On QED corrections to $\bar{B} \to \bar{K}\ell^+\ell^-$ and R_K: Theory vs Experiment

Flavour day at IJCLab 2023

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IJCLab

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2009.00929 [G.Isidori, SN, R.Zwicky]

2205.08635 [D.Lancierini, G.Isidori, SN, R.Zwicky]

Lepton Flavour Universality (LFU) predicted by SM.

One can thus define *lepton flavour universality* ratios, such as R_K:

$$R_K\left[q^2_{\min}, q^2_{\max}\right] = \frac{\int_{q^2_{\min}}^{q^2_{\max}} dq^2 \frac{d\Gamma\left(B\to K\mu^+\mu^-\right)}{dq^2}}{\int_{q^2_{\min}}^{q^2_{\max}} dq^2 \frac{d\Gamma(B\to Ke^+e^-)}{dq^2}},$$

where $q^2 = (\ell^+ + \ell^-)^2$.

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where $q^2 = (\ell^+ + \ell^-)^2$.

Naively expect $R_K = 1 + \mathcal{O}(\frac{\alpha}{\pi})$. LHCb reports [2212.09152]

$$R_K\left[1.1\mathrm{GeV}^2, 6\mathrm{GeV}^2\right] = 0.949^{+0.042+0.022}_{-0.041-0.022}.$$

Now in agreement with SM! (from previous 3.1σ deviation)

QED corrections are expected to be small, since $\frac{\alpha}{\pi} \approx 2{\cdot}10^{-3}$.

Due to kinematic effects however, QED corrections are enhanced to $\mathcal{O}(\frac{\alpha}{\pi}) \ln \hat{m}_\ell \gtrsim 2 - 3\,\%$ [*Note:* $\hat{m}_\ell \equiv \frac{m_\ell}{m_B}$].

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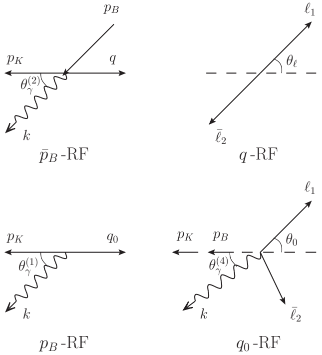
Also, important for the precise determination of CKM matrix elements.



$$\{q_a^2, c_a\} = \begin{cases} q_\ell^2 = (\ell_1 + \ell_2)^2, \qquad c_\ell = -\left(\frac{\vec{\ell_1}\cdot\vec{p}_K}{|\vec{\ell_1}||\vec{p}_K|}\right)_{q-\mathrm{RF}} \qquad [\text{“Hadron collider”}] \ , \\ q_0^2 = (p_B - p_K)^2 \ , \quad c_0 = -\left(\frac{\vec{\ell_1}\cdot\vec{p}_K}{|\vec{\ell_1}||\vec{p}_K|}\right)_{q_0-\mathrm{RF}} \qquad [\text{“B-factory”}] \ , \end{cases}$$

where $q - \mathrm{RF}$ and $q_0 - \mathrm{RF}$ denotes the rest frames of $q \equiv \ell_1 + \ell_2$ and $q_0 \equiv p_B - p_K = q + k$ respectively.

For the *real contribution* to the differential rate, we implement a *physical cut-off on the photon energy* (based on the visible kinematics),

$$\bar{p}_B^2 \equiv {m_B}_{\mathrm{rec}}^2 = (p_B - k)^2 = (\ell_1 + \ell_2 + p_K)^2 \ ,$$

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For the *virtual contribution*, since there is *no photon-emission*, there is no difference between the $\{q^2, c_\ell\}$- and $\{q_0^2, c_0\}$-variables.

IR Divergences

To isolate the IR divergences, we employ the two cut-off *phase space slicing method* [Harris, Owens '01].

We find that

- All soft divergences cancel between real and virtual, independent of the choice of differential variables.
- All hard-collinear divergences (ie. ln $\hat{m}_\ell$ sensitive terms) cancel in the photon-inclusive case AND in the differential variables $\{q_0^2, c_0\}$ (*IR-safe variables*).

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- hc divergences survive in the differential variables $\{q^2, c_\ell\}$, even in the photon-inclusive case.
- hc divergences never cancel as soon as one introduces a cut $\delta_{\rm ex}$ on the photon energy.

Q: Do we miss any $\ln \hat{m}_\ell$ contributions due to structure dependence, by performing an EFT calculation?

A: No, gauge invariance ensures that there are no such additional contributions [Sec. 3.4 in 2009.00929].

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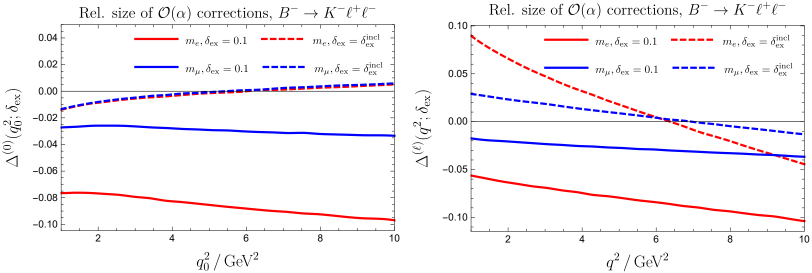
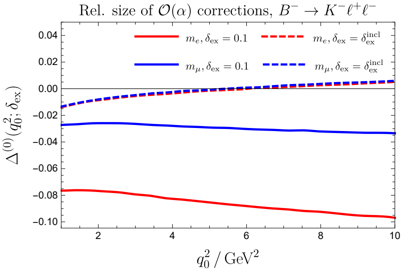
A: No, gauge invariance ensures that there are no such additional contributions [Sec. 3.4 in 2009.00929].

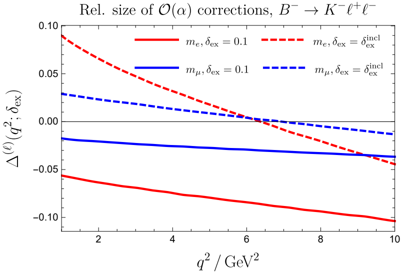
- However, using the EFT analysis, we do not capture *all* of the $\ln \hat{m}_K$ effects, which are a-priori not so small.
- *Structure Dependent Contributions: LCSR approach* [Ongoing].
- See 2209.06925 [SN, R.Zwicky] for the implementation of a charged gauge-invariant interpolating operator.

