Gravitino Dark Matter

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Outline

- DM scenarios in SUSY models and CMSSM
- Gravitino DM models, calculation of gravitino thermal density
- Discussion on SUGRA models that can produce PBH
- Summary

Neutralino (χ) as DM

- The most well studied SUSY DM candidate particle, both theoretically and experimentally
- Motivated by the so-called WIMP miracle, it has been studied in various SUSY models: CMSSM/mSUGRA, NUHM, pMSSM etc
- $\Omega h^2 \sim 0.12$ is achieved in particular regions of the parameter space: coannihilation regions (stop, stau, gauginos), focus point region, A-funnel region
- Possible tension in these regions between direct and indirect DM constraints

Recent study

- **Revisit the χ DM regions in CMSSM, locate the strips**
- **Use indirect constraints from neutrino fluxes from Sun (IceCube data) and gammas from dSph (Fermi-LAT data)**

Soft parameters for Constrained Minimal SUSY SM (CMSSM)

❶ m_0
Common mass for sfermion: sleptons, squarks

❷ $m_{1/2}$
Common mass for gauginos, M_1, M_2, M_3

❸ $\tan\beta$
The ratio of the vev's of two Higgs fields, $\tan\beta = \langle H_2^0 \rangle / \langle H_1^0 \rangle$

❹ A_0
Trilinear parameter, affects the sfermion masses and couplings

❺ $\mathrm{sign}(\mu)$
sign of Higgs mixing parameter μ

Supersymmetric particles

SM

fermions (spin 1/2)

u, d, c, s, t, b

$e, \mu, \tau, \nu_e, \nu_\mu, \nu_\tau$

gauge bosons (spin 1)

$\gamma, Z, W^\pm, g$

Higgs boson (spin 0)

h

sfermions (spin 0)

$\tilde{u}, \tilde{d}, \tilde{c}, \tilde{s}, \tilde{t}, \tilde{b}$

NSP

$\tilde{e}, \tilde{\mu}, \tilde{\tau}, \tilde{\nu}_e, \tilde{\nu}_\mu, \tilde{\nu}_\tau$

gauginos (spin 1/2)

$\tilde{\gamma}, \tilde{Z}, \tilde{W}^\pm, \tilde{g}$

Higgsinos (spin 1/2)

$\tilde{H}_{1,2}$

more Higgs bosons (spin 0)

$H, A, H^\pm$

χ LSP

SUSY and Higgs mass

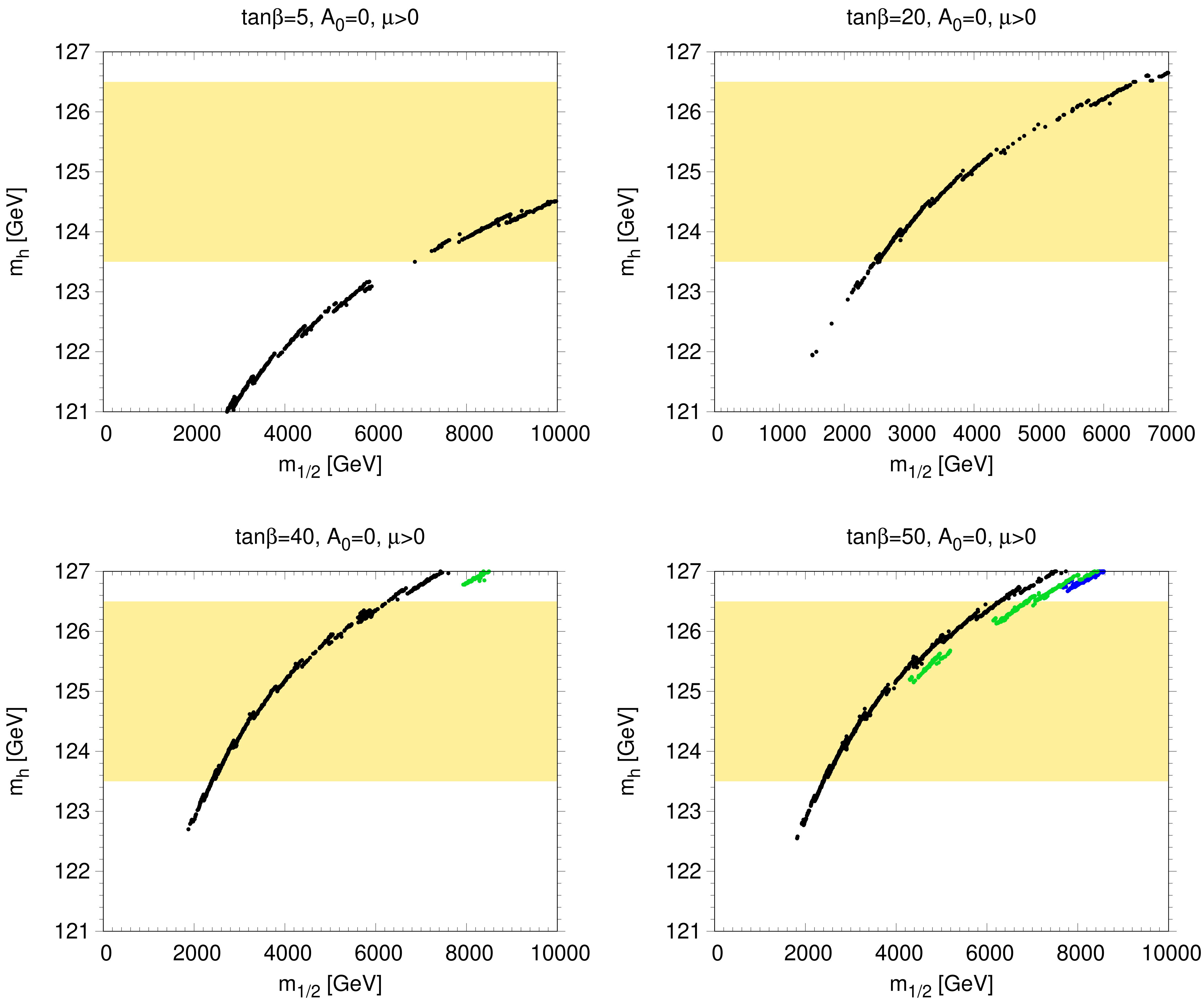
$$m_h = \sqrt{\frac{1}{2}[M_A^2 + M_Z^2 - \sqrt{(M_A^2 + M_Z^2)^2 - 4M_A^2 M_Z^2 \cos^2\beta}]} < M_Z$$

