CLUSTER OF EXCELLENCE
QUANTUM UNIVERSE

Reheating after inflaton fragmentation

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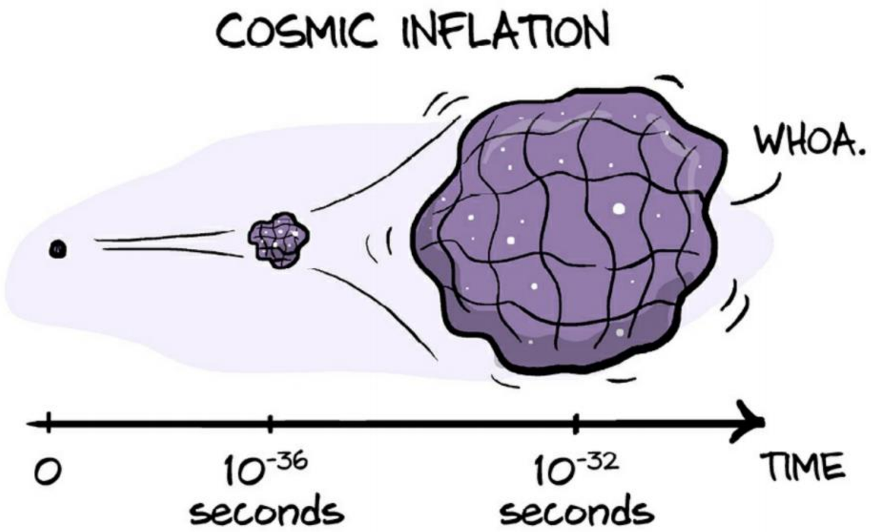
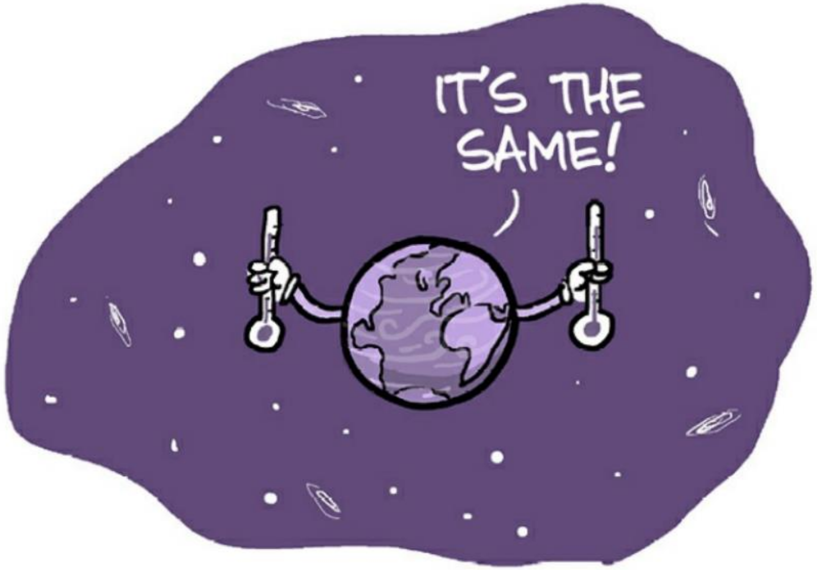
Based on
[**arXiv:2306.08038**] with M. A. G. Garcia
[**arXiv:2308.16231**] with M. A. G. Garcia, M. Gross, Y. Mambrini, K. Olive and J.-H. Yoon



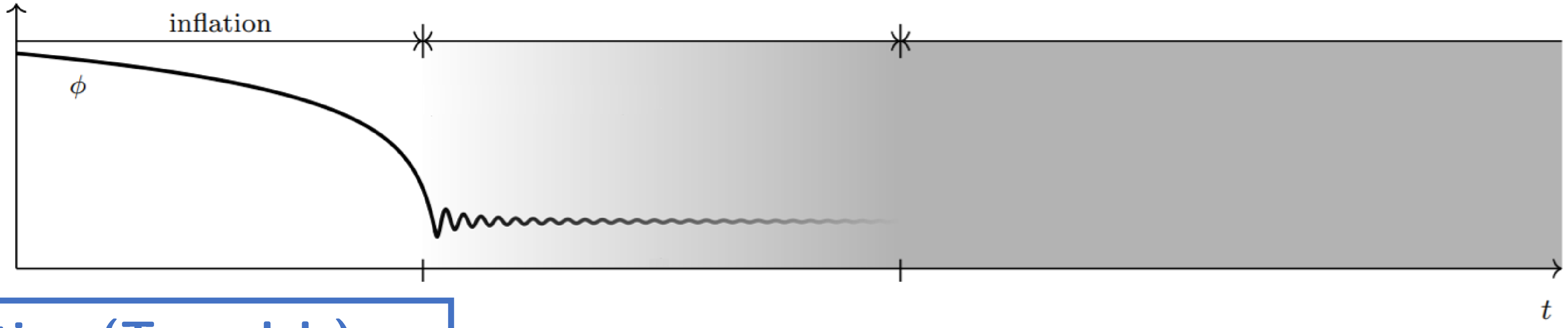
This talk



[Credit: M. A. G. Garcia – talk given at BUAP - 2023]

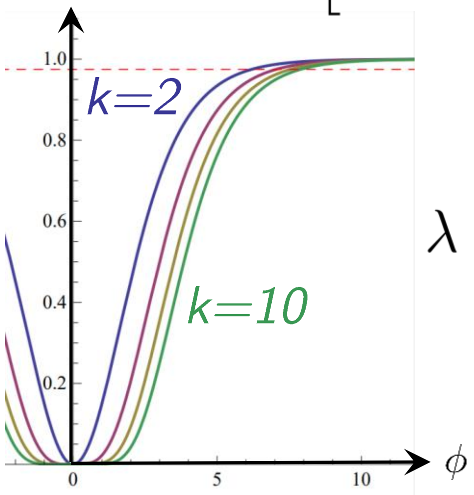
[Credit: J. Cham, D. Whiteson, *We Have No Idea: A Guide to the Unknown Universe.* Riverhead Books, New York, 2017]

The early universe



Inflation (T-models)

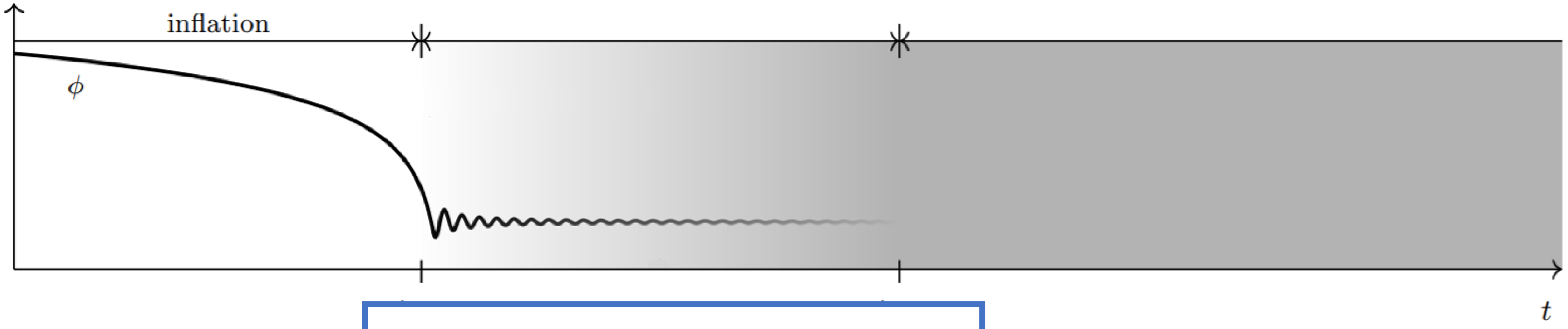
$$V(\phi) = \lambda M_P^4 \left[\sqrt{6}\tanh\left(\frac{\phi}{\sqrt{6}M_P}\right)\right]^k$$



$$\lambda = \frac{18\pi^2 A_s}{6^{k/2} N_*^2}$$

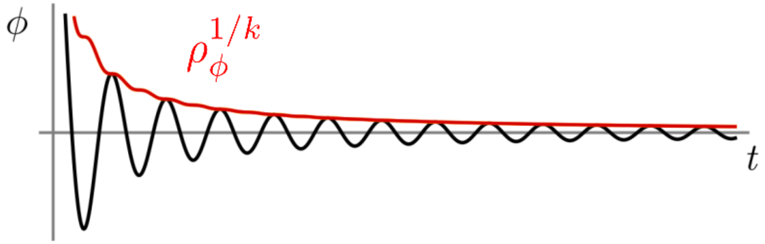
[Kallosh & Linde arXiv:1306.5220]