Results
$B^- \to K^- \ell^+ \ell^-$ in q_a^2

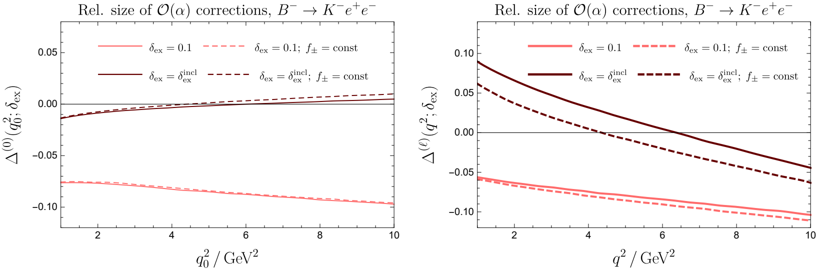


- In photon-inclusive case ($\delta_{ex} = \delta_{ex}^{inc}$, dashed lines), all IR sensitive terms cancel in the q_0^2 variable locally.
- (Approximate) lepton universality on the plots on the left.



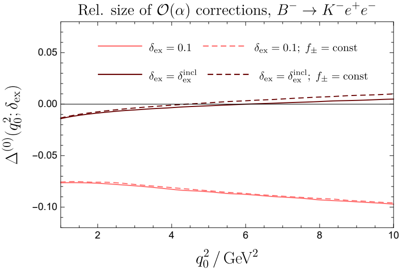


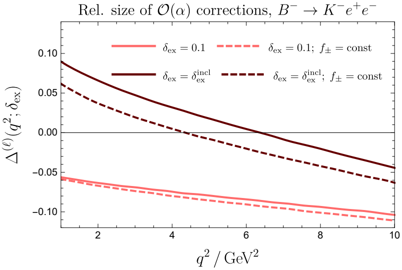
- In photon-inclusive case ($\delta_{\rm ex} = \delta_{\rm ex}^{\rm inc}$, dashed lines), all IR sensitive terms cancel in the q_0^2 variable locally.
- (Approximate) lepton universality on the plots on the left.
- $\delta_{\rm ex}$ effects are sizeable since hard-collinear logs do not cancel in that case. More pronounced for electrons.
- In charged case, we see finite effects of the $\mathcal{O}(2\%)$ due to $\ln \hat{m}_K$ effects which do not cancel.



- Effects are more prominent in the photon-inclusive case ($\delta_{\rm ex} = \delta_{\rm ex}^{\rm inc}$) since there is more phase space for the q^2- and q_0^2-variables to differ.
- *In fact, a fixed q^2 probes the full range of q_0^2 in that case!!*

Results
Distortion of the $\bar{B} \to \bar{K}\ell^+\ell^-$ spectrum





- Effects are more prominent in the photon-inclusive case $(\delta_{\text{ex}} = \delta_{\text{ex}}^{\text{inc}})$ since there is more phase space for the q^2- and q_0^2-variables to differ.
- *In fact, a fixed q^2 probes the full range of q_0^2 in that case!!*
- *Could be problematic for probing R_K in $q^2 \in [1.1, 6]$ GeV^2 range, due to charmonium resonances!*

ℓ	$m_B^{\mathrm{rec}}[\,\mathrm{GeV}]$	δ_{ex}	$(q_0^2)_{\max}$
μ	5.175	0.0486	$q^2 + 1.36\ \mathrm{GeV}^2$
e	4.88	0.146	$q^2 + 4.07\ \mathrm{GeV}^2$

- $(q_0^2)_{\max} = q^2 + \delta_{\mathrm{ex}} m_B^2$ for zero angle between the photon and the radiating particle.
- Photon energy cut-off on the muon is tighter, so the migration of radiation effect is smaller.

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Thus for $q^2 = 6\,\text{GeV}^2$, in the electron case, the system probes the pole location of the first charmonium resonance, but not the second one:

$$m^2_{\Psi(2S)} \approx 13.6\,\text{GeV}^2 > (q_0^2)_{\rm max} > m^2_{J/\Psi} \approx 9.58\,\text{GeV}^2.$$

Based on 2205.08635 [D.Lancierini, G.Isidori, SN, R.Zwicky]

- MC normalised so that the total rate (combining 3-body and 4-body events) when fully photon inclusive, *integrated in a bin of q_0^2*, is equal to the LO rate *(different from previous plots!)*.
- Excellent approximation (checked explicitly), since all log-sensitive terms cancel in that case.

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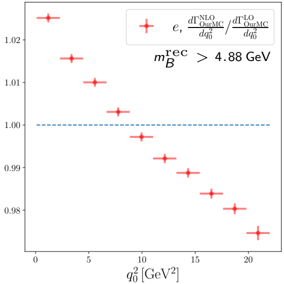
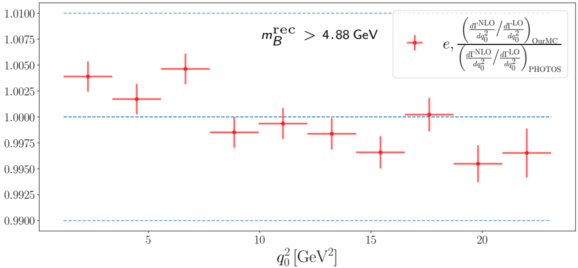
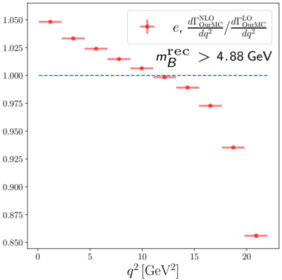
- Excellent approximation (checked explicitly), since all log-sensitive terms cancel in that case.
- Focus on neutral meson case. Full form factor used (*Ball-Zwicky parameterisation*), rather than an expansion.
- Photon energy cuts implemented via m_B^{rec}, 4.88 GeV for electrons, and 5.175 GeV for muons.

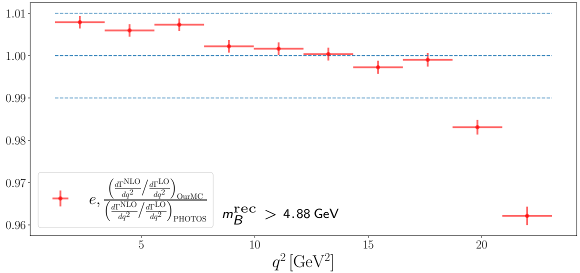




- NLO includes the tree level contributions, unlike in previous plots.
- Excellent agreement with PHOTOS.

