Compact Objects in Scalar-Tensor theories

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AstroParticle Symposium 2023

November 9, 2023

H.F.R.I.
Hellenic Foundation for Research & Innovation

The research project was supported by the Hellenic Foundation for Research and Innovation (H.F.R.I.) under the "3rd Call for H.F.R.I. Research Projects to support Post-Doctoral Researchers" (Project Number: 7212)

Outline

1. Black Holes in General Relativity and the No-Hair Theorem
2. No-Scalar Hair Theorem
3. Black hole solutions in Horndeski and beyond Horndeski
4. Wormholes in General Relativity
5. Wormhole solutions in beyond Horndeski theory

Based on:

JHEP **08** (2022), 055. & *arXiv:2310.11919*

JHEP **04** (2022) 096 & Phys.Rev.D **107** (2023) 12, 124035

JCAP **05** (2022) no.05, 022. *arXiv:2111.09857*

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Black Holes in General Relativity and the No-Hair Theorem

- Black holes in General Relativity may be described only by three physical quantities: Mass, E/M charge and Angular Momentum.
- Black holes are very special objects: Two stars with the same mass are, in general, very different, but two black holes with the same characteristics (M, Q and J) will be identical.
- *No hair theorems:* Uniqueness theorems which state that in General Relativity only four possible solutions for black holes may exist.

No-Scalar Hair Theorem

Adding new matter/energy forms in the theory could lead to new black holes solutions?

The simplest form is a Scalar field coupled to the gravitational field:

$$S = \int d^4x\sqrt{-g}\left[R - \frac{1}{2}\nabla_\mu\Phi\nabla^\mu\Phi - V\left(\Phi\right)\right].$$

Assumptions:

- Asymptotically flatness,
- The scalar field has the same symmetries with the spacetime,
- $V(\Phi) > 0$.
- Minimal coupling.

Under these assumptions black holes with scalar hair do not exist [1].

[1] J. D. Bekenstein, Phys. Rev. Lett. **28** (1972) 452
J. D. Bekenstein, Phys. Rev. D **51** (1995) no.12 R6608

Beyond Horndeski theory

$$S = \int \mathrm{d}^4x\sqrt{-g}\,\left(\mathcal{L}_2 + \mathcal{L}_3 + \mathcal{L}_4 + \mathcal{L}_5 + \mathcal{L}_4^{\mathrm{bH}} + \mathcal{L}_5^{\mathrm{bH}}\right),$$

with

$$X = -\frac{1}{2}\partial_\mu\phi\partial^\mu\phi,$$

$$\mathcal{L}_2 = G_2(\phi, X), \qquad \mathcal{L}_3 = -G_3(\phi, X)\,\Box\phi,$$

$$\mathcal{L}_4 = G_4(\phi, X)R + G_{4X}\left[(\Box\phi)^2 - \nabla_\mu\partial_\nu\phi\,\nabla^\mu\partial^\nu\phi\right],$$

$$\mathcal{L}_5 = G_5(\phi, X)G_{\mu\nu}\nabla^\mu\partial^\nu\phi - \frac{1}{6}\,G_{5X}\Big[(\Box\phi)^3 - 3\,\Box\phi\,\nabla_\mu\partial_\nu\phi\,\nabla^\mu\partial^\nu\phi + 2\,\nabla_\mu\partial_\nu\phi\,\nabla^\nu\partial^\rho\phi\,\nabla_\rho\partial^\mu\phi\Big],$$

$$\mathcal{L}_4^{\mathrm{bH}} = F_4(\phi, X)\varepsilon^{\mu\nu\rho\sigma}\,\varepsilon^{\alpha\beta\gamma}{}_{\sigma}\,\partial_\mu\phi\,\partial_\alpha\phi\,\nabla_\nu\partial_\beta\phi\,\nabla_\rho\partial_\gamma\phi,$$

$$\mathcal{L}_5^{\mathrm{bH}} = F_5(\phi, X)\varepsilon^{\mu\nu\rho\sigma}\,\varepsilon^{\alpha\beta\gamma\delta}\,\partial_\mu\phi\,\partial_\alpha\phi\,\nabla_\nu\partial_\beta\phi\,\nabla_\rho\partial_\gamma\phi\,\nabla_\sigma\partial_\delta\phi,$$

and

$$XG_{5X}F_4 = 3F_5(G_4 - 2XG_{4X}).$$

The Field Equations for the Shift-Symmetric theory

We focus on the shift symmetric case, therefore $G_i = G_i(X)$ and $F_i = F_i(X)$.

