Black holes in modified gravity: Numerical solutions and scalarization

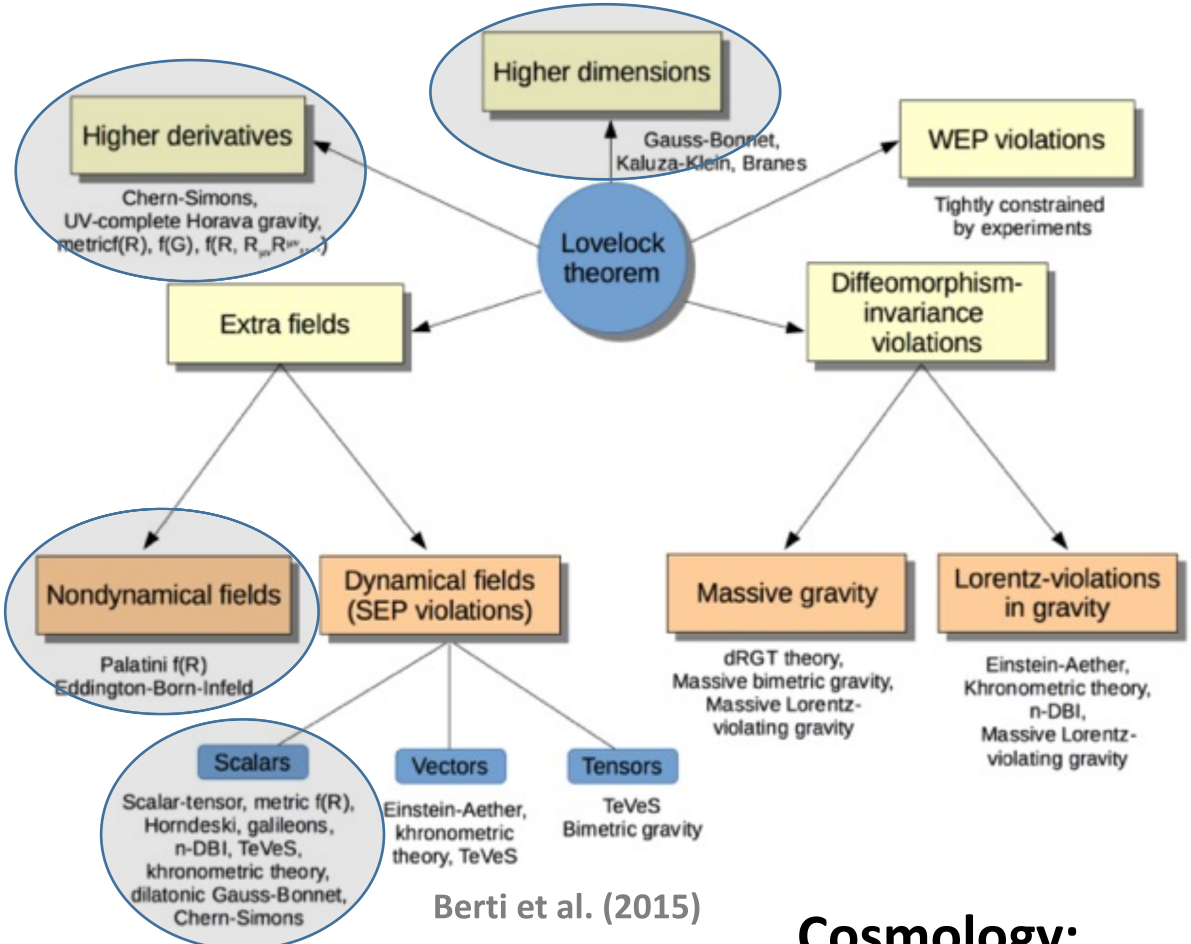
Daniela Doneva

University of Tübingen

Going beyond GR

- Guide mainly through **astrophysical relevant solutions** (asymptotically flat, not charged)
- GR alternatives typically carry **additional degrees of freedom**
- Either directly or effectively described by an **additional scalar field**
- Why **not vector fields**: often **ghost** instabilities are present **Silva at al (2022)**

(from yesterday) Lovelock's theorem



Berti et al. (2015)

Quantum gravity motivated:

- Gauss-Bonnet gravity
- Chern-Simons gravity

Cosmology:

- Ultralight axion dark matter
- Inflation scalar field
- f(R) theories

No-scalar-hair theorems

Bekenstein (1972),Hawking (1972), Heuser (1992,1996)

Assumptions

- ✓ A **(non)minimally coupled scalar field** to Einstein's gravity.

$$S = \frac{1}{4\pi}\int d^4x\sqrt{-g}\left(F(\varphi)R - \frac{1}{2}\nabla_\mu\varphi\nabla^\mu\varphi - V(\varphi)\right)$$

- ✓ Scalar field energy density > 0 (**weak energy condition**)

  $\rho = T^S_{\mu\nu}U^\mu U^\nu > 0,$ where $T^S_{\mu\nu}$ - scalar field energy momentum tensor

- ✓ The scalar field has **the same symmetries as the spacetime**.

$$\partial_t\varphi = \partial_\phi\varphi = 0$$

- The theorems extended also to scalar-tensor theories with non-canonical self-gravitating static **multiple scalar fields** DD, Yazadjiev (2020)
- A nice **review** on the topic Herdeiro&Radu (2015)

Violate the no-scalar-hair theorems

- Add **extra matter fields** (often **not astrophysically viable**): accretions discs out of ”normal” matter **Cardoso et al 2016**, charger black holes with nonlinear electrodynamics **Stefanov et al (2008,2009), DD et al (2010)**, for Einstein-Yang-Mills theory **Bartnik, Mckinnon (1988),** Einstein-Maxwell-scalar models **Herdeiro at al. (2018)**
- Add extra **curvature invarinats** (Gauss-Bonnet, Chern-Simons) **Kanti et al., 1996, Yunes&Stein (2011), Sotiriou&Zhou (2014), Doneva et al (2018), Silva et al (2018), Antoniou at al (2018)**
- **Time dependent scalar field** (or even vector field without ghosts) **Herdeiro et al. (2015); Kleihaus et al. (2015); Babichev et al., (2017); Heisenberg et al. (2017); Herdeiro et al. (2016)**
- Perhaps too exotic or pathological: violate the **weak energy condition** (e.g. well designed scalar field potential)

Violate the no-scalar-hair theorems

- The most general action involving scalar field (single!) and second order field equations: **Hordneski theories**

$$S_H = \int d^4x\sqrt{-g}\,(L_2 + L_3 + L_4 + L_5)$$

$$L_2 = K(\phi, X),$$

$$L_3 = -G_3(\phi, X)\Box\phi,$$

$$L_4 = G_4(\phi, X)R + G_{4X}\left[(\Box\phi)^2 - (\nabla_\mu\nabla_\nu\phi)^2\right],$$

$$L_5 = G_5(\phi, X)G_{\mu\nu}\nabla^\mu\nabla^\nu\phi - \frac{G_{5X}}{6}\left[(\Box\phi)^3 - 3\Box\phi(\nabla_\mu\nabla_\nu\phi)^2 + 2(\nabla_\mu\nabla_\nu\phi)^3\right]$$

- Includes: scalar-tensor theories, Gauss-Bonnet theories, etc.
- Outside of this classification – dynamical-Chern-Simons gravity, multiple scalar fields, Lorenz-violating theories
- **Long lived scalar fields** (e.g. as a results of supperradiance), NOT a solution of the field equations but can “live” long enough

Stellar vs. Supermassive black holes

- Often **different GR modifications for the two regimes**

- **Example**: curvature invariants often decrease very fast from the source and relevant only for small mass black holes with larger curvature

Gauss-Bonnet invariant for the **Schwarzschild** solutions:

$$\mathcal{R}^2_{GB} = R^2 - 4R_{\mu\nu}R^{\mu\nu} + R_{\mu\nu\alpha\beta}R^{\mu\nu\alpha\beta}$$

$R^2_{GB} = \frac{48M^2}{r^6}$, at the horizon $R^2_{GB} \sim \frac{1}{r_h^4}$

- Numerical solutions for **supermassive black holes** – time dependent scalar/vector field around black holes. **Only rotating solutions!**

