PRIMORDIAL BLACK HOLES REHEATING

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Introduction

- **There is yet no evidence for primordial black holes (PBHs), but their existence could be a very interesting possibility : e.g. they can**
  - **have astrophysical implications : e.g. provide insights into the evolution of galaxies over cosmic time, or provide information on the nature of dark, ...**
  - **Shed light on the physics at early universe: e.g. primordial inhomogeneities, scale of inflation, reheating, modify BBN, ...**
- **So far the studies of the reheating of universe considered either :**
  - **the standard scenario: inflaton produces thermal radiation**
  - **PBHs formed after a phase of an instantaneous inflaton reheating**
- **Thus the main goal: discuss the impacts of PBHs on the reheating dynamics, in non-instantaneous decays of inflaton**
  - **the implication on the reheating temperature, and potentially constraint the parameter space**

Outline of the talk:

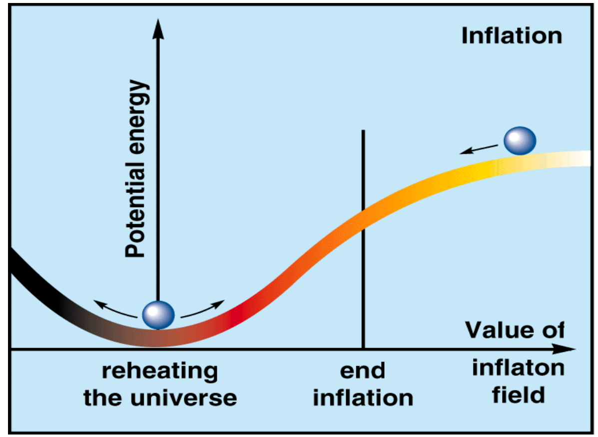
1. Introduce the standard reheating : in the absence of PBHs
2. Discuss PBHs reheating
   - Monochromatic case : Impact of PBHs on the reheating temperature
   - Comment on the extended mass distribution case
3. Conclusions

Standard reheating

- We consider the inflaton potential of the form

$$V(\phi) = \lambda M_P^4 \left(\frac{\phi}{M_P}\right)^n, \quad \text{for even } n$$

- and the reheating is driven by $y_\phi \bar{f} f \phi$ interactions.



(Source: arXiv:astro-ph/9906497)

- Evolution equations:

$$\begin{aligned}
\dot{\rho_\phi} + 3H(1+\omega_\phi)\rho_\phi &= -\Gamma_\phi \rho_\phi (1+\omega_\phi), \\
\dot{\rho}_R + 4H\rho_R &= \Gamma_\phi \rho_\phi (1+\omega_\phi), \\
3H^2 M_P^2 &= \rho_\phi + \rho_R, \; \omega_\phi = \frac{n-2}{n+2},
\end{aligned}$$

$\Gamma_\phi = \phi$ decay rate, H = Hubble parameter, $M_P =$ reduced Planck mass

- The solutions in terms of scale factor a [Garcia, Kaneta, Mambrini and Olive, JCAP 04 (2021), 012]

$$\rho_\phi(a) = \rho_{\rm end} \left(\frac{a}{a_{\rm end}}\right)^{-\frac{6n}{n+2}}, \quad \rho_R(a) = \frac{\lambda^{\frac{1}{n}} y_\phi^2}{8\pi} \left(\frac{\alpha_n}{M_P^4}\right) \left(\frac{\rho_{\rm end}}{M_P^4}\right)^{1-\frac{1}{n}} \left[\left(\frac{a}{a_{\rm end}}\right)^{\frac{6-6n}{n+2}} - \left(\frac{a}{a_{\rm end}}\right)^{-4}\right] M_P^4$$

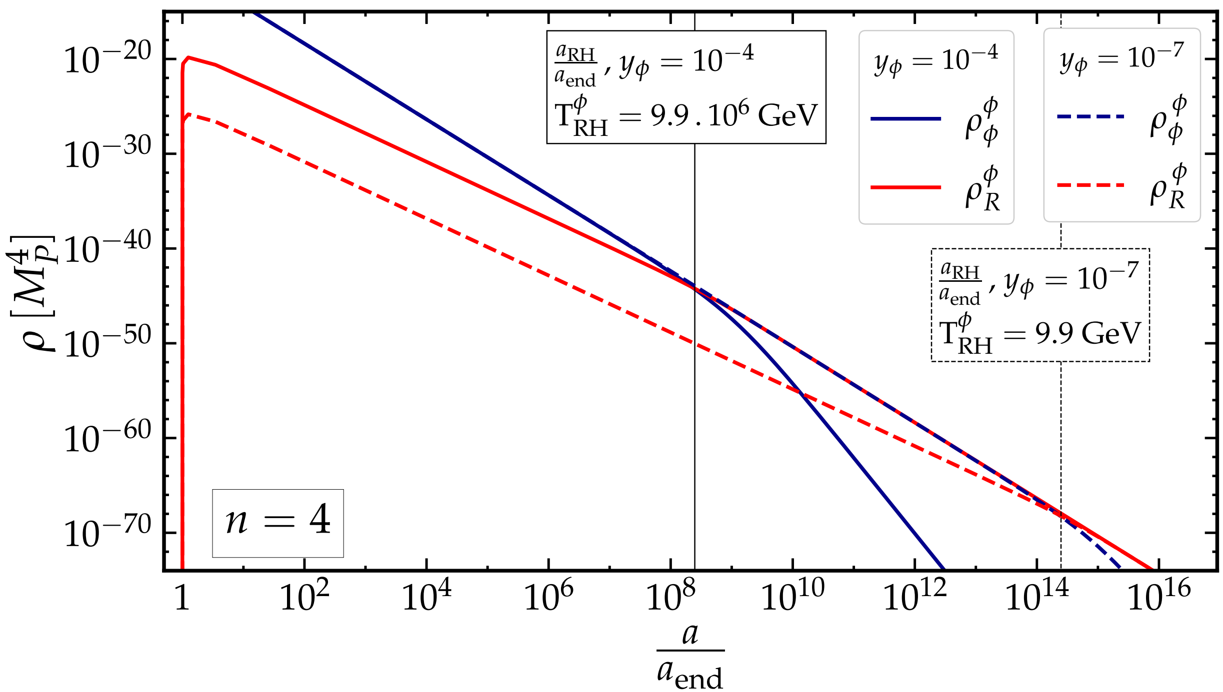
$$\alpha_n = M_P^4 \frac{\sqrt{3n^3(n-1)}}{7-n}, \qquad (n < 7)$$

- **The reheating temperature, $T^{\phi}_{\rm RH}$, defined when $\rho_\phi = \rho_R$ is then:**

$$T^{\phi}_{\rm RH} \simeq \left(\frac{30\lambda}{g_T \pi^2}\right)^{\frac{1}{4}} \left(\frac{y_\phi^2}{8\pi}\right)^{\frac{n}{4}} \left(\frac{\alpha_n}{M_p^4}\right)^{\frac{n}{4}} M_P$$

- **e.g:** $n = 4, \lambda = 5 \times 10^{-11}$, $\rho_{\rm end} = 1.45 \times 10^{63}\ \mathrm{GeV}^4$

$y_\phi = 10^{-4} \Rightarrow T_{\rm RH} \simeq 10^7 \mathrm{GeV}$ **and for** $y_\phi = 10^{-7} \Rightarrow T_{\rm RH} \simeq 10 \mathrm{GeV}$