For a spherically symmetric line-element

$$ds^2 = -h(r)dt^2 + \frac{dr^2}{f(r)} + r^2 d\Omega^2 \,,$$

The field equations of the beyond Horndeski theory are

$$\begin{aligned} X'\mathcal{A} &= 2\left(\frac{h'}{h} - \frac{f'}{f}\right)\mathcal{B}\,, \\ \frac{h'f}{2h}\mathcal{A} &= G_{2X}r^2 + 2G_{4X} - 2rf\phi' G_{3X} - 2fZ_X\,, \\ 2f\frac{h'}{h}\mathcal{B} &= -G_2 r^2 - 2G_4 - 2fZ\,. \end{aligned}$$

with

$$Z(X) = 2XG_{4X} - G_4 + 4X^2F_4\,, \qquad Y(X) = \frac{1}{2}(-2X)^{3/2}G_{5X} + 3(-2X)^{5/2}F_5\,,$$

$$\mathcal{A} = 4rZ_X + \phi'(r^2G_{3X} + G_{5X}) + 2\sqrt{f}Y_X, \qquad \text{and} \qquad \mathcal{B} = rZ + \sqrt{f}Y\,.$$

Parity symmetric theories - Integrability

In theories with Parity Symmetry in ϕ we have $G_3 = G_5 = F_5 = 0$

In this case the first equation may be integrated to

$$Z^2 f = \gamma^2 h,$$

while the remaining equations take the form

$$r^2 (ZG_2)_X + 2(G_4 Z)_X = 0 \qquad \text{and} \qquad 2\gamma^2 (rh)' + Z(G_2 r^2 + 2G_4) = 0\,.$$

A general way to proceed in order to find explicit solutions is to consider an arbitrary function $\mathcal{G} = \mathcal{G}(X)$, such that

$$\mathcal{G}_X = \frac{\alpha r^2 + \beta}{\epsilon r^2 + \delta}$$

and the first equation are compatible.

Compatibility immediately gives the conditions

$$G_2 Z = \epsilon \mathcal{G} - \alpha X + C\,, \quad 2G_4 Z = \delta \mathcal{G} - \beta X + D\,,$$

Parity symmetric theories - Homogeneous solutions ($Z = \gamma$)

We set $Z = \gamma = -1$ which leads to $f = h$.

We also choose $\mathcal{G} = 2\mu X + \zeta$.

then we find

$$G_2 = -\epsilon\mu X^2,$$
$$G_4 = -\frac{\delta\mu}{2}X^2 - \frac{\delta\zeta - \beta}{2}X + 1\,,$$
$$F_4 = -\frac{\beta - \delta\zeta}{8X} + \frac{3\delta\mu}{8}\,.$$

The solution is

$$h(r) = 1 + \frac{(\beta - \delta\zeta)^2}{8\delta\mu}\,\frac{\arctan(\sqrt{\frac{\epsilon}{\delta}}r)}{\sqrt{\frac{\epsilon}{\delta}}r} - \frac{2M}{r}\,, \qquad \text{and} \qquad \phi'(r) = -\frac{(\beta - \delta\zeta)}{\mu(\epsilon r^2 + \delta)}\,\frac{1}{h(r)}\,,$$

Parity symmetric theories - Non-Homogeneous solutions ($Z = \gamma[1 + X]$)

We use $Z(X) = \gamma(1 + X)$, with $\gamma = -1$ and $\mathcal{G} = 2\mu X + \zeta$.

