Spectral techniques for laser plasma interactions modeling

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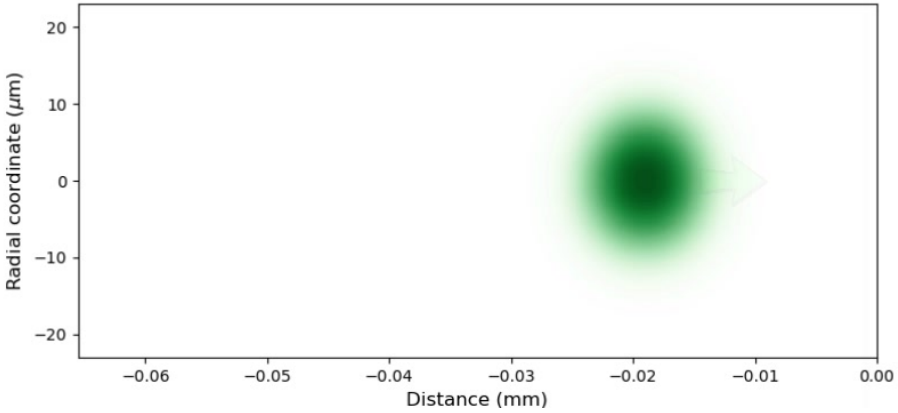
Laser plasma acceleration

Unique source features

- compact
- ultra-fast $\sim fs$
- multi-kA current
- jitterless synchronization



High repetition rate systems (mJ@kHz)

- high compression (few-cycles)
- sharp focusing ($w_0 \sim 1\,\mu$m)
- low e^- energy (few MeVs)
- high average flux
- stable operation
- e^- diffraction, Compton X-rays

High power lasers (TW-PW)

- laser guiding
- multi-GeV electrons
- controllable injection
- low-divergence $\lesssim 1$ mrad
- monoenergetic
- future XFELs and colliders

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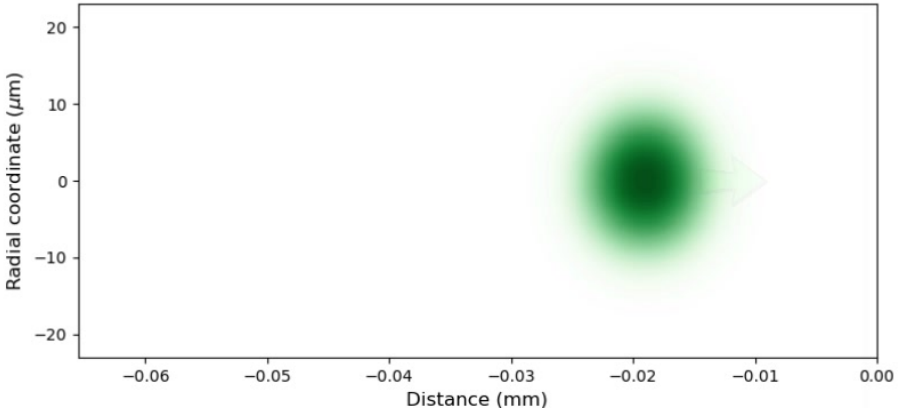

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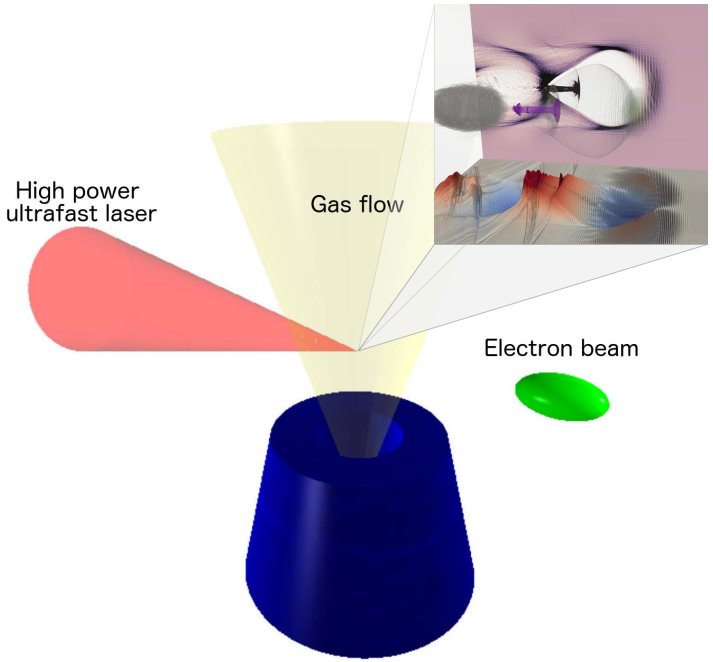
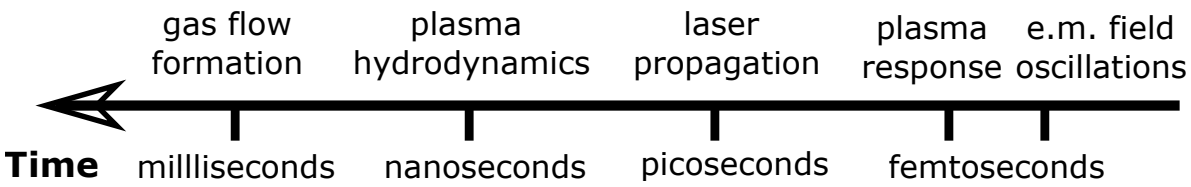
Time-scales in LWFA





- gas flow: (super)sonic, transient/steady-state, turbulent, viscosity
- plasma hydro-dynamics and heat transport: channel/shock formation
- laser spot formation, measurements interpretation
- LWFA: e.m. field, plasma response, propagation

Open-source/on-request solutions

PIC codes

- WARPX[multi-/solv, port]
- Smilei[multi-/solv]
- OSIRIS[multi-/solv]
- PIConGPU[multi-/solv, port]
- EPOCH [multi-/solv]
- FBPIC[RZ, PSATD, GPU]
- ChimeraCL[RZ, PSATD, GPU]
- HiPACE++[QSA]
- QuickPIC[QSA]
- WAND-PIC[QSA]
- Architect[RZ, Fluid]
- Piccante/ALaDyn
- VPIC
- iPic3D