Kerr black holes with synchronized scalar hair

- **GR** action **plus a minimally coupled complex massive scalar field** Φ

$$S = \int \left[\frac{R}{2} - g^{\mu\nu}\partial_\mu \Phi^* \partial_\nu \Phi - 2U(\Phi) \right] \sqrt{-g}\, d^4x \,, \quad \text{with} \quad U = \frac{1}{2}\mu^2 |\Phi|^2$$

- The system is **invariant under U(1) transformations**, $\Phi \rightarrow \Phi e^{i\alpha}$, that leads to the presence of a **conserved current** and thus a **Noether charge**

$$j^\mu = -i(\Phi^* \partial^\mu \Phi - \Phi \partial^\mu \Phi^*) \qquad Q = \int_{\Sigma \setminus \mathcal{H}} j^\mu n_\mu dV$$

- The **Noether charge -> number of particles**.
- Stationarity and axisymmetry of the black hole require that

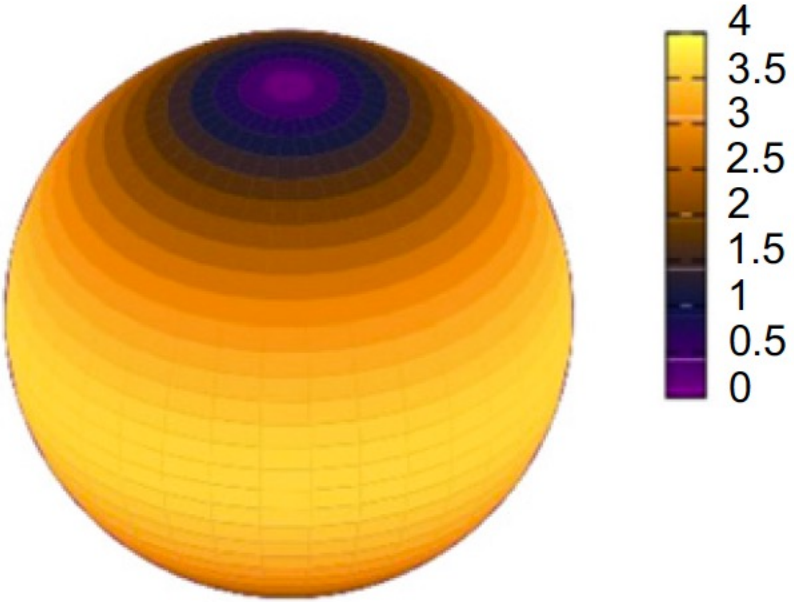
$$\Phi = \phi(r, \theta) e^{i(\omega t + m\varphi)}$$

- The **regularity** at the BH horizon requires $\boldsymbol{\omega = m\Omega_H}$.
- Thus the name – Kerr black holes with synchronized scalar hair

Kerr black holes with synchronized scalar hair

Structure of the solutions

- Complex scalar fields are **angularly excited** upon rotation
- The scalar field distributes itself in a **torus** (similar to boson stars)



Scalar field for **hairy Kerr BH**

Herdeiro, Radu PRL (2015)

Beyond-GR black holes in EFT and scalarization

scalar-Gauss-Bonnet gravity

$$S = \frac{1}{16\pi}\int d^4x\sqrt{-g}\left[R - 2\nabla_\mu\varphi\nabla^\mu\varphi - V(\varphi) + \lambda^2 f(\varphi)\mathcal{R}^2_{GB}\right].$$

Gauss-Bonnet invariant:

$$\mathcal{R}^2_{GB} = R^2 - 4R_{\mu\nu}R^{\mu\nu} + R_{\mu\nu\alpha\beta}R^{\mu\nu\alpha\beta}$$

- Schwarzschild: $R^2_{GB} = \frac{48M^2}{r^6}$
- Field equations :

$$R_{\mu\nu} - \frac{1}{2}Rg_{\mu\nu} + \Gamma_{\mu\nu} = 2\nabla_\mu\varphi\nabla_\nu\varphi - g_{\mu\nu}\nabla_\alpha\varphi\nabla^\alpha\varphi - \frac{1}{2}g_{\mu\nu}V(\varphi),$$

$$\nabla_\alpha\nabla^\alpha\varphi = \frac{1}{4}\frac{dV(\varphi)}{d\varphi} - \frac{\lambda^2}{4}\frac{df(\varphi)}{d\varphi}\mathcal{R}^2_{GB},$$

Scalar field equations – scalarization

- **Scalar field equation** (zero scalar field potential) **:**

$$\Box\varphi = -\frac{\lambda^2}{4}\frac{df}{d\varphi}R^2_{GB}$$

- **Conditions for the existence** of scalarized solutions

$$(\Box - \mu^2_{\text{eff}})\delta\varphi = 0 \text{ with } \mu^2_{\text{eff}} = -\frac{\lambda^2}{4}\frac{d^2 f}{d\varphi^2}(0)R^2_{GB} < 0$$

- If $\boldsymbol{\mu^2_{\text{eff}} < 0}$ a **tachyonic instability** is present leading to a development of the scalar field.

Scalar field coupling $f(\varphi)$ $\nabla_\alpha \nabla^\alpha \varphi = -\frac{\lambda^2}{4}\frac{df(\varphi)}{d\varphi} R^2_{GB}$

Expand $f(\varphi)$ in series around $\varphi = 0$:

$$f(\varphi) = f_0 + f_1\varphi + f_2\varphi^2 + f_3\varphi^3 + f_4\varphi^4 + O(\varphi^5)$$

Type I:

- $f_1 \neq 0$: **shift-symmetric** theory, Schwarzschild is not a solution, $|\varphi| > 0$ always
  Kanti et al PRD(1996), Torii et al (1996), Pani&Cardoso PRD (2009)

Type II:

- $f_1 = 0$, $f_2 > 0$, $R^2_{GB} > 0$: **spontaneous** scalarization, Kerr unstable for **small masses** DD, Yazadjiev PRL (2018), Silva et al PRL (2018), Antoniou et al (2018)
- $f_1 = 0$, $f_2 < 0$, $R^2_{GB} < 0$: **spin-induced** scalarization, Kerr unstable for **large spins**
  Dima et al PRL (2020), DD et al RPD(2020), Berti at al PRL (2021), Herdeiro et al PRL (2021)

Type III:

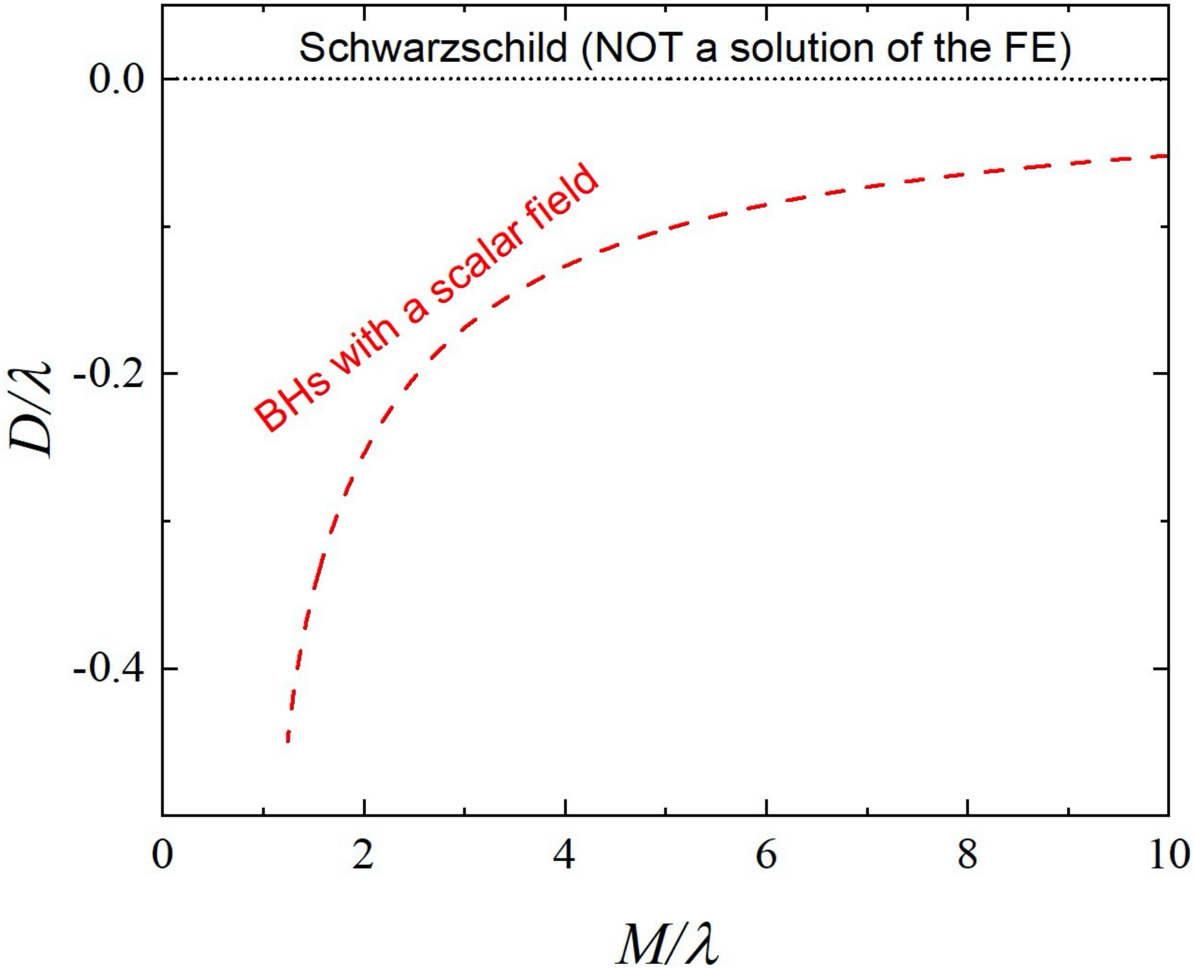
- $f_1 = 0$, $f_2 = 0$: $\mu^2_{\mathrm{eff}} = 0$, **nonlinear** scalarization, Kerr **linearly stable always, nonlinear scalarized phases can co-exist** DD, Yazadjiev, PRD Lett. (2021)

