QCD effects in $b \to s\ell\ell$

Journée de la Saveur 2023 – 02/06/2023

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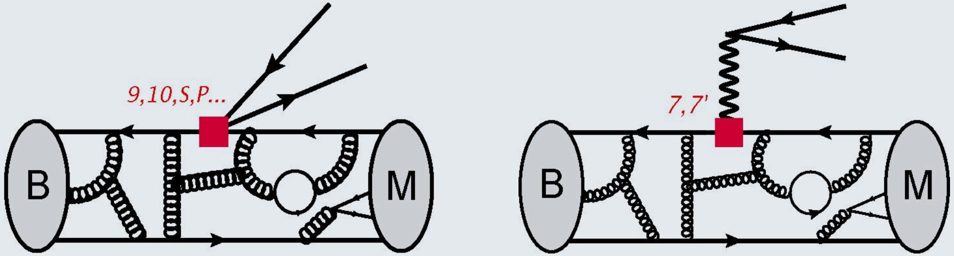
Based on Gubernari, Reboud, van Dyk, JV 2305.06301

Form factors in b → sℓℓ

$$\mathcal{H}(b \to s\ell\ell) = -\frac{4G_F}{\sqrt{2}} V_{tb}V_{ts}^* \sum_{i=1}^{10} C_i(\mu)\mathcal{O}_i(\mu)$$

$$\mathcal{O}_{9(10)} = \frac{e^2}{16\pi^2}\left(\bar{s}_L\gamma_\mu b_L\right)\left(\bar{\ell}\gamma^\mu(\gamma_5)\ell\right)$$

$$\mathcal{O}_7 = \frac{e}{16\pi^2}\left(\bar{s}_L\sigma_{\mu\nu}b_R\right)F^{\mu\nu}$$



$$\mathcal{A}_\lambda^{L,R}(B \to M_\lambda\ell\ell) = \mathcal{N}_\lambda\left\{(C_9 \mp C_{10})\mathcal{F}_\lambda(q^2) + \frac{2m_bM_B}{q^2}\left[C_7\mathcal{F}_\lambda^T(q^2) - 16\pi^2\frac{M_B}{m_b}\mathcal{H}_\lambda(q^2)\right]\right\}$$

- $B \to K^{(*)}\mu\mu$
- $B_s \to \varphi\,\mu\mu$
- $\Lambda_b \to \Lambda^{(*)}\mu\mu$

Local form-factors, involves e.g.

$$\mathcal{F}_\mu(k,q) = \langle\bar{M}(k)|\bar{s}\gamma_\mu b_L|\bar{B}(q+k)\rangle$$

Form factors in $b \to s\ell\ell$

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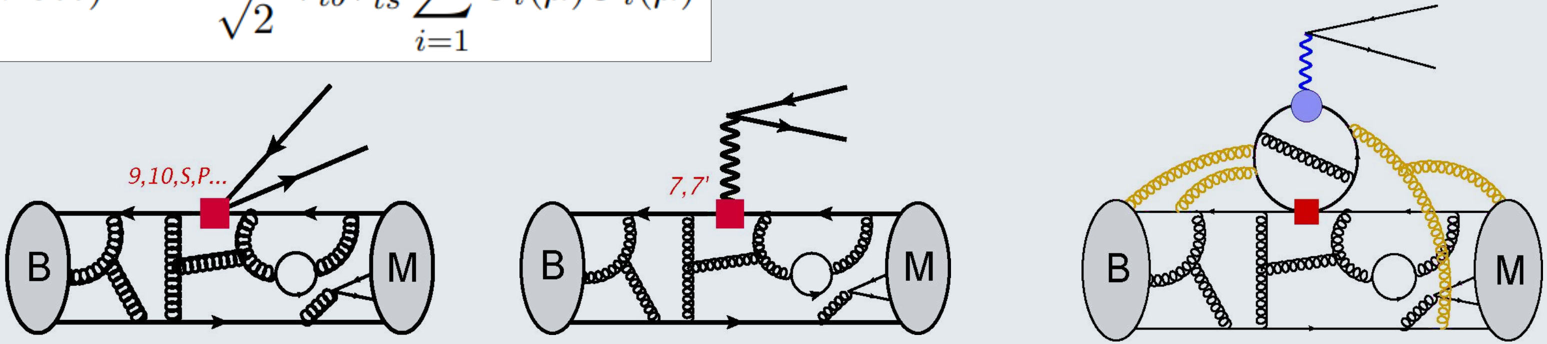

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$$\mathcal{H}_\mu(k,q) = i \int d^4x\, e^{iq\cdot x} \langle \bar{M}(k) | T\{ \mathcal{J}_\mu^{\text{em}}(x), \mathcal{C}_i\mathcal{O}_i \} | \bar{B}(q+k) \rangle$$

Non-local form-factors

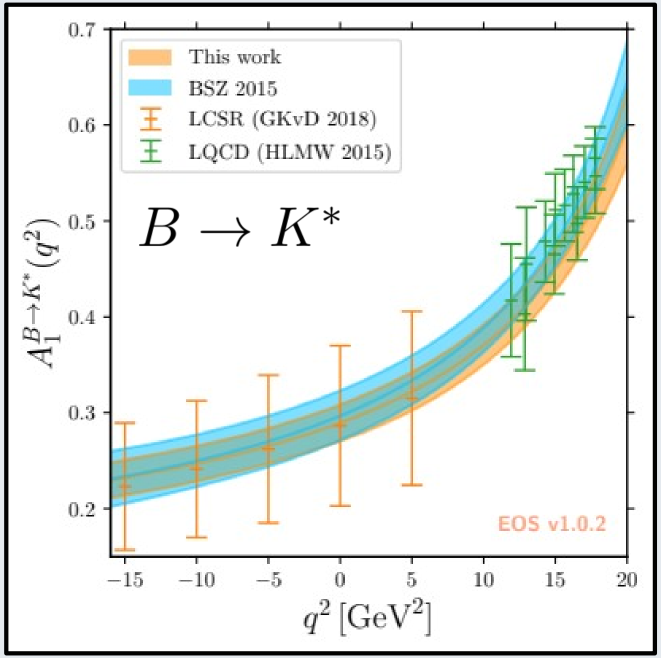
→ Main contributions: the **"charm-loops"** $\mathcal{O}_{2(1)}^c = (\bar{s}_L\gamma_\mu(T^a)c_L)(\bar{c}_L\gamma^\mu(T^a)b_L)$

Local form factors

- **2 main approaches**
  - Lattice QCD → most feasible at **large q^2**
  - Light-cone sum rules → most feasible at **small q^2**, 2 possible LCSRs

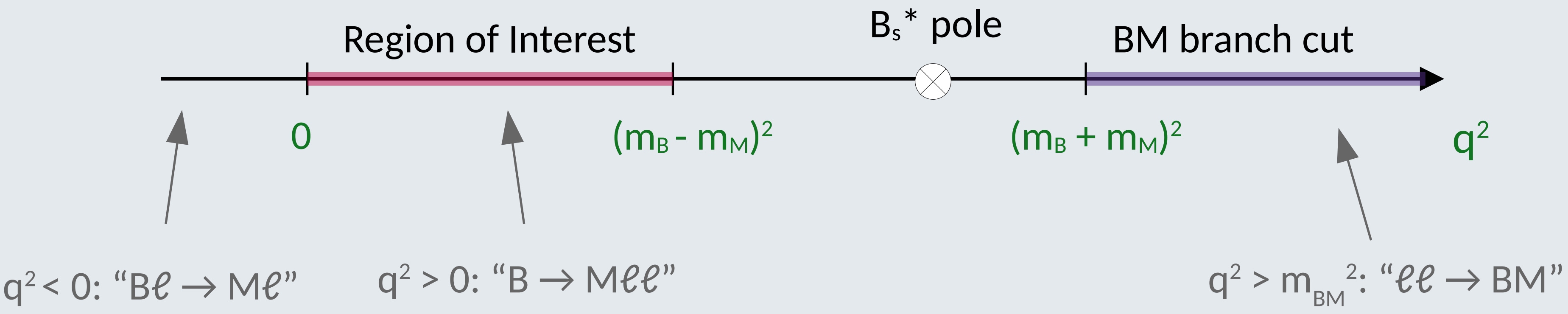
→ **Interpolation/Extrapolation**, depending on the use case