Comparison with PHOTOS
Results: Distributions in q^2 (electron case)





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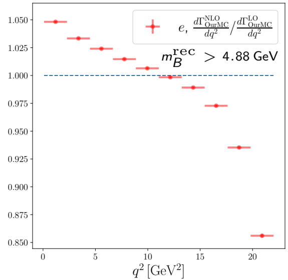

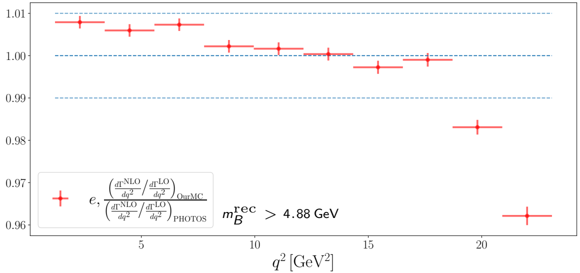

- No problem in the low q^2 region, relevant for R_K.
- At high q^2, disagreements of the order of $3 - 4\%$ observed.
- Can be explained by fixed order result (Our MC) vs resummed soft logs in PHOTOS, which are more pronounced at the end-point.

Charmonium resonances implemented through

$$C_9^{\text{eff}}(q^2) = C_9 + \Delta C_9(q^2)\ ,$$

$$\Delta C_9(q^2) = \Delta C_9(0) + \eta_{J/\psi} e^{i\delta_{J/\psi}} \frac{q^2}{m_{J/\psi}^2} \frac{m_{J/\psi}\Gamma_{J/\psi}}{\left(m_{J/\psi}^2 - q^2\right) - i m_{J/\psi}\Gamma_{J/\psi}}\ ,$$

using single-subtracted dispersion relation (at $q^2 = 0$).

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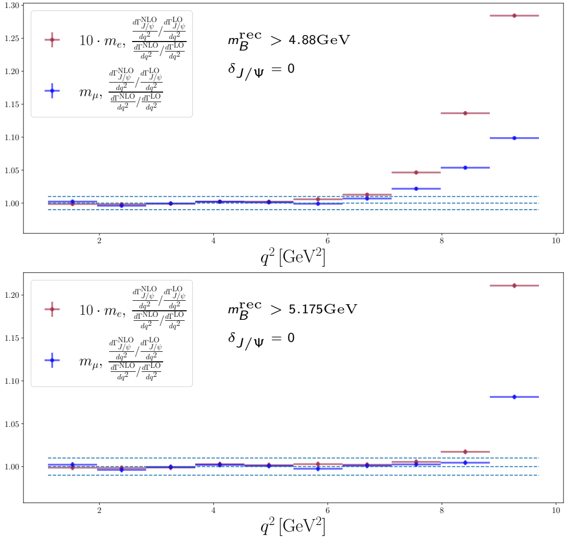
using single-subtracted dispersion relation (at $q^2 = 0$).

- Only interference between rare mode and resonant mode included in the MC study.
- Because of sampling efficiency, replace electron by a lepton with mass of $10\, m_e$.
- $\eta_{J/\psi}$ fixed by using the measured values of the branching fractions $\mathcal{B}(\bar{B} \to \bar{K} J/\psi)$ and $\mathcal{B}(J/\psi \to \mu^+\mu^-)$.

Effect of charmonium resonances

Results: Distributions in q^2 with $\delta_{J/\psi} = 0$ (maximal interference)



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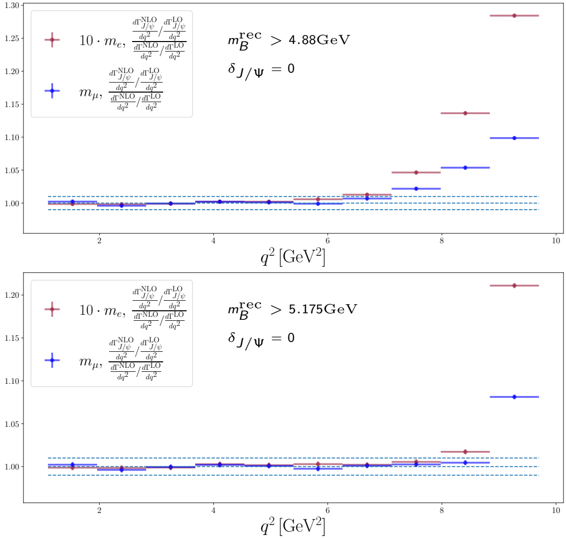

- Only interference effects considered.
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- The interference effect is more pronounced as the SD- and J/Ψ-contribution are not out of phase.
- minimal effect on the $q^2 \in [1.1, 6]\,\mathrm{GeV}^2$ bin.

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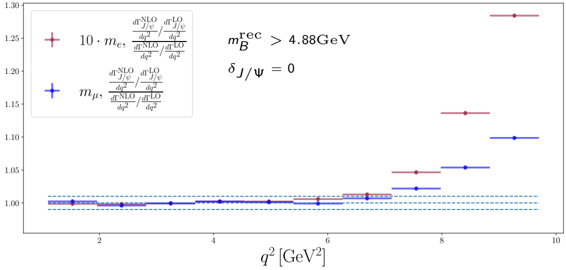