The early universe



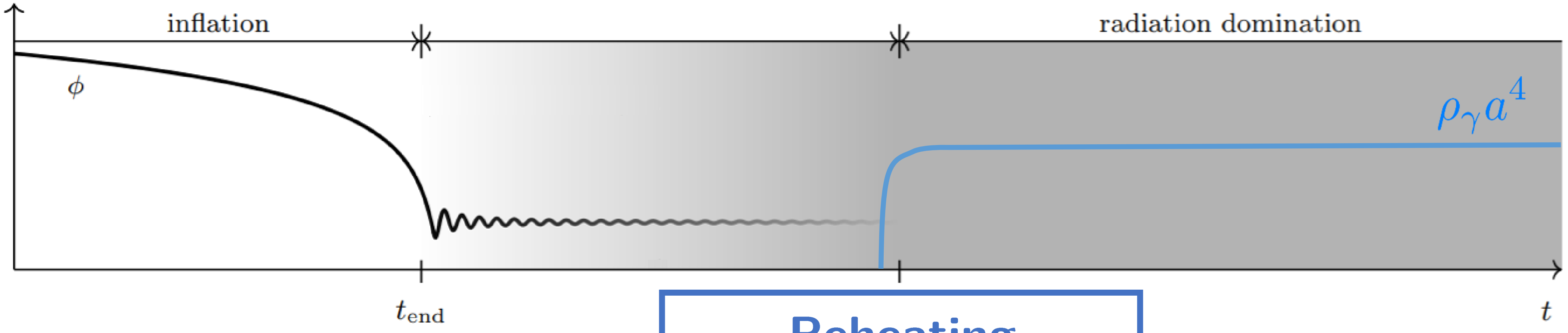
After end of inflation

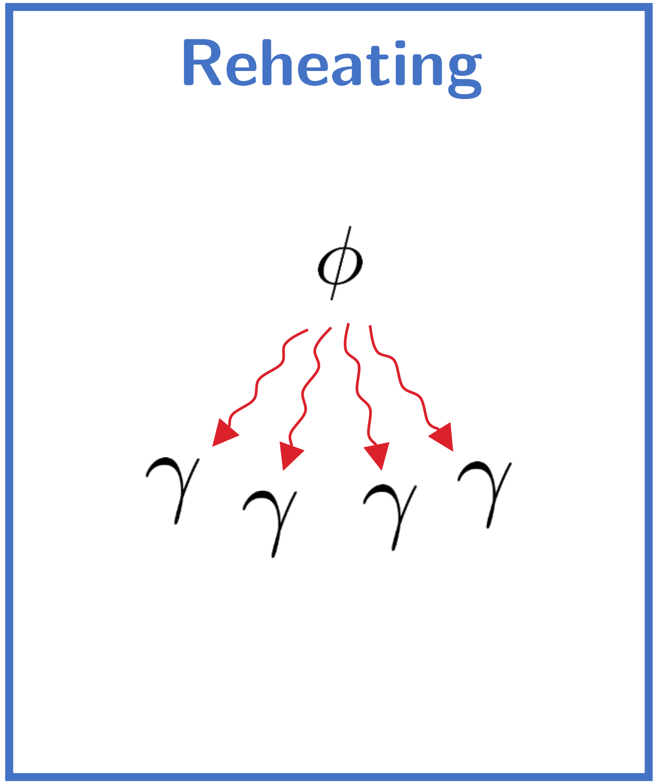
$$V(\phi) \simeq \lambda \frac{\phi^k}{M_P^k}$$



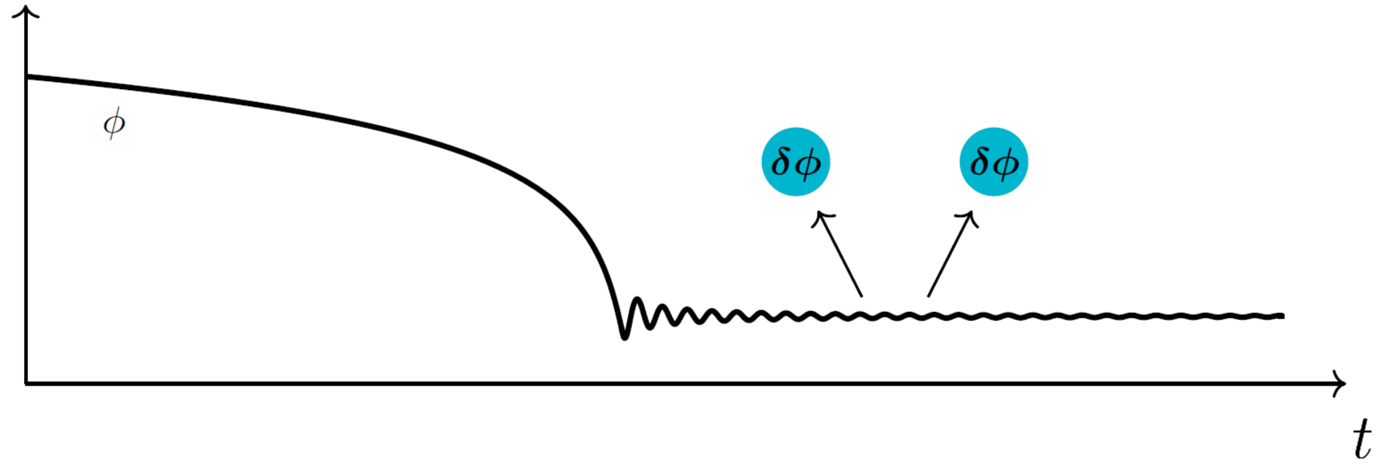
$$\langle w_\phi \rangle \simeq \frac{k-2}{k+2}$$

The early universe





1. Fragmentation



Post-inflation growth of inhomogeneities

Inflaton **inhomogeneities** follow (at **linear order** in perturbation theory)

$$\ddot{\delta\phi} + 3H\dot{\delta\phi} - \frac{\nabla^2\delta\phi}{a^2} + k(k-1)\lambda M_P^2\left(\frac{\phi(t)}{M_P}\right)^{k-2}\delta\phi \;=\; 0$$

Switching to **conformal time** and **quantizing** rescaled fluctuations

$$X(\tau,\boldsymbol{x}) \;\equiv a\delta\phi(\tau,\boldsymbol{x}) \;=\; \int \frac{\mathrm{d}^3\boldsymbol{p}}{(2\pi)^{3/2}} e^{-i\boldsymbol{p}\cdot\boldsymbol{x}}\left[X_p(\tau)\hat{a}_{\boldsymbol{p}} + X_p^*(\tau)\hat{a}^\dagger_{-\boldsymbol{p}}\right]$$

The **equation of motion** reads

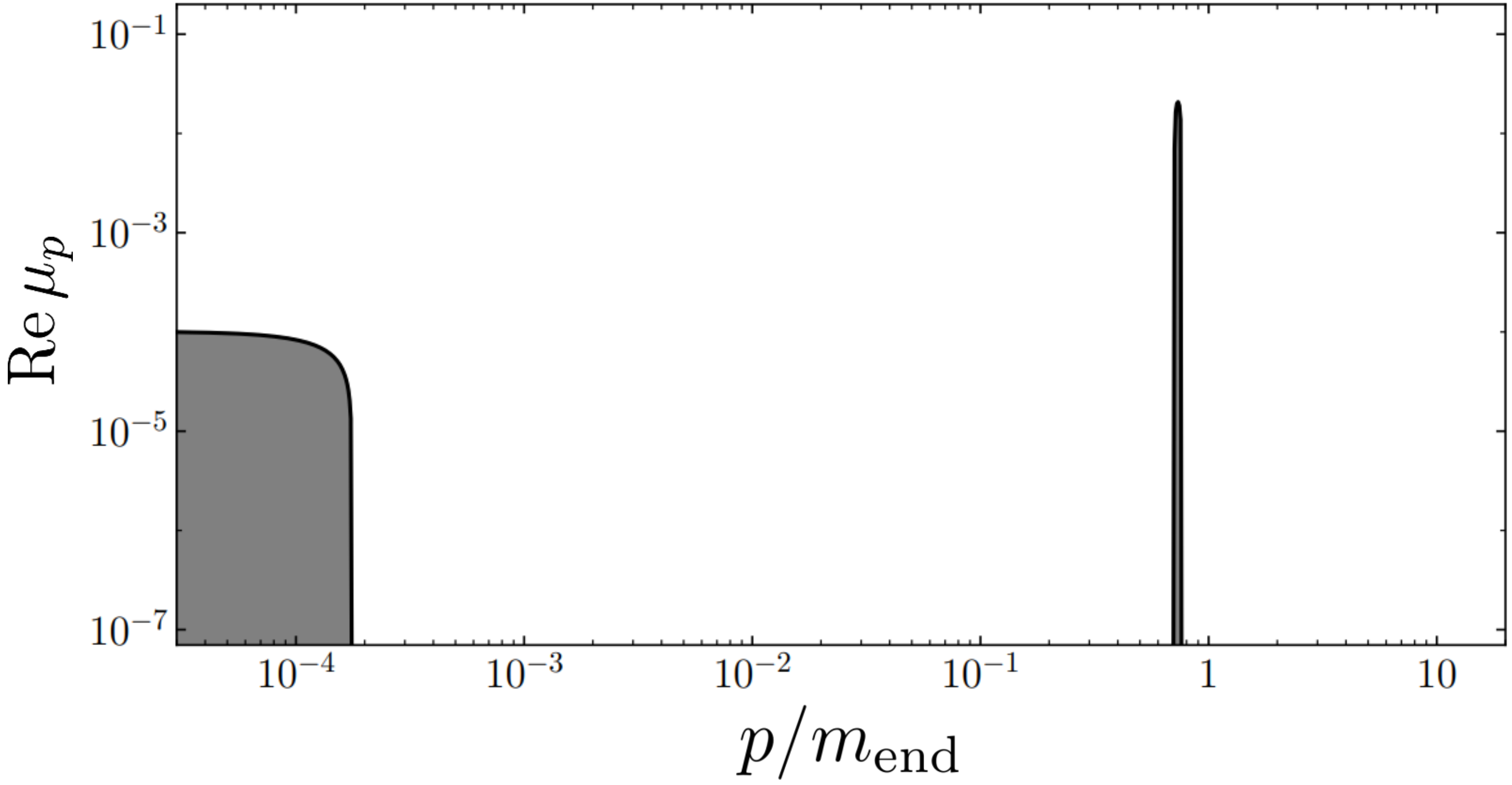
$$X_p'' + \omega_p^2 X_p \;=\; 0\,, \qquad \text{with} \qquad \omega_p^2 \;\equiv\; p^2 - \frac{a''}{a} + k(k-1)\lambda a^2 M_P^2\left(\frac{\phi(t)}{M_P}\right)^{k-2}$$

