Strong Backreaction in Axion-Inflation



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Strong Backreaction Regime in Axion Inflation

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We study the nonlinear dynamics of axion inflation, capturing for the first time the inhomogeneity and full dynamical range during strong backreaction, till the end of inflation. Accounting for inhomogeneous effects leads to a number of new relevant results, compared to spatially homogeneous studies: (i) the number of extra efoldings beyond slow-roll inflation increases very rapidly with the coupling, (ii) oscillations of the inflaton velocity are attenuated, (iii) the tachyonic gauge field helicity spectrum is smoothed out (i.e., the spectral oscillatory features disappear), broadened, and shifted to smaller scales, and (iv) the nontachyonic helicity is excited, reducing the chiral asymmetry, now scale dependent. Our results are expected to impact strongly on the phenomenology and observability of axion inflation, including gravitational wave generation and primordial black hole production.

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Bartolo et al '16, 1610.06481



Bartolo et al '16, 1610.06481



Bartolo et al '16, 1610.06481

PROBLEM: PNG, GW and PBH **Analytical approximations** !

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Let's have a look to the full problem !

$$\left(V(\phi) = \frac{1}{2}m^2\phi^2\right)$$

PROBLEM: PNG, GW and PBH ----- Analytical approximations !

$$\pi_{\phi} \equiv \dot{\phi} , \quad E_{i} \equiv \dot{A}_{i} , \quad B_{i} \equiv \epsilon_{ijk} \partial_{j} A_{k}$$
$$\tilde{\pi}_{\phi} = a^{3} \pi_{\phi} , \quad \tilde{\vec{E}} = a \vec{E} , \ \pi_{a} \equiv \dot{a}$$

Let's have a look to the full problem !

$$\left(V(\phi) = \frac{1}{2}m^2\phi^2\right)$$





















PROBLEM: PNG, GW and PBH **Analytical approximations** !

Can we do better than homogeneous backreaction?



PROBLEM: PNG, GW and PBH **Analytical approximations** !

Yes, we need a full lattice approach



PROBLEM: PNG, GW and PBH **Analytical approximations** !

Yes, we need a full lattice approach



PROBLEM: PNG, GW and PBH **Analytical approximations** !

Yes, we need a full lattice approach



PROBLEM: PNG, GW and PBH **Analytical approximations** !

Let's "latticize" the system of EOM !




- 2. Cont. Limit to $\mathcal{O}(dx^2)$
- 3. Lattice Bianchi Identities: $\Delta_i^-(B_i^{(4)} + B_{i,+\hat{0}}^{(4)}) = 0, \dots$
- 4. Topological Term: $(F_{\mu\nu}\tilde{F}^{\mu\nu})_L = \Delta^+_{\mu}K^{\mu}$ (CS current) $\begin{bmatrix} F_{\mu\nu}\tilde{F}^{\mu\nu} = \partial_{\mu}K^{\mu} \end{bmatrix}$ Exact Shift Sym. on the lattice !

