Inflationary and Post-Inflationary Scalar Dark Matter Production

Marcos A. G. García

+ Mathias Pierre (DESY) and Sarunas Verner (Florida)

2206.08940, 2303.07359, 2305.14446

Universidad Nacional
Autónoma de México

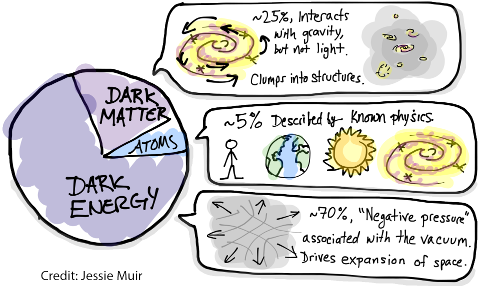
CONAHCYT
CONSEJO NACIONAL DE HUMANIDADES
CIENCIAS Y TECNOLOGÍAS

INSTITUTO
DE FÍSICA

Instituto de Física
UNAM

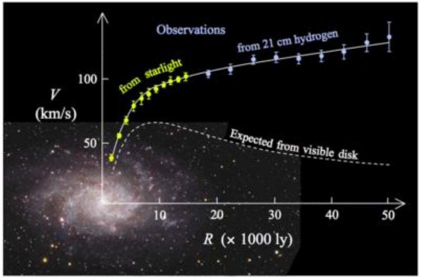
física teórica
IFUNAM

Dark Matter



Credit: Jessie Muir

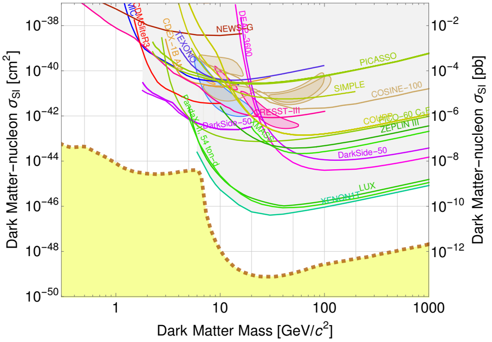
galaxy rotation



collisions of clusters

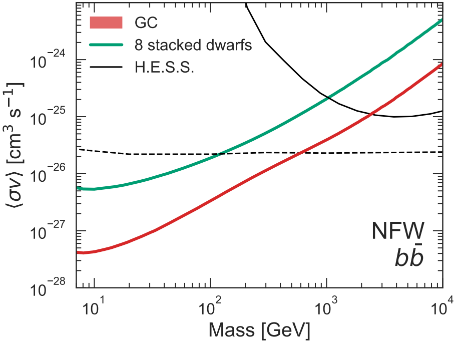
Non-gravitational detection

direct detection

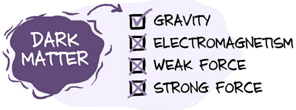


SuperCDMS Dark Matter Limit Plotter

annihilations in the galactic core



K. Abazajian et al., PRD 102 (2020), 043012 (Fermi-LAT)



Minimal scalar dark matter

A simple DM model: scalar χ (spin 0) which only interacts with gravity and/or the inflaton ϕ

$$\mathcal{S} = \int d^4x\sqrt{-g}\Bigg[-\frac{1}{2}(M_P^2 - \overbrace{\xi\chi^2}^{\text{non-minimal coupling}})R + \frac{1}{2}(\partial_\mu\chi)^2 - \frac{1}{2}m_\chi^2\chi^2 - \frac{1}{2}\overbrace{\sigma\phi^2\chi^2}^{\phi\text{-coupling}}$$

$$+\frac{1}{2}(\partial_\mu\phi)^2 - \underbrace{6\lambda M_P^4\tanh^2\left(\frac{\phi}{\sqrt{6}M_P}\right)}_{\text{T-model inflation}} - \underbrace{y\phi\bar{\psi}\psi}_{\text{reheating}} + \mathcal{L}_{\rm SM}\Bigg]$$

Inflationary couplings normalized by

$$\lambda \simeq \frac{3\pi^2 A_{S*}}{N_*^2}\,, \qquad T_{\rm reh} \simeq \left(\frac{9\lambda}{20\pi^4 g_{\rm reh}}\right)^{1/4} y\,M_P$$

QFT in the early universe

Introducing *conformal time*, $dt = a\,d\tau$, and the re-scaled field $X = a\chi$,

$$\left(\partial_\tau^2 - \nabla^2 + a^2 m_{\rm eff}^2\right) X \;=\; 0\,, \qquad m_{\rm eff}^2 \;=\; m_\chi^2 + \sigma\phi^2 + \frac{1}{6}(1-6\xi)R$$