PBHs reheating

- PBHs could have been produced during the early Universe due to various mechanisms
  - The common point for most of them is that the primordial cosmological energy density fluctuations play a crucial role
- One possibility is that PBHs formed in a relatively short period of time
- In this scenario, the mass distribution of primordial black holes would be concentrated, sharply-peaked, or monochromatic around a specific mass given by

$$M_{\rm in} = \gamma M_H = \gamma \frac{4\pi}{3}\frac{\rho_{\rm in}}{H_{\rm in}^3} = 4\pi\gamma\frac{M_P^2}{H_{\rm in}}\,, \quad \text{where } \gamma = \omega^{3/2}$$

ω is the equation of state at formation, $\rho_{\rm in}$ and $H_{\rm in}$ are respectively the total energy density and Hubble parameter at PBHs formation

- **It follows that for Schawrzchild PBH, the subsequent evolution of the mass due to evaporation :**

$$\frac{dM_{\rm BH}}{dt} = -\varepsilon \frac{M_P^4}{M_{\rm BH}^2}, \quad \textbf{where} \quad \varepsilon = \frac{\pi g_*}{480}$$

- **The mass evolution in terms of the scale factor, in a Universe whose expansion is dominated by a fluid with an equation of state $P = \omega\rho$ is :**

$$M_{\rm BH}^3(a) = M_{\rm in}^3 + \frac{2\varepsilon M_P^2 M_{\rm in}}{4\pi\gamma(1+\omega)}\left[1 - \left(\frac{a}{a_{\rm in}}\right)^{\frac{3}{2}(1+\omega)}\right]$$

- **The lifetime $\tau_{\rm BH}$ is:**

$$\tau_{\rm BH} = \frac{M_{\rm in}^3}{3\varepsilon M_P^4}$$

- **In the following, we consider PBHs forming after inflation and evaporating before BBN :**
  - **The lower bound on $M_{\rm in}$ is fixed by the inflationary energy scale $H_{\rm I}^{\rm max} \sim 5\times 10^{13}$ GeV**

    $\rightarrow M_{\rm in} \geq 4\pi\gamma \frac{M_p^2}{H_{\rm I}^{\rm max}} \sim 2.9\times 10^{23}\,{\rm GeV} = 1\,{\rm g}$
  - **The upper bound on $M_{\rm in}$ is set by BBN time scale:** $\tau_{\rm BH} < t_{\rm BBN} - t_{\rm in} \rightarrow M_{\rm in} \lesssim 10^8\,{\rm g}$

- **The initial energy density of PBH, $\rho_{\rm BH}^{\rm in}$, is parameterized by β defined as**

$$\beta = \frac{\rho_{\rm BH}^{\rm in}}{\rho_\phi^{\rm in} + \rho_R^{\rm in}}\,.$$

  - **β can be restricted by the constraints on the GWs generated by the density fluctuation due to the inhomogeneities of the PBHs distribution and that avoids the back-reaction problem**

    [e.g. Papanikolaou, Vennin and Langlois, JCAP 03 (2021), 053]: $\beta < 10^{-4}\left(M_{\rm in}/10^9\,{\rm g}\right)^{-1/4}$

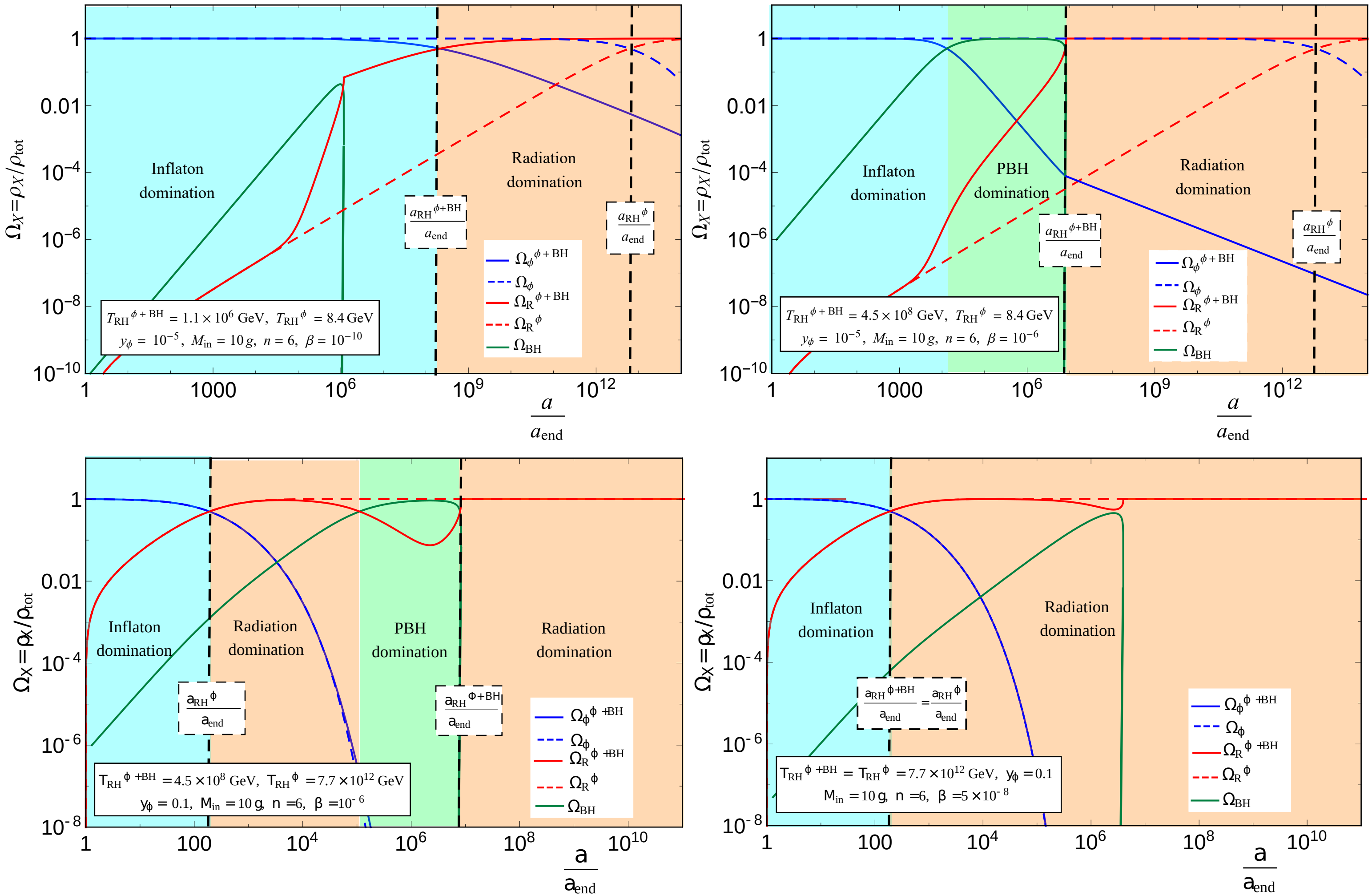
  - **We consider a stronger limit on β that ensures that the amount of generated GWs do not affect BBN constraints on the effective number of species**

    [Domènech, Lin, and Sasaki, JCAP 04 (2021), 062]: $\beta < 1.1\times 10^{-6}\left(\frac{\omega^{3/2}}{0.2}\right)^{-1/2}\left(\frac{M_{\rm in}}{10^4\,{\rm g}}\right)^{-17/24}$