Then we find

$$G_2 = -\frac{\epsilon\mu X^2}{(1 + X)},$$

$$G_4 = -\frac{\delta\mu X^2 + (\delta\zeta - \beta)X - 2}{2(1 + X)},$$

$$F_4 = -\frac{\beta - \delta\zeta + X^2\,(2 - \delta\mu) + X\,(-\beta + 6 + \delta(\zeta - 3\mu))}{8X(X + 1)^2}.$$

The solution is

$$h(r) = 1 + \frac{(\beta - \delta\zeta)^2}{8\delta\mu}\frac{\arctan(\sqrt{\frac{\epsilon}{\delta}}r)}{\sqrt{\frac{\epsilon}{\delta}}r} - \frac{2M}{r}, \qquad f(r) = \frac{h(r)}{(1 + X)^2},$$

with

$$\phi'^2(r) = -\frac{(\beta - \delta\zeta)}{\mu(\epsilon r^2 + \delta)}\,\frac{1}{f(r)}, \qquad \text{and} \qquad X(r) = \frac{\beta - \delta\zeta}{2\mu(\epsilon r^2 + \delta)}$$

Non parity preserving theories

The first two beyond Horndeski equations are

$$X'\mathcal{A} = 2\left(\frac{h'}{h} - \frac{f'}{f}\right)\mathcal{B}\,,$$

$$\frac{h'f}{2h}\mathcal{A} = G_{2X}r^2 + 2G_{4X} - 2rf\phi' G_{3X} - 2fZ_X\,.$$

For a homogeneous solution ($f = h$) the first equation gives $\mathcal{A} = 0$ or $X' = 0$.

For the case $\mathcal{A} = 0$ right hand part in the second equation must be vanish independently or it may be proportional to $\mathcal{A}$

$$G_{2X}r^2 + 2G_{4X} - 2rf\phi' G_{3X} - 2fZ_X = -\sqrt{f}\mathcal{A}Q,$$

where

$$\mathcal{A} = 4rZ_X + \phi'(r^2G_{3X} + G_{5X}) + 2\sqrt{f}Y_X\,,$$

Non parity preserving theories

This leads to the following constraints

$$G_{2X} = -\sqrt{-2X} Q G_{3X} = -2Q^2 Z_X , \qquad 2G_{4X} = -\sqrt{-2X} Q G_{5X} , \qquad Z_X = Q Y_X ,$$

$$Z = QY \left(1 - \frac{G_4}{2XG_{4X}}\right) \qquad Q_X Y = \left(QY \frac{G_4}{2XG_{4X}}\right)_{,X} .$$

In this case there are only two independent coupling functions: The coupling functions G_2, G_3, G_5, F_4 and F_5 are given in terms of G_4, Q

The field equations have the form

$$X' \mathcal{A} = 2\left(\frac{h'}{h} - \frac{f'}{f}\right)\mathcal{B} ,$$

$$\mathcal{A}\left(\frac{h'\sqrt{f}}{h} + 2Q\right) = 0 ,$$

$$2f\frac{h'}{h}\mathcal{B} + G_2 r^2 + 2G_4 + 2fZ = 0 ,$$

Homogeneous black holes ($f = h$) in non parity preserving theories

We use $G_4 = 1 + \alpha(-2X)^n$ and $Q = \sqrt{-2X}$. For this case

$$Y = -G_{4X}\sqrt{-2X}, \qquad Z = -\left(G_4 - 2XG_{4X}\right), \qquad G_2 = -2\alpha n(2n-1)\frac{(-2X)^{n+1}}{n+1},$$

$$G_3 = 2\alpha(2n-1)\left(-2X\right)^n, \qquad G_{5X} = 4\alpha n\left(-2X\right)^{n-2}.$$

From equation $\mathcal{A} = 0$ we find

$$\phi' = \frac{1-\sqrt{(2n-1)f}}{r\sqrt{(2n-1)f}},$$

while the last equation we get an algebraic equation

$$(n+1)\left(2n-1\right)^n r^{2n-1}\left[(2n-1)(2M-r) + rF^2\right] + \alpha\left(1-F\right)^{2n}\left(1+2nF+F^2\right) = 0,$$

where $F^2(r) \equiv (2n-1)f > 0$.

A special case (n=1)

For the special case $n = 1$ with the redefinition $\alpha \to 2\alpha$ we may find an analytic solution of the algebraic equation[2]

$$h(r) = f(r) = 1 + \frac{r^2}{2\alpha}\left(1 - \sqrt{1 + \frac{8\alpha M}{r^3}}\right), \quad \text{and} \quad \phi' = \frac{\sqrt{h} - 1}{r\sqrt{h}},$$

The coupling functions are

$$G_2 = 8\alpha X^2,\ G_3 = -8\alpha X,\ G_4 = 1 + 4\alpha X,\ G_5 = -4\alpha \ln|X|.$$

[2] H. Lu and Y. Pang, Phys. Lett. B **809** (2020), 135717. [arXiv:2003.11552 [gr-qc]].