Quantize as a superposition of oscillators

$$\hat{X}(\tau,\boldsymbol{x}) \;=\; \int \frac{d^3\boldsymbol{k}}{(2\pi)^{3/2}}\, e^{-i\boldsymbol{k}\cdot\boldsymbol{x}}\left[X_k(\tau)\hat{a}_{\boldsymbol{k}} + X_k^*(\tau)\hat{a}^\dagger_{-\boldsymbol{k}}\right]\,, \qquad [\hat{a}_{\boldsymbol{k}}, \hat{a}^\dagger_{\boldsymbol{k}'}] = \delta(\boldsymbol{k}-\boldsymbol{k}')\,, \quad \hat{a}_{\boldsymbol{k}}|0\rangle \;=\; 0$$

obtaining

$$X_k'' + \omega_k^2 X_k \;=\; 0\,, \quad \text{with} \quad \omega_k^2 \;=\; k^2 + a^2 m_{\rm eff}^2$$

QFT in the early universe

Introducing *conformal time*, $dt = a\,d\tau$, and the re-scaled field $X = a\chi$,

$$\left(\partial_\tau^2 - \nabla^2 + a^2 m_{\rm eff}^2\right) X \;=\; 0\,, \qquad m_{\rm eff}^2 \;=\; m_\chi^2 + \sigma\phi^2 + \frac{1}{6}(1-6\xi)R$$

Quantize as a superposition of oscillators

$$\hat{X}(\tau, \mathbf{x}) \;=\; \int \frac{d^3\boldsymbol{k}}{(2\pi)^{3/2}}\, e^{-i\boldsymbol{k}\cdot\mathbf{x}} \left[X_k(\tau)\hat{a}_{\boldsymbol{k}} + X_k^*(\tau)\hat{a}^\dagger_{-\boldsymbol{k}}\right]\,, \qquad [\hat{a}_{\boldsymbol{k}}, \hat{a}^\dagger_{\boldsymbol{k}'}] = \delta(\boldsymbol{k} - \boldsymbol{k}')\,, \quad \hat{a}_{\boldsymbol{k}}|0\rangle \;=\; 0$$

obtaining

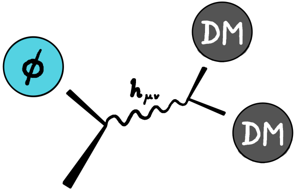
$$X_k'' + \omega_k^2 X_k \;=\; 0\,, \quad \text{with} \quad \omega_k^2 \;=\; k^2 + a^2 m_{\rm eff}^2$$

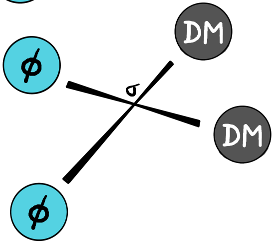
For a mode inside the horizon,

$$\omega_k^2 \;=\; \underbrace{k^2}_{\text{free particle}} + \,\mathcal{O}\left(\underbrace{\frac{a^2 H^2}{k^2}}_{\text{interactions}}\right) \;>\; 0$$

Perturbative DM production

The perturbative picture: inflaton, gravity and dark matter as (quasi)particles





The solution of the Boltzmann equation

$$\frac{\partial f_\chi}{\partial t} - H|\boldsymbol{P}|\frac{\partial f_\chi}{\partial|\boldsymbol{P}|} = \frac{\pi|\mathcal{M}|^2}{2m_\phi^2}\delta(|\boldsymbol{P}| - m_\phi)$$

with

$$|\mathcal{M}|^2 = \frac{1}{8}\frac{\rho_\phi^2}{m_\phi^4}\left[\sigma - \lambda(1-6\xi)\right]^2$$

is the following Phase Space Distribution:

$$f_\chi(q,t) = \frac{\sqrt{3}\pi\hat{\sigma}^2\rho_{\rm end}^{3/2}M_P}{16m_\phi^7}\left(\frac{H_{\rm end}}{m_\phi}q\right)^{-9/2}\theta(q-1)\,\theta\left(\frac{a(t)}{a_{\rm end}} - \frac{H_{\rm end}}{m_\phi}q\right)$$

with $q \equiv \dfrac{|\boldsymbol{P}|}{T_\star}\left(\dfrac{a}{a_{\rm end}}\right)$, $T_\star \equiv H_{\rm end}$

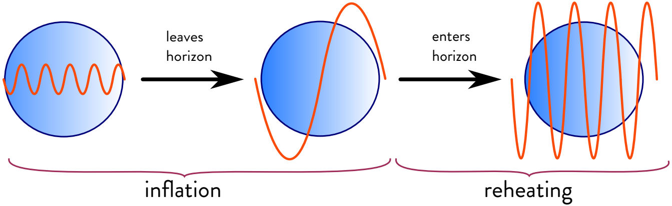
Gravitational particle production during inflation

Light scalar fields are unstable during inflation

$$X_k'' + \omega_k^2 X_k \; = \; 0 \, , \quad \text{with} \quad \omega_k^2 \; = \; k^2 + 2(aH)^2 \left[\frac{m_\chi^2}{2H^2} + \frac{\sigma \phi^2}{2H^2} - 1 + 6\xi \right]$$

