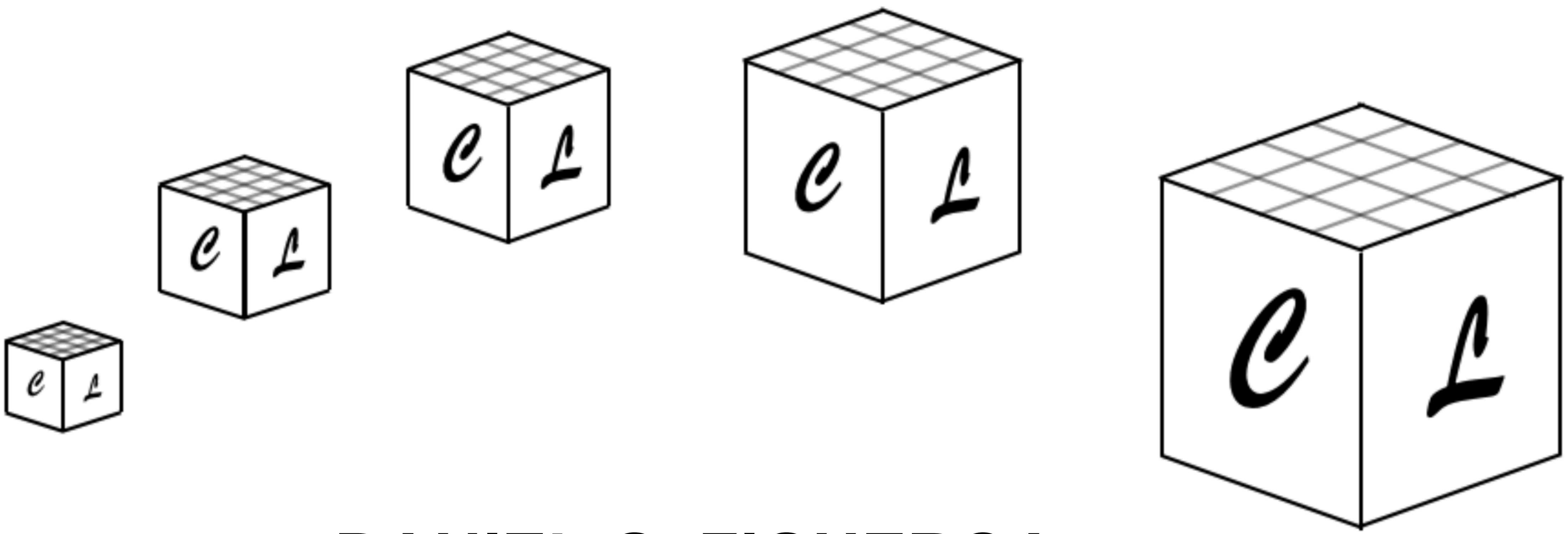
Strong Backreaction in Axion-Inflation



DANIEL G. FIGUEROA
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AstroParticle Symposium 2023, Institute Pascal, Université Paris-Saclay

PHYSICAL REVIEW LETTERS **131,** 151003 (2023)

Strong Backreaction Regime in Axion Inflation

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We study the nonlinear dynamics of axion inflation, capturing for the first time the inhomogeneity and full dynamical range during strong backreaction, till the end of inflation. Accounting for inhomogeneous effects leads to a number of new relevant results, compared to spatially homogeneous studies: (i) the number of extra efoldings beyond slow-roll inflation increases very rapidly with the coupling, (ii) oscillations of the inflaton velocity are attenuated, (iii) the tachyonic gauge field helicity spectrum is smoothed out (i.e., the spectral oscillatory features disappear), broadened, and shifted to smaller scales, and (iv) the nontachyonic helicity is excited, reducing the chiral asymmetry, now scale dependent. Our results are expected to impact strongly on the phenomenology and observability of axion inflation, including gravitational wave generation and primordial black hole production.

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(e-Print: 2303.17436 [astro-ph.CO])

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First exact* calculation of non-linear dynamics of axion inflation (till the end of inflation)

(*Inhomogeneity & full dynamical range)

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Axion-Inflation

Freese, Frieman, Olinto '90; . . .

Shift symmetry $\phi \to \phi + const.$

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$$\frac{\phi}{4\Lambda} F\tilde{F}$$

(Shift Symm.)

$$\phi \longrightarrow \phi + const\,.$$

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ϕ

Axion = Inflaton

Gauge Sector

Axion-Inflation

Freese, Frieman, Olinto '90; . . .

Shift symmetry $\phi \to \phi + const.$

Potential

$$V(\phi) + \frac{\phi}{4\Lambda} F\tilde{F}$$

(Shift Symm.)

$$\phi \longrightarrow \phi + const\,.$$

Due to
Non-Perturbative
effects

Axion-Inflation

Freese, Frieman, Olinto '90; . . .

Shift symmetry $\phi \to \phi + const.$

$$V(\phi) + \frac{\phi}{4\Lambda} F\tilde{F} \qquad \Rightarrow \qquad A''_{\pm} + \left(k^2 \pm \frac{k\dot{\phi}}{\Lambda H \tau}\right) A_{\pm} = 0$$

(Shift Symm.)

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Axion-Inflation

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L.Sorbo et al 2006-2012

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Chiral instability

$$A_+ \propto e^{\pi \xi}\,, \quad |A_-| \ll |A_+|$$

A_+ exponentially amplified

Axion-Inflation

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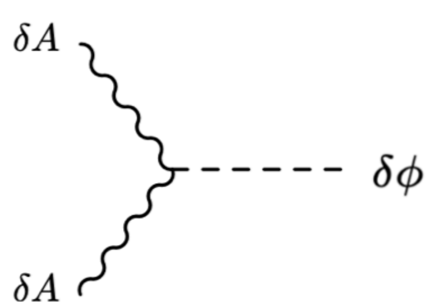
Inflaton perturbations $\delta\phi$ through inverse decay (highly non-Gaussian)



Barnaby, Peloso '10
Planck '15

Axion-Inflation

Freese, Frieman, Olinto '90; . . .

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Amplitude $\delta\phi$ must be bounded

Otherwise too many

Primordial Black Holes (PBH) !

Inflaton perturbations $\delta\phi$
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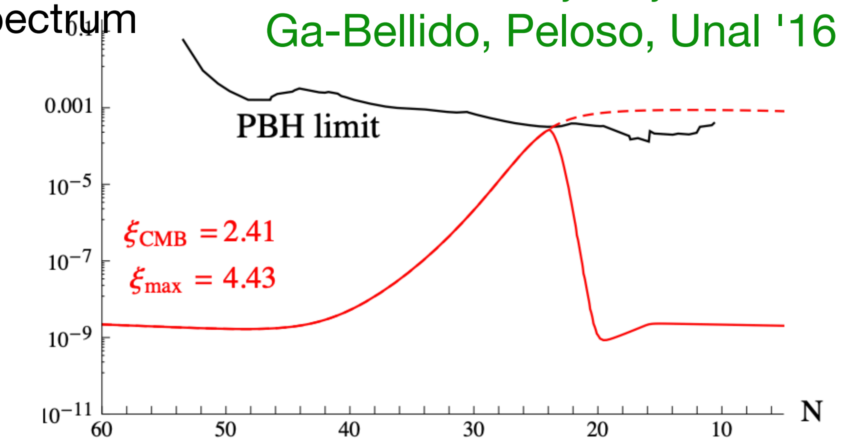
Power spectrum

Linde, Mooij, Pajer '13
Ga-Bellido, Peloso, Unal '16



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$(\xi \propto \dot{\phi})$

Only one chirality of gauge field

then… chiral GWs !

$$\{E_iE_j + B_iB_j\}^{\mathrm{TT}}$$

$h_L \quad , \quad \cancel{h_R}$

Cook & Sorbo '11
Amber & Sorbo '12

Inflaton perturbations $\delta\phi$ through inverse decay (highly non-Gaussian)

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A_+ exponentially amplified

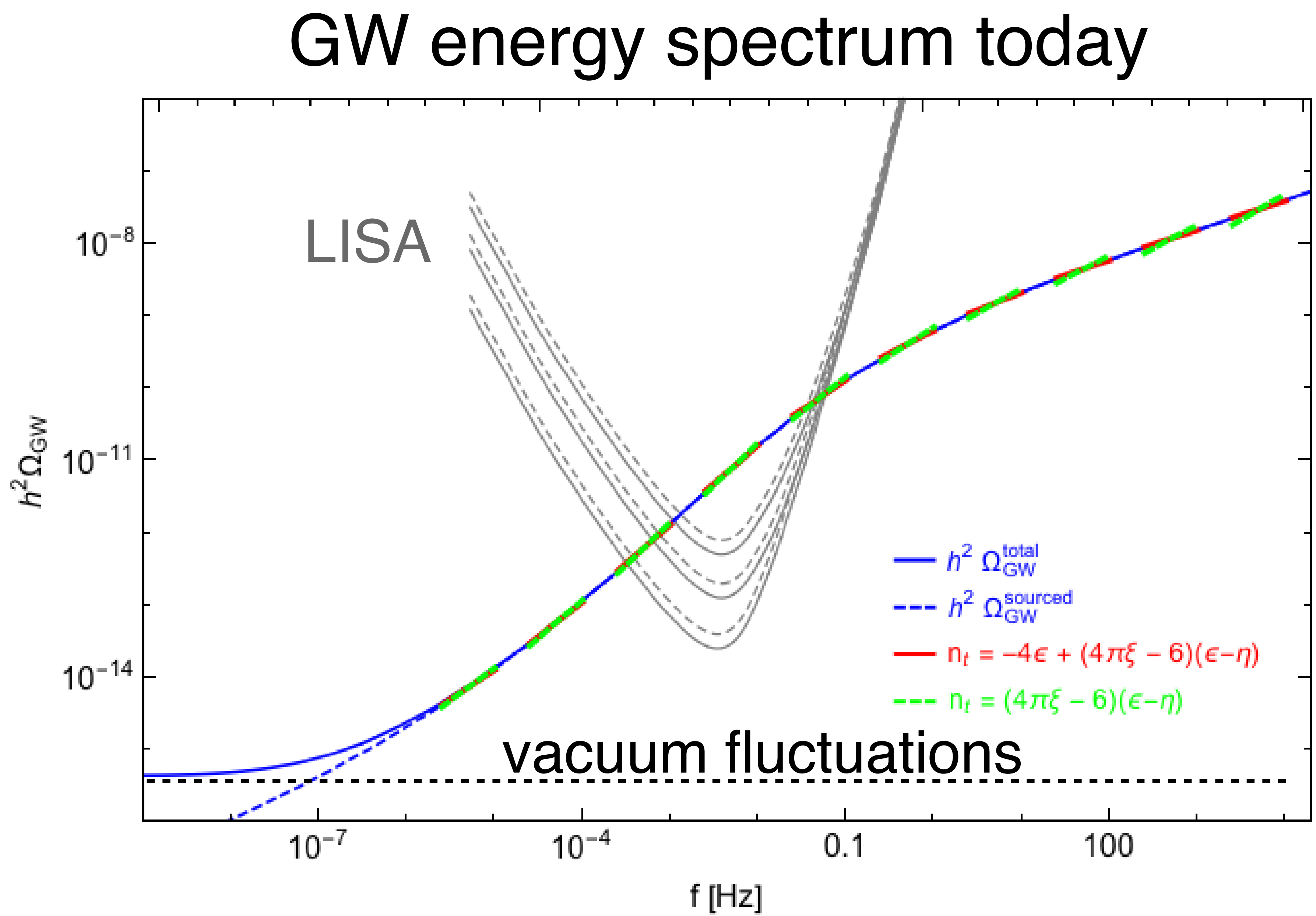
$(\xi \propto \dot{\phi})$

Only one chirality of gauge field
then… chiral GWs !

$$h''_{ij} + 2\mathcal{H}h'_{ij} - \nabla^2 h_{ij} = 16\pi G \Pi^{\mathrm{TT}}_{ij} \propto \{E_i E_j + B_i B_j\}^{TT}$$

GW left-chirality only ! $\Leftarrow$ A_μ Chiral

Axion-Inflation



Blue-Tilted

+ Chiral

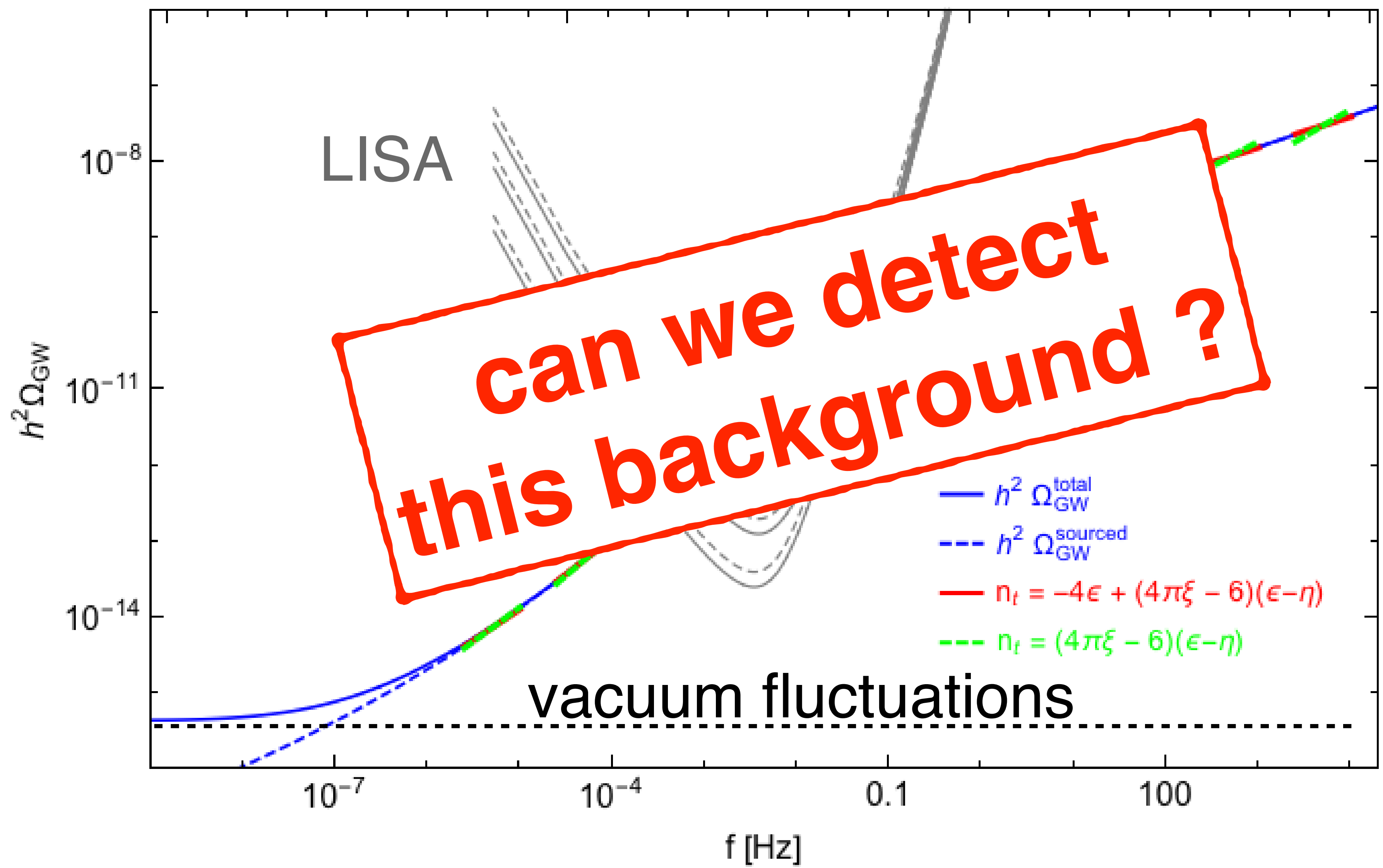
+ Non-G

GW background

Bartolo et al '16, 1610.06481

Axion-Inflation

GW energy spectrum today



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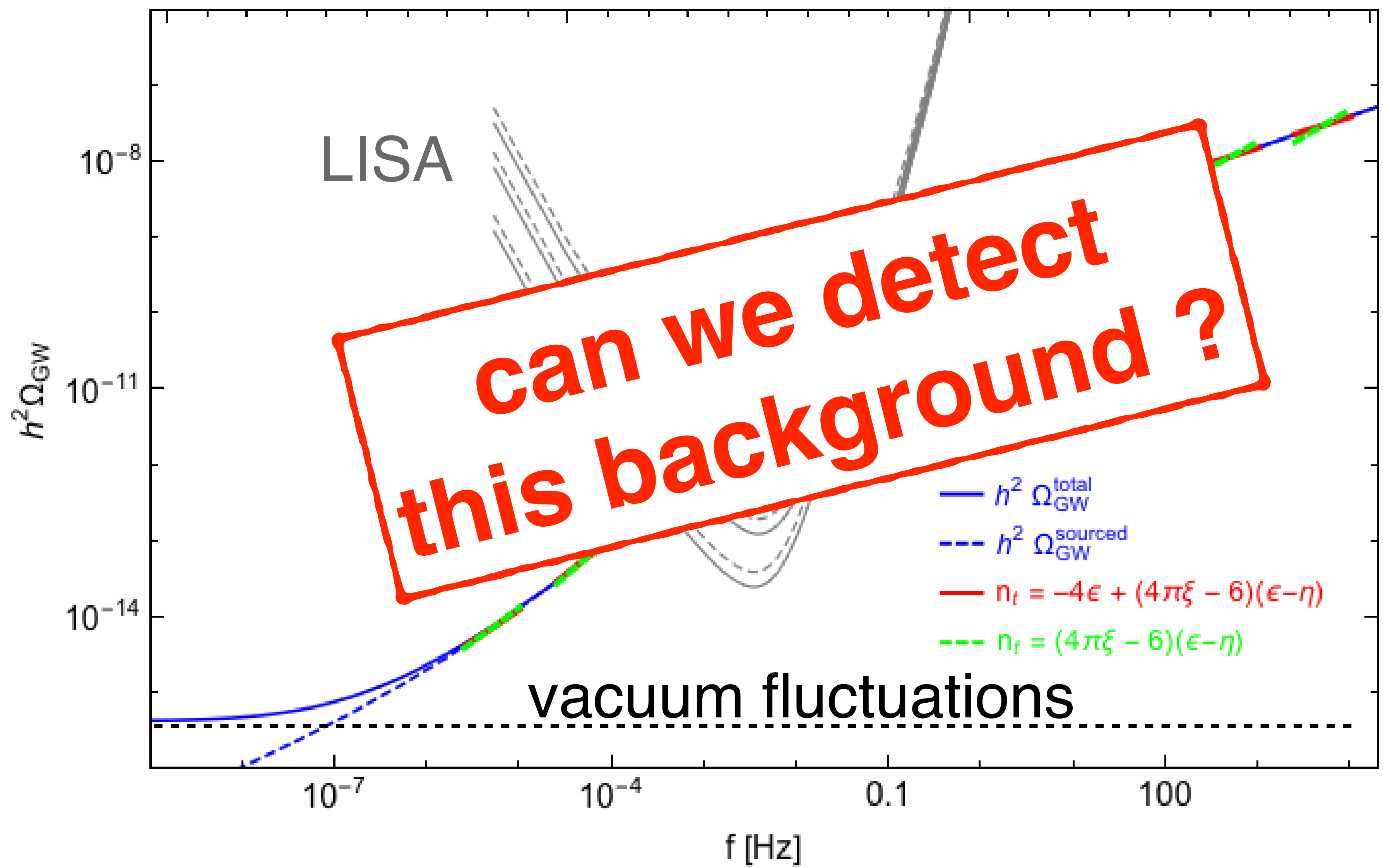

Blue-Tilted

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GW background

As $A_+ \propto e^{\dot{\phi}}$, GWs very sensitive to choice of $V(\phi)$ and calculation details

Bartolo et al '16, 1610.06481

Axion-Inflation

PROBLEM: PNG, GW and PBH $\longrightarrow$ **Analytical approximations !**

Axion-Inflation

PROBLEM: PNG, GW and PBH → **Analytical approximations !**

Let's have a look to the full problem !

$$\left(V(\phi) = \frac{1}{2} m^2 \phi^2 \right)$$

Axion-Inflation

PROBLEM: PNG, GW and PBH $\longrightarrow$ **Analytical approximations !**

$$\pi_\phi \equiv \dot\phi \ , \quad E_i \equiv \dot A_i \ , \quad B_i \equiv \epsilon_{ijk}\partial_j A_k$$

$$\tilde\pi_\phi = a^3\pi_\phi \ , \quad \tilde{\vec{E}} = a\vec{E} \ , \ \pi_a \equiv \dot a$$

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EoM

Local EoM $(\vec{\mathbf{x}}, \mathbf{t})$

$$\begin{cases} \dot{\tilde{\pi}}_\phi = a\vec{\nabla}^2\phi - a^3 m^2 \phi + \dfrac{1}{a\Lambda}\tilde{\vec{E}} \cdot \vec{B}\ , \\[2em] \dot{\tilde{\vec{E}}} = -\dfrac{1}{a}\vec{\nabla} \times \vec{B} - \dfrac{1}{a^3\Lambda}\tilde{\pi}_\phi \vec{B} + \dfrac{1}{a\Lambda}\vec{\nabla}\phi \times \tilde{\vec{E}}\ . \end{cases}$$

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Hom. EoM $(\mathbf{t})$

$$\dot{\pi}_a = -\frac{a}{6m_{pl}^2}(\rho + 3p) = \frac{a}{3m_{pl}^2}\langle \underbrace{-2K_\phi}_{\text{(Kin)}} + \underbrace{V}_{\text{(Pot)}} - \underbrace{K_A}_{\text{(Elec)}} - \underbrace{G_A}_{\text{(Mag)}} \rangle$$

$$\left(\begin{array}{c} K_\phi \equiv \frac{1}{2}\pi_\phi^2 = \frac{1}{2a^6}\tilde{\pi}_\phi^2\ , \quad G_\phi \equiv \frac{1}{2a^2}\sum(\partial_i\phi)^2\ , \quad V \equiv \frac{1}{2}m^2\phi^2\ , \\ K_A \equiv \frac{1}{2}\sum_i \frac{E_i^2}{a^2} = \frac{1}{2}\sum_i \frac{\tilde{E}_i^2}{a^4}\ , \quad G_A \equiv \frac{1}{2}\sum_i \frac{B_i^2}{a^4} \end{array} \right)$$