Black hole solutions in Modified Gravity

A general gravitation theory has the form

$$S = \int d^4x\sqrt{-g}\left[R - \frac{1}{2}\nabla_\mu\Phi\,\nabla^\mu\Phi - V(\Phi) + \alpha\mathcal{L}_i\left(g_{\mu\nu},\Phi\right)\right],$$

If we break the assumptions of the no scalar hair theorem, the $\mathcal{L}_i$ term usually contains non-minimal couplings

For example the EsGB theory accepts asymptotically flat black hole solutions for $V(\Phi) < 0$. [3]

$$S = \int d^4x\sqrt{-g}\left[R - \frac{1}{2}\nabla_\mu\Phi\,\nabla^\mu\Phi - V(\Phi) + \alpha f(\Phi){R^2}_{GB}\right],$$

[3] A. B, P. Kanti and N. Pappas, Phys. Rev. D **101** (2020) no.8, 084059 (arXiv:2003.02473).

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$$S = \int d^4x\sqrt{-g}\left[R - \frac{1}{2}\nabla_\mu\Phi\,\nabla^\mu\Phi - V(\Phi)\right],$$

If we switch off the $\mathcal{L}_i$ term ($\alpha \to 0$), the background solution is not the Schwarzschild but instead depends on the potential $V(\Phi)$.

[3] A. B, P. Kanti and N. Pappas, Phys. Rev. D **101** (2020) no.8, 084059 (arXiv:2003.02473).

The field equations

We assume a spherically symmetric form for the line-element:

$$ds^2 = -e^{A(r)}\, B(r)\, dt^2 + \frac{dr^2}{B(r)} + r^2\left(d\theta^2 + \sin^2\theta\, d\varphi^2\right),$$

The field equations are

$$A'(r) = \frac{r}{2}\left[\Phi'(r)\right]^2\,,$$

$$B''(r) + \frac{3}{2}\,A'(r)B'(r) + \left\{A''(r) + \frac{A'(r)}{r} + \frac{\left[A'(r)\right]^2}{2} - \frac{2}{r^2}\right\}B(r) = -\frac{2}{r^2}\,,$$

$$V(\Phi) = \frac{2}{r^2} - \frac{2}{r}\,A'(r)B(r) - \frac{2B(r)}{r^2} + \frac{1}{2}\left[\Phi'(r)\right]^2 B(r) - \frac{2B'(r)}{r}\,.$$

Black holes with a Coulombic scalar field

We assume a Coulombic form for the scalar field $\Phi(r) = \dfrac{q}{r}$ and we find the solution[4]:

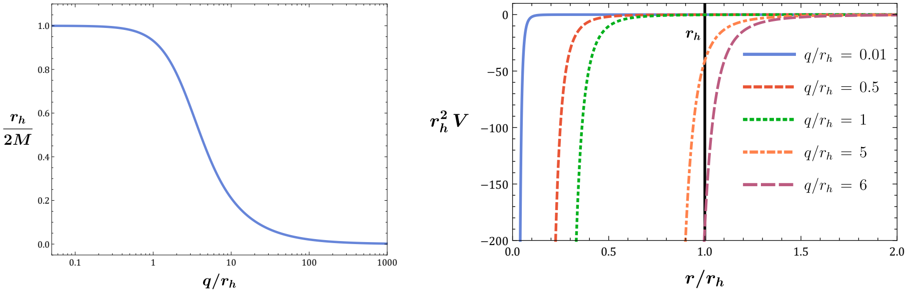
$$A(r) = -\frac{q^2}{4r^2}\,, \qquad B(r) = 1 - \frac{2m(r)}{r}$$

$$m(r) = \frac{r}{2} + \frac{4r^3}{q^2} + \frac{e^{\frac{q^2}{8r^2}}\,r^2}{q^2}\left[-12M + \sqrt{2\pi}\,q\,\mathrm{erf}\left(\frac{q}{2\sqrt{2}\,r}\right)\right]$$
$$- \frac{e^{\frac{q^2}{4r^2}}\,r^3}{q^3}\left\{4q - 12\sqrt{2\pi}\,M\,\mathrm{erf}\left(\frac{q}{2\sqrt{2}\,r}\right) + \pi q\left[\mathrm{erf}\left(\frac{q}{2\sqrt{2}\,r}\right)\right]^2\right\},$$

