Modèles économiques et thermodynamique

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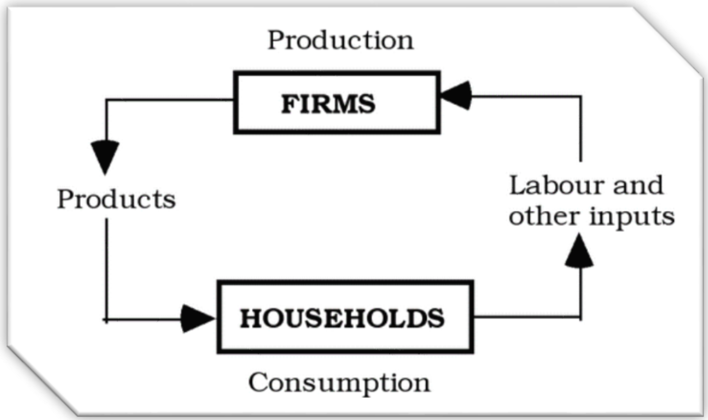
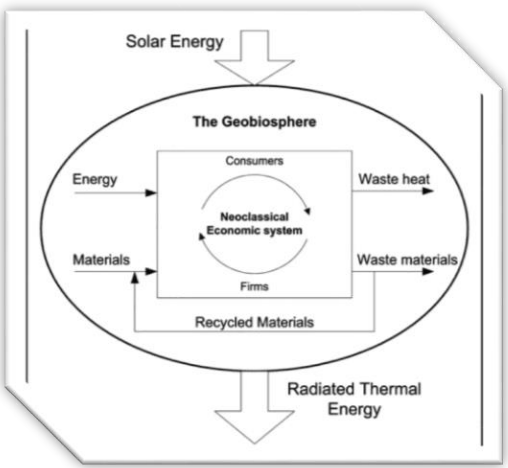
EcoClim 2023

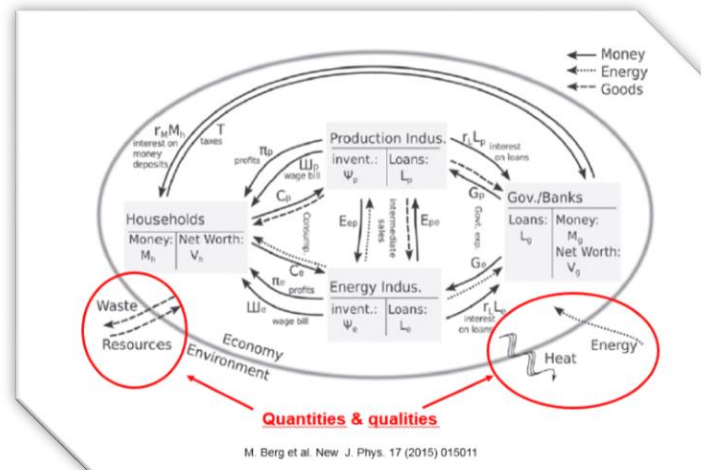
What is the system we are talking about?



Perpetual motion et al…



Glucina et al. 2010
Hall et al. 1986



Berg et al. 2015

« Celui qui croit qu'une croissance exponentielle peut
continuer indéfiniment dans un monde fini
est soit un fou, soit un économiste. »

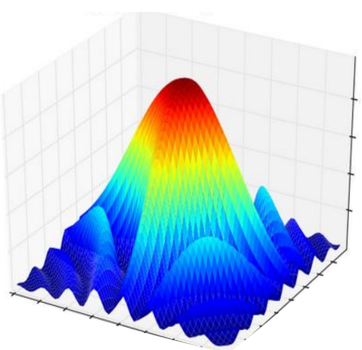
Kenneth E. Boulding, cité dans *Jump the Curve* (Jack Uldrich, 2008)

Few words of history

Economics IS Mechanics!..............?

Or quand l'économique veut s'acquitter de sa tâche en disant quelle est la cause du fait de la valeur d'échange et prétend la trouver, grâce à la méthode mathématique, dans la *rareté* ou *l'intensité du dernier besoin satisfait*, les économistes non-mathématiciens se récrient, et même les mathématiciens refusent d'accepter "qu'une satisfaction puisse être mesurée". Avec les premiers il serait oiseux de discuter: eux et nous ne parlons pas la même langue. Mais avec les mathématiciens il en est autrement: nous pouvons nous expliquer et peut-être nous entendre.

"Et il me semble facile de faire voir aux mathématiciens, par deux exemples décisifs, que sa manière de procéder est rigoureusement identique à celle de deux sciences physico-mathématique des plus avancées et des plus incontestées: **la mécanique rationnelle et la mécanique céleste.** Quand nous serons d'accord sur ce point, le procès sera jugé." (Walras 1909)



… and Herman Laurent wrote to Léon Walras….

(about the integrability of the model)

From Gibbs to Samuelson

"Schumpeter has suggested that it would be particularly well for me to give as **I gave last time a general theory of equilibrium** such as this is understood by physical chemists including the phase systems of Willard Gibbs. **Most of our equilibrium theory in economics really has for its background the notions of equilibrium which arise in mechanics**. Although Pareto was certainly quite familiar with the types of equilibrium which arise in physical chemistry and are necessary in fact for the study of the steam engine he doesn't use this line of thought in economics." (Wilson 1938)

"Now if one wishes to postulate the derivatives including the second derivatives in an absolutely definite quadratic form one doesn't need to talk about the limits of stability because the definiteness of the quadratic form means that one has stability. I wonder whether you can't make it clearer or can't come nearer following the general line of ideas that **Gibbs has given in his Equilibrium of Heterogeneous Substances**, equation 133." (Wilson 1938)

$$G = U - TS + PV - M_1 m_1 - M_2 m_2 ... - M_n m_n$$

[8]Willard Gibbs, (1876): "Internal Stability of Homogeneous Fluids as indicated by Fundamental Equations"

Similar to Thermodynamics: Von Neumann

"A direct interpretation of the function $\Phi(X, Y)$ (*related to the production of goods*) would be highly desirable. **Its role appears to be similar to that of thermodynamic potentials** in phenomenological thermodynamics; it can be surmised that the similarity will persist in its full phenomenological generality (independent of our restrictive idealizations." (Von Neumann 1932)

"The analogies with thermodynamics are probably misleading. The authors think that the 'amortization' is analogous to 'entropy'. It seems to me, that if this analogy can be worked out at all, the analogon of 'entropy' must be sought in the direction of 'liquidity'. To be more specific: if the analogon of 'energy' is 'value of the estate of an economical subject, then analogon of its thermodynamic 'free energy' should be its 'cash value'".(Von Neumann 1934)

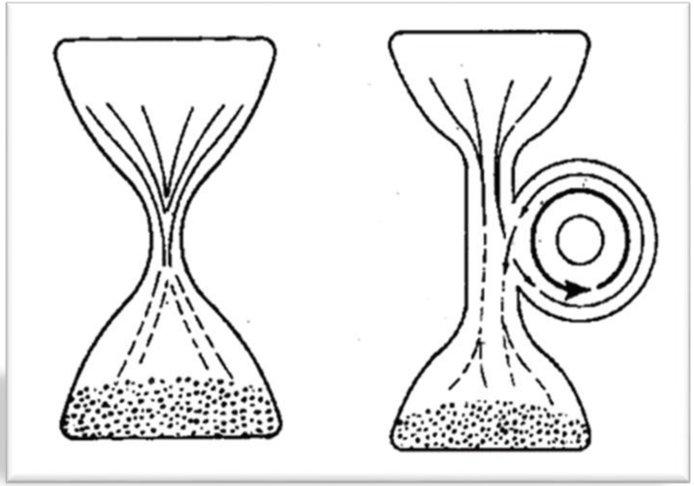
....Who's who?

When Schumpeter meets Onsager and Prigogine

"Walras arrived at his unique equilibrium by starting from a "*prix crié par hasard*" and allowing people to say what quantities they would be willing to demand and to supply at that price without actually buying or selling until that initial price is "*par tâtonnement*" so adjusted as to equate quantity supplied and quantity demanded...**The presence of friction, will, of course, always entail an equilibrium different from that which would otherwise be reached**, as well as slow up progress toward equilibrium..."(Schumpeter 1939)

"The first puts into its proper light our former statement, that disturbances of equilibrium arising from innovation **cannot be currently and smoothly absorbed**. In fact, it is now easy to realize that those disturbances must necessarily be "big," in the sense that they will disrupt the existing system and enforce a distinct process of adaptation. **This is independent either of the size of the innovating firm** or of the importance of the immediate effects their action would in itself entail." (Schumpeter 1939)

The Roegen assumption: « matter matters too»



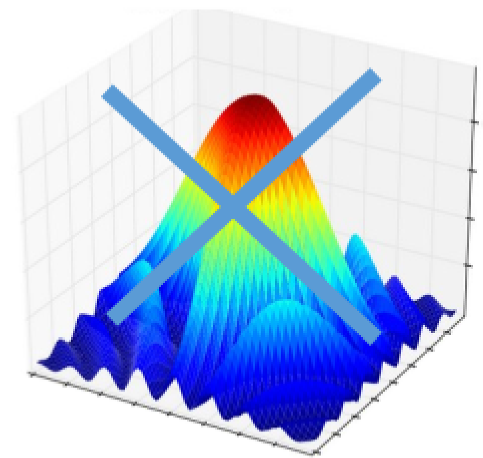
The far out of equilibrium assumption is ok,
but
there is no need of a fourth thermodynamics principle for matter.