Hydrodynamics

- OpenFOAM
- FluidX3D
- COOLFluiD

Plasma MHD

- FLASH
- CASTRO
- FRONT3D

Bunch Transport

- ELEGANT[ALL]
- ASTRA[RK,SC,SCR]
- OCELOT[MTRX,SC,SCR]
- AT[MTRX]
- Beta[MTRX]
- Synergia[PIC]

SR

- SynchRad[SR,GPU]
- AxiProp[Prop,GPU]
- SRW[SR,Prop]
- XRT[SR,Prop,GPU]
- CHIMERA[SR]
- Shadow3 (OASYS)*
- OPC[Prop]

FEL

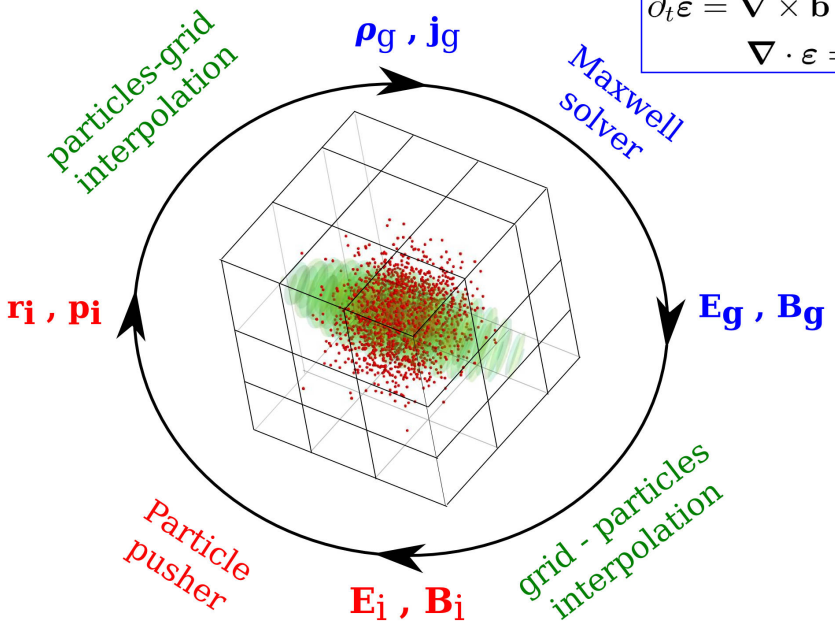
- GENESIS[3D,TD]
- PUFFIN[3D,TD, unav]
- CHIMERA[3D,PSATD,TD, unav]

Bremsstrahlung / K-α

- GEANT4

PIC BOOSTING TECHNIQUES AND CHALLENGES

$$\partial_t \boldsymbol{\varepsilon} = \boldsymbol{\nabla} \times \mathbf{b} - \mathbf{j}\,, \quad \partial_t \mathbf{b} = -\boldsymbol{\nabla} \times \boldsymbol{\varepsilon}\,,$$
$$\boldsymbol{\nabla} \cdot \boldsymbol{\varepsilon} = n\,, \quad \boldsymbol{\nabla} \cdot \boldsymbol{b} = 0\,,$$



Challenges: typically $10^5 - 10^6$ steps, numerical artefacts

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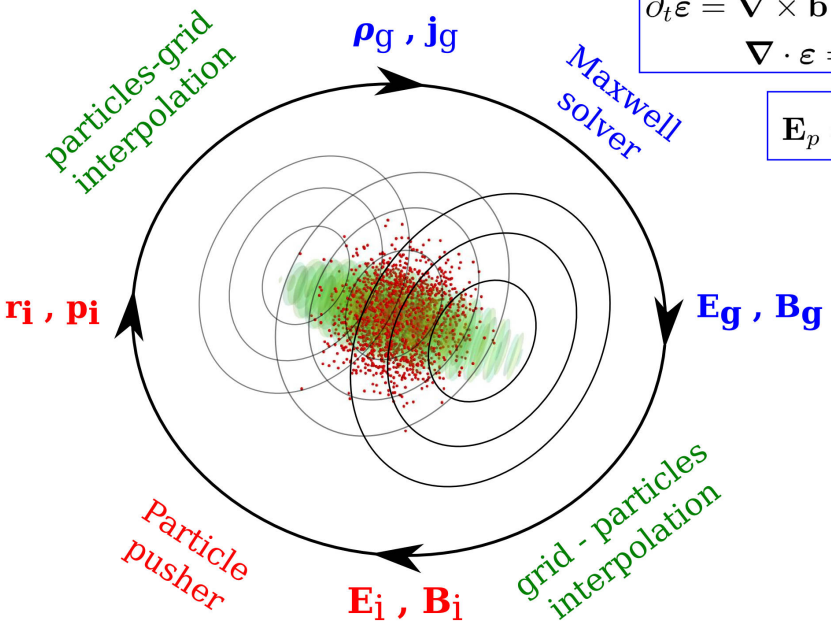

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$$\mathbf{E}_p = \sum_m \mathrm{e}^{im\theta_p} \mathbf{E}_p^{(m)}$$

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- Quasi-cylindrical: $\times 2\, N_\perp / N_m$