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Non-Linear dynamics

Hom. EoM $(\mathbf{t})$

$$\dot{\pi}_a = -\frac{a}{6m_{pl}^2}(\rho + 3p) = \frac{a}{3m_{pl}^2}\langle -2\underset{\text{(Kin)}}{K_\phi} + \underset{\text{(Pot)}}{V} - \underset{\text{(Elec)}}{K_A} - \underset{\text{(Mag)}}{G_A}\rangle$$

$$\left(\begin{gathered} K_\phi \equiv \frac{1}{2}\pi_\phi^2 = \frac{1}{2a^6}\tilde{\pi}_\phi^2\ , \quad G_\phi \equiv \frac{1}{2a^2}\sum(\partial_i\phi)^2\ , \quad V \equiv \frac{1}{2}m^2\phi^2\ , \\ K_A \equiv \frac{1}{2}\sum_i \frac{E_i^2}{a^2} = \frac{1}{2}\sum_i \frac{\tilde{E}_i^2}{a^4}\ , \quad G_A \equiv \frac{1}{2}\sum_i \frac{B_i^2}{a^4} \end{gathered}\right)$$

Axion-Inflation

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$$\pi_\phi \equiv \dot\phi \ , \quad E_i \equiv \dot A_i \ , \quad B_i \equiv \epsilon_{ijk}\partial_j A_k$$

$$\tilde\pi_\phi = a^3\pi_\phi \ , \quad \vec{\tilde E} = a\vec E \ , \ \pi_a \equiv \dot a$$

EoM

Linear Regime

Local EoM $(\vec{\mathbf{x}}, \mathbf{t})$

$$\begin{cases} \dot{\tilde\pi}_\phi = \cancel{a\vec\nabla^2\phi} - a^3 m^2\phi + \cancel{\frac{1}{a\Lambda}\vec{\tilde E}\cdot\vec B} \ , \\ \dot{\vec{\tilde E}} = -\frac{1}{a}\vec\nabla\times\vec B - \frac{1}{a^3\Lambda}\tilde\pi_\phi\vec B + \cancel{\frac{1}{a\Lambda}\vec\nabla\phi\times\vec{\tilde E}} \ . \end{cases}$$

Interaction

Hom. EoM $(\mathbf{t})$

$$\dot\pi_a = -\frac{a}{6m_{pl}^2}(\rho + 3p) = \frac{a}{3m_{pl}^2}\left\langle -2\underset{\text{(Kin)}}{K_\phi} + \underset{\text{(Pot)}}{V} - \underset{\text{(Elec)}}{K_A} - \underset{\text{(Mag)}}{G_A}\right\rangle$$

$$\left(\begin{array}{c} K_\phi \equiv \frac{1}{2}\pi_\phi^2 = \frac{1}{2a^6}\tilde\pi_\phi^2 \ , \quad G_\phi \equiv \frac{1}{2a^2}\sum(\partial_i\phi)^2 \ , \quad V \equiv \frac{1}{2}m^2\phi^2 \ , \\ K_A \equiv \frac{1}{2}\sum_i\frac{E_i^2}{a^2} = \frac{1}{2}\sum_i\frac{\tilde E_i^2}{a^4} \ , \quad G_A \equiv \frac{1}{2}\sum_i\frac{B_i^2}{a^4} \end{array} \right)$$

Axion-Inflation

PROBLEM: PNG, GW and PBH ⟶ **Analytical approximations !**

$$\pi_\phi \equiv \dot{\phi}\ , \quad E_i \equiv \dot{A}_i\ , \quad B_i \equiv \epsilon_{ijk}\partial_j A_k$$

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EoM

Linear Regime

Local EoM $(\vec{\mathbf{x}}, \mathbf{t})$

$$\begin{cases} \dot{\tilde{\pi}}_\phi = \cancel{a\vec{\nabla}^2\phi} - a^3 m^2 \phi \cancel{+ \frac{1}{a\Lambda}\tilde{\vec{E}}\cdot\vec{B}}\ , \\ \dot{\tilde{\vec{E}}} = -\frac{1}{a}\vec{\nabla}\times\vec{B} - \frac{1}{a^3\Lambda}\tilde{\pi}_\phi\vec{B} \cancel{+ \frac{1}{a\Lambda}\vec{\nabla}\phi\times\tilde{\vec{E}}}\ . \end{cases}$$

Interaction

Hom. EoM $(\mathbf{t})$

$$\dot{\pi}_a = -\frac{a}{6m_{pl}^2}(\rho + 3p) = \frac{a}{3m_{pl}^2}\Big\langle -2\underbrace{K_\phi}_{\text{(Kin)}} + \underbrace{V}_{\text{(Pot)}} \cancel{- \underbrace{K_A}_{\text{(Elec)}} - \underbrace{G_A}_{\text{(Mag)}}} \Big\rangle$$

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$$\pi_\phi \equiv \dot\phi\,, \quad E_i \equiv \dot A_i\,, \quad B_i \equiv \epsilon_{ijk}\partial_j A_k$$

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EoM

Local EoM $(\vec{\mathbf{x}},\mathbf{t})$

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Back-Reaction

Hom. EoM $(\mathbf{t})$

$$\dot\pi_a = -\frac{a}{6m_{pl}^2}(\rho+3p) = \frac{a}{3m_{pl}^2}\Big\langle -2\underset{\text{(Kin)}}{K_\phi} + \underset{\text{(Pot)}}{V} - \underset{\text{(Elec)}}{K_A} - \underset{\text{(Mag)}}{G_A}\Big\rangle$$

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Axion-Inflation

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Hom. (t) Approx.

Back-Reaction (Homog. Approx.)

Hom. EoM $(\mathbf{t})$

$$\dot\pi_a = -\frac{a}{6m_{pl}^2}(\rho+3p) = \frac{a}{3m_{pl}^2}\Big(\big\langle \underbrace{-2K_\phi}_{\text{(Kin)}} + \underbrace{V}_{\text{(Pot)}}\big\rangle - \big\langle \underbrace{K_A}_{\text{(Elec)}} + \underbrace{G_A}_{\text{(Mag)}}\big\rangle\Big)$$

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Axion-Inflation

PROBLEM: PNG, GW and PBH $\longrightarrow$ **Analytical approximations !**

$$\langle \vec{E}\vec{B}\rangle = -\frac{\lambda}{a^4}\int \frac{dk}{4\pi}k^3\frac{d}{d\tau}\left|A_{-\lambda}(\tau,\vec{k})\right|^2 \quad ;$$

$$\langle K_A + G_A\rangle \equiv \left\langle \frac{E^2+B^2}{2}\right\rangle = \frac{1}{a^4}\int\frac{dk}{4\pi^2}k^2\left(\left|\frac{dA_{-\lambda}(\tau,\vec{k})}{d\tau}\right|^2 + k^2\left|A_{-\lambda}(\tau,\vec{k})\right|^2\right) \quad ;$$

$$\lambda = \pm \begin{cases} \lambda = +\,,\ \text{if}\,\dot\phi > 0 \\ \lambda = -\,,\ \text{if}\,\dot\phi < 0 \end{cases}$$

EoM

Local EoM $(\vec{\mathbf{x}}, \mathbf{t})$

$$\begin{cases} \dot{\tilde{\pi}}_\phi = \cancel{a\vec\nabla^2\phi} - a^3m^2\phi + \dfrac{1}{a\Lambda}\langle\tilde{\vec{E}}\cdot\vec{B}\,\rangle \\[2ex] \dot{\tilde{\vec{E}}} = -\dfrac{1}{a}\vec\nabla\times\vec{B} - \dfrac{1}{a^3\Lambda}\tilde\pi_\phi\vec{B} + \cancel{\dfrac{1}{a\Lambda}\vec\nabla\phi\times\tilde{\vec{E}}}\,. \end{cases}$$

Hom. (t) Approx.

Back-Reaction (Homog. Approx.)

Hom. EoM (t)

$$\dot\pi_a = -\frac{a}{6m_{pl}^2}(\rho+3p) = \frac{a}{3m_{pl}^2}\Big(\langle -2K_\phi + V\rangle - \langle K_A + G_A\rangle\Big)$$

(Kin) (Pot) (Elec) (Mag)

$$\left(\begin{array}{c} K_\phi \equiv \frac{1}{2}\pi_\phi^2 = \frac{1}{2a^6}\tilde\pi_\phi^2\,, \quad G_\phi \equiv \frac{1}{2a^2}\sum(\partial_i\phi)^2\,, \quad V \equiv \frac{1}{2}m^2\phi^2\,, \\ K_A \equiv \frac{1}{2}\sum_i\frac{E_i^2}{a^2} = \frac{1}{2}\sum_i\frac{\tilde{E}_i^2}{a^4}\,, \quad G_A \equiv \frac{1}{2}\sum_i\frac{B_i^2}{a^4} \end{array}\right)$$

Axion-Inflation

PROBLEM: PNG, GW and PBH $\longrightarrow$ **Analytical approximations !**

$$\langle \vec{E}\vec{B} \rangle = -\frac{\lambda}{a^4}\int \frac{dk}{4\pi} k^3 \frac{d}{d\tau}\left|A_{-\lambda}(\tau,\vec{k})\right|^2 \quad ;$$

$$\langle K_A + G_A \rangle \equiv \left\langle \frac{E^2+B^2}{2} \right\rangle = \frac{1}{a^4}\int \frac{dk}{4\pi^2} k^2 \left(\left|\frac{dA_{-\lambda}(\tau,\vec{k})}{d\tau}\right|^2 + k^2 \left|A_{-\lambda}(\tau,\vec{k})\right|^2 \right) \quad ;$$

$\lambda = \pm$: $\lambda = +\,, \text{if}\,\dot\phi > 0$; $\lambda = -\,, \text{if}\,\dot\phi < 0$

EoM

Local EoM $(\vec{\mathbf{x}}, \mathbf{t})$

$$\begin{cases} \dot{\tilde{\pi}}_\phi = \cancel{a\vec{\nabla}^2\phi} - a^3 m^2 \phi + \dfrac{1}{a\Lambda}\langle \tilde{\vec{E}}\cdot\vec{B} \rangle \\[2ex] \dfrac{d^2 A_\pm(\tau,\vec{k})}{d\tau^2} + \left[k^2 \pm 2\lambda\xi k a H\right] A_\pm(\tau,\vec{k}) = 0 \; \cancel{\dfrac{1}{a\Lambda}\vec{\nabla}\phi \times \tilde{\vec{E}}}\,. \end{cases}$$

Hom. (t) Approx.

Back-Reaction (Homog. Approx.)

Hom. EoM (t)

$$\dot{\pi}_a = -\frac{a}{6m_{pl}^2}(\rho + 3p) = \frac{a}{3m_{pl}^2}\Big(\langle -2K_\phi + V\rangle - \langle K_A + G_A\rangle\Big)$$

(Kin) (Pot) (Elec) (Mag)

DallAgata et al 2019, Domcke et 2020 $\longrightarrow$ **Elaborated Iterative scheme !**

Axion-Inflation

PROBLEM: PNG, GW and PBH → **Analytical approximations !**

$$\left\langle \vec{E}\vec{B} \right\rangle = -\frac{\lambda}{a^4}\int \frac{dk}{4\pi}k^3\frac{d}{d\tau}\left|A_{-\lambda}(\tau,\vec{k})\right|^2 \quad ;$$

$$\left\langle K_A + G_A \right\rangle \equiv \left\langle \frac{E^2+B^2}{2} \right\rangle = \frac{1}{a^4}\int \frac{dk}{4\pi^2}k^2\left(\left|\frac{dA_{-\lambda}(\tau,\vec{k})}{d\tau}\right|^2 + k^2\left|A_{-\lambda}(\tau,\vec{k})\right|^2\right) \quad ;$$

$\lambda = \pm$ → $\lambda = +\,,\ \text{if}\,\dot\phi > 0$; $\lambda = -\,,\ \text{if}\,\dot\phi < 0$

EoM

Local EoM $(\vec{\mathbf{x}},\mathbf{t})$

$$\begin{cases} \dot{\tilde{\pi}}_\phi = \cancel{a\vec{\nabla}^2\phi} - a^3m^2\phi + \dfrac{1}{a\Lambda}\left\langle \tilde{\vec{E}}\cdot\vec{B} \right\rangle \\[2ex] \dfrac{d^2A_\pm(\tau,\vec{k})}{d\tau^2} + \left[k^2 \pm 2\lambda\xi k a H\right]A_\pm(\tau,\vec{k}) = 0 \ \cancel{\dfrac{1}{a\Lambda}\vec{\nabla}\phi\times\tilde{\vec{E}}}\ . \end{cases}$$

Hom. (t) Approx.

Back-Reaction (Homog. Approx.)

Hom. EoM $(\mathbf{t})$

$$\dot{\pi}_a = -\frac{a}{6m_{pl}^2}(\rho+3p) = \frac{a}{3m_{pl}^2}\Big(\big\langle \underbrace{-2K_\phi}_{\text{(Kin)}} + \underbrace{V}_{\text{(Pot)}}\big\rangle - \big\langle \underbrace{K_A}_{\text{(Elec)}} + \underbrace{G_A}_{\text{(Mag)}}\big\rangle\Big)$$

Gorbar et al 2021 → **Correlator Gradient Expansion**

Axion-Inflation

PROBLEM: PNG, GW and PBH $\longrightarrow$ **Analytical approximations !**

Can we do better than homogeneous backreaction ?

EoM

Local EoM $(\vec{\mathbf{x}}, \mathbf{t})$

$$\begin{cases} \dot{\tilde{\pi}}_\phi = \cancel{a\vec{\nabla}^2\phi} - a^3 m^2\phi + \dfrac{1}{a\Lambda}\langle \tilde{\vec{E}}\cdot\vec{B}\,\rangle \\ \dfrac{d^2A_\pm(\tau,\vec{k})}{d\tau^2} + \left[k^2 \pm 2\lambda\xi k a H\right]A_\pm(\tau,\vec{k}) = 0\;\cancel{\dfrac{1}{a\Lambda}\vec{\nabla}\phi\times\tilde{\vec{E}}}\,. \end{cases}$$

Hom. (t) Approx.

Back-Reaction (Homog. Approx.)

Hom. EoM $(\mathbf{t})$

$$\dot{\pi}_a = -\frac{a}{6m_{pl}^2}(\rho+3p) = \frac{a}{3m_{pl}^2}\Big(\langle -2K_\phi + V\rangle - \langle K_A + G_A\rangle\Big)$$

(Kin) (Pot) (Elec) (Mag)

Axion-Inflation

PROBLEM: PNG, GW and PBH → **Analytical approximations !**

Yes, we need a full lattice approach

EoM

Local EoM $(\vec{\mathbf{x}}, \mathbf{t})$

$$\begin{cases} \dot{\tilde{\pi}}_\phi = \cancel{a\vec{\nabla}^2\phi} - a^3 m^2 \phi + \dfrac{1}{a\Lambda}\vec{\tilde{E}}\cdot\vec{B} \\[2ex] \dfrac{d^2 A_\pm(\tau,\vec{k})}{d\tau^2} + \left[k^2 \pm 2\lambda\xi k a H\right] A_\pm(\tau,\vec{k}) = 0 \;\cancel{\dfrac{1}{a\Lambda}\vec{\nabla}\phi\times\vec{\tilde{E}}}\,. \end{cases}$$

Back-Reaction (Source InHom.)

Hom. EoM $(\mathbf{t})$

$$\dot{\pi}_a = -\frac{a}{6m_{pl}^2}(\rho + 3p) = \frac{a}{3m_{pl}^2}\left\langle \underset{\text{(Kin)}}{-2K_\phi} + \underset{\text{(Pot)}}{V} - \underset{\text{(Elec)}}{K_A} - \underset{\text{(Mag)}}{G_A} \right\rangle$$

Axion-Inflation

PROBLEM: PNG, GW and PBH ⟶ **Analytical approximations !**

Yes, we need a full lattice approach

EoM

Local EoM $(\vec{\mathbf{x}}, \mathbf{t})$

$$
\begin{cases}
\dot{\tilde{\pi}}_\phi = \cancel{a\vec{\nabla}^2\phi} - a^3 m^2 \phi + \dfrac{1}{a\Lambda}\vec{\tilde{E}}\cdot\vec{B} \\[2ex]
\dot{\vec{\tilde{E}}} = -\dfrac{1}{a}\vec{\nabla}\times\vec{B} - \dfrac{1}{a^3\Lambda}\tilde{\pi}_\phi\vec{B} + \cancel{\dfrac{1}{a\Lambda}\vec{\nabla}\phi\times\vec{\tilde{E}}}\,.
\end{cases}
$$

Back-Reaction (Source InHom.)

Hom. EoM $(\mathbf{t})$

$$
\dot{\pi}_a = -\frac{a}{6m_{pl}^2}(\rho + 3p) = \frac{a}{3m_{pl}^2}\Big\langle -2\underset{\text{(Kin)}}{K_\phi} + \underset{\text{(Pot)}}{V} - \underset{\text{(Elec)}}{K_A} - \underset{\text{(Mag)}}{G_A} \Big\rangle
$$

Axion-Inflation

PROBLEM: PNG, GW and PBH → **Analytical approximations !**

Yes, we need a full lattice approach

EoM

Local EoM $(\vec{\mathbf{x}}, \mathbf{t})$

$$\begin{cases} \dot{\tilde{\pi}}_\phi = a\vec{\nabla}^2\phi - a^3 m^2 \phi + \dfrac{1}{a\Lambda}\tilde{\vec{E}} \cdot \vec{B} \\[2ex] \dot{\tilde{\vec{E}}} = -\dfrac{1}{a}\vec{\nabla} \times \vec{B} - \dfrac{1}{a^3\Lambda}\tilde{\pi}_\phi \vec{B} + \dfrac{1}{a\Lambda}\vec{\nabla}\phi \times \tilde{\vec{E}}\,. \end{cases}$$

Back-Reaction (Fully InHom.)

Hom. EoM $(\mathbf{t})$

$$\dot{\pi}_a = -\frac{a}{6m_{pl}^2}(\rho + 3p) = \frac{a}{3m_{pl}^2}\langle \underbrace{-2K_\phi}_{\text{(Kin)}} + \underbrace{V}_{\text{(Pot)}} - (\underbrace{K_A}_{\text{(Elec)}} + \underbrace{G_A}_{\text{(Mag)}})\rangle$$

Axion-Inflation

PROBLEM: PNG, GW and PBH $\longrightarrow$ **Analytical approximations !**

Let's "latticize" the system of EOM !

EoM

Local EoM $(\vec{\mathbf{x}}, \mathbf{t})$

$$\begin{cases} \dot{\tilde{\pi}}_\phi = a\vec{\nabla}^2\phi - a^3 m^2 \phi + \dfrac{1}{a\Lambda}\vec{\tilde{E}} \cdot \vec{B} \\ \dot{\vec{\tilde{E}}} = -\dfrac{1}{a}\vec{\nabla} \times \vec{B} - \dfrac{1}{a^3\Lambda}\tilde{\pi}_\phi \vec{B} + \dfrac{1}{a\Lambda}\vec{\nabla}\phi \times \vec{\tilde{E}} \,. \end{cases}$$

Back-Reaction (Fully InHom.)