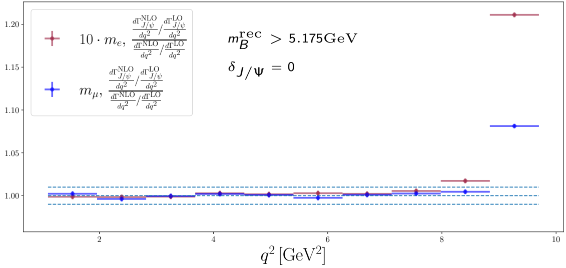

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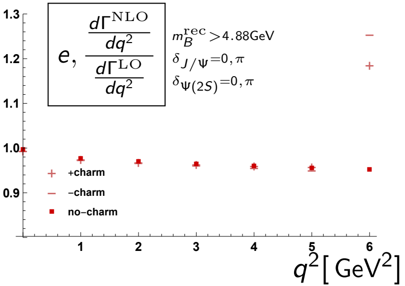
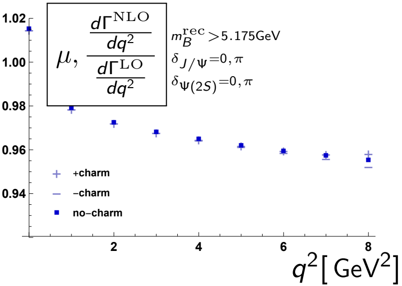
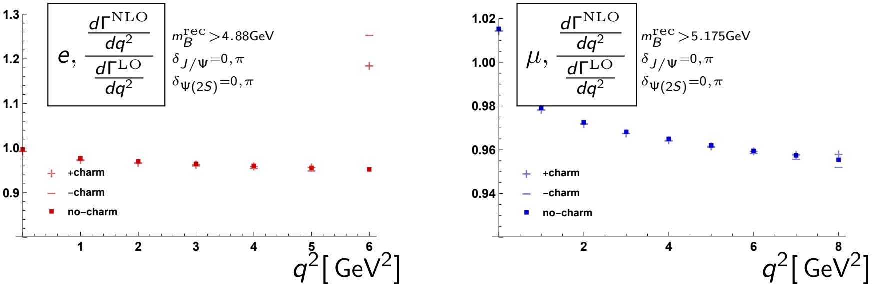
$\Longrightarrow$ *R_K safe wrt interference between LD and SD amplitudes!*





- In the semi-analytic approach (using the splitting function), we include the contribution from the modulus squared part of the J/ψ resonance, as well as the $\psi(2S)$ resonance.

Effect of charmonium resonances:
Results (Semi-analytic)



- In the semi-analytic approach (using the splitting function), we include the contribution from the modulus squared part of the J/ψ resonance, as well as the $\psi(2S)$ resonance.
- With an electron-like photon energy cut-off, the peak of the J/ψ is probed at $q^2 = 6\,\text{GeV}^2$, due to migration of radiation effects.

Summary

Take-home messages

- EFT analysis captures all hard collinear logs ln m_ℓ. No further contribution from structure dependence.
- Our MC, based on EFT analysis, is consistent with PHOTOS.

- EFT analysis captures all hard collinear logs $\ln m_\ell$. No further contribution from structure dependence.
- Our MC, based on EFT analysis, is consistent with PHOTOS.
- R_K is safe as far as the interference effects of charmonium resonances is concerned.
  $\Longrightarrow$ *this also applies to other LFU ratios by extension.*

Outlook

- Fixing ambiguities in the UV counterterms, and structure-dependent corrections (including $\ln \hat{m}_K$ contributions) 2209.06925 [SN, R.Zwicky], [Ongoing].
- Analysis of moments of the angular distribution [Ongoing].

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- Analysis of moments of the angular distribution [Ongoing].
- Charged-current semileptonic decays ($\bar{B} \to D\ell\nu$). Unidentified neutrino in final state makes it hard to reconstruct B meson and to apply a cut-off on photon energy.

BACKUP SLIDES

Bordone et al. [1605.07633] already performed a calculation to estimate QED corrections in $\bar{B} \to \bar{K}^{(*)}\ell^+\ell^-$ and $R_{K^{(*)}}$, working in single differential in q^2.

In our work,

- Results at the *full (double)* differential level are given, and hence they can be used for angular analysis (moments). Moreover, knowledge of the lepton angles are necessary for *applying kinematical cuts* on the photon energy.
- We work with *full matrix elements*, starting from an *EFT Lagrangian description*. Hence, we can capture effects beyond collinear $\ln \hat{m}_\ell$ terms, such as $\ln \hat{m}_K$ *(except structure dependent contributions)* which are not necessarily so small.
- We present a *detailed discussion on IR divergences*, and demonstrate explicitly the conditions under which they cancel.

We use an *EFT*, for $\bar{B}(p_B) \to \bar{K}(p_K)\,\ell^+(\ell_2)\,\ell^-(\ell_1)$.

$$\mathcal{L}_{\rm int}^{\rm EFT} = g_{\rm eff}\, L^\mu V_\mu^{\rm EFT} + {\rm h.c.}\ ,$$

$$V_\mu^{\rm EFT} = \sum_{n\geq 0} \frac{f_\pm^{(n)}(0)}{n!}(-D^2)^n[(D_\mu B^\dagger)K \mp B^\dagger(D_\mu K)]\ ,$$

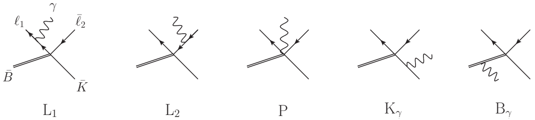
where D_μ is the QED covariant derivative and $f_\pm^{(n)}(0)$ denotes the $n^{\rm th}$ derivative of the *$B \to K$ form factor $f_\pm(q^2)$*.

$$\begin{aligned} H_0^\mu(q_0^2) \equiv \langle \bar{K}|V_\mu|\bar{B}\rangle &= f_+(q_0^2)(p_B+p_K)^\mu + f_-(q_0^2)(p_B-p_K)^\mu \\ &= \langle \bar{K}|V_\mu^{\rm EFT}|\bar{B}\rangle + \mathcal{O}(e)\ , \end{aligned}$$

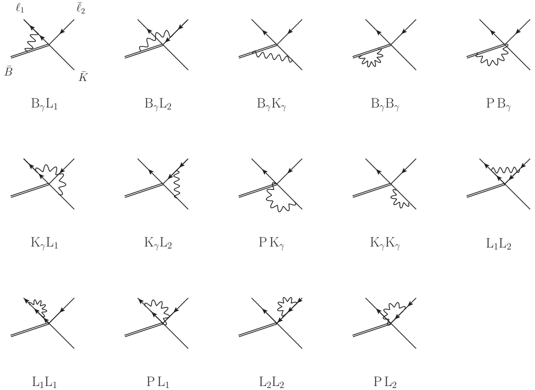
$$L_\mu \equiv \bar{\ell}_1\Gamma^\mu\ell_2\,, \quad V_\mu \equiv \bar{s}\gamma_\mu(1-\gamma_5)b\,,$$

$$g_{\rm eff} \equiv \frac{G_F}{\sqrt{2}}\lambda_{\rm CKM}, \qquad \Gamma^\mu \equiv \gamma^\mu(C_V + C_A\gamma_5)\ , \qquad C_{V(A)} = \alpha\frac{C_{9(10)}}{2\pi}\ .$$