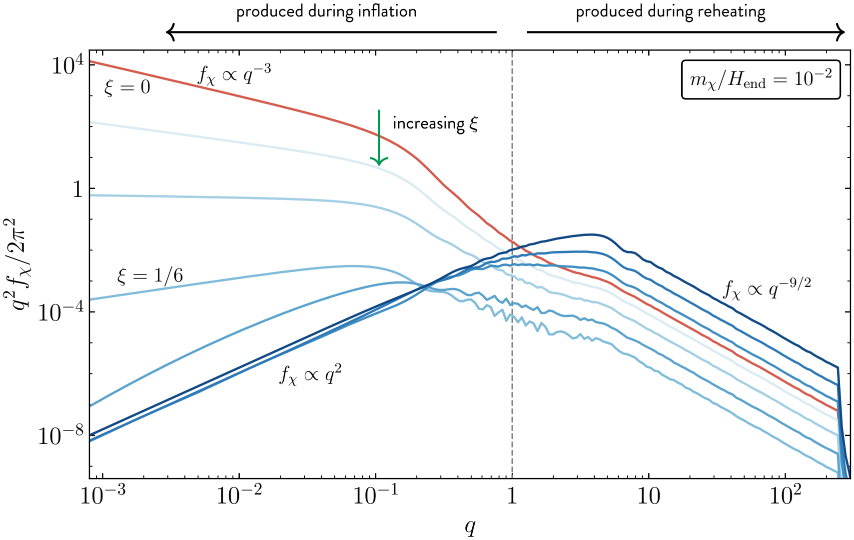
For a mode that is outside the horizon ($k/aH \ll 1$),

$$\omega_k^2 \; < \; 0 \qquad \text{if} \qquad m_\chi^2 < 2H^2 \, , \; \sigma/\lambda \ll 1, \quad \text{and} \quad \xi < 1/6 \qquad \text{(tachyonic instability)}$$

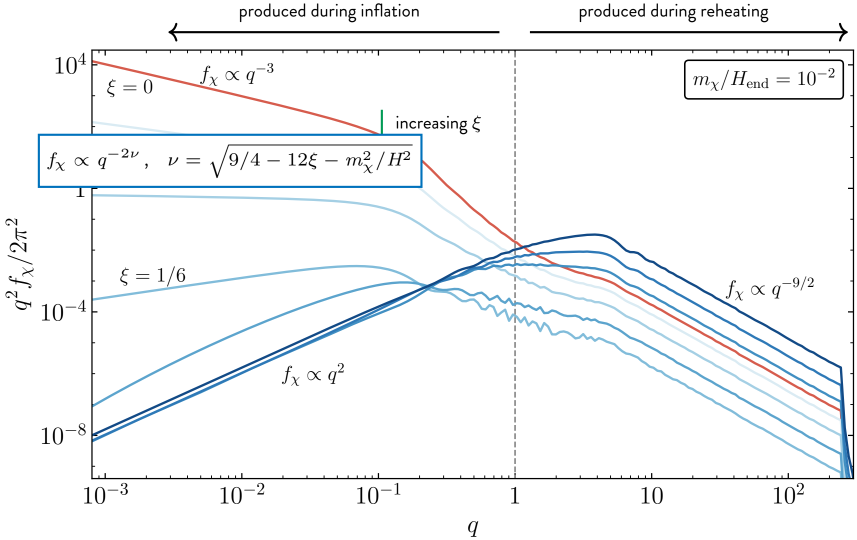


No free particle state during inflation ⇒ no perturbative picture

Gravitational production

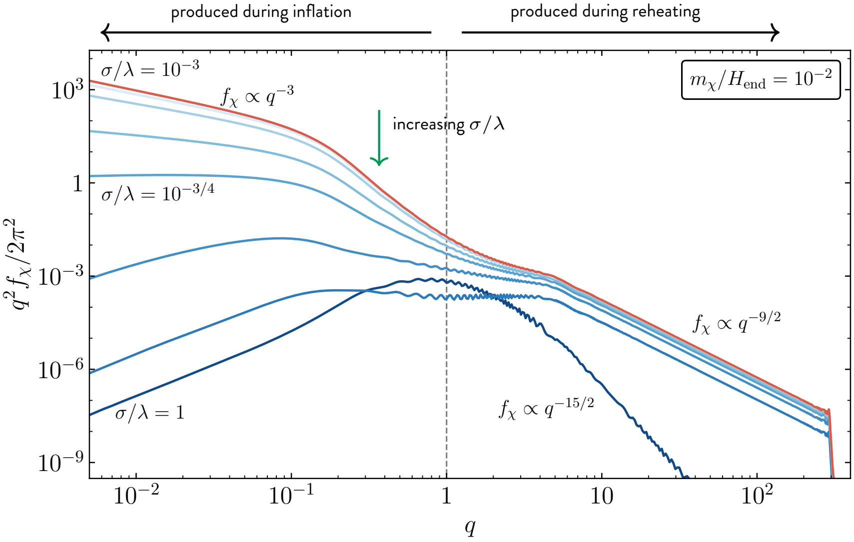


Gravitational production



$$\boxed{f_\chi \propto q^{-2\nu}\,, \quad \nu = \sqrt{9/4 - 12\xi - m_\chi^2/H^2}}$$