$$\tan\beta = \frac{v_2}{v_1}$$

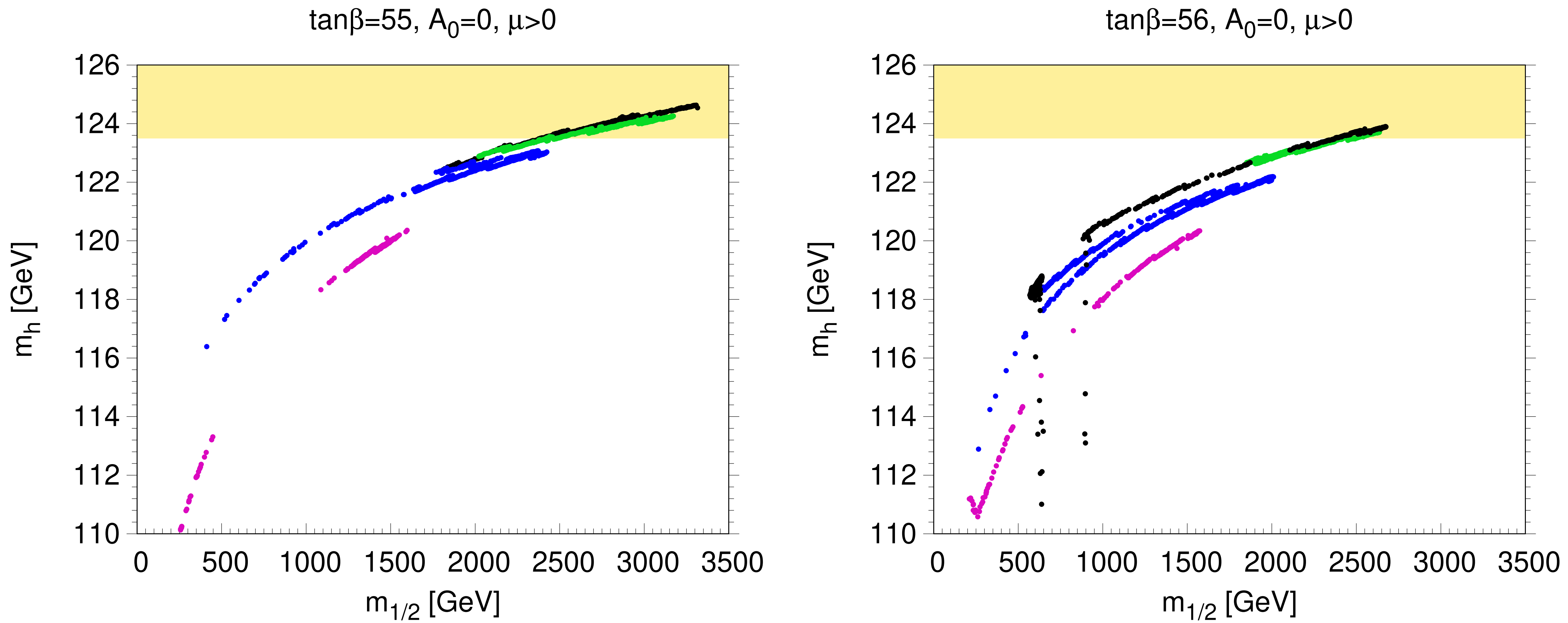
$$\Delta m_h^2 \simeq \frac{3}{4\pi^2}\frac{m_t^4}{v^2}\left[\log\left(\frac{m_{\tilde{t}}^2}{m_t^2}\right) + \frac{X_t^2}{m_{\tilde{t}}^2} - \frac{X_t^4}{12 m_{\tilde{t}}^4}\right]$$
$$- \frac{3}{48\pi^2}\frac{m_b^4}{v^2}\,\frac{t_\beta^4}{(1+\epsilon_b t_\beta)^4}\,\frac{\mu^4}{m_{\tilde{b}}^4}$$
$$- \frac{1}{48\pi^2}\frac{m_\tau^4}{v^2}\,\frac{t_\beta^4}{(1+\epsilon_\ell t_\beta)^4}\,\frac{\mu^4}{m_{\tilde{\tau}}^4}$$