$$V(\Phi) = \frac{2(24+\Phi^2)}{q^2} - \frac{12\,\Phi\,e^{\Phi^2/8}}{q^3}\left[12M - \sqrt{2\pi}\,q\,\mathrm{erf}\left(\frac{\Phi}{2\sqrt{2}}\right)\right]$$
$$+ \frac{(\Phi^2-12)e^{\Phi^2/4}}{q^3}\left\{4q - 12\sqrt{2\pi}\,M\,\mathrm{erf}\left(\frac{\Phi}{2\sqrt{2}}\right) + \pi q\left[\mathrm{erf}\left(\frac{\Phi}{2\sqrt{2}}\right)\right]^2\right\}.$$

[4] A. B. and T. Nakas, JHEP **04** (2022), 096 (arXiv:2107.05656).

Black holes with Coulombic scalar field



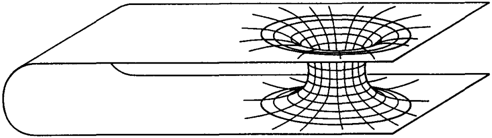
For small values of the ratio q/r_h, the fraction $r_h/(2M)$ is equal to unity and therefore $r_h = 2M$ as in the Schwarzschild geometry.

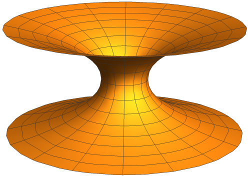
As the value of q/r_h increases, the value of $r_h/(2M)$ decreases leading to ultra-compact black holes.

For more information see Theodoros Nakas' poster.

Wormholes in General Relativity

A wormhole is a solution of the Einstein's field equations which has the property to connect two distant regions in spacetime.





A wormhole may connect:

- Two distant regions of our universe (intra-universe wormholes)[5].
- Two different universes (inter-universe wormhole).

The difference between the two kind of wormholes is topological. An observer who makes measurements near the wormhole cannot identify the class of the wormhole.

[5] The figure of the intra-universe wormhole is from the following book: C. W. Misner, K. S. Thorne and J. A. Wheeler, "*Gravitation*", San Francisco, 1973

Traversable wormholes - Morris & Thorne wormholes

- Morris and Thorne[6] suggest that we may construct traversable wormholes using an "*engineering-like*" technique.

- We start with a metric which describes a traversable wormhole and by solving the Einstein field equation in the reverse direction we find the associate energy-momentum tensor.

$$ds^2 = -e^{2\phi(r)}dt^2 + \left(1 - \frac{b(r)}{r}\right)^{-1} dr^2 + r^2 d\Omega^2.$$

- A traversable wormhole violates the Energy Conditions.

[6] M. S. Morris and K. S. Thorne, Am. J. Phys. **56**, 395 (1988).
M. Visser, "*Lorentzian wormholes: From Einstein to Hawking*", Woodbury, USA: AIP (1995)

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- A traversable wormhole violates the Energy Conditions.

We need Exotic Matter in order to keep the throat open!

[6] M. S. Morris and K. S. Thorne, Am. J. Phys. **56**, 395 (1988).
M. Visser, "*Lorentzian wormholes: From Einstein to Hawking*", Woodbury, USA: AIP (1995)

Wormholes in Einstein Scalar Gauss-Bonnet Theory

$$S = \int d^4x \sqrt{-g}\left[R - \frac{1}{2}\nabla_\mu\phi\,\nabla^\mu\phi + f(\phi){R^2}_{GB}\right],$$

- Wormhole solutions with real scalar field and no need for exotic matter[7].
- They are traversable and may have a single or a double throat.
- Stability[8]?
- For phantom scalar field the system accepts the Ellis-Bronikov wormhole as a stealth solution[9] (See Nikos Chatzifotis' poster).

[7] P. Kanti, B. Kleihaus, and J. Kunz, Phys. Rev. Lett. **107** (2011) 271101.
G. Antoniou, A. B., P. Kanti, B. Kleihaus, and J. Kunz, Phys. Rev. D **101**, (2020) 024033.