- **Thus the evolution of the system is determined by solving the following set of equations:**

$$\begin{aligned}
\dot{\rho_\phi} + 3(1+\omega_\phi)H\rho_\phi &= -(1+\omega_\phi)\Gamma_\phi\rho_\phi\,,\\
\dot{\rho_{\rm BH}} + 3H\rho_{\rm BH} &= \frac{\rho_{\rm BH}}{M_{\rm BH}}\frac{dM_{\rm BH}}{dt}\,\theta(t-t_{\rm in})\,\theta(t_{\rm ev}-t)\,,\\
\dot{\rho}_R + 4H\rho_R &= (1+\omega_\phi)\Gamma_\phi\rho_\phi - \frac{\rho_{\rm BH}}{M_{\rm BH}}\frac{dM_{\rm BH}}{dt}\,\theta(t-t_{\rm in})\,\theta(t_{\rm ev}-t)\,,\\
\rho_\phi + \rho_R + \rho_{\rm BH} &= 3H^2M_P^2
\end{aligned}$$

- **Depending on y_ϕ, $M_{\rm in}$, and β, several scenarios evolution can be distinguished: e.g. $n = 6$**

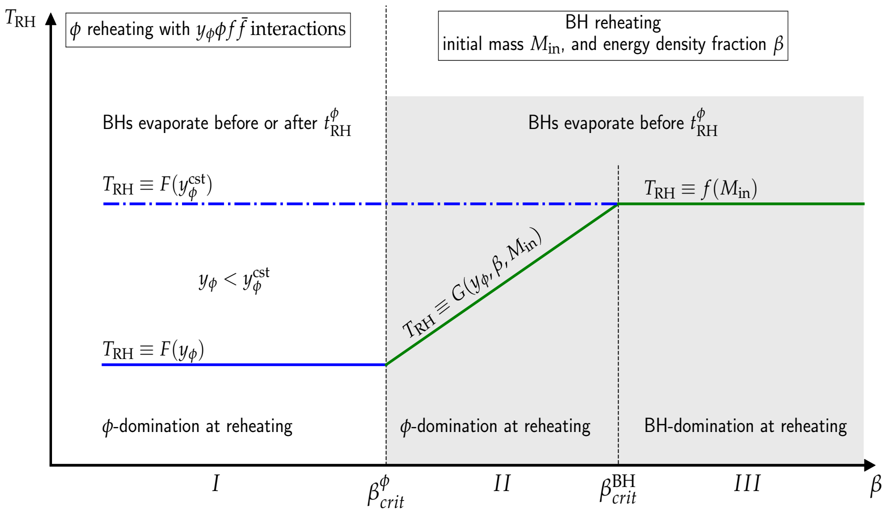


Even if PBHs do not dominate, their evaporation can dominate the reheating process

• Results: Qualitatively, there two classes of solutions depending on y_ϕ

• **First class:** $y_\phi < y^{\rm crit}$**, PBH effects lead to** $T_{\rm RH} \geq T_{\rm RH}^{\phi}$

Case $T_{\rm RH}^{\phi} < T_{\rm RH}^{\phi+\rm BH}$: $t_{\rm RH}^{\phi}$ = time scale of reheating point without BHs (standard scenario)



$$y_\phi^{\rm crit}|_{n<7} = \sqrt{8\pi}\left(\frac{M_P^4}{\alpha_n}\right)^{\frac{1}{2}}\left(\frac{3\varepsilon^2}{\lambda}\right)^{\frac{1}{2n}}\left(\frac{M_P}{M_{\rm in}}\right)^{\frac{3}{n}}$$

$$\beta_{crit}^{\phi} = \delta_n \left(\frac{y_\phi^2}{8\pi}\right)^{\frac{6\omega_\phi-2}{3-3\omega_\phi}}\left(\frac{M_P}{M_{\rm in}}\right)^{\frac{2-2\omega_\phi}{1+\omega_\phi}}\lambda^{\frac{3\omega_\phi-1}{3\omega_\phi+3}}$$

$$\beta_{crit}^{\rm BH} = \left(\frac{\varepsilon}{(1+\omega_\phi)2\pi\gamma}\right)^{\frac{2\omega_\phi}{1+\omega_\phi}}\left(\frac{M_P}{M_{\rm in}}\right)^{\frac{4\omega_\phi}{1+\omega_\phi}}$$

$$\mathrm{I}: \quad \beta < \beta_{crit}^{\phi} \quad \rightarrow \quad T_{\rm RH} \sim \left(\frac{30\lambda}{g_T\pi^2}\right)^{\frac{1}{4}}\left(\frac{y_\phi^2}{8\pi}\right)^{\frac{n}{4}}\left(\frac{\alpha_n}{M_P^4}\right)^{\frac{n}{4}} M_P$$

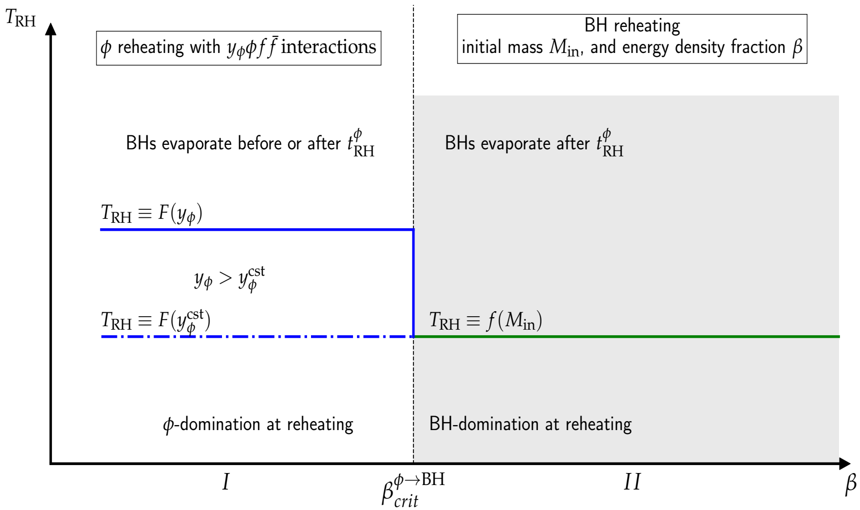
$$\mathrm{II}: \quad \beta_{crit}^{\phi} < \beta < \beta_{crit}^{\rm BH} \quad \rightarrow \quad T_{\rm RH} \sim M_P\left(\frac{M_P}{M_{\rm in}}\right)^{\frac{3-3\omega_\phi}{2-6\omega_\phi}}\beta^{\frac{3+3\omega_\phi}{12\omega_\phi-4}}$$

$$\mathrm{III}: \quad \beta > \beta_{crit}^{\rm BH} \quad \rightarrow \quad T_{\rm RH} \sim \left(\frac{90\varepsilon^2}{g_T\pi^2}\right)^{\frac{1}{4}}\left(\frac{M_P}{M_{\rm in}}\right)^{\frac{3}{2}} M_P$$

• **For** $n = 4$, β_{crit}^{ϕ} **represents also** $\beta_{crit}^{\rm BH}$**, value at which PBH dominate at reheating**

- **Second class: $y_\phi > y^{\rm crit}$, PBH effects lead to $T_{\rm RH} \leq T_{\rm RH}^{\phi}$**

Case $T_{\rm RH}^{\phi} > T_{\rm RH}^{\phi+{\rm BH}}$: $t_{\rm RH}^{\phi}$ = time scale of reheating point without BHs (standard scenario)



$$\beta_{crit}^{\phi\to{\rm BH}} = \delta \left(\frac{y_\phi^2}{8\pi}\right)^{\frac{6\omega-2}{3-3\omega}} \left(\frac{M_P}{M_{\rm in}}\right)^{\frac{2-2\omega}{1+\omega}} \lambda^{\frac{3\omega-1}{3\omega+3}}$$