Type I - Schwarzschild is not a solution

- Einstein-dilaton-Gauss-Bonnet

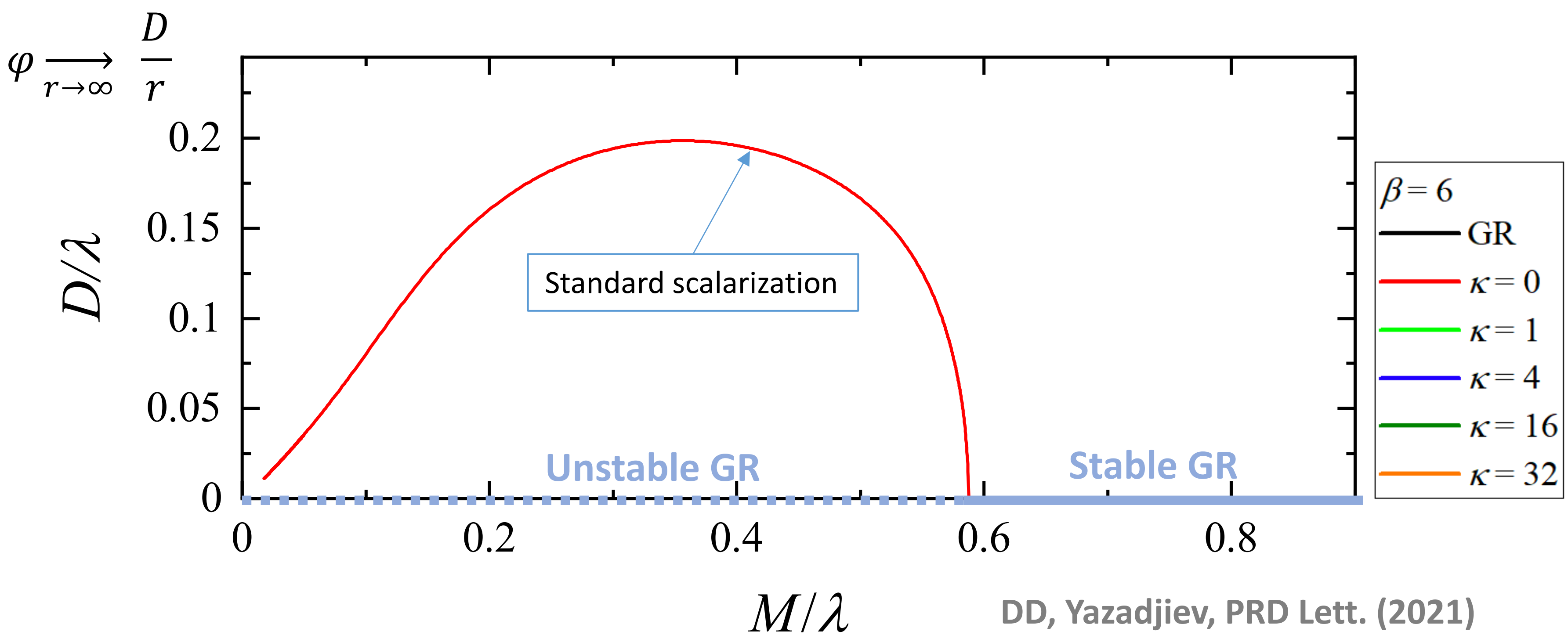
$$f(\varphi) = e^{\alpha\varphi}$$

$$\varphi \xrightarrow[r\to\infty]{} \frac{D}{r}$$



Type II – Spontaneous scalarization

$$f(\varphi) = \frac{1}{2\beta}\left(1 - e^{-\beta(\varphi^2 + \kappa\varphi^4)}\right)$$

