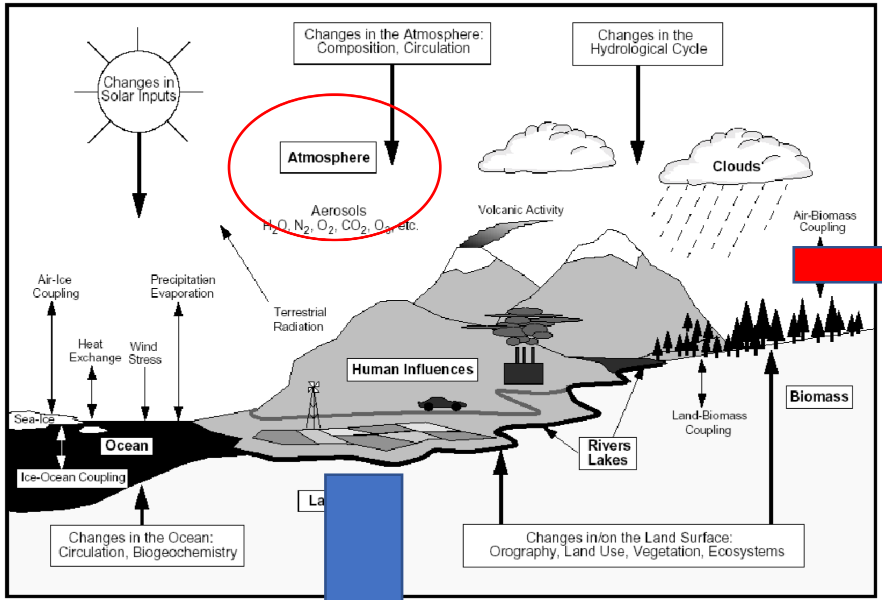
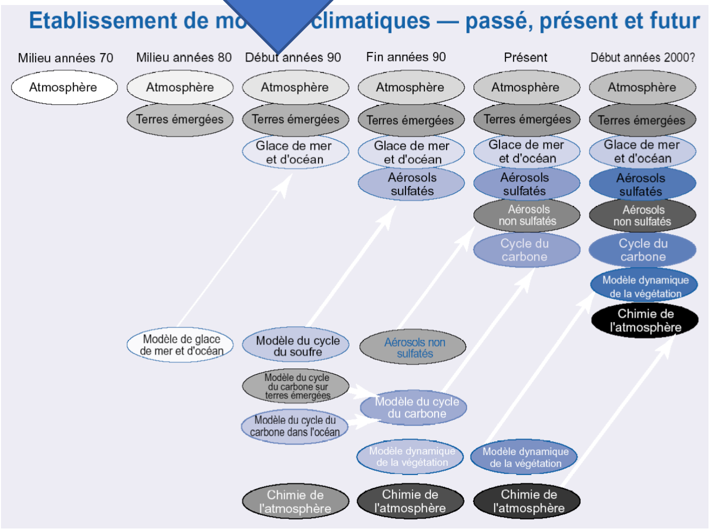
Modelling geophysical flows: how to go beyond honey?

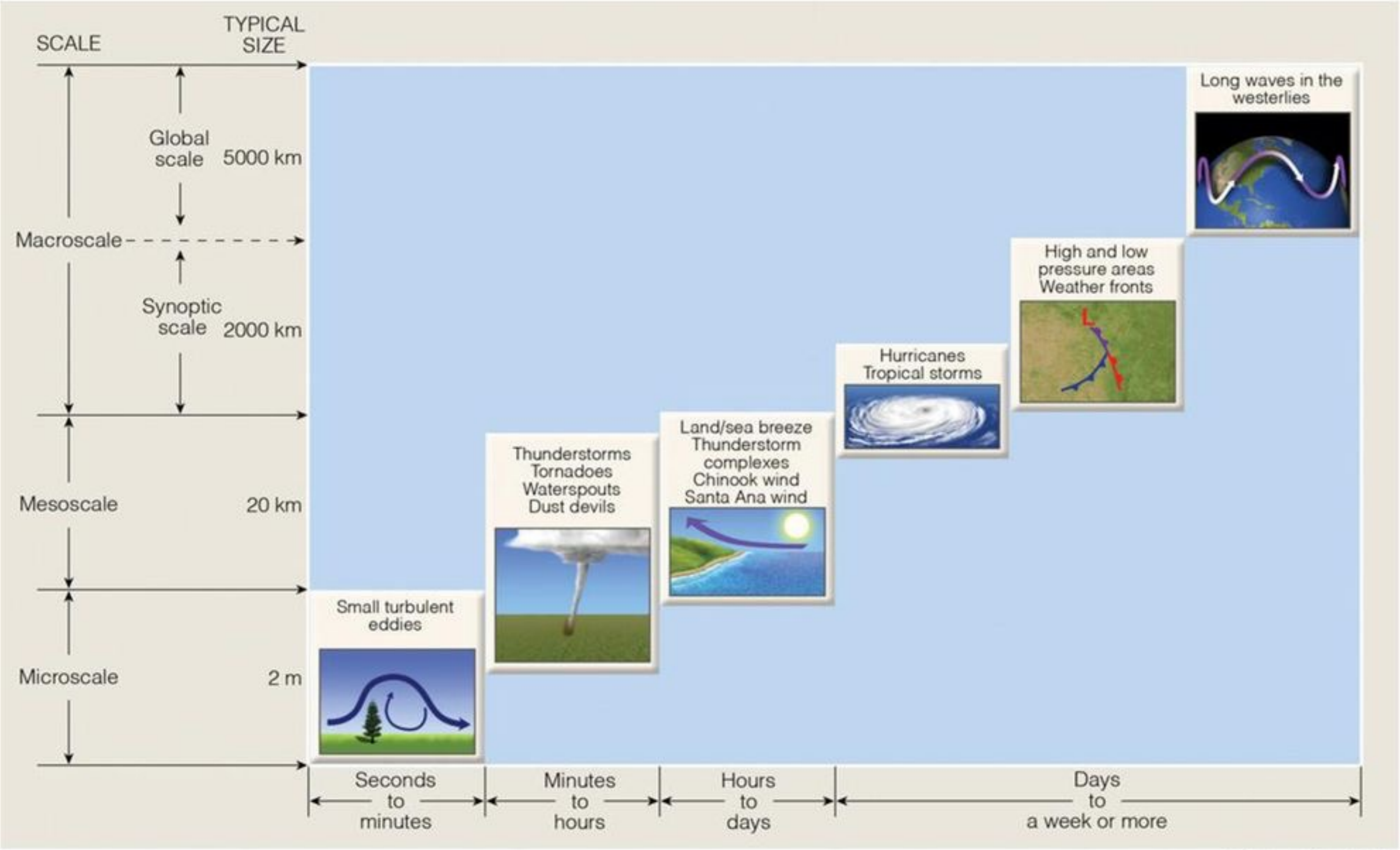
B. Dubrulle
CEA Saclay/SPEC/SPHYNX
CNRS UMR 3680

Climate model and degrees of freedom



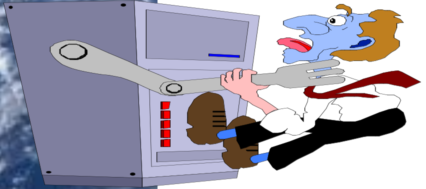


Scales of Motion



(pp. 230-231)

Climate scales, climate model and degrees of freedom

Number of degrees of freedom

$$N = \left(\frac{L}{\eta}\right)^3$$

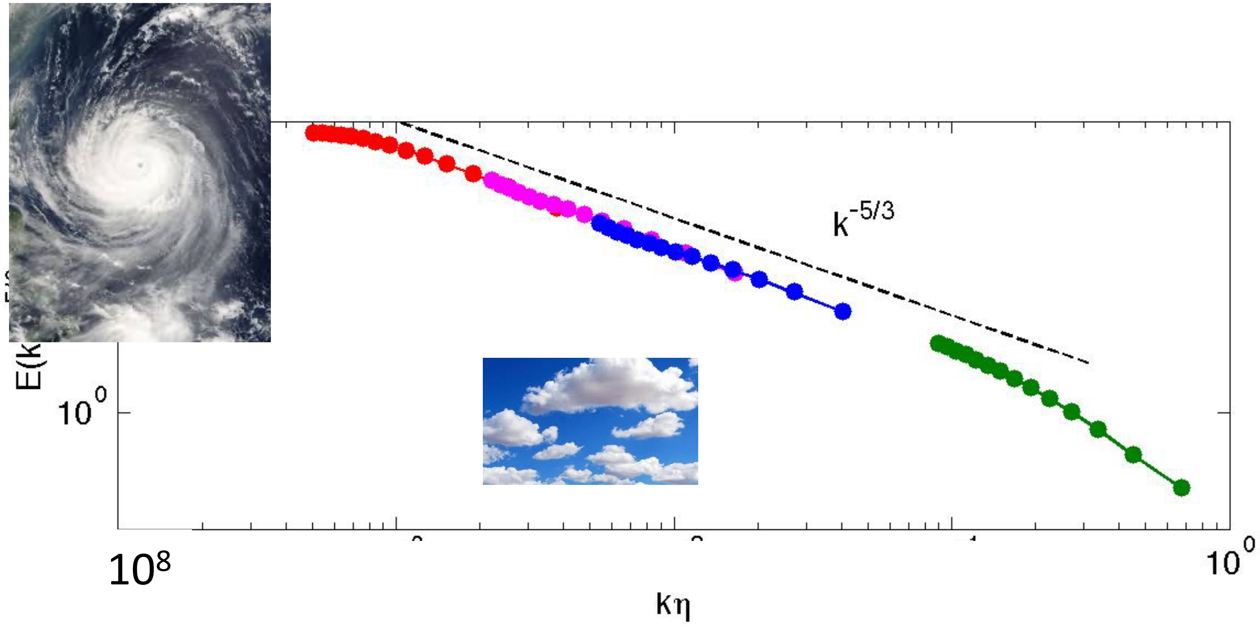
Air

L=1000 km
H=100 km
η=10 mm
Δt=1 s

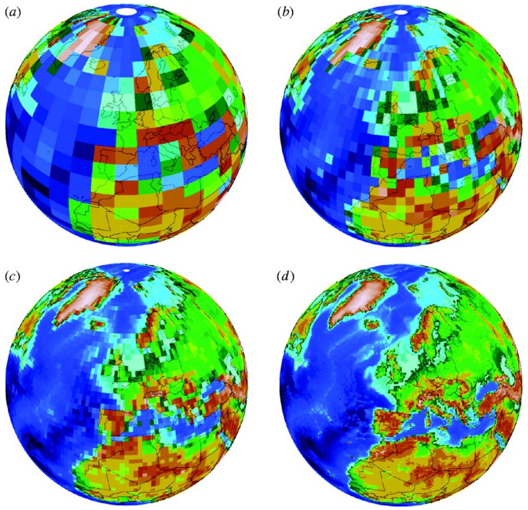
Horizontal: $N=10^{16}$

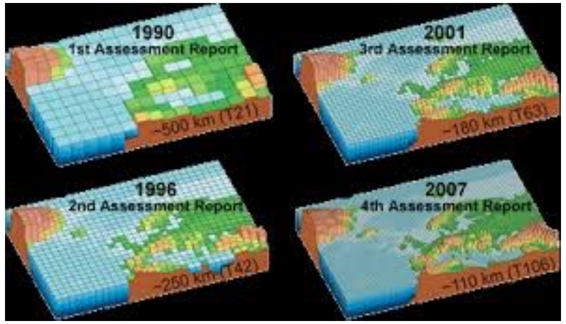
Vertical: $N=10^{7}$

Volume: $N=10^{23}$



Peut-on simuler la turbulence/ le climat ?

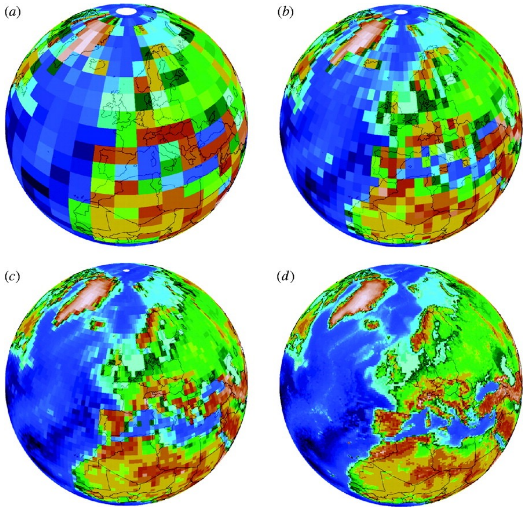


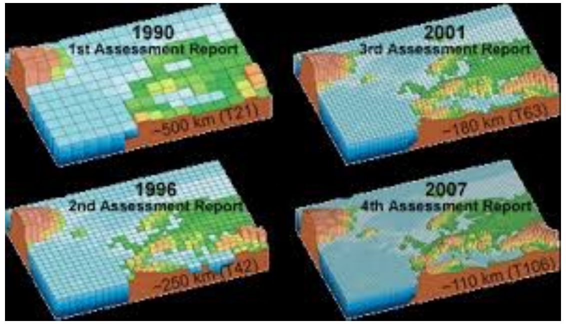


Plus l'ordinateur est gros,
mieux on peu discrétiser
Plus on peut prendre en compte
De degrés de liberté

Combien y a t-il de degrés de liberté pertinents et combien ça coute de les simuler?

Peut-on simuler la turbulence/ le climat ?