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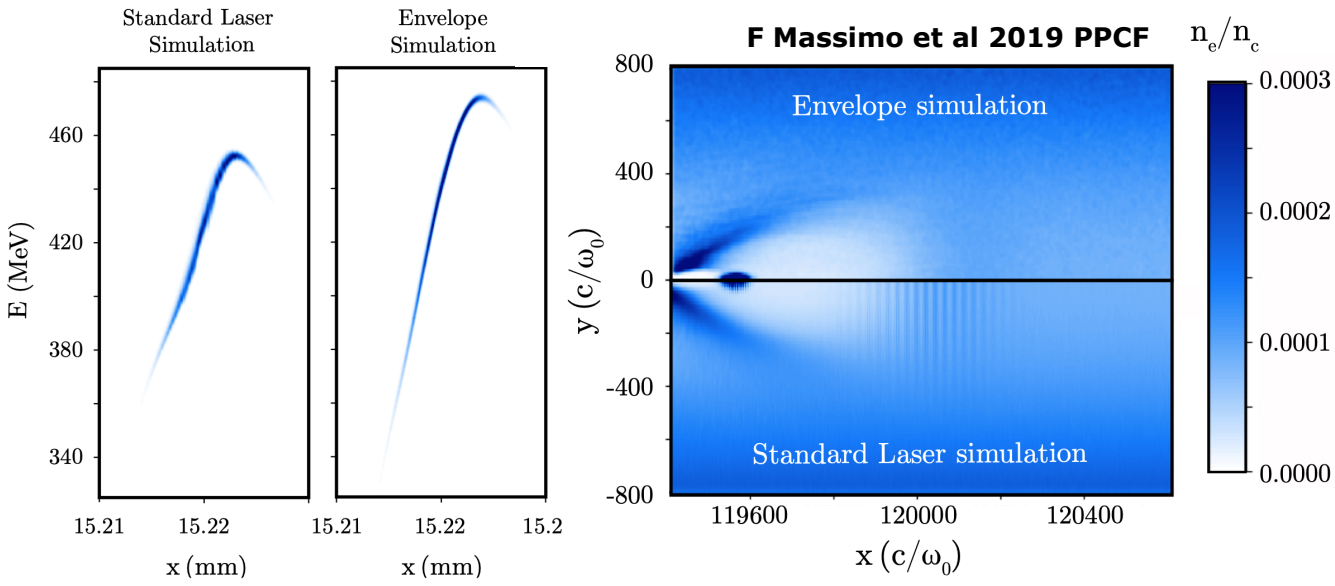

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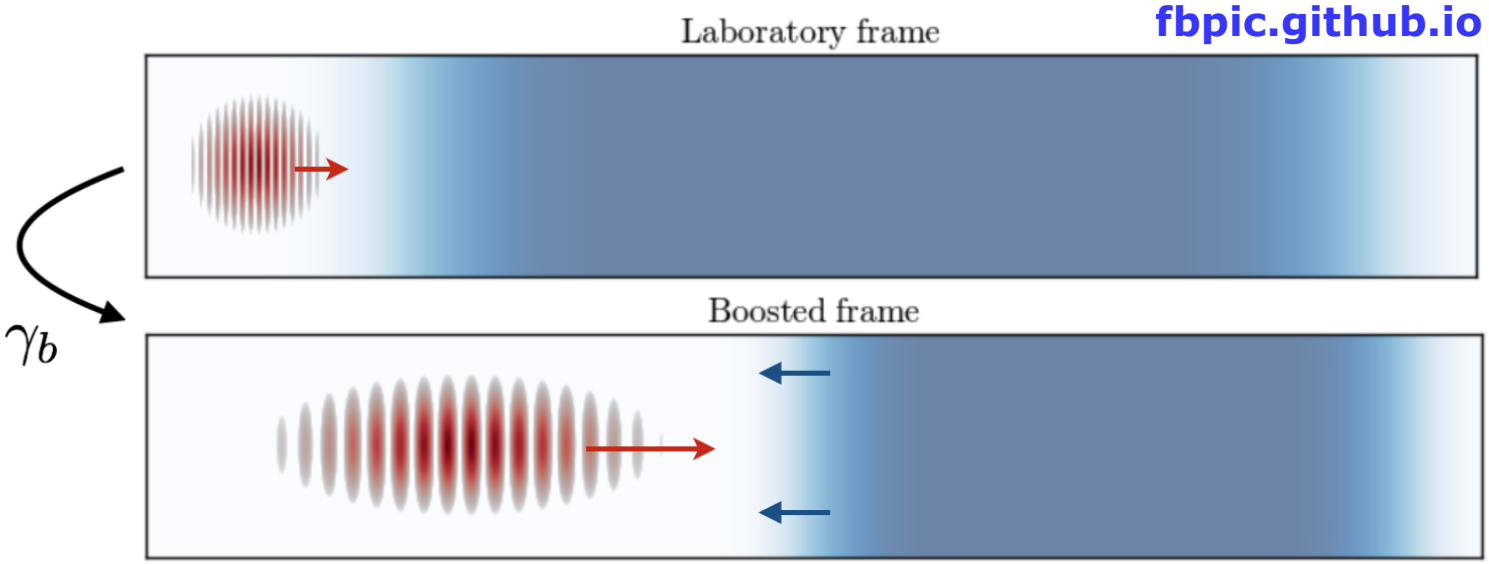
- Laser field envelope: $\times n_c / n_{pe}$