'The idea that the economic process is not a mechanical analogue, but an entropic, unidirectional transformation began to turn over in my mind long ago, as I witnessed the oil wells of the Ploesti field of both World Wars' fame becoming dry one by one and as I grew aware of the Romanian peasant's struggle against the deterioration of their farming soil by continuous use and by rains as well. **However, it was the new representation of a process that enabled me to crystallize my thoughts in describing for the first time the economic process as the entropic transformation of valuable natural resources (low entropy) into valueless waste (high entropy)**." (Roegen 1971)

Conclusion on thermostatics

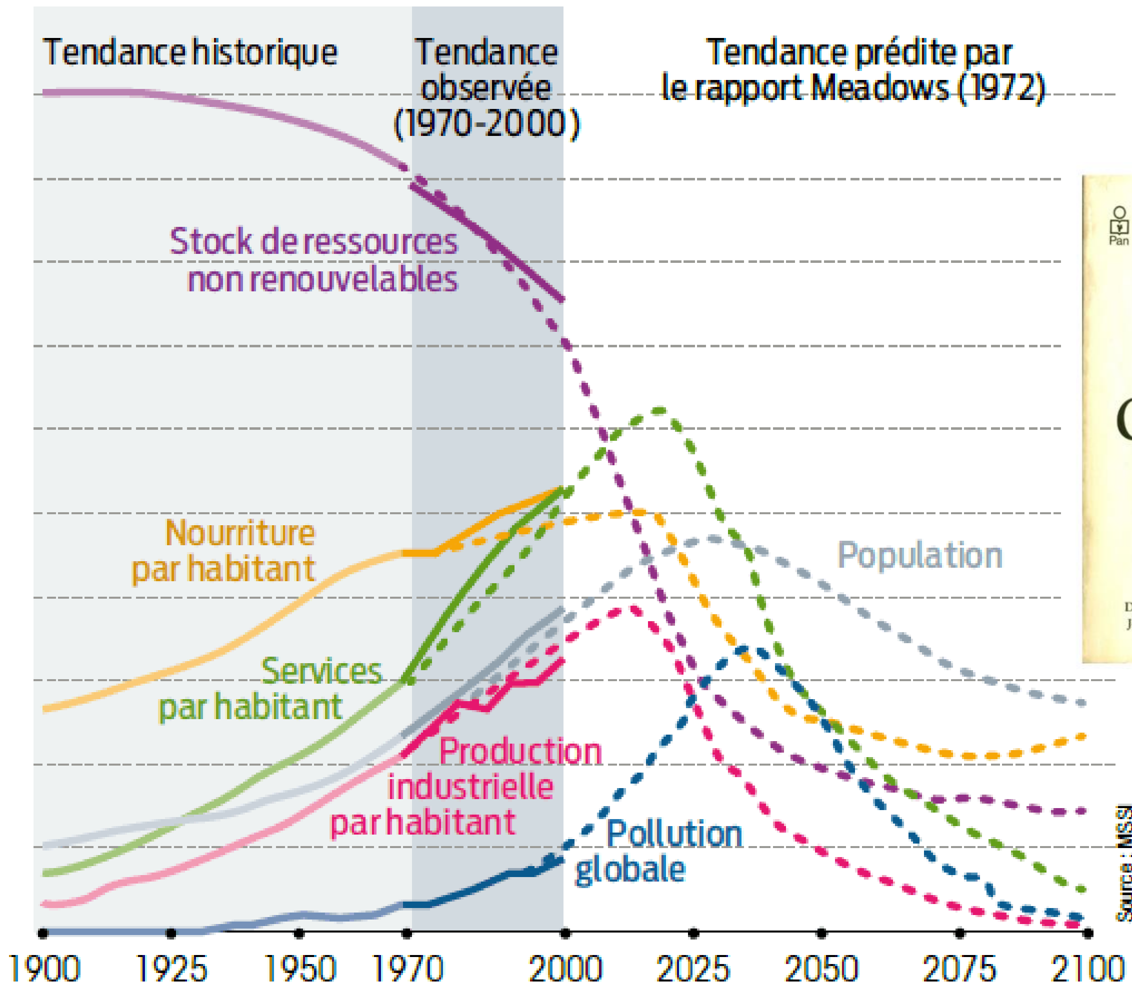
- Economics do not have the structure of thermodynamics
- No general optimization (but locally)
- No fundamental equation // variational principle
- No ergodicity
- No (?) emerging properties (as T, P...)
- No hyper surface of utilities

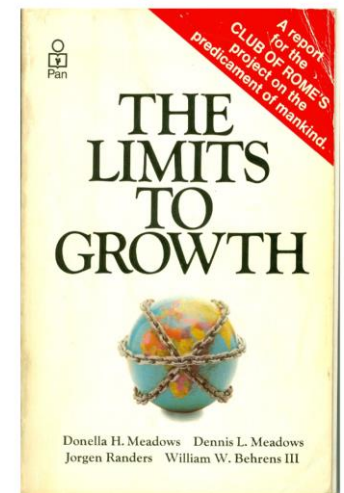
- And economic systems are not in equilibrium !



When Matter matters

The Meadows report (1972)





https://www.alternatives-economiques.fr/previsions-rapport-meadows-se-realisent-0102201988382.html

Cobb-Douglas *Production* / *Utility* function

$$Y = c\,K^{\alpha}\,L^{1-\alpha} \text{ with } \left|\begin{array}{l} Y \text{ the total production} \\ K \text{ capital input} \\ L \text{ work input} \end{array}\right.$$

Elasticities

$$\epsilon_K = \frac{K}{Y}\frac{\delta Y}{\delta K} = \alpha$$

$$\epsilon_L = \frac{L}{Y}\frac{\delta Y}{\delta L} = 1 - \alpha$$

Marginal productivity

$$\frac{\delta Y}{\delta K} = c\ \alpha\ \left(\frac{K}{L}\right)^{\alpha-1}$$

$$\frac{\delta Y}{\delta L} = c\ (1-\alpha)\ \left(\frac{K}{L}\right)^{\alpha}$$

Solow & Stiglitz: Thermodynamics fisrt law: yes, second law: ?

https://hal.science/hal-02332485/document

Natural Resources in the Theory of Production:
The Georgescu-Roegen/Daly versus Solow/Stiglitz Controversy

$$\alpha + \beta + \gamma = 1$$

Quentin Couix[a]

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Solow 1973: $Q = K^{\alpha} R^{\beta} L^{\gamma}$

Stiglitz 1974: $Q = e^{\lambda t} K^{\alpha} R^{\beta} L^{\gamma}$

If a>b then the productivity of resources Q/R can tend towards infinity and the economy can continue to grow:

- Through capital accumulation and K-R substitution (Solow)
- Through technical change λ (Stiglitz)

(Stiglitz 1974)

The fact that natural resources are limited and necessary for production does not necessarily mean that the economy must eventually stagnate and then decline. Two countervailing forces have been identified: technical change and capital accumulation. Even in the absence of technical change, capital accumulation can compensate for the effects of declining natural resource inputs, as long as capital is 'more important' than natural resources, i.e. the share of capital is greater than that of natural resources. **With technical change, at any positive rate, we can easily find paths along which overall production does not fall.**

But what do we mean by production Q ?

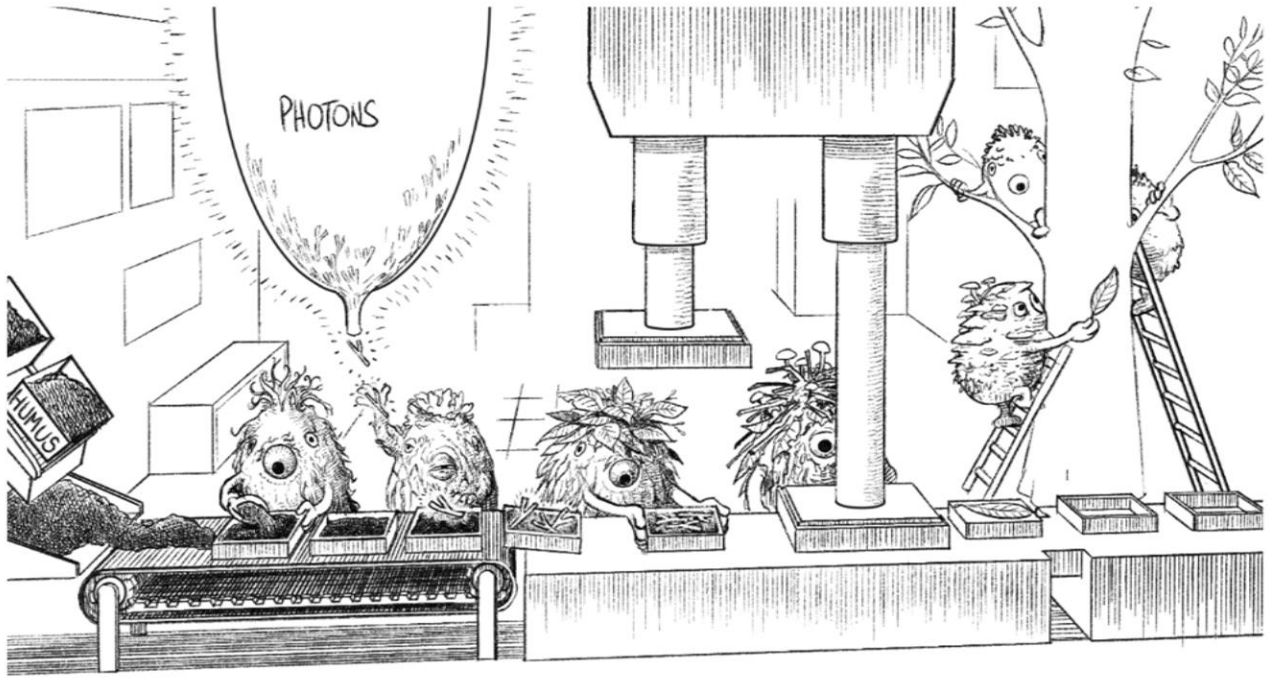
Unbounded resources productivity : How to make a (big) cake ?