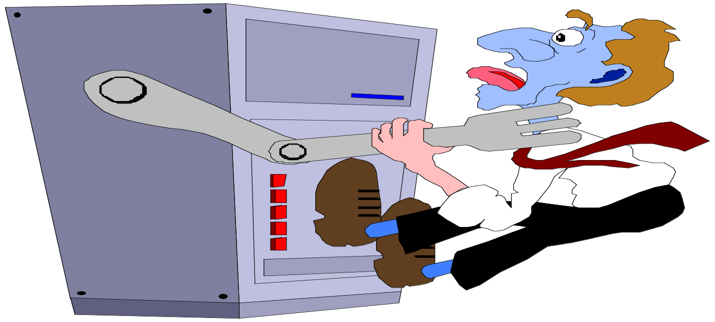
Plus l'ordinateur est gros,
mieux on peu discrétiser
Plus on peut prendre en compte
De degrés de liberté

Calcul:

6 10^9 nœuds --> 1 semaine de CPU
sur Blue Gene

Stokage:
10^8 nœuds- -> 2Tb= 1 disk

Les échelles du système



$Re=10^6$
L=10 cm
η=0,01 mm
Δt=1 ms

$N=10^{12}$

Eau

10 ans de cpu
10 000 disks

L=1000 km
H=100 km
η=10 mm
Δt=1 s

Horizontal: $N=10^{16}$

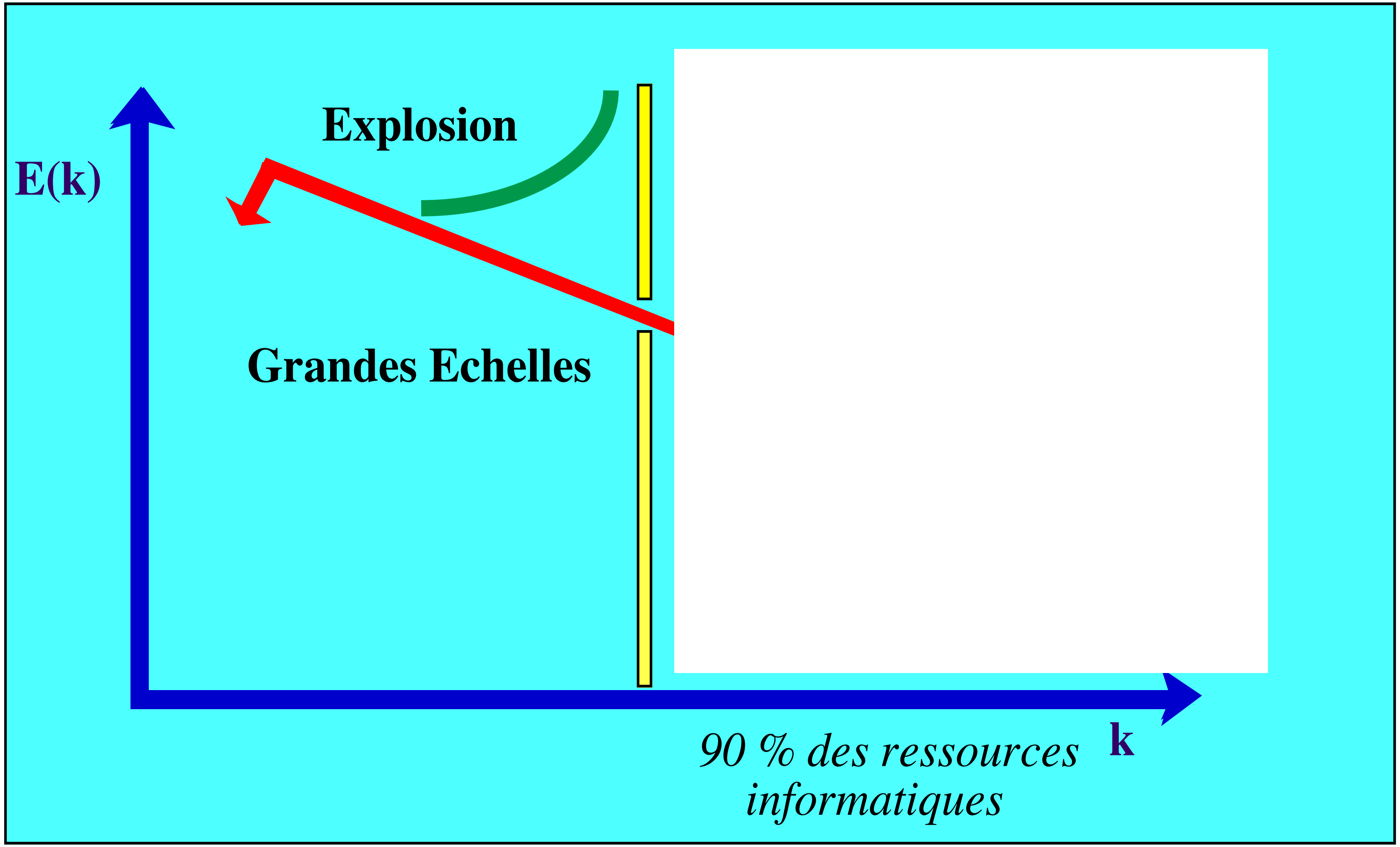
Vertical: $N=10^7$

Volume: $N=10^{23}$

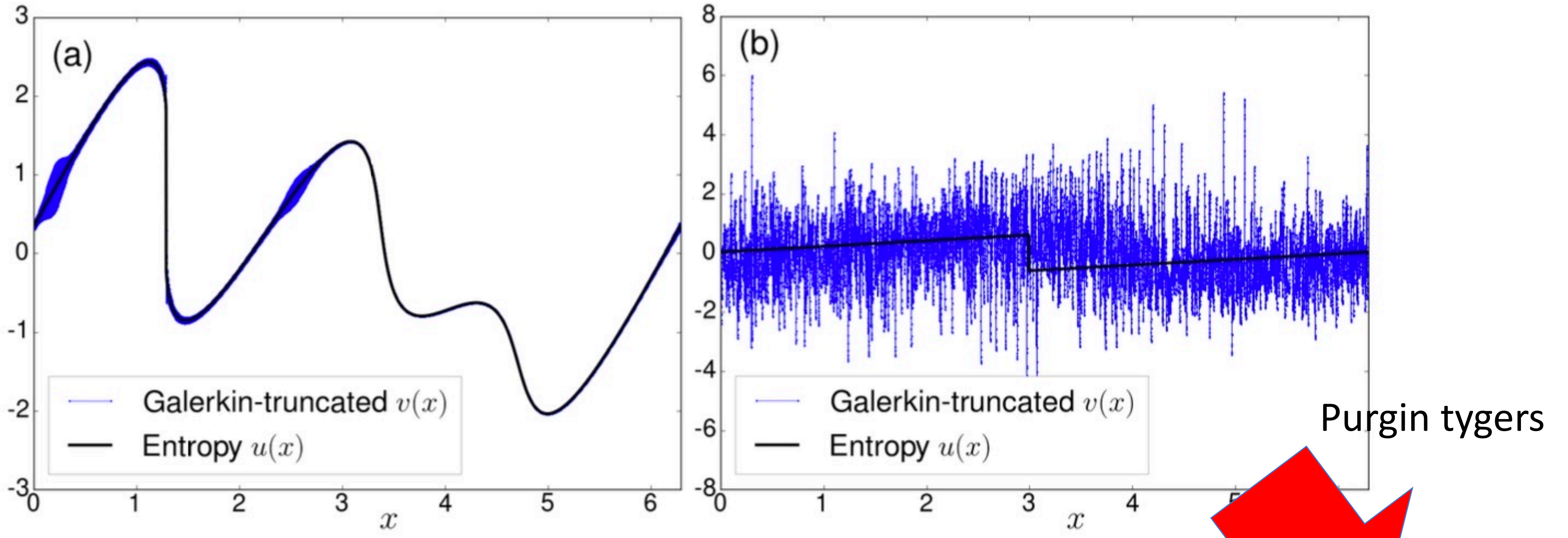
10^{11} ans de cpu
10^{15} disks

Air

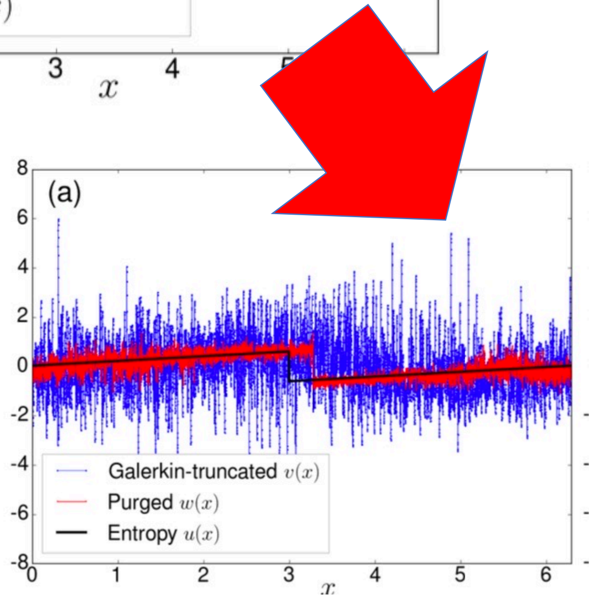
What can be done? Truncate?