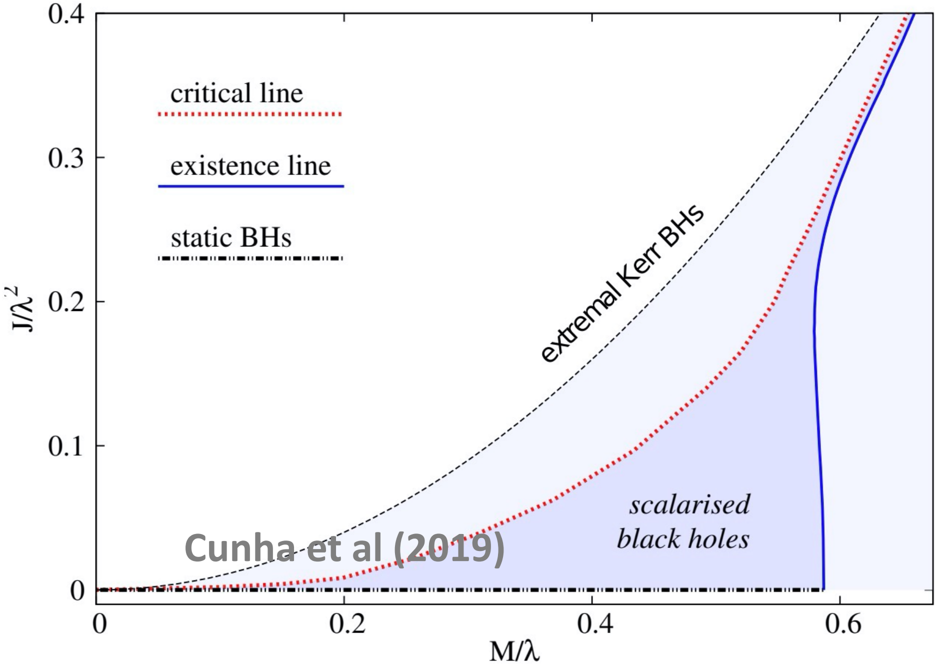


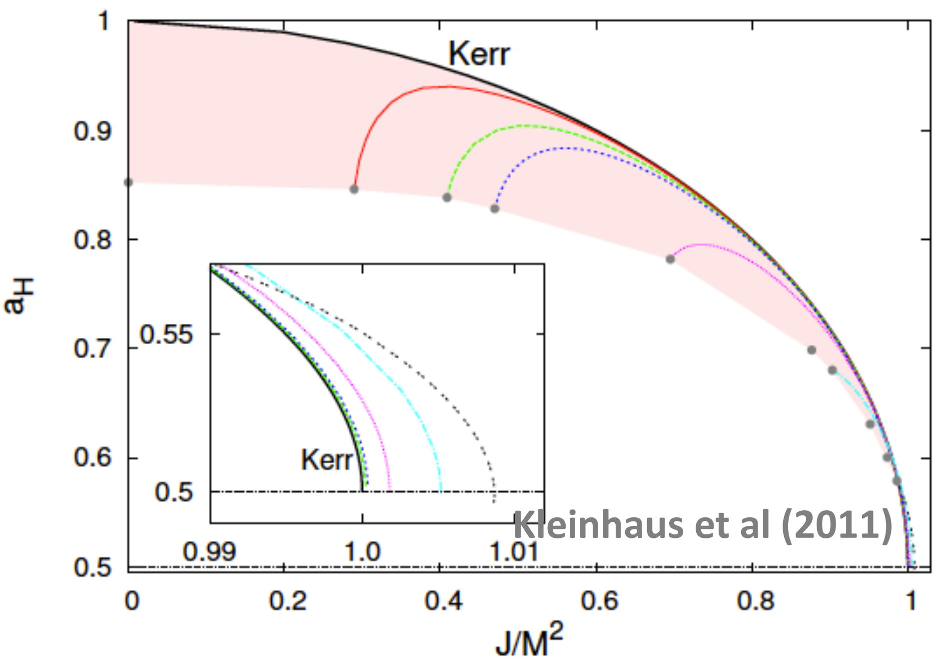
DD, Yazadjiev, PRD Lett. (2021)

What about rotation?

- Rotation suppresses scalarization

$$f(\varphi) = \frac{1}{2\beta}\left(1 - e^{-\beta(\varphi^2 + \kappa\varphi^4)}\right) \qquad f(\varphi) = \frac{1}{2\beta}\left(1 - e^{-\alpha\varphi}\right)$$



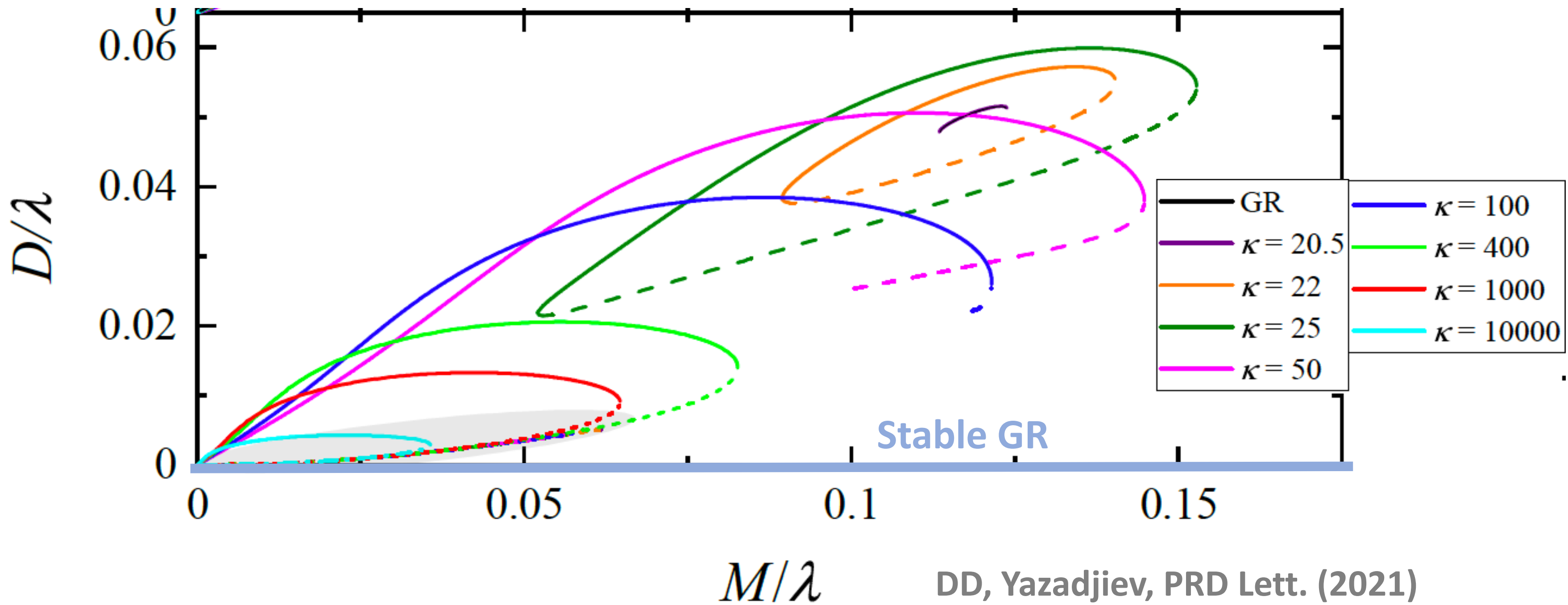


- Above $\frac{J}{M^2} > 0.5$: **spin induced scalarization** Dima at al, PRL (2020), DD et al., PRD (2020), Herdeiro et al., PRL (2020), Berti et al., PRL (2020)

Type III – Nonlinear scalarization

$$f(\varphi) = \frac{1}{2\beta}\left(1 - e^{-\beta(\varphi^2+\kappa\varphi^4)}\right)$$

$$\varphi \xrightarrow[r\to\infty]{} \frac{D}{r}$$



DD, Yazadjiev, PRD Lett. (2021)

Mixed II + III: Standard + nonlinear scalarization

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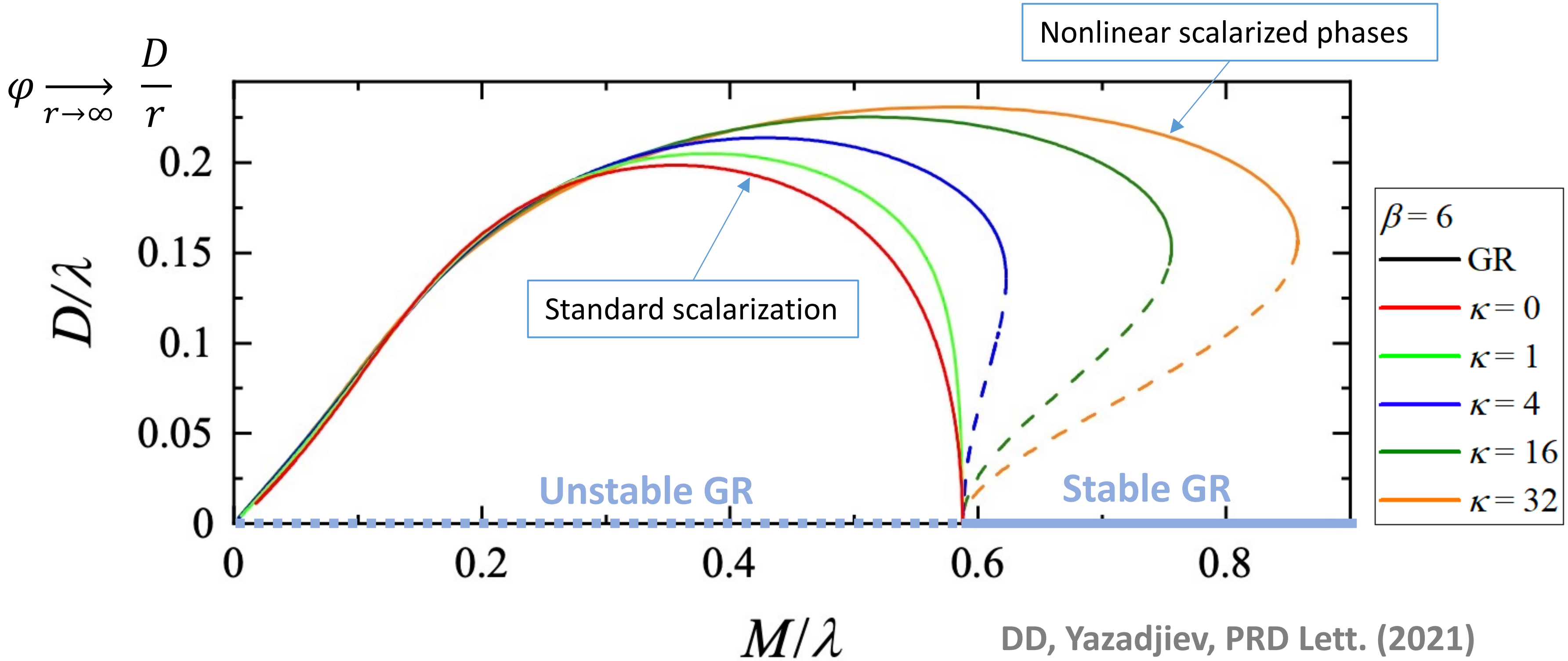