Hom. EoM $(\mathbf{t})$

$$\dot{\pi}_a = -\frac{a}{6m_{pl}^2}(\rho + 3p) = \frac{a}{3m_{pl}^2}\langle \underbrace{-2K_\phi}_{\text{(Kin)}} + \underbrace{V}_{\text{(Pot)}} - (\underbrace{K_A}_{\text{(Elec)}} + \underbrace{G_A}_{\text{(Mag)}}) \rangle$$

LATTICE FORMULATION of $\phi F\tilde{F}$

EoM

Inflaton EoM

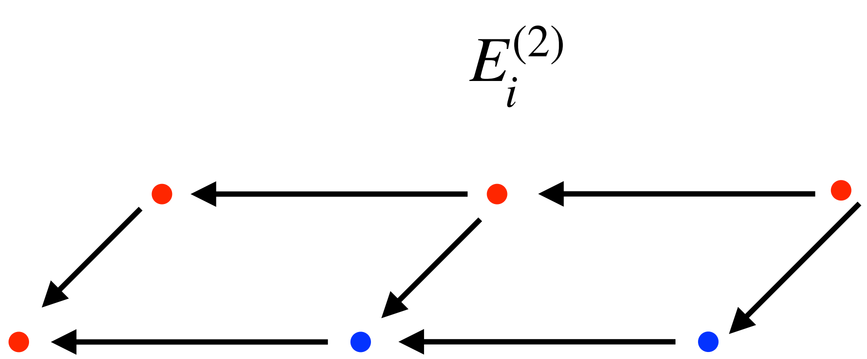
$$\Delta_0^+\left(a^3\pi_\phi\right) = a_{+\frac{\hat{0}}{2}}\sum_i \Delta_i^-\Delta_i^+\phi_{+\frac{\hat{0}}{2}} - a^3_{+\frac{\hat{0}}{2}} m^2\phi_{+\frac{\hat{0}}{2}} + \frac{1}{\Lambda}\sum_i \frac{1}{2} E^{(2)}_{i,+\frac{\hat{0}}{2}}\left(B_i^{(4)} + B_{i,+\hat{0}}^{(4)}\right)$$

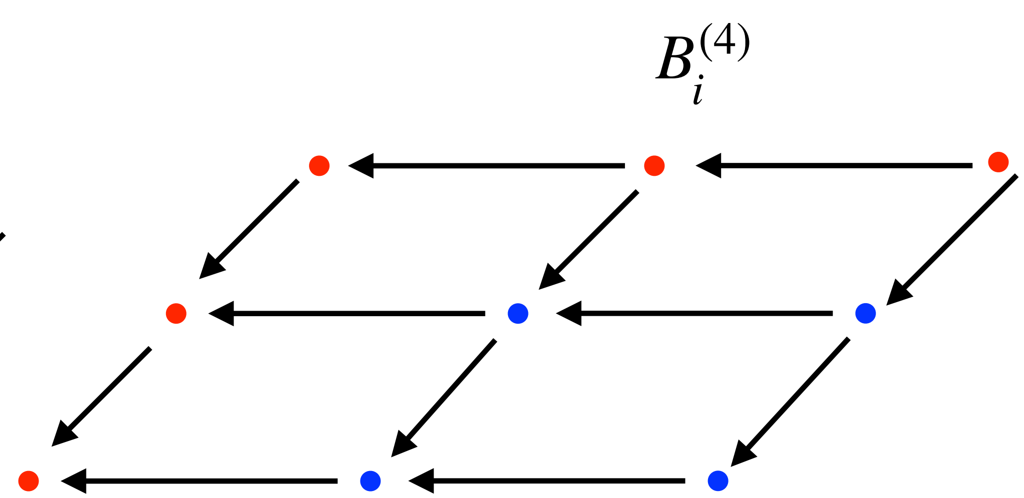
Gauge Fld EoM

$$\begin{aligned}\Delta_0^-\left(a_{+\frac{\hat{0}}{2}}E_{i,+\frac{\hat{0}}{2}}\right) = &-\frac{1}{a}\sum_{j,k}\epsilon_{ijk}\Delta_j^- B_k - \frac{1}{2\Lambda}\left(\pi_\phi B_i^{(4)} + \pi_{\phi,+i}B^{(4)}_{i,+i}\right)\\ &+\frac{1}{8\Lambda}(2+dx\Delta_i^+)\sum_\pm\sum_{j,k}\left\{\epsilon_{ijk}[(\Delta_j^\pm\phi)E^{(2)}_{k,\pm j}]_{+\frac{\hat{0}}{2}} + [(\Delta_j^\pm\phi)E^{(2)}_{k,\pm j}]_{-\frac{\hat{0}}{2}}\right\}\end{aligned}$$

$$a_{+\frac{\hat{0}}{2}}\sum_i \Delta_i^- E_{i,+\frac{\hat{0}}{2}} = -\frac{1}{4\Lambda}\sum_\pm\sum_i\left(\Delta_i^\pm\phi_{+\frac{\hat{0}}{2}}\right)\left(B_i^{(4)} + B_{i,+\hat{0}}^{(4)}\right)_{\pm i}\ ,$$ (Gauss Law)

DGF, Shaposhnikov 2017
Canivete, DGF 2018





Lattice gauge techniques

LATTICE FORMULATION of $\phi F\tilde{F}$

EoM

Inflaton EoM

$$\Delta_0^+\left(a^3\pi_\phi\right) = a_{+\frac{\hat{0}}{2}}\sum_i \Delta_i^-\Delta_i^+\phi_{+\frac{\hat{0}}{2}} - a^3_{+\frac{\hat{0}}{2}}m^2\phi_{+\frac{\hat{0}}{2}} + \frac{1}{\Lambda}\sum_i \frac{1}{2}E^{(2)}_{i,+\frac{\hat{0}}{2}}\left(B_i^{(4)} + B_{i,+\hat{0}}^{(4)}\right)$$

Gauge Fld EoM

$$\Delta_0^-\left(a_{+\frac{\hat{0}}{2}}E_{i,+\frac{\hat{0}}{2}}\right) = -\frac{1}{a}\sum_{j,k}\epsilon_{ijk}\Delta_j^- B_k - \frac{1}{2\Lambda}\left(\pi_\phi B_i^{(4)} + \pi_{\phi,+i}B_{i,+i}^{(4)}\right)$$
$$+\frac{1}{8\Lambda}(2 + dx\Delta_i^+)\sum_\pm\sum_{j,k}\left\{\epsilon_{ijk}[(\Delta_j^\pm\phi)E^{(2)}_{k,\pm j}]_{+\frac{\hat{0}}{2}} + [(\Delta_j^\pm\phi)E^{(2)}_{k,\pm j}]_{-\frac{\hat{0}}{2}}\right\}$$

$$a_{+\frac{\hat{0}}{2}}\sum_i \Delta_i^- E_{i,+\frac{\hat{0}}{2}} = -\frac{1}{4\Lambda}\sum_\pm\sum_i\left(\Delta_i^\pm\phi_{+\frac{\hat{0}}{2}}\right)\left(B_i^{(4)} + B_{i,+\hat{0}}^{(4)}\right)_{\pm i}\ ,$$ (Gauss Law)

DGF, Shaposhnikov 2017
Canivete, DGF 2018

1. Lattice Gauge Inv: $A_\mu \longrightarrow A_\mu + \Delta_\mu^+\alpha$

2. Cont. Limit to $\mathcal{O}(dx^2)$

3. Lattice Bianchi Identities: $\Delta_i^-(B_i^{(4)} + B_{i,+\hat{0}}^{(4)}) = 0\,,\ldots$

4. Topological Term: $(F_{\mu\nu}\tilde{F}^{\mu\nu})_L = \Delta_\mu^+ K^\mu$ (**CS current**)
   $[\,F_{\mu\nu}\tilde{F}^{\mu\nu} = \partial_\mu K^\mu\,]$

Exact Shift Sym. on the lattice !

LATTICE FORMULATION of $\phi F\tilde{F}$

EoM

Inflaton EoM

$$\Delta_0^+\left(a^3\pi_\phi\right) = a_{+\frac{\hat{0}}{2}}\sum_i \Delta_i^-\Delta_i^+\phi_{+\frac{\hat{0}}{2}} - a^3_{+\frac{\hat{0}}{2}} m^2\phi_{+\frac{\hat{0}}{2}} + \frac{1}{\Lambda}\sum_i \frac{1}{2}E^{(2)}_{i,+\frac{\hat{0}}{2}}\left(B_i^{(4)} + B^{(4)}_{i,+\hat{0}}\right)$$

Gauge Fld EoM

$$\Delta_0^-\left(a_{+\frac{\hat{0}}{2}}E_{i,+\frac{\hat{0}}{2}}\right) = -\frac{1}{a}\sum_{j,k}\epsilon_{ijk}\Delta_j^- B_k - \frac{1}{2\Lambda}\left(\pi_\phi B_i^{(4)} + \pi_{\phi,+i}B^{(4)}_{i,+i}\right)$$

$$+\frac{1}{8\Lambda}(2 + dx\Delta_i^+)\sum_{\pm}\sum_{j,k}\left\{\epsilon_{ijk}[(\Delta_j^\pm\phi)E^{(2)}_{k,\pm j}]_{+\frac{\hat{0}}{2}} + [(\Delta_j^\pm\phi)E^{(2)}_{k,\pm j}]_{-\frac{\hat{0}}{2}}\right\}$$

$$a_{+\frac{\hat{0}}{2}}\sum_i \Delta_i^- E_{i,+\frac{\hat{0}}{2}} = -\frac{1}{4\Lambda}\sum_{\pm}\sum_i\left(\Delta_i^\pm\phi_{+\frac{\hat{0}}{2}}\right)\left(B_i^{(4)} + B^{(4)}_{i,+\hat{0}}\right)_{\pm i},$$ (Gauss Law)

DGF, Shaposhnikov 2017
Canivete, DGF 2018

$$\dot{\pi}_\phi = -3H\pi_\phi + \frac{1}{a^2}\vec{\nabla}^2\phi - m^2\phi + \frac{1}{a^3\Lambda}\vec{E}\cdot\vec{B},$$

$$\dot{\vec{E}} = -H\vec{E} - \frac{1}{a^2}\vec{\nabla}\times\vec{B} - \frac{1}{a\Lambda}\pi_\phi\vec{B} + \frac{1}{a\Lambda}\vec{\nabla}\phi\times\vec{E}$$

$$\vec{\nabla}\cdot\vec{E} = -\frac{1}{a\Lambda}\vec{\nabla}\phi\cdot\vec{B}$$ (Gauss Law)

EoM Continuum

LATTICE FORMULATION of $\phi F\tilde{F}$

EoM

Inflaton EoM

$$\Delta_0^+\left(a^3\pi_\phi\right) = a_{+\frac{\hat{0}}{2}}\sum_i \Delta_i^-\Delta_i^+\phi_{+\frac{\hat{0}}{2}} - a^3_{+\frac{\hat{0}}{2}} m^2\phi_{+\frac{\hat{0}}{2}} + \frac{1}{\Lambda}\sum_i \frac{1}{2}E^{(2)}_{i,+\frac{\hat{0}}{2}}\left(B_i^{(4)} + B_{i,+\hat{0}}^{(4)}\right)$$

Gauge Fld EoM

$$\Delta_0^-\left(a_{+\frac{\hat{0}}{2}}E_{i,+\frac{\hat{0}}{2}}\right) = -\frac{1}{a}\sum_{j,k}\epsilon_{ijk}\Delta_j^- B_k - \frac{1}{2\Lambda}\left(\pi_\phi B_i^{(4)} + \pi_{\phi,+i}B_{i,+i}^{(4)}\right)$$

$$+\frac{1}{8\Lambda}(2 + dx\Delta_i^+)\sum_{\pm}\sum_{j,k}\left\{\epsilon_{ijk}[(\Delta_j^\pm\phi)E^{(2)}_{k,\pm j}]_{+\frac{\hat{0}}{2}} + [(\Delta_j^\pm\phi)E^{(2)}_{k,\pm j}]_{-\frac{\hat{0}}{2}}\right\}$$

$$a_{+\frac{\hat{0}}{2}}\sum_i \Delta_i^- E_{i,+\frac{\hat{0}}{2}} = -\frac{1}{4\Lambda}\sum_{\pm}\sum_i\left(\Delta_i^\pm\phi_{+\frac{\hat{0}}{2}}\right)\left(B_i^{(4)} + B_{i,+\hat{0}}^{(4)}\right)_{\pm i}\,,$$ (Gauss Law)

DGF, Shaposhnikov 2017
Canivete, DGF 2018

$$\dot{\pi}_\phi = -3H\pi_\phi + \frac{1}{a^2}\vec{\nabla}^2\phi - m^2\phi + \frac{1}{a^3\Lambda}\vec{E}\cdot\vec{B},$$

$$\dot{\vec{E}} = -H\vec{E} - \frac{1}{a^2}\vec{\nabla}\times\vec{B} - \frac{1}{a\Lambda}\pi_\phi\vec{B} + \frac{1}{a\Lambda}\vec{\nabla}\phi\times\vec{E}$$

$$\vec{\nabla}\cdot\vec{E} = -\frac{1}{a\Lambda}\vec{\nabla}\phi\cdot\vec{B}$$ (Gauss Law)

EoM Continuum

LATTICE FORMULATION of $\phi F\tilde{F}$

EoM

Inflaton EoM

$$\Delta_0^+\left(a^3\pi_\phi\right) = a_{+\frac{\hat{0}}{2}}\sum_i \Delta_i^-\Delta_i^+\phi_{+\frac{\hat{0}}{2}} - a^3_{+\frac{\hat{0}}{2}}m^2\phi_{+\frac{\hat{0}}{2}} + \frac{1}{\Lambda}\sum_i \frac{1}{2}E^{(2)}_{i,+\frac{\hat{0}}{2}}\left(B_i^{(4)} + B^{(4)}_{i,+\hat{0}}\right)$$

Gauge Fld EoM

$$\Delta_0^-\left(a_{+\frac{\hat{0}}{2}}E_{i,+\frac{\hat{0}}{2}}\right) = -\frac{1}{a}\sum_{j,k}\epsilon_{ijk}\Delta_j^-B_k - \frac{1}{2\Lambda}\left(\pi_\phi B_i^{(4)} + \pi_{\phi,+i}B^{(4)}_{i,+i}\right)$$

$$+\frac{1}{8\Lambda}(2+dx\Delta_i^+)\sum_\pm\sum_{j,k}\left\{\epsilon_{ijk}[(\Delta_j^\pm\phi)E^{(2)}_{k,\pm j}]_{+\frac{\hat{0}}{2}} + [(\Delta_j^\pm\phi)E^{(2)}_{k,\pm j}]_{-\frac{\hat{0}}{2}}\right\}$$

$$a_{+\frac{\hat{0}}{2}}\sum_i \Delta_i^-E_{i,+\frac{\hat{0}}{2}} = -\frac{1}{4\Lambda}\sum_\pm\sum_i\left(\Delta_i^\pm\phi_{+\frac{\hat{0}}{2}}\right)\left(B_i^{(4)} + B^{(4)}_{i,+\hat{0}}\right)_{\pm i}\ ,$$ (Gauss Law)

DGF, Shaposhnikov 2017
Canivete, DGF 2018

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$$\vec{\nabla}\cdot\vec{E} = -\frac{1}{a\Lambda}\vec{\nabla}\phi\cdot\vec{B}$$ (Gauss Law)

EoM Continuum

LATTICE FORMULATION of $\phi F\tilde{F}$

EoM

Inflaton EoM

$$\Delta_0^+\left(a^3\pi_\phi\right) = a_{+\frac{\hat{0}}{2}}\sum_i \Delta_i^-\Delta_i^+\phi_{+\frac{\hat{0}}{2}} - a^3_{+\frac{\hat{0}}{2}}m^2\phi_{+\frac{\hat{0}}{2}} + \frac{1}{\Lambda}\sum_i \frac{1}{2}E^{(2)}_{i,+\frac{\hat{0}}{2}}\left(B_i^{(4)} + B^{(4)}_{i,+\hat{0}}\right)$$

Gauge Fld EoM

$$\Delta_0^-\left(a_{+\frac{\hat{0}}{2}}E_{i,+\frac{\hat{0}}{2}}\right) = -\frac{1}{a}\sum_{j,k}\epsilon_{ijk}\Delta_j^- B_k - \frac{1}{2\Lambda}\left(\pi_\phi B_i^{(4)} + \pi_{\phi,+i}B^{(4)}_{i,+i}\right)$$

$$+\frac{1}{8\Lambda}(2+dx\Delta_i^+)\sum_\pm\sum_{j,k}\left\{\epsilon_{ijk}[(\Delta_j^\pm\phi)E^{(2)}_{k,\pm j}]_{+\frac{\hat{0}}{2}} + [(\Delta_j^\pm\phi)E^{(2)}_{k,\pm j}]_{-\frac{\hat{0}}{2}}\right\}$$

$$a_{+\frac{\hat{0}}{2}}\sum_i \Delta_i^- E_{i,+\frac{\hat{0}}{2}} = -\frac{1}{4\Lambda}\sum_\pm\sum_i\left(\Delta_i^\pm\phi_{+\frac{\hat{0}}{2}}\right)\left(B_i^{(4)} + B^{(4)}_{i,+\hat{0}}\right)_{\pm i}\ ,$$ (Gauss Law)

DGF, Shaposhnikov 2017
Canivete, DGF 2018

$$\dot{\pi}_\phi = -3H\pi_\phi + \frac{1}{a^2}\vec{\nabla}^2\phi - m^2\phi + \frac{1}{a^3\Lambda}\vec{E}\cdot\vec{B},$$

$$\dot{\vec{E}} = -H\vec{E} - \frac{1}{a^2}\vec{\nabla}\times\vec{B} - \frac{1}{a\Lambda}\pi_\phi\vec{B} + \frac{1}{a\Lambda}\vec{\nabla}\phi\times\vec{E}$$

$$\vec{\nabla}\cdot\vec{E} = -\frac{1}{a\Lambda}\vec{\nabla}\phi\cdot\vec{B}$$ (Gauss Law)

EoM Continuum

LATTICE FORMULATION of $\phi F\tilde{F}$

Lattice Formulation

EoM

$$
\begin{aligned}
\Delta_0^+\left(a^3\pi_\phi\right) &= a_{+\frac{\hat{0}}{2}}\sum_i \Delta_i^-\Delta_i^+\phi_{+\frac{\hat{0}}{2}} - a^3_{+\frac{\hat{0}}{2}}m^2\phi_{+\frac{\hat{0}}{2}} \\
&\quad +\frac{1}{\Lambda}\sum_i \frac{1}{2}E^{(2)}_{i,+\frac{\hat{0}}{2}}\left(B_i^{(4)}+B^{(4)}_{i,+\hat{0}}\right), \\
\Delta_0^-\left(a_{+\frac{\hat{0}}{2}}E_{i,+\frac{\hat{0}}{2}}\right) &= -\frac{1}{a}\sum_{j,k}\epsilon_{ijk}\Delta_j^- B_k - \frac{1}{2\Lambda}\left(\pi_\phi B_i^{(4)} + \pi_{\phi,+i}B^{(4)}_{i,+i}\right) \\
&\quad +\frac{1}{8\Lambda}(2+dx\Delta_i^+)\sum_{\pm}\sum_{j,k}\left\{\epsilon_{ijk}[(\Delta_j^\pm\phi)E^{(2)}_{k,\pm j}]_{+\frac{\hat{0}}{2}} + [(\Delta_j^\pm\phi)E^{(2)}_{k,\pm j}]_{-\frac{\hat{0}}{2}}\right\} \\
a_{+\frac{\hat{0}}{2}}\sum_i \Delta_i^- E_{i,+\frac{\hat{0}}{2}} &= -\frac{1}{4\Lambda}\sum_{\pm}\sum_i\left(\Delta_i^\pm\phi_{+\frac{\hat{0}}{2}}\right)\left(B_i^{(4)}+B^{(4)}_{i,+\hat{0}}\right)_{\pm i}, \quad \text{(Gauss Law)}
\end{aligned}
$$

Expansion

$$
\left(\Delta_0^+ a_{-\hat{0}/2}\right)^2 = \frac{a^2}{3m_{\rm pl}^2}\rho_L\,,
$$

$$
\Delta_0^-\Delta_0^+ a_{+\hat{0}/2} = -\frac{a_{+\hat{0}/2}}{6m_{\rm pl}^2}(\rho_L + 3p_L)_{+\hat{0}/2}
$$

LATTICE FORMULATION of $\phi F\tilde{F}$

Lattice Formulation

EoM

$$
\begin{aligned}
\Delta_0^+\left(a^3\pi_\phi\right) &= a_{+\frac{\hat{0}}{2}}\sum_i \Delta_i^-\Delta_i^+\phi_{+\frac{\hat{0}}{2}} - a^3_{+\frac{\hat{0}}{2}}m^2\phi_{+\frac{\hat{0}}{2}} \\
&\quad + \frac{1}{\Lambda}\sum_i \frac{1}{2}E^{(2)}_{i,+\frac{\hat{0}}{2}}\left(B_i^{(4)} + B^{(4)}_{i,+\hat{0}}\right), \\
\Delta_0^-\left(a_{+\frac{\hat{0}}{2}}E_{i,+\frac{\hat{0}}{2}}\right) &= -\frac{1}{a}\sum_{j,k}\epsilon_{ijk}\Delta_j^-B_k - \frac{1}{2\Lambda}\left(\pi_\phi B_i^{(4)} + \pi_{\phi,+i}B^{(4)}_{i,+i}\right) \\
&\quad + \frac{1}{8\Lambda}(2 + dx\Delta_i^+)\sum_\pm\sum_{j,k}\left\{\epsilon_{ijk}[(\Delta_j^\pm\phi)E^{(2)}_{k,\pm j}]_{+\frac{\hat{0}}{2}} + [(\Delta_j^\pm\phi)E^{(2)}_{k,\pm j}]_{-\frac{\hat{0}}{2}}\right\} \\
a_{+\frac{\hat{0}}{2}}\sum_i \Delta_i^-E_{i,+\frac{\hat{0}}{2}} &= -\frac{1}{4\Lambda}\sum_\pm\sum_i\left(\Delta_i^\pm\phi_{+\frac{\hat{0}}{2}}\right)\left(B_i^{(4)} + B^{(4)}_{i,+\hat{0}}\right)_{\pm i}, \quad \text{(Gauss Law)}
\end{aligned}
$$

Expansion

$$
\begin{aligned}
\left(\Delta_0^+ a_{-\hat{0}/2}\right)^2 &= \frac{a^2}{3m_{\rm pl}^2}\rho_L\,, \\
\Delta_0^-\Delta_0^+ a_{+\hat{0}/2} &= -\frac{a_{+\hat{0}/2}}{6m_{\rm pl}^2}(\rho_L + 3p_L)_{+\hat{0}/2}
\end{aligned}
$$

$$
\begin{aligned}
\rho_L &= \bar{H}^{\rm kin} + \frac{1}{a^2}\frac{1}{2}(\bar{H}^{\rm grad}_{-\hat{0}/2} + \bar{H}^{\rm grad}_{+\hat{0}/2}) + \frac{1}{2}(\bar{H}^{\rm pot}_{-\hat{0}/2} + \bar{H}^{\rm pot}_{+\hat{0}/2}) + \frac{1}{a^2}\frac{1}{2}(\bar{H}^E_{-\hat{0}/2} + \bar{H}^E_{+\hat{0}/2}) + \frac{1}{a^4}\bar{H}^B\,, \\
(\rho_L + 3p_L)_{+\hat{0}/2} &= 2(\bar{H}^{\rm kin} + \bar{H}^{\rm kin}_{+\hat{0}}) - 2\bar{H}^{\rm pot}_{+\hat{0}/2} + \frac{2}{a^2_{+\hat{0}/2}}\bar{H}^E + \frac{1}{a^4_{+\hat{0}/2}}(\bar{H}^B + \bar{H}^B_{+\hat{0}})\,,
\end{aligned}
$$

$$
\left(
\begin{aligned}
&\bar{H}^{\rm kin} = \left\langle\frac{1}{N^3}\sum_{\vec{n}}\frac{\pi_\phi^2}{2}\right\rangle \qquad \bar{H}^{\rm grad} = \left\langle\frac{1}{N^3}\sum_{\vec{n}}\frac{1}{2}\sum_i(\Delta_i^+\phi_{+\frac{\hat{0}}{2}})^2\right\rangle, \qquad \bar{H}^{\rm pot} = \left\langle\frac{1}{N^3}\sum_{\vec{n}}\frac{1}{2}m^2\phi^2_{+\frac{\hat{0}}{2}}\right\rangle \\
&\bar{H}^E = \left\langle\frac{1}{N^3}\sum_{\vec{n}}\sum_i\frac{1}{2}E^2_{i,+\frac{\hat{0}}{2}}\right\rangle \quad \bar{H}^B = \left\langle\frac{1}{N^3}\sum_{\vec{n}}\sum_{i,j}\frac{1}{4}(\Delta_i^+A_j - \Delta_j^+A_i)^2\right\rangle
\end{aligned}
\right)
$$