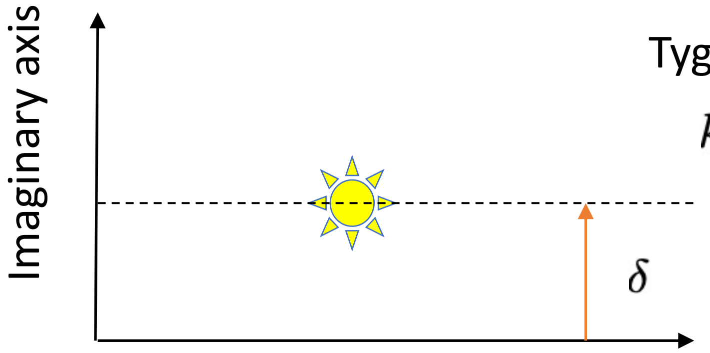


What happens when we truncate? 1D



Purgin tygers





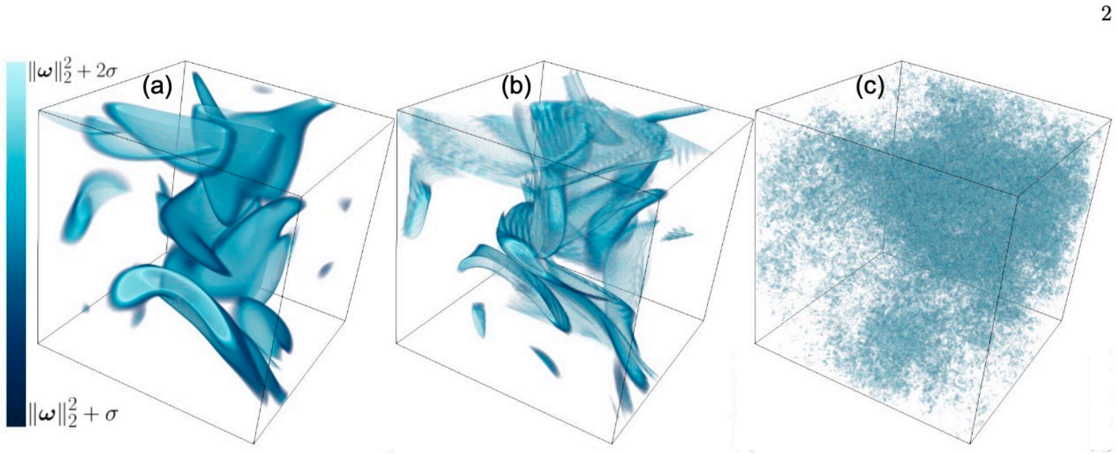
Tygers form when

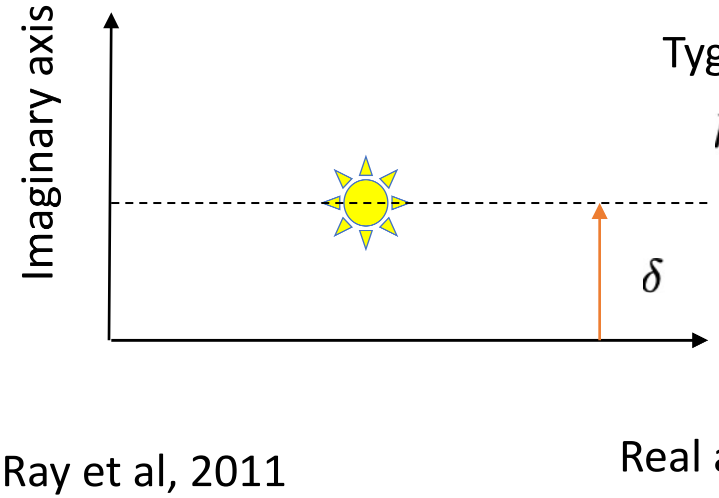
$$k_{max}\delta <\sim 1$$

Ray et al, 2011

Murugan et al, 2022

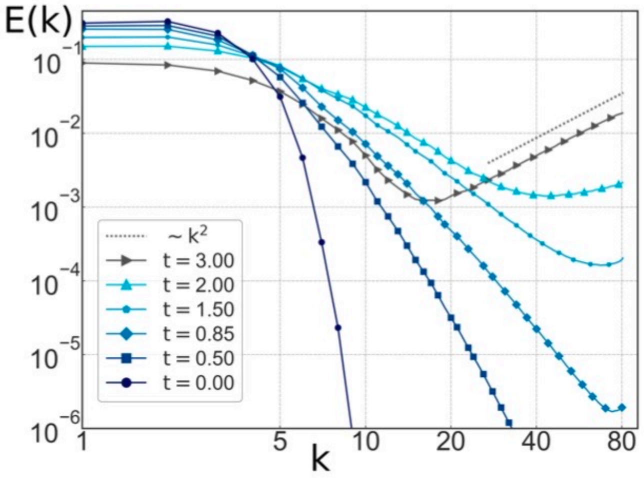
What happens when we truncate? 3D





Tygers form when

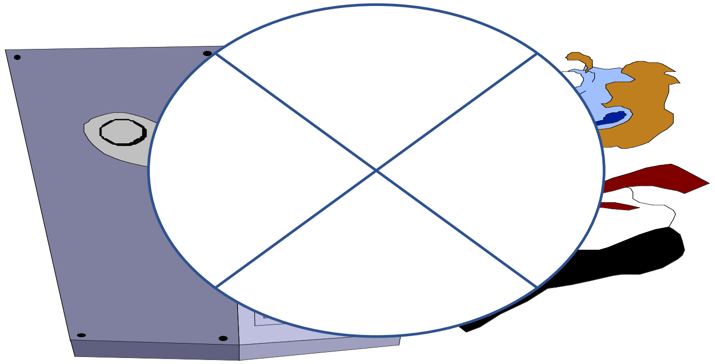
$$k_{max}\delta <\sim 1$$



Ray et al, 2011

Murugan et al, 2022

Les échelles du système



10 ans de cpu
10 000 disks

Re=10^6
L=10 cm
η=0,01 mm
Δt=1/1000 s
N=10^{12}

Eau

L=1000 km
H=100 km
η=10 mm
Δt=1 s

Horizontal: N=10^{16}

Vertical: N=10^7

1 mn de cpu
Moins d'un disk

Re=600
L=10 cm
η=10 mm
Δt=1/10 s
N=10^3

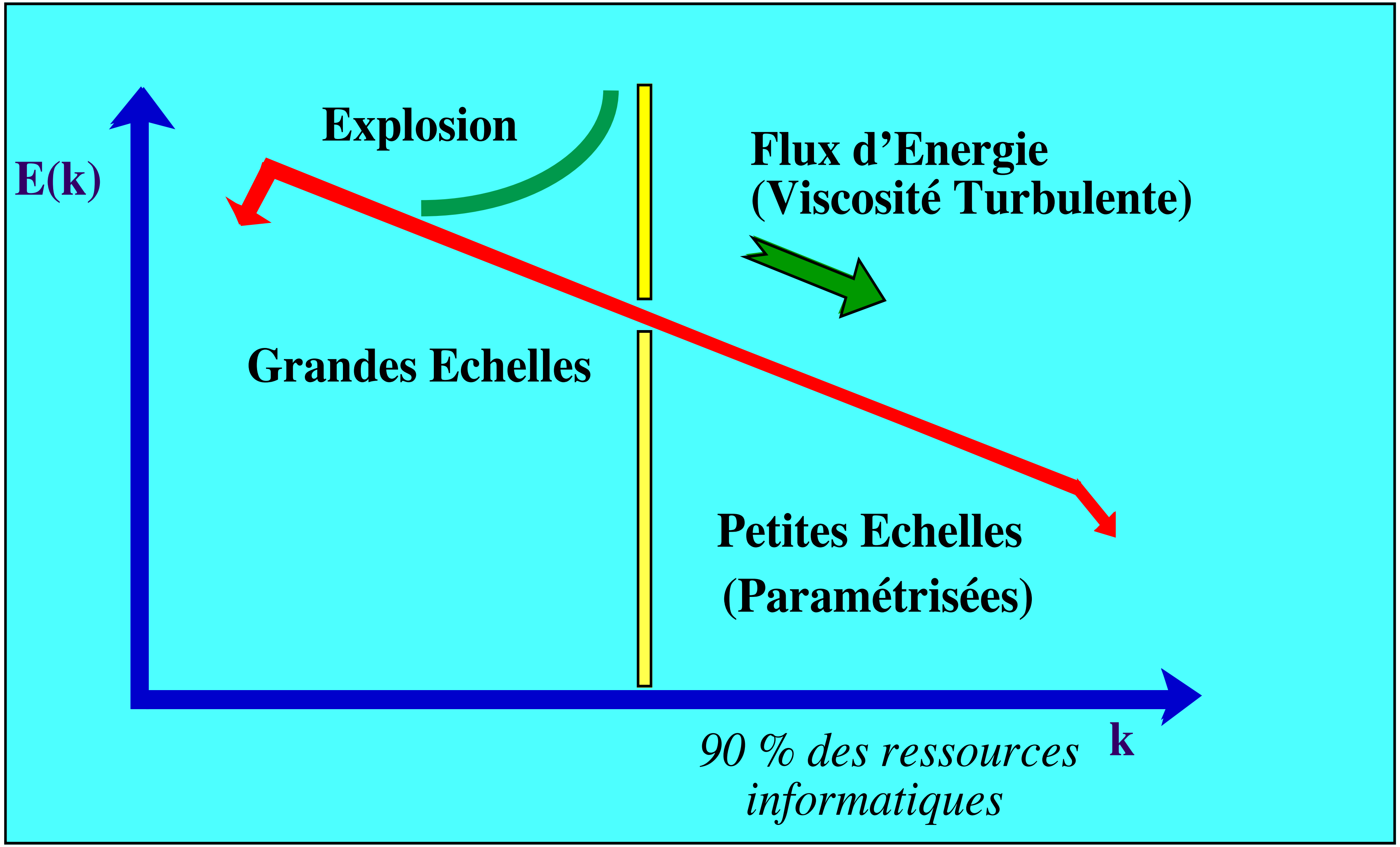
Glycerol

Volume: N=10^{23}

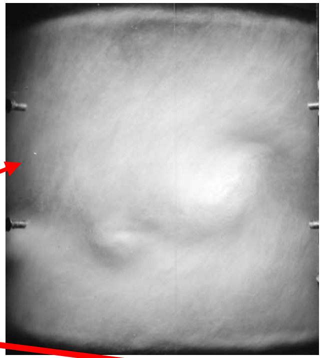
10^{11} ans de cpu
10^{15} disks

Air

What can be done?

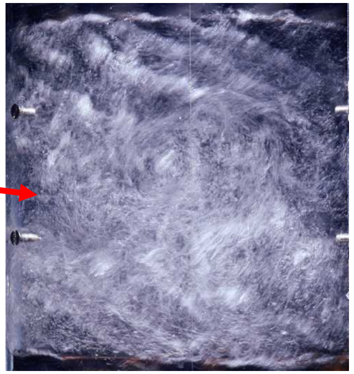


Two ways to cut the scale space

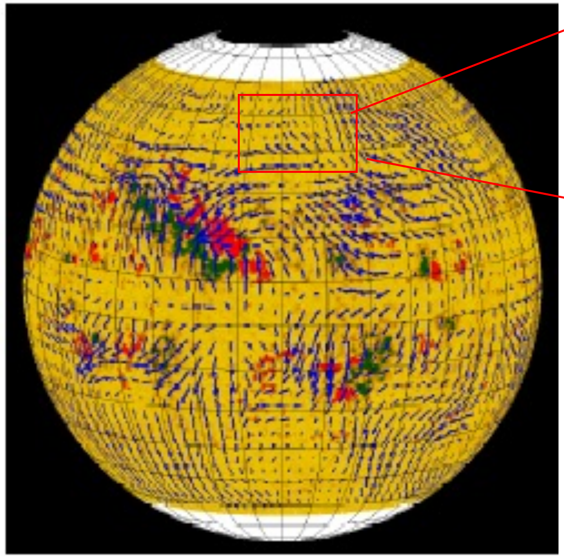


Mean Flow

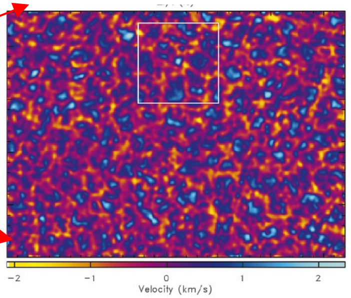
Fluctuations



RANS:
You keep the mean
Flow
Parametrize
fluctuations



Large scale



Small
scales

LES:
You keep the large scale
Parametrize the small

Mathematical translation

$$\partial_t u_i + u_j \nabla_j u_i = -\nabla_i p + \frac{1}{\mathrm{Re}} \Delta u_i + f_i$$

$$u = \overline{u} + u'$$

- Spatial filter for LES
- Ensemble average for RANS

$$\partial_t \overline{u}_i + \overline{u}_j \nabla_j \overline{u}_i = -\nabla_i \overline{p} + \frac{1}{\mathrm{Re}} \Delta \overline{u}_i + \overline{f}_i - \nabla_j \tau_{ij}$$

Reynolds stress

$$\tau_{ij} = \overline{\overline{u}_i \overline{u}_j} - \overline{u}_i \overline{u}_j + \overline{\overline{u}_i u'_j} + \overline{u'_i \overline{u}_j} + \overline{u'_i u'_j} \qquad \text{LES}$$

$$\tau_{ij} = +\overline{u'_i u'_j} \qquad \text{RANS}$$

Parametrization: RANS

Issue: Reynolds stress parametrization

$$\tau_{ij} = +\overline{u'_i u'_j}$$
$$= -\alpha_{ijk}\overline{u_k} - \beta_{ijkl}\nabla_k \overline{u_l}$$

AKA effect	Turbulent Viscosity
Helicity effect	4 order tensor
Influence on mean flow (breaks Galilean invariance)	Can be « negative » (instabilities)
Produces large scale-instabilities (cf dynamo effect)	
Sulem, Frisch, She	*Dubrulle&Frisch*

Parametrization: RANS
AKA effect

Use to explain: Solar Granulation (Kishan, MNRAS, 1991)

Galaxy Clustering (Kishan, MNRAS, 1993)

Large-scale vortices in disks
(Kitchatinov et al, A&A, 1994)

Liitle (not?) used in general turbulence

No general theory

Analogy with dynamo:

$$\alpha_{ijk} = \frac{1}{3}\frac{\overline{\vec{u}'\bullet(\nabla\times\vec{u}')}}{\tau}\varepsilon_{ijk}$$

3D isotropic

Parametrization: RANS Viscosity

Not necessarily isotrop(cf shear flows) (Dubrulle&Frisch,

Isotropic Case $\beta_{ijkl} = \nu_T \delta_{jk} \delta_{il}$

Dimensional analysis

$$\nu_T = K V L$$

Characteristic lenght

Constant

Characteristic velocity

Kolmogorov theory $V = (\varepsilon L)^{1/3}$

$$\nu_T = K \varepsilon^{1/3} L^{4/3}$$

Example : Smagorinski

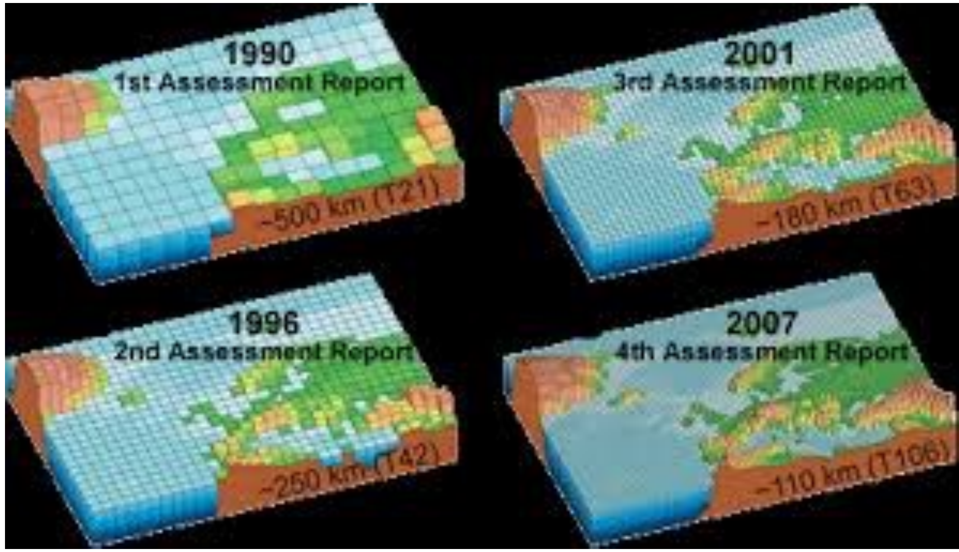
Viscosity written function of mean gradients

$$\nu_T = \left(c_s \Delta\right)^2 \sqrt{\nabla_j \overline{u}_i \nabla_j \overline{u}_i}$$

Adjustable Constant (c_s)

Mesh size (Δ)

Climate model and degrees of freedom



L=1000 km
H=100 km
$\eta=10$ mm
$\Delta t=1$ s

Horizontal: $N=10^{16}$

Vertical: $N=10^{7}$

Volume: $N=10^{23}$

Air

Viscosity x 10^{6} !

L=1 000 km
$\Delta L=100$ km

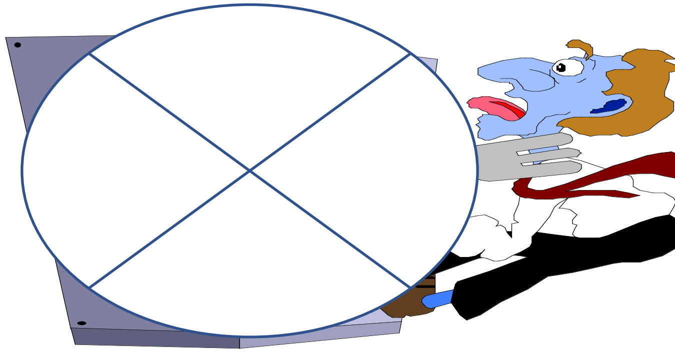
H=100 km
$\Delta H=5$ km
$\Delta t=1000$ s

Horizontal: $N=10^{2}$

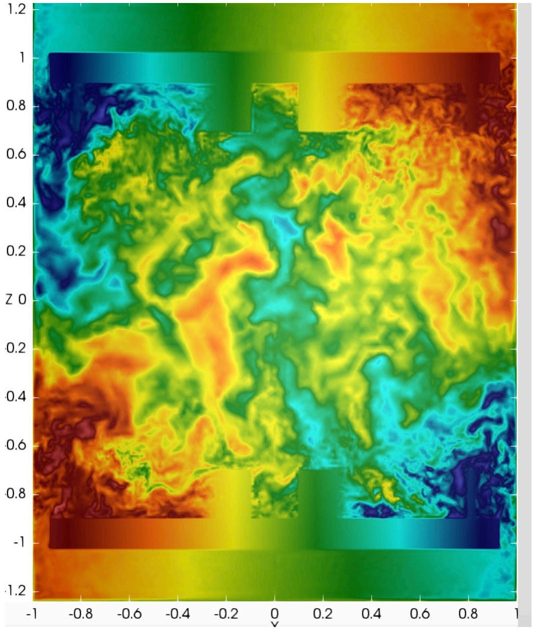
Vertical: N=20

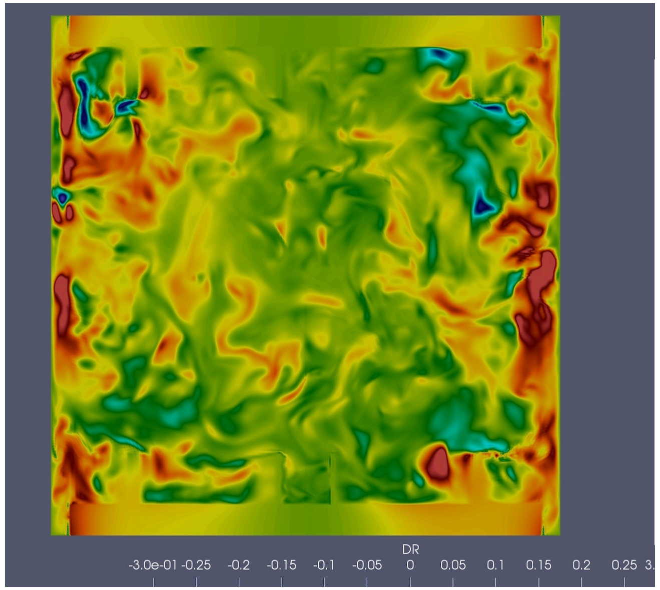
Volume: $N=2\times10^{3}$

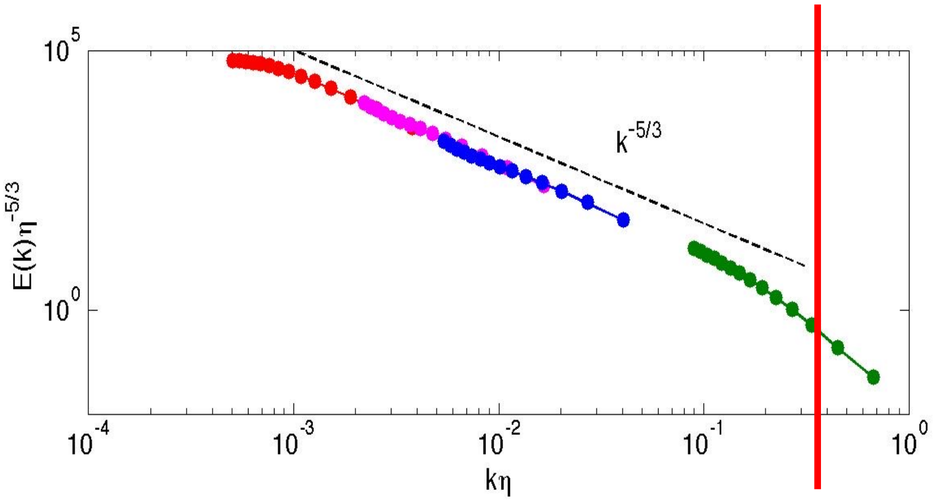
Peanut Butter



How could we observe this new paradigm?