Weak inflaton coupling



Strong inflaton coupling

Linear regime: The inflaton remains a condensate ⇒ Hartree aproximation

$$\ddot{\phi} + 3H\dot{\phi} + V_\phi + \sigma\langle\chi^2\rangle\phi = 0$$

$$\langle\chi^2\rangle = \frac{1}{(2\pi)^3 a^2}\int d^3\boldsymbol{p}\left(|X_p|^2 - \frac{1}{2\omega_p}\right)$$

L. Kofman, A. Linde, A. Starobinsky, PRD 56 (1997) 3258

MG, K. Kaneta, Y. Mambrini, K. Olive, S. Verner, JCAP 03 (2022) 016

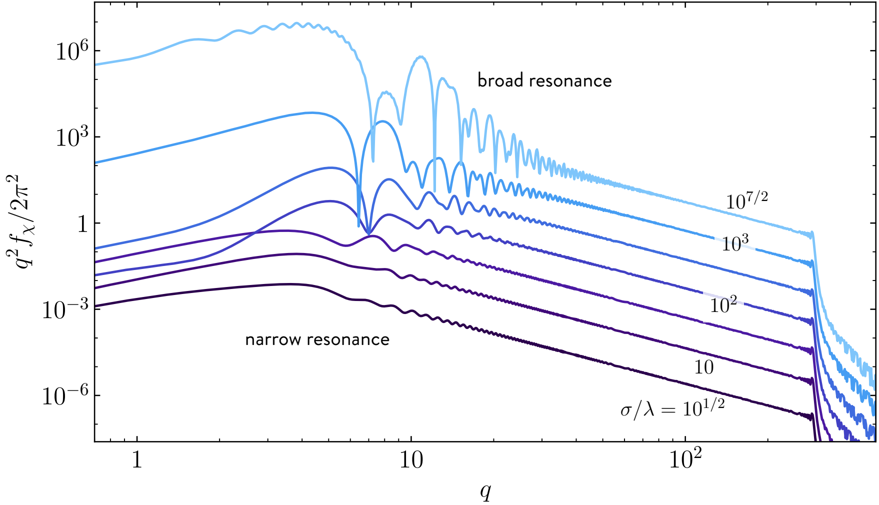
Non-linear regime: The inflaton is fragmented ⇒ ($\mathcal{C}$osmo)$\mathcal{L}$attice

$$\ddot{\phi} + 3H\dot{\phi} - \frac{\nabla^2\phi}{a^2} + V_{,\phi} = 0$$

$$\ddot{\chi} + 3H\dot{\chi} - \frac{\nabla^2\chi}{a^2} + V_{,\chi} = 0$$

D. Figueroa, et al., Comput. Phys. Commun. 283, 108586 (2023)

Linear regime

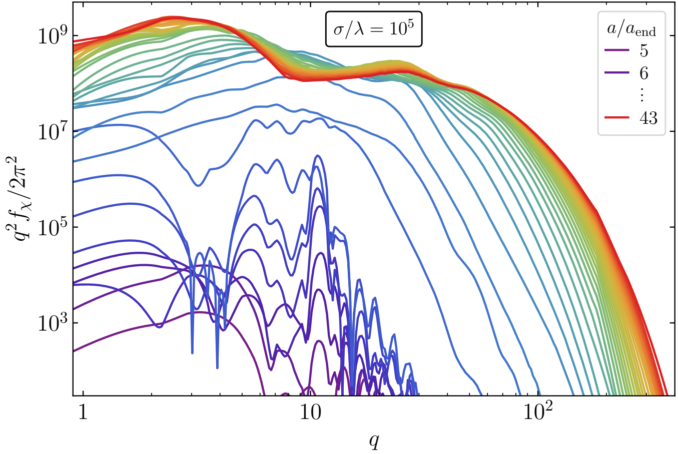


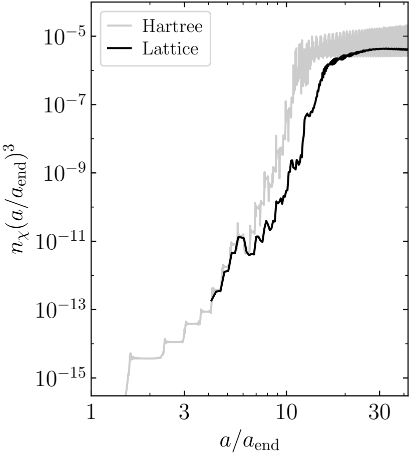
Parametric resonance

$f_\chi(p) \sim e^{2\mu_p m_\phi t}$

Non-linear regime

Re-scattering leads to a broader distribution with pseudo-thermal tail for ϕ and χ, $f_\chi \sim e^{-\alpha(\sigma/\lambda;t)q}$