In the Solow-Stiglitz variant [of production function], to make a cake we need not only the cook and his kitchen, but also some non-zero amount of flour, sugar, eggs, etc.

This seems a great step forward until we realize that we could make our cake a thousand times bigger with no extra ingredients, if we simply would stir faster and use bigger bowls and ovens.

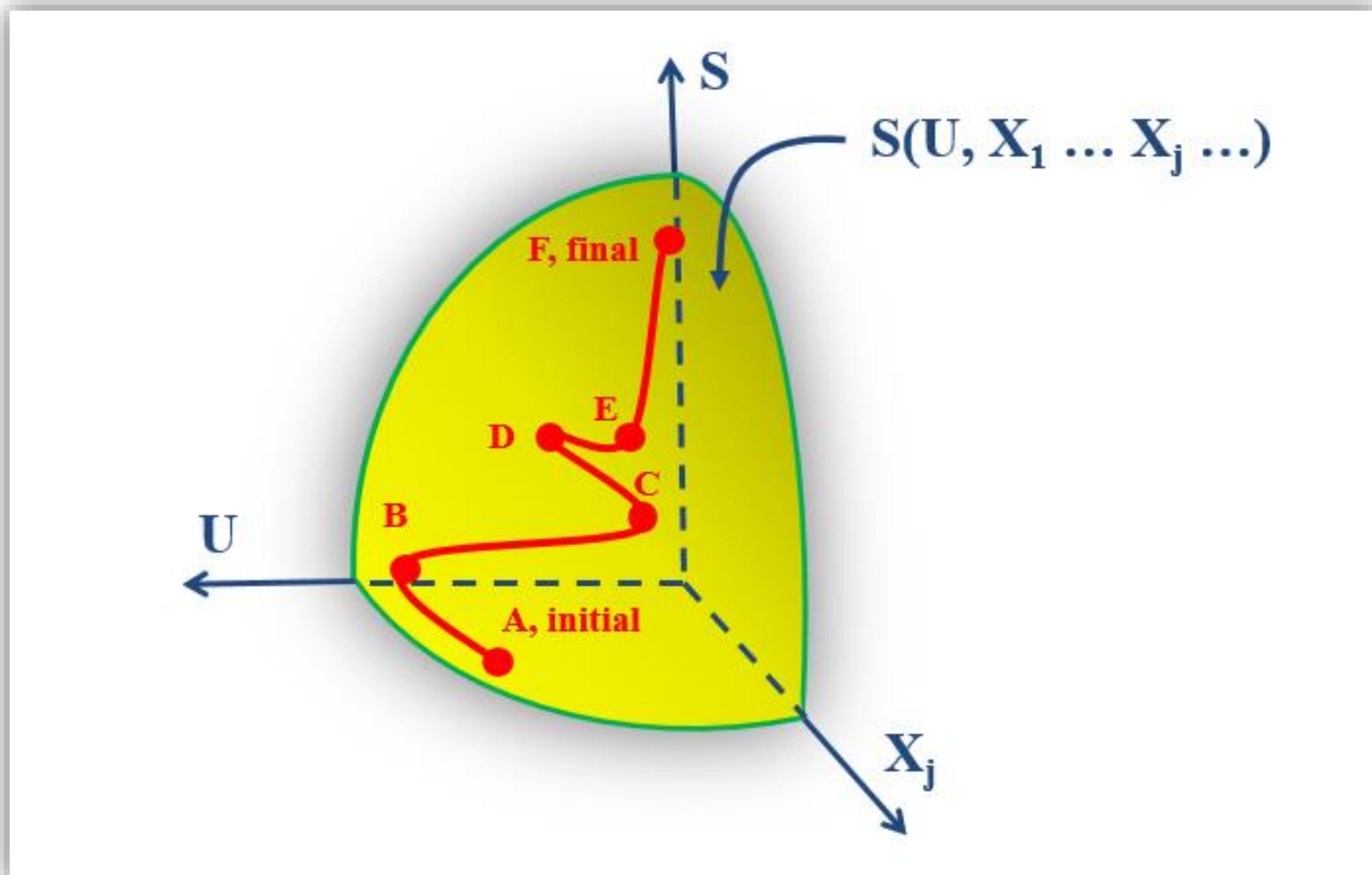
— Daly 1997 —

Matter, Thermodynamics & economy



Photosynthèse : Allégorie

Equilibrium thermodynamics: Callen approach



General structure

(adapted from Tania Sousa et al. Physica A 371 2006: 492-512)

- Let be an extensive function of state $Y(X_1 ... X_n)$
- Let be $z = 1...m$ constraints expressions $G_z = 0$. (in particuler the equations of state)
- Y is a function of the n variables X_i, which are extensive quantities.

$$L = Y + \sum_{z=1}^{m} \lambda_z G_z$$

At equilibrium, $\frac{\partial L}{\partial x_i} = 0$ and $\frac{\partial L}{\partial \lambda_z} = 0$
which is equivalent to

$$\pi_i = - \sum_{z=1}^{m} \lambda_z \frac{\partial G_z}{\partial X_i} \equiv \frac{\partial Y}{\partial X_i}$$

where π_i intensive quantities.
And the equations of state are $\pi_i = \pi_i(X_1 ... X_n)$ functions

Thermodynamics

- The Y function is the entropy $S = S(U, V, N)$
- The X_i variables are the classical extensive quantities U, V, N...
- The constraints $G_z = 0$ are the equations of state and the initial and boundary conditions

Then equilibrium there no gradient of any potential then, $\pi_i = \frac{\partial S}{\partial X_i} = \frac{1}{T}, \frac{P}{T}, -\frac{\mu}{T}$
$\Rightarrow$ Gibbs equation and Euler fundamental equation

$$dS = \frac{\partial S}{\partial U} dU + \frac{\partial S}{\partial V} dV + \frac{\partial S}{\partial N} dN + \mathbf{0}$$

$$S = S(U, V, N) = \frac{1}{T} U + \frac{P}{T} V - \frac{\mu}{T} N$$

+ the equation(s) of state: ex: perfect gas: $\frac{P}{T} = \frac{Nk}{V}$

Economics

- The Y function is the utility $Ut = Ut(X_1 ... X_n)$
- The X_i variables are the purchased goods.
- The prices *density* are purchased goods: p_i
- $\frac{\partial Ut}{\partial X_i}$: marginal utilities
- Constraints: state equation? + external constraints

Equilibrium $\frac{1}{p_1}\frac{\partial Ut}{\partial X_1} = \frac{1}{p_i}\frac{\partial Ut}{\partial X_i} ... = \frac{1}{p_n}\frac{\partial Ut}{\partial X_n}$

$\Rightarrow$ Gibbs equation and Euler fundamental equation[11]

$$dUt = \frac{\partial Ut}{\partial X_1} dX_1 + \frac{\partial Ut}{\partial X_2} dX_2 + ... + \frac{\partial Ut}{\partial X_n} dX_n + \mathbf{0}$$

$$Ut = \frac{\partial Ut}{\partial X_1} X_1 + \frac{\partial Ut}{\partial X_2} X_2 + ... + \frac{\partial Ut}{\partial X_n} X_n$$

Do marginal utilities $\frac{\partial Ut}{\partial X_1}$ can have the statut of potentials as $\frac{1}{T}, \frac{P}{T}, -\frac{\mu}{T}$ have!

Thermoelastic coefficients, elasticities...

The thermoelastic coefficients are properties of the thermodynamics "material" (fluid, condensed matter...):

$$C_P = T\left(\frac{\partial S}{\partial T}\right)_{P,N} \qquad C_V = T\left(\frac{\partial S}{\partial T}\right)_{V,N}$$

$$\kappa_T = -\frac{1}{V}\left(\frac{\partial V}{\partial P}\right)_{T,N} \qquad \alpha = \frac{1}{V}\left(\frac{\partial V}{\partial T}\right)_{P,N}$$

Economic elasticities follow from the same derivation: $\varepsilon_{ij} = \frac{\pi_j}{X_i}\left(\frac{\partial X_i}{\partial \pi_j}\right)_{X,\Pi}$

IF the state equations are known $\pi_i = \pi_i(X_1, X_2 ... X_n)$, so does the fundamental equation...then it becomes integrable. (Laurent is still there!)