$$D_\ell(\mathbf{u}) = \frac{1}{4}\int_{\mathcal{V}} d^3r\, (\boldsymbol{\nabla} G_\ell)(\mathbf{r}) \cdot \delta\mathbf{u}(\mathbf{r})\, |\delta\mathbf{u}(\mathbf{r})|^2,$$

Problem

Navier-Stokes Equations:

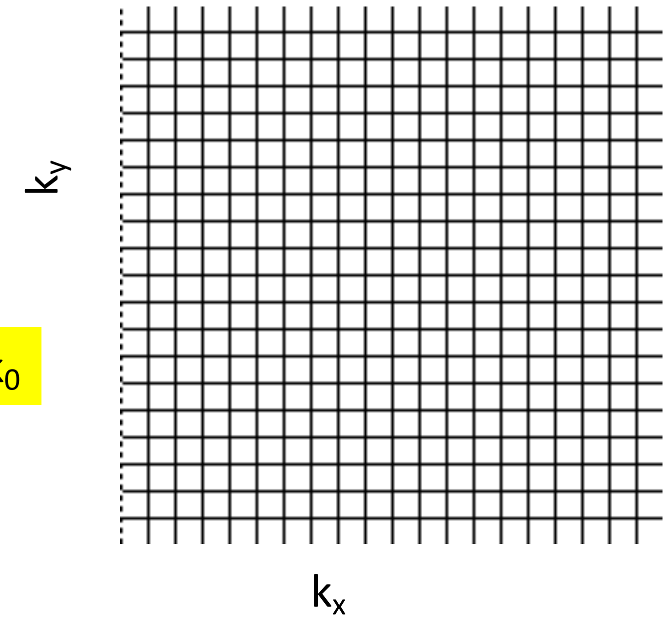
$$\partial_t u + u \bullet \nabla u = -\nabla p + \nu \Delta u + f$$

When we truncate the scale space
we truncate energy transfer and impede the building of large fluctuations->
necessity to go to at least Kolmogorov scale to get them

From DNS to log-lattices

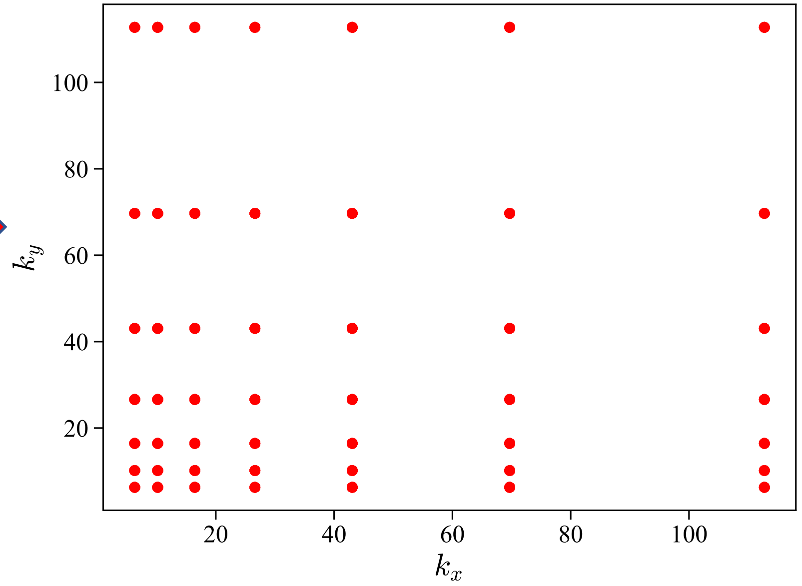
$$\partial_t \hat{u}_i = P_{ij}\left(-ik_q\hat{u}_q * \hat{u}_j + \hat{f}_j\right) - \nu_r k^2 \hat{u}_i,$$

Fourier grid



$K_n = n\, k_0$

Log grid



$K_n = 2^n\, k_0$

$$u * v \qquad m = n + q,\ (\mathrm{m,n,q}) \in \mathbb{Z}^3$$

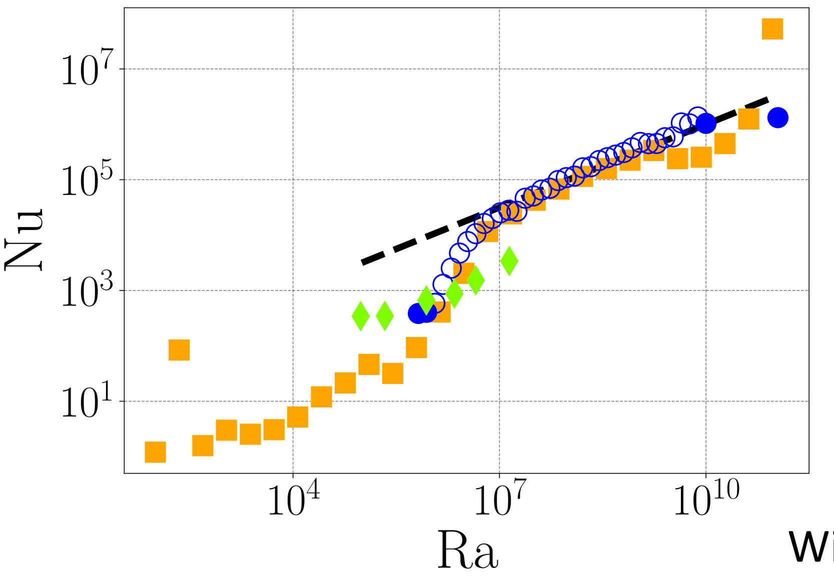
$$\lambda^m = \lambda^n + \lambda^q,\ (\mathrm{m,n,q}) \in \mathbb{Z}^3$$

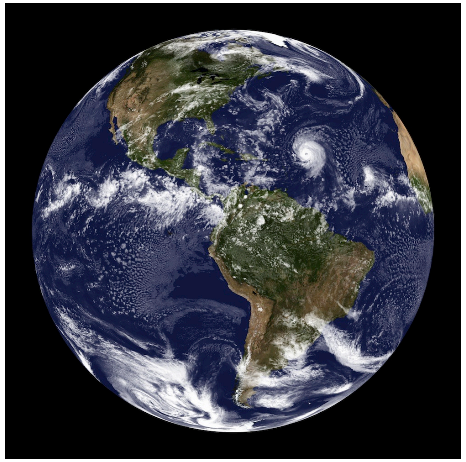
$$\lambda = 2 \quad (z = 3^D).$$
$$\lambda = \sigma \approx 1.325 \quad (z = 12^D)$$
$$\lambda = \Phi \approx 1.618 \quad (z = 6^D)$$
$$1 = \lambda^b - \lambda^a, 0 < a < b$$

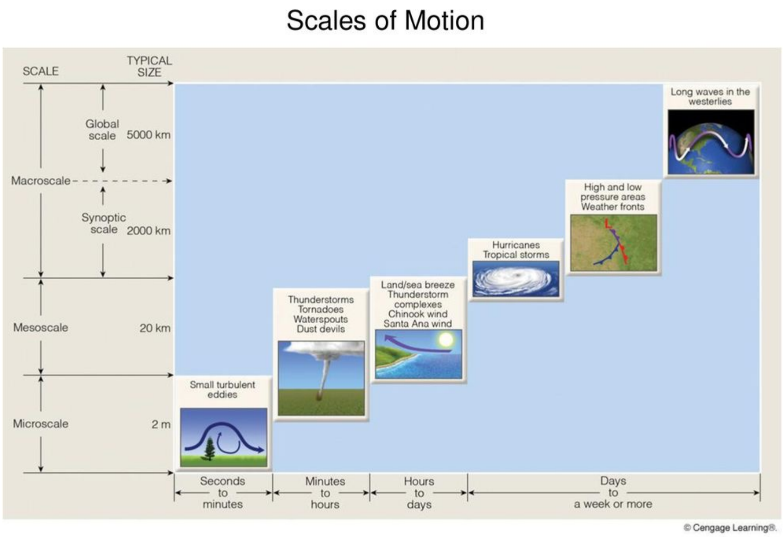
Campolina&Mailybaev, 2018

Generalization to Convection

Barral&Dubrulle, 2023

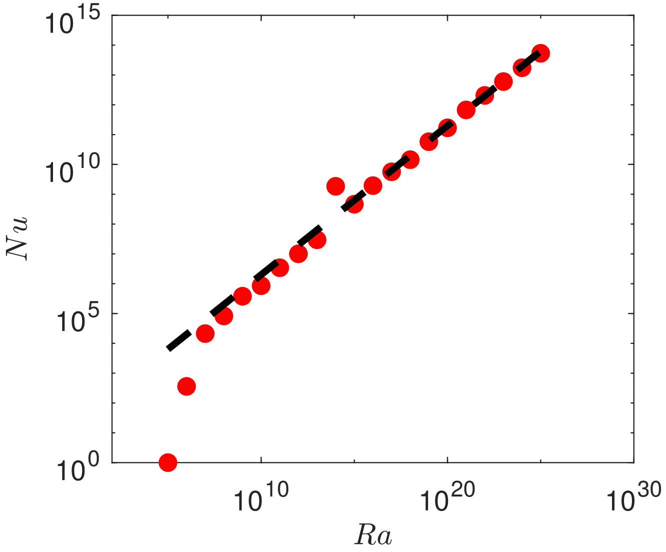






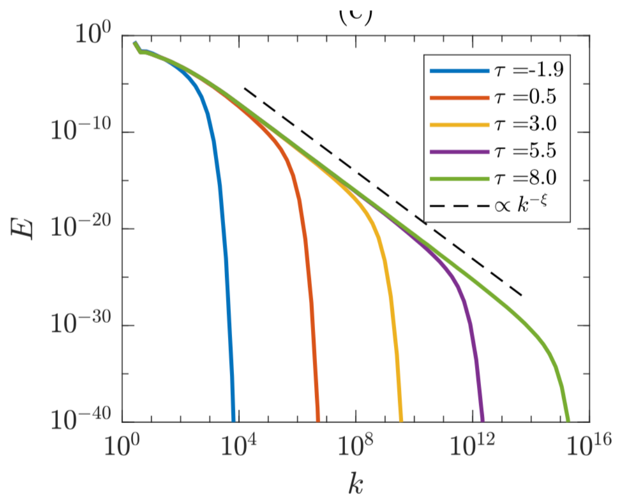
(pp. 230-231)

With 80 modes, we can simulate atmospheric convection with parameters equivalent to that of the atmosphere

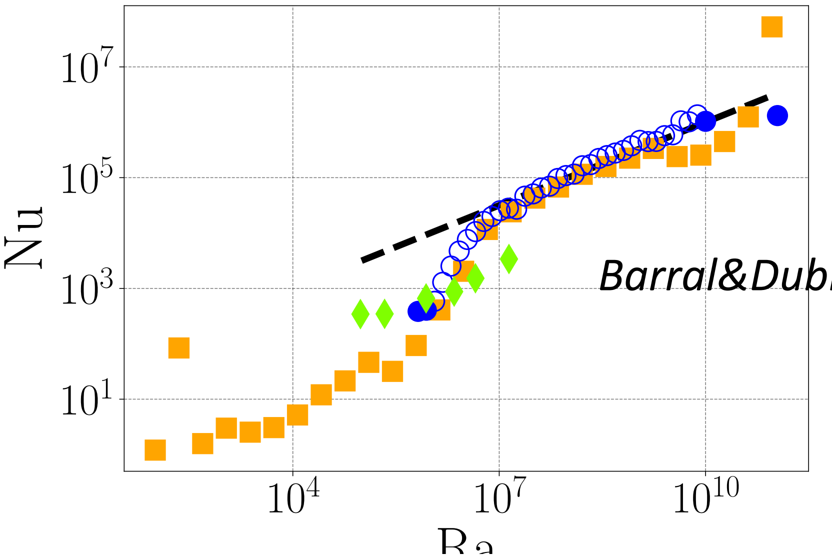


$$\partial_t u + u \cdot \nabla u + \nabla p = Pr(\nabla^2 u + Ra\theta \vec{z}),$$
$$\partial_t \theta + u \cdot \nabla \theta = \nabla^2 \theta + u_z,$$

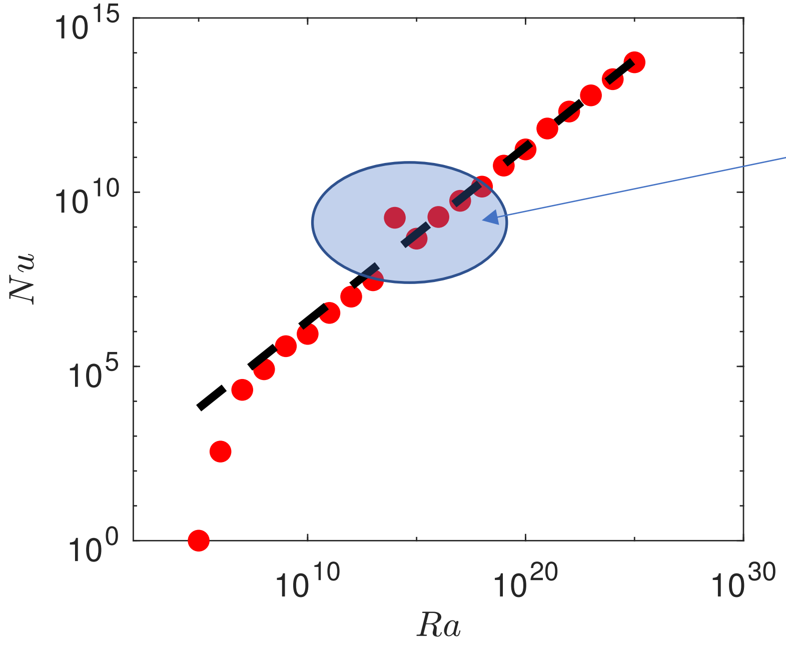
80 modes!!!!