PIC boosting techniques and challenges



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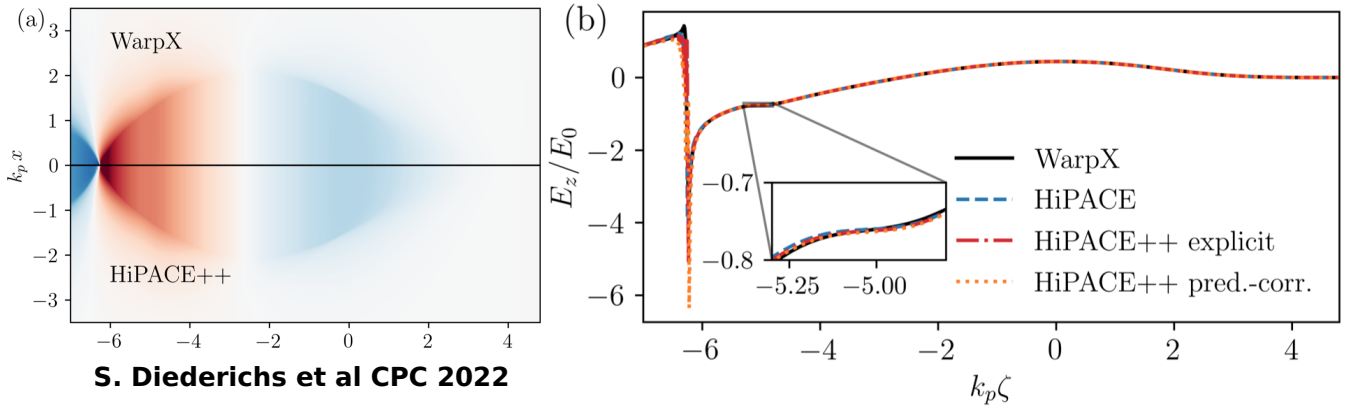
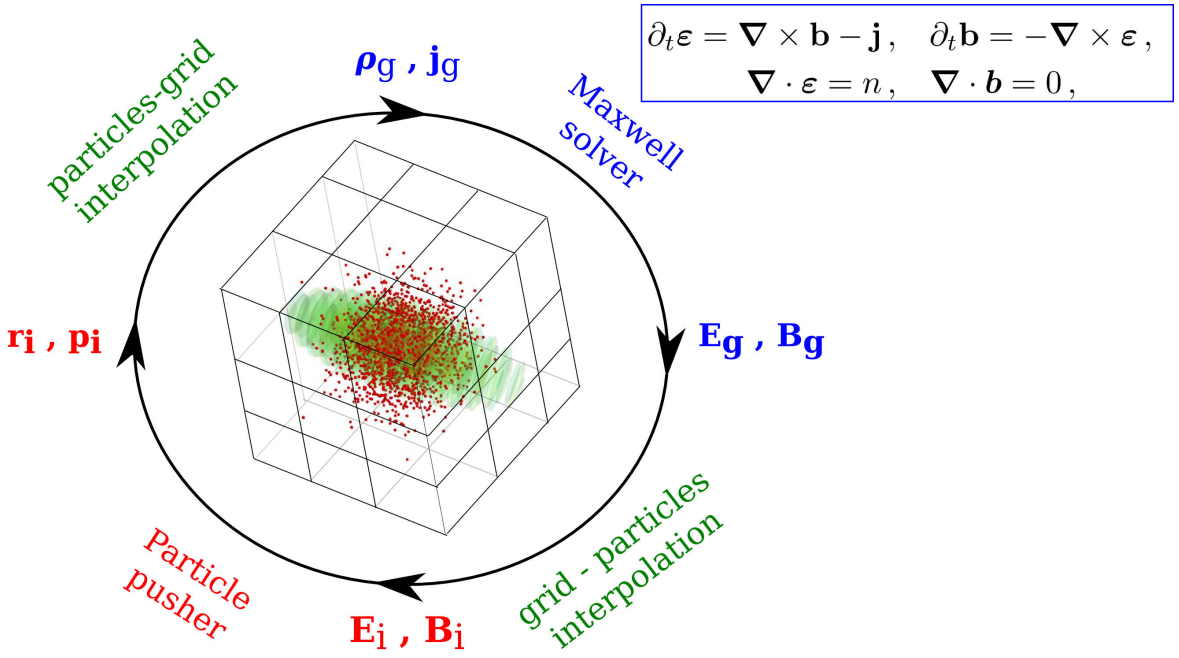


S. Diederichs et al CPC 2022

Challenges: typically $10^5 - 10^6$ steps, numerical artefacts

- Quasi-cylindrical: $\times 2\, N_\perp / N_m$
- Laser field envelope: $\times n_c / n_{pe}$
- Lorentz boosted-frame: $\times n_c / n_{pe}$
- Quasi-static + envelope: $\times \sqrt{2\,\gamma_b}\; n_c / n_{pe}$

PIC BOOSTING TECHNIQUES AND CHALLENGES



Challenges: typically $10^5 - 10^6$ steps, numerical artefacts

- 2D, 3D, RZ
- FDTD: Yee, NDFX, DS, CKC
- PSATD: FFT2D/3D, FFT+Hankel RZ, Gallilean frame
- Envelope: 2D, 3D, RZ
- current deposition: direct, Esirkepov, ZigZag
- pushers: Boris, Vay, Higuera-Cary
- QSA: predic-corr, explicit

Spectral Maxwell solvers

$$\left(\nabla^2 - \frac{1}{c^2}\partial_t^2\right)\mathbf{E} = \frac{1}{\epsilon_0}\left(\frac{1}{c^2}\partial_t\mathbf{J} + \nabla\rho\right) \quad \Longrightarrow \quad \partial_z^2\hat{\mathbf{E}} = -\left(\frac{\omega^2}{c^2} - k_\perp^2\right)\hat{\mathbf{E}} - i\mu_0\omega\left(\hat{\mathbf{J}} - \frac{\mathbf{k}c}{\omega}c\hat{\rho}\right)$$

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Classic wave optics: scalar field, no plasma waves

Temporal-spatial spectra:

- **TXY** : $E(t,x,y) = \sum_{\omega,k_x,k_y} \hat{E}(\omega,k_x,k_y)\ \mathrm{e}^{-\mathrm{i}(\omega/c\ t - k_x x - k_y y)}$
- **TRΘ**: $E(t,r,\theta) = \sum_{\omega,k_\perp,m} \hat{E}(\omega,k_\perp,m)\ J_m(k_\perp r)\ \mathrm{e}^{-\mathrm{i}(\omega/c\ t - m\theta)}$

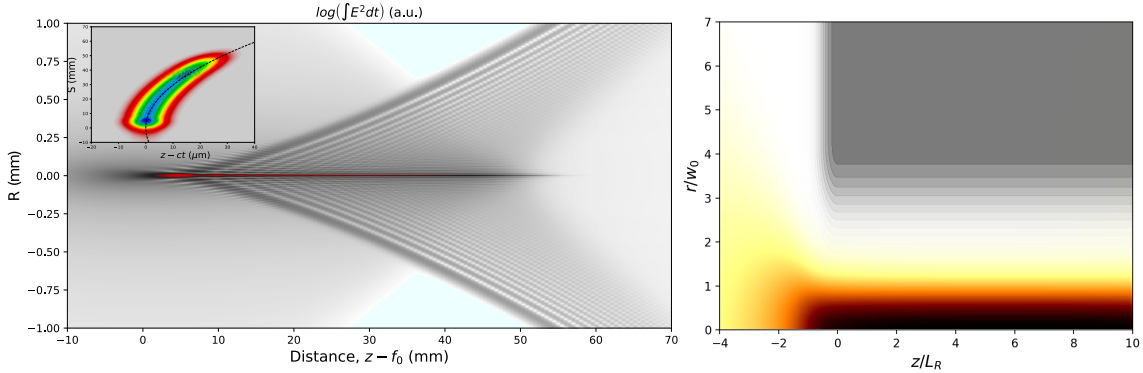
Are used to propagate optical field:

$$\hat{E}(z) = \hat{E}_0\ \mathrm{e}^{\mathrm{i}k_\parallel z\sqrt{1-\tilde{n}}}$$

with $k_\parallel^2 = \omega^2/c^2 - k_\perp^2 - k_p^2$, and $\tilde{n}$ corrects for non-linear conductivity $\hat{J}/\hat{E} \neq$const