- Integrability and fundamental function
- Maxwell relations
- Lechatelier-Braun theorem.

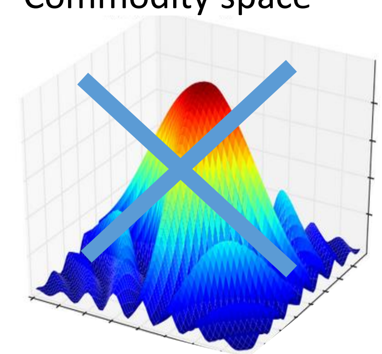
Is there an Economic Material ?

Partial conclusion: There may be economic materials

1. Economics do not have the structure of thermodynamics.
2. There is no problem with doing local optimization…until we don't claim it is GENERAL.
3. There is no fundamental equation for the economy, nor variational principle (hopefully)
4. There is no ergodicity.
5. T, P…are emerging properties, are there any in economics?

GENERAL EQUILIBRIUM

Commodity space

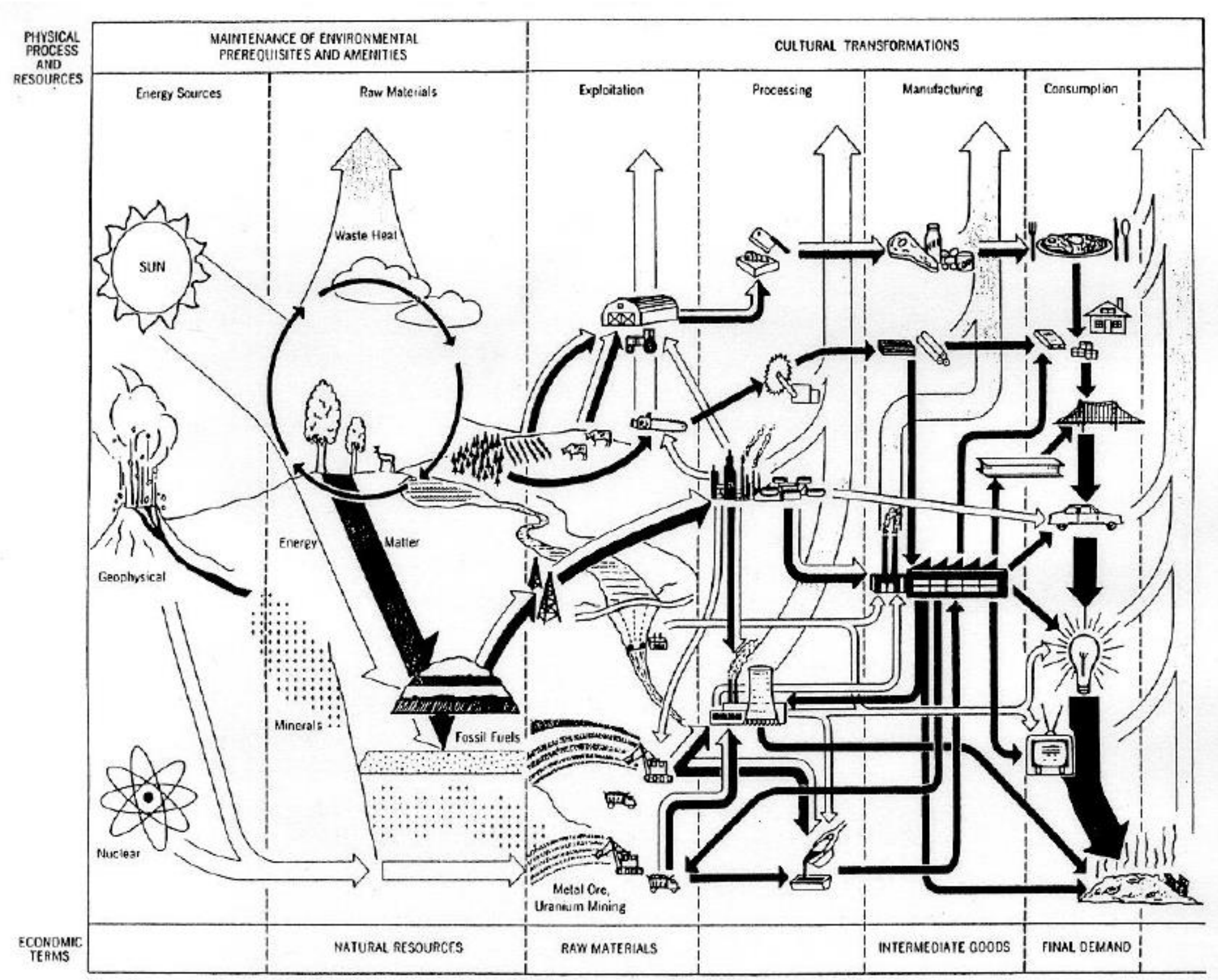


And….Economic systems are not equilibrium systems!

Can we extract some « Thermodynamic categories »

- **Conservation** laws (no full substitutability)
- **Non conservation** of some physical quantities (utility...)
- Difference between **extensive** (quantity: matter, energy) and **intensive** (quality) physical quantities
- Intensive physical quantities are the **potentials** from which we can derive forces·
- Intensive physical **quantities** are the potentials that give a possible measurement of **quality**·
- There is NO economic entropy but we can consider quantity and quality approach·
- Give up any full thermodynamic paradigm (Integrability, Equation of state...)
- Time does matter BUT, there is no variational principle in out-of-equilibrium systems

We almost all agree with this, but…



Hall et al. 2006

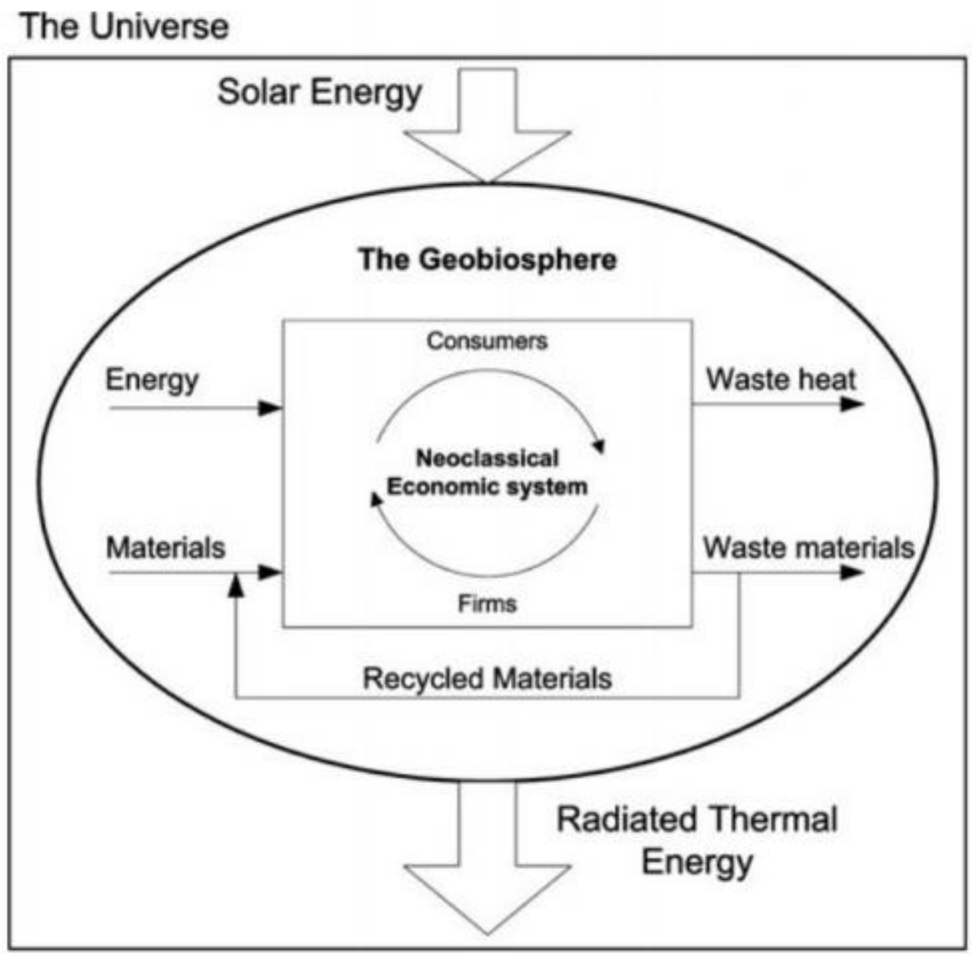
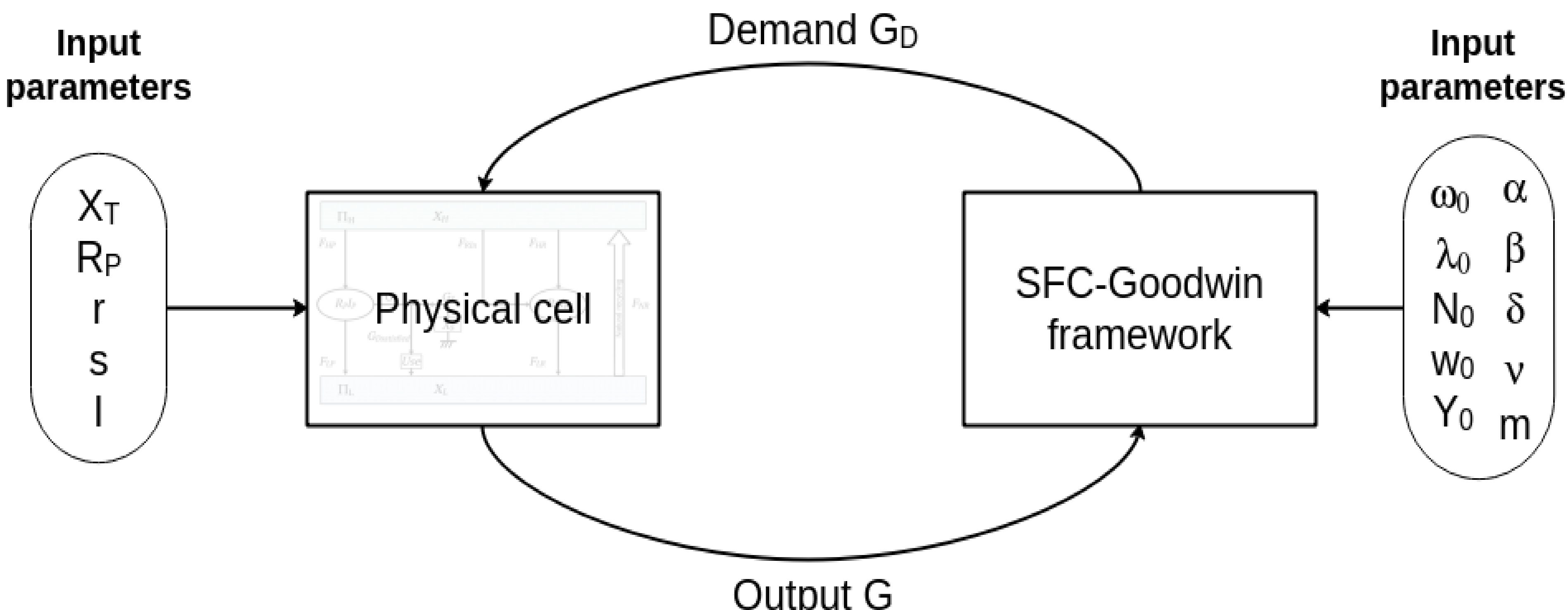