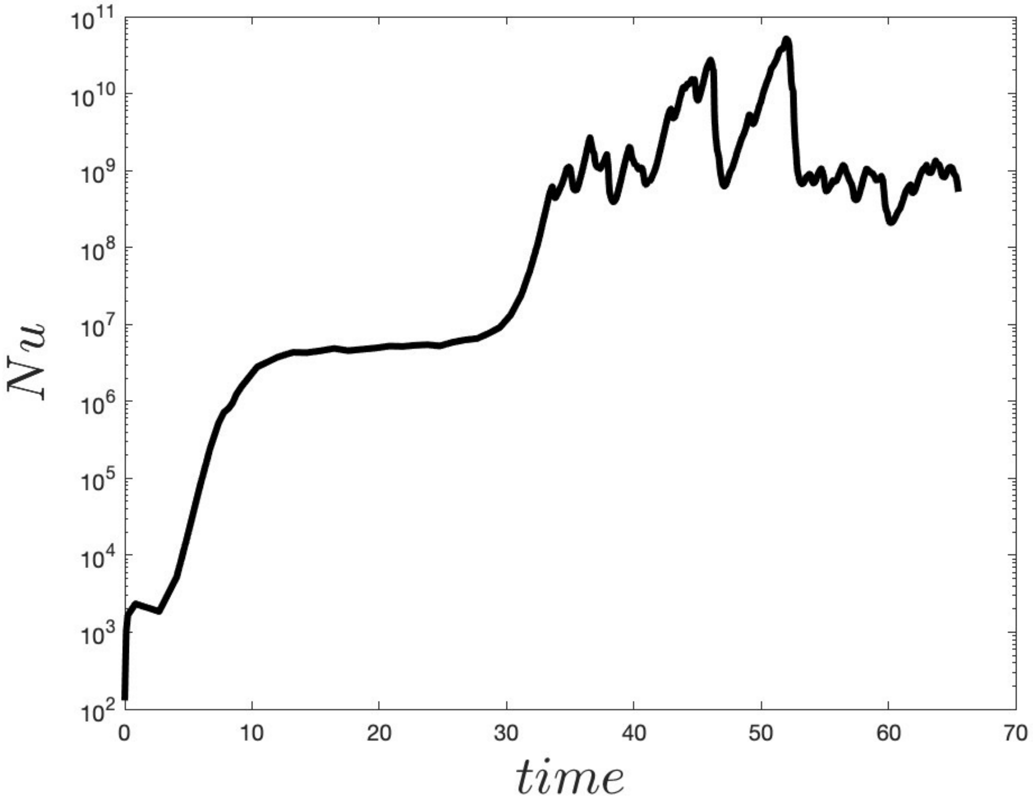
Generalization to Convection



Barral&Dubrulle, 2023

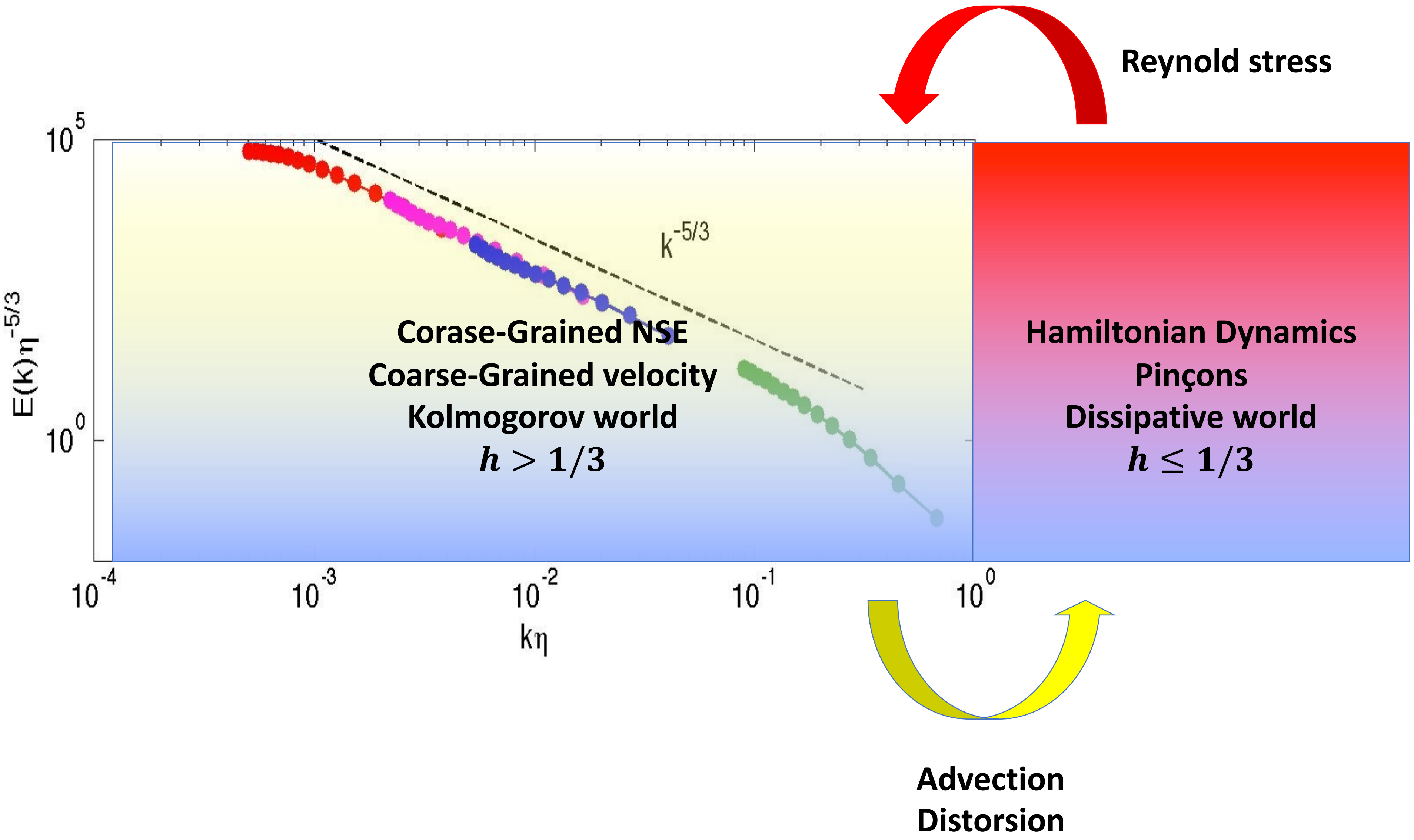


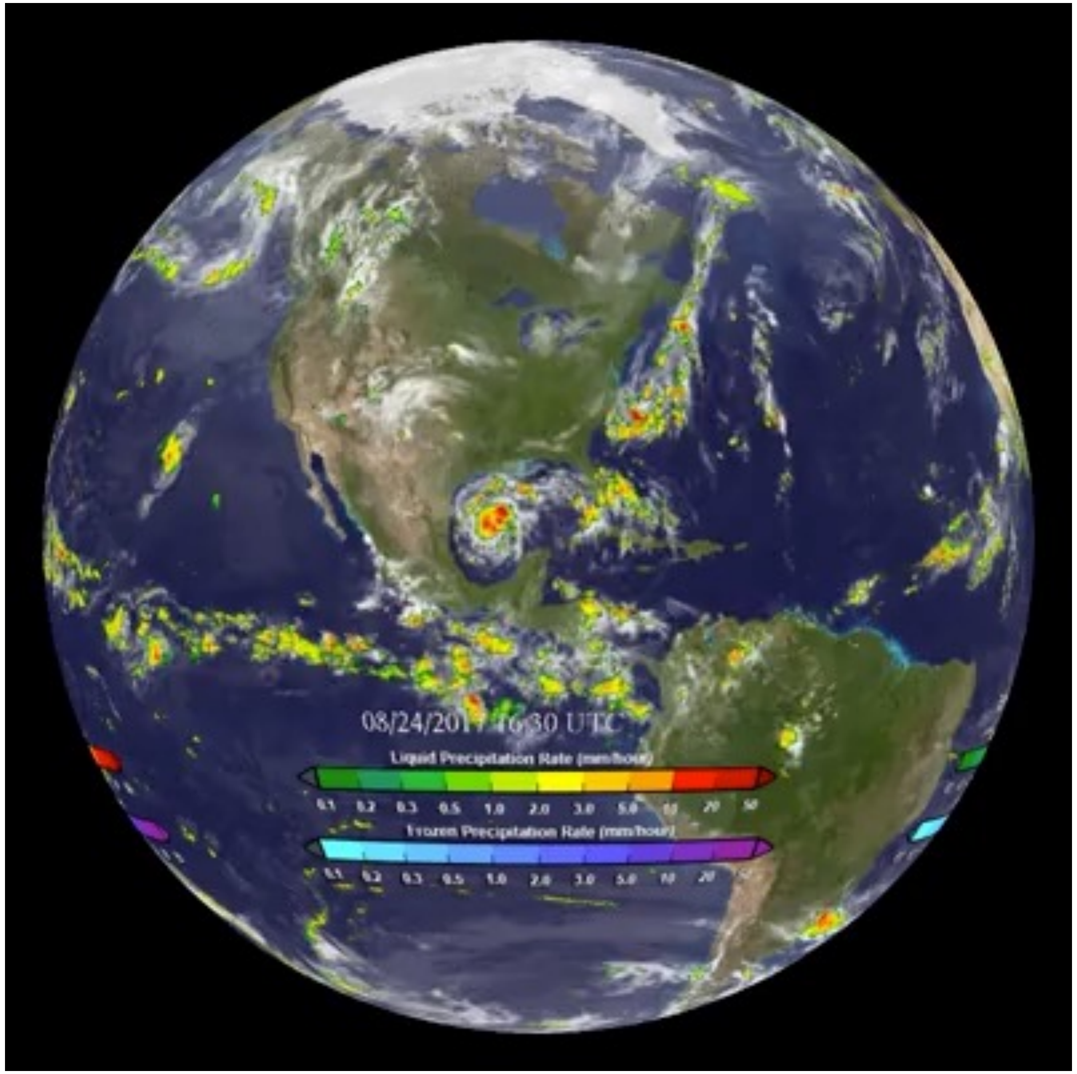
What is going on there

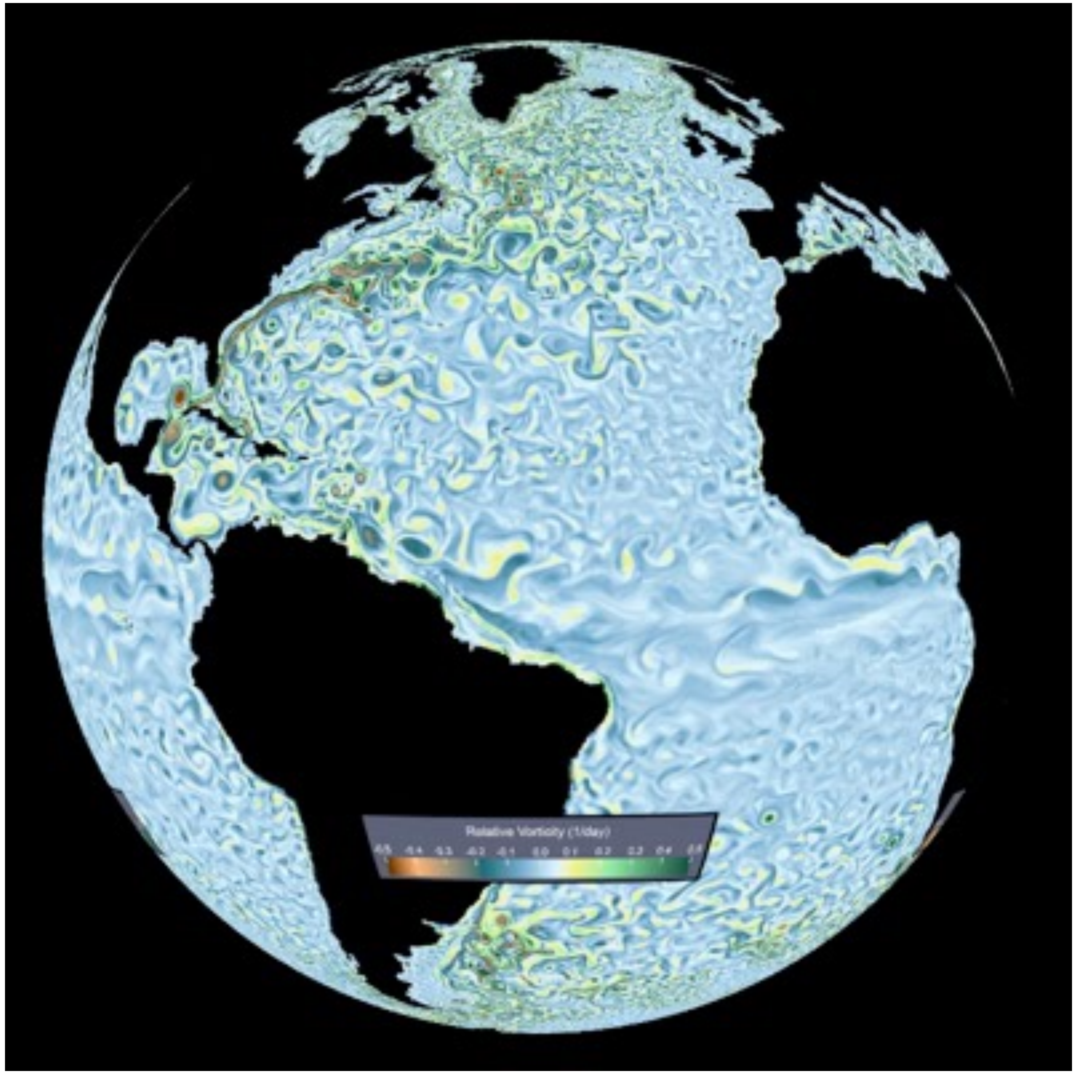


Bifurcation to an intermittent regime

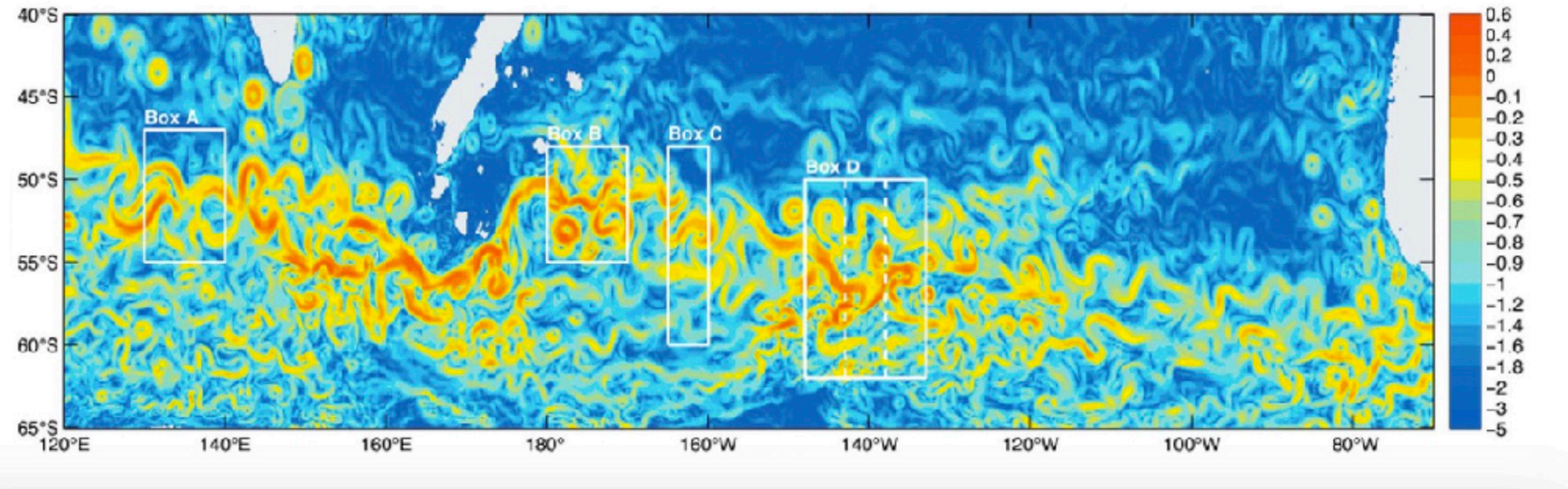
Two-fluids model of turbulence



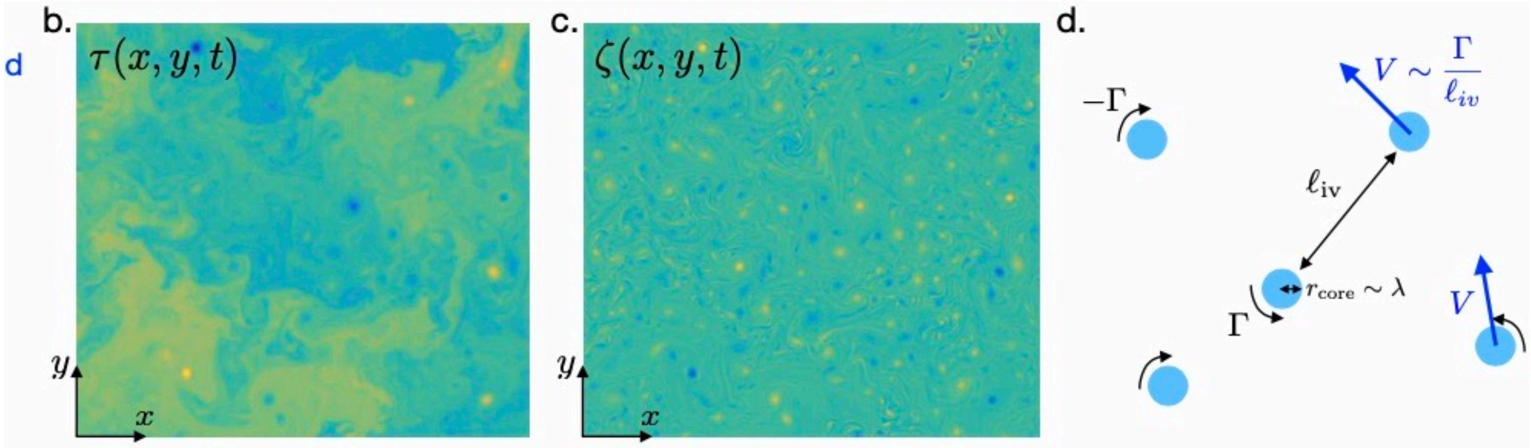

Liquid Precipitation Rate (mm/hour)
0.1 0.2 0.3 0.5 1.0 2.0 3.0 5.0 10 20 50
Frozen Precipitation Rate (mm/hour)
0.1 0.2 0.3 0.5 1.0 2.0 3.0 5.0 10 20


Relative Vorticity (1/day)

2D case: point vortices in the Ocean

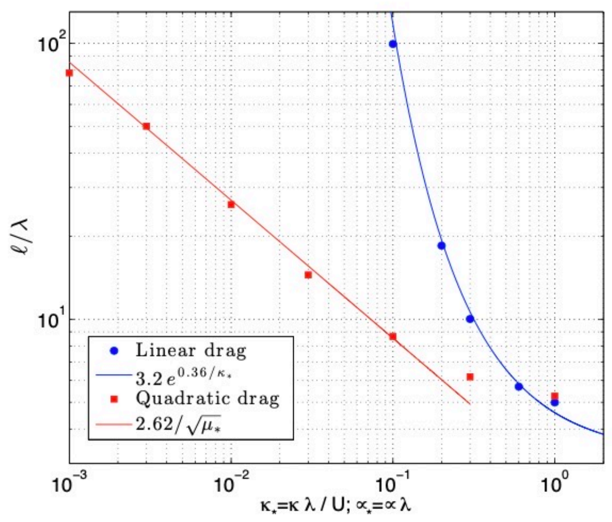


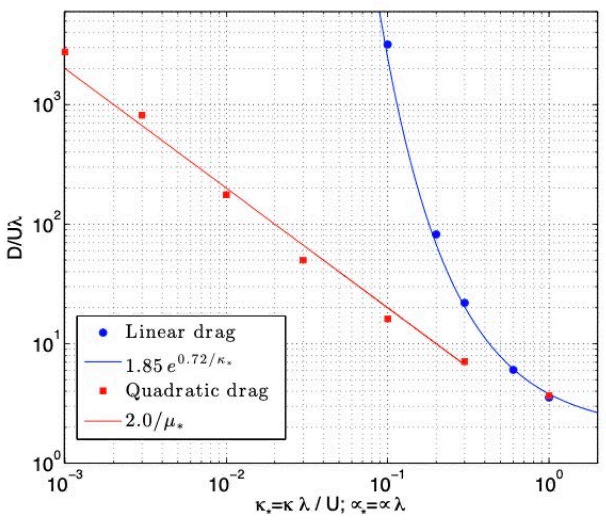