$$\begin{split} & \text{LATTICE FORMULATION of } \phi F \tilde{F} \\ & \text{Eom} \\ \Delta_{0}^{+} \left(a^{3} \pi_{\phi} \right) = a_{+\frac{0}{2}} \sum_{i} \Delta_{i}^{-} \Delta_{i}^{+} \phi_{+\frac{0}{2}} - a_{+\frac{0}{2}}^{3} m^{2} \phi_{+\frac{0}{2}} + \frac{1}{\Lambda} \sum_{i} \frac{1}{2} E_{i,+\frac{0}{2}}^{(2)} \left(B_{i}^{(4)} + B_{i,+0}^{(4)} \right) \\ & \Delta_{0}^{-} \left(a_{+\frac{0}{2}} E_{i,+\frac{0}{2}} \right) = -\frac{1}{a} \sum_{j,k} \epsilon_{ijk} \Delta_{j}^{-} B_{k} - \frac{1}{2\Lambda} \left(\pi_{\phi} B_{i}^{(4)} + \pi_{\phi,+i} B_{i,+i}^{(4)} \right) \\ & + \frac{1}{8\Lambda} (2 + dx \Delta_{i}^{+}) \sum_{\pm} \sum_{j,k} \left\{ \epsilon_{ijk} [(\Delta_{j}^{\pm} \phi) E_{k,\pm j}^{(2)}]_{+\frac{0}{2}} + [(\Delta_{j}^{\pm} \phi) E_{k,\pm j}^{(2)}]_{-\frac{0}{2}} \right\} \\ & a_{+\frac{0}{2}} \sum_{i} \Delta_{i}^{-} E_{i,+\frac{0}{2}} = -\frac{1}{4\Lambda} \sum_{\pm} \sum_{i} \left(\Delta_{i}^{\pm} \phi_{+\frac{0}{2}} \right) \left(B_{i}^{(4)} + B_{i,+0}^{(4)} \right)_{\pm i} , \quad \text{(Gauss Law)} \\ & \text{Def. Shapostnikov 2017} \\ & \dot{\pi}_{\phi} = -3H \pi_{\phi} + \frac{1}{a^{2}} \vec{\nabla}^{2} \phi - m^{2} \phi + \frac{1}{a^{3}\Lambda} \vec{E} \cdot \vec{B}, \\ & \dot{\vec{E}} = -H \vec{E} - \frac{1}{a^{2}} \vec{\nabla} \times \vec{B} - \frac{1}{a\Lambda} \pi_{\phi} \vec{B} + \frac{1}{a\Lambda} \vec{\nabla} \phi \times \vec{E} \\ & \vec{\nabla} \cdot \vec{E} = -\frac{1}{a\Lambda} \vec{\nabla} \phi \cdot \vec{B} \text{(Gauss Law)} \\ & \text{EoM} \\ & \text{Continuum} \end{aligned}$$

$$\begin{split} & \text{LATTICE FORMULATION of } \phi F \tilde{F} \\ & \text{Eom} \\ & \Delta_{0}^{+} \left(a^{3} \pi_{\phi} \right) = a_{+\frac{0}{2}} \sum_{i} \Delta_{i}^{-} \Delta_{i}^{+} \phi_{+\frac{0}{2}} - a_{+\frac{0}{2}}^{3} m^{2} \phi_{+\frac{0}{2}} + \frac{1}{\Lambda} \sum_{i} \frac{1}{2} E_{i,+\frac{0}{2}}^{(2)} \left(B_{i}^{(4)} + B_{i,+\hat{0}}^{(4)} \right) \\ & \Delta_{0}^{-} \left(a_{+\frac{0}{2}} E_{i,+\frac{0}{2}} \right) = -\frac{1}{a} \sum_{j,k} \epsilon_{ijk} \Delta_{j}^{-} B_{k} - \frac{1}{2\Lambda} \left(\pi_{\phi} B_{i}^{(4)} + \pi_{\phi,+i} B_{i,+i}^{(4)} \right) \\ & + \frac{1}{8\Lambda} (2 + dx \Delta_{i}^{+}) \sum_{\pm} \sum_{j,k} \left\{ \epsilon_{ijk} [(\Delta_{j}^{\pm} \phi) E_{k,\pm j}^{(2)}]_{+\frac{0}{2}} + [(\Delta_{j}^{\pm} \phi) E_{k,\pm j}^{(2)}]_{-\frac{0}{2}} \right\} \\ & a_{+\frac{0}{2}} \sum_{i} \Delta_{i}^{-} E_{i,+\frac{0}{2}} = -\frac{1}{4\Lambda} \sum_{\pm} \sum_{i} \left(\Delta_{i}^{\pm} \phi_{+\frac{0}{2}} \right) \left(B_{i}^{(4)} + B_{i,+\hat{0}}^{(4)} \right)_{\pm i} , \quad \text{(Gauss Law)} \\ & \text{For provide the second sec$$