Real Amplitudes:



Virtual Amplitudes:



$\Longrightarrow$ *Explicit gauge invariance*

IR Divergences

The real integrals are split into *IR sensitive parts* which can be done *analytically* and a necessarily regular part which is dealt with numerically.

$$\mathcal{F}_{ij}^{(a)}(\delta_{\rm ex}) = \frac{d^2\Gamma^{\rm LO}}{dq^2 dc_\ell}\tilde{\mathcal{F}}_{ij}^{(s)}(\omega_s) + \tilde{\mathcal{F}}_{ij}^{(hc)(a)}(\underline{\delta}) + \Delta\mathcal{F}_{ij}^{(a)}(\underline{\delta}) \,,$$

with $\tilde{\mathcal{F}}_{ij}^{(s)}$ ($\tilde{\mathcal{F}}_{ij}^{(hc)(a)}$) containing all *soft* (*hard-collinear*) singularities, whereas $\Delta\mathcal{F}$ is regular.

We adopt the *phase space slicing method*, which requires the introduction of two auxiliary (unphysical) cut-offs $\omega_{s,c}$,

$$\omega_s \ll 1 \,, \quad \frac{\omega_c}{\omega_s} \ll 1 \,.$$

[*Note: Hard-collinear* $\equiv \ln \hat{m}_\ell$ *sensitive terms.*]

Phase Space slicing conditions

$$\bar{p}_B^2 \geq m_B^2\,(1-\omega_s) \iff E_\gamma^{p_B-\mathrm{RF}} \leq \frac{\omega_s m_B}{2},$$

$$k\cdot\ell_{1,2} \leq \omega_c m_B^2 \;.$$

All soft divergences cancel between real and virtual, independent of the choice of differential variables.

In the collinear limit $(k||\ell_1)$, the matrix element squared factorises:

$$|\mathcal{A}^{(1)}_{\ell_1||\gamma}|^2 = \frac{e^2}{(k\cdot\ell_1)}\hat{Q}^2_{\ell_1}\tilde{P}_{f\to f\gamma}(z)|\mathcal{A}^{(0)}(q_0^2, c_0)|^2 + \mathcal{O}(m^2_{\ell_1}) \ ,$$

where $|\mathcal{A}^{(0)}(q_0^2, c_0)|^2 = |\mathcal{A}^{(0)}_{\bar{B}\to\bar{K}\ell_{1\gamma}\bar{\ell}_2}|^2$ and $\tilde{P}_{f\to f\gamma}(z)$ is the collinear part of the splitting function for a fermion to a photon

$$\tilde{P}_{f\to f\gamma}(z) \equiv \left(\frac{1+z^2}{1-z}\right) \ .$$

z gives the momentum fraction of the photon and lepton.

$$\ell_1 = z\ell_{1\gamma}, \quad k = (1-z)\,\ell_{1\gamma} \ ,$$

which then implies

$$q^2 = zq_0^2 \ .$$

Lower limit on z integration: Depends on the cut-off δ_{ex}.

In $\{q_0^2, c_0\}$ variables, *when fully photon inclusive*,

$$\left.\frac{d^2\Gamma}{dq_0^2 dc_0}\right|_{\ln \hat{m}_{\ell_1}} = \frac{d^2\Gamma^{\,\mathrm{LO}}}{dq_0^2 dc_0}\left(\frac{\alpha}{\pi}\right)\hat{Q}_{\ell_1}^2 \ln \hat{m}_{\ell_1} \times C_{\ell_1}^{(0)} \ ,$$

where

$$C_{\ell_1}^{(0)} = \left[\frac{3}{2} + 2\ln \bar{z}(\omega_s)\right]_{\tilde{\mathcal{F}}^{(hc)}} + \left[-1 - 2\ln \bar{z}(\omega_s)\right]_{\tilde{\mathcal{F}}^{(s)}} + \left[\frac{3}{2} - 2\right]_{\tilde{\mathcal{H}}} = 0 \ .$$

On the other hand, in $\{q^2, c_\ell\}$ variables,

$$\left.\frac{d^2\Gamma}{dq^2 dc_\ell}\right|_{\mathrm{hc}} = \frac{\alpha}{\pi}\big(\hat{Q}_{\ell_1}^2 K_{\mathrm{hc}}(q^2, c_\ell)\ln \hat{m}_{\ell_1} + \hat{Q}_{\ell_2}^2 K_{\mathrm{hc}}(q^2, -c_\ell)\ln \hat{m}_{\ell_2}\big) \ ,$$

where $K_{\mathrm{hc}}(q^2, c_\ell)$ is a non-vanishing function.

After integration over q^2 and c_ℓ, the above vanishes.

However, with a cut-off δ_{ex}, collinear logs survive in both differential variables!

The real amplitude can be decomposed,

$$\mathcal{A}^{(1)} = \hat{Q}_{\ell_1} a_{\ell_1}^{(1)} + \delta\mathcal{A}^{(1)} ,$$

into a term $\hat{Q}_{\ell_1} a_{\ell_1}^{(1)}$ with all terms proportional to $\hat{Q}_{\ell_1}$, and the remainder $\delta\mathcal{A}^{(1)}$.

$$a_{\ell_1}^{(1)} = -e g_{\text{eff}} \bar{u}(\ell_1) \left[\frac{2\epsilon^* \cdot \ell_1 + \not{\epsilon}^* \not{k}}{2k \cdot \ell_1} \Gamma \cdot H_0(q_0^2) \right] v(\ell_2) ,$$

which contains all $1/(k \cdot \ell_1)$-terms.

The structure-dependence of this term is encoded in the form factor H_0.

The amplitude square is given by

$$\sum_{\text{pol}} |\mathcal{A}^{(1)}|^2 = \sum_{\text{pol}} |\delta\mathcal{A}^{(1)}|^2 - \hat{Q}_{\ell_1}^2 \sum_{\text{pol}} |a_{\ell_1}^{(1)}|^2 + 2\hat{Q}_{\ell_1} \mathrm{Re}[\sum_{\text{pol}} \mathcal{A}^{(1)} a_{\ell_1}^{(1)*}] \,,$$

where it will be important that $\mathcal{A}^{(1)}$ is gauge invariant.