`Axiprop`: open-source CPU-GPU portable library [github/hightower8083/axiprop]

- paraxial, non-paraxial
- **TXY**, **TRΘ**
- plasma response (linear+perturbation)
- ***LASY*** integration



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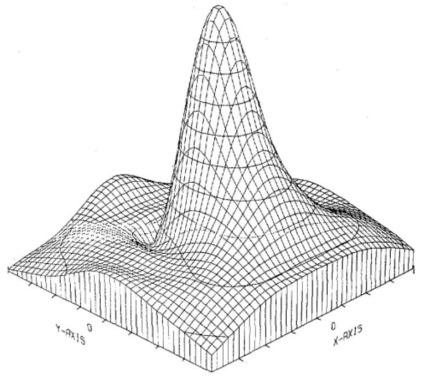
Interactions with particles: FEL modeling

- TEM spectral decomposition with envelope (SVEA)

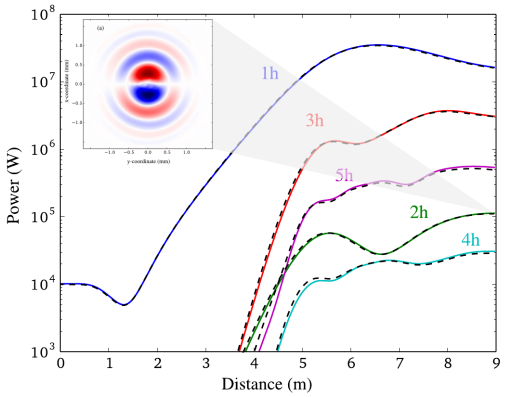
Gaussian-Hermite or Gaussian-Laguerre
[Tang, Sprangle IEEE J. Quantum Electron. 1985], [Freund, Yu NIMA 1997] etc



- Full spectral decomposition:

3D Fourier, Fourier-Bessel, non-averaged, NUFT $z \Leftrightarrow \omega$
[Andriyash et al JCP 2016]

PSATD implemented in multiphysics code CHIMERA
[github/hightower8083/chimera]



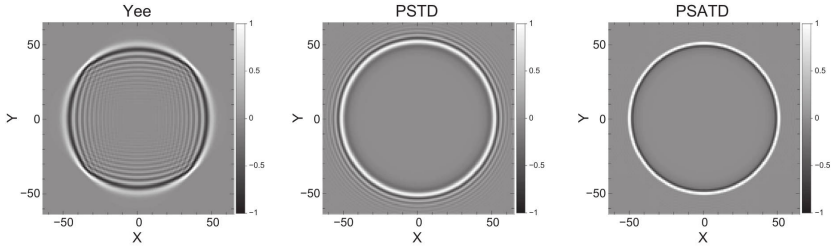
Spectral Maxwell solvers

Full electromagnetic PIC

$$\nabla \times \mathbf{E} = -\mu_0 \partial_t \mathbf{H}\,,$$
$$\nabla \times \mathbf{H} = \epsilon_0\, \partial_t \mathbf{E} + \mathbf{J}\,,$$
$$\nabla \cdot \mathbf{E} = \rho/\epsilon_0$$

$$\Longrightarrow$$

$$\partial_t \hat{\mathbf{B}} = -i\mathbf{k} \times \hat{\mathbf{E}}\,,$$
$$c^{-2}\, \partial_t \hat{\mathbf{E}} = i\mathbf{k} \times \hat{\mathbf{B}} - \mu_0 \hat{\mathbf{J}}\,,$$
$$i\mathbf{k} \cdot \hat{\mathbf{E}} = \hat{\rho/\epsilon_0}$$



- Cartesian geometry

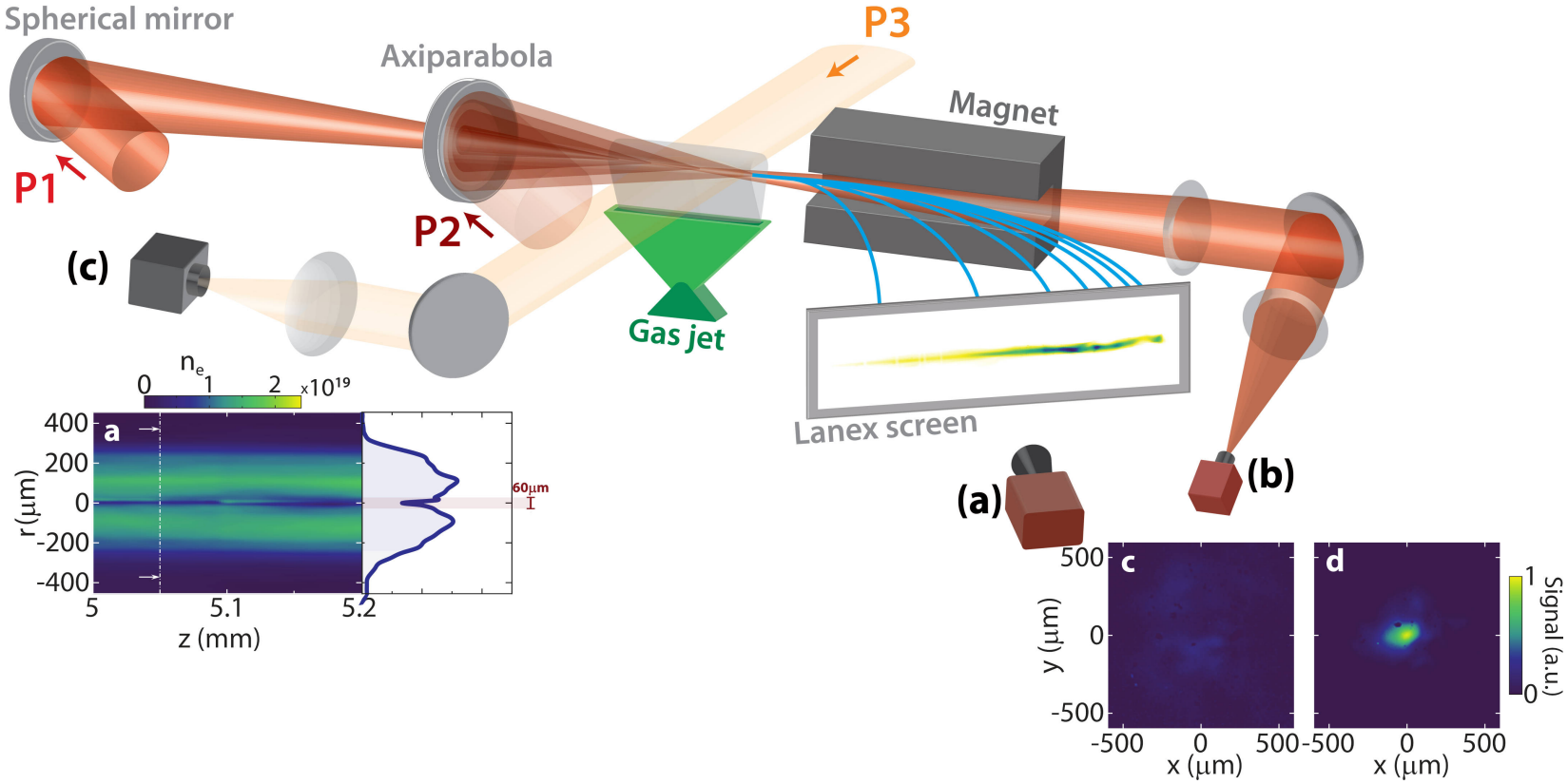
- $[x, y, z] \Leftrightarrow [k_x, k_y, k_z]$ via 3D FFT
- MPI domain[Vay CPC 2013]
- stencil order [Vincenti CPC 2016]
- Warp $\rightarrow$ WarpX