$$\begin{split} & \text{LATTICE FORMULATION of } \phi F \tilde{F} \\ & \text{Eom} \\ \Delta_{0}^{+} \left(a^{3} \pi_{\phi} \right) = a_{+\frac{0}{2}} \sum_{i} \Delta_{i}^{-} \Delta_{i}^{+} \phi_{+\frac{0}{2}} - a_{+\frac{0}{2}}^{3} m^{2} \phi_{+\frac{0}{2}} + \frac{1}{\Lambda} \sum_{i} \frac{1}{2} E_{i,+\frac{0}{2}}^{(2)} \left(B_{i}^{(4)} + B_{i,+\hat{0}}^{(4)} \right) \\ & \Delta_{0}^{-} \left(a_{+\frac{0}{2}} E_{i,+\frac{0}{2}} \right) = -\frac{1}{a} \sum_{j,k} \epsilon_{ijk} \Delta_{j}^{-} B_{k} - \frac{1}{2\Lambda} \left(\pi_{\phi} B_{i}^{(4)} + \pi_{\phi,+i} B_{i,+i}^{(4)} \right) \\ & + \frac{1}{8\Lambda} (2 + dx \Delta_{i}^{+}) \sum_{\pm} \sum_{j,k} \left\{ \epsilon_{ijk} [(\Delta_{j}^{\pm} \phi) E_{k,\pm j}^{(2)}]_{+\frac{0}{2}} + [(\Delta_{j}^{\pm} \phi) E_{k,\pm j}^{(2)}]_{-\frac{0}{2}} \right\} \\ & a_{+\frac{0}{2}} \sum_{i} \Delta_{i}^{-} E_{i,+\frac{0}{2}} = -\frac{1}{4\Lambda} \sum_{\pm} \sum_{i} \left(\Delta_{i}^{\pm} \phi_{+\frac{0}{2}} \right) \left(B_{i}^{(4)} + B_{i,+\hat{0}}^{(4)} \right)_{\pm i} , \quad \text{(Gauss Law)} \\ & \text{For } M \pi_{\phi} + \frac{1}{a^{2}} \vec{\nabla}^{2} \phi - m^{2} \phi + \frac{1}{a^{3}\Lambda} \vec{E} \cdot \vec{B}, \\ & \vec{E} = -H \vec{E} - \frac{1}{a^{2}} \vec{\nabla} \times \vec{B} - \frac{1}{a\Lambda} \pi_{\phi} \vec{B} + \frac{1}{a\Lambda} \vec{\nabla} \phi \times \vec{E} \\ & \vec{\nabla} \cdot \vec{E} = -\frac{1}{a\Lambda} \vec{\nabla} \phi \cdot \vec{B} \text{(Gauss Law)} \\ & \text{Fom } \text{Continuum} \end{aligned}$$

$$\begin{split} & \text{LATTICE FORMULATION of } \phi F \tilde{F} \\ & \text{EoM} \\ & \Delta_{0}^{+} \left(a^{3} \pi_{\phi} \right) = a_{+\frac{0}{2}} \sum_{i} \Delta_{i}^{-} \Delta_{i}^{+} \phi_{+\frac{0}{2}} - a_{+\frac{0}{2}}^{3} m^{2} \phi_{+\frac{0}{2}} + \frac{1}{\Lambda} \sum_{i} \frac{1}{2} E_{i,+\frac{0}{2}}^{(2)} \left(B_{i}^{(4)} + B_{i,+\hat{0}}^{(4)} \right) \\ & \Delta_{0}^{-} \left(a_{+\frac{0}{2}} E_{i,+\frac{0}{2}} \right) = -\frac{1}{a} \sum_{j,k} \epsilon_{ijk} \Delta_{j}^{-} B_{k} - \frac{1}{2\Lambda} \left(\pi_{\phi} B_{i}^{(4)} + \pi_{\phi,+i} B_{i,+i}^{(4)} \right) \\ & + \frac{1}{8\Lambda} (2 + dx \Delta_{i}^{+}) \sum_{\pm} \sum_{j,k} \frac{\left\{ \epsilon_{ijk} [(\Delta_{j}^{\pm} \phi) E_{k,\pm j}^{(2)}]_{+\frac{0}{2}} + [(\Delta_{j}^{\pm} \phi) E_{k,\pm j}^{(2)}]_{-\frac{0}{2}} \right\} \\ & a_{+\frac{0}{2}} \sum_{i} \Delta_{i}^{-} E_{i,+\frac{0}{2}} = -\frac{1}{4\Lambda} \sum_{\pm} \sum_{i} \left(\Delta_{i}^{\pm} \phi_{+\frac{0}{2}} \right) \left(B_{i}^{(4)} + B_{i,+\hat{0}}^{(4)} \right)_{\pm i} , \quad \text{(Gauss Law)} \\ & \text{DGF, Shaposhnikov 2017 Canivete, DGF 2018} \\ & \dot{\pi}_{\phi} = -3H\pi_{\phi} + \frac{1}{a^{2}} \vec{\nabla}^{2} \phi - m^{2} \phi + \frac{1}{a^{3}\Lambda} \vec{E} \cdot \vec{B}, \\ & \dot{\vec{E}} = -H\vec{E} - \frac{1}{a^{2}} \vec{\nabla} \times \vec{B} - \frac{1}{a\Lambda} \pi_{\phi} \vec{B} + \frac{1}{a\Lambda} \vec{\nabla} \phi \times \vec{E} \\ & \vec{\nabla} \cdot \vec{E} = -\frac{1}{a\Lambda} \vec{\nabla} \phi \cdot \vec{B} \text{(Gauss Law)} \\ & \text{EoM} \\ & \text{Continuum} \\ \end{array}$$