Starting from **Bunch-Davies** initial conditions $X_p(\tau_0) \;=\; \dfrac{1}{\sqrt{2\omega_p}}\,, \quad X_p'(\tau_0) \;=\; -\dfrac{i\omega_p}{\sqrt{2\omega_p}}\,,$

Growth of inhomogeneities: quartic case $k = 4$

Switching to **dimensionless time** $z \equiv m_{\rm end}(\tau - \tau_{\rm end})$ the EOM reduces to

$$\frac{\mathrm{d}^2 X_p}{\mathrm{d}z^2} + \left[\left(\frac{p}{m_{\rm end}}\right)^2 + \underbrace{\mathrm{sn}^2}_{\text{Jacobi elliptic function}}\left(\frac{z}{\sqrt{6}}, -1\right)\right] X_k \;=\; 0 \qquad \text{with} \qquad m_{\rm end}^2 \;\equiv\; V_{\phi\phi}(\phi_{\rm end})$$



Solutions given in terms of **Floquet index**

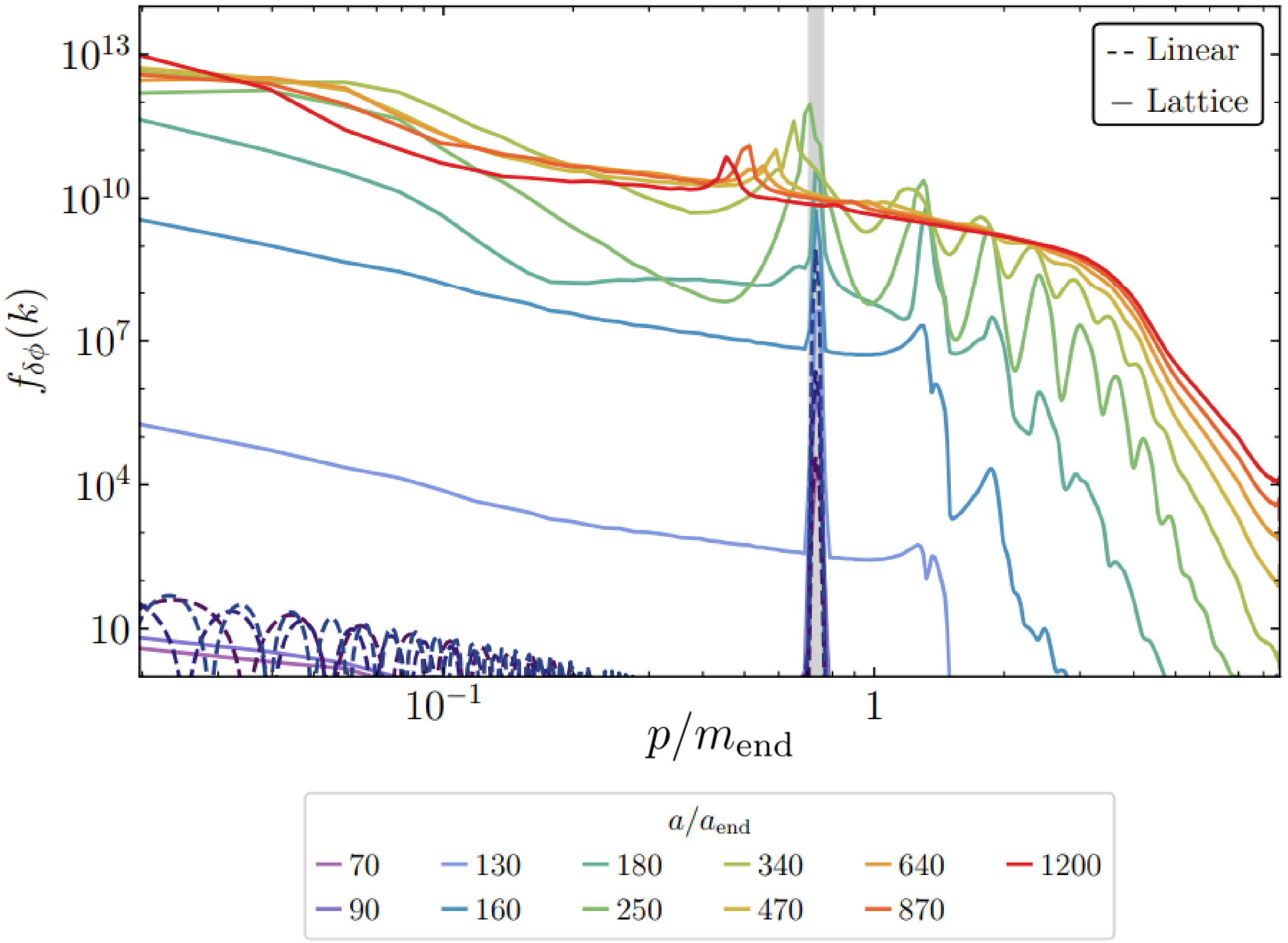
$$X_p(\tau) \;=\; e^{\mu_p \tau} g_1(\tau) + e^{-\mu_p \tau} g_2(\tau)$$

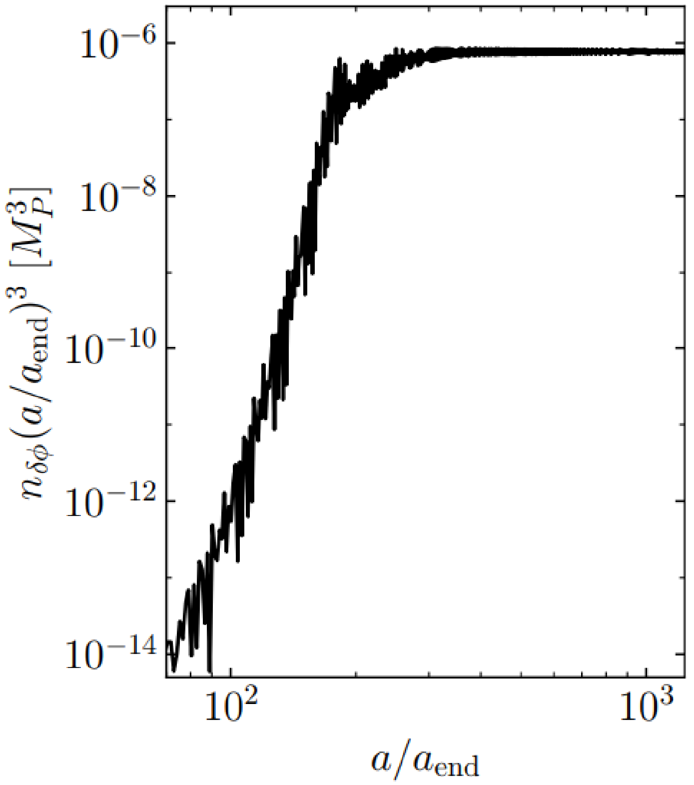
Once $\delta\phi \sim \phi$, **inhomogeneities** are **large**, non-linearities captured by solving the **full** EOM on the **lattice**

$$\ddot{\phi} + 3H\dot{\phi} - \frac{\nabla^2 \phi}{a^2} + V_\phi \;=\; 0$$

Growth of inhomogeneities: quartic case $k = 4$

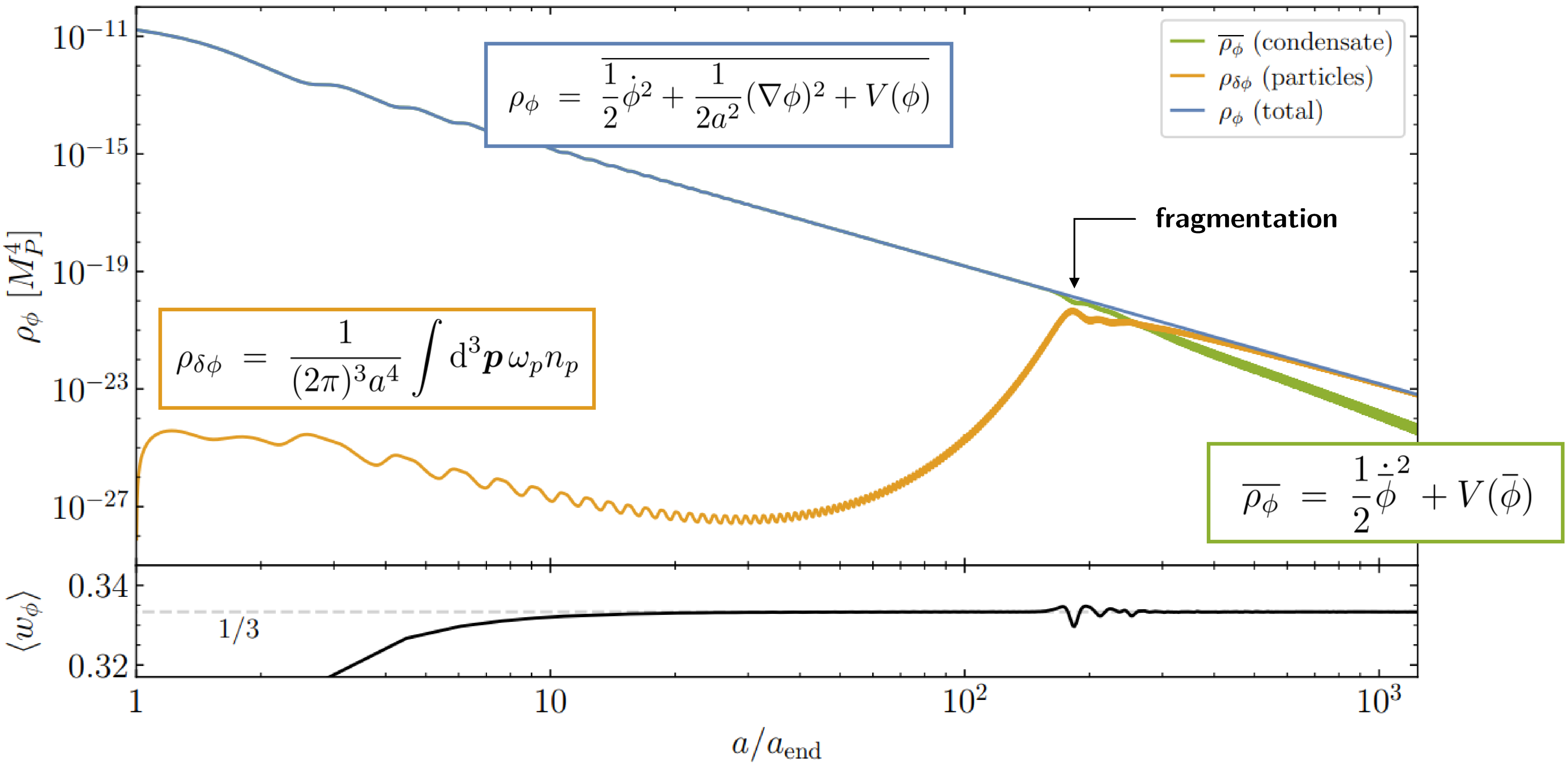
- **Simulations** with $\mathcal{C}$osmo$\mathcal{L}$attice [D. G. Figueroa, A. Florio, F. Torrenti, and W. Valkenburg, "*CosmoLattice*" arXiv:2102.01031]
- Estimate the **occupation number** (PSD) $f_{\delta\phi}(p,t) = n_p = \dfrac{1}{2\omega_p}\left|\omega_p X_p - iX_p'\right|^2$