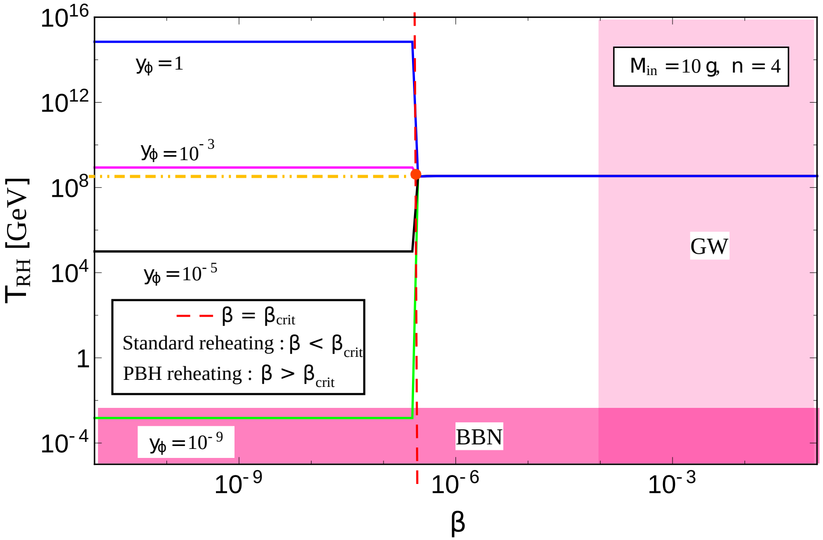
$$\mathrm{I}: \quad \beta < \beta_{crit}^{\phi\to{\rm BH}} \quad \rightarrow \quad T_{\rm RH} \sim \left(\frac{30\lambda}{g_T \pi^2}\right)^{\frac{1}{4}} \left(\frac{y_\phi^2}{8\pi}\right)^{\frac{n}{4}} \left(\frac{\alpha_n}{M_P^4}\right)^{\frac{n}{4}} M_P$$

$$\mathrm{II}: \quad \beta > \beta_{crit}^{\phi\to{\rm BH}} \quad \rightarrow \quad T_{\rm RH} \sim \left(\frac{90\varepsilon^2}{g_T \pi^2}\right)^{\frac{1}{4}} \left(\frac{M_P}{M_{\rm in}}\right)^{\frac{3}{2}} M_P$$

• $T_{\rm RH}$ versus β for $n = 4$

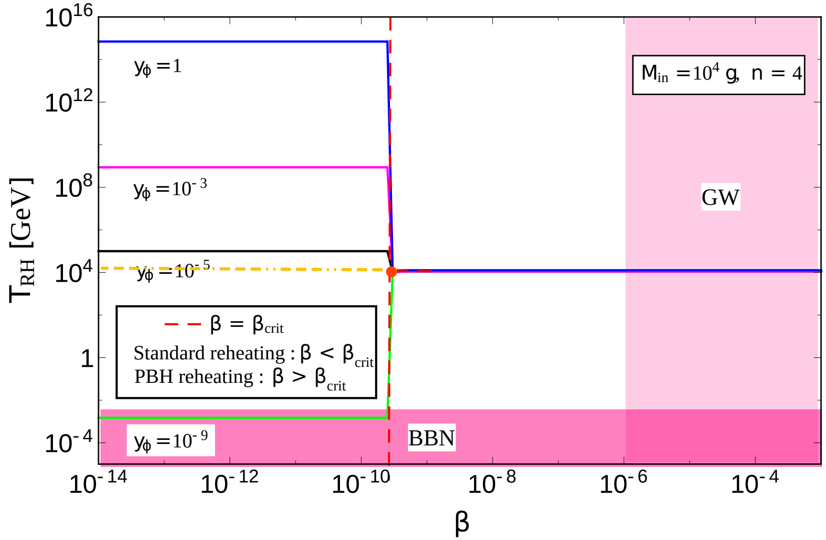
- $M_{\rm in} = 10\,g$:

  $y_\phi^{\rm cst} = 6 \times 10^{-4}$, $\beta_{crit}^{\rm BH} \sim 3 \times 10^{-7}$



- $M_{\rm in} = 10^4\,g$:

  $y_\phi^{\rm cst} = 3.3 \times 10^{-6}$, $\beta_{crit}^{\rm BH} \sim 3 \times 10^{-10}$



- $T_{\rm RH}$ **versus** β **for** $n = 6$

- $M_{\rm in} = 10\, g$:

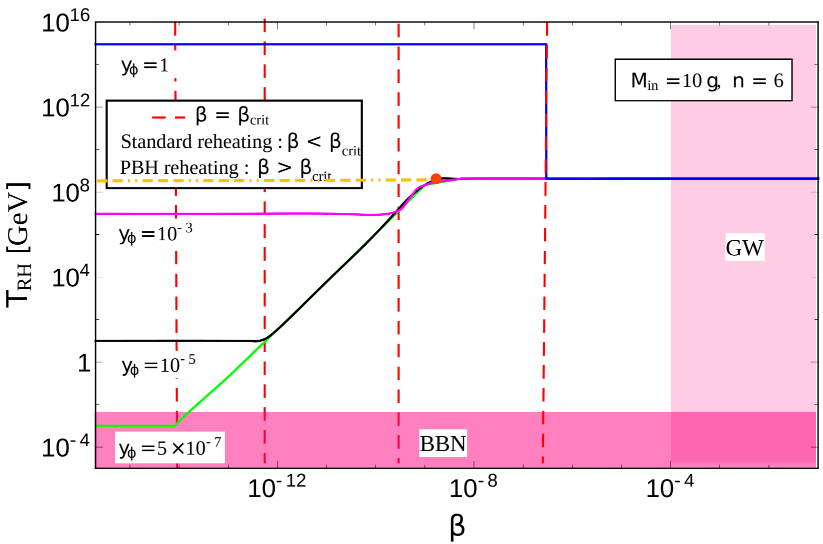
  $y_\phi^{\rm cst} \simeq 3.3 \times 10^{-3}$, $\quad \beta_{crit}^{\rm BH} \sim 1.2 \times 10^{-9}$

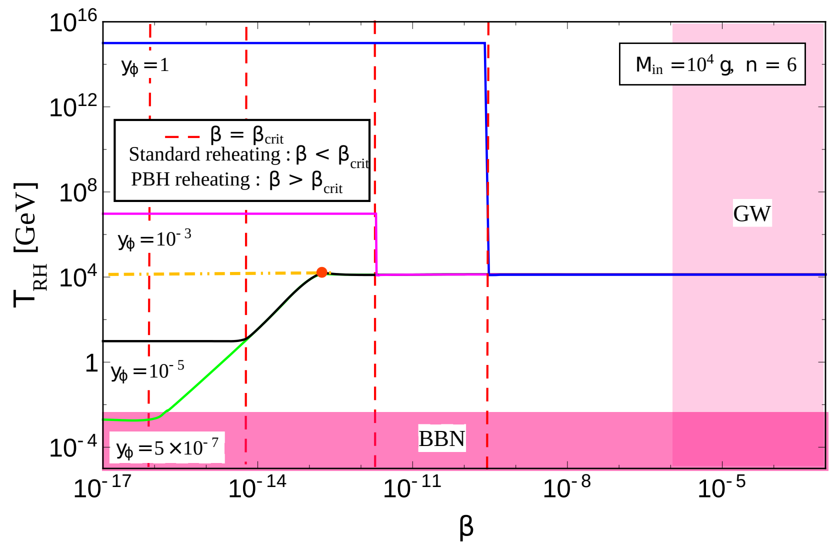
  $\beta_{crit}^{\phi} \propto y_\phi^{\frac{12\omega-4}{3-3\omega}}$