LATTICE FORMULATION of $\phi F \tilde{F}$

Lattice Formulation

EoM

$$\begin{aligned}
\Delta_0^+ \left(a^3 \pi_\phi\right) &= a_{+\frac{\hat{0}}{2}} \sum_i \Delta_i^- \Delta_i^+ \phi_{+\frac{\hat{0}}{2}} - a^3_{+\frac{\hat{0}}{2}} m^2 \phi_{+\frac{\hat{0}}{2}} \\
&\quad + \frac{1}{\Lambda} \sum_i \frac{1}{2} E^{(2)}_{i,+\frac{\hat{0}}{2}} \left(B_i^{(4)} + B_{i,+\hat{0}}^{(4)}\right) , \\
\Delta_0^- \left(a_{+\frac{\hat{0}}{2}} E_{i,+\frac{\hat{0}}{2}}\right) &= -\frac{1}{a} \sum_{j,k} \epsilon_{ijk} \Delta_j^- B_k - \frac{1}{2\Lambda} \left(\pi_\phi B_i^{(4)} + \pi_{\phi,+i} B_{i,+i}^{(4)}\right) \\
&\quad + \frac{1}{8\Lambda} (2 + dx \Delta_i^+) \sum_{\pm} \sum_{j,k} \left\{ \epsilon_{ijk} [(\Delta_j^\pm \phi) E^{(2)}_{k,\pm j}]_{+\frac{\hat{0}}{2}} + [(\Delta_j^\pm \phi) E^{(2)}_{k,\pm j}]_{-\frac{\hat{0}}{2}} \right\} \\
a_{+\frac{\hat{0}}{2}} \sum_i \Delta_i^- E_{i,+\frac{\hat{0}}{2}} &= -\frac{1}{4\Lambda} \sum_{\pm} \sum_i \left(\Delta_i^\pm \phi_{+\frac{\hat{0}}{2}}\right) \left(B_i^{(4)} + B_{i,+\hat{0}}^{(4)}\right)_{\pm i} , \quad \text{(Gauss Law)}
\end{aligned}$$

Expansion

$$\begin{aligned}
\left(\Delta_0^+ a_{-\hat{0}/2}\right)^2 &= \frac{a^2}{3 m_{\rm pl}^2} \rho_L , \\
\Delta_0^- \Delta_0^+ a_{+\hat{0}/2} &= -\frac{a_{+\hat{0}/2}}{6 m_{\rm pl}^2} (\rho_L + 3 p_L)_{+\hat{0}/2}
\end{aligned}$$

Now I will show **our work**

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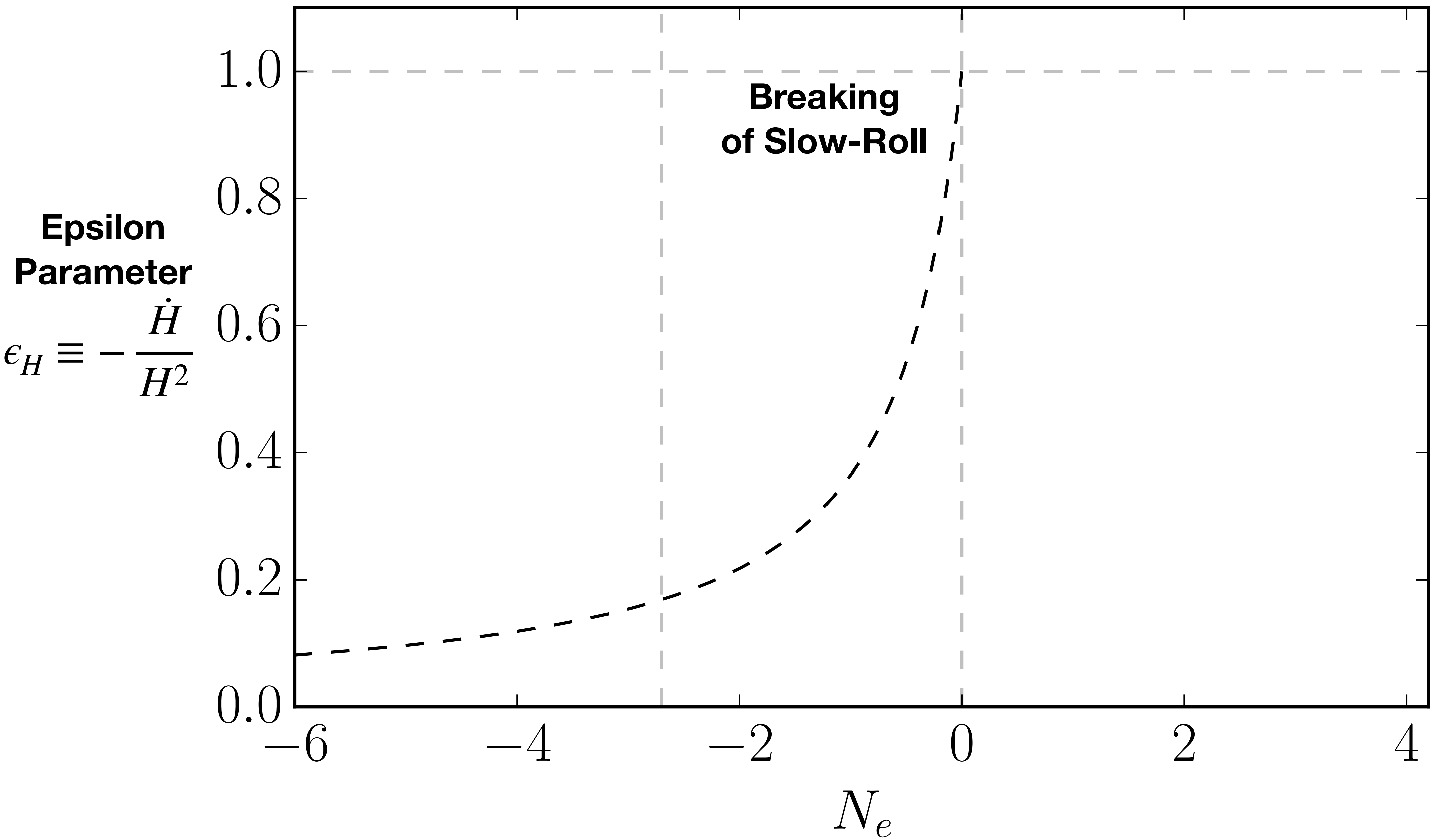
CosmoLattice

$$V(\phi) = \frac{1}{2}m^2\phi^2 \;;\; \frac{\phi}{4\Lambda}F\tilde{F} \;;\; \Lambda = \frac{m_p}{18}$$

Axion-Inflation($V(\phi) = \frac{1}{2}m^2\phi^2 \; ; \; \frac{\phi}{4\Lambda}F\tilde{F} \; ; \; \Lambda = \frac{m_p}{18}$)

Linear regime (– – –)

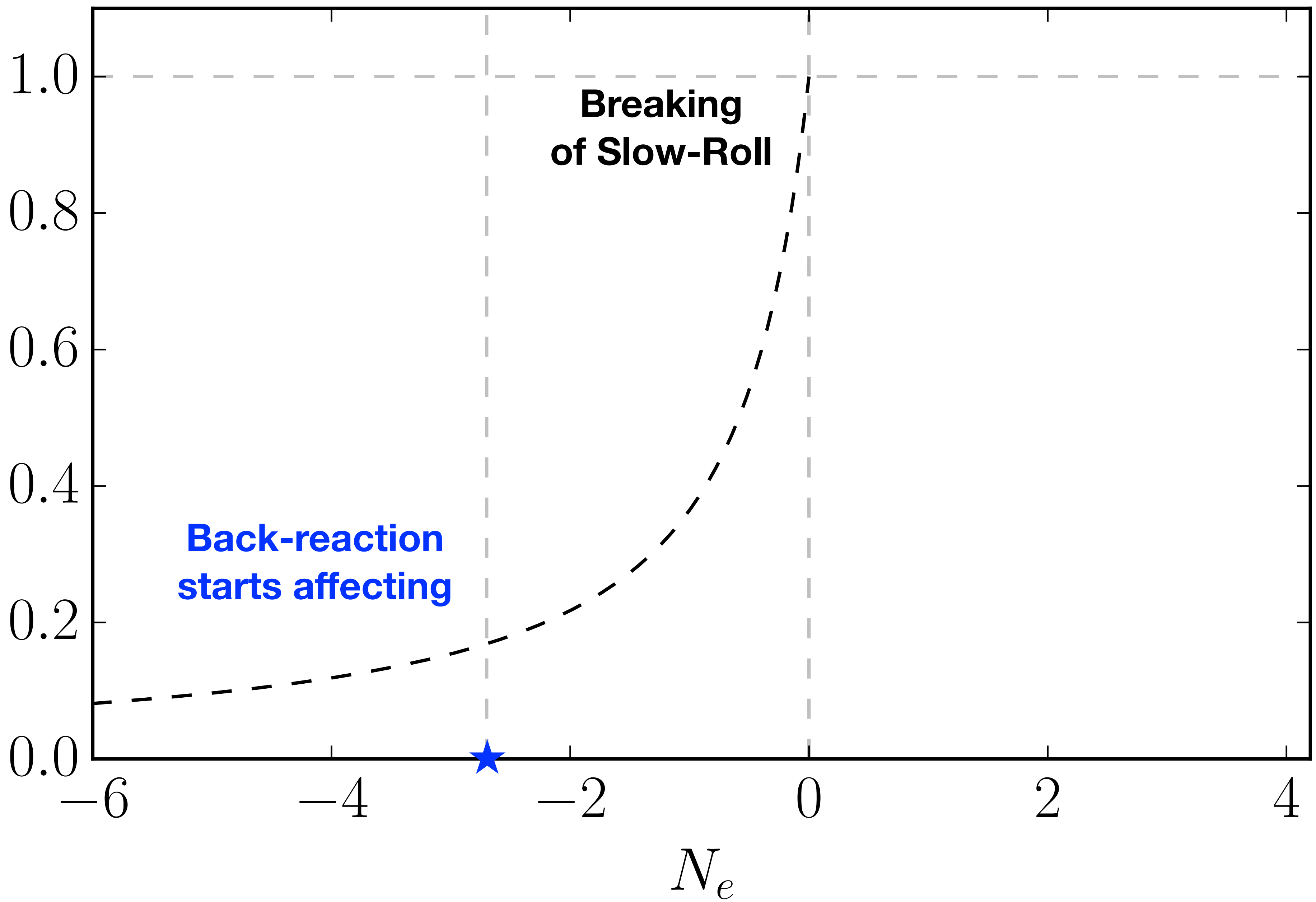


Axion-Inflation($V(\phi) = \frac{1}{2}m^2\phi^2\ ;\ \frac{\phi}{4\Lambda}F\tilde{F}\ ;\ \Lambda = \frac{m_p}{18}$)

Linear regime (– – –)

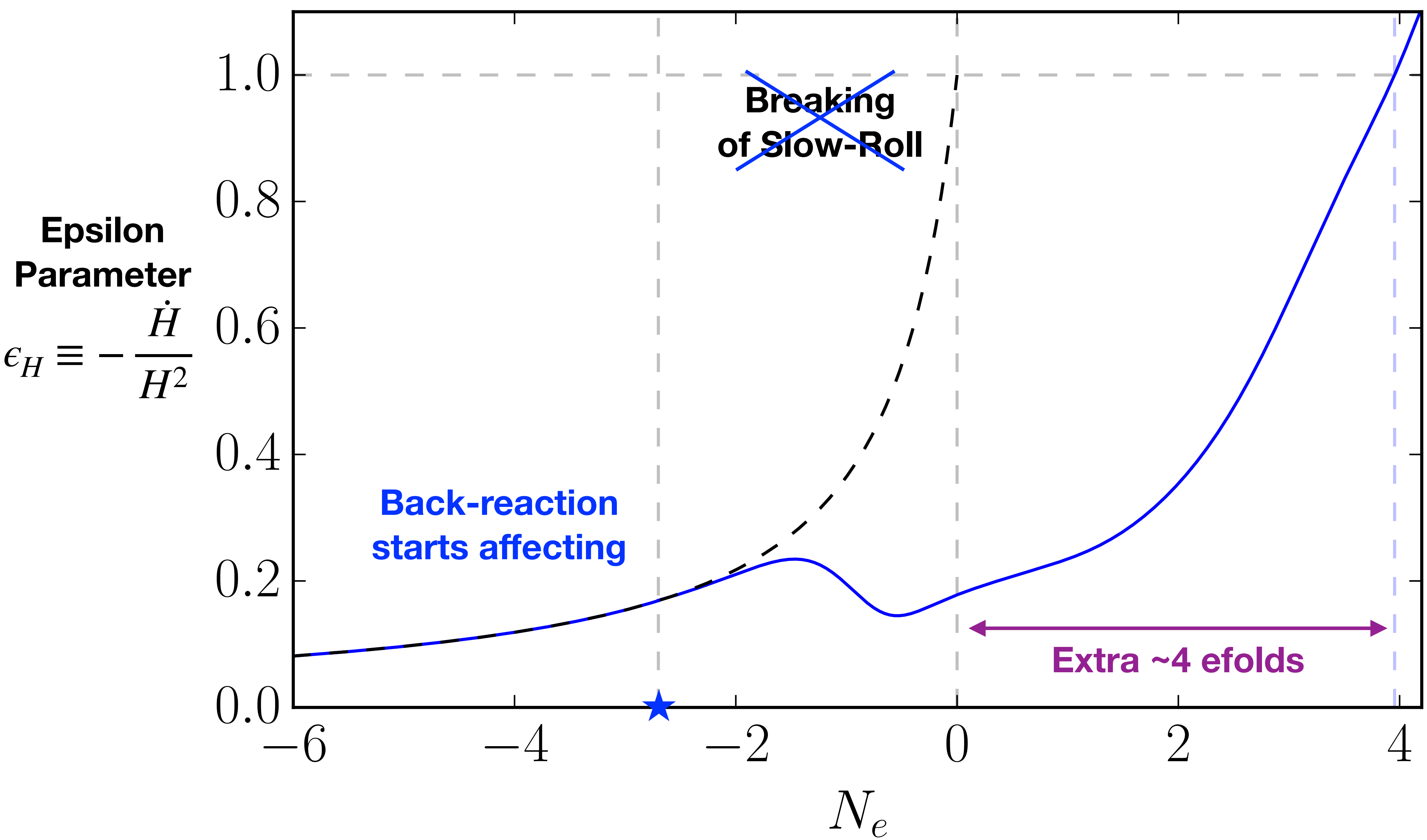
Epsilon Parameter

$\epsilon_H \equiv -\frac{\dot{H}}{H^2}$

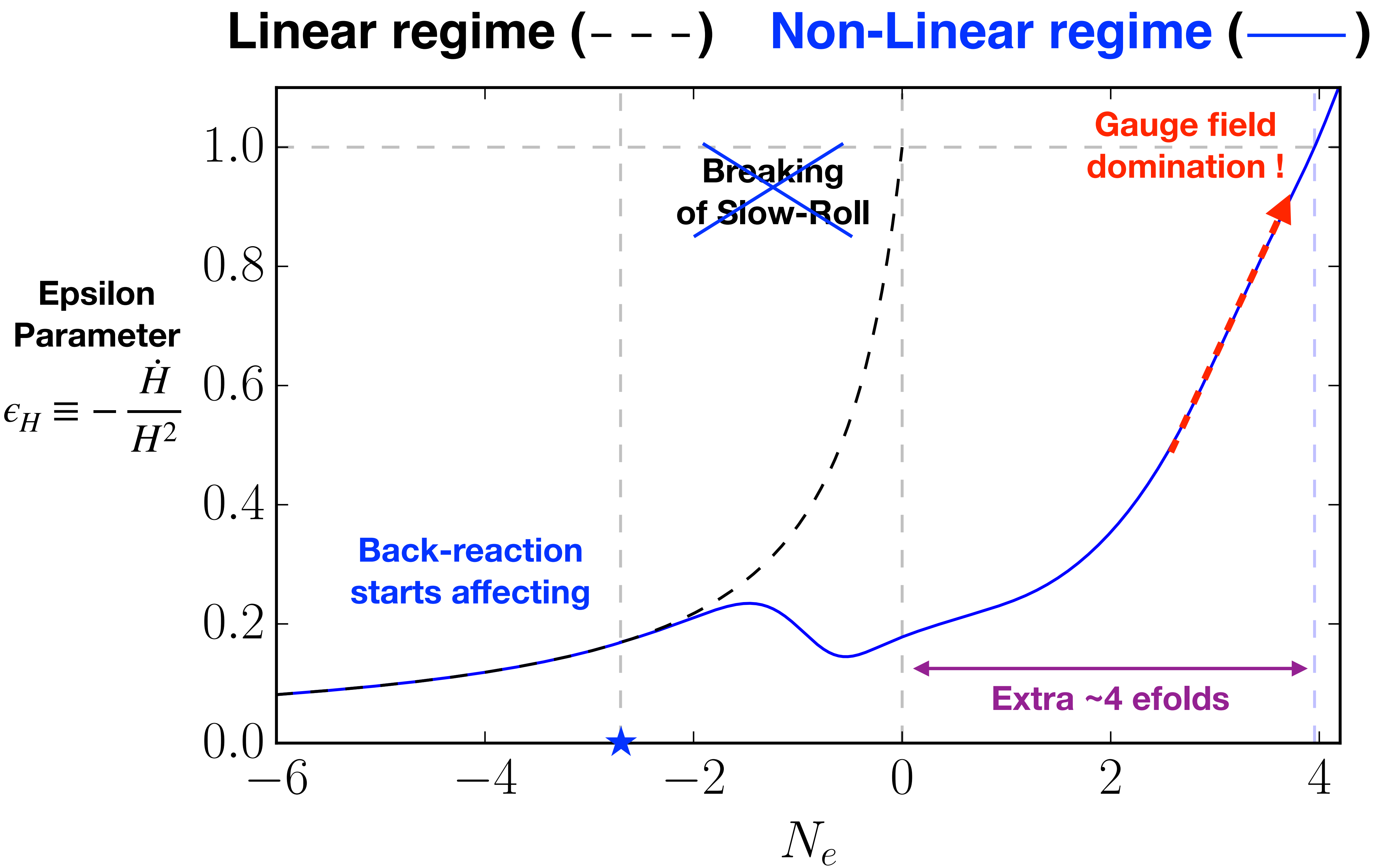


Axion-Inflation($V(\phi) = \frac{1}{2}m^2\phi^2$; $\frac{\phi}{4\Lambda}F\tilde{F}$; $\Lambda = \frac{m_p}{18}$)

Linear regime (– – –) **Non-Linear regime (——)**



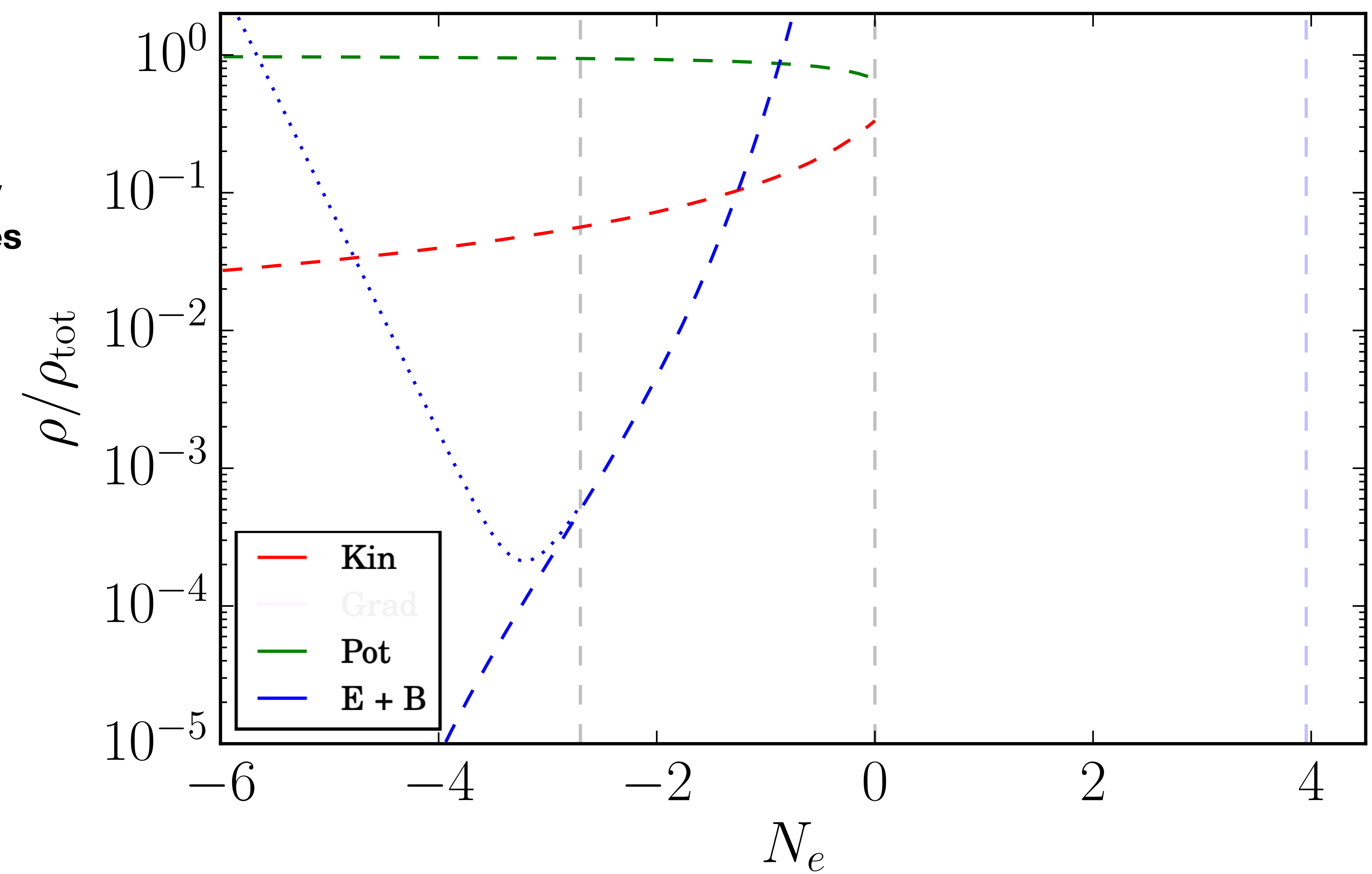
Axion-Inflation$\left(V(\phi) = \frac{1}{2}m^2\phi^2 \ ; \ \frac{\phi}{4\Lambda}F\tilde{F} \ ; \ \Lambda = \frac{m_p}{18}\right)$