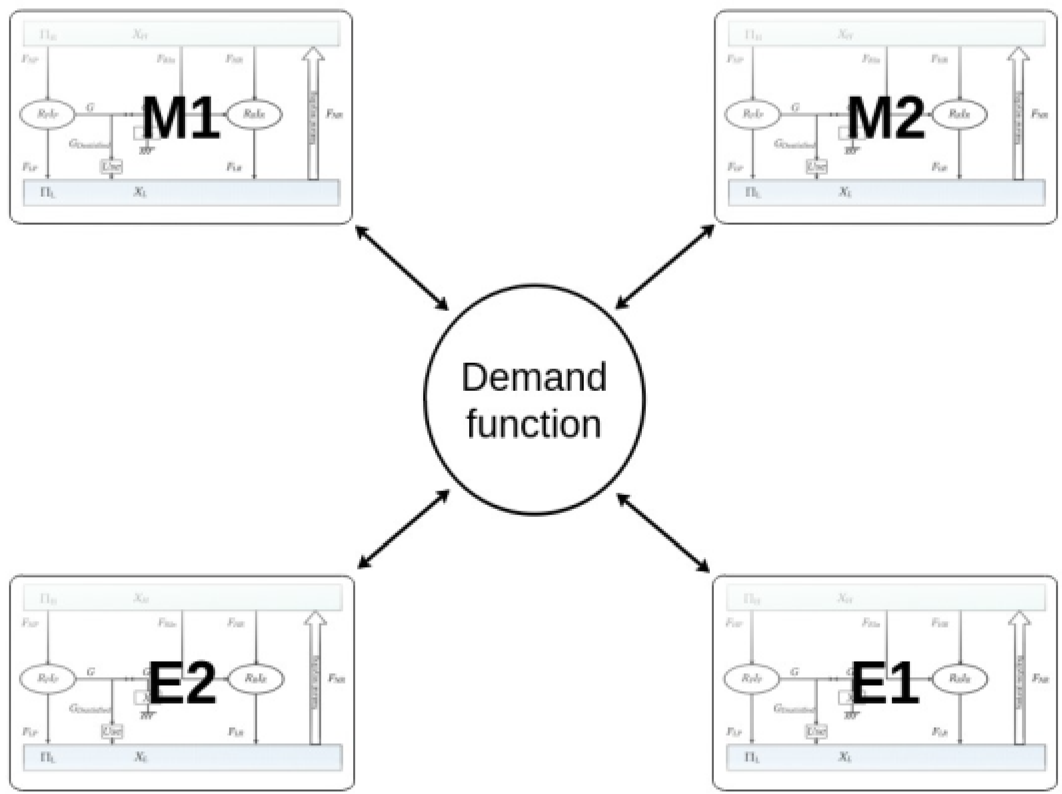
Figure 3. The ecological economic model, or preanalytic vision. In thermodynamic terms, the economy is an open system inside the Earth, which is a closed system. (Based on Hall *et al.*[99])

From *Connecting thermodynamics and economics*, Glucina and Mayumi — 2010 —

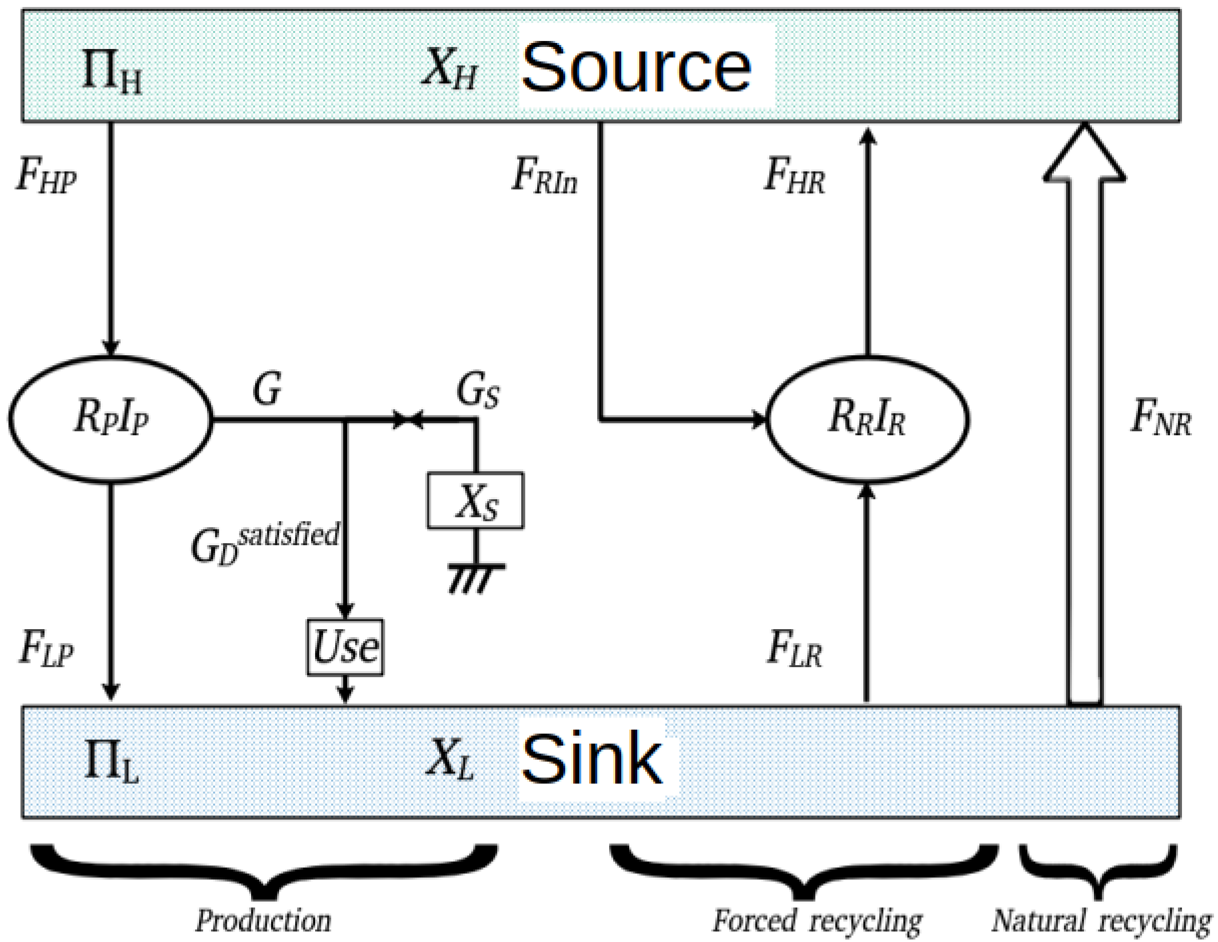
Distinction between Physical & Ecomonic cell