- Quasi-cylindrical geometry

- $[r, z, \theta] \Leftrightarrow [k_z, k_r, m]$ Fourier-Bessel
- FBPIC [Lehe et al CPC 2016]
- CHIMERA(CL) [Andriyash et al PoP 2016]
- Galilean frame [Kirchen et al PoP 2016]
- Finite-order stencil [Jalas et al PoP 2017]

Guided LWFA at LOA: Experiment

[Oubrerie et al Light Sci. Appl. 2022]



Laser, 30fs, 810 nm

- P2: $f_0 = 200$ mm, axiparabola $\delta = 30$ mm, 1.46 mJ
- P1: $f_0 = 1.5$ m, 1.7 J (60% in main peak), + 2ns

Target

- Slit nozzle 15mm 40 bars $\rightarrow$ $n_{pe} = 1.4 \times 10^{19}$ cm^{-3}
- Injection: ionization (H_2+1%N_2), shock (H_2)
- motorised blade to produce shock

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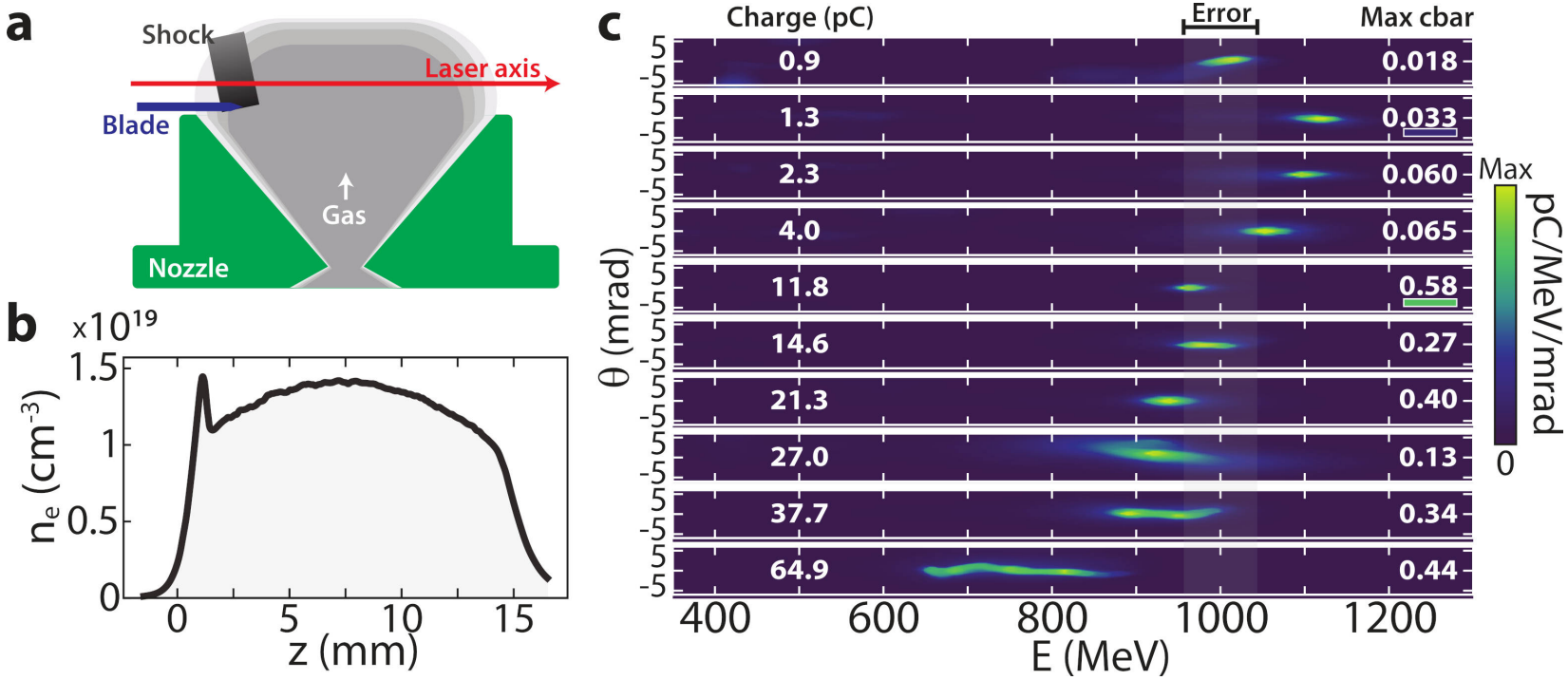