Model of barocline vortices by
Gas of point vortices



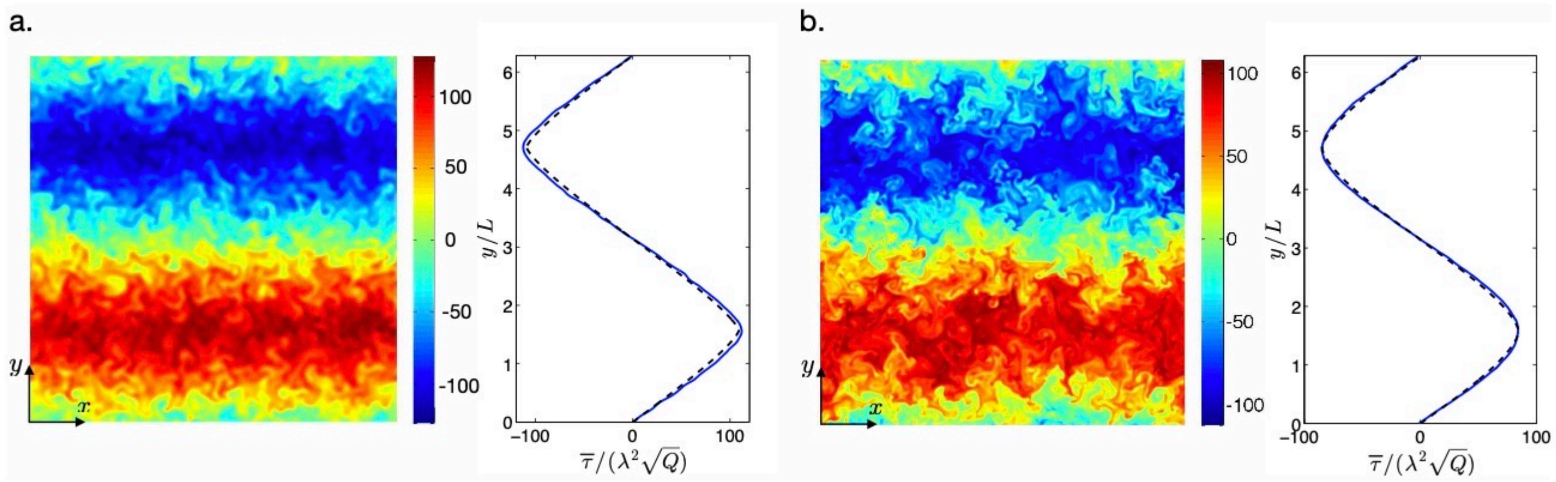
Gallet&Ferrari, PNAS 2020

2D case: point vortices in the Ocean



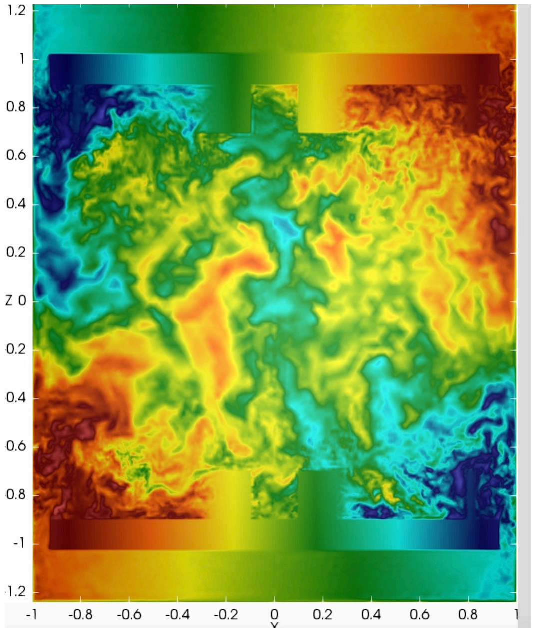


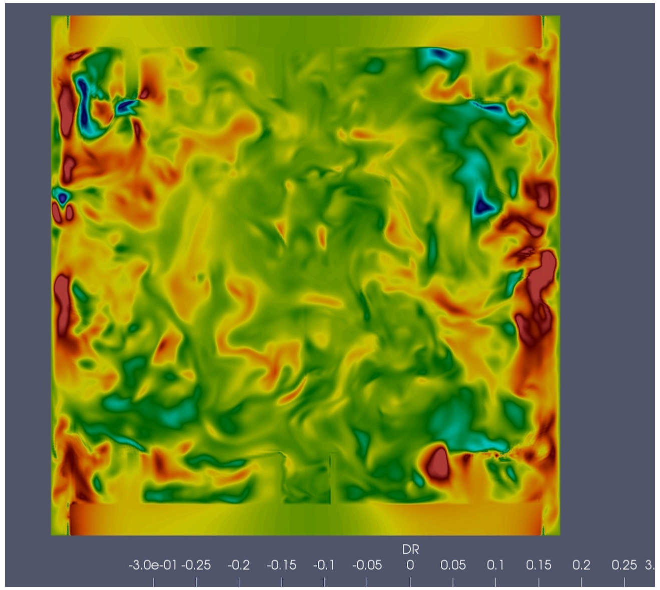
$$\nu_T = K V L$$



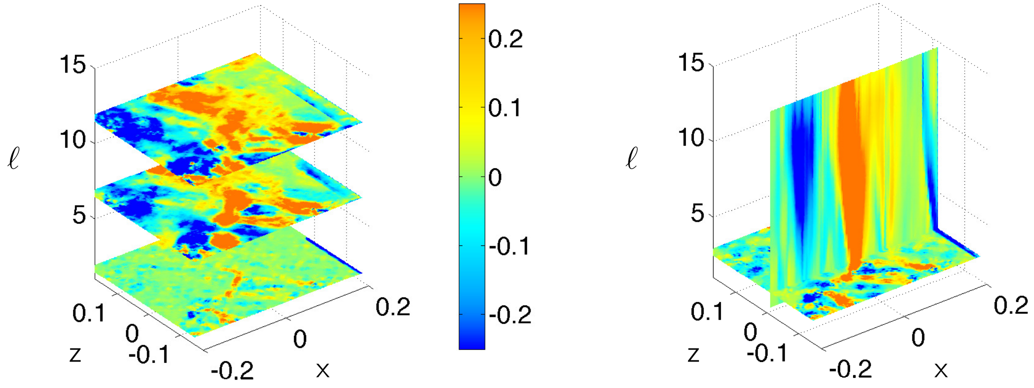
Gallet&Ferrari, PNAS 2020

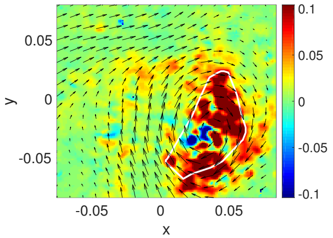
3D case: quasi-singularities





High fluctuations are built through
Local energy transfer through
Smaller scale





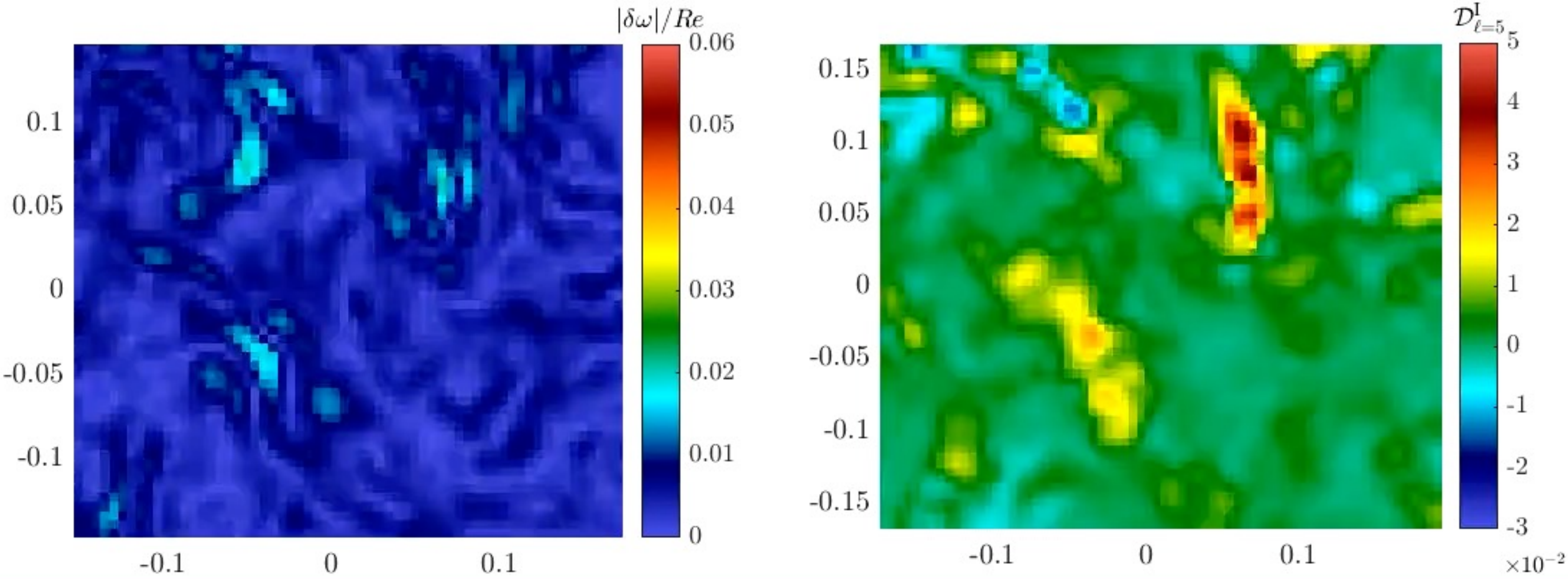
$$D_\ell(\mathbf{u}) = \frac{1}{4}\int_{\mathcal{V}} d^3r\, (\boldsymbol{\nabla} G_\ell)(\mathbf{r}) \cdot \delta\mathbf{u}(\mathbf{r})\, |\delta\mathbf{u}(\mathbf{r})|^2,$$