- $M_{\rm in} = 10^4\, g$:

  $y_\phi^{\rm cst} \simeq 10^{-4}$, $\quad \beta_{crit}^{\rm BH} \sim 1.2 \times 10^{-13}$

  $\beta_{crit}^{\phi} \propto y_\phi^{\frac{12\omega-4}{3-3\omega}}$





Extended mass case:

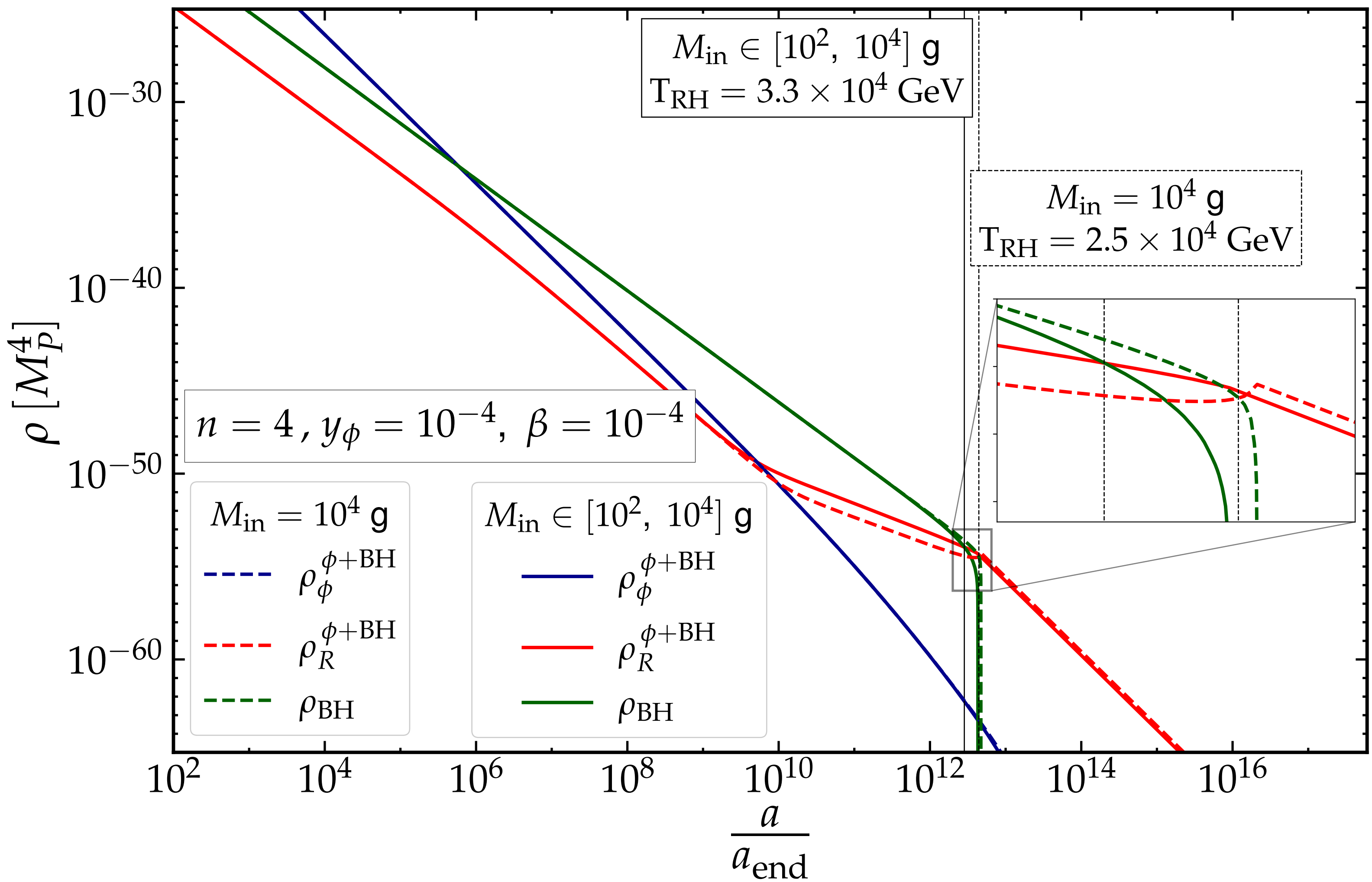
- **PBHs could be formed over a prolonged period of time, thus would have an extended mass distribution**
- **We consider the power-law distribution**

$$f_{\rm BH}(M_i,t_i)=\begin{cases} CM_i^{-\alpha}, & \textbf{for } M_{\rm min}\leq M_i\leq M_{\rm in}\,, \\ 0, & \textbf{otherwise}\,. \end{cases}, \quad \textbf{where} \quad \alpha=\frac{2+4\omega}{1+\omega}$$

- **The energy densities evolve according to :**

$$\begin{aligned}
\dot{\rho_\phi}+3(1+\omega_\phi)H\rho_\phi &= -(1+\omega_\phi)\Gamma_\phi\rho_\phi\,,\\
\dot{\rho}_{\rm BH}+3H\rho_{\rm BH} &= \frac{a_{\rm in}^3}{a^3}\int_{\widetilde{M}}^{M_{\rm in}}\frac{dM}{dt}f_{\rm BH}(M_i,t_i)dM_i,\\
\dot{\rho}_R+4H\rho_R &= (1+\omega_\phi)\Gamma_\phi\rho_\phi-\frac{a_{\rm in}^3}{a^3}\int_{\widetilde{M}}^{M_{\rm in}}\frac{dM}{dt}f_{\rm BH}(M_i,t_i)dM_i\,,\\
\rho_\phi+\rho_R+\rho_{\rm BH} &= 3H^2M_P^2\,.
\end{aligned}$$

- **There no substantial difference in the results for monochromatic and extended mass distribution**
  - **Although the evolution of energy densities might be different, the $T_{\rm RH}$ depends mostly on the evaporation point, $\tau_{\rm BH}\to$ depends on largest mass which is $M_{\rm in}$ in both cases**



$M_{\rm in} \in [10^2,\ 10^4]$ g
$T_{\rm RH} = 3.3 \times 10^4$ GeV
$M_{\rm in} = 10^4$ g
$T_{\rm RH} = 2.5 \times 10^4$ GeV
$n = 4\,,\ y_\phi = 10^{-4},\ \beta = 10^{-4}$
$M_{\rm in} = 10^4$ g
$\rho_\phi^{\phi+\rm BH}$
$\rho_R^{\phi+\rm BH}$
$\rho_{\rm BH}$
$M_{\rm in} \in [10^2,\ 10^4]$ g
$\rho_\phi^{\phi+\rm BH}$
$\rho_R^{\phi+\rm BH}$
$\rho_{\rm BH}$
$\rho\ [M_P^4]$
10^{-30}
10^{-40}
10^{-50}
10^{-60}
10^2
10^4
10^6
10^8
10^{10}
10^{12}
10^{14}
10^{16}
$\frac{a}{a_{\rm end}}$

Conclusions

- **We discussed how PBHs can affect the post-inflation reheating dynamics where the radiation bath is simultaneously generated by :**
  - ► **decaying inflaton via $y_\phi \phi \bar{f} f$ interactions, for potential $V(\phi) \propto \phi^n$, $n \geq 4$,**
  - ► **evaporating PBHs with initial energy fraction β, and mass $M_{\rm in}$**
- **We analyzed the parameter space (y_ϕ, β, $M_{\rm in}$) and shown that there is a extremely rich phenomenology of different evolution scenarios ...**
- **PBHs can dominate the reheating process, and the energy budget of the universe $\rightarrow$ $T_{\rm RH}$ can change drastically in the presence of PBH**
  - ► **One important feature is that $T_{\rm RH}$ can be significantly modified without PBHs ever dominating the energy budget of the Universe.**

THANK YOU FOR YOUR ATTENTION !