$$X_t = A_t + \mu/\tan\beta \approx A_t$$

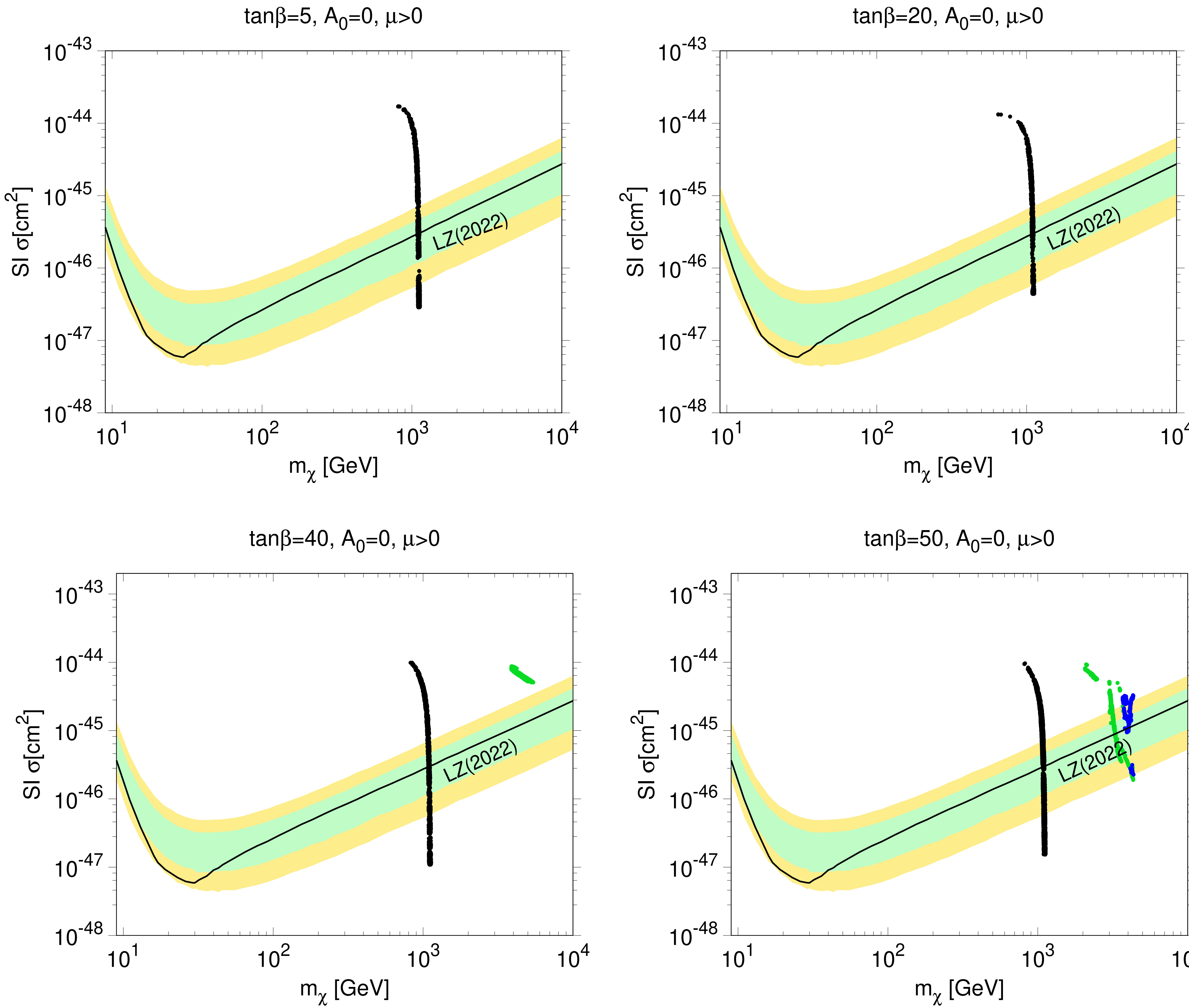
- **Higgs Mass = 125 GeV => Heavy Spectrum**