Navier-Stokes Equations:

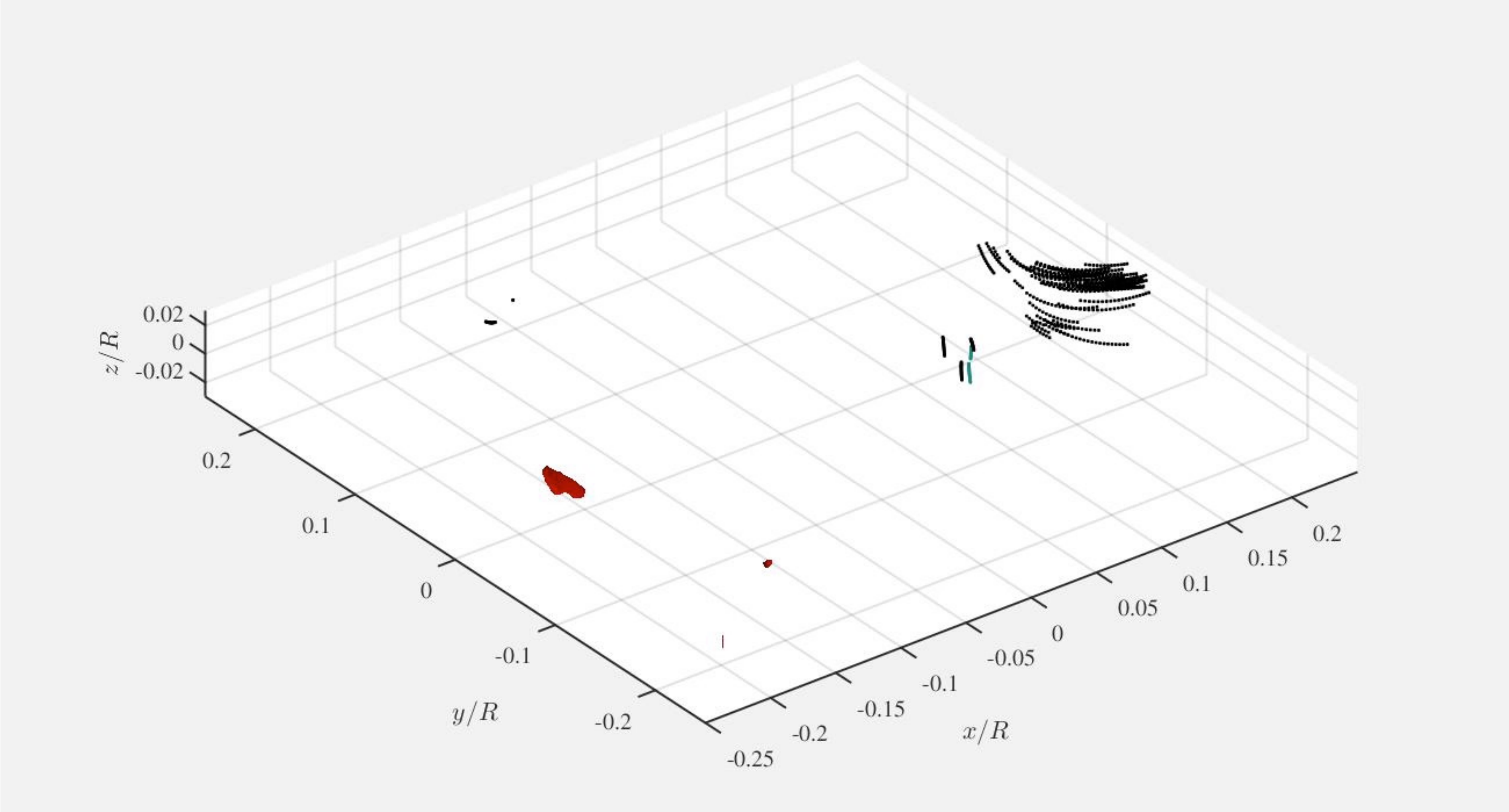
$$\partial_t u + u \bullet \nabla u = -\nabla p + \nu \Delta u + f$$

Building of quasi-singular events

Dynamics of intense energy transfers



Dynamics of quasi singularities



Reconnexion?

Eulerian

Lagrangian

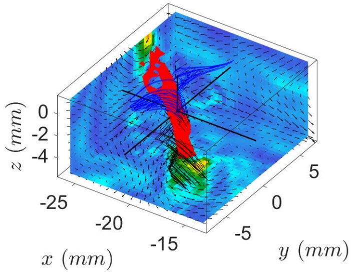
Cheminet et al, PRL 2022

Model of NS singularity: homogeneous solution of NS of degree -1

Recaling Symmetry for h=-1 $(t, x, u) \longrightarrow (\gamma^2\, t, \gamma x, \gamma^{-1} u)(\,\nu \neq 0)$

$u(\gamma^2\, t, \gamma x) = \gamma^{-1} u(t, x)$ **homogeneous solutions of NS of degree -1**

Stationary: only solution=Axisymmetric: (Sverak, xx)
Landau –Squire solutions



$$
\begin{aligned}
\nabla \cdot \mathbf{U} &= 0, \\
(\mathbf{U} \cdot \nabla)\mathbf{U} + \frac{\nabla p}{\rho} - \nu \Delta \mathbf{U} &= \nu^2 \delta(\mathbf{x}) \mathbf{F},
\end{aligned}
\qquad (2.1)
$$

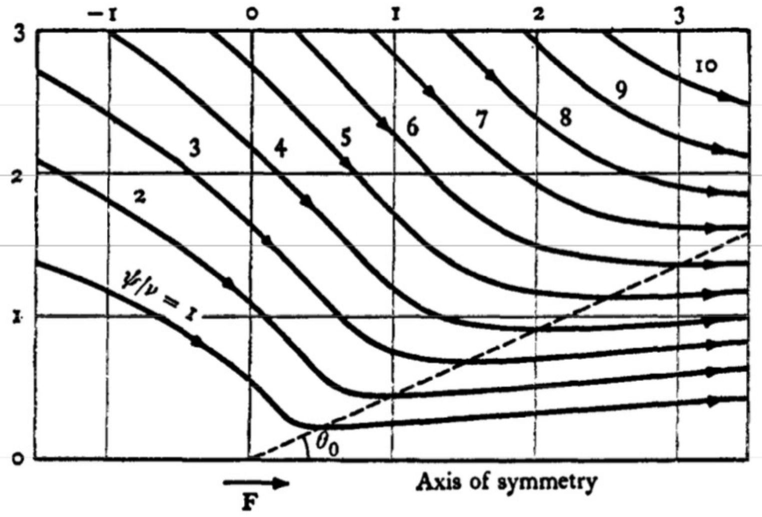


Figure 4.6.1. Streamlines of the flow for $c = 0{\cdot}1$, $\theta_0 = 24^\circ\, 37'$.
(The units for ψ/ν and r are consistent.)

Landau, (1944)
Batchelor, 1951
H. Faller et al, (2021)

Model of singularity: homogeneous solution of NS of degree -1

Stationary solutions of NSE with a force at the origin

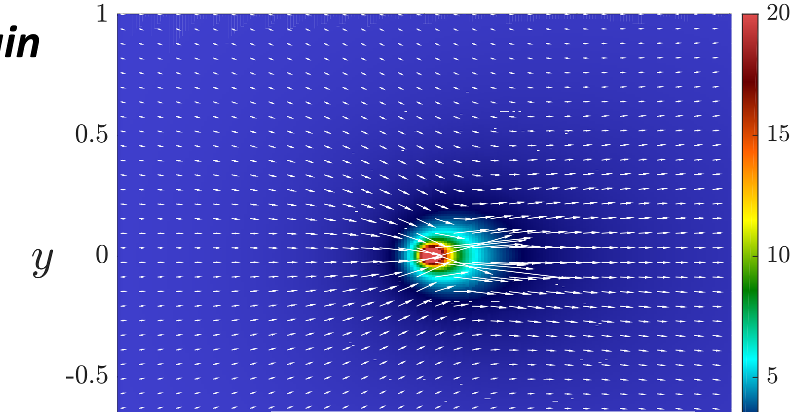
$$
\begin{aligned}
\nabla \cdot \mathbf{U} &= 0, \\
(\mathbf{U} \cdot \nabla)\mathbf{U} + \frac{\nabla p}{\rho} - \nu \Delta \mathbf{U} &= \nu^2 \delta(\mathbf{x}) \mathbf{F},
\end{aligned}
$$

General form

$$
\begin{aligned}
\phi(\mathbf{x}, \boldsymbol{\gamma}) &= \|\mathbf{x}\| - \boldsymbol{\gamma} \cdot \mathbf{x}, \qquad \gamma < 1 \\
\mathbf{U} &= -2\nabla(\ln \phi) + 2\mathbf{x}\Delta \ln(\phi),
\end{aligned}
$$

and

$$
\begin{aligned}
\mathbf{F} &= F(||\boldsymbol{\gamma}||)\frac{\boldsymbol{\gamma}}{||\boldsymbol{\gamma}||}, \\
F(\gamma) &= 4\pi \left[\frac{4}{\gamma} - \frac{2}{\gamma^2} \ln\left(\frac{1+\gamma}{1-\gamma} \right) + \frac{16}{3} \frac{\gamma}{1-\gamma^2} \right].
\end{aligned}
$$



Pinçon

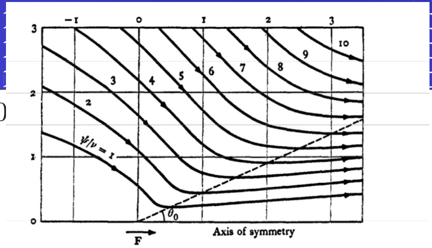
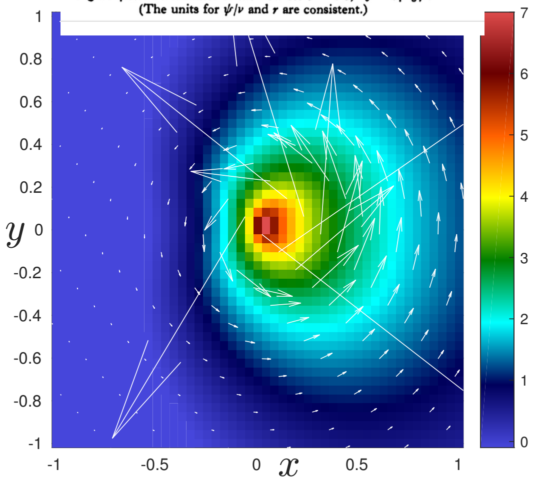


Figure 4.6.1. Streamlines of the flow for $c = 0{\cdot}1$, $\theta_0 = 24^\circ 37'$. (The units for ψ/ν and r are consistent.)



H. Faller et al submitted (2021)

Model of singularity: homogeneous solution of NS of degree -1

Stationary solutions of NSE with a force at the origin

$$\begin{aligned} \nabla\cdot\mathbf{U} &= 0, \\ (\mathbf{U}\cdot\nabla)\mathbf{U} + \frac{\nabla p}{\rho} - \nu\Delta\mathbf{U} &= \nu^2\delta(\mathbf{x})\mathbf{F}, \end{aligned}$$

General form

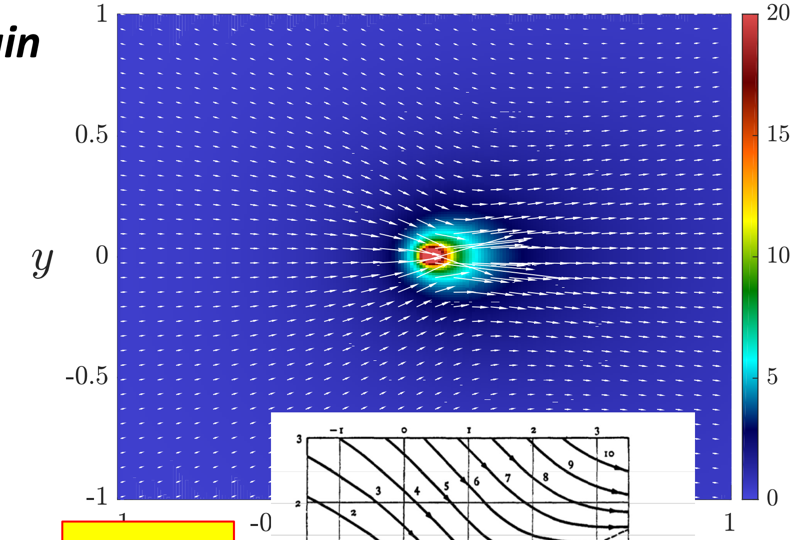
$$\phi(\mathbf{x},\boldsymbol{\gamma}) = \|\mathbf{x}\| - \boldsymbol{\gamma}\cdot\mathbf{x}, \qquad \gamma < 1$$

$$\mathbf{U} = -2\nabla(\ln\phi) + 2\mathbf{x}\Delta\ln(\phi),$$

and

$$\mathbf{F} = F(||\boldsymbol{\gamma}||)\frac{\boldsymbol{\gamma}}{||\boldsymbol{\gamma}||},$$

$$F(\gamma) = 4\pi\left[\frac{4}{\gamma} - \frac{2}{\gamma^2}\ln\left(\frac{1+\gamma}{1-\gamma}\right) + \frac{16}{3}\frac{\gamma}{1-\gamma^2}\right].$$



Pinçon

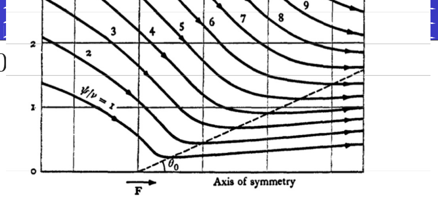
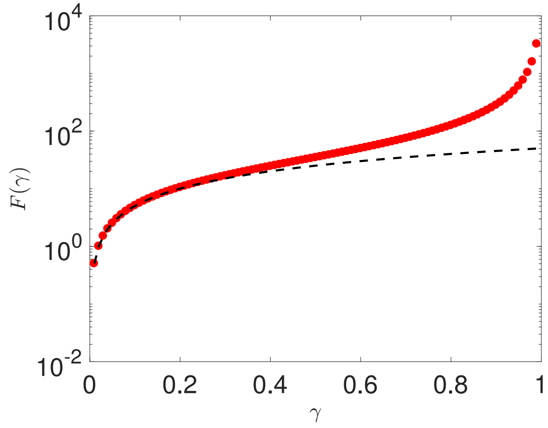


Figure 4.6.1. Streamlines of the flow for $c = 0{\cdot}1$, $\theta_0 = 24°\,37'$. (The units for ψ/ν and r are consistent.)