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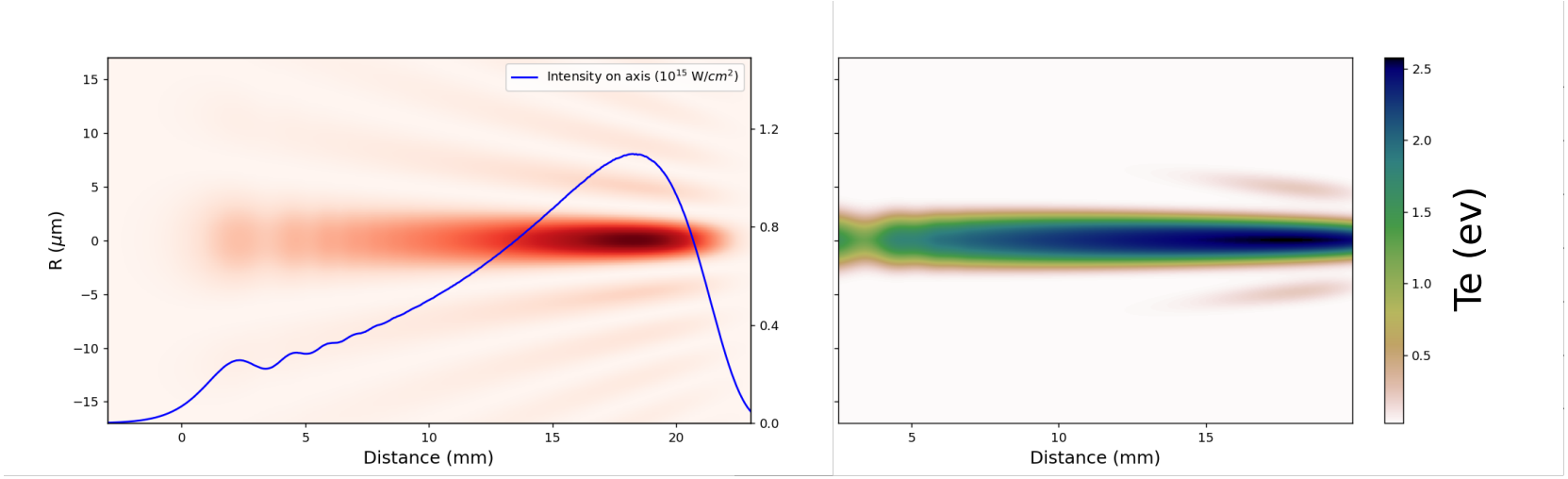
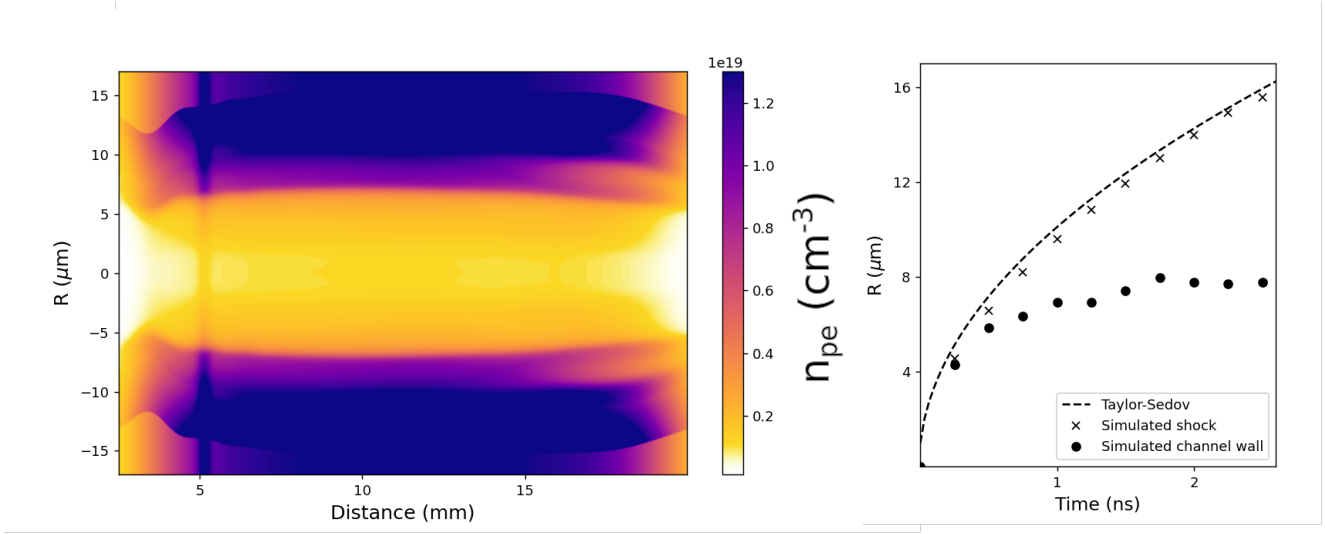
Guided LWFA at LOA: Modeling

- Optical propagation (`AxiProp`) → OFI heating (`home-made`)



- Channel formation (`FRONT3D`)