Physical Cells with four sector interconnected

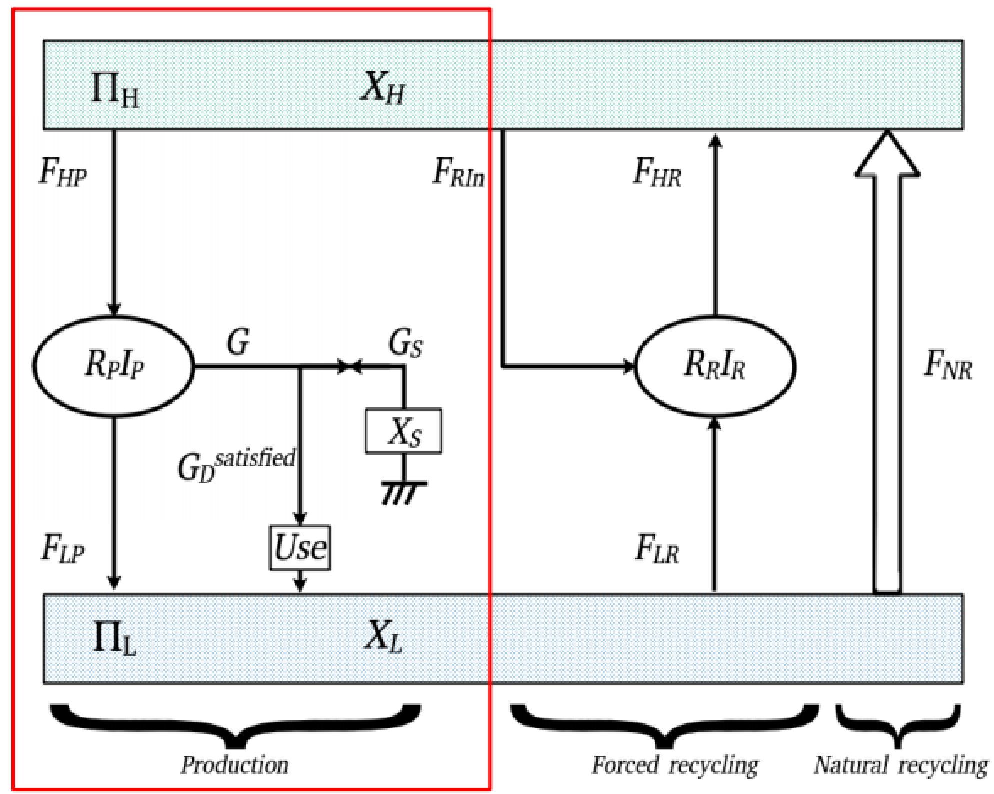


Global scheme of resource economy



Balance equations: production zone

$$
\begin{aligned}
X_T &= X_H + X_L + X_S \\
I_P &= \beta_P I \\
F_{HP} &= \Pi_H I_P \\
F_{LP} &= \Pi_L I_P + R_P I_P{}^2 \\
G &= F_{HP} - F_{LP} \\
G &= G_D^{\text{satisfied}} + G_S
\end{aligned}
$$



Balance equations: recycling zone

Forced recycling

$I_R = n_R\, I$

$F_{HR} = \Pi_H\, I_R$

$F_{LR} = \Pi_L\, I_R - R_R\, {I_R}^2$

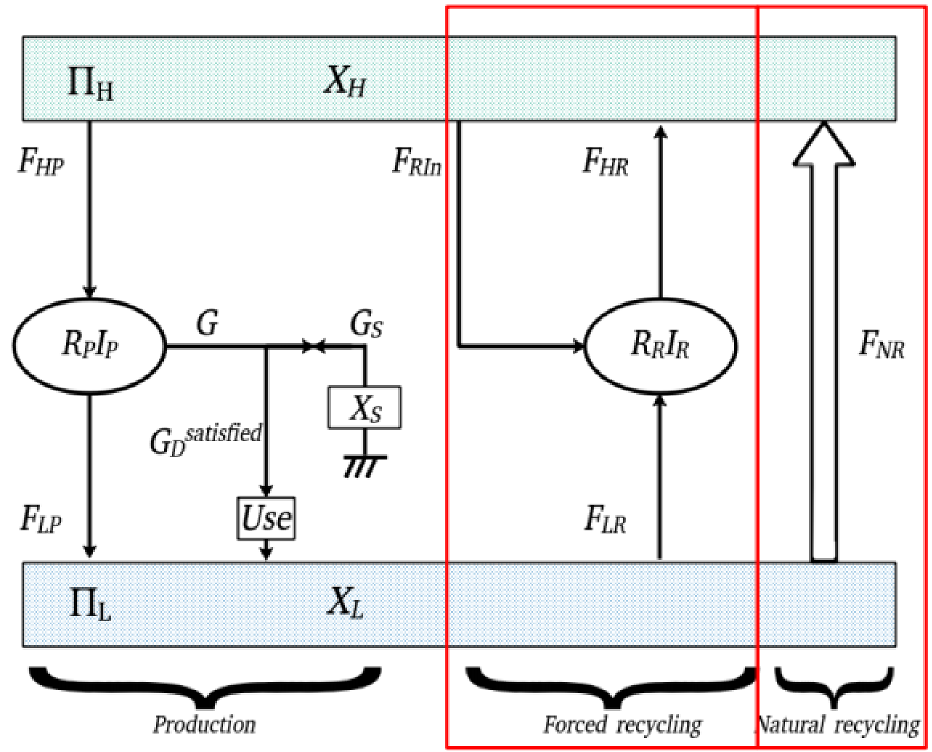
$F_{RIn} = F_{HR} - F_{LR}$

Natural recycling

$F_{NR} = r\; X_H\;\; (1 - T_H)\;\; \left(\frac{T_H}{s} - 1\right)$

$T_H = \frac{X_H}{X_T}$

$r\;:$ regeneration rate



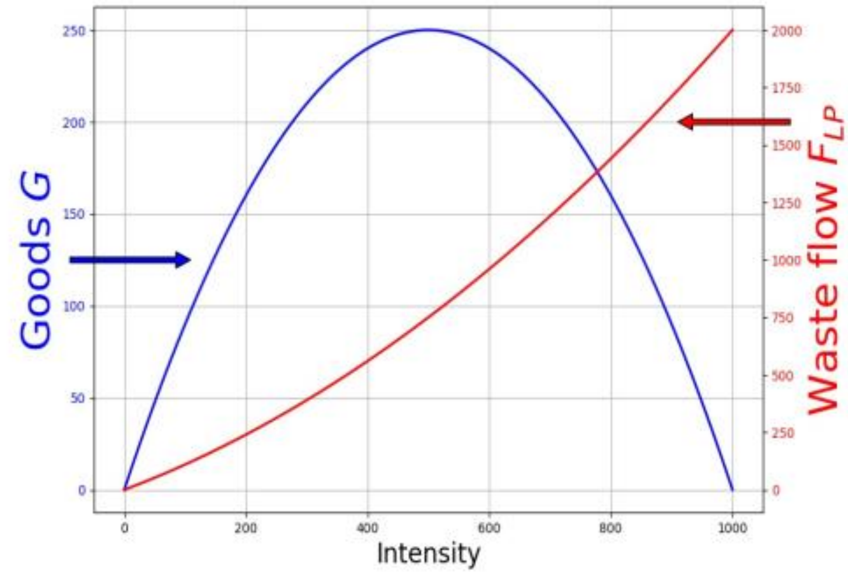
Case studies (no economic engine)