→ How to control extrapolation uncertainties



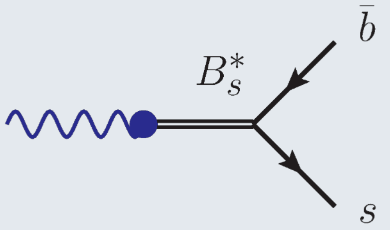
Form Factor Properties

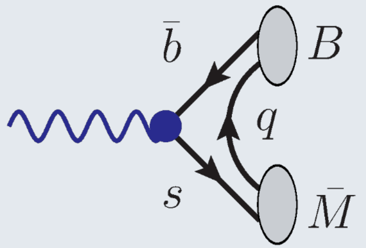
$$\mathcal{F}_\mu(k,q) = \langle \bar{M}(k) | \bar{s}\gamma_\mu b_L | \bar{B}(q+k) \rangle$$



Analytic properties of the form factors:

- Pole due to **$\bar{b}s$ bound state**
- **Branch cut** due to on-shell BM production





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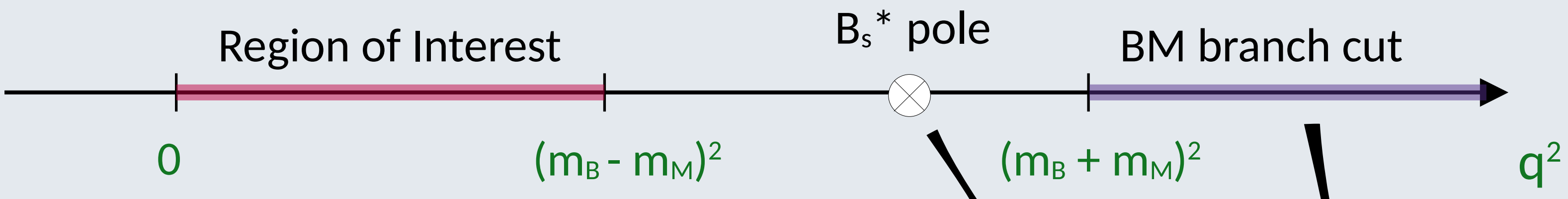

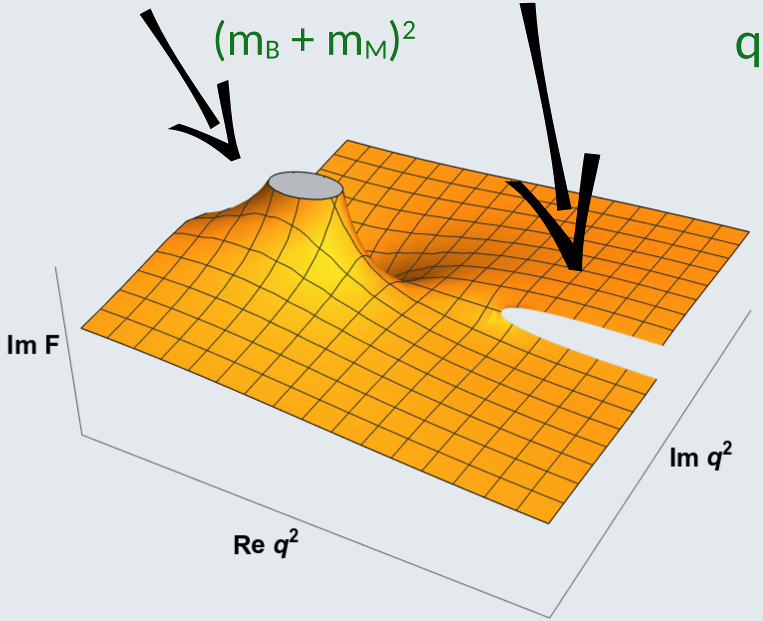

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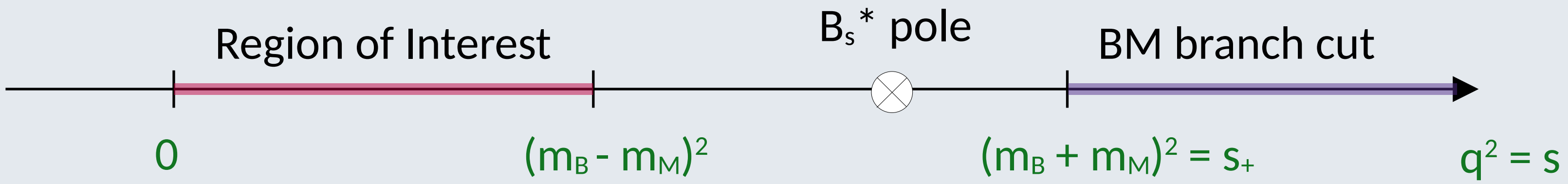
Form Factor Parametrization



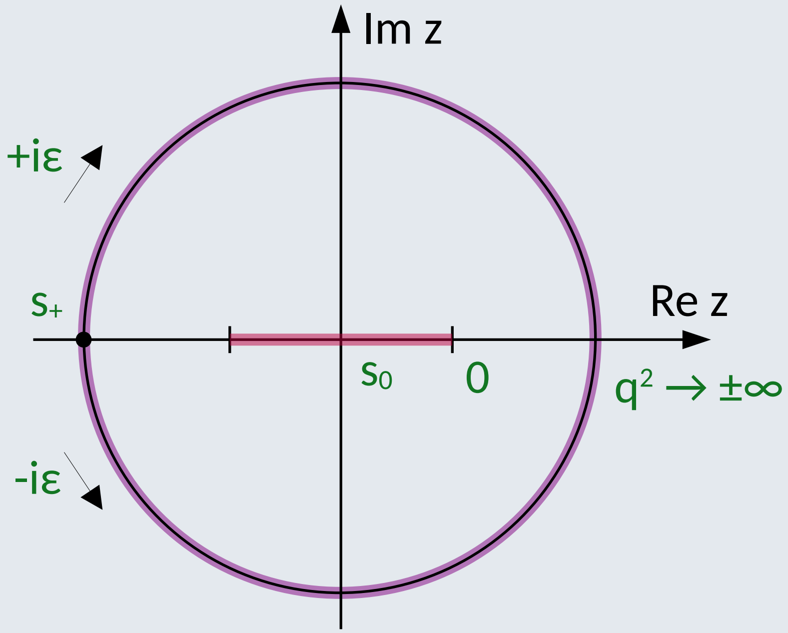
Conformal mapping [Boyd, Grinstein, Lebed '97]

$$z(s) \equiv \frac{\sqrt{s_+ - s} - \sqrt{s_+ - s_0}}{\sqrt{s_+ - s} + \sqrt{s_+ - s_0}}$$

Simplified Series expansion [Bourrely, Caprini, Lellouch, '08; Bharucha, Feldmann, Wick '10]

$$\mathcal{F}_\lambda^{(T)}(q^2) = \frac{1}{q^2 - m_{B_s^*}^2} \sum_{k=0}^{N} \alpha_{\lambda,k} z^k$$

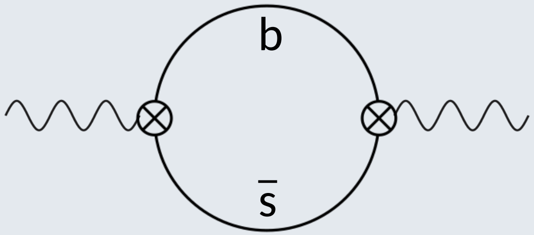
What is the uncertainty due to the truncation order N?