Relic abundance, gravitational production

If $m_\chi \ll H_{\text{end}}$ and $\xi \ll 1$,

$$\begin{aligned}\Omega_{\text{DM}} &\simeq \frac{\rho_\chi}{\rho_c} \\ &\propto \frac{m_\chi T_{\text{reh}}}{M_P^2} \underbrace{\int dq\, q^2 f_\chi(q)}_{\propto\ m_\chi^{-1}}\end{aligned}$$

$20\ \text{GeV}$

m_χ :

Relic abundance, gravitational production

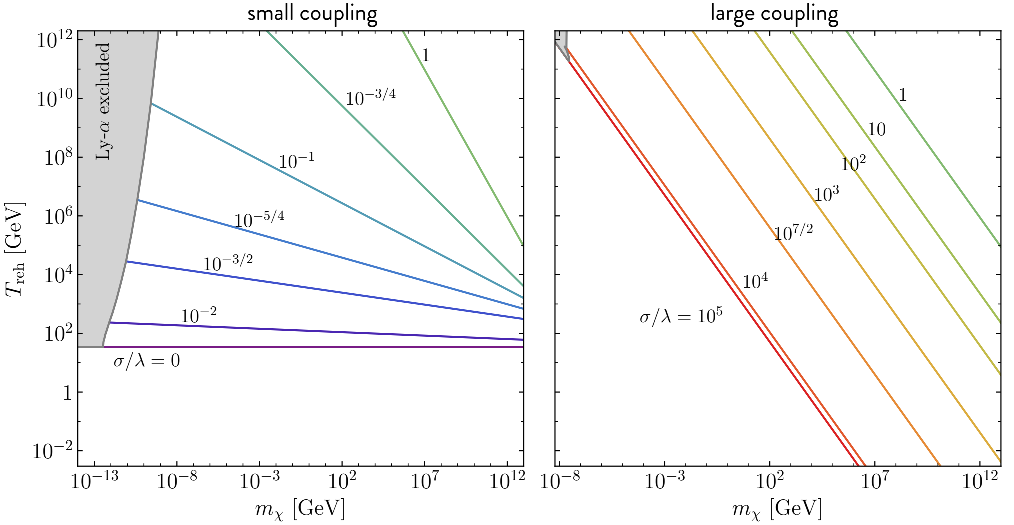
If $m_\chi \ll H_{\rm end}$ and $\xi \gtrsim 1/6$,

$$\begin{aligned}\Omega_{\rm DM} &\simeq \frac{\rho_\chi}{\rho_c} \\ &\propto \frac{m_\chi T_{\rm reh}}{M_P^2} \underbrace{\int dq\, q^2 f_\chi(q)}_{F(\xi)}\end{aligned}$$

increasing m_χ ↓

m_χ :

Relic abundance, inflaton decay



Isocurvature in the CMB

CDI: cold dark matter density isocurvature

NDI: neutrino density isocurvature

NVI: neutrino velocity isocurvature

However, they have not been detected,

$$\beta_{\rm iso} = \frac{\mathcal{P}_{\mathcal{S}}}{\mathcal{P}_{\mathcal{R}} + \mathcal{P}_{\mathcal{S}}} < \begin{cases} 2.5\% \text{ (CDI)} \\ 7.4\% \text{ (NDI)} \\ 6.8\% \text{ (NVI)} \end{cases}$$

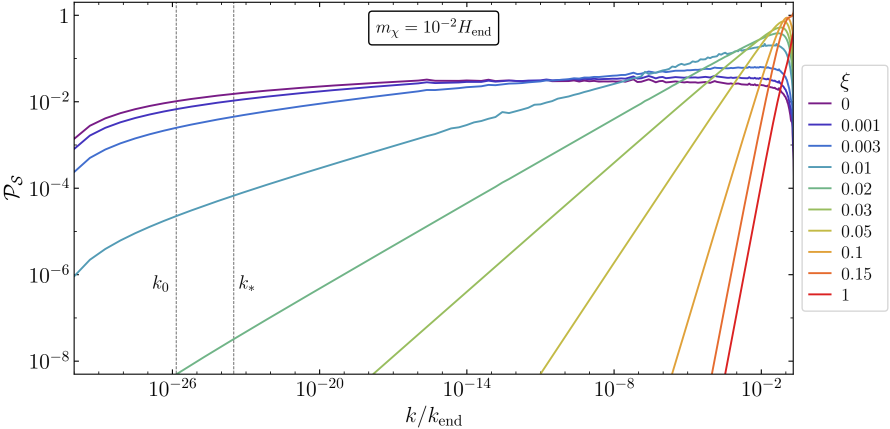
This constraint applies only at

large scales ($k_* = 0.002\,\mathrm{Mpc}^{-1}$)