- λ **is constrained by CBM [Drewes, Kang and Mun, JHEP 11 (2017), 072]**

$$\lambda = \quad \alpha_1^n \left(\frac{3\pi^2 r A_{\mathscr{R}}}{2}\right)^4 \left[\frac{n^2 + n + \sqrt{n^2 + 3\alpha(2+n)(1-n_s)}}{n(2+n)}\right]^n$$

$A_{\mathscr{R}} \sim 2.19 \times 10^{-9}$ **is the amplitude of the scalar perturbations,** r **the tensor-to-scalar ratio, and** n_s **the spectral index.**

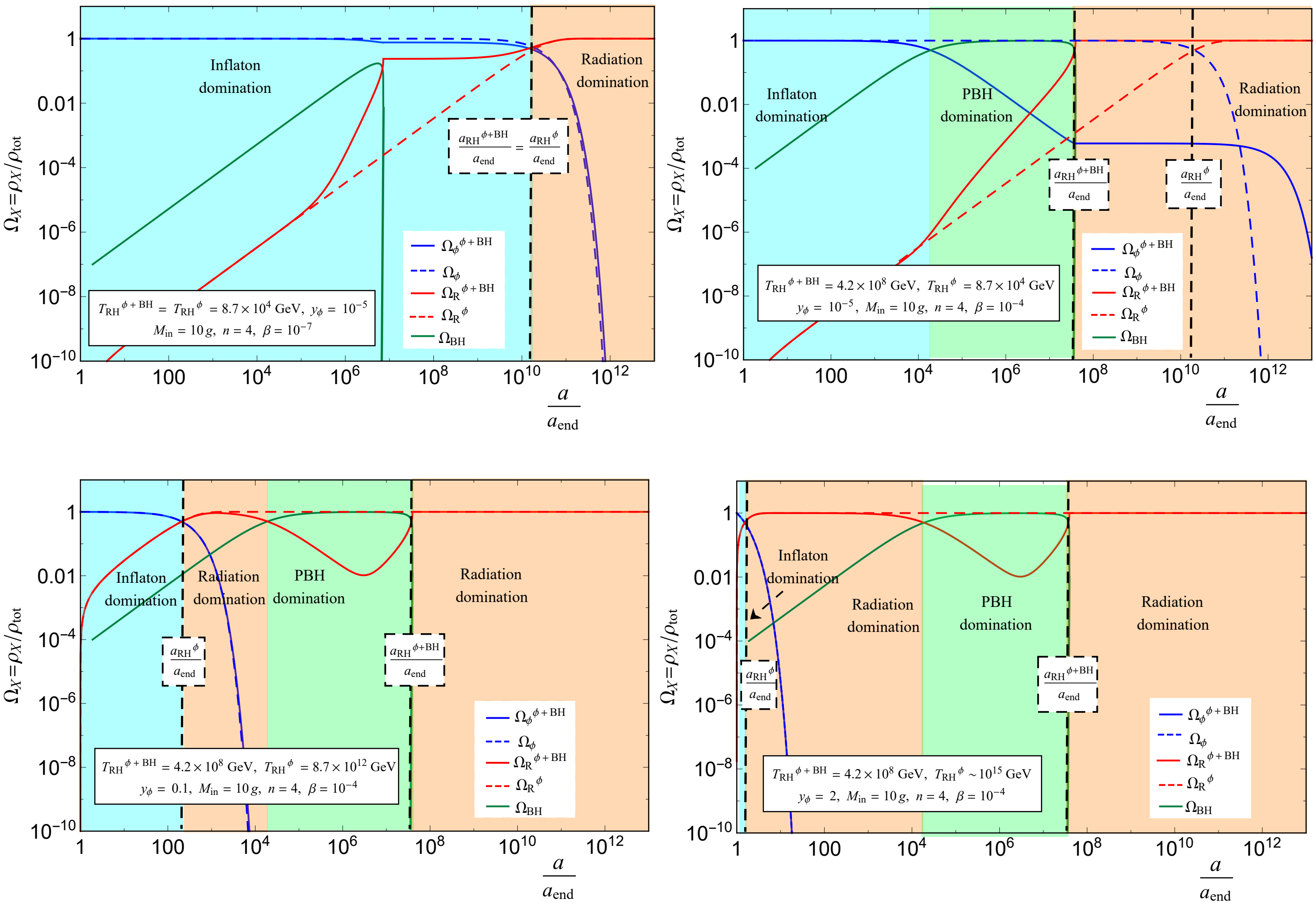
- **The field value at the end of the inflation can be written as**

$$\phi_{\rm end} = \frac{M_P}{\alpha_1} \ln\left(\frac{n}{\sqrt{3\alpha}} + 1\right).$$

$$V(\phi_{\rm end}) = \frac{\lambda\, M_P^4}{\alpha_1^4} \left(\frac{n}{n + \sqrt{3\alpha}}\right)^n$$

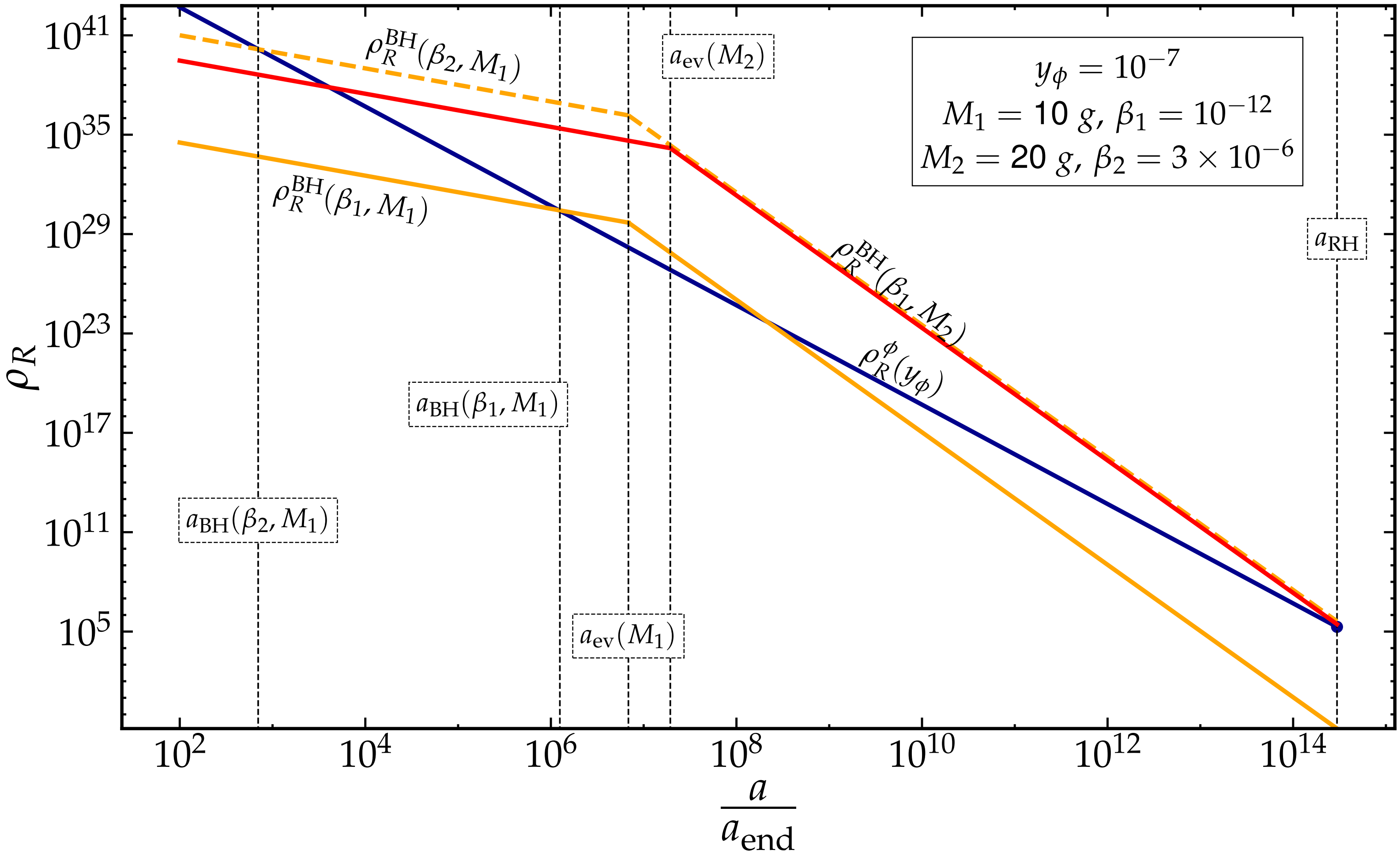
- **Energy density at the end of inflation is then**