DD, Yazadjiev, PRD Lett. (2021)

- Transition from **stable scalarized to GR** happens with a **jump**
- For a similar effect for charged BH see Blázquez-Salcedo et al. PLB (2020)

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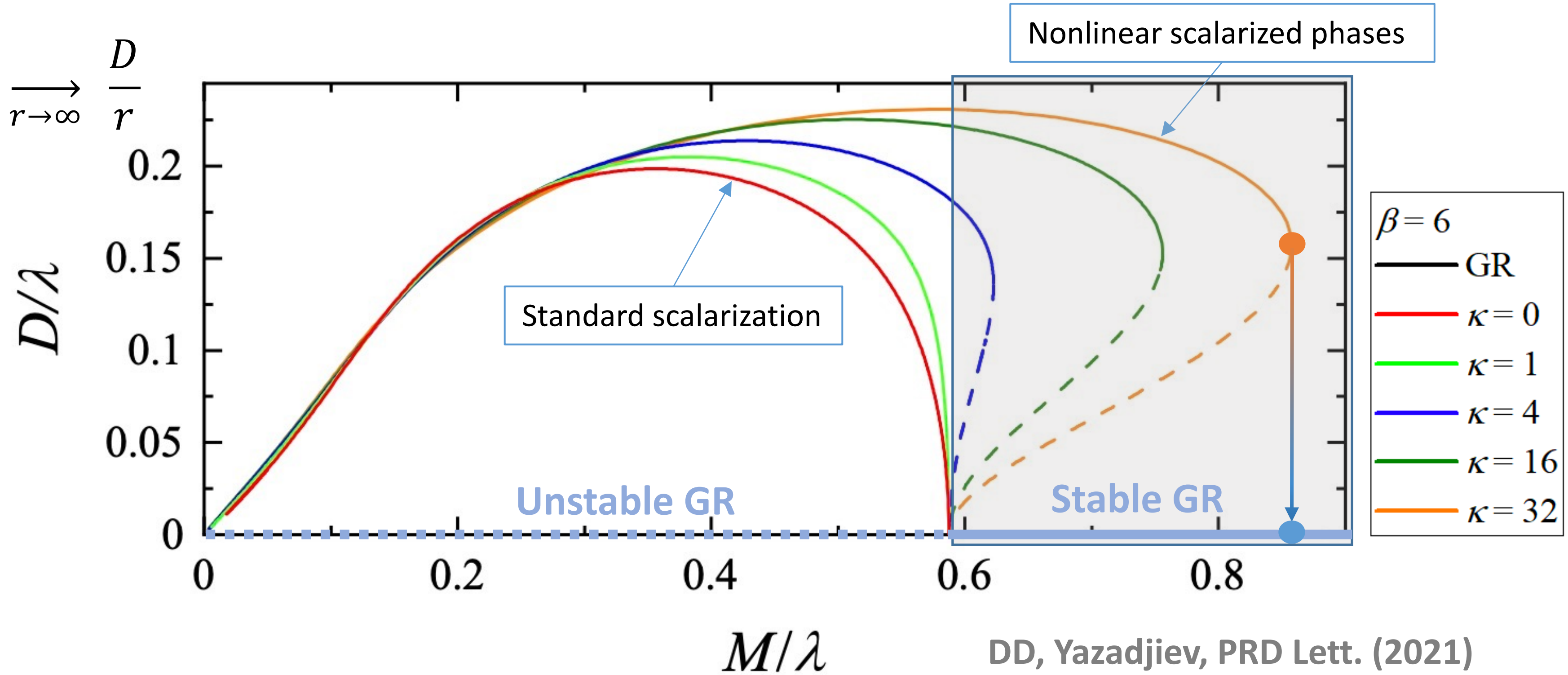

DD, Yazadjiev, PRD Lett. (2021)

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Well-posedness

Studying hyperbolicity

- **Principle symbol** – a matrix assembled by the coefficients in front of the leading (2nd) order derivative in the differential equation

- Scalar-Gauss-Bonnet gravity: Can be describen in terms of an **effective metric** Real PRD (2021), Areste-Salo et al PRL (2022), PRD (2022)

$$g_{\text{eff}}^{\mu\nu} = g^{\mu\nu} - \Omega^{\mu\nu}$$

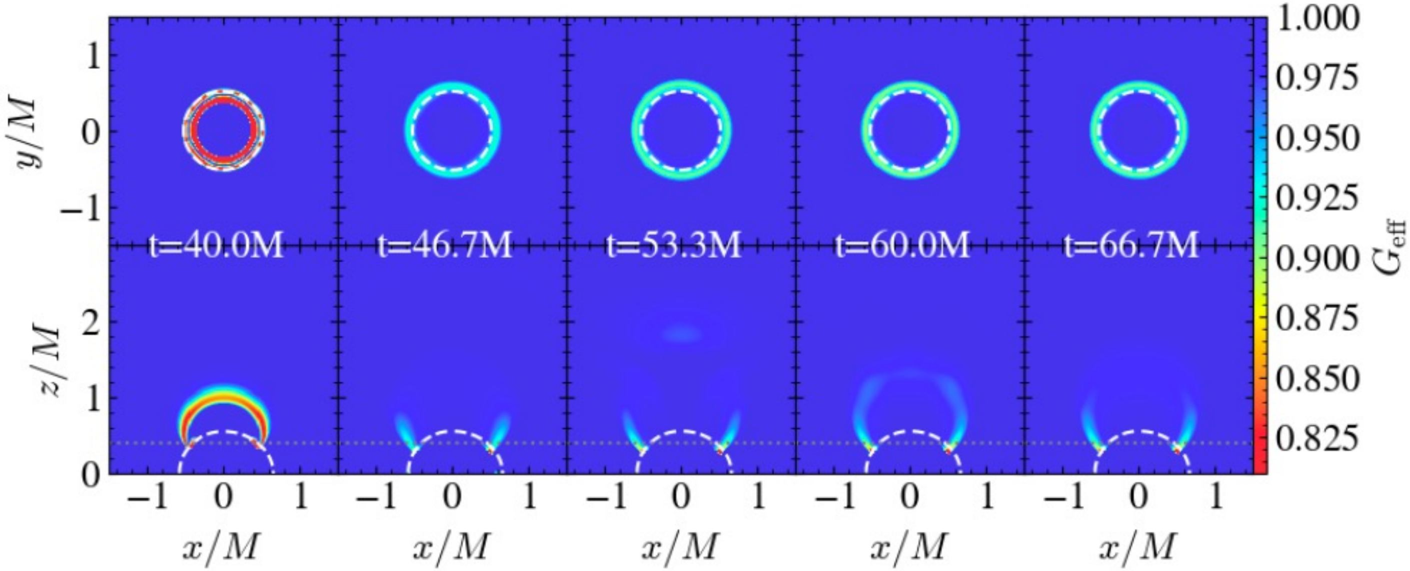
$$\Omega_{\mu\nu} = \lambda \nabla_{\mu}\nabla_{\nu} f(\varphi)$$

- **Hyperbolicity loss** when the determinant of the effective metric < 0

East&Ripley PRL (2021), Areste-Salo et al PRL (2022), Hegade et al PRD (2023), Corman at al. PRD (2023)