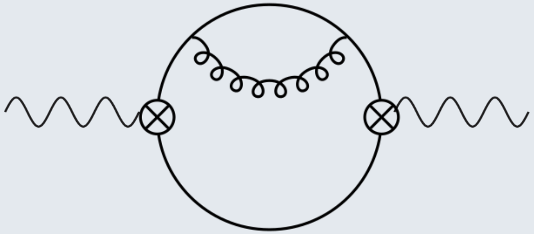
Dispersive bounds

- **Main idea:** Compute the inclusive $e^+e^- \to \bar{b}s$ cross-section and relate it to the form factors [Bharucha, Feldmann, Wick '10]

Insertion of a scalar, vector or tensor current



+ other diagrams: loops, quark and gluon condensates…



Dispersive bounds

- **Main idea:** Compute the inclusive $e^+e^- \to \bar{b}s$ cross-section and relate it to the form factors [Bharucha, Feldmann, Wick '10]

- Usually, the correlator $\Pi^{\mu\nu}_\Gamma(q) \equiv i \int d^4x\, e^{iq\cdot x} \langle 0|\mathcal{T}\left\{J^\mu_\Gamma(x) J^{\dagger,\nu}_\Gamma(0)\right\}|0\rangle$

  is decomposed as:

$$\Pi^{\mu\nu}_\Gamma(q) \equiv \frac{q^\mu q^\nu}{q^2}\Pi_\Gamma^{(J=0)} + \frac{1}{D-1}\left(\frac{q^\mu q^\nu}{q^2} - g^{\mu\nu}\right)\Pi_\Gamma^{(J=1)}$$

Dispersive bounds

- **Main idea:** Compute the inclusive $e^+e^- \to \bar{b}s$ cross-section and relate it to the form factors [Bharucha, Feldmann, Wick '10]

- We suggest the more generic decomposition:

$$\Pi_\Gamma^{\mu\nu}(q) \equiv \sum_{\lambda,\lambda'} \epsilon^\mu(\lambda)\epsilon^{\nu*}(\lambda')\,\Pi_\Gamma^{(\lambda,\lambda')}(q^2)$$

Polarization vectors

Dispersive bounds

- **In equations:**
  - This is the bound used in the literature:

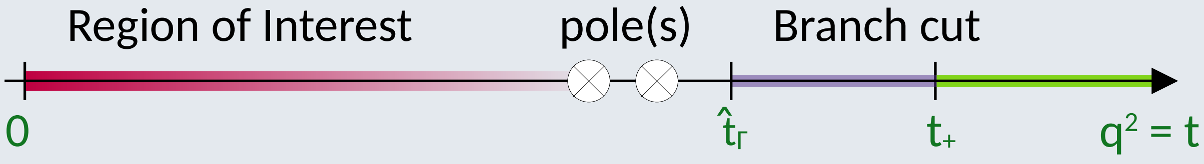
$$\chi_A^{(J=1)}\Big|_{BK^*} = \frac{\eta^{B\to K^*}}{24\pi^2}\int_{(M_B+M_{K^*})^2}^{\infty} ds \frac{\lambda_{\text{kin}}^{1/2}}{s^2(s-Q^2)^3}\left[s\,(M_B+M_{K^*})^2 \boxed{|A_1^{B\to K^*}|^2} + 32\, M_B^2 M_{K^*}^2 \boxed{|A_{12}^{B\to K^*}|^2}\right]$$

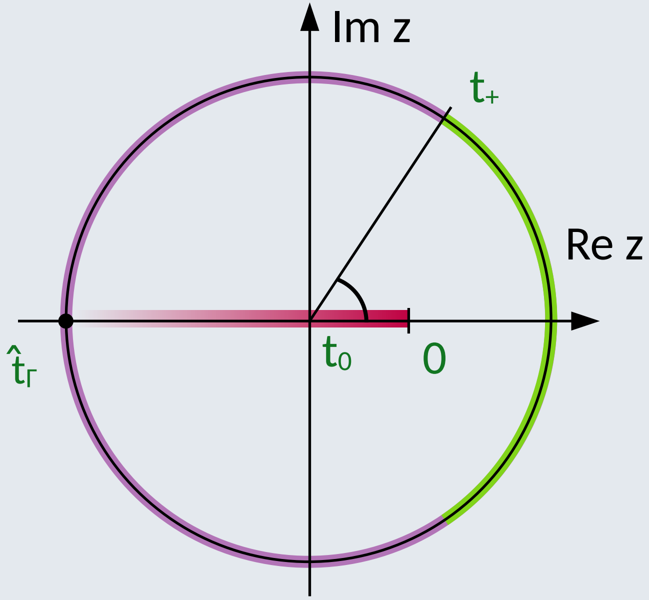
  - And this is what we propose:

$$\chi_A^{(0)}\Big|_{\bar{B}K^*} = \frac{\eta^{B\to K^*}}{\pi^2}\int_{(M_B+M_{K^*})^2}^{\infty} ds \frac{\lambda_{\text{kin}}^{1/2}}{s^2(s-Q^2)^3} 4\, M_B^2 M_{K^*}^2 \boxed{|A_{12}^{B\to K^*}|^2},$$

$$\chi_A^{(\parallel)}\Big|_{\bar{B}K^*} = \frac{\eta^{B\to K^*}}{8\pi^2}\int_{(M_B+M_{K^*})^2}^{\infty} ds \frac{\lambda_{\text{kin}}^{1/2}}{s^2(s-Q^2)^3} s\,(M_B + M_{K^*})^2 \boxed{|A_1^{B\to K^*}|^2},$$

- Our parametrization
  - Implements **sub-threshold branch cuts**





  - Diagonalizes the dispersive bounds:

$$\hat{\mathcal{F}}(\hat{z}) = \sum_{n=0}^{\infty} \beta_n \, p_n(\hat{z}) \qquad \sum_{n=0}^{\infty} |\beta_n|^2 < 1$$

Orthonormal polynomials of the arc of the unit circle

Local form factors fit

- With this framework we perform the **first simultaneous fit** of $B \to K$, $B \to K^*$ and $B_s \to \varphi$ LCSR and lattice QCD inputs:
  - $B \to K$:
    - [HPQCD ’13 and ’22; FNAL/MILC ’17]
    - ([Khodjamiriam, Rusov ’17]) → large uncertainties, not used in the fit
  - $B \to K^*$:
    - [Horgan, Liu, Meinel, Wingate ’15]
    - [Gubernari, Kokulu, van Dyk ’18] (B-meson LCSRs)
  - $B_s \to \varphi$:
    - [Horgan, Liu, Meinel, Wingate ’15]
    - [Gubernari, van Dyk, Virto ’20] (B-meson LCSRs)
- Baryonic decays should be added, but there are currently only few constraints

Setup

- Bayesian analysis using EOS eos.github.io
- Implementation of the dispersive bound:

$$
-2\log P(r) = \begin{cases} 0 & \text{if } r < 1\,, \\ \frac{(r-1)^2}{\sigma^2} & \text{otherwise}\,, \end{cases}
$$

10% uncertainty on the OPE calculation
[Bharucha, Feldmann, Wick '10]

- **Stability criterion**: truncate the series expansion to N = 2, 3, 4 and compare the form factor uncertainties

Setup

- All the samples are considered to be **correlated only via the dispersive bounds**
- Since $B \to K$ and ($B \to K^*$, $B_s \to \varphi$) are decoupled, we perform **3 separated fits**
- $B \to K^*$ and $B_s \to \varphi$ samples are combined with a weighting procedure:

$$w(r_{\Gamma,\lambda}^{B\to K^*}) = \int dr\, p_{\Gamma,\lambda}^{B_s\to\phi}(r) \times P\left(\text{dispersive bound for } \Gamma, \lambda \,|\, r_{\Gamma,\lambda}^{\text{1pt}} + r_{\Gamma,\lambda}^{B\to K^*} + r\right)$$