[8] M. A. Cuyubamba, R. A. Konoplya and A. Zhidenko, Phys. Rev. D **98** (2018) no.4, 044040.
V. A. Rubakov, Theor. Math. Phys. **188** (2016) no.2, 1253-1258.
O. A. Evseev and O. I. Melichev, Phys. Rev. D **97** (2018) no.12, 124040.
S. Mironov, V. Rubakov and V. Volkova, Class. Quant. Grav. **36** (2019) no.13, 135008.
G. Franciolini, L. Hui, R. Penco, L. Santoni and E. Trincherini, JHEP **01** (2019), 221.

[9] A.B, N. Chatzifotis, C. Erices, and E. Papantonopoulos arXiv:2306.16768

The disformal transformation

A disformal transformation D depending on X takes a solution of Horndeski theory to a solution of beyond Horndeski theory.

$$g_{\mu\nu} = \bar{g}_{\mu\nu} - D(\bar{X})\,\partial_\mu\phi\,\partial_\nu\phi\,.$$

For a spherically symmetric solution $\bar{\phi} = \phi$, $\bar{h} = h$ and

$$f = \frac{\bar{f}}{W(\bar{X})}, \qquad X = \frac{\bar{X}}{W(\bar{X})}\,, \qquad W(\bar{X}) = 1 + 2D\bar{X}\,.$$

A homogeneous solution in Horndeski is transformed to a non-homogeneous in beyond Horndeski.

Wormhole solution

We start with a homogeneous black hole solution in Horndeski with horizon radius r_h i.e.$(h(r_h) = f(r_h) = 0)$.

- If $W(\bar{X})^{-1}$ is everywhere finite we get a non-homogeneous black hole solution.
- If $W(\bar{X})^{-1}$ has a root $W(\bar{X})^{-1} = 0|_{r=r_0}$ with $r_0 > r_h$ we get a wormhole $(f(r_0) = 0$ and $h(r_0) \neq 0)$.

We will apply the disformal transformation to the Lu-Pang black hole solution.
(See also N Chatzifotis, E. Papantonopoulos & C. Vlachos arXiv:2111.08773)

$$\bar{h}(r) = \bar{f}(r) = 1 + \frac{r^2}{2\alpha}\left(1 - \sqrt{1 + \frac{8\alpha M}{r^3}}\right), \quad \text{and} \quad \bar{\phi}' = \frac{\sqrt{\bar{h}} - 1}{r\sqrt{\bar{h}}},$$

$$\bar{X} = -\frac{1}{2}\,\bar{h}\bar{\phi}'^2 = -\frac{1}{2}\,\frac{(\sqrt{\bar{h}} - 1)^2}{r^2}\,.$$

Wormhole Solution

We use the transformation

$$W(\bar{X})^{-1} = 1 - b_1 \sqrt{-2\bar{X}} = 1 - \frac{b_1}{r}\left(1 - \sqrt{h}\right).$$

By setting $b_1 = r_0/\lambda$ we find

$$h(r) = \bar{h}(r), \quad f(r) = h(r)\left(1 - \frac{r_0}{\lambda r}\left(1 - \sqrt{h}\right)\right), \qquad \text{and} \qquad \phi' = \frac{\sqrt{h} - 1}{r\sqrt{h}}.$$

The nature of the compact object is determined from the roots of the metric functions:

$$f(r_0) = 0 \Longrightarrow h(r_0) = (1-\lambda)^2 \quad \begin{cases} 0 < \lambda < 1 \longrightarrow \text{Wormhole} \\ \lambda = 1 \longrightarrow \text{Black hole} \end{cases}$$

where

$$r_0 = \frac{M + \sqrt{M^2 - \alpha\lambda^3\,(2-\lambda)^3}}{\lambda(2-\lambda)}, \qquad \text{and} \qquad \alpha \leq \frac{M^2}{\lambda^3\,(2-\lambda)^3}.$$

At infinity we find:

$$h(r) = 1 - \frac{2M}{r} + \frac{4\alpha M^2}{r^4} + \mathcal{O}\left(r^{-5}\right), \qquad f(r) = 1 + \frac{2M}{r} + \frac{b_1 M + 4M^2}{r^2} + \mathcal{O}\left(r^{-3}\right),$$

$$\phi'(r) = -\frac{M}{r^2} + \mathcal{O}\left(r^{-3}\right).$$

The coordinate r covers only a half of the wormhole spacetime.