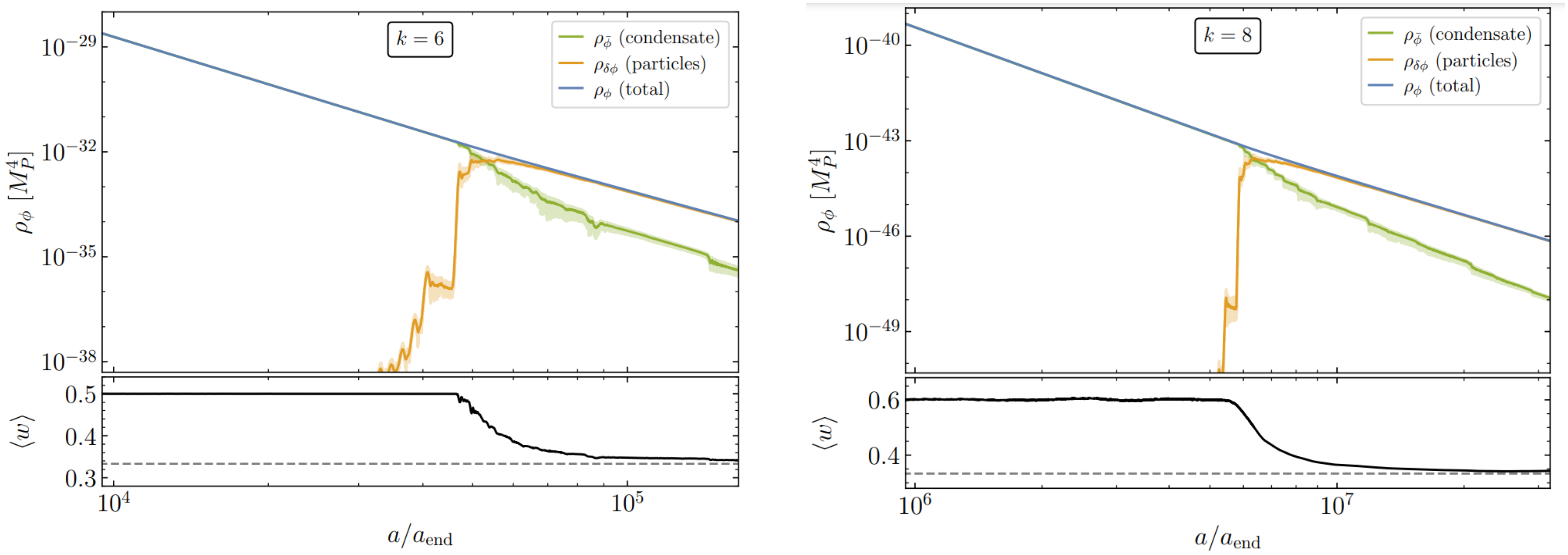
$$n_{\delta\phi} = \frac{1}{(2\pi)^3 a^3}\int \mathrm{d}^3\boldsymbol{p}\, n_p$$

Fragmentation: quartic case $k = 4$



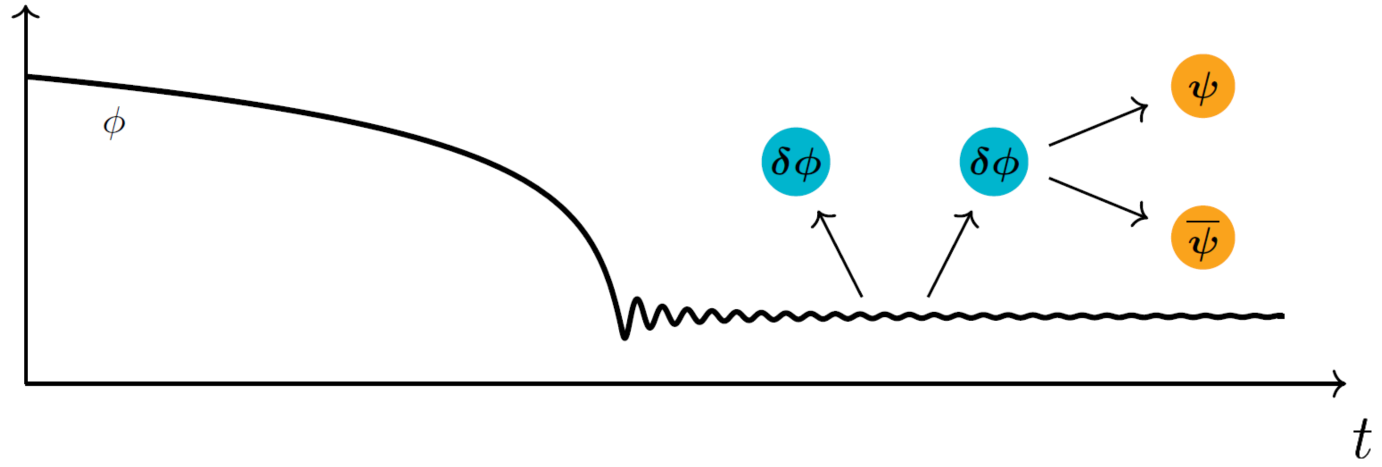
[D. G. Figueroa, A. Florio, F. Torrenti, and W. Valkenburg, "*CosmoLattice*" arXiv:2102.01031]

Fragmentation: $k > 4$



- Transition to **radiation-like** era at the onset of **fragmentation**
- Fragmentation takes **longer** for larger ***k*** but the condensate **subsists**!

2. Reheating

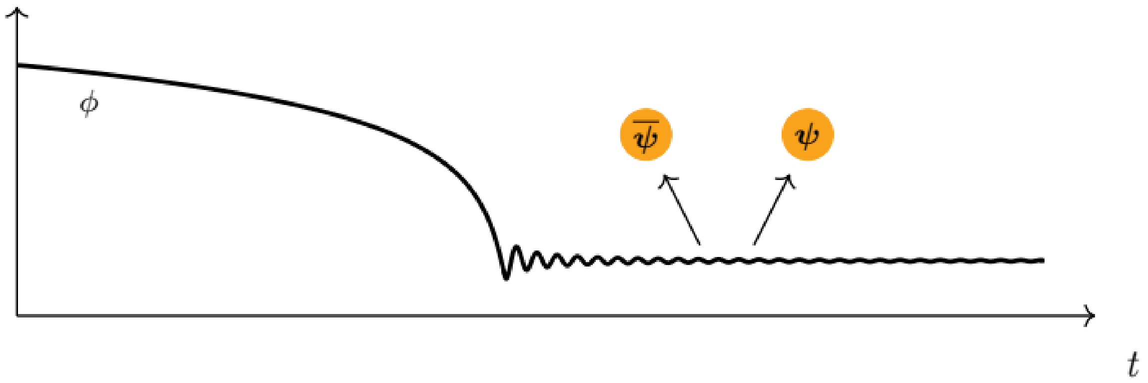


Reheating

Consider **coupling to fermions** $\mathcal{L} \supset -y\phi\bar{\psi}\psi$

$$\frac{\partial f_\psi}{\partial t} - H|\boldsymbol{P}|\frac{\partial f_\psi}{\partial|\boldsymbol{P}|} = \mathcal{C}[f_\psi] \quad \xrightarrow{\int \mathrm{d}^3\boldsymbol{P}} \quad \begin{aligned} \dot{\rho}_\psi + 4H\rho_\psi &= \overbrace{R_\phi + R_{\delta\phi}}^{\text{production rate}} \\ \dot{\rho}_\phi + 3H(1+w_\phi)\rho_\phi &= -(R_\phi + R_{\delta\phi}) \end{aligned}$$