Current-specific weight

Integration over the $B_s \to \varphi$ saturations PDF

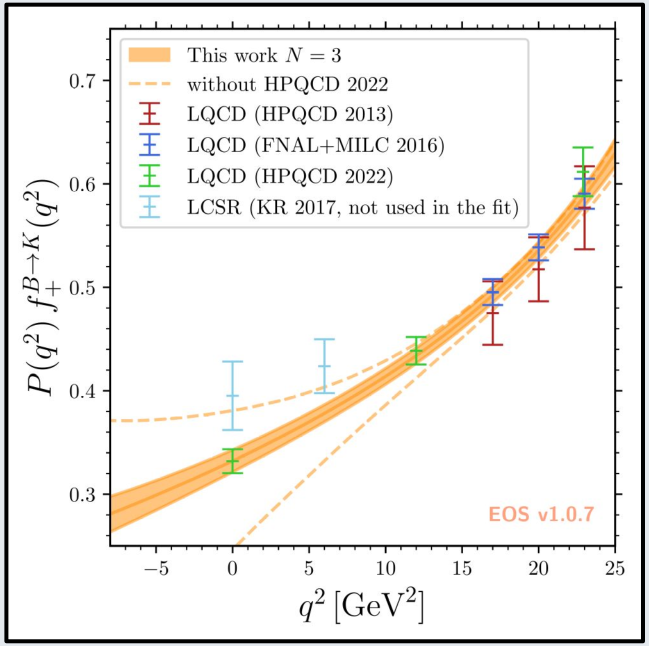
Dispersive bound of the previous slide

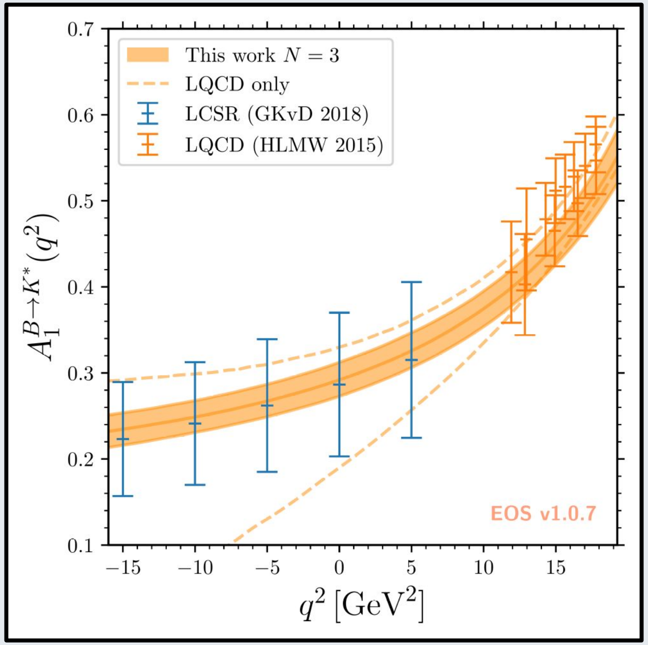
$$w^{B\to K^*} = \prod w(r_{\Gamma,\lambda}^{B\to K^*})$$

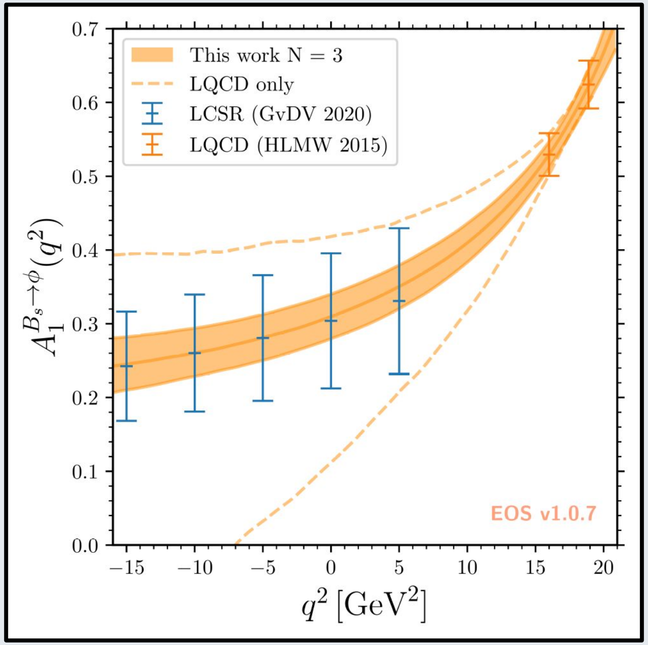
Results

Main conclusions:

- Fits are very good already at N = 2 (p-values > 77%)
- LCSR and LQCD combine nicely and still dominate the uncertainties
- Progresses in LQCD will eventually make LCSR irrelevant (?)

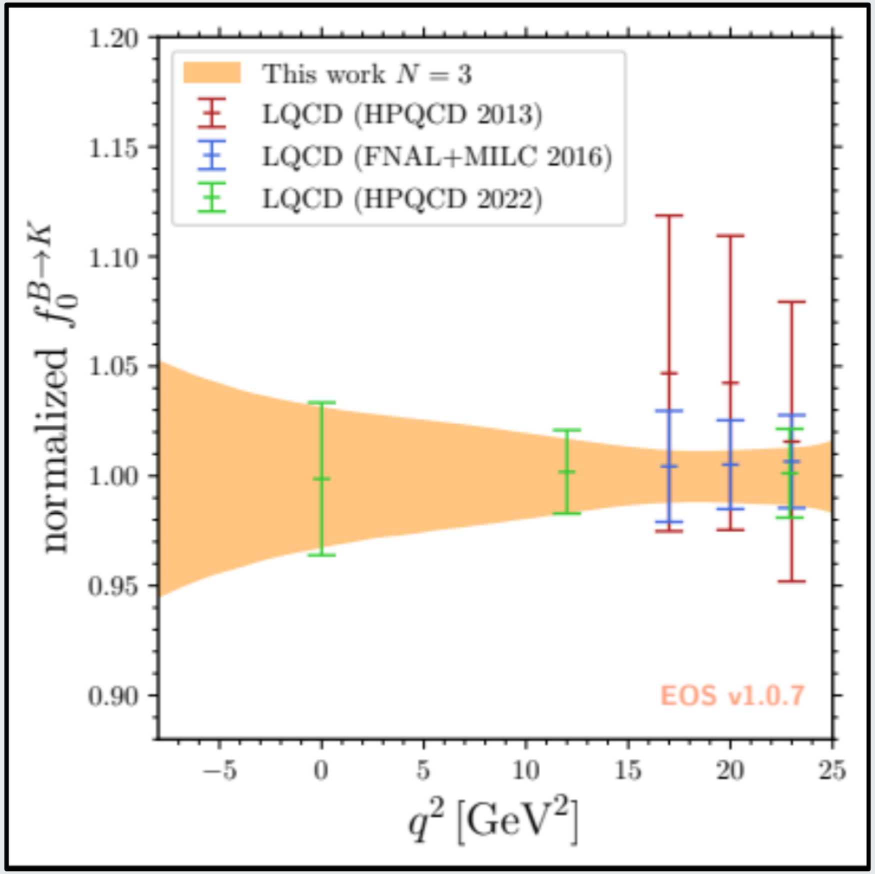






Comparison plots

- For comparison purposes I normalize the form factors to our N = 3 best-fit point
- Uncertainties for B → K are now well below 5% in the physical region



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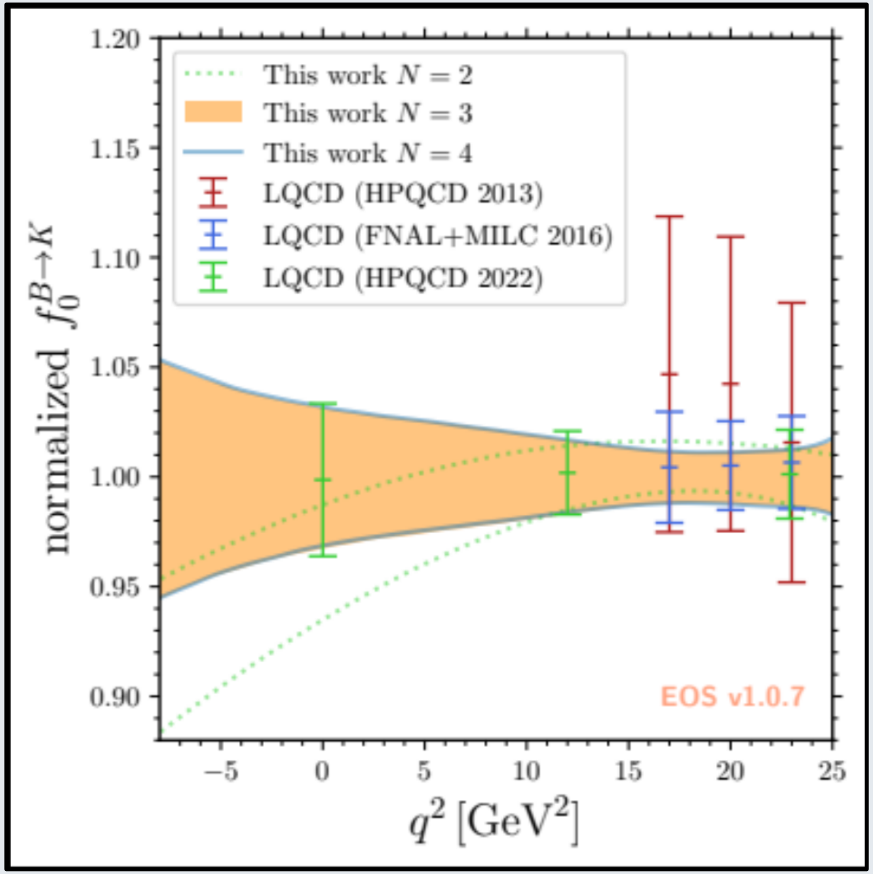