The *first term* is manifestly free from hard-collinear logs $\ln m_{\ell_1}$.

We use *gauge invariance* and set $\xi = 1$ under which the polarisation sum

$$\sum_{\text{pol}} \epsilon_\mu^* \epsilon_\nu = (-g_{\mu\nu} + (1-\xi) k_\mu k_\nu / k^2) \to -g_{\mu\nu} \,,$$

collapses to the metric term only.

The *second term* evaluates to

$$\int d\Phi_\gamma \, \hat{Q}^2_{\ell_1} \sum_{\text{pol}} |a^{(1)}_{\ell_1}|^2 = \int d\Phi_\gamma \, \hat{Q}^2_{\ell_1} \frac{\mathcal{O}(m^2_{\ell_1}) + \mathcal{O}(k \cdot \ell_1)}{(k \cdot \ell_1)^2} = \mathcal{O}(1) \, \hat{Q}^2_{\ell_1} \ln m_{\ell_1}$$

where we used $k - \ell_1 = \mathcal{O}(m^2_{\ell_1})$, valid in the collinear region.

We now turn to the *third term*.

Using anticommutation relations, $k - \ell_1 = \mathcal{O}(m_{\ell_1}^2)$ in the collinear limit, and the EoMs, we rewrite $a_{\ell_1}^{(1)}$ as

$$a_{\ell_1}^{(1)} = -e g_{\text{eff}} \bar{u}(\ell_1) \left[\frac{4\epsilon^* \cdot \ell_1 + m_{\ell_1} \not{\epsilon}^*}{2k \cdot \ell_1} \Gamma \cdot H_0(q_0^2) \right] v(\ell_2) \, .$$

Gauge invariance $k \cdot \mathcal{A}^{(1)} = 0$ implies $\ell_1 \cdot \mathcal{A}^{(1)} = \mathcal{O}(m_{\ell_1}^2)$ in the collinear region.

Therefore, the first part of $a_{\ell_1}^{(1)}$ contributes to

$$\hat{Q}_{\ell_1}\mathrm{Re}[\sum_{\mathrm{pol}}\mathcal{A}^{(1)}a_{\ell_1}^{(1)*}] \to c_1\,\hat{Q}_{\ell_1}^2\,\frac{\mathcal{O}(m_{\ell_1}^2)}{(k\cdot\ell_1)^2} + c_2\,\hat{Q}_{\ell_1}\,\hat{Q}_X\,\frac{\mathcal{O}(m_{\ell_1}^2)}{(k\cdot\ell_1)}\ ,$$

where $X \in \{\bar{B}, \bar{K}, \bar{\ell}_2\}$.

The second part of $a_{\ell_1}^{(1)}$ contributes to

$$\hat{Q}_{\ell_1}\mathrm{Re}[\sum_{\mathrm{pol}}\mathcal{A}^{(1)}a_{\ell_1}^{(1)*}] \to c_1'\,\hat{Q}_{\ell_1}^2\,\frac{\mathcal{O}(m_{\ell_1}^2)}{(k\cdot\ell_1)^2} + c_2'\,\hat{Q}_{\ell_1}\,\hat{Q}_X\,\frac{\mathcal{O}(m_{\ell_1})}{(k\cdot\ell_1)}\ ,$$

Thus, using gauge invariance, one concludes that $\delta\mathcal{A}^{(1)}$ (indicated by terms $\propto \hat{Q}_X$ in the above) does not lead to collinear logs.

Results

We consider *relative* corrections. For a single differential in $\frac{d}{dq_a^2}$,

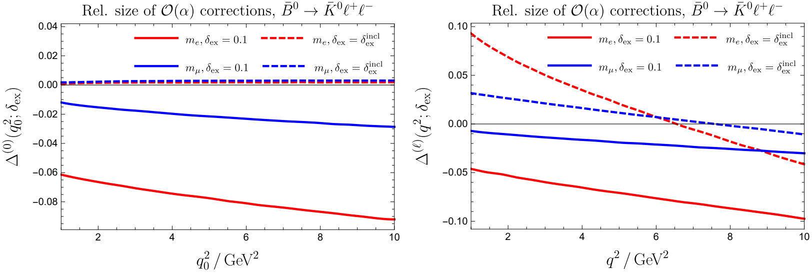
$$\Delta^{(a)}(q_a^2;\delta_{\rm ex}) = \left(\frac{d\Gamma^{\rm LO}}{dq_a^2}\right)^{-1} \frac{d\Gamma(\delta_{\rm ex})}{dq_a^2}\Bigg|_{\alpha},$$

where the numerator and denominator are integrated separately over $\int_{-1}^{1} dc_a$ respectively.

It is important to integrate the QED correction and the LO separately as this corresponds to the experimental situation.

QED corrections are taken into account in the experimental analysis by using PHOTOS.

$\Longrightarrow$ *Second part of my talk!*



- In photon-inclusive case ($\delta_{\rm ex} = \delta_{\rm ex}^{\rm inc}$, dashed lines), all IR sensitive terms cancel in the q_0^2 variable locally.
- (Approximate) lepton universality on the plots on the left.
- Effects due to the photon energy cuts are sizeable since hard-collinear logs do not cancel in that case. More pronounced for electrons.

We consider *relative* QED corrections. For a single differential in $\frac{d}{dq_a^2}$,

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where the numerator and denominator are integrated separately over $\int_{-1}^{1} dc_a$ respectively. In addition, we define the single differential in $\frac{d}{dc_a}$

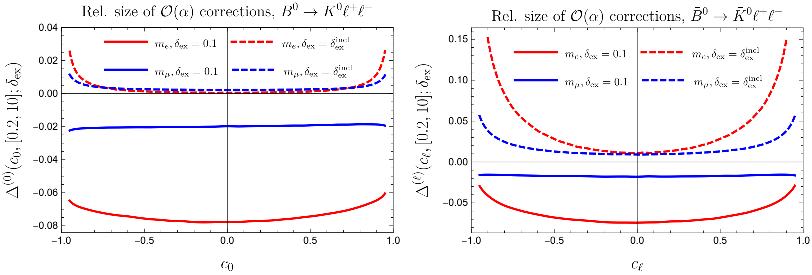
$$\Delta^{(a)}(c_a,[q_1^2,q_2^2];\delta_{\rm ex}) = \left(\int_{q_1^2}^{q_2^2} \frac{d^2\Gamma^{\rm LO}}{dq_a^2 dc_a} dq_a^2\right)^{-1} \int_{q_1^2}^{q_2^2} \frac{d^2\Gamma(\delta_{\rm ex})}{dq_a^2 dc_a} dq_a^2\Big|_\alpha \,,$$

where the non-angular variable is binned.