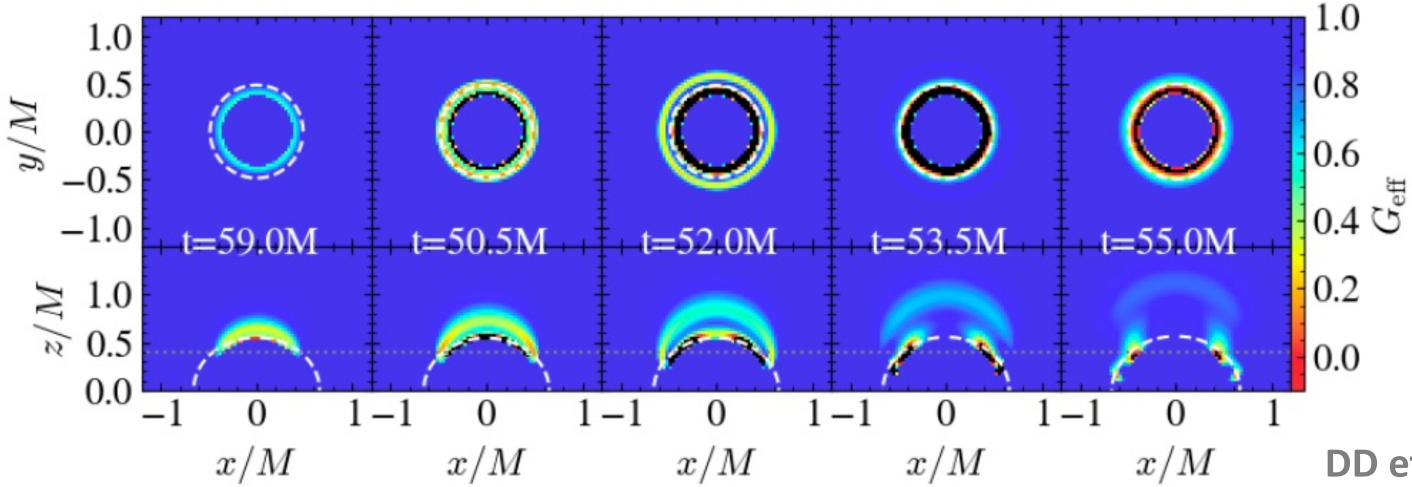
- **Modified harmonic gauge** in Gauss-Bonnet theory – the system remains **hyperbolic in weak coupling** limit Kovacs&Real PRL (2021)

Normalized determinant – spin-induced black hole



Hyperbolic evolution

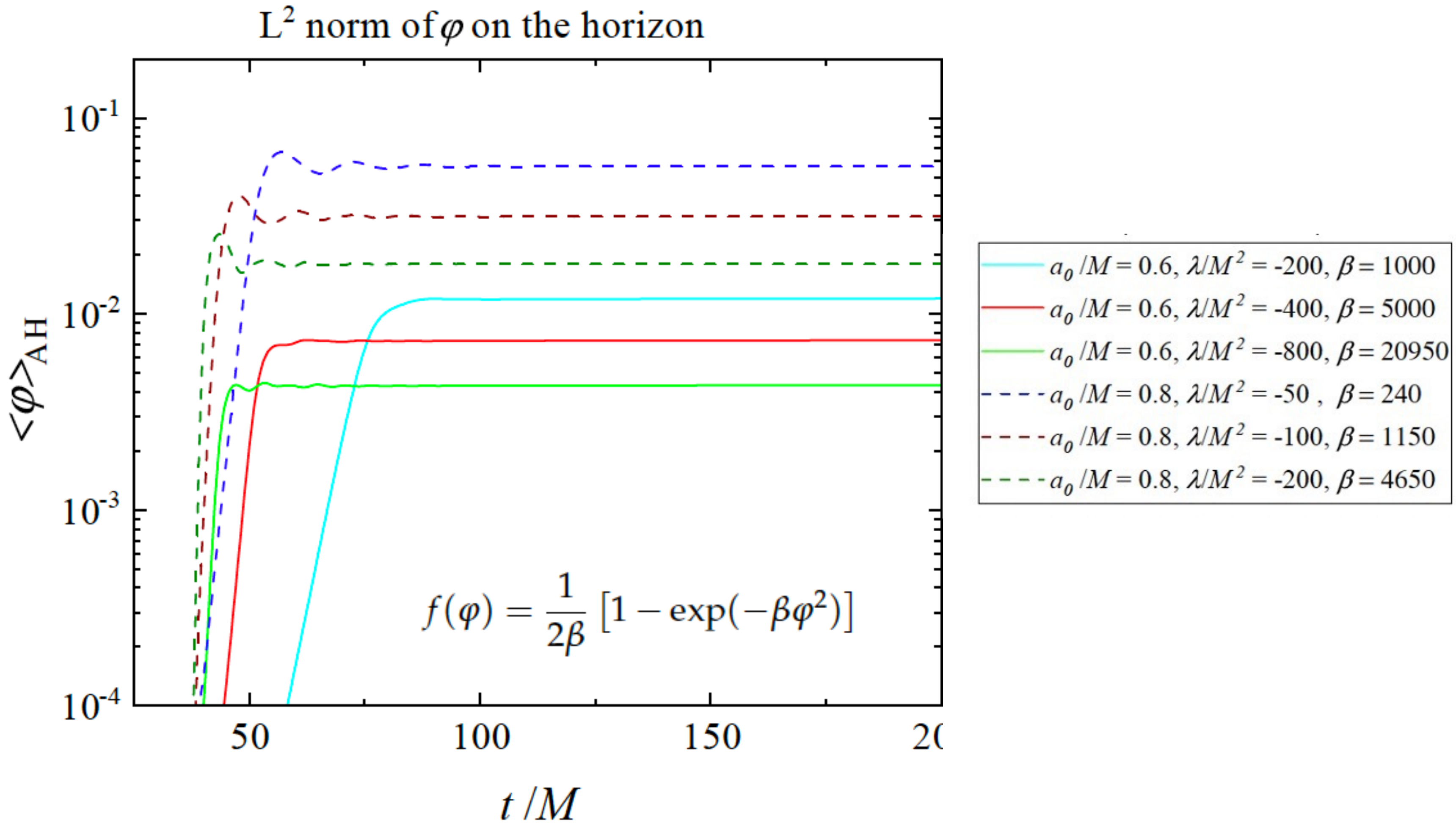
VS.



Hyperbolicity loss

DD et al. PRD (2023)

Limitting models for hyperbolicity loss



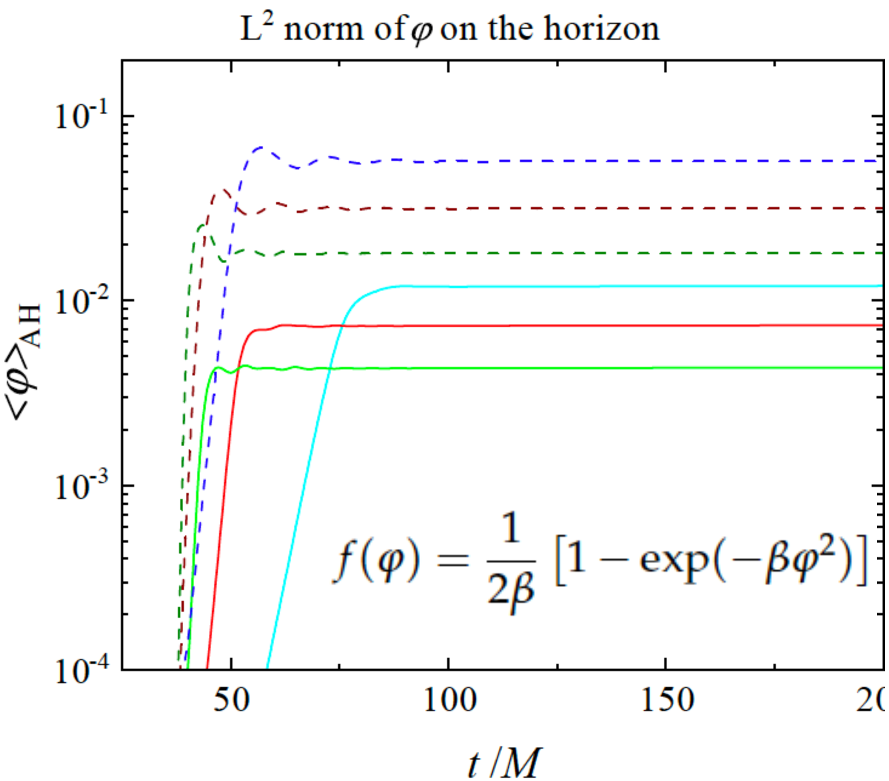
DD et al. PRD (2023)