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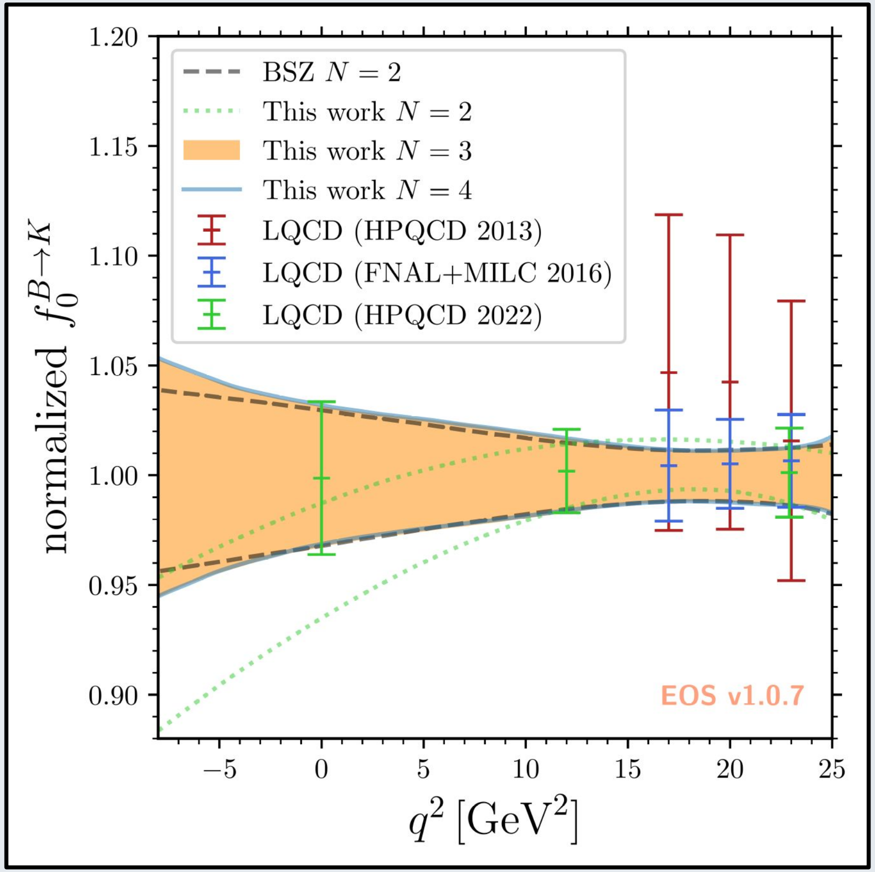
- We compare the different values of the truncation order N
- I also add the result of a usual Simplified Series Expansion *à la* [Bharucha, Feldmann, Wick '10; Bharucha, Straub, Zwicky '15]

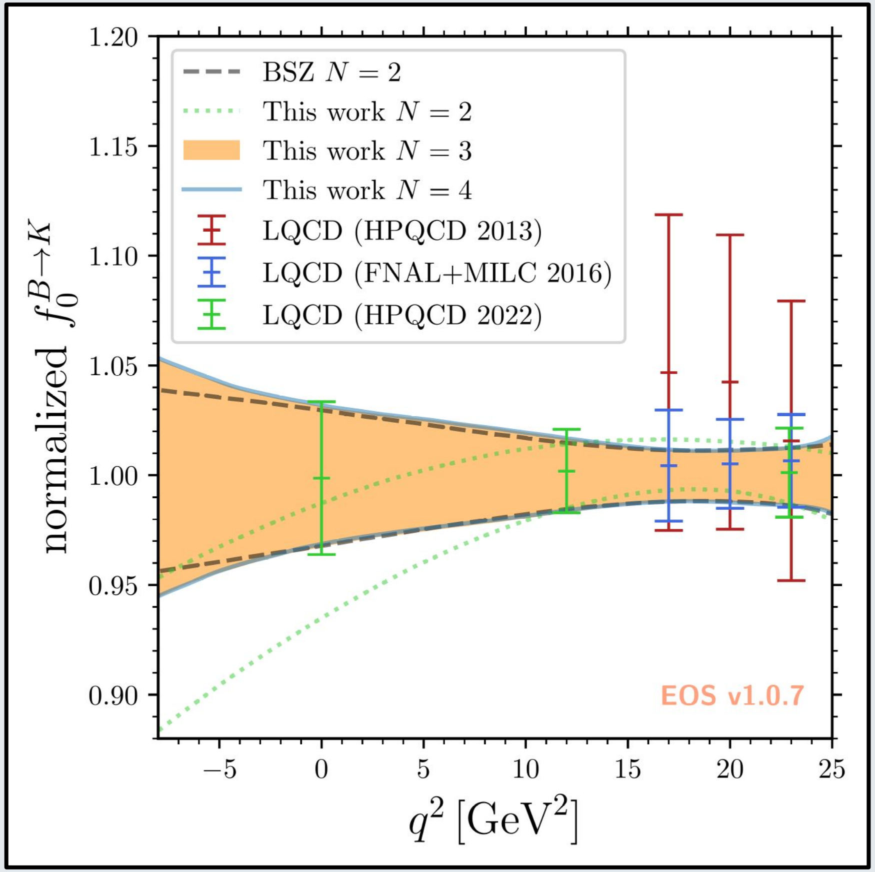
$$f(t) = \frac{1}{P(t)} \sum_k \tilde{\alpha}_k \, z^k(t, t_0)$$