- *Demand is arb. chosen*
- *I* is chosen in order to satisfy *G*

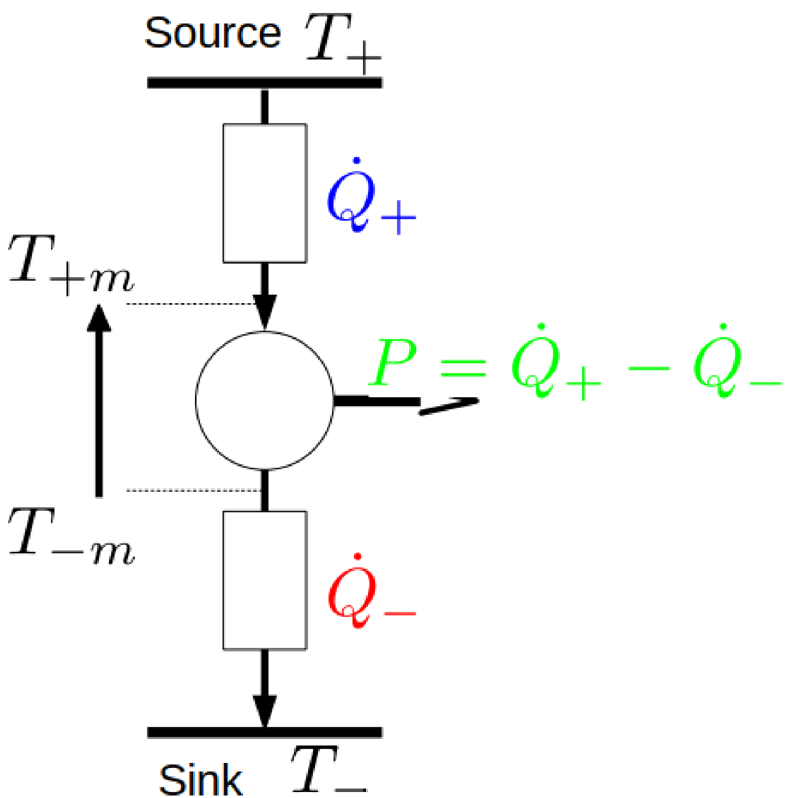
$$-R_P I^2 + \Delta\Pi I - G_D = 0$$

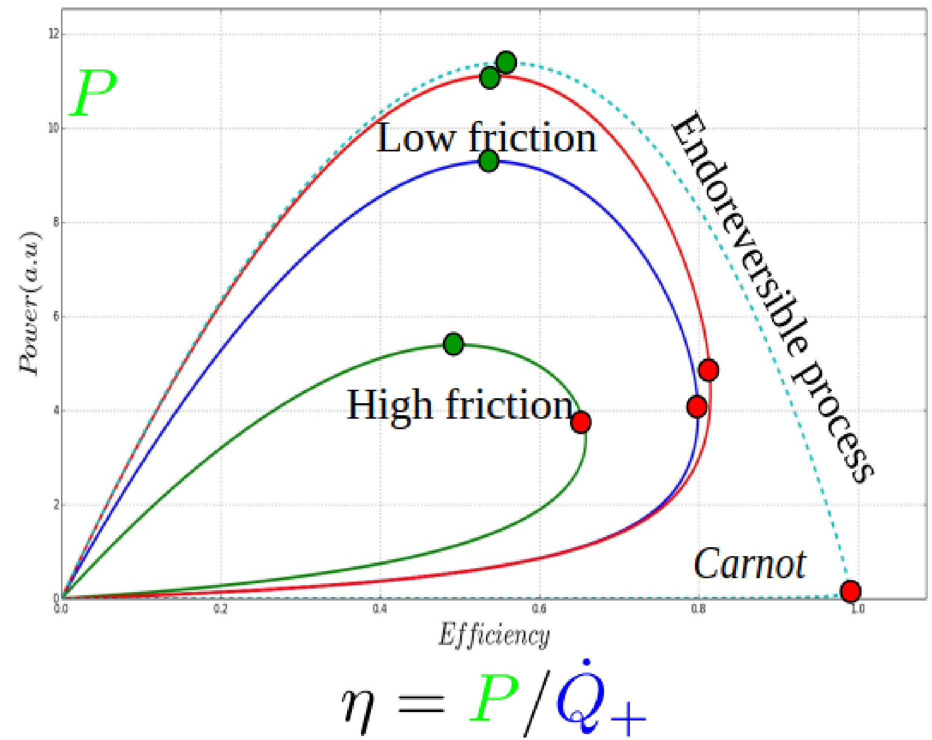
- Fixed forced recycling intensity
- Optimum intensity satisfies the demand with minimum waste
- Maximum quantity of goods possible to produce

$$I_P^{max} = \frac{\Delta\Pi}{2R_P}$$

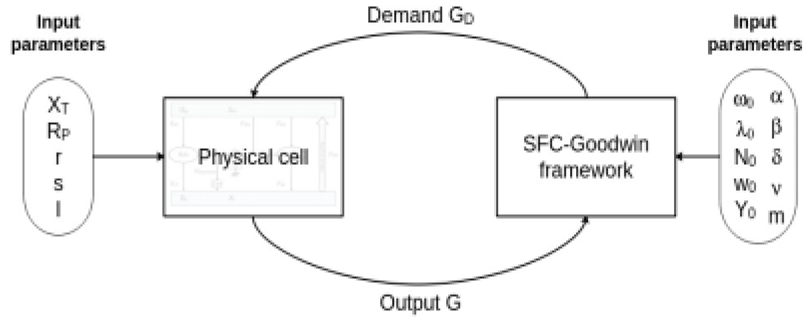


Energy conversion engine





Iterative procedure



- Resource is a global resource
- Starting from Y_0 the initial demand

$$G_D = Y_0$$

- The physical cell takes into account the demand and depend on resource, friction and recycling
- We obtain G_D
- We then try to reach $G=G_D$ by modifying *I* in

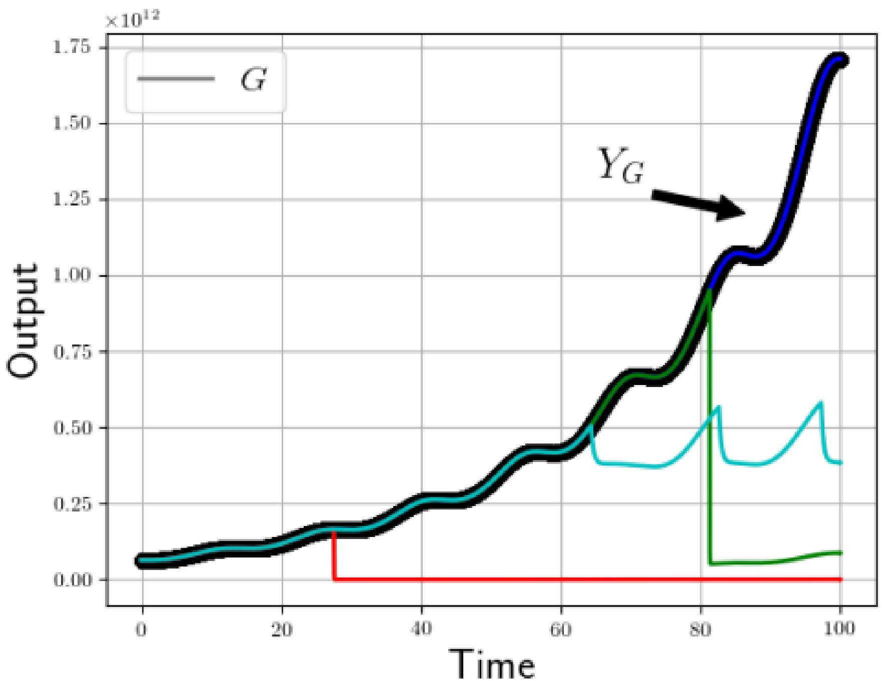
$$-R_P I^2 + \Delta\Pi I - G_D = 0$$

- Hence Intensity depends on G_D, R_P, and $\Delta\pi$

Five production scenarios

Scenario	**G**	1	2	3	4
X_T	-	$\mathbf{10^8}$	100.0	100.0	100.0
R_P	-	0.001	0.001	0.001	**0.1**
Features	Goodwin pathway	Infinite resources	Standard example	No Forced recycling	High friction