[Ellis, Olive, VCS, Stamou (2023)]



tanβ=55, A0=0, μ>0
tanβ=56, A0=0, μ>0
mh [GeV]
m1/2 [GeV]
126
124
122
120
118
116
114
112
110
0
500
1000
1500
2000
2500
3000
3500



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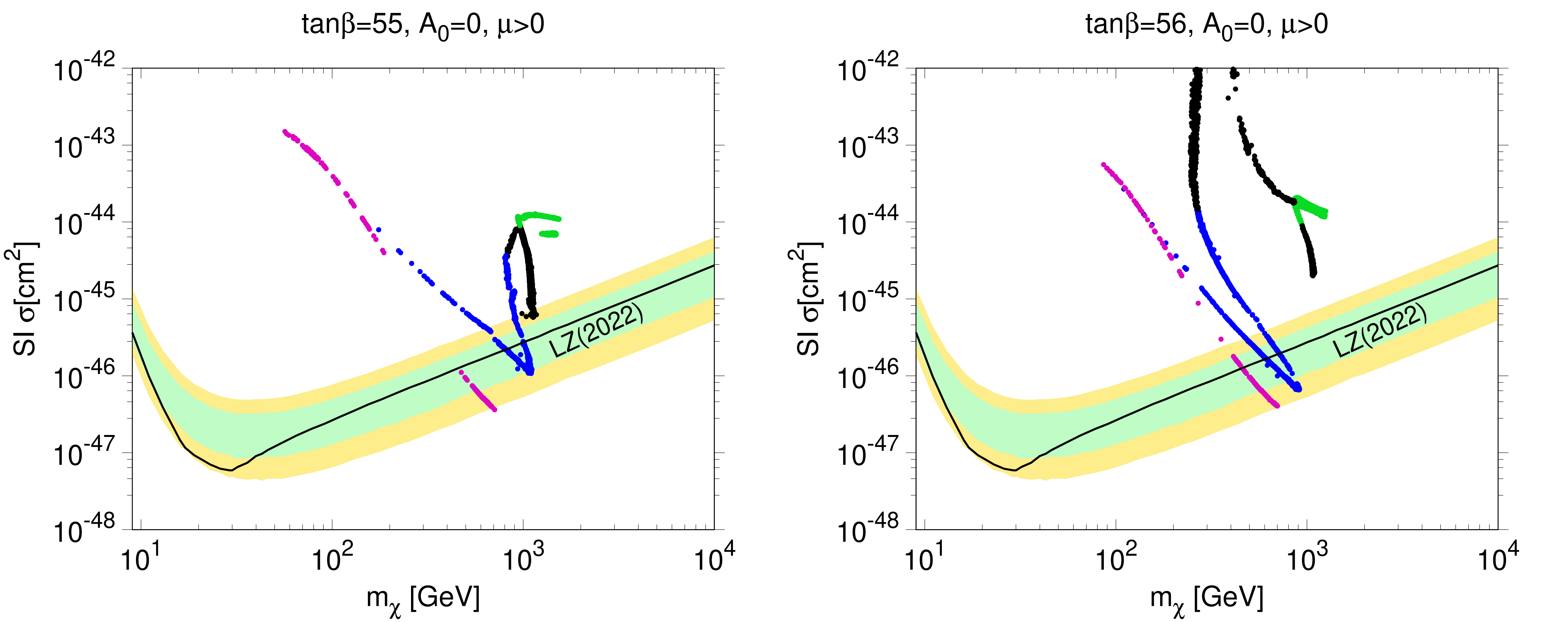
tanβ=20, A0=0, μ>0
tanβ=40, A0=0, μ>0
tanβ=50, A0=0, μ>0
SI σ[cm2]
mχ [GeV]
LZ(2022)
10-43
10-44
10-45