Condensate contribution



- **Express** $\phi(t) \simeq \phi_0(t) \underbrace{\sum_{n=-\infty}^{\infty} \mathcal{P}_n e^{-in\omega_\phi t}}_{\text{oscillating function}}$

  enveloppe

- Fermion **production** from each **oscillation mode** $E_n = n\omega_\phi$

$$\Gamma_\phi = \frac{1}{8\pi(1+w_\phi)\rho_\phi}\sum_{n=1}^{\infty}\langle|\mathcal{M}_n|^2 E_n\beta_n\rangle \simeq \alpha^2\frac{y^2}{8\pi}m_\phi(t)$$

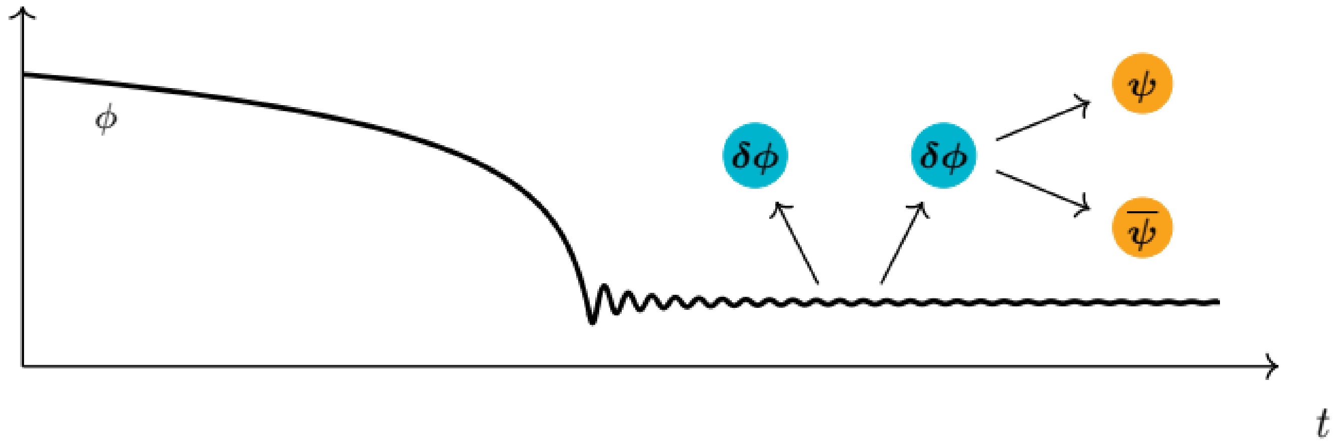
kinematic factor   efficiency factor

$$R_\phi = \frac{4}{3}\Gamma_\phi\overline{\rho_\phi}$$

[M. A. G. Garcia, K. Kaneta, Y. Mambrini, K. A. Olive, JCAP 04, 012 (2021)]

Reheating: production from fragmented quanta

Quanta contribution



- **Mass term** induced by **leftover condensate: allow** quanta to decay
- Estimate **number density** from the **lattice**

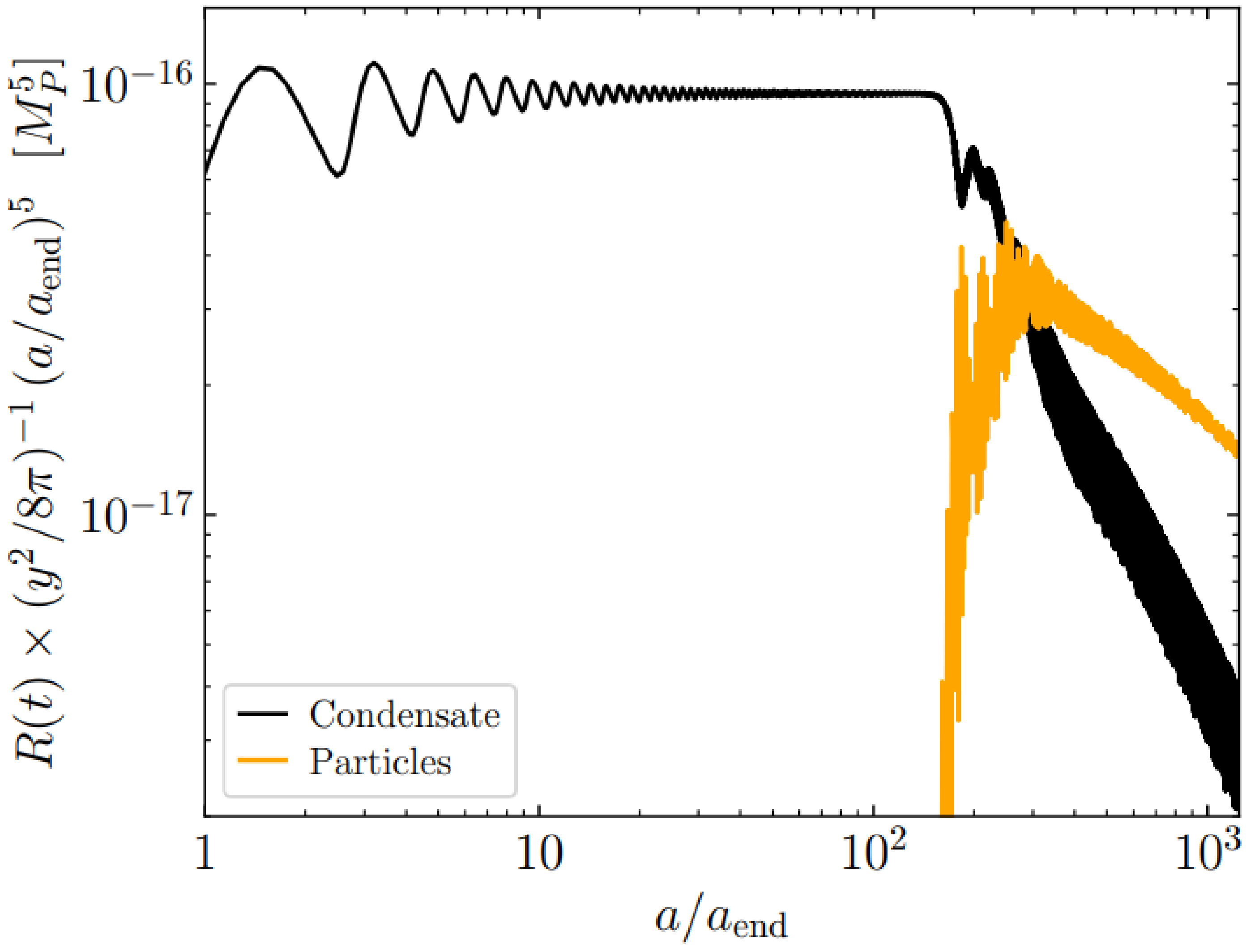
$$\mathcal{C}_{\delta\phi}[f_\psi] = \frac{1}{P^0}\int \frac{\mathrm{d}^3\boldsymbol{K}}{(2\pi)^3 2K^0}\frac{\mathrm{d}^3\boldsymbol{P}'}{(2\pi)^3 2P^{0\prime}}(2\pi)^4\delta^{(4)}(K-P-P')|\mathcal{M}_{\delta\phi\to\bar{\psi}\psi}|^2 f_{\delta\phi}(K)$$

$$\Gamma_{\delta\phi} = \frac{|\mathcal{M}_{\delta\phi\to\bar{\psi}\psi}|^2}{16\pi m_\phi}\sqrt{1-\frac{4m_\psi^2}{m_\phi^2}} \simeq \frac{y^2}{8\pi}m_\phi(t)$$

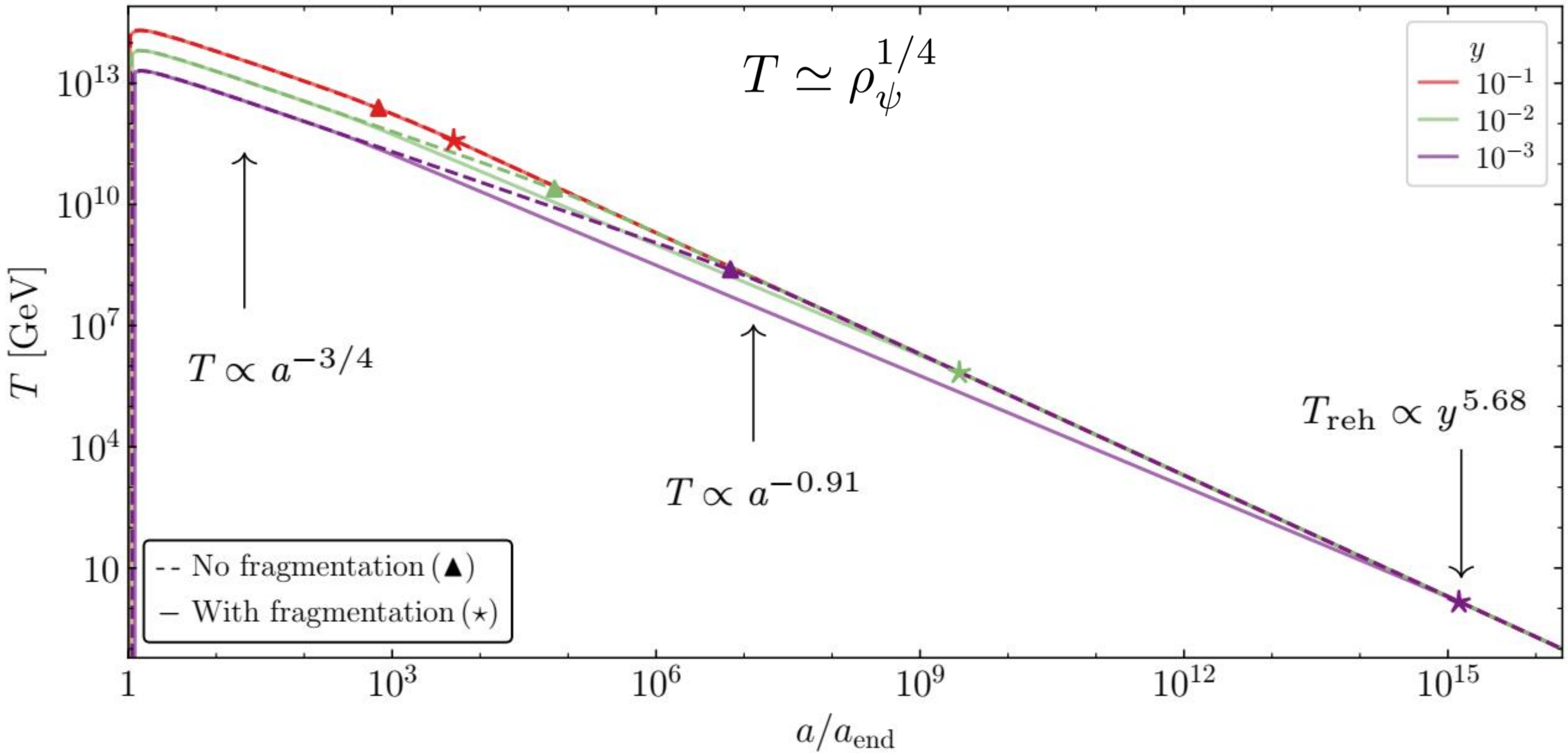
$$R_{\delta\phi}(t) = \Gamma_{\delta\phi} m_\phi n_{\delta\phi}$$