H. Faller et al submitted (2021)

3D: Interaction between a regular field and a pinçon

Consider the case where a pinçon , located at $\mathbf{x}_\alpha$ is embedded in a regular velocity field. What is going on?

The system is solution of NSE provided the two sets of equations are satisfied

For the field

$$\partial_t \overline{\mathbf{v}_R}^\ell + (\overline{\mathbf{v}_R}^\ell \cdot \nabla)\overline{\mathbf{v}_R}^\ell + \frac{\nabla \overline{p_r}^\ell}{\rho} - \nu \Delta \overline{\mathbf{v}_R}^\ell = \tau^\ell - \frac{\nu^2}{\ell^3}\psi\left(\frac{\mathbf{x}-\mathbf{x}_\alpha}{\ell}\right)\mathbf{F},$$

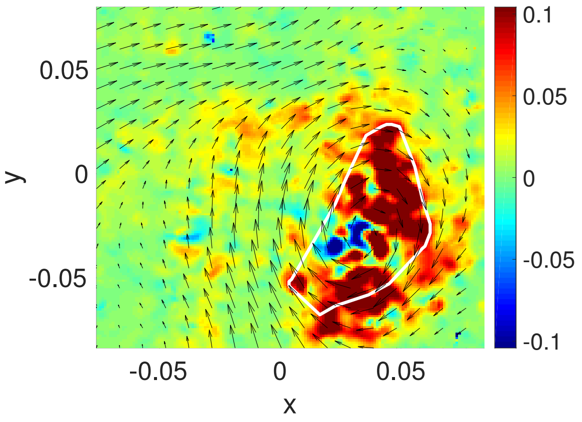
The two contributions are equal at the Kolmogorov scale

where $\tau^\ell = \nabla \cdot (\overline{\mathbf{v}_R}^\ell \overline{\mathbf{v}_R}^\ell - \overline{\mathbf{v}_R \mathbf{v}_R}^\ell)$ is the Reynolds stress.

For the pinçon

$$\dot{\mathbf{x}}_\alpha = \mathbf{v}_R(\mathbf{x}_\alpha)$$

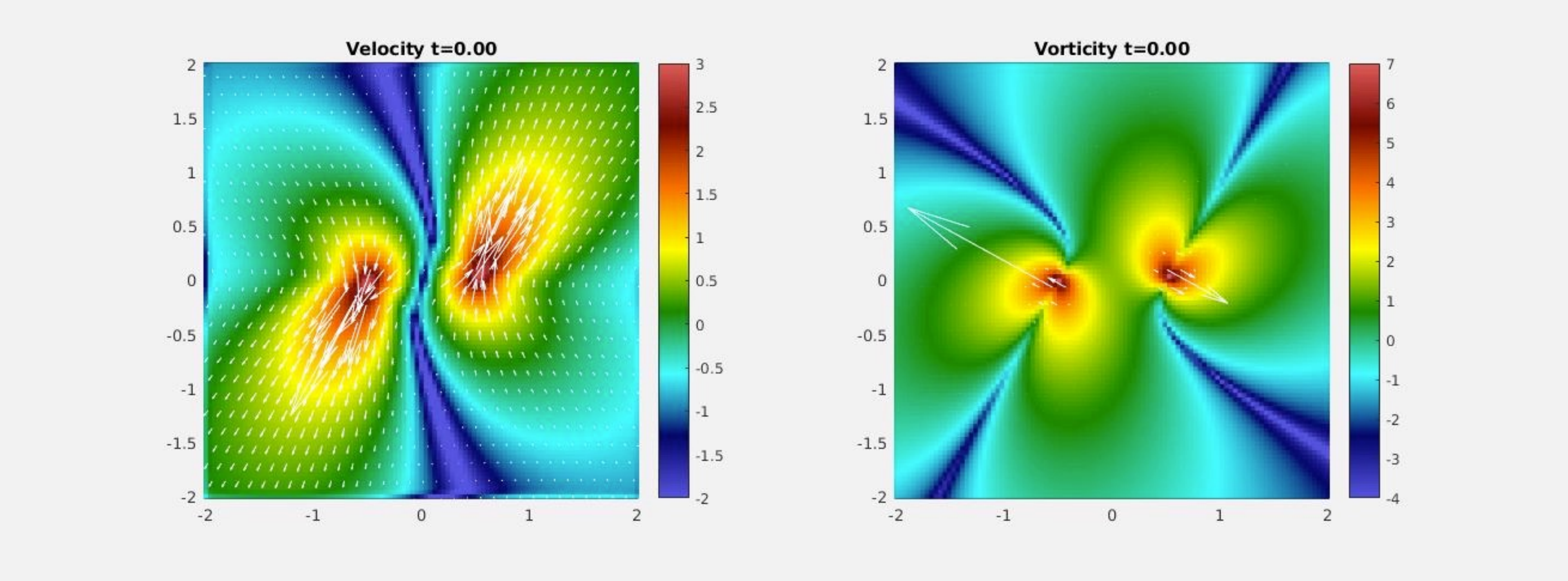
$$\dot{\gamma}\overline{\nabla_\gamma \mathbf{U}}^\ell = -(\overline{\mathbf{U}}^\ell \cdot \nabla)\mathbf{v}_R.$$

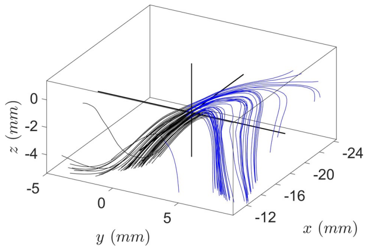


The pinçon moves with the fluid velocity and is sheared by the regular field

H. Faller et al submitted (2021)

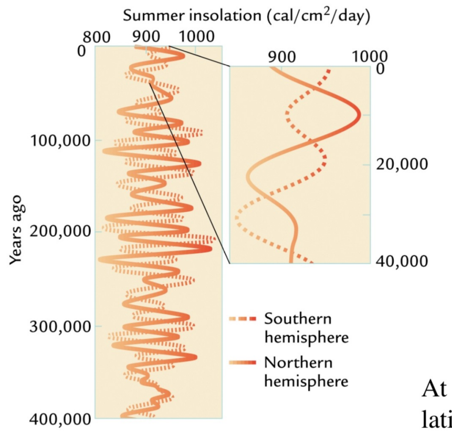
Interaction between a dipole of pinçons



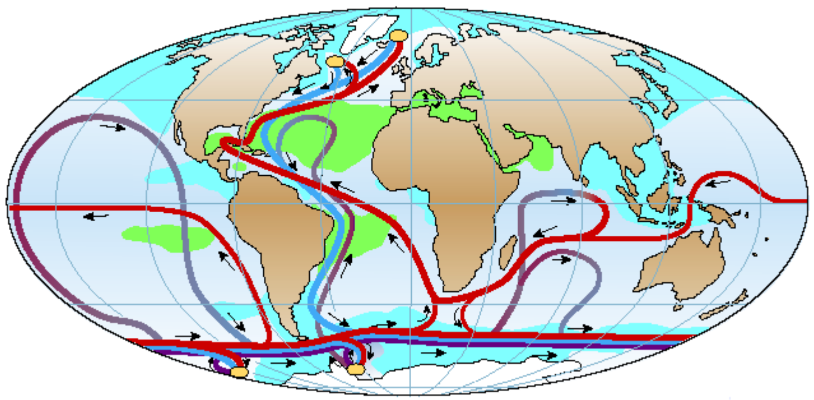


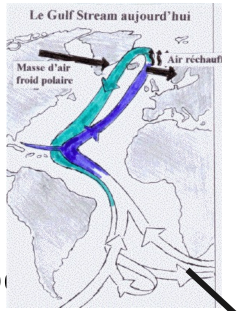
H. Faller et al entropy (2021)

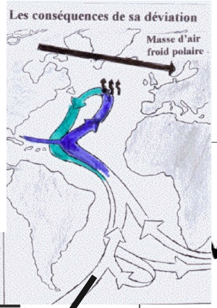
Climate Bifurcations or Tipping points



At 65 ° latitude

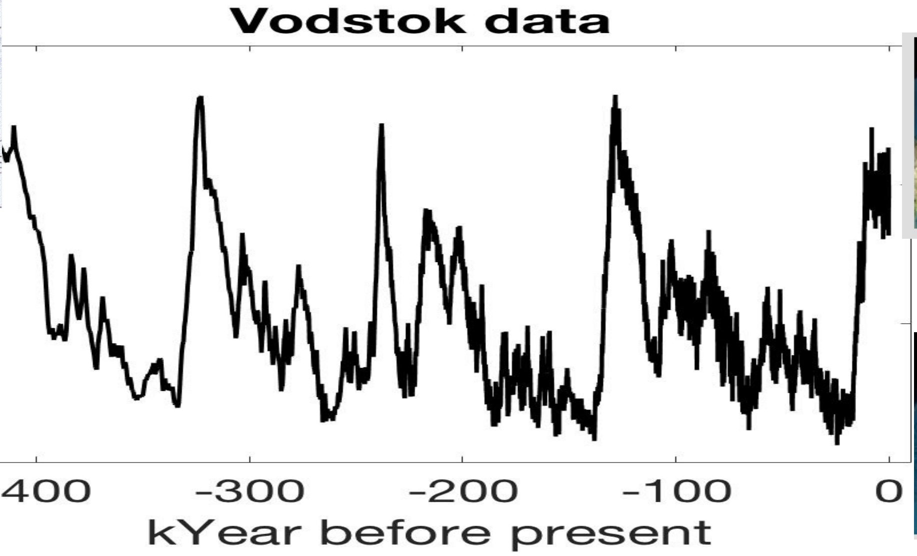






Sp

Heinrich and Dansgaard–Oeschger events



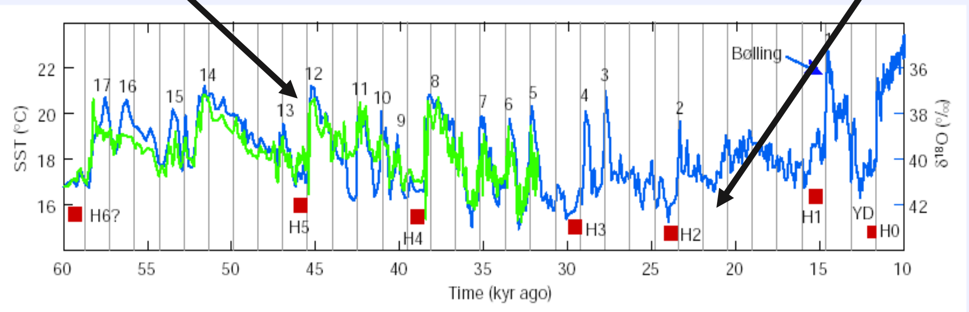


Figure 3 Temperature reconstructions from ocean sediments and Greenland ice. Proxy data from the subtropical Atlantic[86] (green) and from the Greenland ice core GISP2 (ref. 87; blue) show several Dansgaard–Oeschger (D/O) warm events (numbered). The timing of Heinrich events is marked in red. Grey lines at intervals of 1,470 years illustrate the tendency of D/O events to occur with this spacing, or multiples thereof.

Forced bifurcation

Can we predict climate bifurcations????

L=1000 km
H=100 km
η=10 mm

Horizontal: N=10^{16}

Vertical: N=10^7

Volume: N=10^{23}

ↄ^2

H=100 km
ΔH=5 km
Δt=1000 s

Vertical: N=20

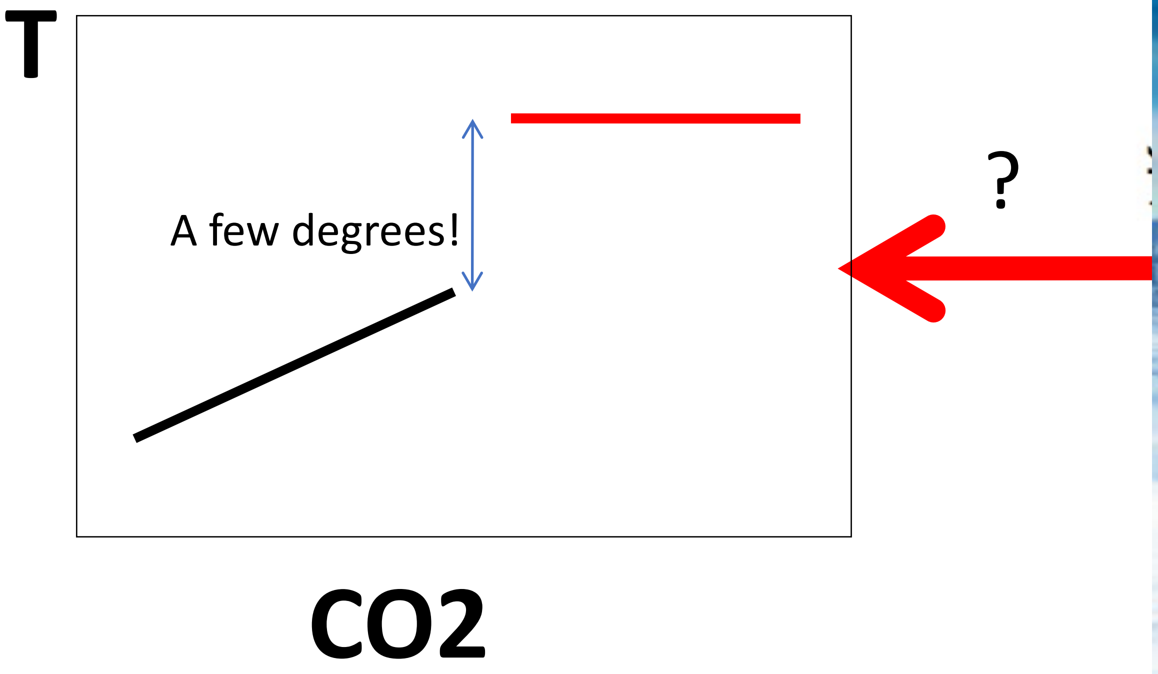
Volume: N=2x10^3

Air

Viscosity x 10^6 !

Peanut Butter

…… Caveat!!!!!!



What could really be observed in IPCC simulations when increasing resolution?
Problem of climate change might be even more worrysome!!!! (no more « adaptibility! »)