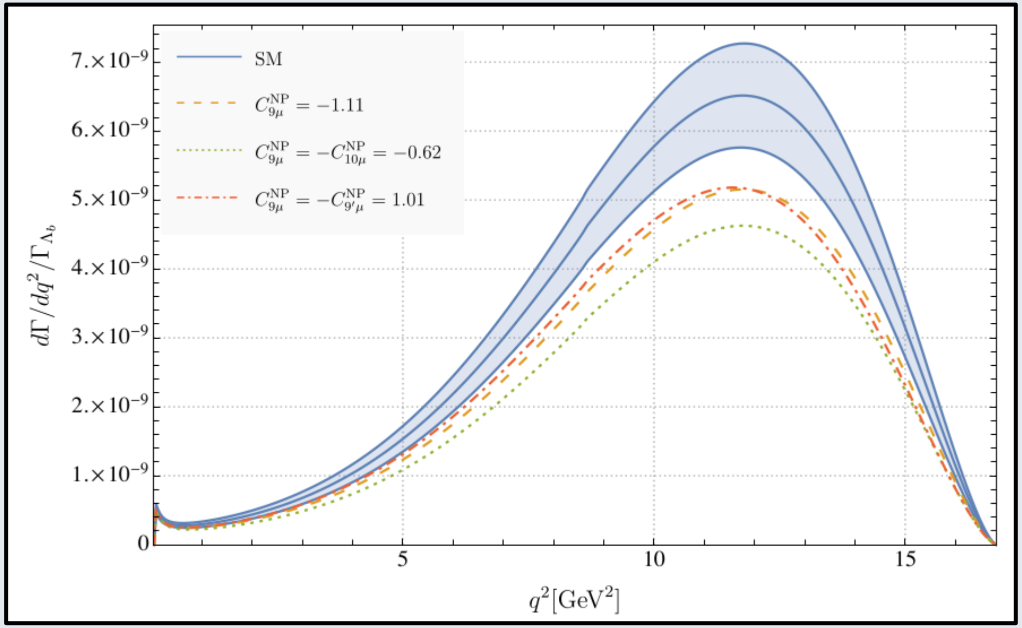
[LHCb 1507.03414]

- The resonance structure way richer than the one in $B \to J/\psi \pi K$
- Λ(1520) is the narrowest observed resonance
- Narrow-width approximation can be applied

Comparison with the literature: BR

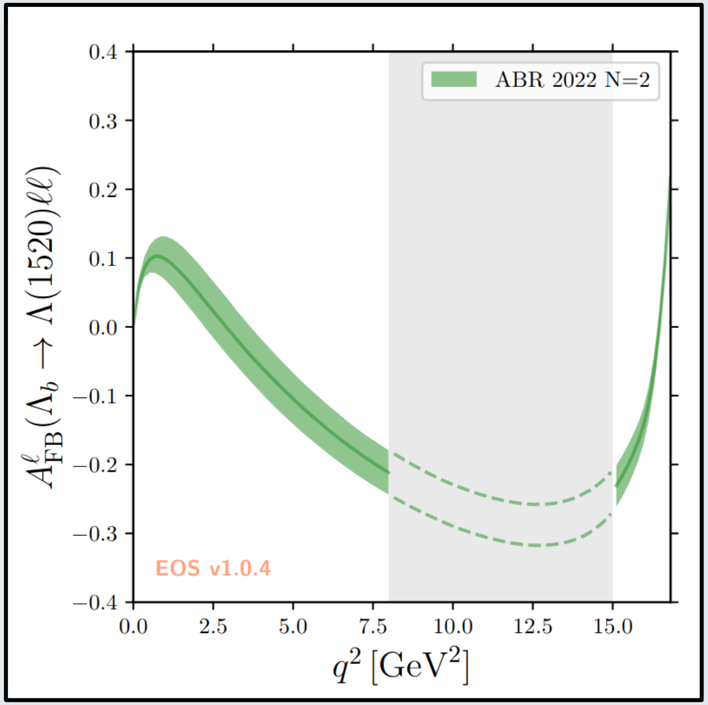
- Branching ratio changes with respect to [Descotes-Genon, M. Novoa-Brunet '19] → this is due to the Quark Model they used for the form factors, which is in large tension both with SCET relations and LQCD results

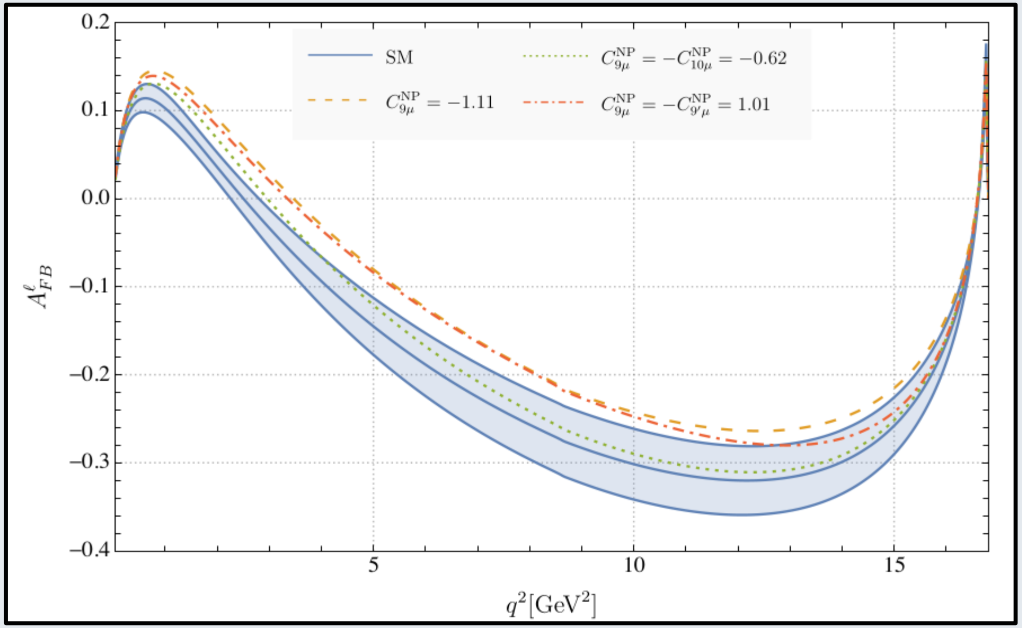




Comparison with the literature: A_FB

- The forward-backward asymmetry is not impacted





Details on the fit procedure

- The fit is performed in two steps…
  - Preliminary fits:
    - **Local** form factors:
      - BSZ parametrization (**8 + 19 + 19 parameters**)
      - Constrained on LCSR and LQCD calcultations
    - **Non-local** form factors:
      - order 5 GRvDV parametrization (**12 + 36 + 36 parameters**)
      - 4 points at negative q^2 + $B \to M\, J/\psi$ data

    → **130 nuisance parameters**
  - 'Proof of concept' fit to the WET's **Wilson coefficients**
- … using **EOS**: eos.github.io