LATTICE FORMULATION of $\phi F \tilde{F}$ **Lattice Formulation**

		$\mathbf{x} + (3)$ $\mathbf{x} = \mathbf{x} + (3)$ $\mathbf{x} = \mathbf{x} + (3)$
		$\Delta_{0}^{+}\left(a^{3}\pi_{\phi} ight)=a_{+rac{\hat{0}}{2}}\sum_{i}\Delta_{i}^{-}\Delta_{i}^{+}\phi_{+rac{\hat{0}}{2}}-a^{3}_{+rac{\hat{0}}{2}}m^{2}\phi_{+rac{\hat{0}}{2}}$
		$+rac{1}{\Lambda}\sum_{i}^{i}rac{1}{2}E_{i,+rac{\hat{0}}{2}}^{(2)}\left(B_{i}^{(4)}+B_{i,+\hat{0}}^{(4)} ight),$
	Δ_0^-	$\left(a_{+rac{\hat{0}}{2}}E_{i,+rac{\hat{0}}{2}} ight) = -rac{1}{a}\sum_{j,k}\epsilon_{ijk}\Delta_{j}^{-}B_{k} - rac{1}{2\Lambda}\left(\pi_{\phi}B_{i}^{(4)} + \pi_{\phi,+i}B_{i,+i}^{(4)} ight)$
		$+\frac{1}{8\Lambda}(2+dx\Delta_{i}^{+})\sum_{\pm}\sum_{j,k}\left\{\epsilon_{ijk}[(\Delta_{j}^{\pm}\phi)E_{k,\pm j}^{(2)}]_{+\frac{\hat{0}}{2}}+[(\Delta_{j}^{\pm}\phi)E_{k,\pm j}^{(2)}]_{-\frac{\hat{0}}{2}}\right\}$
	$a_{+{\hat 0\over 2}}$	$\sum_{i} \Delta_{i}^{-} E_{i,+\frac{\hat{0}}{2}} = -\frac{1}{4\Lambda} \sum_{\pm} \sum_{i} \left(\Delta_{i}^{\pm} \phi_{+\frac{\hat{0}}{2}} \right) \left(B_{i}^{(4)} + B_{i,+\hat{0}}^{(4)} \right)_{\pm i} , \text{(Gauss Law)}$

	Expansion	
L	•	2
	$\left(\Delta_0^+ a_{-\hat{0}/2} ight)^2$	$^{2}=rac{a^{2}}{3m_{ m pl}^{2}} ho_{L},$
	$\Delta_0^-\Delta_0^+a_{+\hat{0}/2}$	$a_2 = -rac{a_{+\hat{0}/2}}{6m_{ m pl}^2}(ho_L+3p_L)_{+\hat{0}/2}$