$$\rho_{\rm end} = \frac{3}{2} V(\phi_{\rm end}) = \frac{3\,\lambda\, M_P^4}{2\,\alpha_1^4} \left(\frac{n}{n + \sqrt{3\alpha}}\right)^n.$$



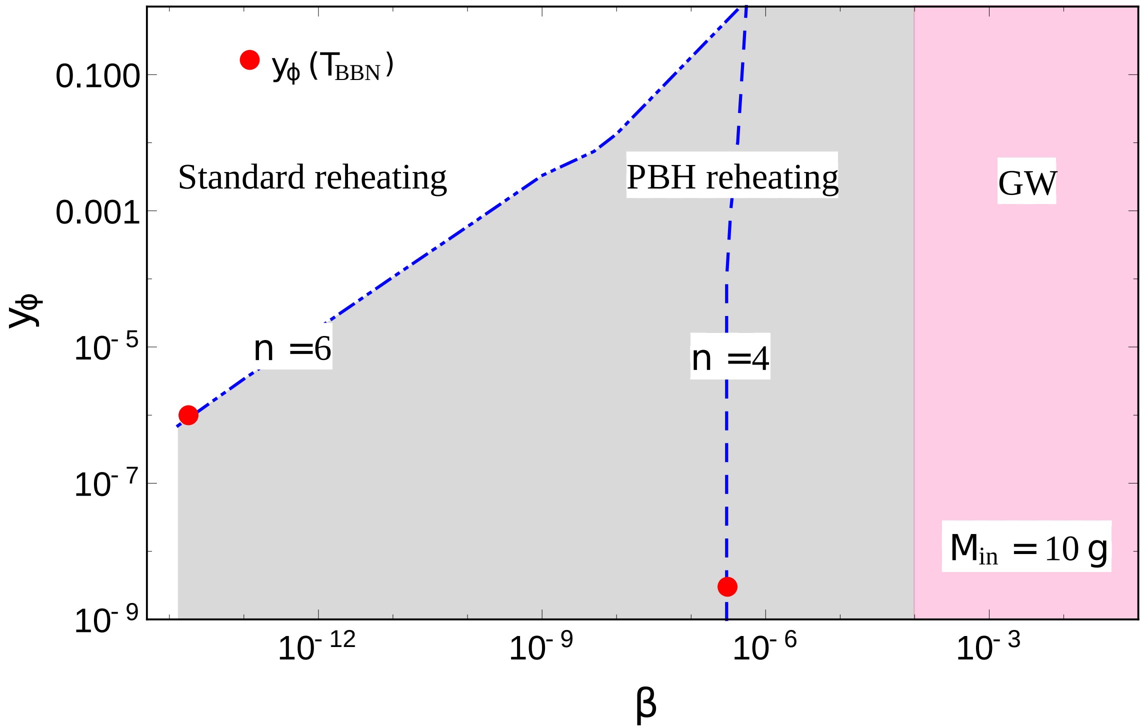
Inflaton domination
Radiation domination
$a_{RH}^{\phi+BH}/a_{end} = a_{RH}^{\phi}/a_{end}$
$T_{RH}^{\phi+BH} = T_{RH}^{\phi} = 8.7\times10^4$ GeV, $y_\phi = 10^{-5}$
$M_{in} = 10\,g$, $n = 4$, $\beta = 10^{-7}$
$\Omega_\phi^{\phi+BH}$
Ω_ϕ
$\Omega_R^{\phi+BH}$
Ω_R^{ϕ}
Ω_{BH}
$\Omega_X = \rho_X/\rho_{tot}$
a/a_{end}
Inflaton domination
PBH domination
Radiation domination
$a_{RH}^{\phi+BH}/a_{end}$
a_{RH}^{ϕ}/a_{end}
$T_{RH}^{\phi+BH} = 4.2\times10^8$ GeV, $T_{RH}^{\phi} = 8.7\times10^4$ GeV
$y_\phi = 10^{-5}$, $M_{in} = 10\,g$, $n = 4$, $\beta = 10^{-4}$
Inflaton domination
Radiation domination
PBH domination
Radiation domination
$T_{RH}^{\phi+BH} = 4.2\times10^8$ GeV, $T_{RH}^{\phi} = 8.7\times10^{12}$ GeV
$y_\phi = 0.1$, $M_{in} = 10\,g$, $n = 4$, $\beta = 10^{-4}$
Inflaton domination
Radiation domination
PBH domination
Radiation domination
$T_{RH}^{\phi+BH} = 4.2\times10^8$ GeV, $T_{RH}^{\phi} \sim 10^{15}$ GeV
$y_\phi = 2$, $M_{in} = 10\,g$, $n = 4$, $\beta = 10^{-4}$

- **Even when PBH do not dominate, their evaporation can dominate the reheating process**
  - **e.g: for quartic potential, $\beta \lesssim 3 \times 10^{-6}$ implies PBHs never dominate, BUT ...**