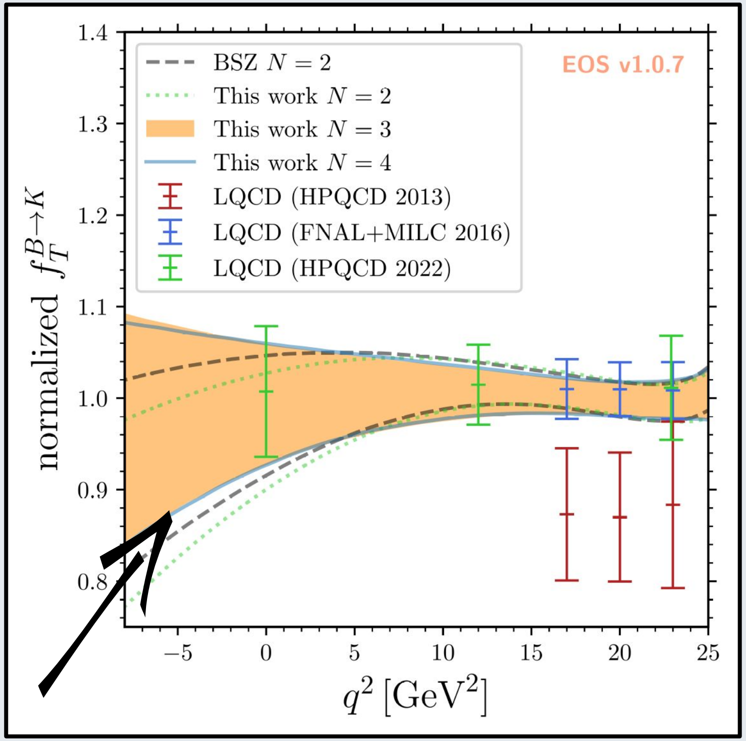
Generic result

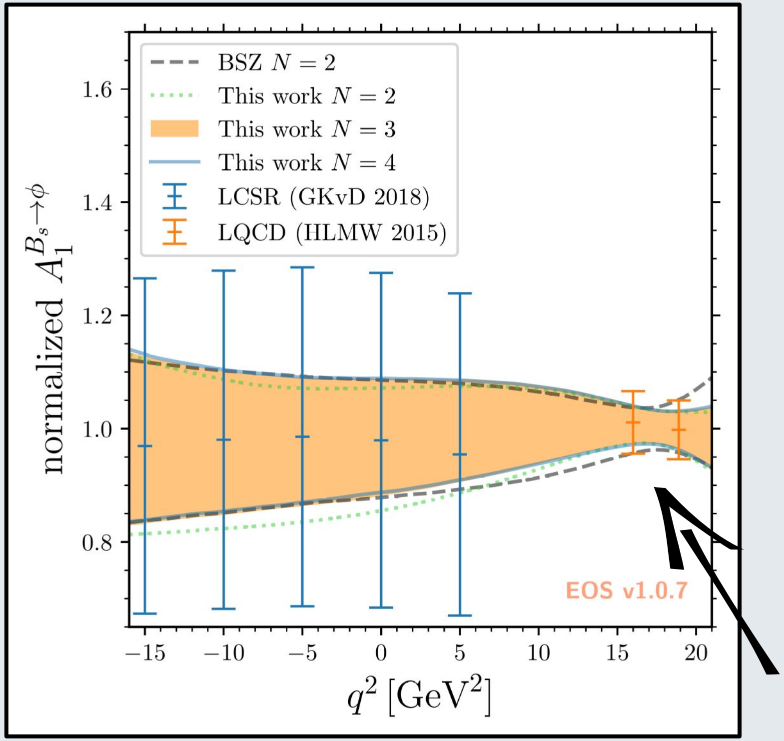
This is the generic result, namely:

- N = 2 shows a peculiar behaviour
- For N > 2 the uncertainties are stable
- BSZ is a good approximation in the physical range, but underestimates the uncertainties at negative q^2



Specific cases





→ The dispersive bounds stabilizes regions of the phase space with few theory constraints
→ This is particularly useful at negative q^2 to estimate the non-local form factors

Additional plots can be found in the paper: 2305.06301

Where to find our results

- All the plots are available here: https://doi.org/10.5281/zenodo.7919635
- We also added
  - the updated posterior distributions for N = 2 in our parametrization and using a SSE as YAML files
  - All the tools/documentation to reproduce our results
- These results are also available in **EOS v1.0.7**:
  - /eos/constraints/B-to-P-form-factors.yaml
  - /eos/constraints/B-to-V-form-factors.yaml

Conclusion

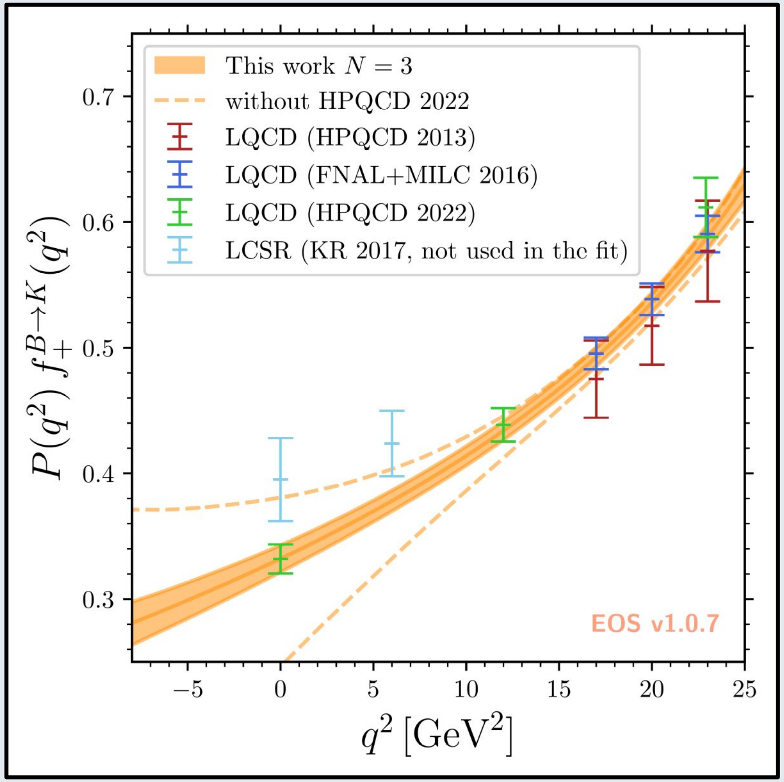
Discussing BSM models requires a solid understanding of the hadronic physics:

- **Local form factors** uncertainties can be controlled and reduced by using improved dispersive bound and a *appropriate* parametrization
  - This is the first global analysis of b → s form factors
  - It is reassuring as it confirms channel-specific analyses...
  - ... and promising as dispersive bounds start to affect the results
- **Non-local form factors** can also be constrained by theory calculation and experimental measurements

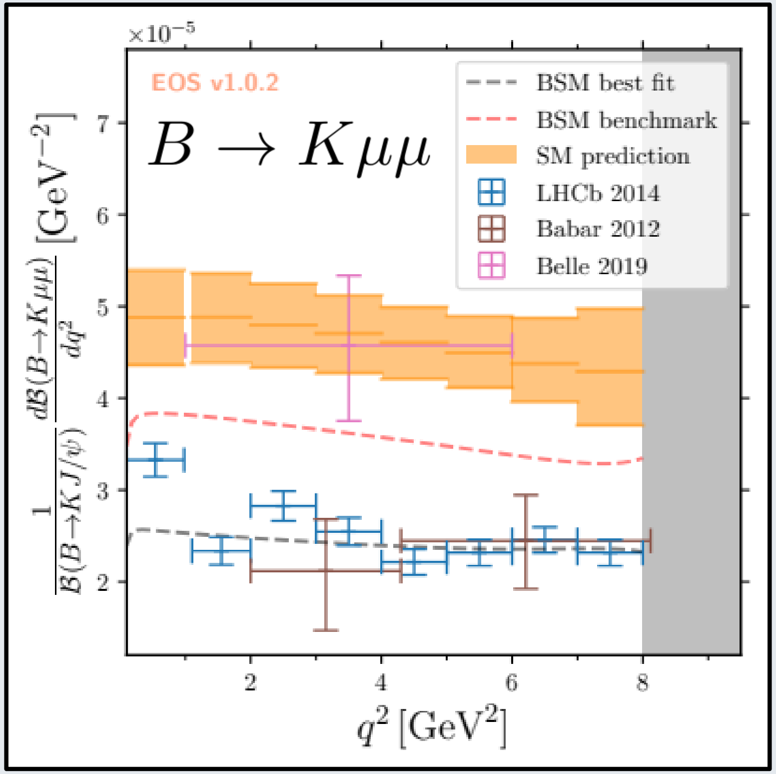
  → In both cases:
  - Uncertainties are still large, but controlled by dispersive bounds
  - Our approach is systematically improvable