At smaller scales, ¯_(ツ)_/¯

Y. Akrami et al. [Planck], Astron. Astrophys. 641, A10 (2020)

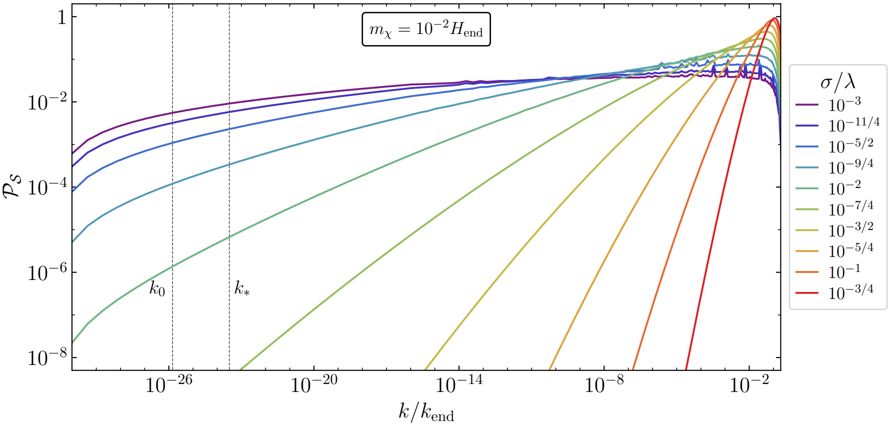
Isocurvature in gravitational production



$$\mathcal{P}_S(k) = \frac{k^3}{2\pi^2 \rho_\chi^2} \int d^3\mathbf{x} \, \langle \delta\rho_\chi(\mathbf{x}) \delta_\chi(0) \rangle \, e^{-i\mathbf{k}\cdot\mathbf{x}} \quad \Rightarrow \quad \begin{array}{l} m_\chi \gtrsim 0.54 \, H_{\rm inf} \\ \xi \gtrsim 0.03 \end{array}$$

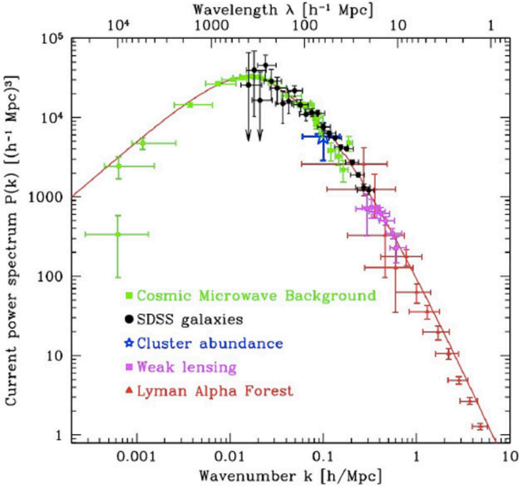
D. Chung, E. Kolb, A. Riotto, L. Senatore, PRD 72, 023511 (2005)

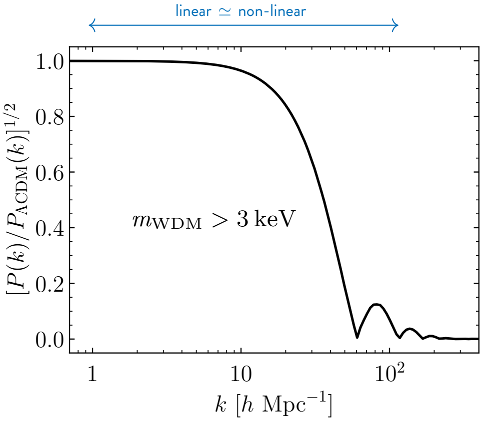
Isocurvature in production from inflaton decay



$$\mathcal{P}_{\mathcal{S}}(k) \; = \; \frac{k^3}{2\pi^2\rho_\chi^2}\int d^3\mathbf{x}\,\langle\delta\rho_\chi(\mathbf{x})\delta_\chi(0)\rangle\, e^{-i\mathbf{k}\cdot\mathbf{x}} \qquad \Rightarrow \qquad \begin{aligned} m_\chi \;&\gtrsim\; 0.54\ H_{\rm inf} \\ \sigma/\lambda \;&\gtrsim\; 0.02 \end{aligned}$$