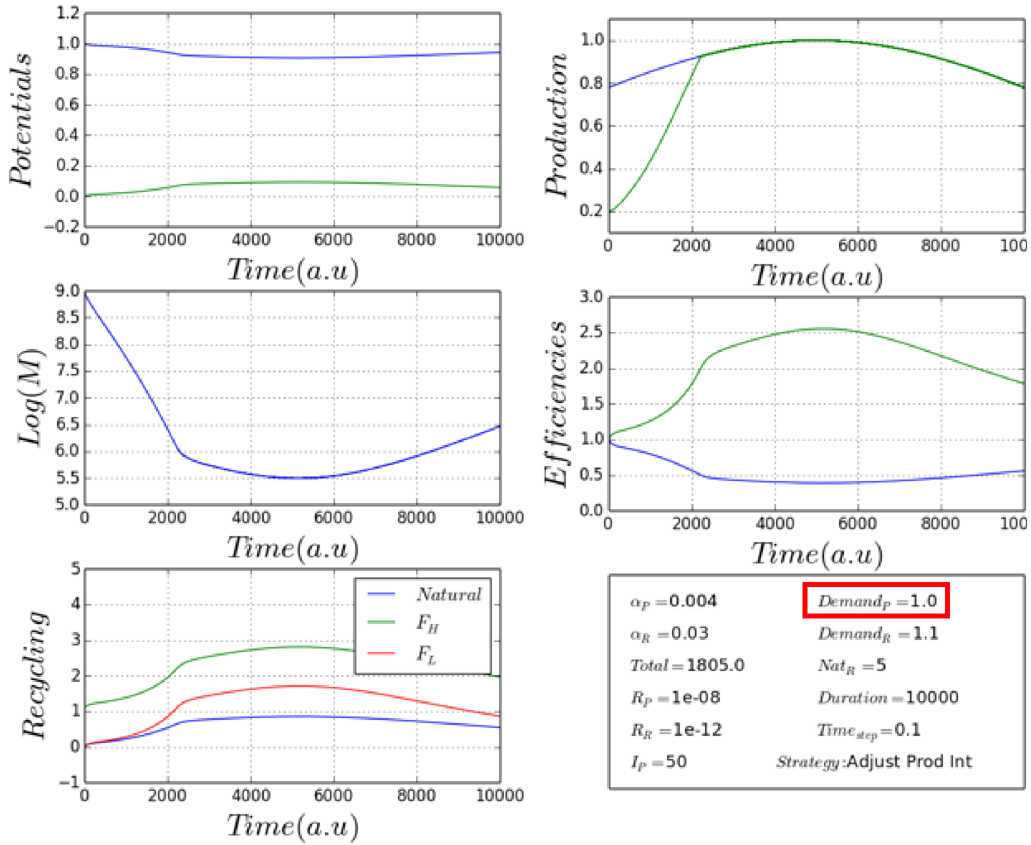
Production of output

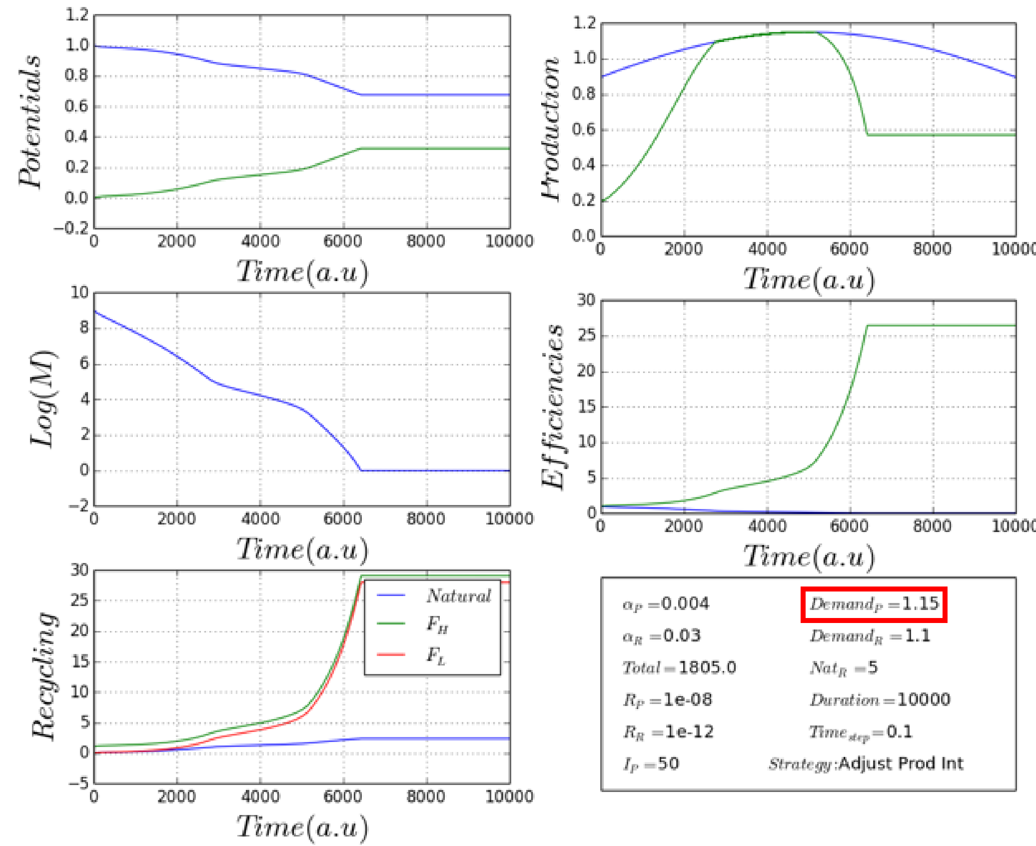


- Goodwin = infinite resources
- Finite resources => collapse
- Natural recycling => early collapse + low output
- High friction => output saturation then oscillation (due to capital dependency of Imax) higher than with lower Rp !

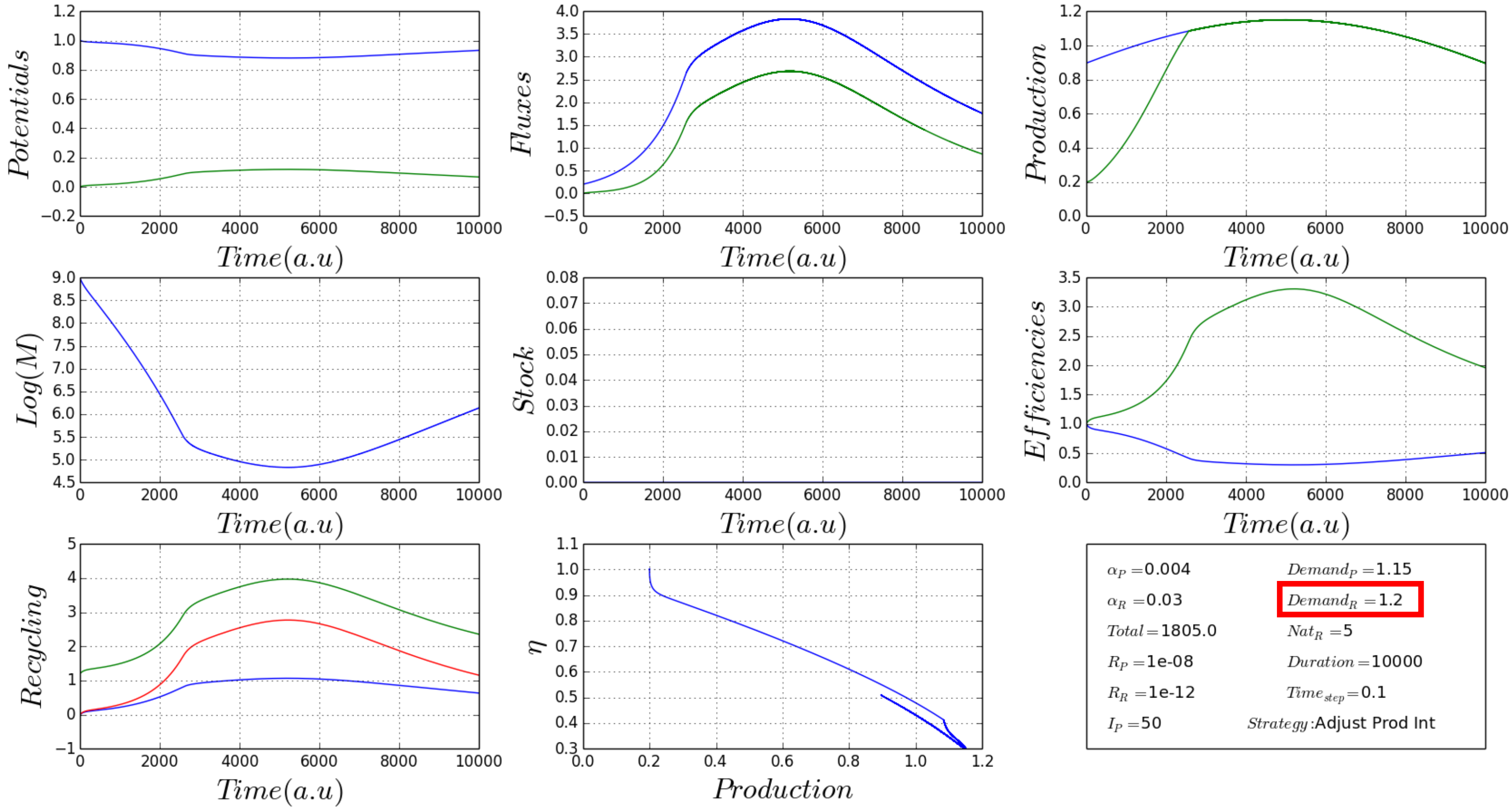
Scenario	**G**	1	2	3	4
Features	Goodwin pathway	Infinite resources	Standard example	No Forced recycling	High friction

Physical exemple: low level recycling





Physical exemple: high level recycling



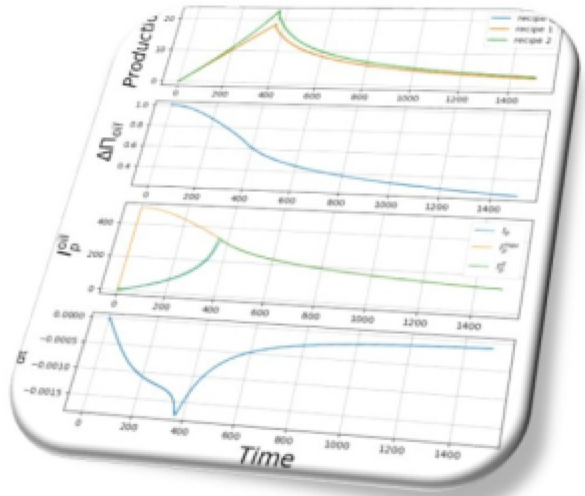
Conclusion

https://github.com/dyco-uparis/EcoDyco

https://arxiv.org/abs/2204.02038

- Clearly separate physical / economical spheres
- Built on quantity / quality
- Built on a modular structure
- Allows for an arbitrary number of resources
- Agnostic concerning the economic engine

  => calibration on a realistic system.



The worst in an interdisciplinary discussion

If you should want to continue this strand of research (which I would find highly worthwhile), I recommend that you look at **what has already been done** on how thermodynamics constrains economic activity, make a clearly defined and targeted contribution (rather than; making broad and sweeping statements, claiming that you have a **theory of everything**), and that in communicating this contribution you write it up for the people to whom you want to make this contribution (rather than: writing a physics paper in physics jargon and style, and then submitting it to a non-physics journal).

Stefan Baumgärtner

Editor-in-Chief, Ecological Economics, january 2023