LATTICE FORMULATION of ϕFF **Lattice Formulation**

EoM $\Delta_{0}^{+}(a^{3}\pi_{\phi}) = a_{\pm 0} \sum \Delta_{i}^{-} \Delta_{i}^{+} \phi_{\pm 0} - a_{\pm 0}^{3} m^{2} \phi_{\pm 0}$	Expansion
$ \begin{split} & =_{0} \left(a^{\pm} u \phi \right)^{-} = a_{\pm \frac{0}{2}} \sum_{i} -i -i + \frac{1}{2} \sum_{i} a_{\pm \frac{0}{2}} e^{-i + \frac{1}{2}} + \frac{1}{2} \\ & + \frac{1}{\Lambda} \sum_{i} \frac{1}{2} E_{i,\pm \frac{0}{2}}^{(2)} \left(B_{i}^{(4)} + B_{i,\pm 0}^{(4)} \right) , \\ & \Delta_{0}^{-} \left(a_{\pm \frac{0}{2}} E_{i,\pm \frac{0}{2}} \right) = -\frac{1}{a} \sum_{j,k} \epsilon_{ijk} \Delta_{j}^{-} B_{k} - \frac{1}{2\Lambda} \left(\pi_{\phi} B_{i}^{(4)} + \pi_{\phi,\pm i} B_{i,\pm i}^{(4)} \right) \\ & + \frac{1}{8\Lambda} (2 + dx \Delta_{i}^{+}) \sum_{\pm} \sum_{j,k} \left\{ \epsilon_{ijk} [(\Delta_{j}^{\pm} \phi) E_{k,\pm j}^{(2)}]_{\pm \frac{0}{2}} + [(\Delta_{j}^{\pm} \phi) E_{k,\pm j}^{(2)}]_{-\frac{0}{2}} \right\} \\ & a_{\pm 0} \sum_{i} \Delta_{i}^{-} E_{i,\pm 0} = -\frac{1}{44} \sum_{i} \sum_{i} \sum_{i} \left(\Delta_{i}^{\pm} \phi_{\pm 0} \right) \left(B_{i}^{(4)} + B_{i,\pm 0}^{(4)} \right) , \text{(Gauss Law)} \end{split} $	$\begin{split} \left(\Delta_0^+ a_{-\hat{0}/2} \right)^2 &= \frac{a^2}{3m_{\rm pl}^2} \rho_L , \\ \Delta_0^- \Delta_0^+ a_{+\hat{0}/2} &= -\frac{a_{+\hat{0}/2}}{6m_{\rm pl}^2} (\rho_L + 3p_L)_{+\hat{0}/2} \end{split}$

$$\begin{split} \rho_L &= \bar{H}^{\rm kin} + \frac{1}{a^2} \frac{1}{2} (\bar{H}^{\rm grad}_{-\hat{0}/2} + \bar{H}^{\rm grad}_{+\hat{0}/2}) + \frac{1}{2} (\bar{H}^{\rm pot}_{-\hat{0}/2} + \bar{H}^{\rm pot}_{+\hat{0}/2}) + \frac{1}{a^2} \frac{1}{2} (\bar{H}^E_{-\hat{0}/2} + \bar{H}^E_{+\hat{0}/2}) + \frac{1}{a^4} \bar{H}^B \,, \\ (\rho_L + 3p_L)_{+\hat{0}/2} &= 2 (\bar{H}^{\rm kin} + \bar{H}^{\rm kin}_{+\hat{0}}) - 2 \bar{H}^{\rm pot}_{+\hat{0}/2} + \frac{2}{a^2_{+\hat{0}/2}} \bar{H}^E + \frac{1}{a^4_{+\hat{0}/2}} (\bar{H}^B + \bar{H}^B_{+\hat{0}}) \,, \end{split}$$

$$\begin{pmatrix} \bar{H}^{\rm kin} = \left\langle \frac{1}{N^3} \sum_{\vec{n}} \frac{\pi_{\phi}^2}{2} \right\rangle \qquad \bar{H}^{\rm grad} = \left\langle \frac{1}{N^3} \sum_{\vec{n}} \frac{1}{2} \sum_{i} (\Delta_i^+ \phi_{+\frac{\hat{0}}{2}})^2, \quad \bar{H}^{\rm pot} = \left\langle \frac{1}{N^3} \sum_{\vec{n}} \frac{1}{2} m^2 \phi_{+\frac{\hat{0}}{2}}^2 \right\rangle \\ \bar{H}^E = \left\langle \frac{1}{N^3} \sum_{\vec{n}} \sum_{i} \frac{1}{2} E_{i,+\frac{\hat{0}}{2}}^2 \right\rangle \quad \bar{H}^B = \left\langle \frac{1}{N^3} \sum_{\vec{n}} \sum_{i,j} \frac{1}{4} (\Delta_i^+ A_j - \Delta_j^+ A_i)^2 \right\rangle$$