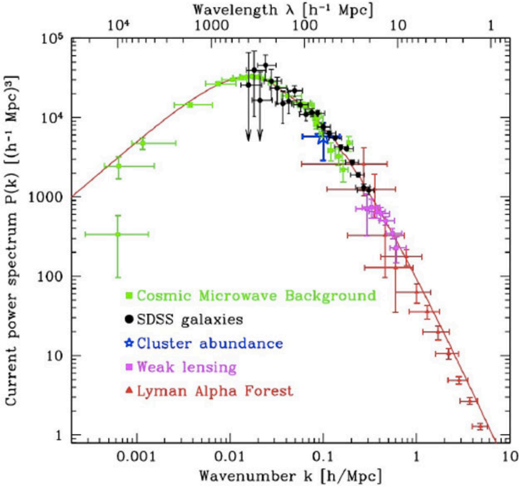
Structure formation constraints

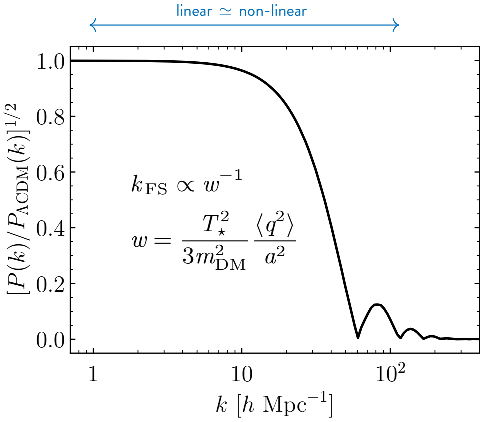




R. Murgia, V. Iršič and M. Viel, PRD 98 (2018), 083540

Structure formation constraints

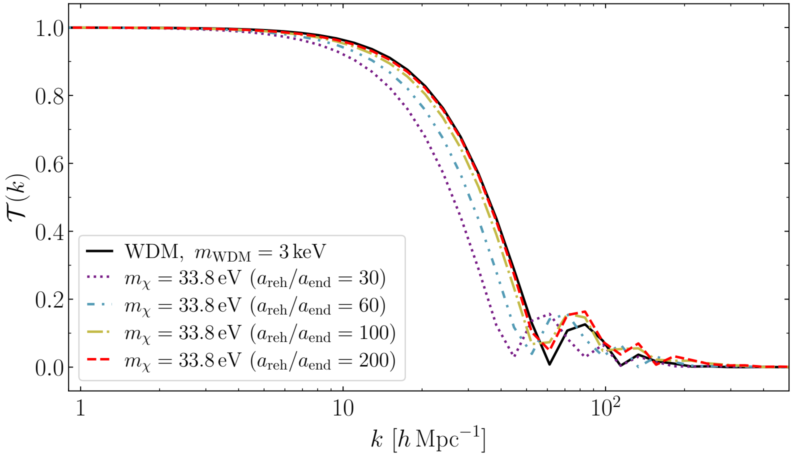




$$m_{\mathrm{DM}} = m_{\mathrm{WDM}}\left(\frac{T_\star}{T_{\mathrm{WDM}}}\right)\sqrt{\frac{\langle q^2\rangle}{\langle q^2\rangle_{\mathrm{WDM}}}}$$

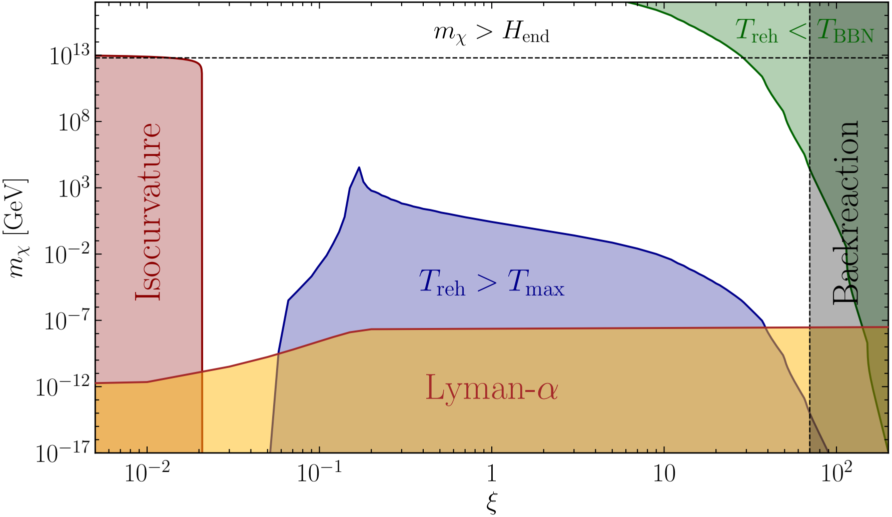
G. Ballesteros, MG and M. Pierre, JCAP 03 (2021), 101

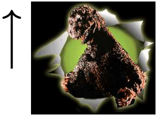
Light, but cold enough, dark matter



$$\langle q^2\rangle \propto \begin{cases} (a_{\mathrm{reh}}/a_{\mathrm{end}})^{1/2}\,, & \xi \gtrsim 1/4 \\ m_\chi^\alpha\,, & \xi \lesssim 1/4 \end{cases} \quad \Rightarrow \quad m_\chi \gtrsim \begin{cases} 33\,\mathrm{eV}\,, & \xi \gtrsim 1/4 \\ 0.2\,\mathrm{meV}\,, & \xi \lesssim 1/4 \end{cases}$$