Axion-Inflation($V(\phi) = \frac{1}{2}m^2\phi^2$; $\frac{\phi}{4\Lambda}F\tilde{F}$; $\Lambda = \frac{m_p}{18}$)

Linear (– – –)

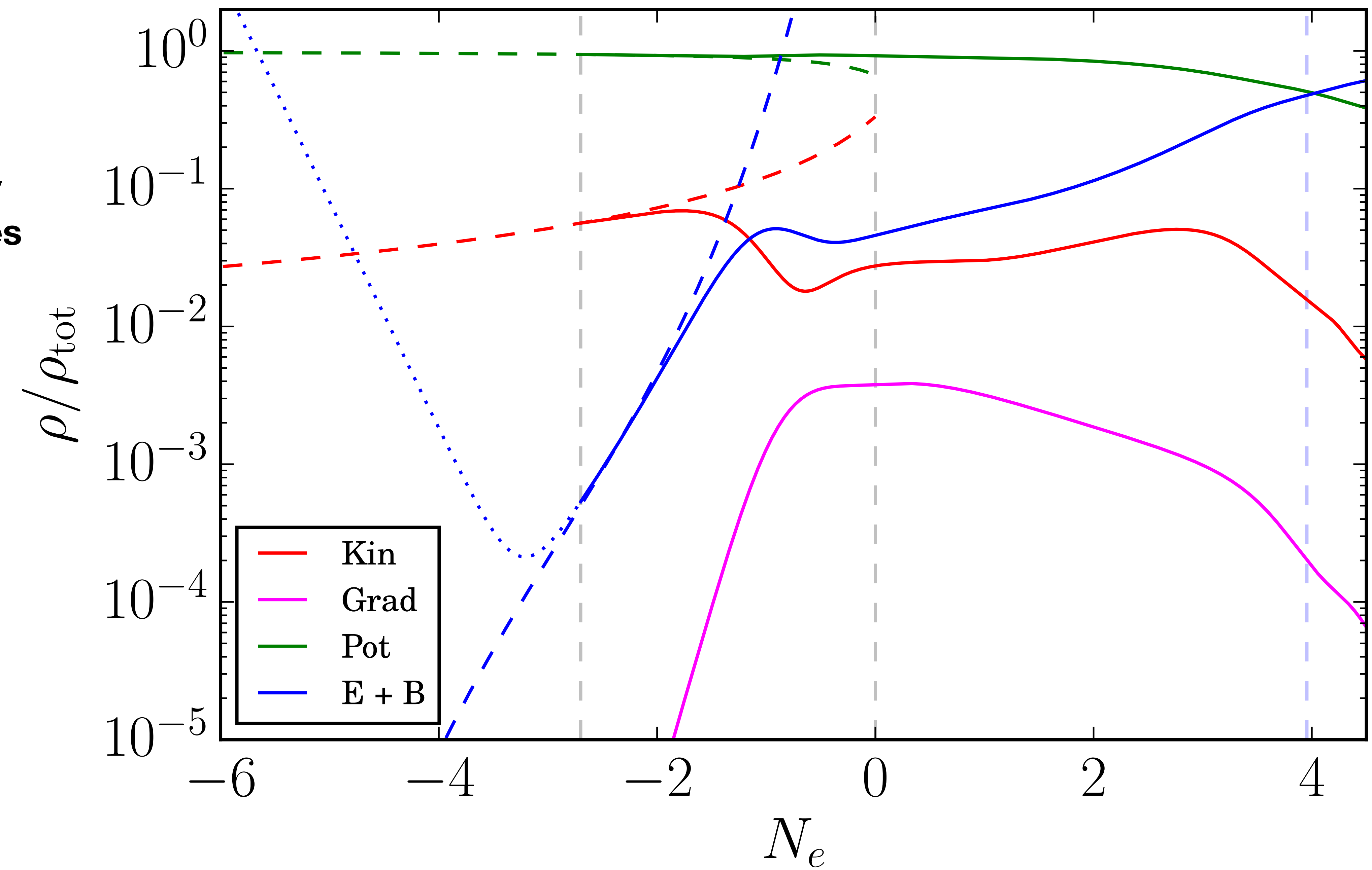
Energy Densities



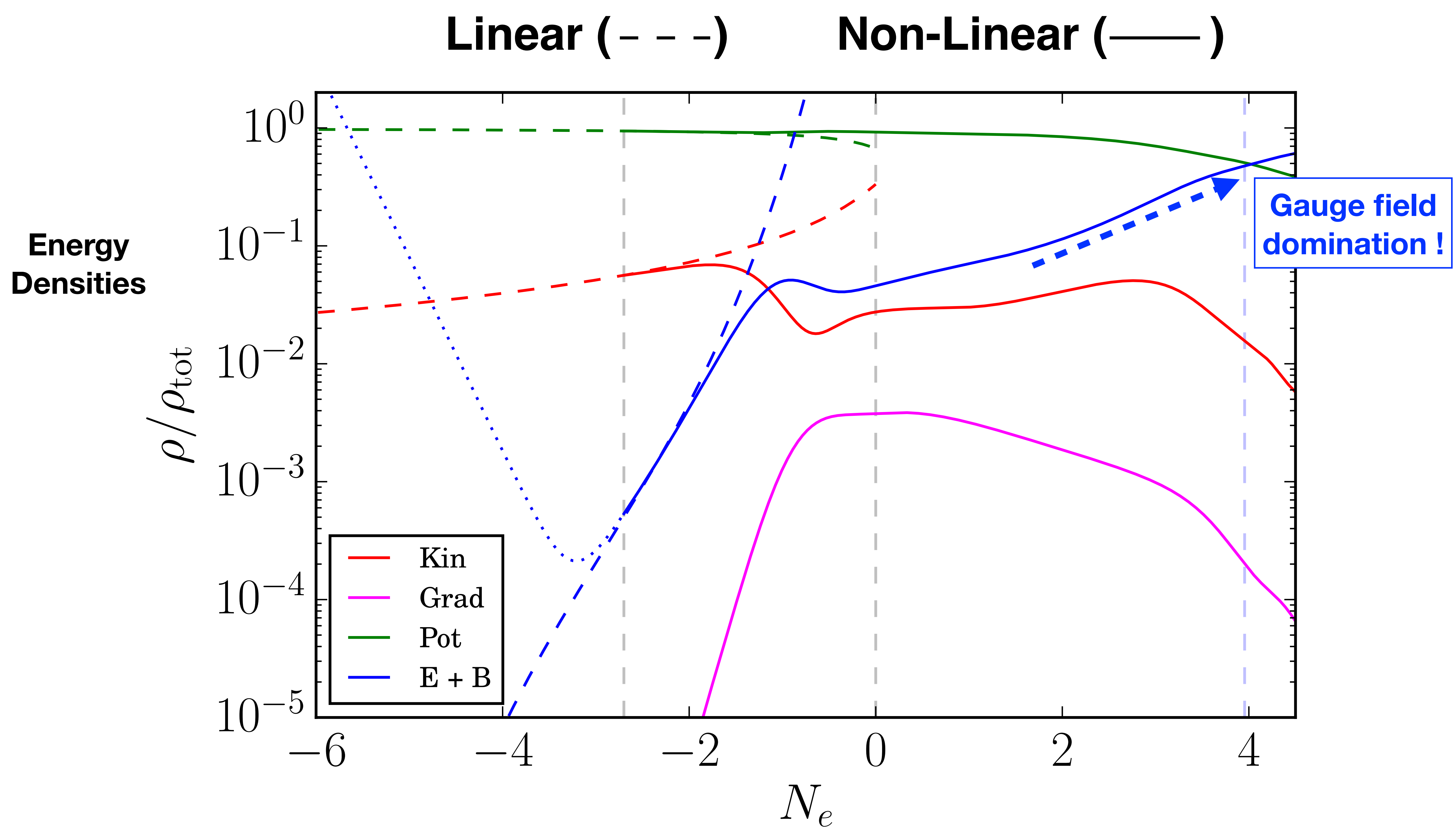
Axion-Inflation $\left(V(\phi) = \frac{1}{2}m^2\phi^2 \; ; \; \frac{\phi}{4\Lambda}F\tilde{F} \; ; \; \Lambda = \frac{m_p}{18}\right)$

Linear (– – –) **Non-Linear (——)**

Energy Densities

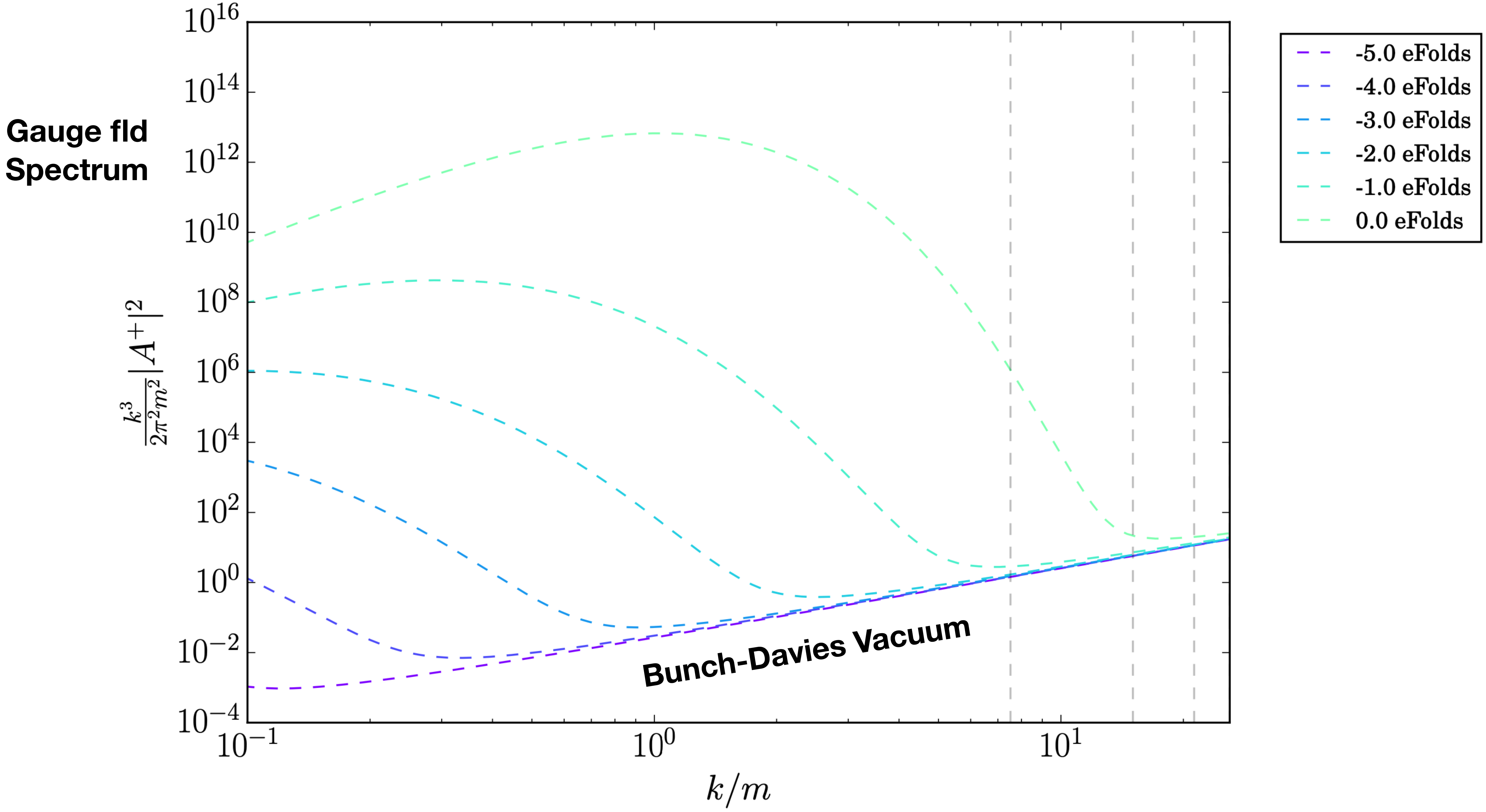


Axion-Inflation($V(\phi) = \frac{1}{2}m^2\phi^2 \; ; \; \frac{\phi}{4\Lambda}F\tilde{F} \; ; \; \Lambda = \frac{m_p}{18}$)

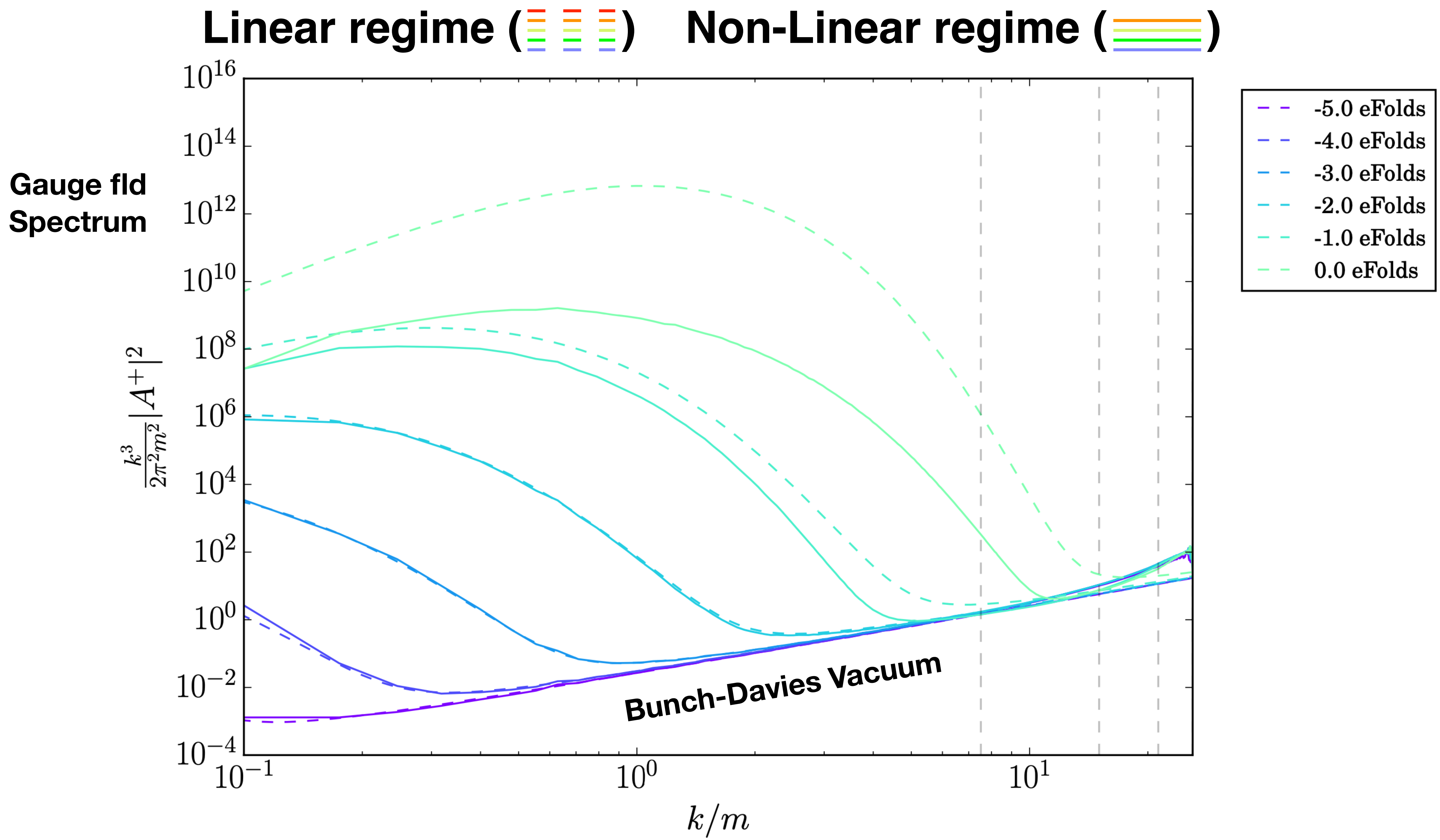


Axion-Inflation($V(\phi) = \frac{1}{2}m^2\phi^2$; $\frac{\phi}{4\Lambda}F\tilde{F}$; $\Lambda = \frac{m_p}{18}$)

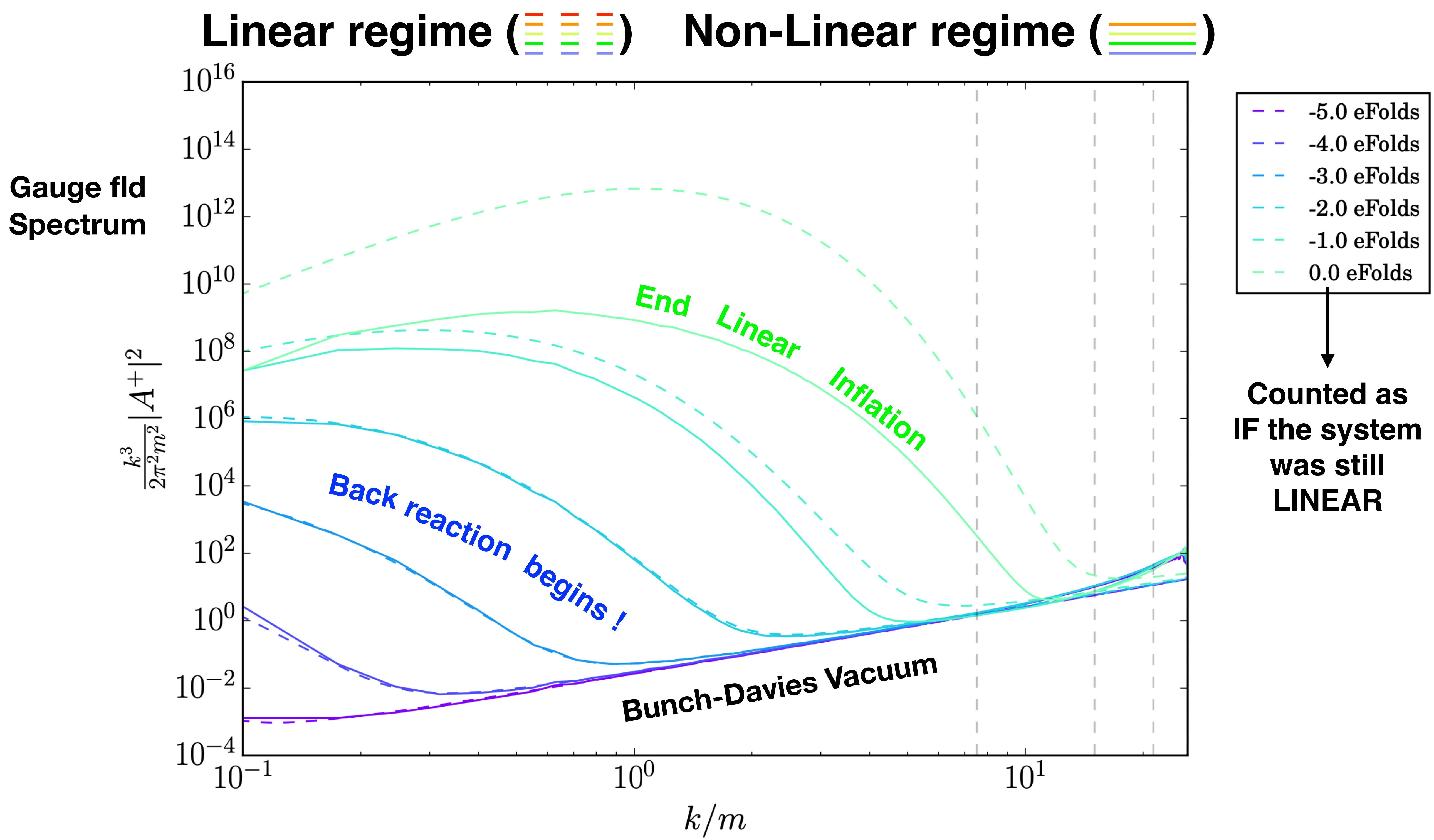
Linear regime ( )