Back-up

Effect of HPQCD 2022

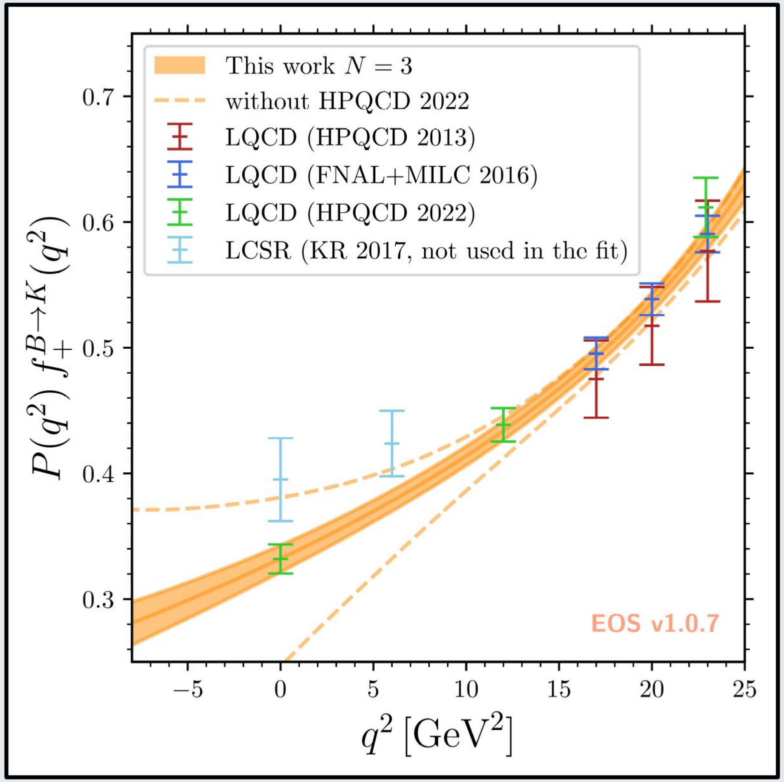


With Khodjamirian-Rusov 2017

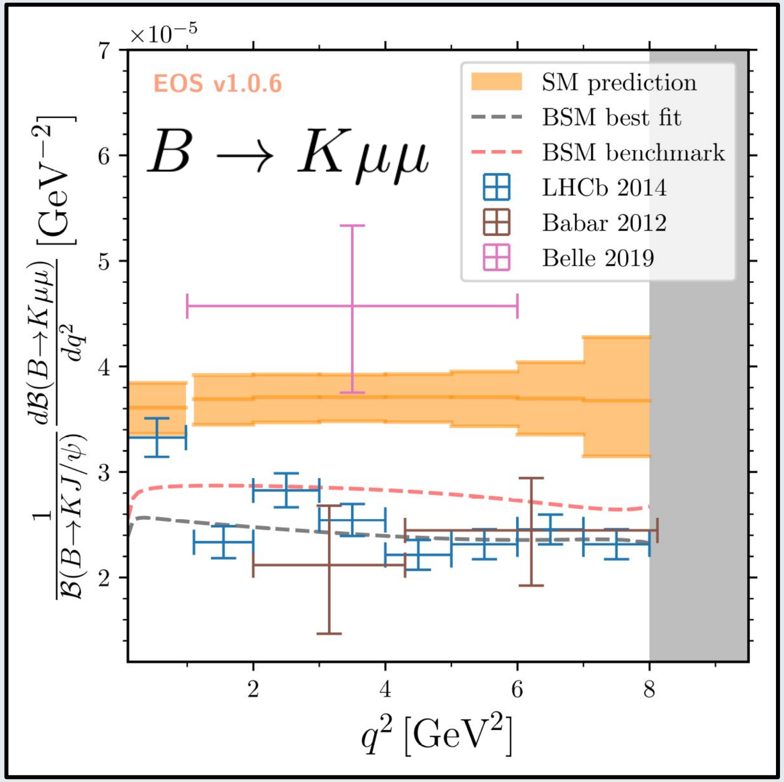


[Gubernari, MR, van Dyk, Virto ‘22]

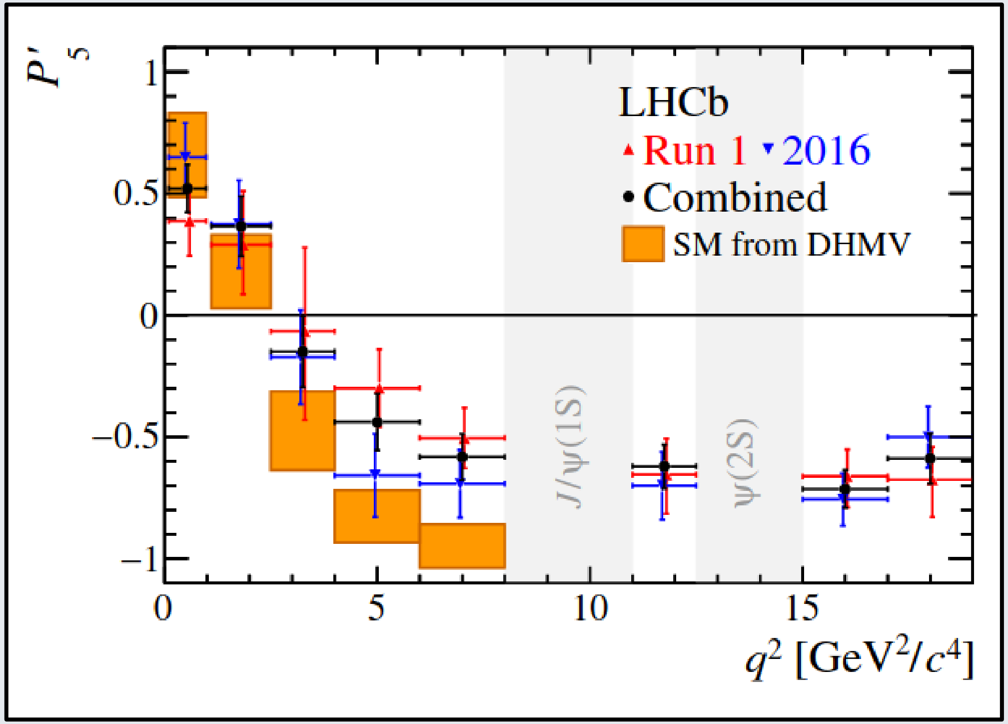
Effect of HPQCD 2022



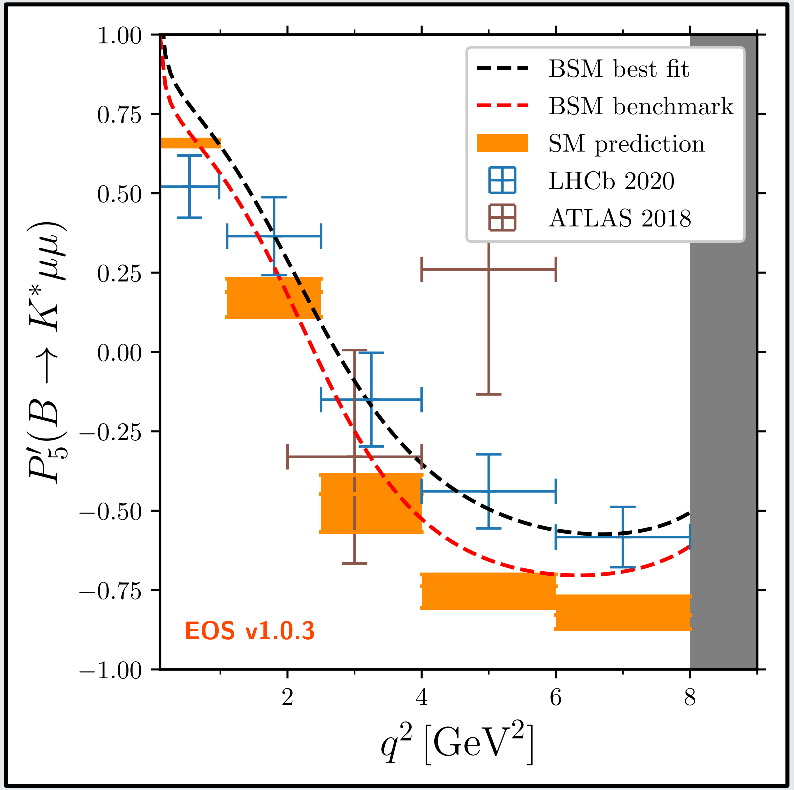
With HPQCD 2022



$B \to K^* P'_5$



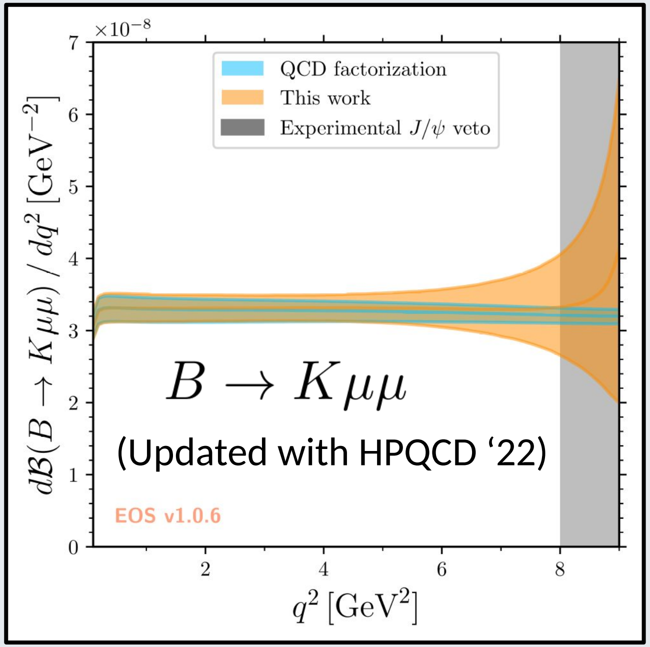
P'_5
LHCb
Run 1
2016
Combined
SM from DHMV
$J/\psi(1S)$
$\psi(2S)$
q^2 [GeV^2/c^4]