We may describe the solutions in both asymptotically flat regions using the coordinate transformation $r^2 = l^2 + r_0^2$ with $l \in (-\infty, +\infty)$.

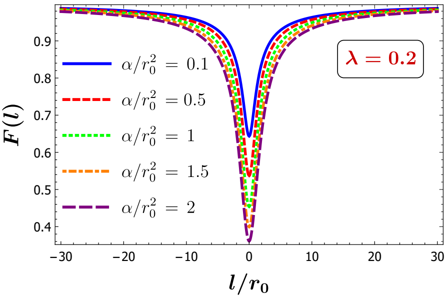
$$ds^2 = -H(l)dt^2 + \frac{1}{F(l)}dl^2 + (l^2 + r_0^2)d\Omega^2,$$

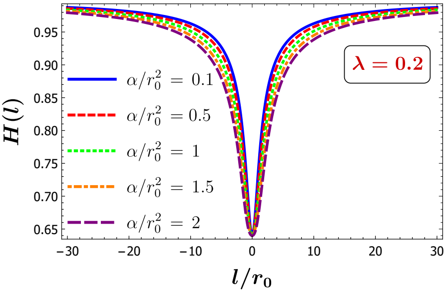
where

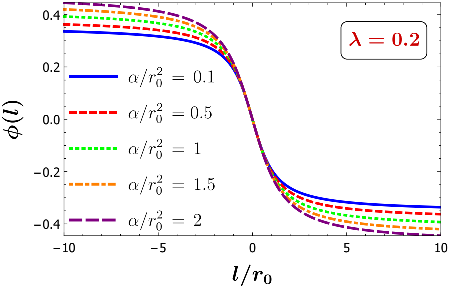
$$H(l) = h(r(l)), \quad \text{and} \quad F(l) = \frac{f(r(l))\,(l^2 + r_0^2)}{l^2}.$$

The new metric functions are continuous at the throat

$$H(l) = h_0 + h_1 l^2 + \mathcal{O}(l^4), \qquad F(l) = f_0 + f_1 l^2 + \mathcal{O}(l^4), \qquad \phi(l) = \phi_0 + \phi_1 l + \mathcal{O}(l^3).$$



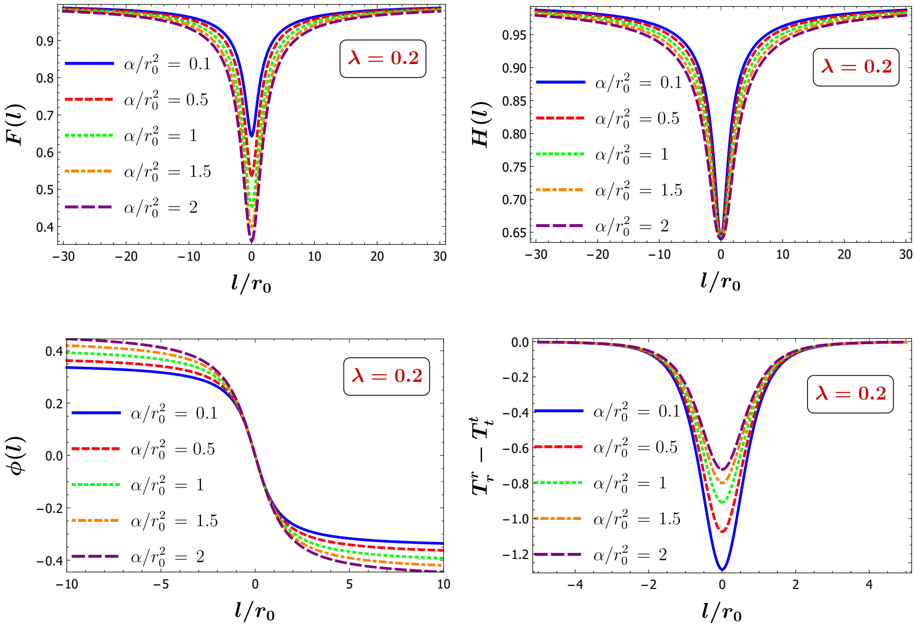




For a spherical symmetric spacetime the null energy condition (NEC) has the form

$$-T^t_t + T^r_r \geq 0, \quad \text{and} \quad -T^t_t + T^\theta_\theta \geq 0,$$

where $T_{\mu\nu}$ is the effective energy momentum tensor due to the scalar field defined from the equation $G_{\mu\nu} = T_{\mu\nu}$.