Results: y_ϕ versus β

- **Summarizing, the dynamics is determined by:** y_ϕ, β, **and** $M_{\rm in}$



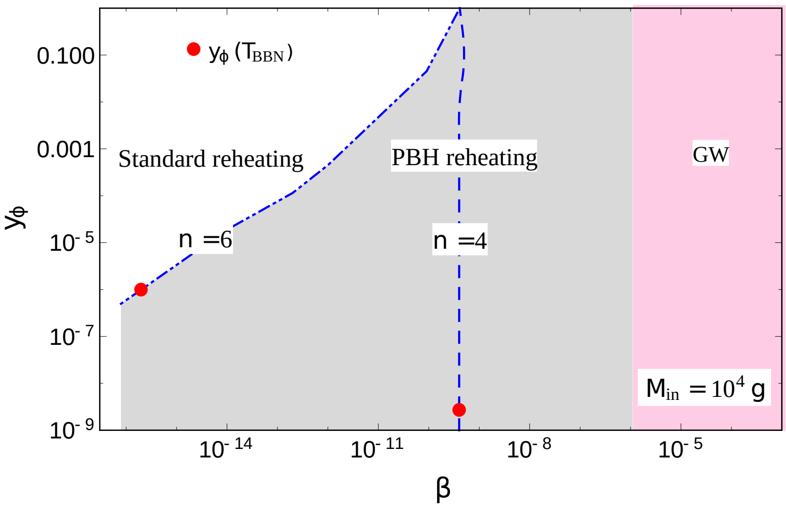
- $M_{\rm in} = 10\,g$

  - $n = 4:\ y_\phi^{\rm cst} = 6 \times 10^{-4}$

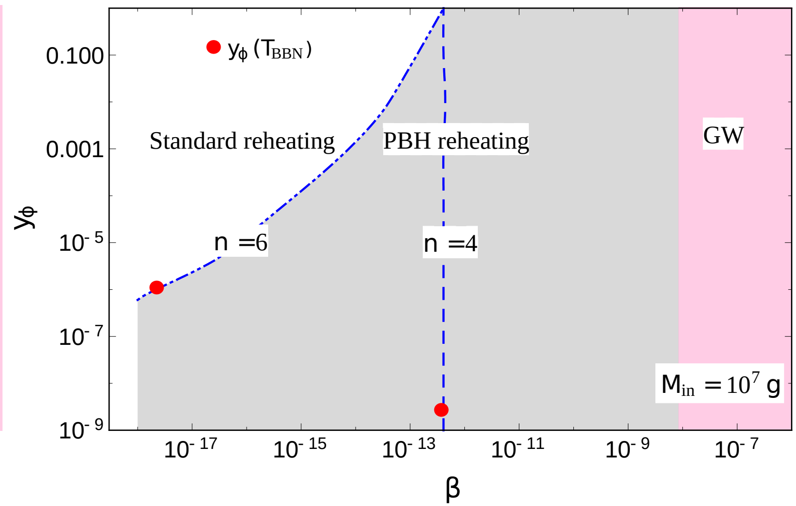
    $\beta_{crit}^{\phi} \simeq \beta_{crit}^{\rm BH} \sim 3 \times 10^{-7}$

  - $n = 6:\ y_\phi^{\rm cst} \sim 3.3 \times 10^{-3}$

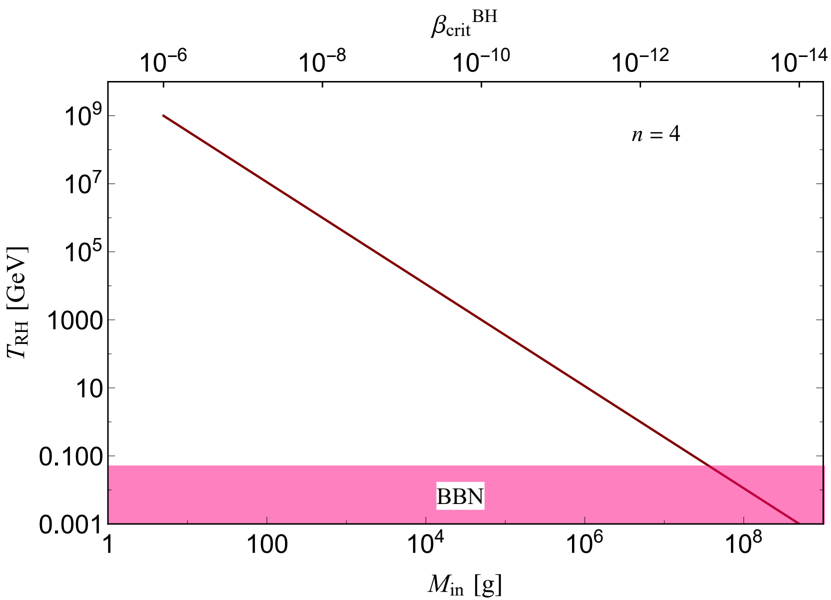
    $\beta_{crit}^{\phi} \propto y_\phi^{\frac{4}{3}}$, and $\beta_{crit}^{\rm BH} \sim 1.2 \times 10^{-9}$



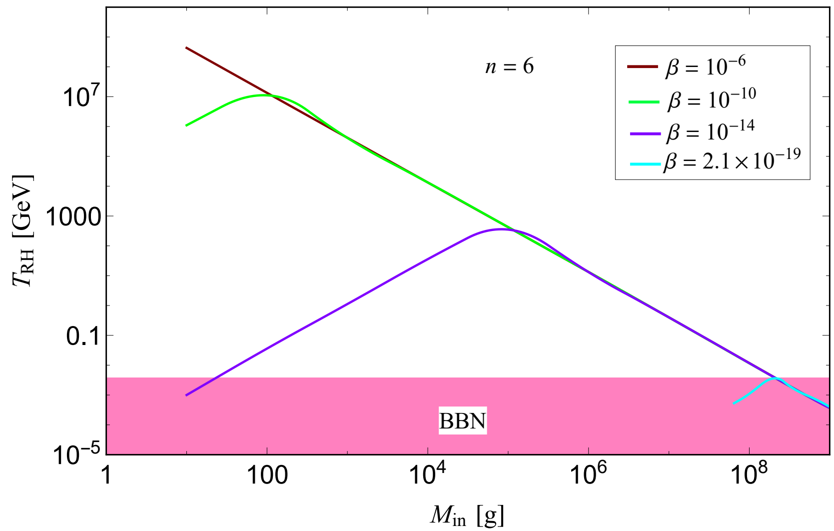
$y_\phi(T_{BBN})$
Standard reheating
PBH reheating
GW
n =6
n =4
$M_{in} = 10^4$ g
0.100
0.001
10^{-5}
10^{-7}
10^{-9}
y_ϕ
10^{-14}
10^{-11}
10^{-8}
10^{-5}
β



$y_\phi(T_{BBN})$
Standard reheating
PBH reheating
GW
n =6
n =4
$M_{in} = 10^7$ g
0.100
0.001
10^{-5}
10^{-7}
10^{-9}
y_ϕ
10^{-17}
10^{-15}
10^{-13}
10^{-11}
10^{-9}
10^{-7}
β



$\beta_{\rm crit}{}^{\rm BH}$
10^{-6}
10^{-8}
10^{-10}
10^{-12}
10^{-14}
$n = 4$
$T_{\rm RH}$ [GeV]
BBN
$M_{\rm in}$ [g]



$n = 6$
$\beta = 10^{-6}$
$\beta = 10^{-10}$
$\beta = 10^{-14}$
$\beta = 2.1 \times 10^{-19}$
$T_{\rm RH}$ [GeV]
BBN
$M_{\rm in}$ [g]

Extended Mass:

- **The comoving number density of PBHs with initial masses within an infinitesimal range of $[M_i, M_i + dM_i]$ remains constant until the time when they completely evaporate, resulting in a drop of the number density to zero:**

$$a^3 f_{\rm BH}(M,t)dM = a_{\rm in}^3 f_{\rm BH}(M_i,t_i)dM_i$$

- $\widetilde{M}(a)$ **can be estimated as**

$$\widetilde{M}(a) = M_i \left(\frac{2\sqrt{3}\varepsilon}{1+\omega}\right)^{1/3} \left(\frac{M_P^5}{M_i^3\sqrt{\rho_{\rm end}}}\right)^{1/3} \left(\frac{a}{a_{\rm in}}\right)^{\frac{1}{2}(1+\omega)} .$$