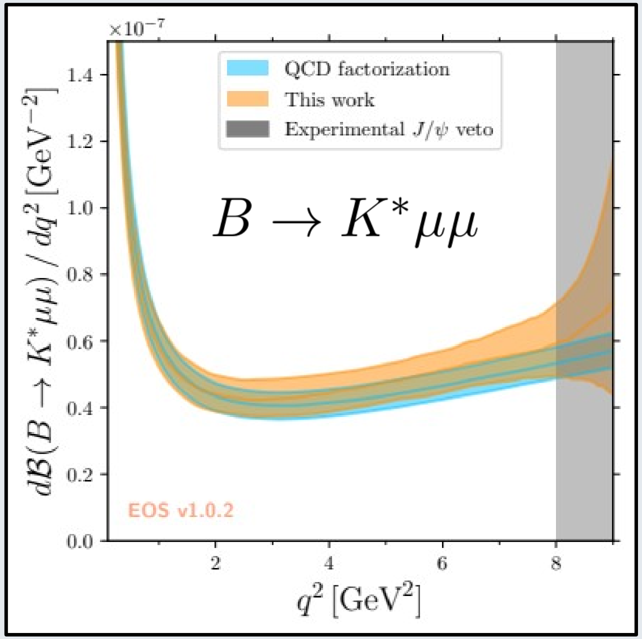


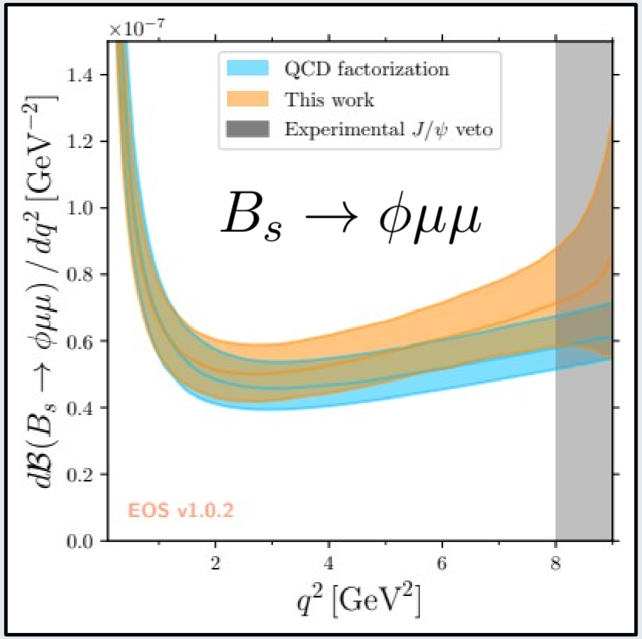
BSM best fit
BSM benchmark
SM prediction
LHCb 2020
ATLAS 2018
EOS v1.0.3
$P'_5(B \to K^* \mu\mu)$
q^2 [GeV^2]

- Good overall agreement with previous theoretical approaches [Beneke, Feldman, Seidel '01 & '04]
  - Small deviation in the slope of $B_s \to \phi\mu\mu$
- Larger but controlled uncertainties especially near the J/ψ

  → The approach is **systematically improvable** (new channels, ψ(2S) data...)



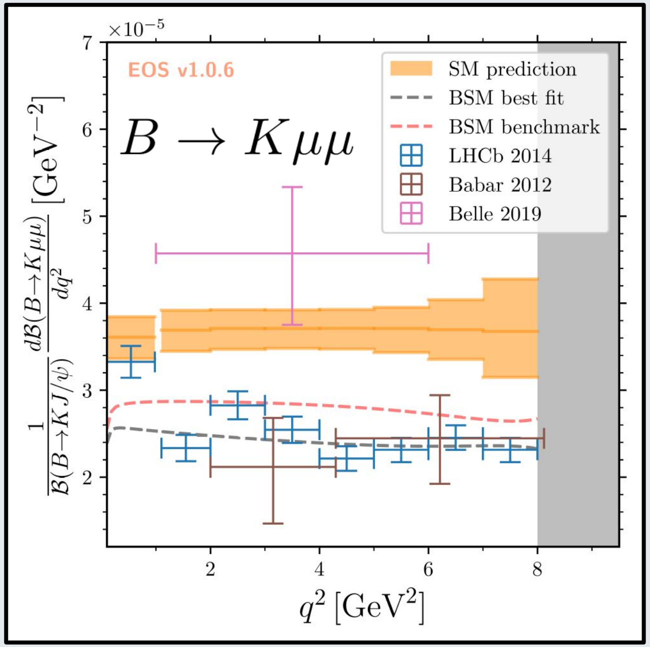


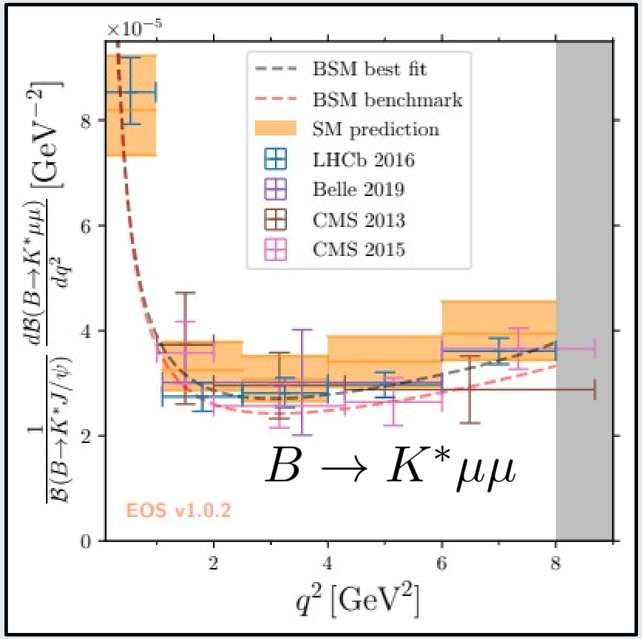


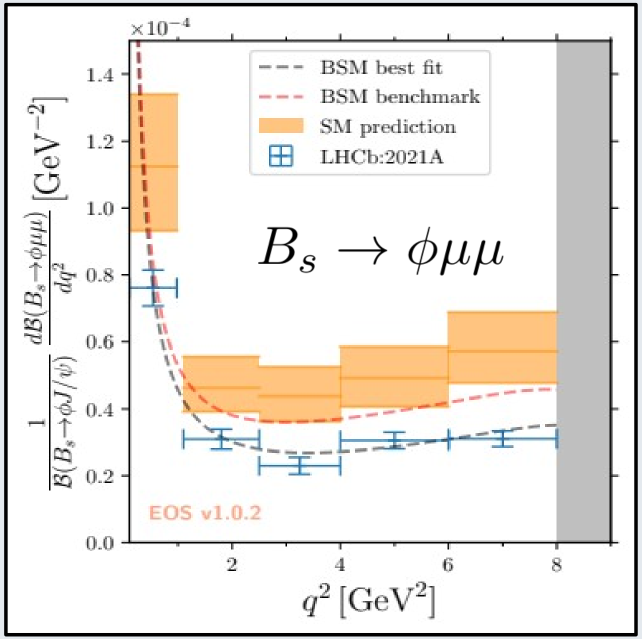
Comparison with data

- Conservatively accounting for the non-local form factors does not solve the b → sμμ anomalies
- The largest source of theoretical uncertainty at low q^2 still comes from local form factors

Experimental results:
[Babar: 1204.3933; Belle: 1908.01848, 1904.02440; ATLAS: 1805.04000, CMS: 1308.3409, 1507.08126, 2010.13968, LHCb: 1403.8044, 2012.13241, 2003.04831, 1606.04731, 2107.13428]







Additional plots can be found in the paper: 2206.03797