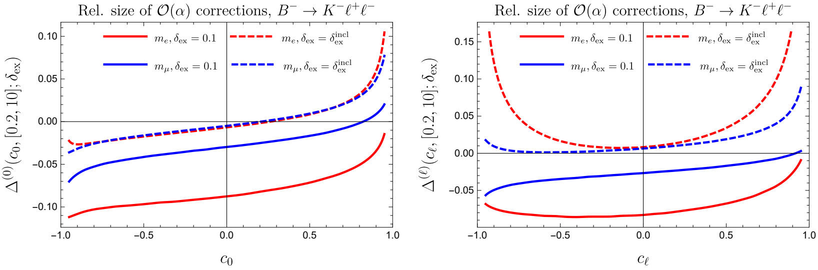
It is important to integrate the QED correction and the LO separately as this corresponds to the experimental situation.

Results

c_a distribution in neutral meson mode



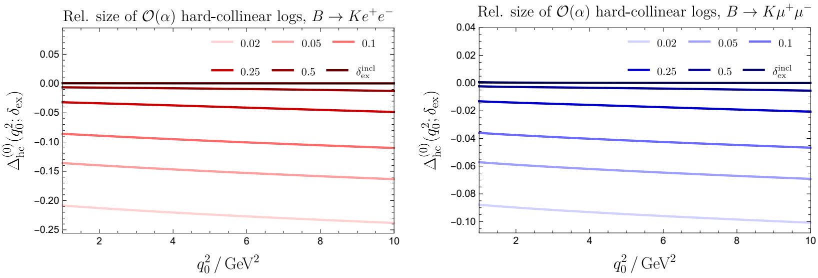
- Enhanced effect towards the endpoints $\{-1, 1\}$ is partly due to the special behaviour of the LO differential rate which behaves like $\propto (1 - c_\ell^2) + \mathcal{O}(m_\ell^2)$ and explains why the effect is less pronounced for muons.
- *Even in c_ℓ. Almost even in c_0 (up to non-collinear effects), since c_0 measured wrt to ℓ_1 in q_0-RF.*



- Same comments as before apply.
- More enhanced than the neutral meson case.

Results

Hard collinear $\ln \hat{m}_\ell$ contributions in q_a^2



- Cancellation of hc $\ln \hat{m}_\ell$ in fully inclusive case $(\delta_{\rm ex} = \delta_{\rm ex}^{\rm inc})$.
- Tighter cut $\implies$ larger corrections.
- Electron and muon cases are scaled by a factor $\approx \frac{\ln \hat{m}_e}{\ln \hat{m}_\mu} \approx 2.36$.

Tighter cut on electrons than muons $\implies$ Partial compensation $\implies$ QED corrections to R_K 'relatively' small.

To understand the distortion better, consider the following analysis in the collinear region:

$$|\mathcal{A}^{(0)}(q_0^2, c_0)|^2 \propto f_+(q_0^2)^2 = f_+(q^2/z)^2.$$

Since $z < 1$ in general, it is clear that momentum transfers of a higher range are probed.

For example, when $c_\ell = -1$, maximising the effect, one gets

$$z_{\delta_{\rm ex}}(q^2)\Big|_{c_\ell=-1} = \frac{q^2}{q^2 + \delta_{\rm ex} m_B^2} \ , \quad (q_0^2)_{\rm max} = q^2 + \delta_{\rm ex} m_B^2 \ ,$$

For $\delta_{\rm ex} = 0.15$, $q^2 = 6\,\text{GeV}^2$ one has $(q_0^2)_{\rm max} = 10.18\,\text{GeV}^2$.

$\Longrightarrow$ *Problematic for probing R_K in $q^2 \in [1.1, 6]$ GeV^2 range, due to charmonium resonances!*

Furthermore, in photon-inclusive case, the lower boundary for z becomes $z_{\rm inc}(c_\ell)|_{m_K \to 0} = \hat{q}^2$ such that $(q_0^2)_{\rm max} = m_B^2$.

$\Longrightarrow$ *Entire spectrum is probed for any fixed value of q^2.*

The net QED correction that should be applied to R_K according to our analysis amounts to

$$\Delta_{\mathrm{QED}} R_K \approx \left.\frac{\Delta\Gamma_{K\mu\mu}}{\Gamma_{K\mu\mu}}\right|_{q_0^2\in[1.1,6]\,\mathrm{GeV}^2}^{m_B^{\mathrm{rec}}=5.175\,\mathrm{GeV}} - \left.\frac{\Delta\Gamma_{Kee}}{\Gamma_{Kee}}\right|_{q_0^2\in[1.1,6]\,\mathrm{GeV}^2}^{m_B^{\mathrm{rec}}=4.88\,\mathrm{GeV}} \approx +1.7\%\ .$$

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$\Longrightarrow$ *Well below the current experimental error reported by LHCb.*

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However, effect of cuts can be significant. In Bordone et al. '16, in addition to the above energy cuts, a tight angle cut was also used, and they reported a correction to R_K of

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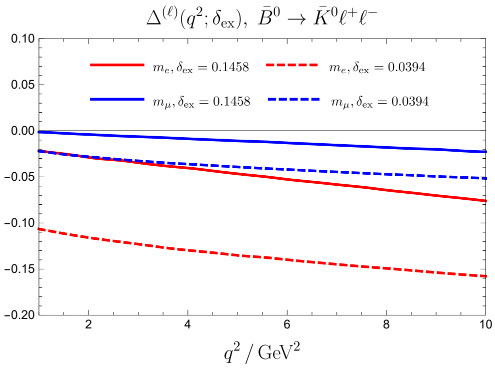
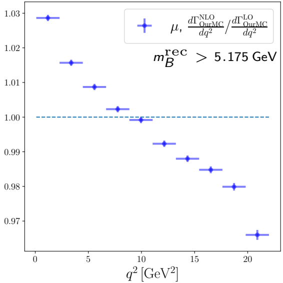
$\Longrightarrow$ *Highlights the importance of building a MC to cross-check the experimental analysis: PHOTOS.*

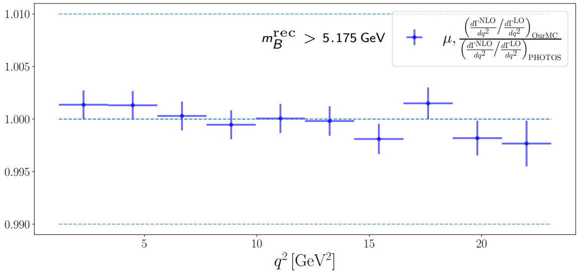


- The different photon energy cuts for the electron and the muon cases causes the shift in R_K due to QED corrections to be relatively low.