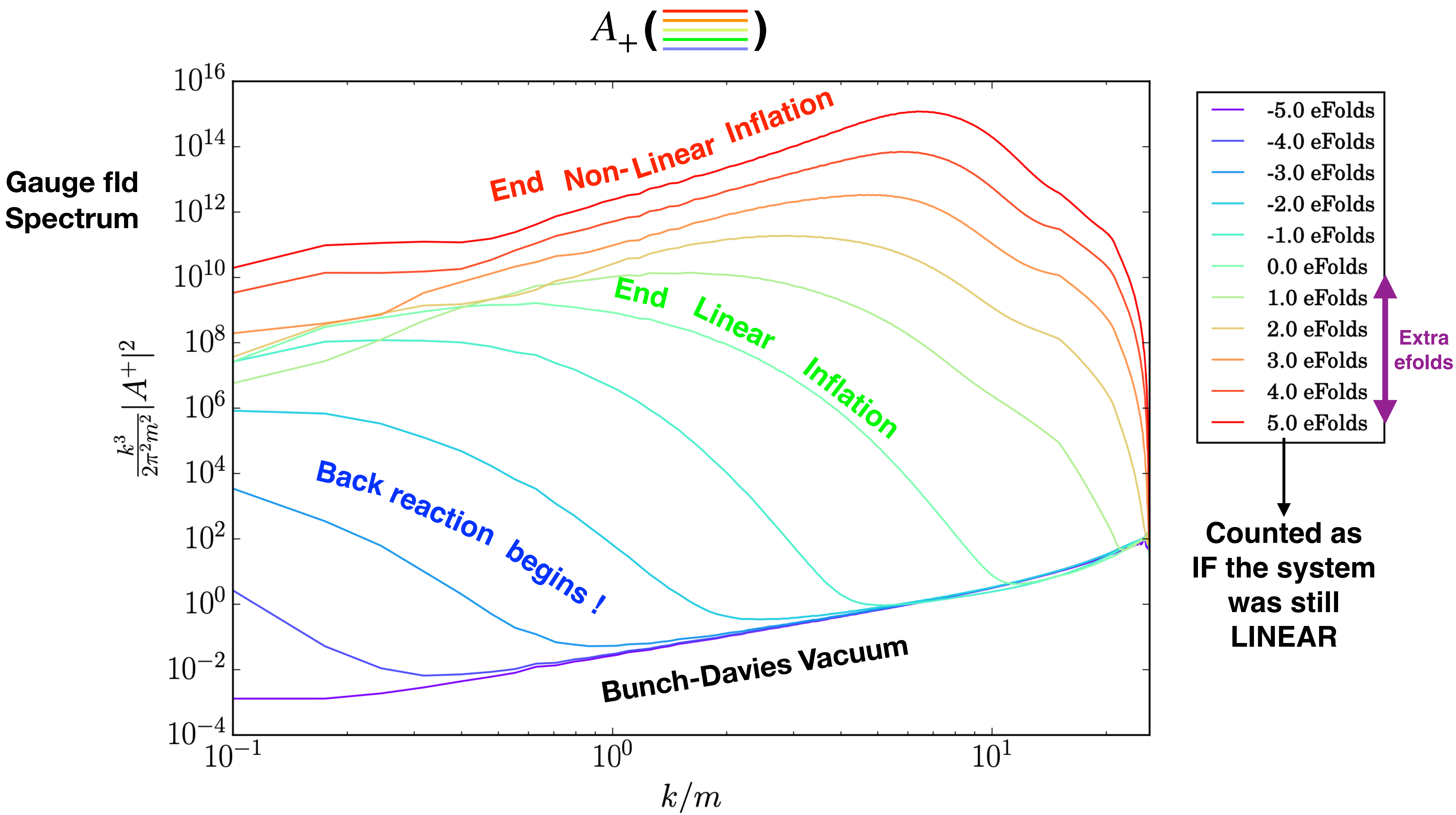
Axion-Inflation($V(\phi) = \frac{1}{2}m^2\phi^2$; $\frac{\phi}{4\Lambda}F\tilde{F}$; $\Lambda = \frac{m_p}{18}$)



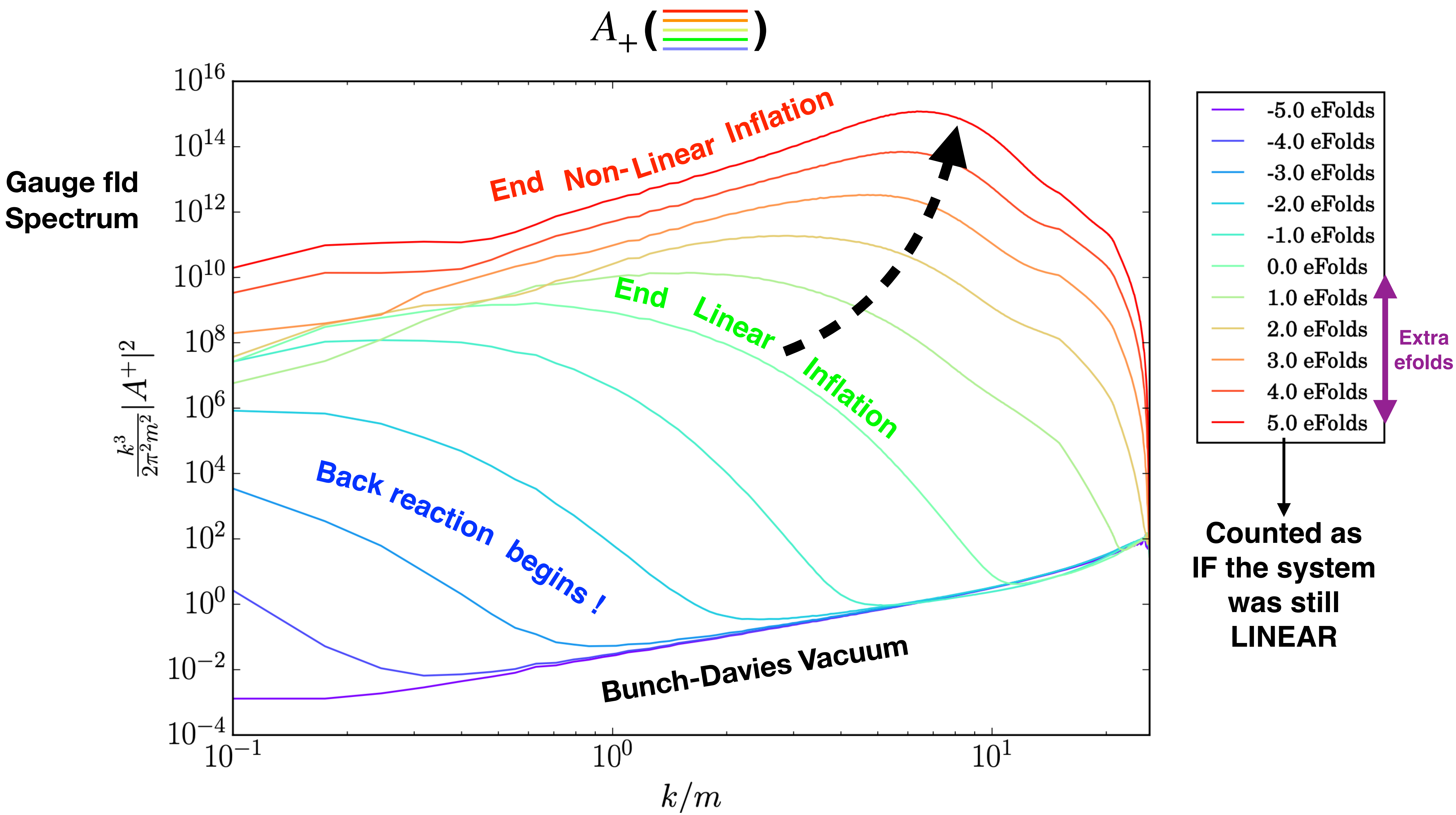
Axion-Inflation$\left(V(\phi)=\frac{1}{2}m^2\phi^2\ ;\ \frac{\phi}{4\Lambda}F\tilde{F}\ ;\ \Lambda=\frac{m_p}{18}\right)$



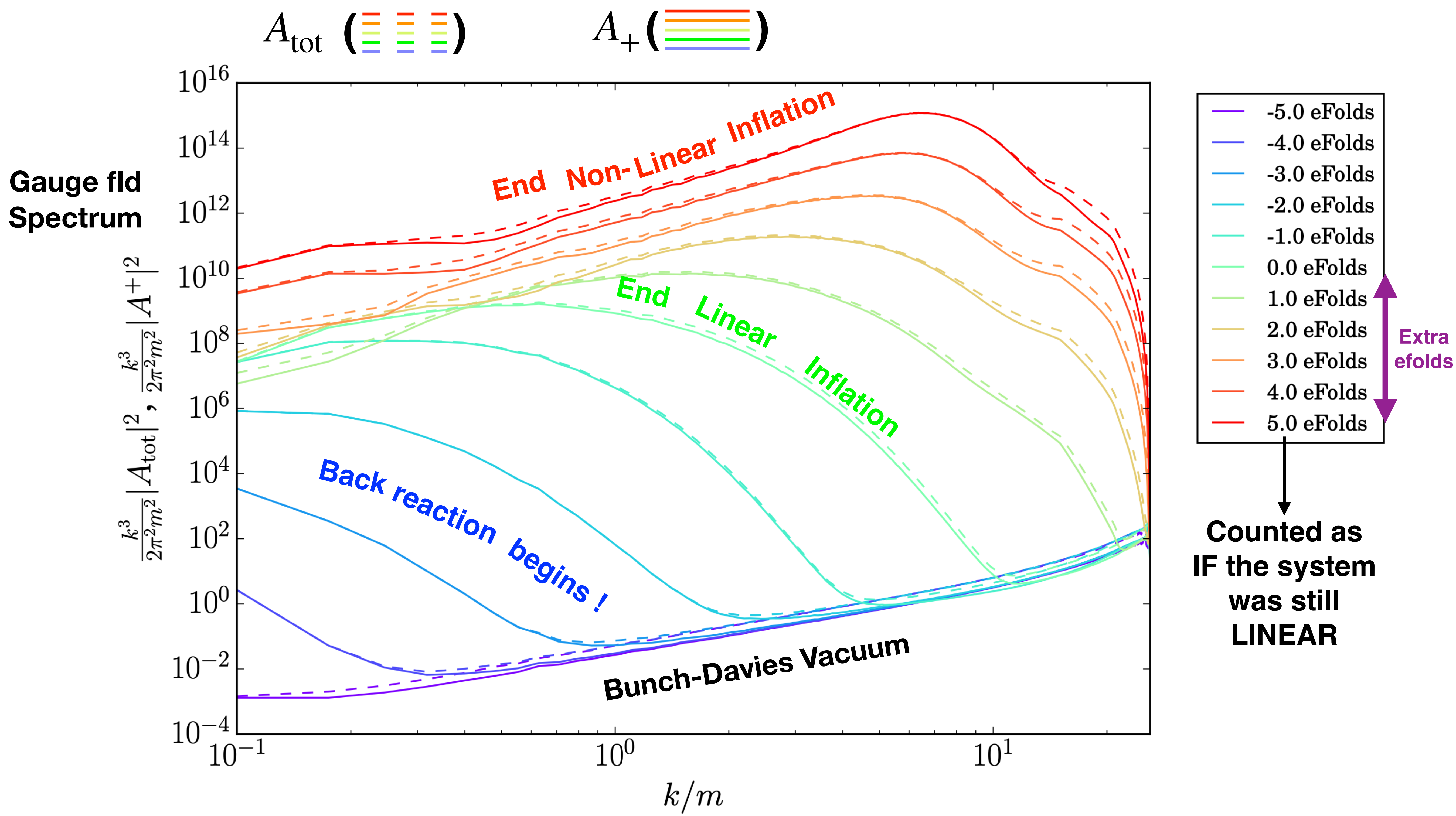
Axion-Inflation$\left(V(\phi) = \frac{1}{2}m^2\phi^2 \;;\; \frac{\phi}{4\Lambda}F\tilde{F} \;;\; \Lambda = \frac{m_p}{18}\right)$



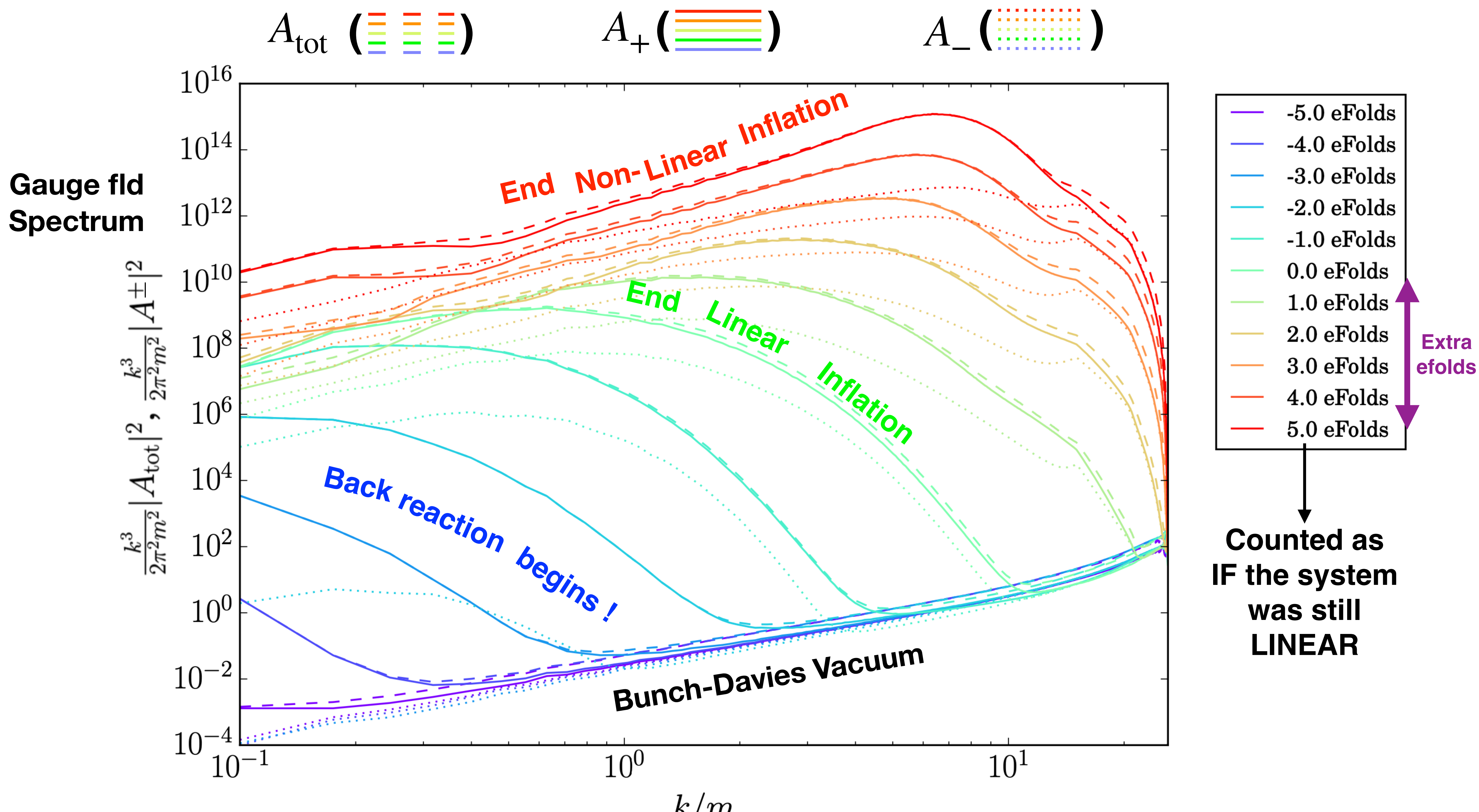
Axion-Inflation$\left(V(\phi) = \frac{1}{2}m^2\phi^2 \; ; \; \frac{\phi}{4\Lambda}F\tilde{F} \; ; \; \Lambda = \frac{m_p}{18}\right)$



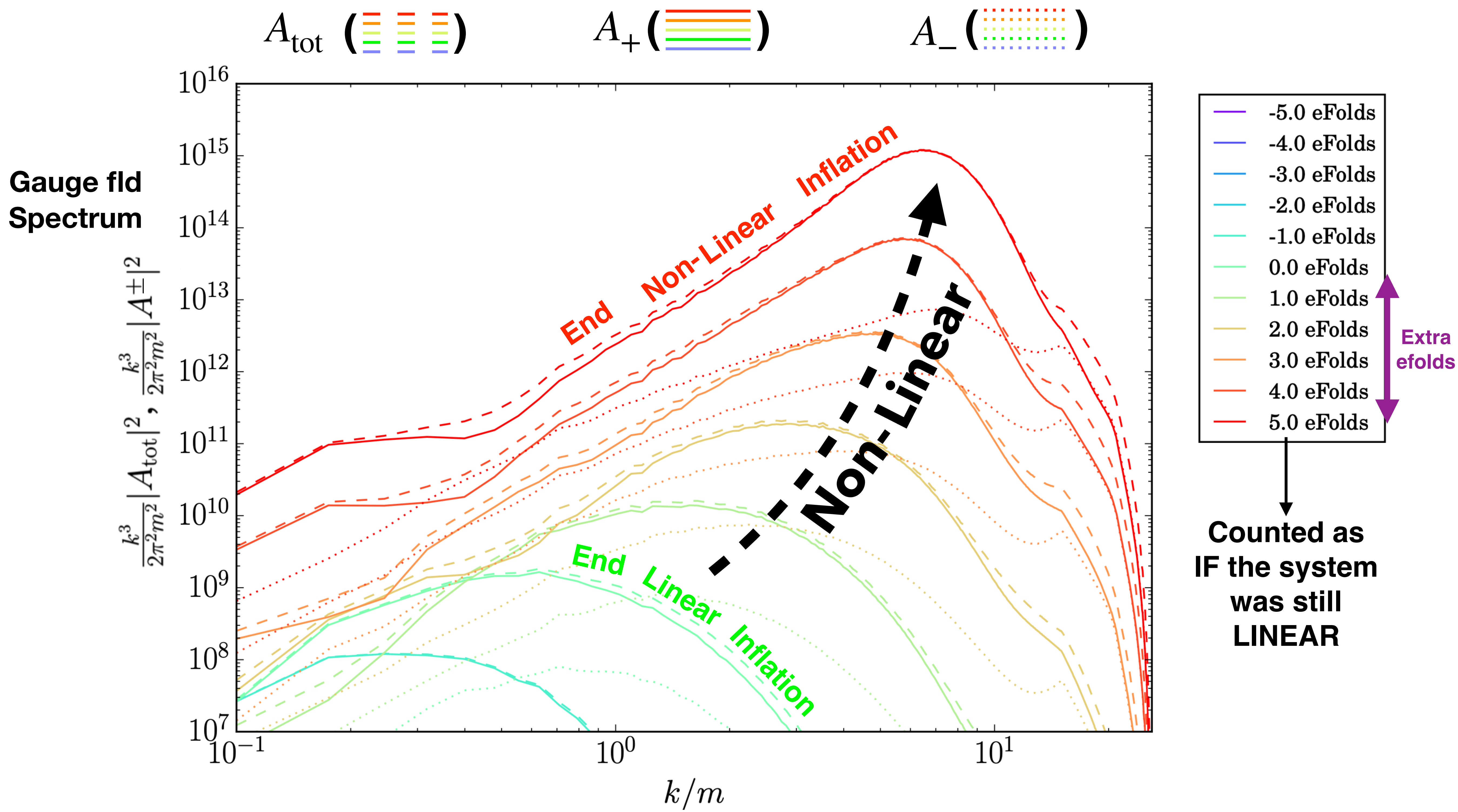
Axion-Inflation$\left(V(\phi) = \frac{1}{2}m^2\phi^2 \; ; \; \frac{\phi}{4\Lambda}F\tilde{F} \; ; \; \Lambda = \frac{m_p}{18}\right)$



Axion-Inflation$\left(V(\phi)=\frac{1}{2}m^2\phi^2\ ;\ \frac{\phi}{4\Lambda}F\tilde{F}\ ;\ \Lambda=\frac{m_p}{18}\right)$

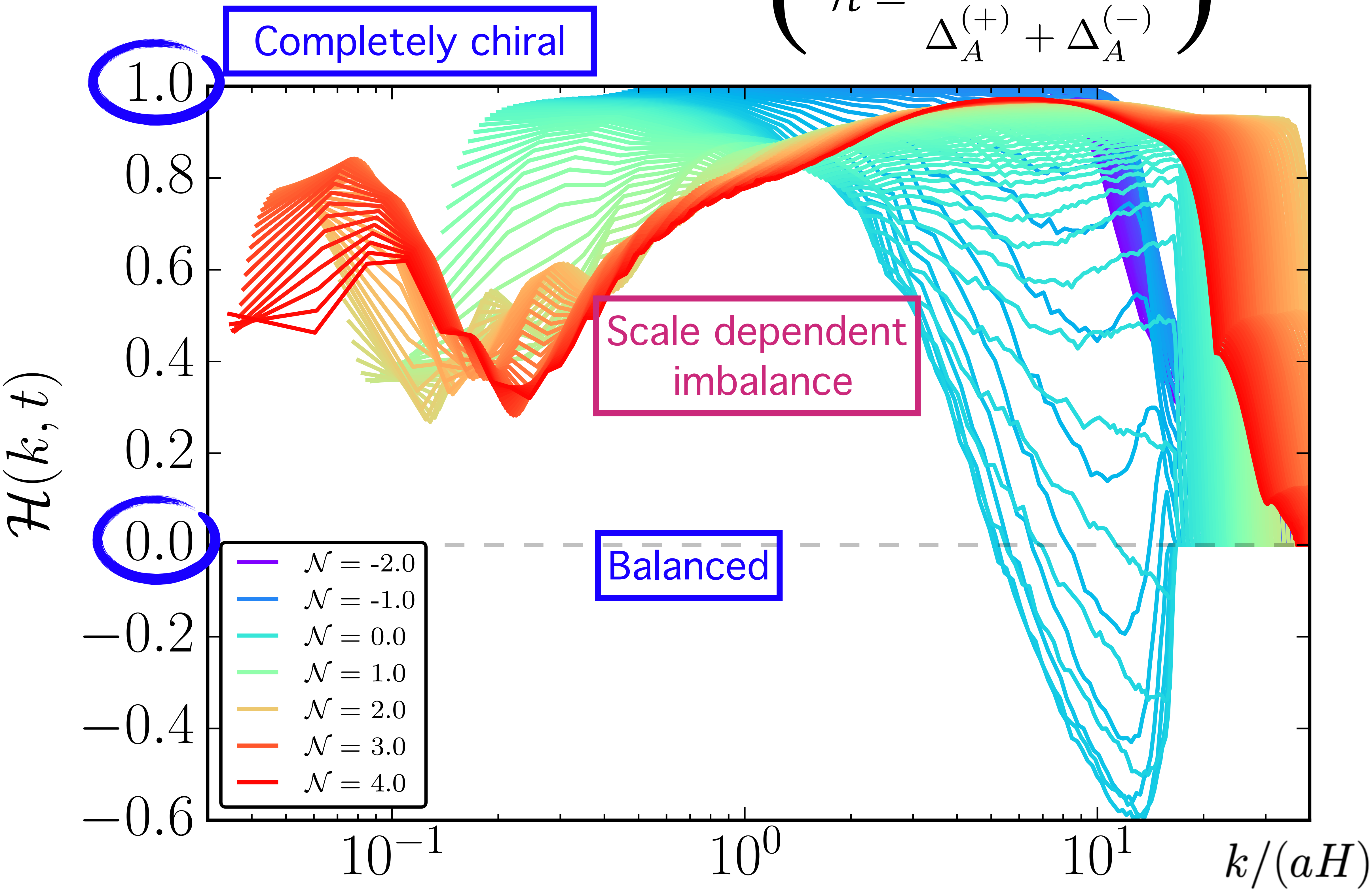


Axion-Inflation$\left(V(\phi) = \frac{1}{2}m^2\phi^2 \; ; \; \frac{\phi}{4\Lambda}F\tilde{F} \; ; \; \Lambda = \frac{m_p}{18}\right)$