Reheating: total production rate for $k = 4$

$$R = R_\phi + R_{\delta\phi}$$

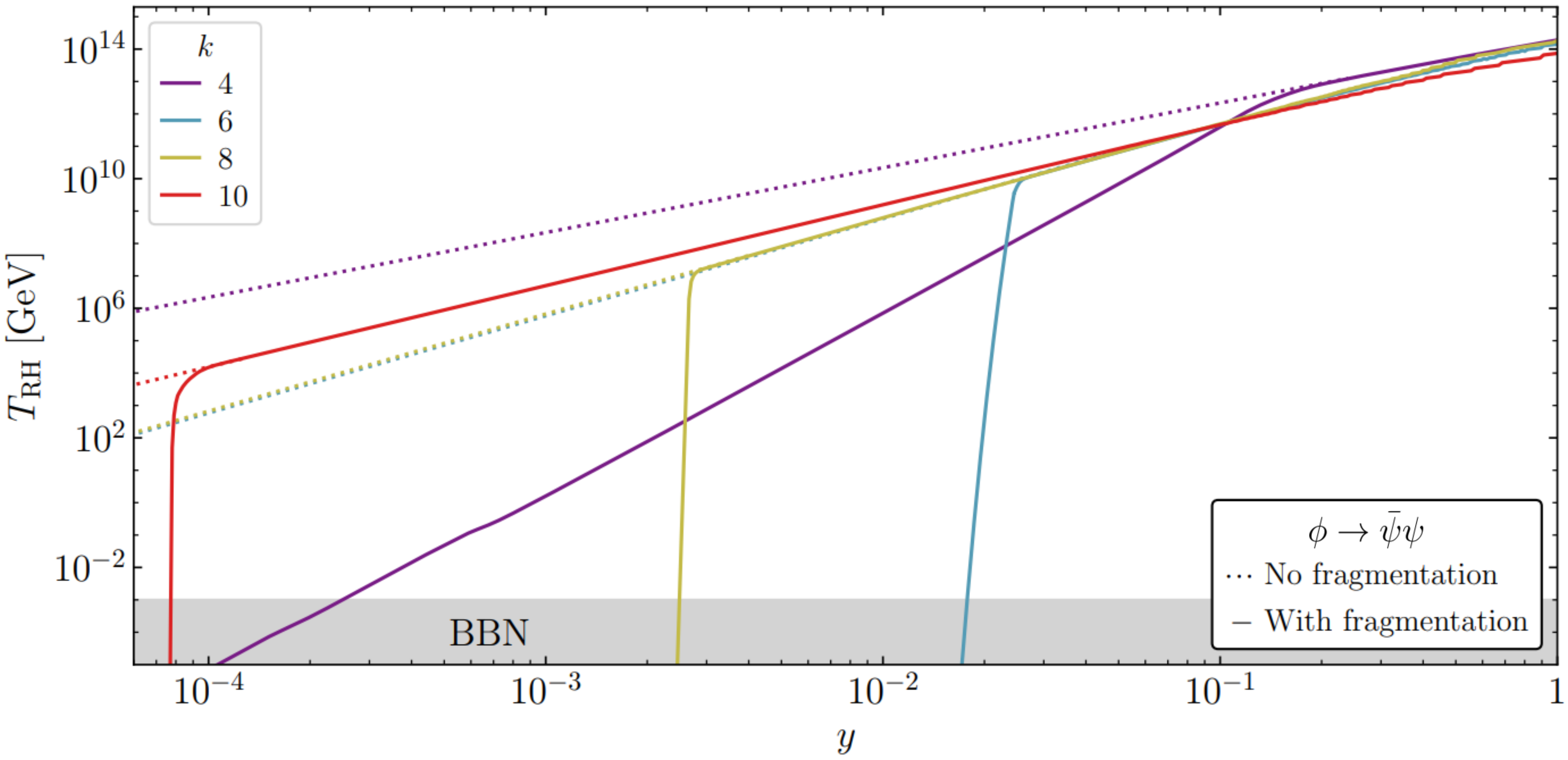


Effect on reheating temperature for $k = 4$



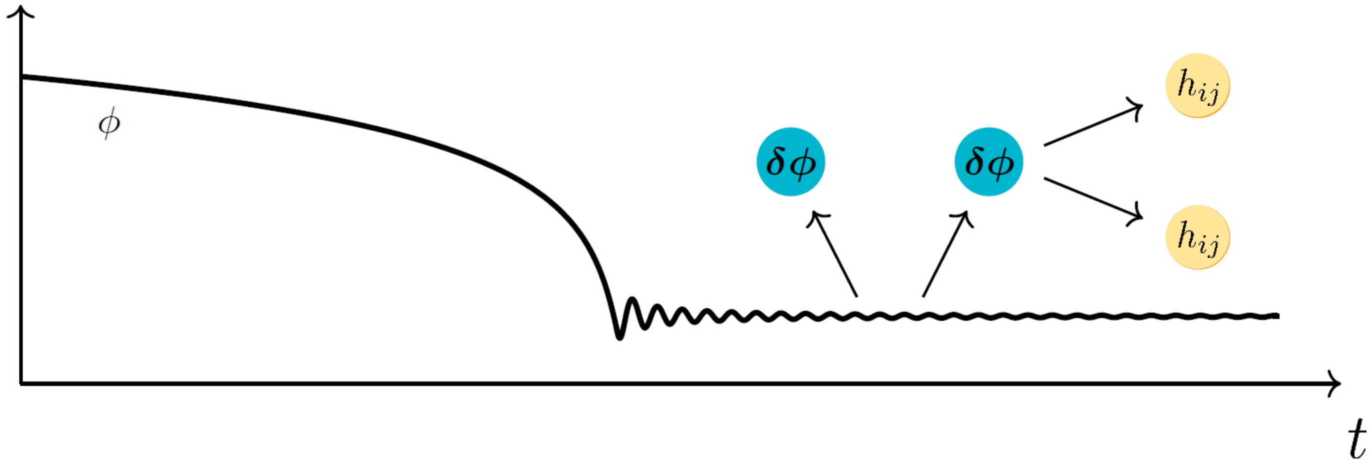
- Fragmentation **suppresses efficiency** of **reheating** process

Effect on reheating temperature



- **Large** (non-perturbative) **couplings** required
- At large ***k***, **post-fragmentation** decays **do not occur**

3. Gravitational waves



Gravitational waves: quartic case $k = 4$

- **Tensor** perturbations of the metric $\mathrm{d}s^2 = a(\tau)^2\left[\mathrm{d}\tau^2 - \left(\delta_{ij} + h_{ij}\right)\mathrm{d}x^i\,\mathrm{d}x^j\right]$

- Sourced by Tranceverse-Traceless (TT) scalar **inhomogeneities**

$$h''_{ij}(\boldsymbol{p},\tau) + 2\mathcal{H}h'_{ij}(\boldsymbol{p},\tau) + k^2 h_{ij}(\boldsymbol{p},\tau) = \frac{2}{M_P^2}\left[\int \frac{\mathrm{d}^3\boldsymbol{q}}{(2\pi)^{3/2}}\, q_i\, q_j\, \phi(\boldsymbol{q},\tau)\,\phi(\boldsymbol{p}-\boldsymbol{q},\tau)\right]^{\mathrm{TT}}$$

- Use Boltzmann **approach** to predict spectrum of inflaton fluctuations $\phi \to \delta\phi\,\delta\phi$

$$f_{\delta\phi}(|\boldsymbol{p}|,t) \simeq \frac{\pi}{c^2}\left(\frac{m_{\mathrm{end}}}{H_{\mathrm{end}}}\right)\left(\frac{a(t)}{a_{\mathrm{end}}} - 1\right)\sum_{n=1}^{\infty}\frac{|\hat{\mathcal{P}}_n|^2}{n^2\beta_n}\delta\left(\frac{|\boldsymbol{p}|}{m_{\mathrm{end}}} - \frac{1}{2}nc\beta_n\right)$$

series of peaks!