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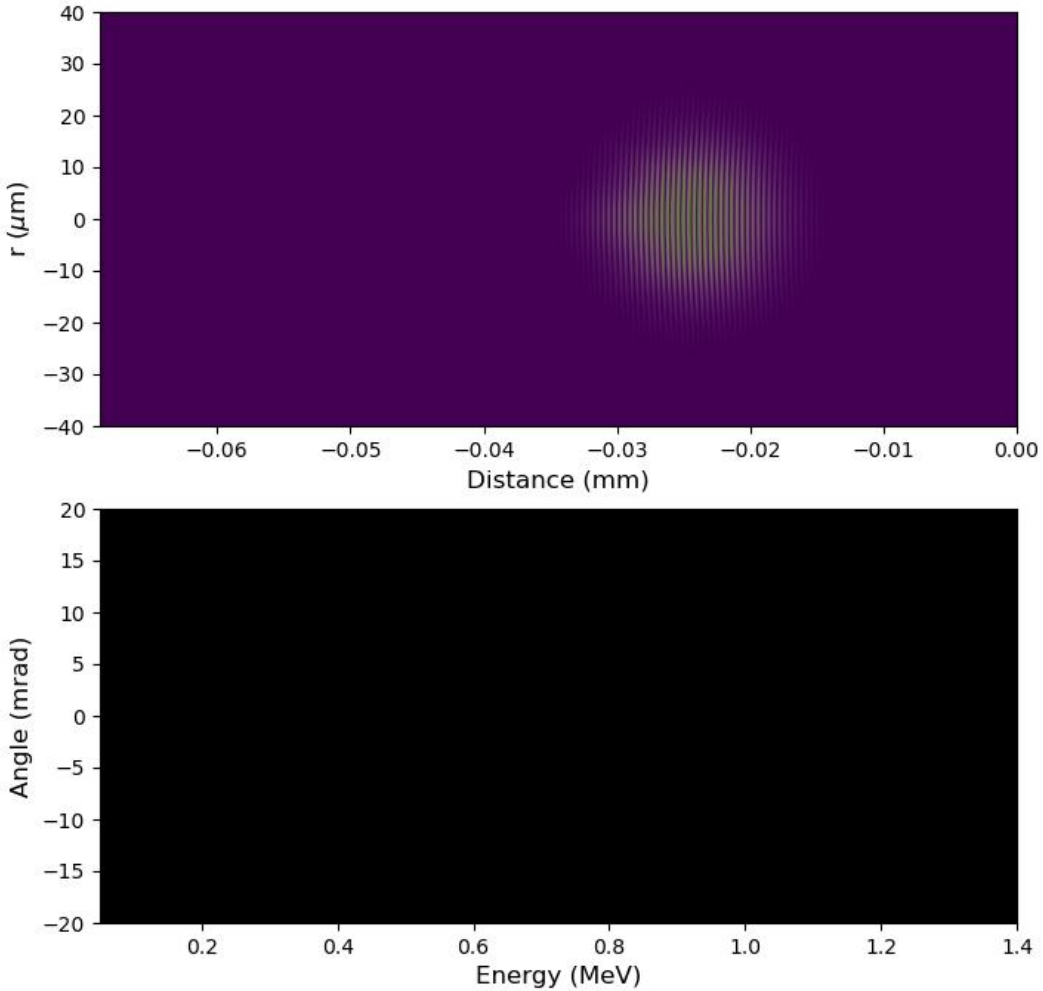

- Lorentz-boosted PIC (`FBPIC`)

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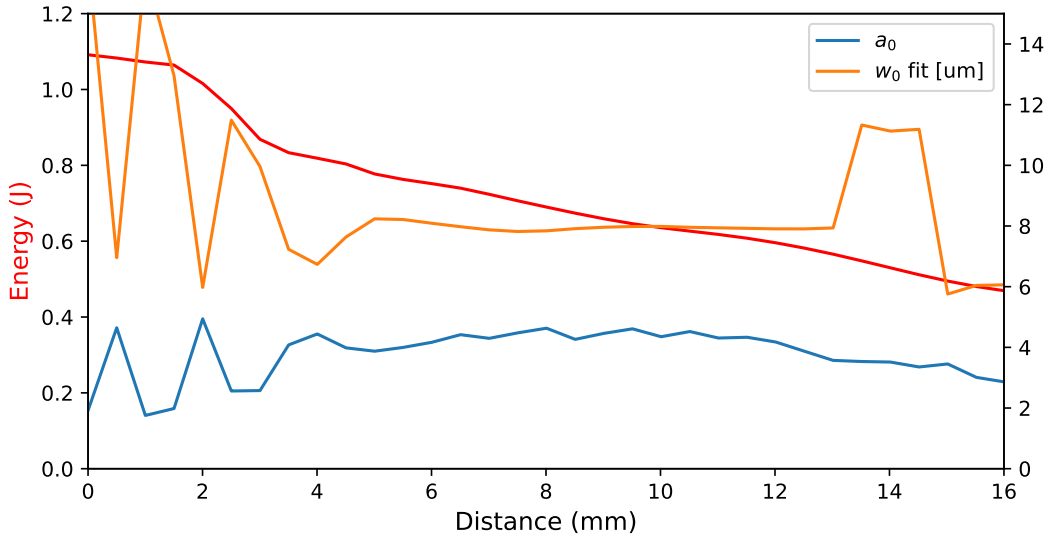

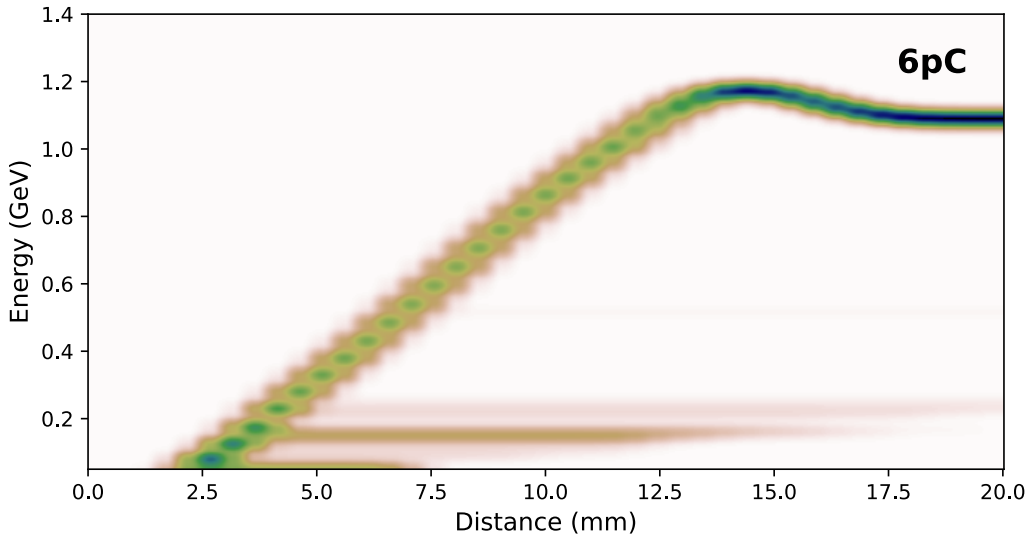

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