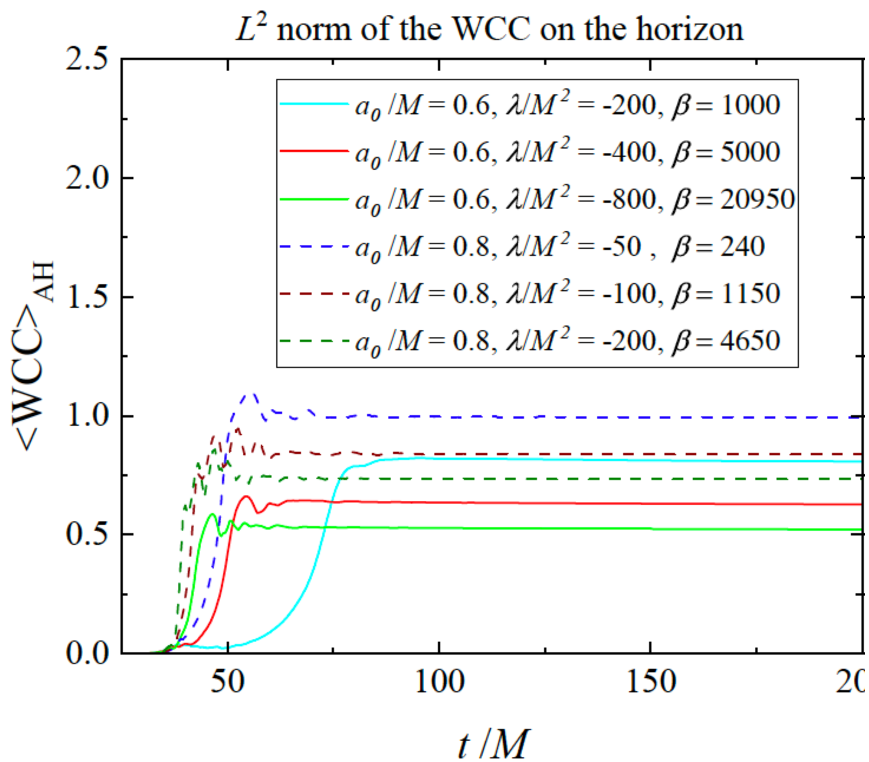
Limitting models and weak coupling condition

- **Weak coupling condition**

$$\sqrt{|\lambda f'(\varphi)|}/L \ll 1$$

$$L^{-1} = \max\{|R_{ij}|^{1/2}, |\nabla_\mu \varphi|, |\nabla_\mu \nabla_\nu \varphi|^{1/2}, |\mathcal{R}^2_{GB}|^{1/4}\}$$





DD et al. PRD (2023)

Resolving the problem

- **Gauge change** – not very likely to help, hyperbolicity loss due to eigenvalues of physical modes becoming imaginary **Areste-Salo et al PRL (2022), PRD (2022), DD et al. PRD (2023)**

- **Fixing approach** **Franchini et al PRD (2022), Cayuso at al PRL (2023)**
  - ✓ A prescription to control the high frequency behaviour of an EFT
  - ✓ Modify in an *ad hoc* way the higher-order contributions to the field equations
  - ✓ Add a driver equation to let the solution relax to its correct value

- **Addition interactions** in the action can mitigate the hyperbolicity loss

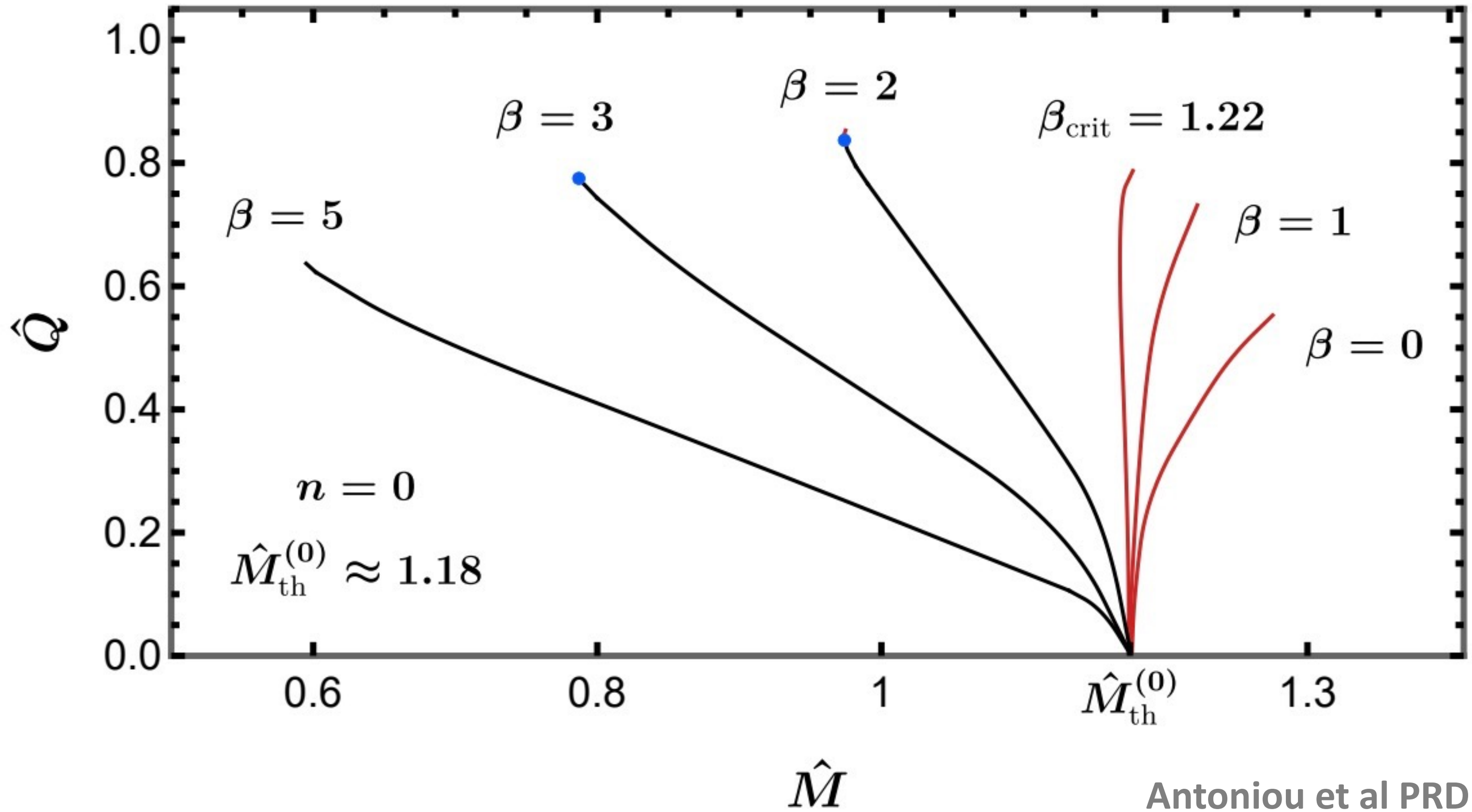
$$S = \frac{1}{16\pi}\int d^4x\,\sqrt{-g}\,\left[R - \frac{1}{2}\nabla_\mu\varphi\nabla^\mu\varphi + \frac{1}{4}\lambda^{GB} f(\varphi)R^{GB} - \beta(\varphi)R\right]$$

Ricci scalar coupling

Ricci scalar coupling

$$f(\varphi) \sim \varphi^2, \qquad \beta(\varphi) \sim \beta\varphi^2$$

- Previously unstable solution **turn stable** Antoniou et al PRD (2021)
- Loss of hyperbolicity is **mitigated** in 1D simulation Thaalba et al (2023)