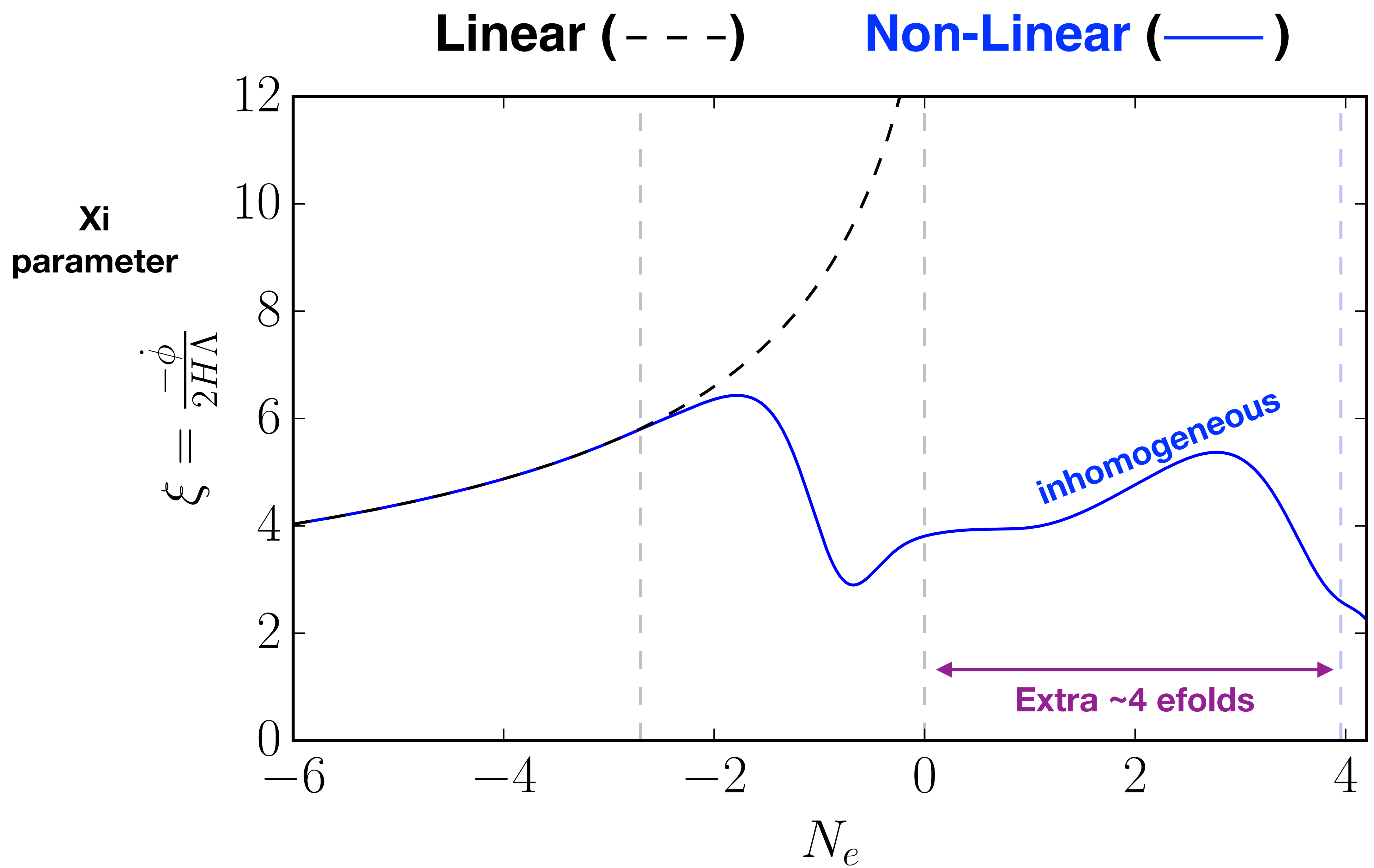


Axion-Inflation($V(\phi) = \frac{1}{2}m^2\phi^2\ ;\ \frac{\phi}{4\Lambda}F\tilde{F}\ ;\ \Lambda = \frac{m_p}{18}$)

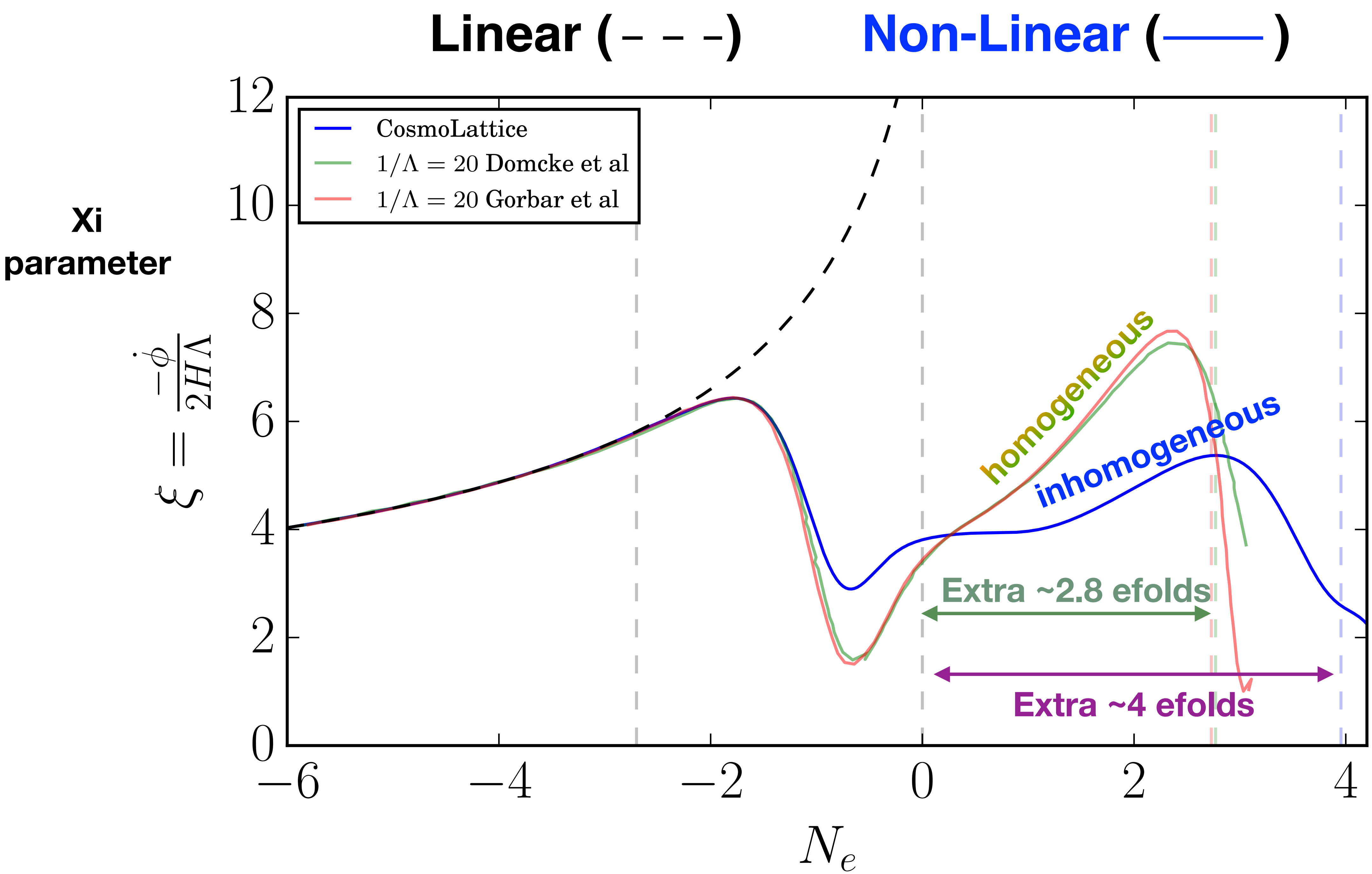
$$\left(\mathcal{H} = \frac{\Delta_A^{(+)} - \Delta_A^{(-)}}{\Delta_A^{(+)} + \Delta_A^{(-)}} \right)$$



Axion-Inflation($V(\phi) = \frac{1}{2}m^2\phi^2$; $\frac{\phi}{4\Lambda}F\tilde{F}$; $\Lambda = \frac{m_p}{18}$)



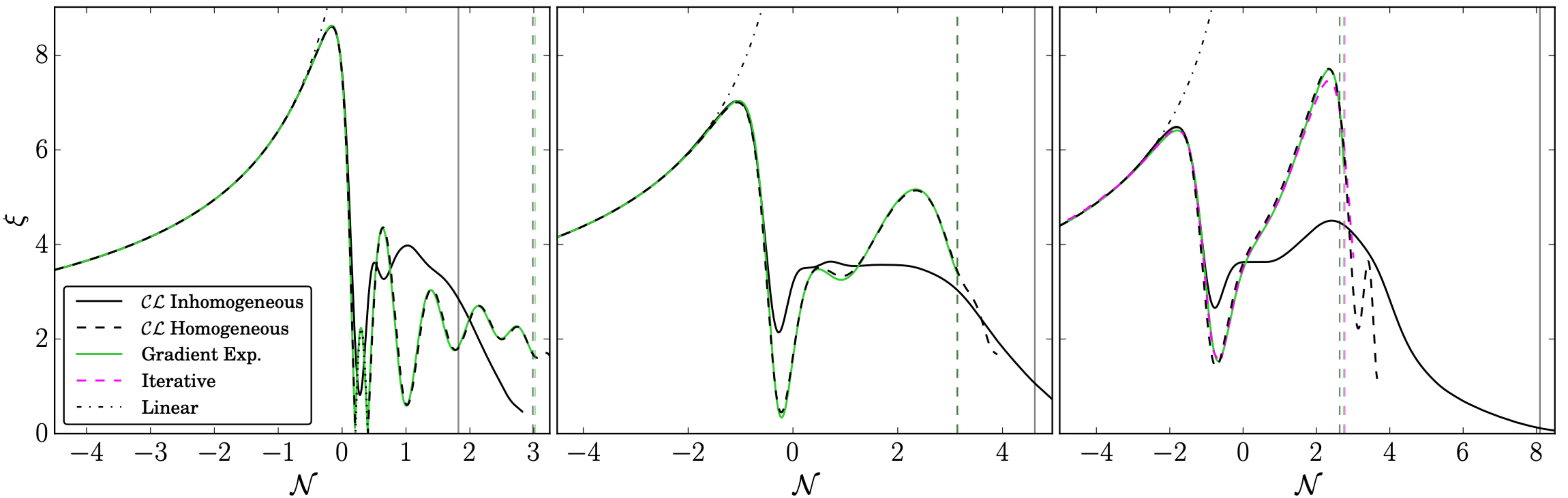
Axion-Inflation($V(\phi) = \frac{1}{2}m^2\phi^2$; $\frac{\phi}{4\Lambda}F\tilde{F}$; $\Lambda = \frac{m_p}{18}$)



Axion-Inflation$\left(V(\phi) = \frac{1}{2}m^2\phi^2 \; ; \; \frac{\phi}{4\Lambda}F\tilde{F} \; ; \; \Lambda = \frac{m_p}{\alpha}\right)$

$(\alpha = 15, 18, 20)$

$\alpha_\Lambda = 15$ | $\alpha_\Lambda = 18$ | $\alpha_\Lambda = 20$



$\Delta\mathcal{N}_{Hom} \simeq 3$ | $\Delta\mathcal{N}_{Hom} \simeq 3$ | $\Delta\mathcal{N}_{Hom} \simeq 3$

$\Delta\mathcal{N}_{InH} \simeq 1.8$ | $\Delta\mathcal{N}_{InH} \simeq 4.5$ | $\Delta\mathcal{N}_{InH} \simeq 8$

Axion-Inflation$\left(V(\phi) = \frac{1}{2}m^2\phi^2 \; ; \; \frac{\phi}{4\Lambda}F\tilde{F} \; ; \; \Lambda = \frac{m_p}{X}\right)$

$(X = 15, 20, 25)$

Summary

* ξ Controls the Gauge field excitation

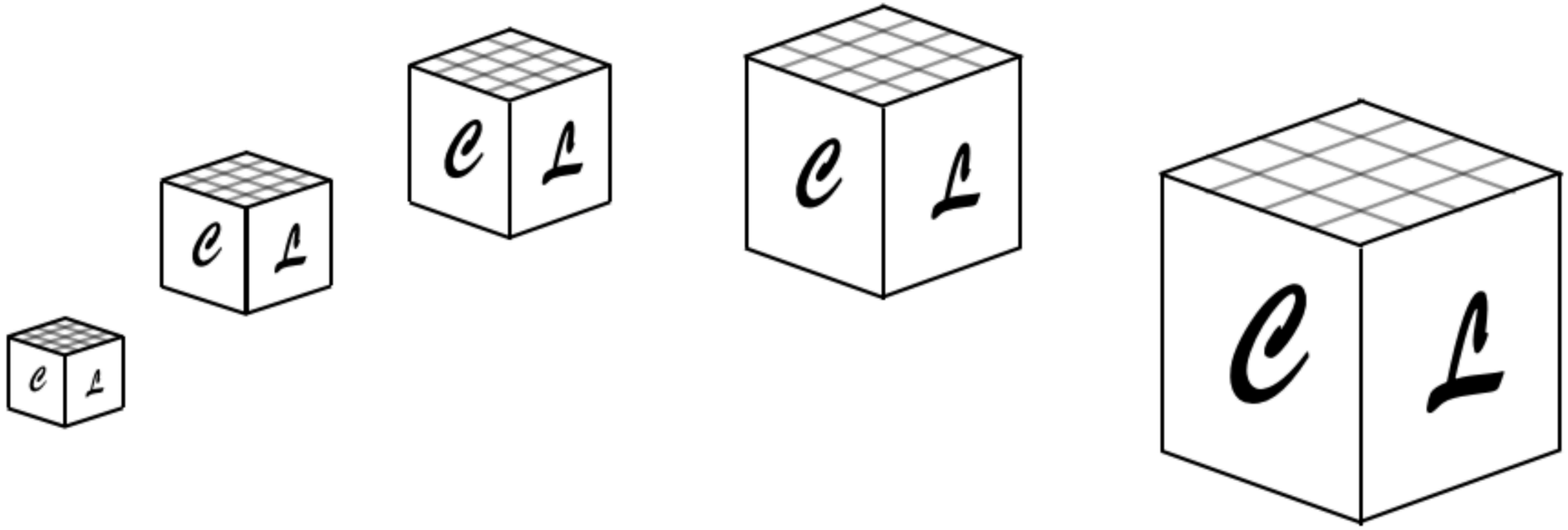
* Linear change in ξ : exponential response in A_μ

* Predictions/constraints (PNG, PBH and GWs) depend crucially on ξ : we will re-assess real observability !

* Adding Schwinger pair production easy via $\vec{J} = \sigma \vec{E}$

* Other phenomena: BAU, Magnetogenesis, …

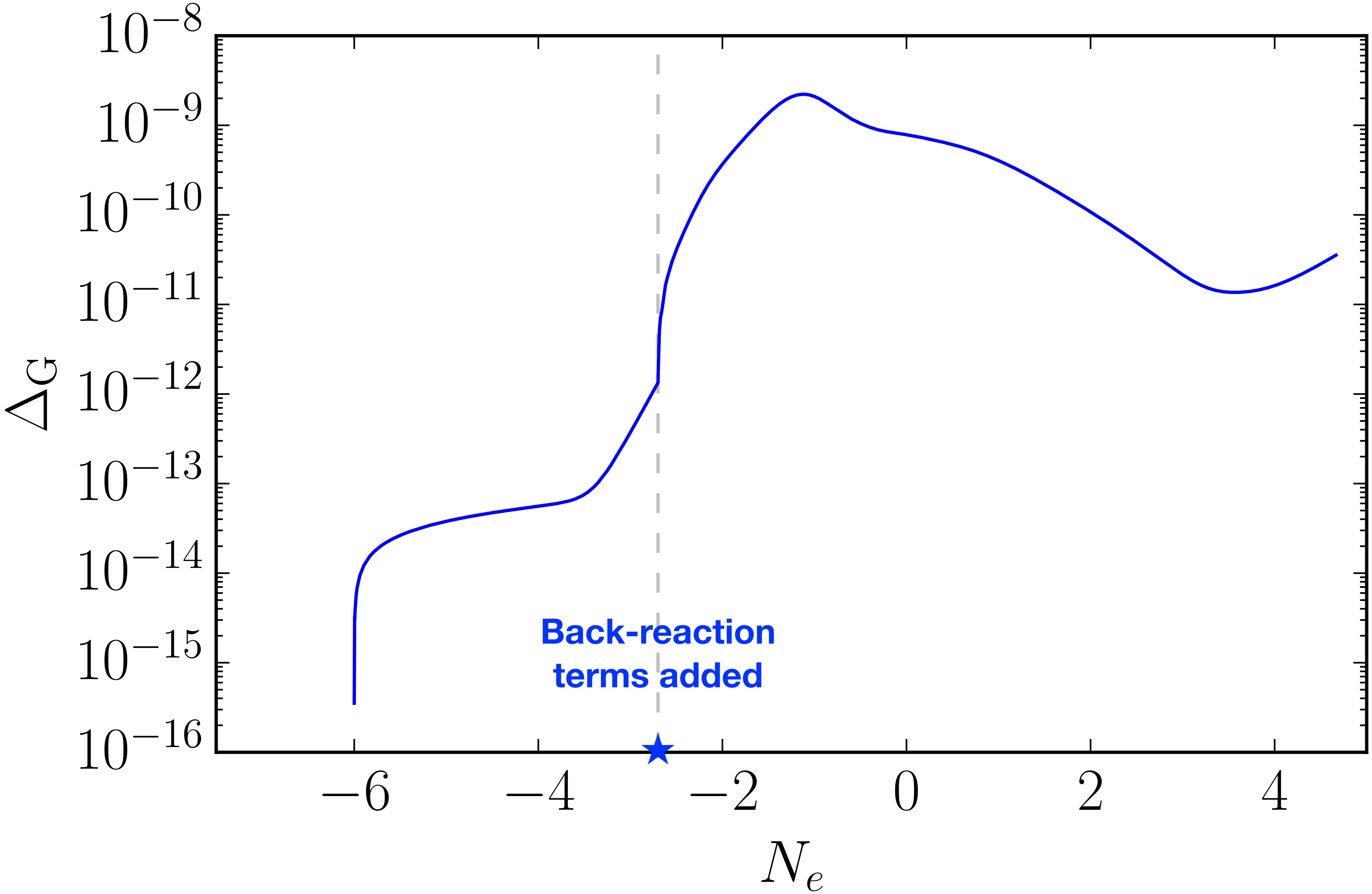
Thanks for your attention !