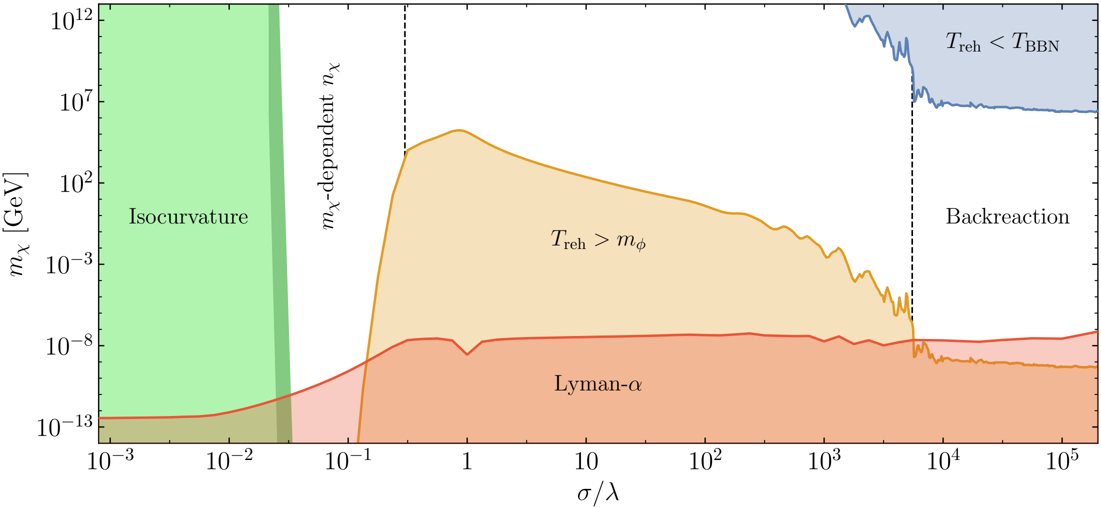
Parameter space for gravitational production





WIMPZILLAS

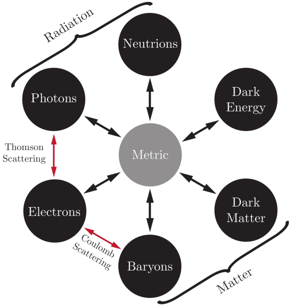
Parameter space for production from inflaton decay



Additional constraints?

Distortions of the CMB frequency spectrum

$$f(E) = \frac{1}{e^{(E-\mu)/T} - 1}$$



Energy injected into the CMB at different times results in a spectrum that mixes regions at different temperatures

FIRAS: $|\mu| < 9 \times 10^{-5}$

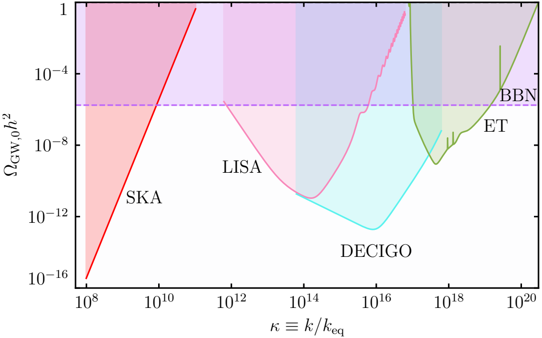
PIXIE: $|\mu| < 10^{-9}$

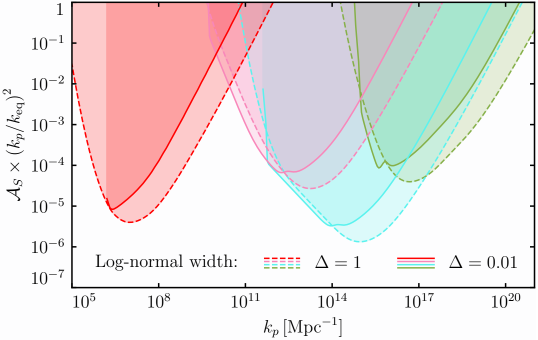
D. Fixsen et al., Astrophys. J. 473 (1996), 576

Additional constraints?

Isocurvature-induced gravitational waves

G. Domènech, S. Passaglia and S. Renaux-Petel, JCAP 03 (2022), 023





$$h_{ij}'' + 2\mathcal{H}h_{ij}' + \nabla^2 h_{ij} = \frac{2}{M_P^2}\left[\partial_i\psi\partial_j\psi\right]^{\rm TT}$$

$$\Omega_{\rm GW,c}(k) = \frac{2}{3}\int_0^\infty dv \int_{|1-v|}^{1+v} du \left(\frac{4v^2-(1-u^2+v^2)^2}{4uv}\right)^2 \overline{I^2(x_c,k,u,v)}\mathcal{P}_S(ku)\mathcal{P}_S(kv)$$

Additional constraints?

Blue tilted isocurvature: Ultra Compact Mini Halos (and PBHs)

M. Ricotti and A. Gould, Astrophys. J. 707 (2009), 979; T. Bringmann et al., PRD 85 (2012), 125027

Thank you