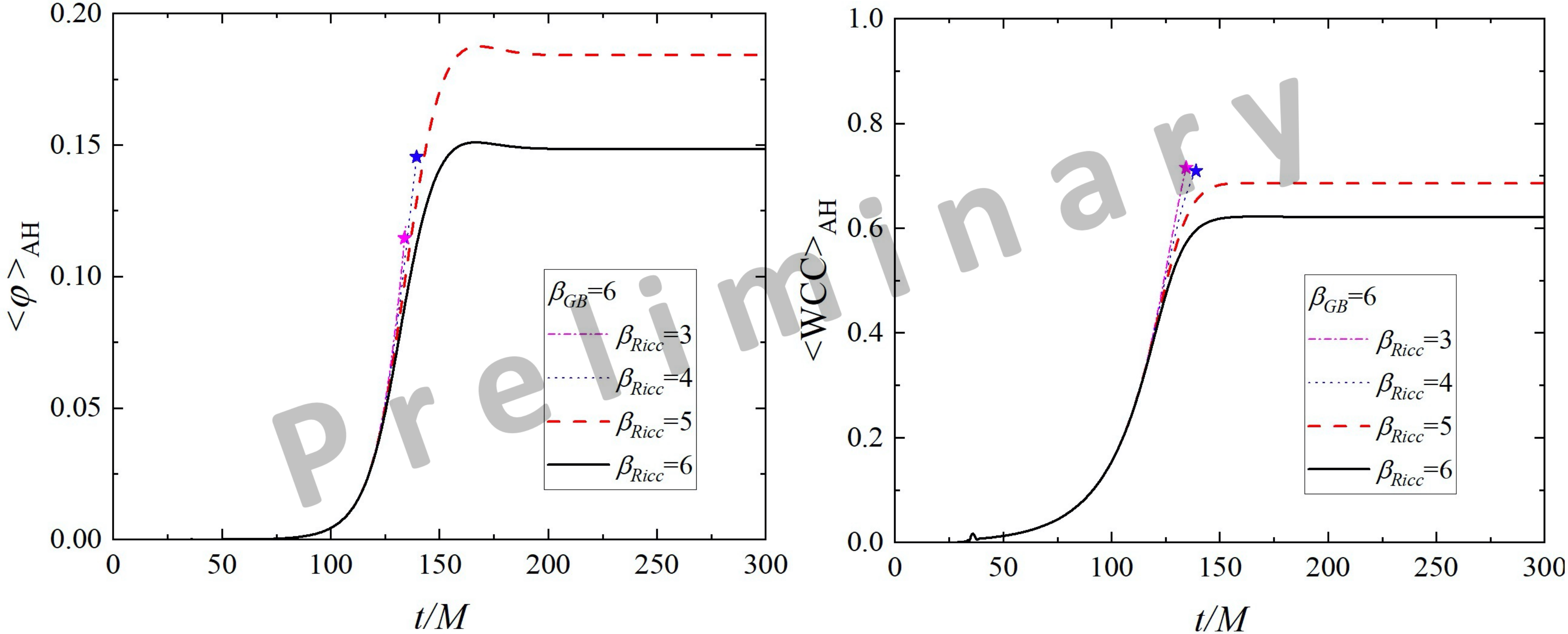


Antoniou et al PRD (2021)

Ricci scalar coupling – 3D simulations

$$f(\varphi) \sim e^{-\beta_{GB}\varphi^2}, \qquad \beta(\varphi) \sim e^{-\beta_{Ricc}\varphi^2}$$

- Evolution of a single nonrotating black hole

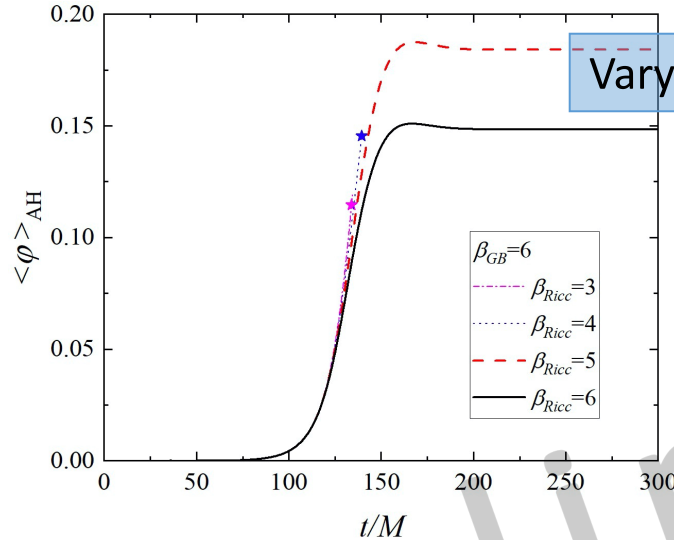


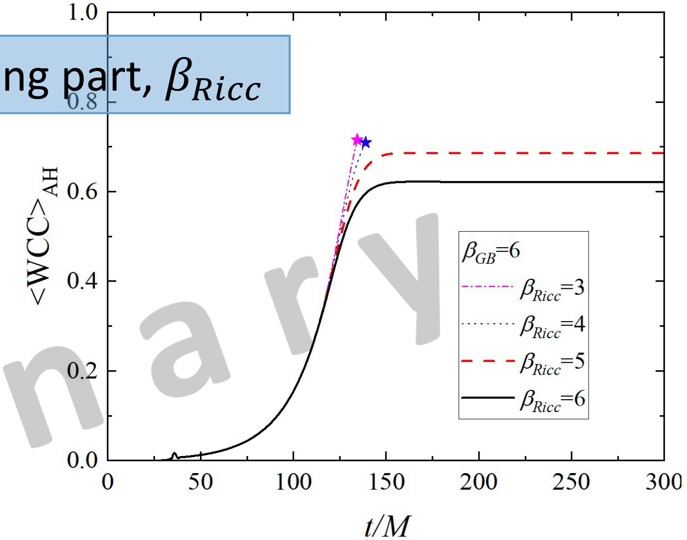
DD, Yazadjiev, at al., in prep.

Ricci coupling vs. GB coupling variation

$$f(\varphi) \sim e^{-\beta_{GB}\varphi^2}, \qquad \beta(\varphi) \sim e^{-\beta_{Ricc}\varphi^2}$$

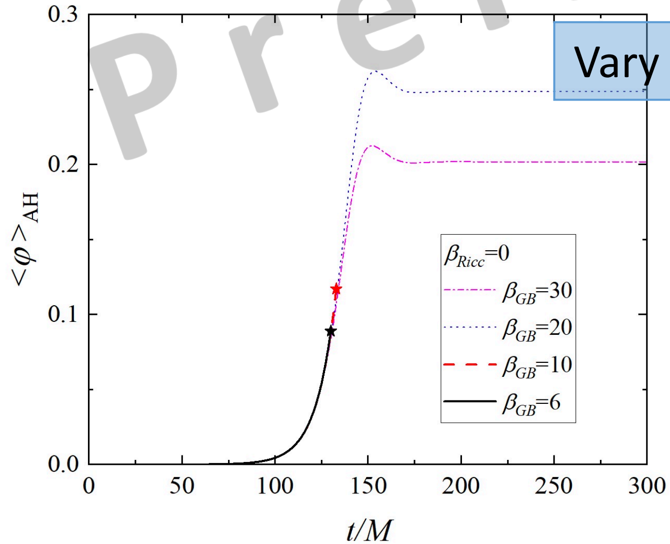
Vary Ricci coupling part, β_{Ricc}

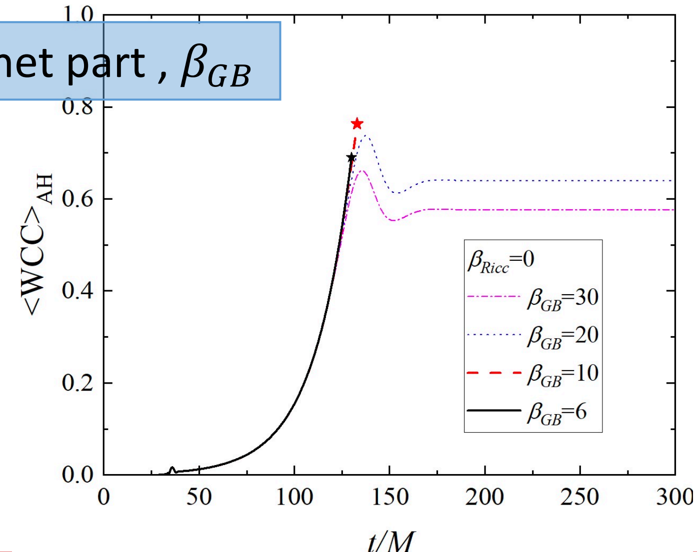




VS.

Vary Gauss-Bonnet part , β_{GB}





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