Comparison with PHOTOS

Preliminary Results: Distributions in q^2 (muon case)

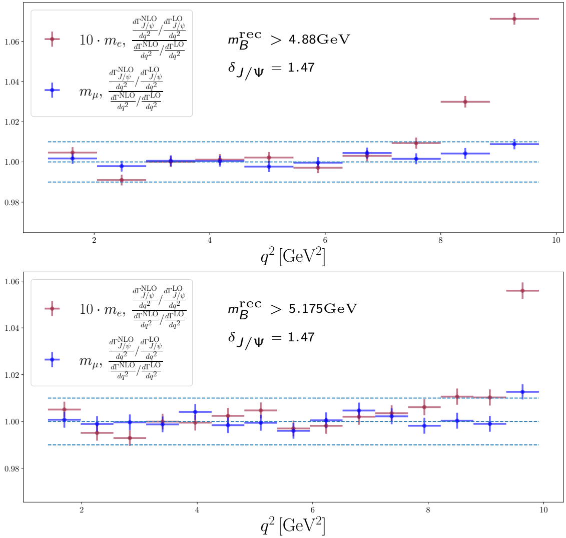




- Again, excellent agreement with PHOTOS here.
- A photon energy cut-off of $m_B^{rec} > 5.175\,\text{GeV}$ is used.

Effect of charmonium resonances

Preliminary Results: Distributions in q^2 with $\delta_{J/\psi} = 1.47$



- Only interference effects considered.
- Difference between $10 m_e$ and m_μ follows the expected $\ln m_\ell$ scaling.
- With $\delta_{J/\Psi} \approx \pi/2$, short distance and charmonium mode are out of phase.
- Minimal effect on the $q^2 \in [1, 6]\,\mathrm{GeV}^2$ bin.

Splitting function formalism

Focussing on collinear logs

Master equation for collinear divergences ($k||\ell_1$)

$$\Delta^{(\ell)}_{\rm hc}(\hat{q}_0^2, c_0) = \frac{\alpha}{\pi}\hat{Q}^2_{\ell_1}\left(\frac{d^2\Gamma^{\rm LO}}{d\hat{q}_0{}^2 dc_0}\right)^{-1}\left(\int_{z_{\ell_1}^{\delta_{\rm ex}}}^{1} dz P_{f\to f\gamma}(z)\frac{d^2\Gamma^{\rm LO}}{d\hat{q}_0{}^2 dc_0}\right)\ln\frac{\mu_{\rm hc}}{m_\ell} ,$$

where $\mu^2_{\rm hc} = \mathcal{O}(m_B^2) \approx 6q_0^2$, and

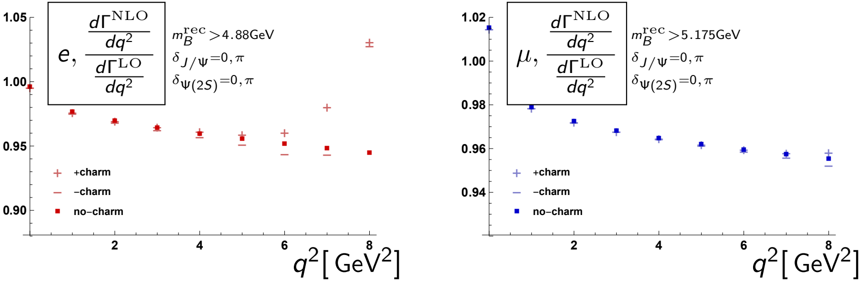
$$P_{f\to f\gamma}(z) \; = \; \lim_{z^*\to 0}\left[\frac{1+z^2}{(1-z)}\theta((1-z^*)-z) + (\frac{3}{2} + 2\ln z^*)\delta(1-z)\right] ,$$

is the splitting function of a fermion to a photon.

Recall: z is the momentum fraction of the photon-lepton system carries by the lepton ($q^2 = zq_0^2$).

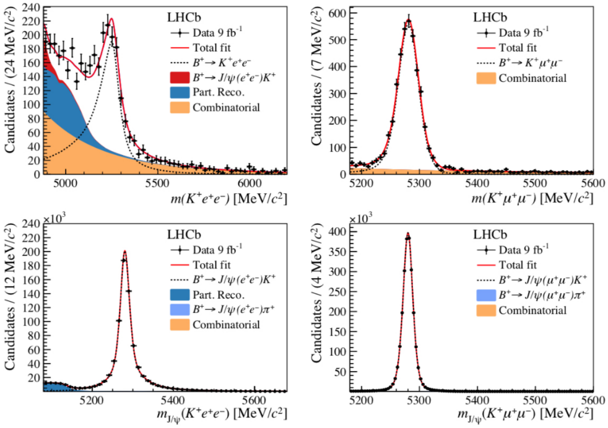
The differential rate factorises from the z-integration in the above variables.

Effect of charmonium resonances:
Results (Semi-analytic)



- In the semi-analytic approach (using the splitting function), we include the contribution from the modulus squared part of the J/ψ resonance, as well as the $\psi(2S)$ resonance.
- Peak of the resonance (only modulus squared part) eliminated through a window $\Delta\omega^2 = 0.1\,\text{GeV}^2$ around it.
- For $q^2 < 6\,\text{GeV}^2$, interference effects are small, even in the electron case.

LHCb plot



- Resonant mode has 10^3 more events than non-resonant mode.
- For the electron case, the non-resonant mode has contributions from $\bar{B} \to J/\psi(e^+e^-)\bar{K}$ due to QED, and loose photon energy cut.