$$\beta_n \equiv \sqrt{1 - \frac{4m_\phi^2}{n^2\omega_\phi^2}} = \sqrt{1 - \left(\frac{2}{nc}\right)^2}$$

$$c \equiv \sqrt{\frac{2\pi}{3}}\frac{\Gamma(3/4)}{\Gamma(1/4)}$$

energy levels of inflaton potential

Gravitational waves: quartic case $k = 4$

- **Tensor** perturbations of the metric $\quad \mathrm{d}s^2 = a(\tau)^2\left[\mathrm{d}\tau^2 - \left(\delta_{ij} + h_{ij}\right)\mathrm{d}x^i\,\mathrm{d}x^j\right]$

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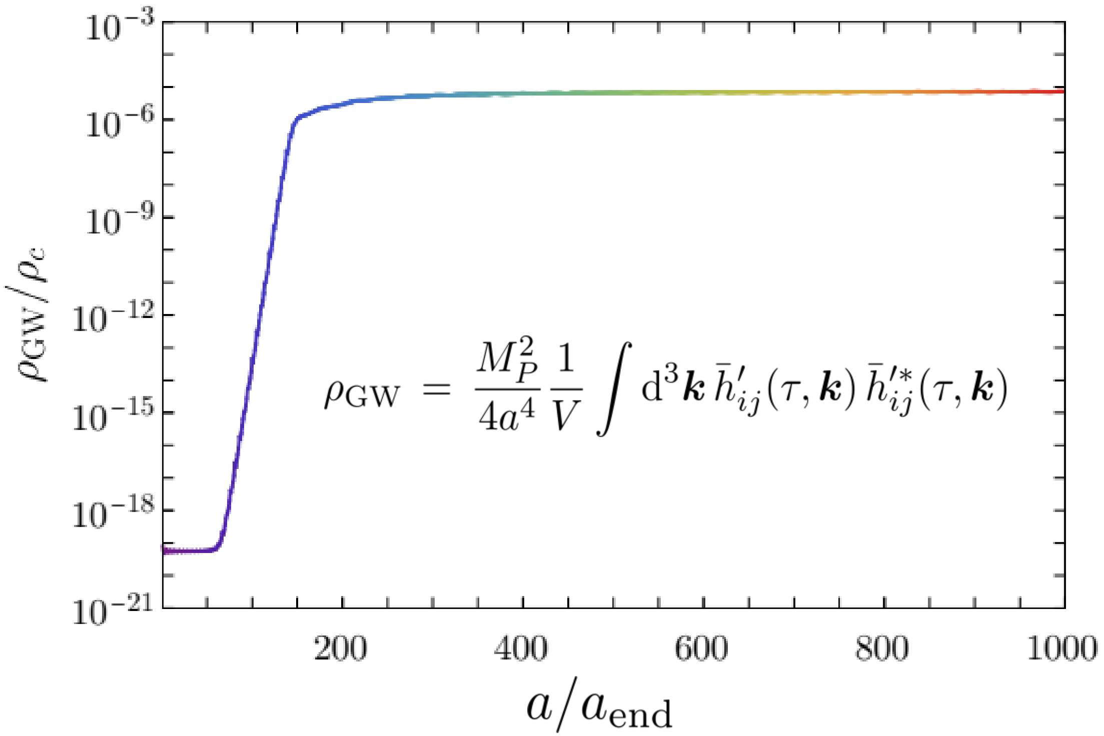
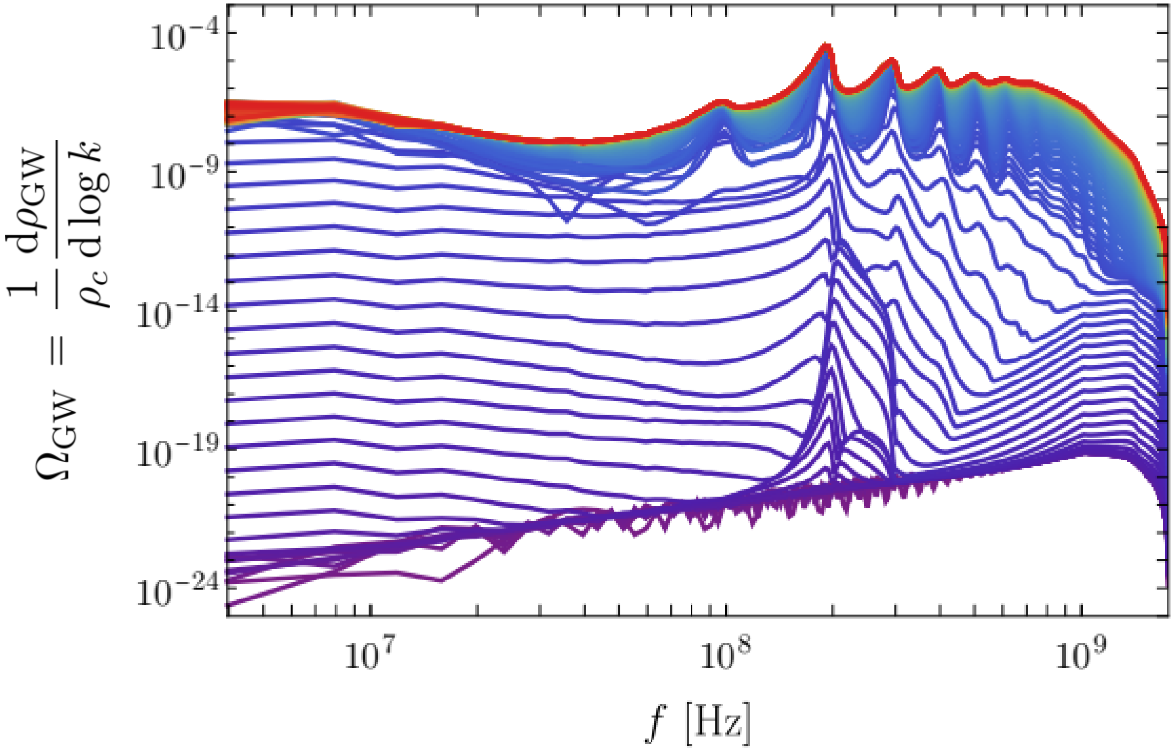
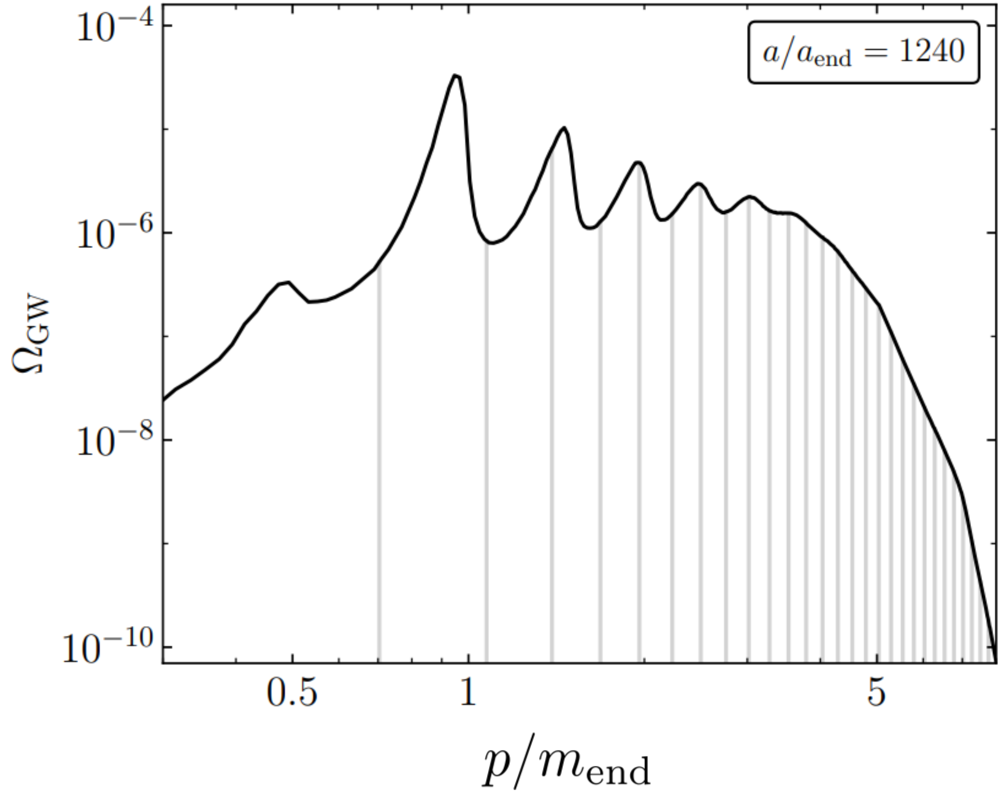
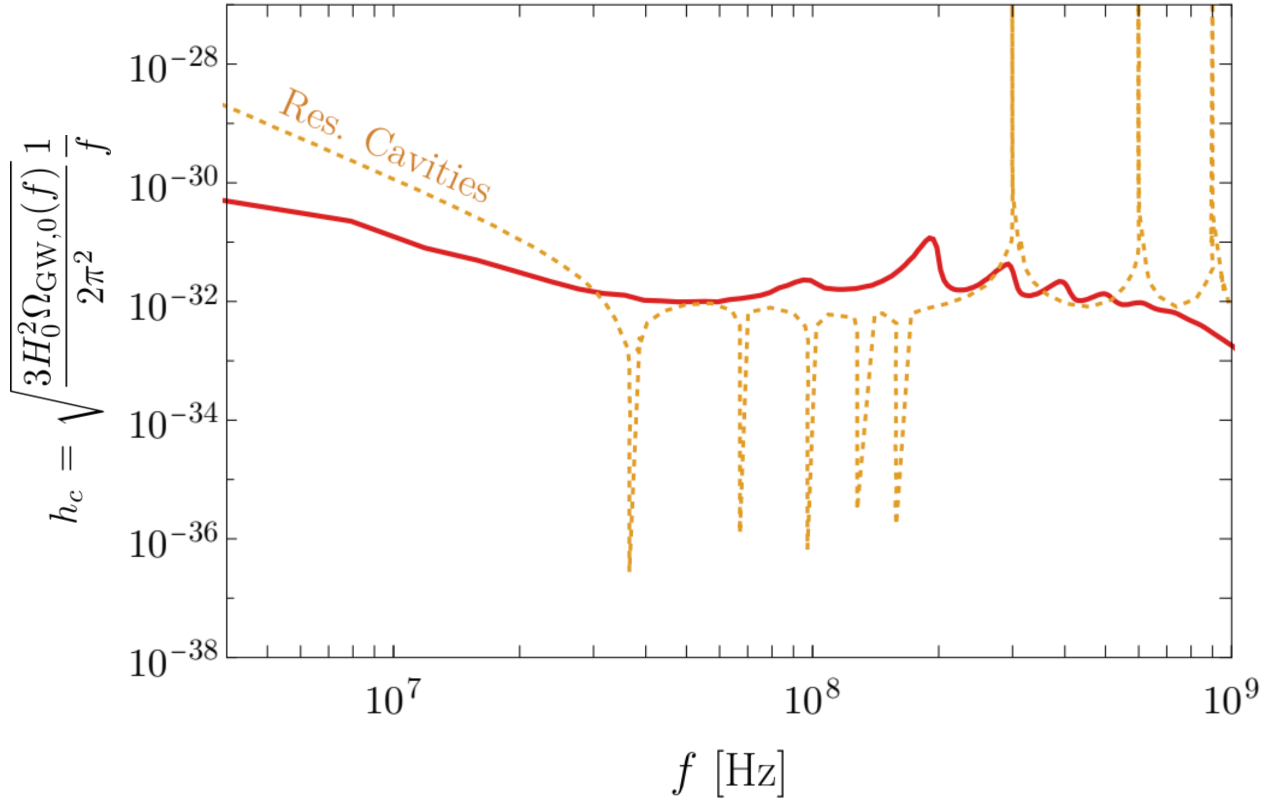
$$h''_{ij}(\boldsymbol{p},\tau) + 2\mathcal{H}h'_{ij}(\boldsymbol{p},\tau) + k^2 h_{ij}(\boldsymbol{p},\tau) = \frac{2}{M_P^2}\left[\int \frac{\mathrm{d}^3\boldsymbol{q}}{(2\pi)^{3/2}}\, q_i\, q_j\, \phi(\boldsymbol{q},\tau)\,\phi(\boldsymbol{p}-\boldsymbol{q},\tau)\right]^{\mathrm{TT}}$$