Axion-inflation extra stuff

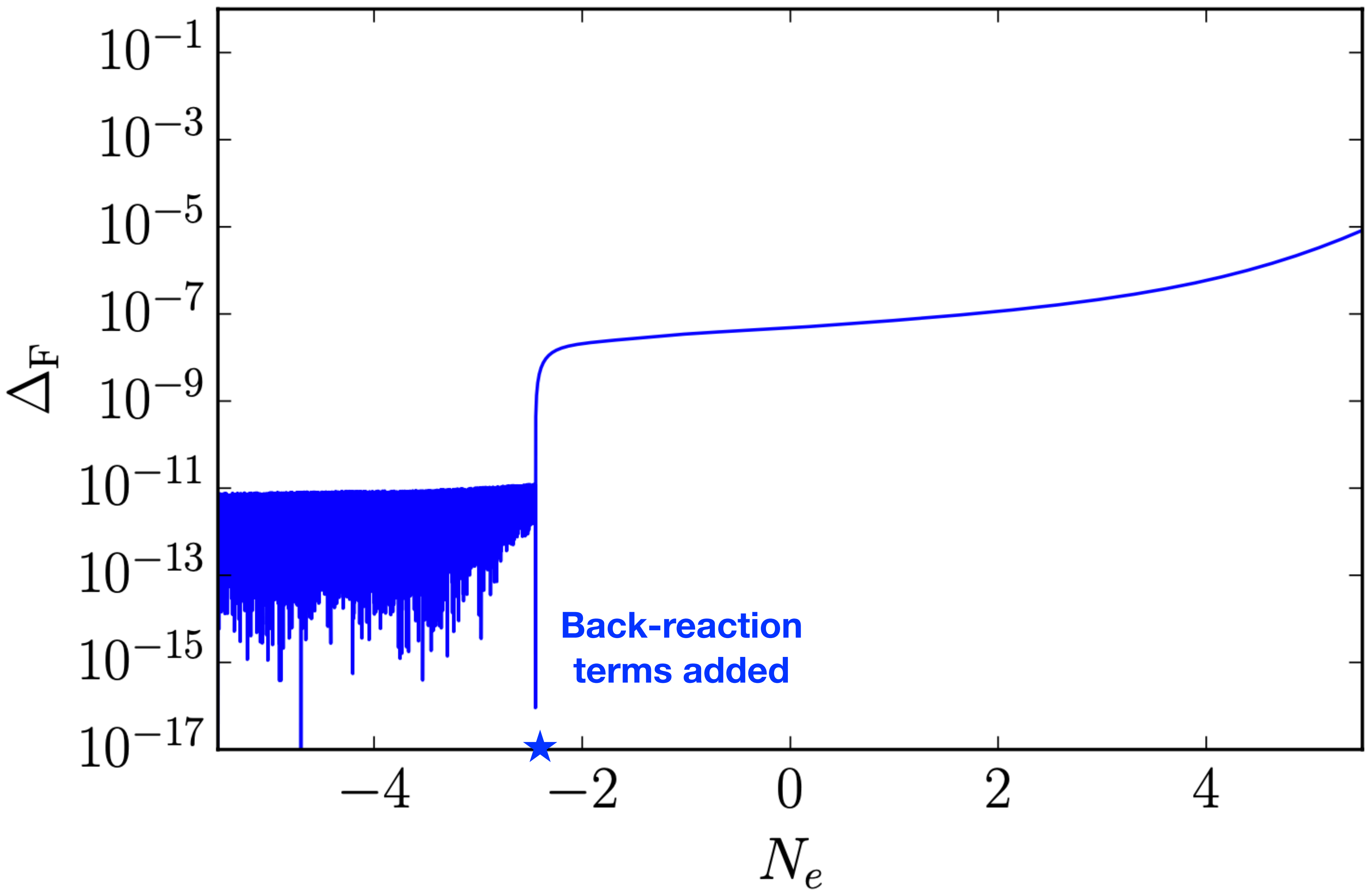
Gauss Constraint

$$\vec{\nabla} \cdot \vec{E} = -\frac{1}{a\Lambda} \vec{\nabla}\phi \cdot \vec{B}$$



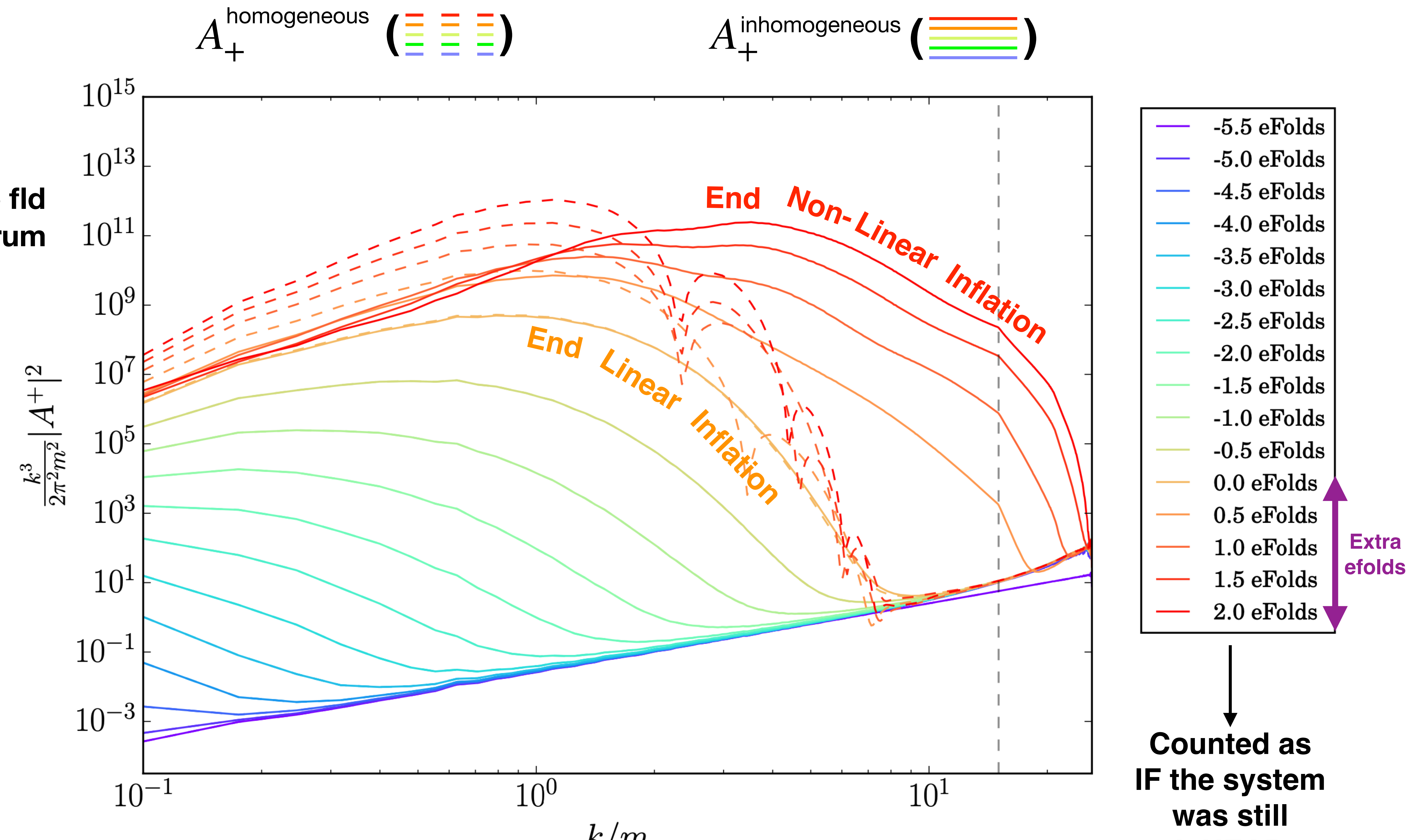
Hubble Constraint

$$\pi_a^2 = \frac{a^2}{3m_{pl}^2}(K_\phi + G_\phi + V + K_A + G_A) \quad ; \quad \pi_a \equiv \dot{a}$$



Axion-Inflation$\left(V(\phi) = \frac{1}{2}m^2\phi^2 \; ; \; \frac{\phi}{4\Lambda}F\tilde{F} \; ; \; \Lambda = \frac{m_p}{15}\right)$

Gauge fld Spectrum

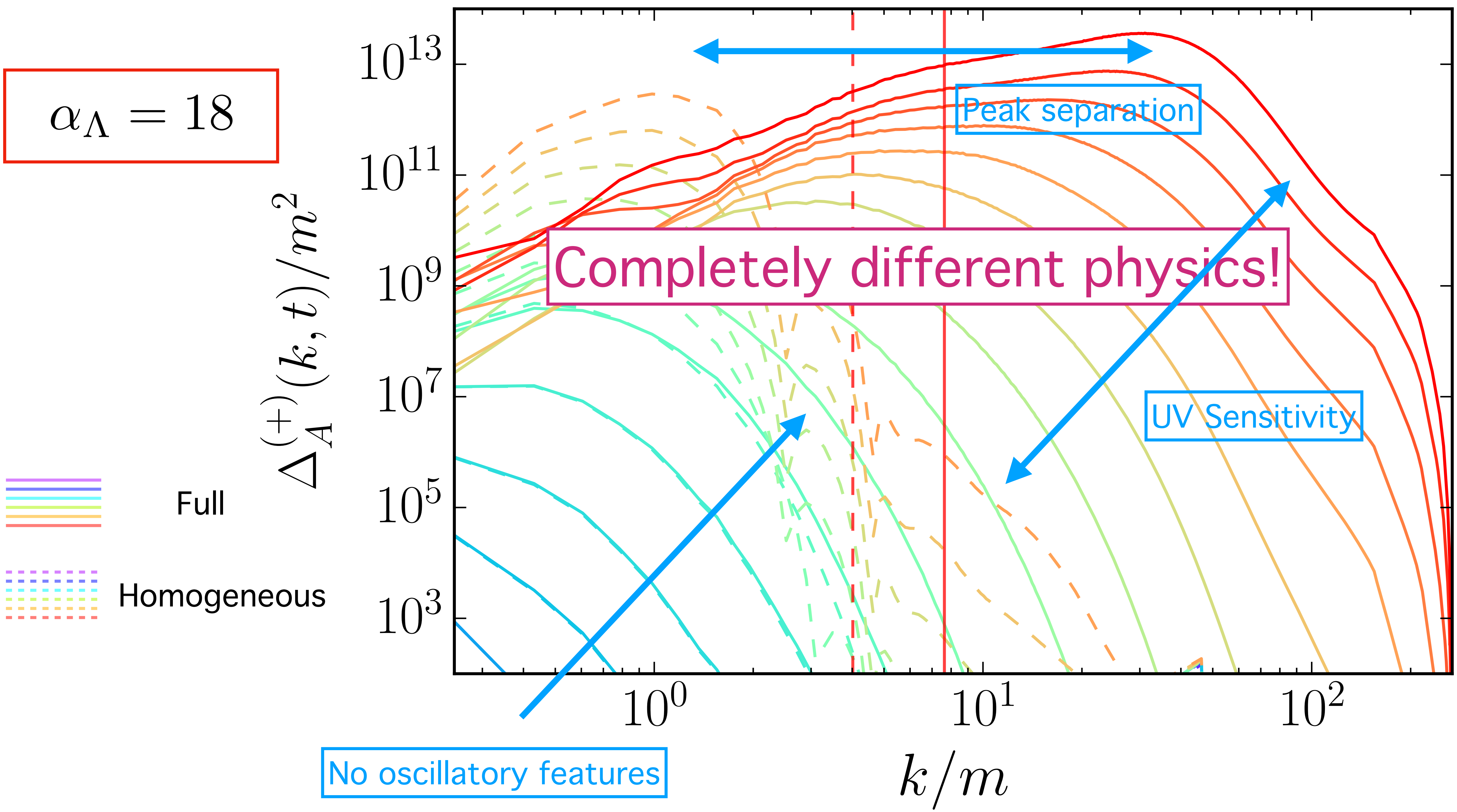


Counted as IF the system was still LINEAR

Zoom

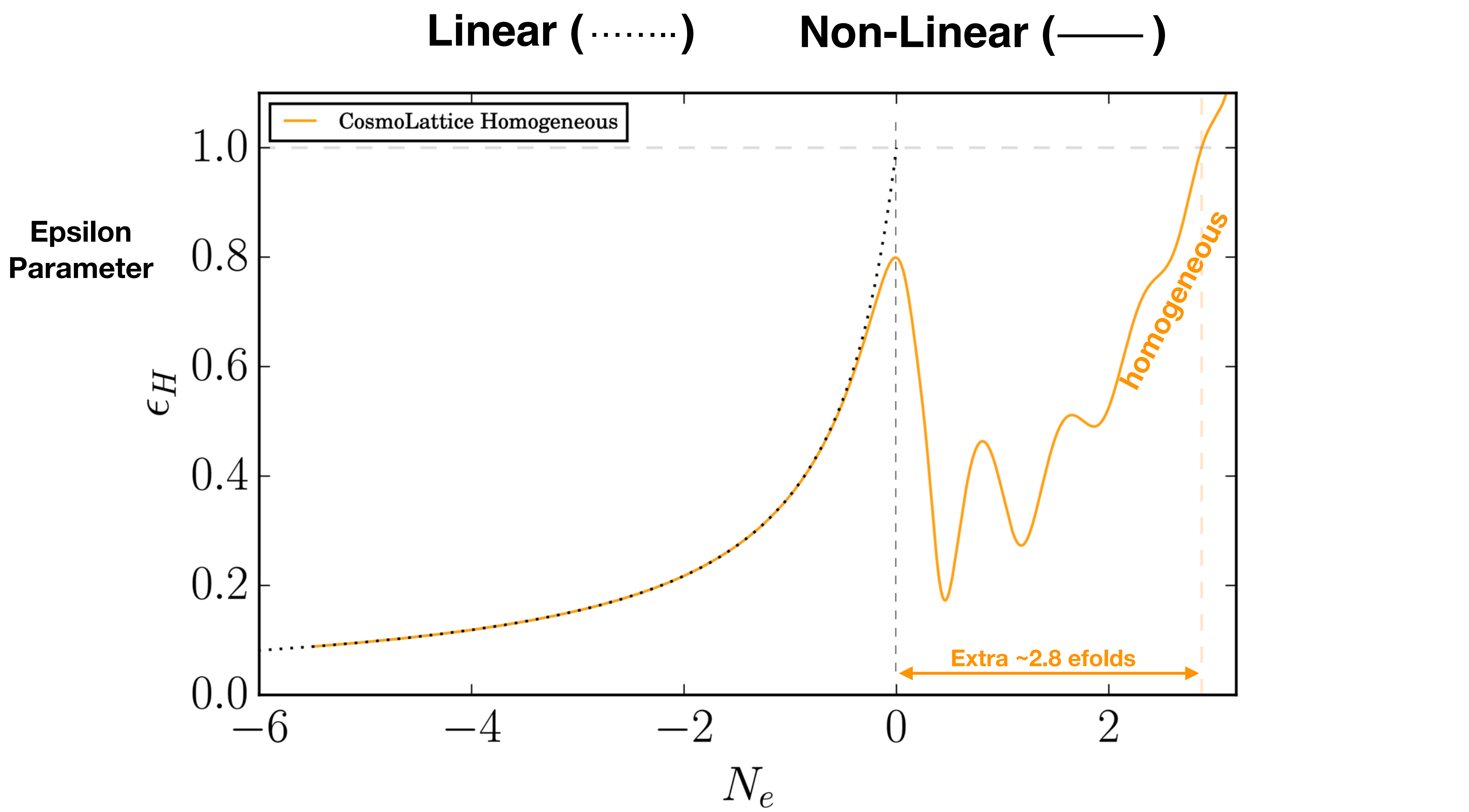
Homogeneous vs Inhomogeneous Backreaction spectrum evolution

$\alpha_\Lambda = 18$



$$V(\phi) = \frac{1}{2}m^2\phi^2 \ ; \ \frac{\phi}{4\Lambda}F\tilde{F} \ ; \boxed{\Lambda = \frac{m_p}{15}}$$

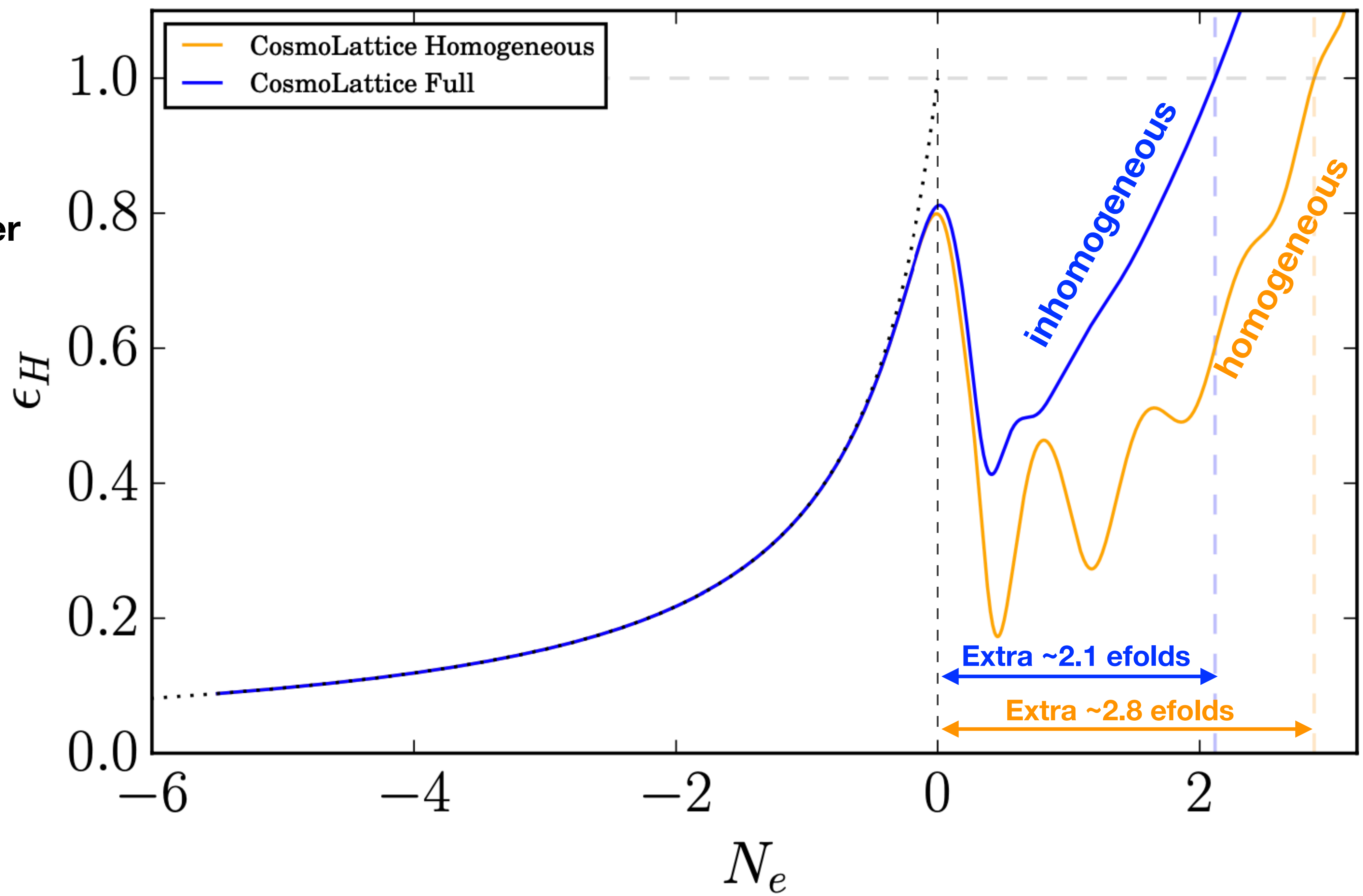
Axion-Inflation($V(\phi) = \frac{1}{2}m^2\phi^2$; $\frac{\phi}{4\Lambda}F\tilde{F}$; $\Lambda = \frac{m_p}{15}$)



Axion-Inflation($V(\phi) = \frac{1}{2}m^2\phi^2$; $\frac{\phi}{4\Lambda}F\tilde{F}$; $\Lambda = \frac{m_p}{15}$)

Linear (········) **Non-Linear (——)**

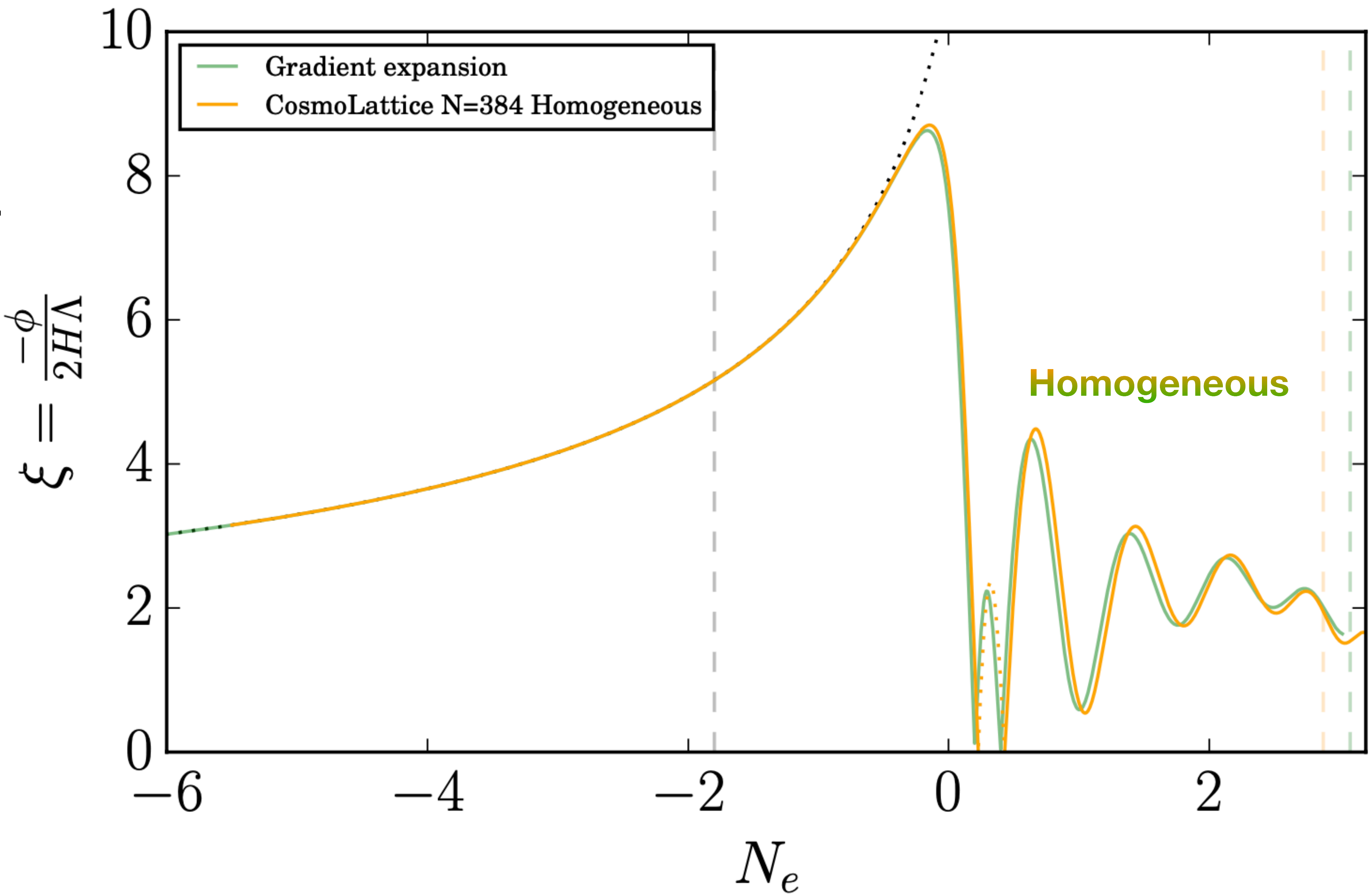
Epsilon Parameter



Axion-Inflation($V(\phi) = \frac{1}{2}m^2\phi^2$; $\frac{\phi}{4\Lambda}F\tilde{F}$; $\Lambda = \frac{m_p}{15}$)

Homogeneous (——)

Xi parameter



Axion-Inflation($V(\phi) = \frac{1}{2}m^2\phi^2$; $\frac{\phi}{4\Lambda}F\tilde{F}$; $\Lambda = \frac{m_p}{15}$)