$F(l)$
$H(l)$
$\phi(l)$
$T^r_r - T^t_t$
l/r_0
$\lambda = 0.2$
$\alpha/r_0^2 = 0.1$
$\alpha/r_0^2 = 0.5$
$\alpha/r_0^2 = 1$
$\alpha/r_0^2 = 1.5$
$\alpha/r_0^2 = 2$

Conclusions

- In the case of Parity preserving theories the set of field equations is integrable and may lead to a variety of black-hole solutions.
- In the case of non-parity preserving theories, although intergrability seems to be lost, we developed a technique for a subclass of theories that allow us to solve the field equations.
- Regular analytic wormhole solutions were found for a class of the beyond Horndeski theories.

Thank You!

Transformation of the coupling functions

The coupling functions are transformed as

$$G_2 = \frac{\bar{G}_2}{(1+2\bar{X}D)^{1/2}}\,, \qquad G_{3X} = \bar{G}_{3\bar{X}}\frac{(1+2\bar{X}D)^{5/2}}{1-2\bar{X}^2 D_{\bar{X}}}\,,$$

$$G_4 = \frac{\bar{G}_4}{(1+2\bar{X}D)^{1/2}}\,, \qquad G_{5X} = \frac{\bar{G}_{5\bar{X}}(1+2\bar{X}D)^{5/2}}{1-2\bar{X}^2 D_{\bar{X}}}\,,$$

$$F_4 = (\bar{G}_4 - 2\bar{X}\bar{G}_{4\bar{X}})\frac{D_{\bar{X}}(1+2\bar{X}D)^{5/2}}{2(1-2\bar{X}^2 D_{\bar{X}})}\,, \qquad F_5 = \bar{X}\bar{G}_{5\bar{X}}\frac{D_{\bar{X}}(1+2\bar{X}D)^{7/2}}{6(1-2\bar{X}^2 D_{\bar{X}})}$$

and

$$Z = (1+2\bar{X}D)^{1/2}\bar{Z}\,, \quad Y = (1+2\bar{X}D)^{1/2}\bar{Y}\,, \quad \mathcal{B} = (1+2\bar{X}D)^{1/2}\bar{\mathcal{B}}\,,$$

$$\mathcal{A} = \frac{(1+2\bar{X}D)^{5/2}}{1-2\bar{X}^2 D_{\bar{X}}}\bar{\mathcal{A}} + 4\frac{(1+2\bar{X}D)^{3/2}}{1-2\bar{X}^2 D_{\bar{X}}}(D+\bar{X}D_{\bar{X}})\,\bar{\mathcal{B}}\,.$$

$$G_2 = \frac{4\ 2^{3/4}\alpha y^4}{\sqrt{\frac{1}{\sqrt{2}-2b_1 y}}}, \qquad G_{3X} = \frac{16\sqrt[4]{2}\alpha\sqrt{\frac{1}{\sqrt{2}-2b_1 y}}}{3\sqrt{2}b_1 y - 2},$$

$$G_4 = \frac{1-4\alpha y^2}{\sqrt[4]{2}\sqrt{\frac{1}{\sqrt{2}-2b_1 y}}}, \qquad G_{5X} = -\frac{8\sqrt[4]{2}\alpha\sqrt{\frac{1}{\sqrt{2}-2b_1 y}}}{y^2\left(3\sqrt{2}b_1 y - 2\right)},$$

$$F_4 = \frac{b_1\left(\sqrt{2}-4b_1 y\right)\left(\frac{1}{\sqrt{2}-2b_1 y}\right)^{5/2}\left(4\alpha y^2+1\right)}{2^{3/4}y^3\left(3\sqrt{2}b_1 y - 2\right)},$$

$$F_5 = \frac{2\ 2^{3/4}\alpha b_1\left(\sqrt{2}-4b_1 y\right)\left(\frac{1}{\sqrt{2}-2b_1 y}\right)^{7/2}}{3y^3\left(2-3\sqrt{2}b_1 y\right)},$$

where,

$$X = y^2(-1+\sqrt{2}b_1 y).$$