Gravitational waves: quartic case $k = 4$



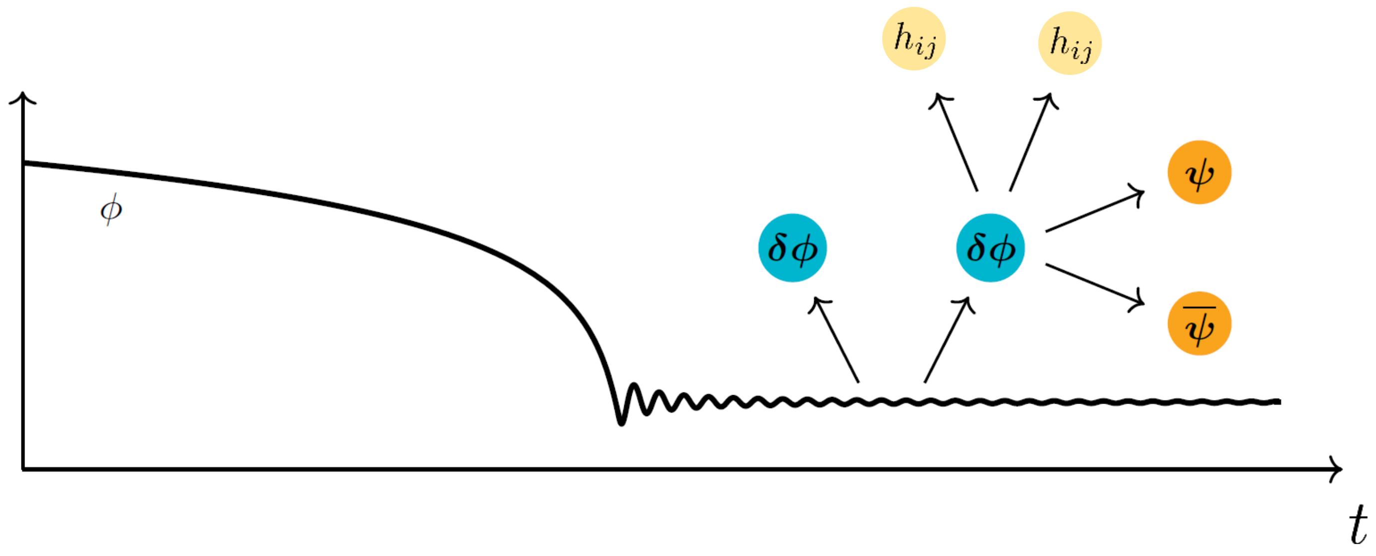


- Peak **interval** from Boltzmann **approach** accurate!
- Detectable via future resonant EM cavities?

[N. Herman, L. Lehoucq, A. Fűzfa, arXiv:2203.15668]

Inflaton spectroscopy

Summary and conclusion



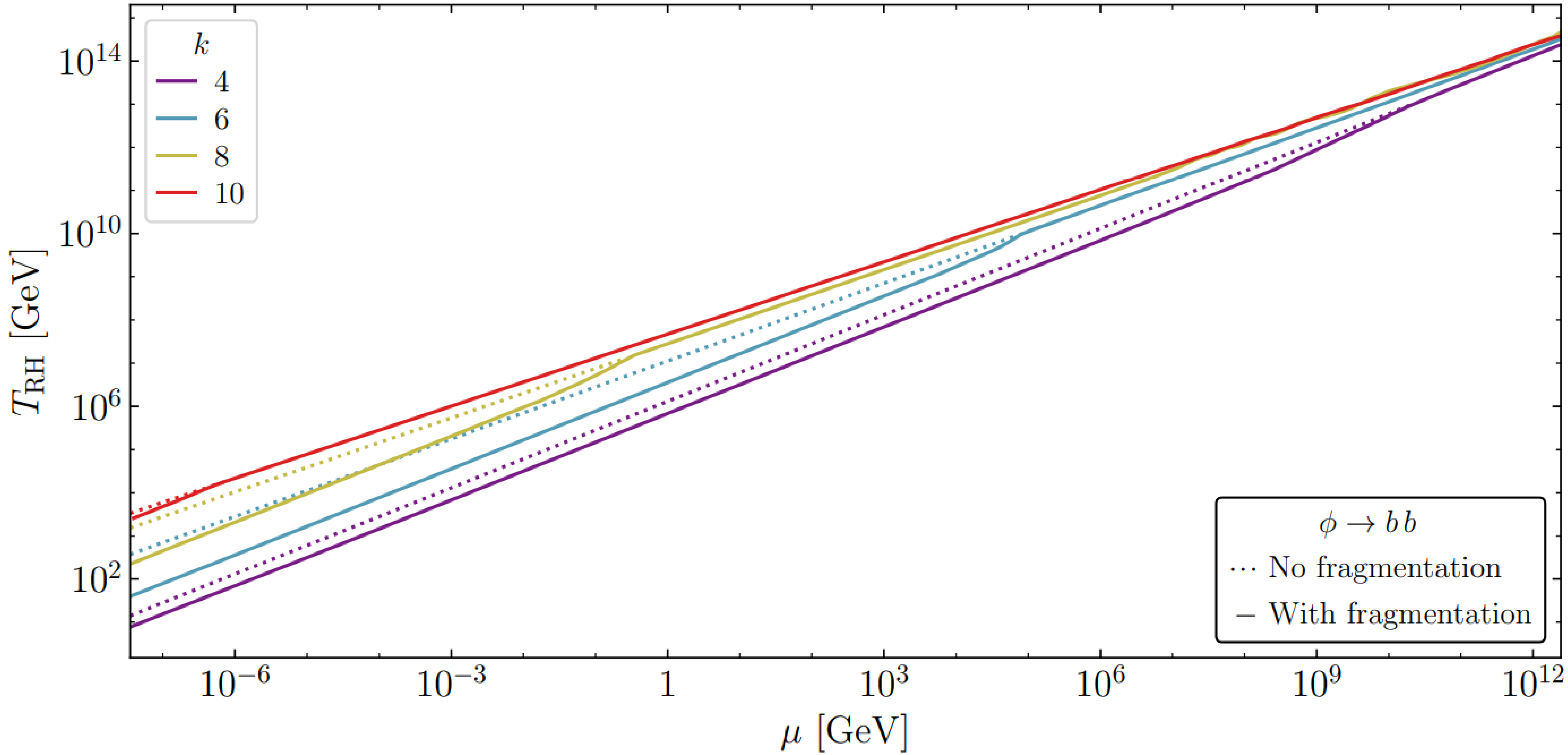
- Parametric **resonances** give rise to large **post-inflation inhomogeneities** leading to **fragmentation** of the inflaton condensate

- **Reheating** can be strongly **altered** by fragmentation

- **Fragmentation** can source **interesting GW signal**

Back-up slides

Reheating to bosons



Reheating to bosons