LATTICE FORMULATION of $\phi F \tilde{F}$ **Lattice Formulation**

		$A + (3)$ $\sum A - A + (3)$ $2 $
		$\Delta_{0}^{+}\left(a^{3}\pi_{\phi} ight)=a_{+rac{\hat{0}}{2}}\sum_{i}\Delta_{i}^{-}\Delta_{i}^{+}\phi_{+rac{\hat{0}}{2}}^{-}-a^{3}_{+rac{\hat{0}}{2}}m^{2}\phi_{+rac{\hat{0}}{2}}^{-}$
		$+rac{1}{\Lambda}\sum_i rac{1}{2}E^{(2)}_{i,+rac{\hat{0}}{2}}\left(B^{(4)}_i+B^{(4)}_{i,+\hat{0}} ight),$
	Δ_0^-	$\left(a_{+rac{\hat{0}}{2}}E_{i,+rac{\hat{0}}{2}} ight) = -rac{1}{a}\sum_{j,k}\epsilon_{ijk}\Delta_{j}^{-}B_{k} - rac{1}{2\Lambda}\left(\pi_{\phi}B_{i}^{(4)} + \pi_{\phi,+i}B_{i,+i}^{(4)} ight)$
		$+\frac{1}{8\Lambda}(2+dx\Delta_{i}^{+})\sum_{\pm}\sum_{j,k}\left\{\epsilon_{ijk}[(\Delta_{j}^{\pm}\phi)E_{k,\pm j}^{(2)}]_{+\frac{0}{2}}+[(\Delta_{j}^{\pm}\phi)E_{k,\pm j}^{(2)}]_{-\frac{0}{2}}\right\}$
	$a_{+{\hat 0\over 2}}$	$\sum_{i} \Delta_{i}^{-} E_{i,+\frac{\hat{0}}{2}} = -\frac{1}{4\Lambda} \sum_{\pm} \sum_{i} \left(\Delta_{i}^{\pm} \phi_{+\frac{\hat{0}}{2}} \right) \left(B_{i}^{(4)} + B_{i,+\hat{0}}^{(4)} \right)_{\pm i} , \text{(Gauss Law)}$

Expansion	
$\left(\Delta_0^+a_{-\hat{0}/2} ight)$ $\Delta_0^-\Delta_0^+a_{+\hat{0}/2}$	$egin{aligned} & a^2 & = rac{a^2}{3m_{ m pl}^2} ho_L, \ & a_2 & = -rac{a_{+\hat{0}/2}}{6m_{ m pl}^2} (ho_L + 3p_L)_{+\hat{0}/2} \end{aligned}$

Now I will show our work *Phys.Rev.Lett.* 131 (2023) 15, 151003 e-Print: 2303.17436 [astro-ph.CO]



CosmoLattice

 $V(\phi) = \frac{1}{2}m^2\phi^2 \; ; \; \frac{\phi}{4\Lambda}F\tilde{F} \; ; \; \Lambda = \frac{m_p}{18}$





































Axion-Inflation $\left(V(\phi) = \frac{1}{2}m^2\phi^2; \frac{\phi}{4\Lambda}F\tilde{F}; \Lambda = \frac{m_p}{\alpha}\right)$ $(\alpha = 15, 18, 20)$



Summary

- * ξ Controls the Gauge field excitation
- * Linear change in ξ : exponential response in A_{μ}
- * Predictions/constraints (PNG, PBH and GWs) depend crucially on ξ : we will re-assess real observability !
- * Adding Schwinger pair production easy via $\vec{J} = \sigma \vec{E}$
- * Other phenomena: BAU, Magnetogenesis, ...

Thanks for your attention !



Axion-inflation extra stuff

Gauss Constraint





Hubble Constraint



0

 N_e

2

4

-2

4

Axion-Inflation $\left(V(\phi) = \frac{1}{2}m^2\phi^2; \frac{\phi}{4\Lambda}F\tilde{F}; \Lambda = \frac{m_p}{15}\right)$



Zoom

Homogeneous vs Inhomogeneous Backreaction spectrum evolution


$V(\phi) = \frac{1}{2}m^2\phi^2 \; ; \; \frac{\phi}{4\Lambda}F\tilde{F} \; ; \; \Lambda = \frac{m_p}{15}$









Homogeneous (—) In-Homogeneous (—)