10-46
10-47
10-48
101
102
103
104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tanβ=56, A0=0, μ>0
SI σ[cm2]
mχ [GeV]
LZ(2022)
10-42
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10-44
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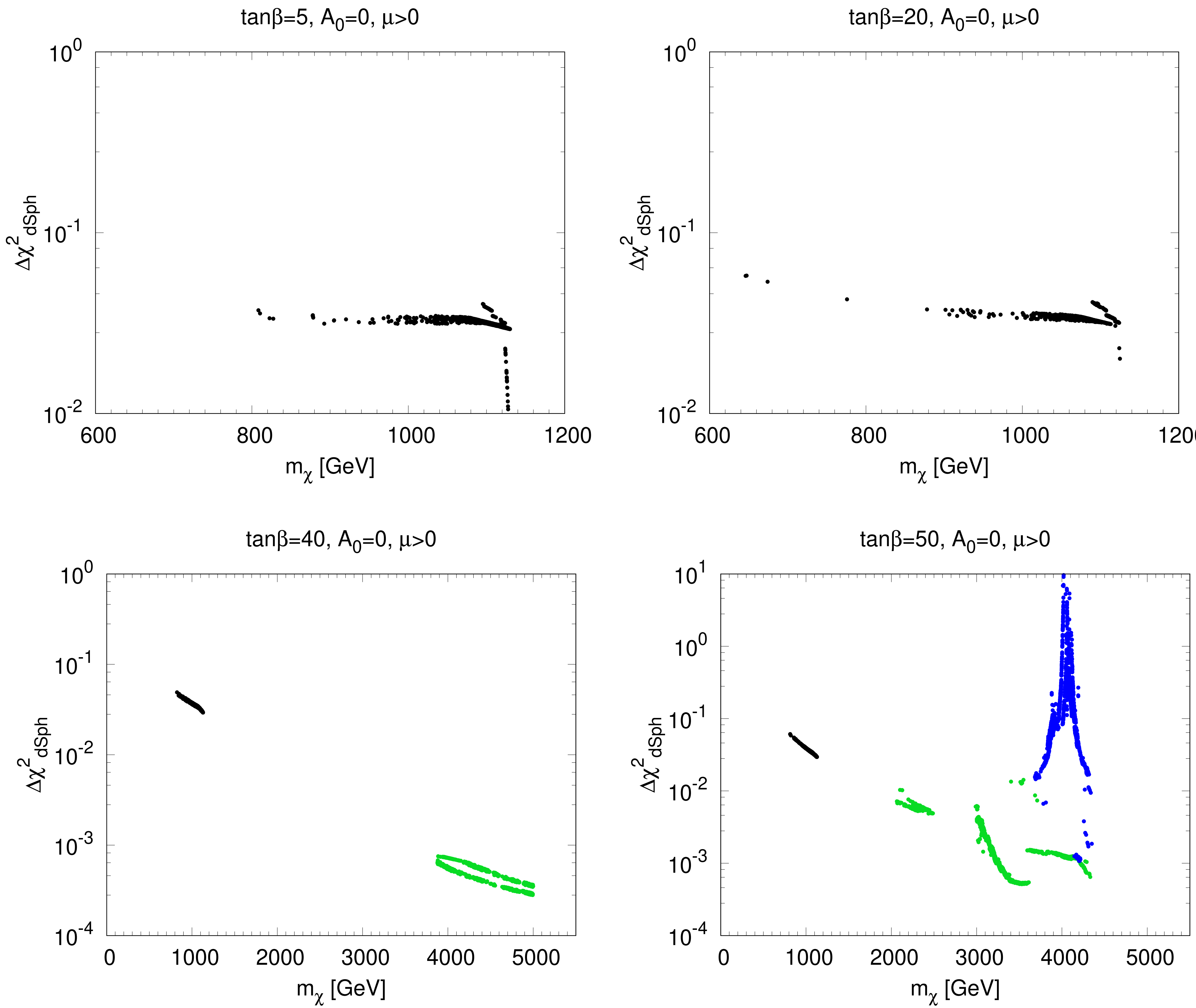
101
102
103
104



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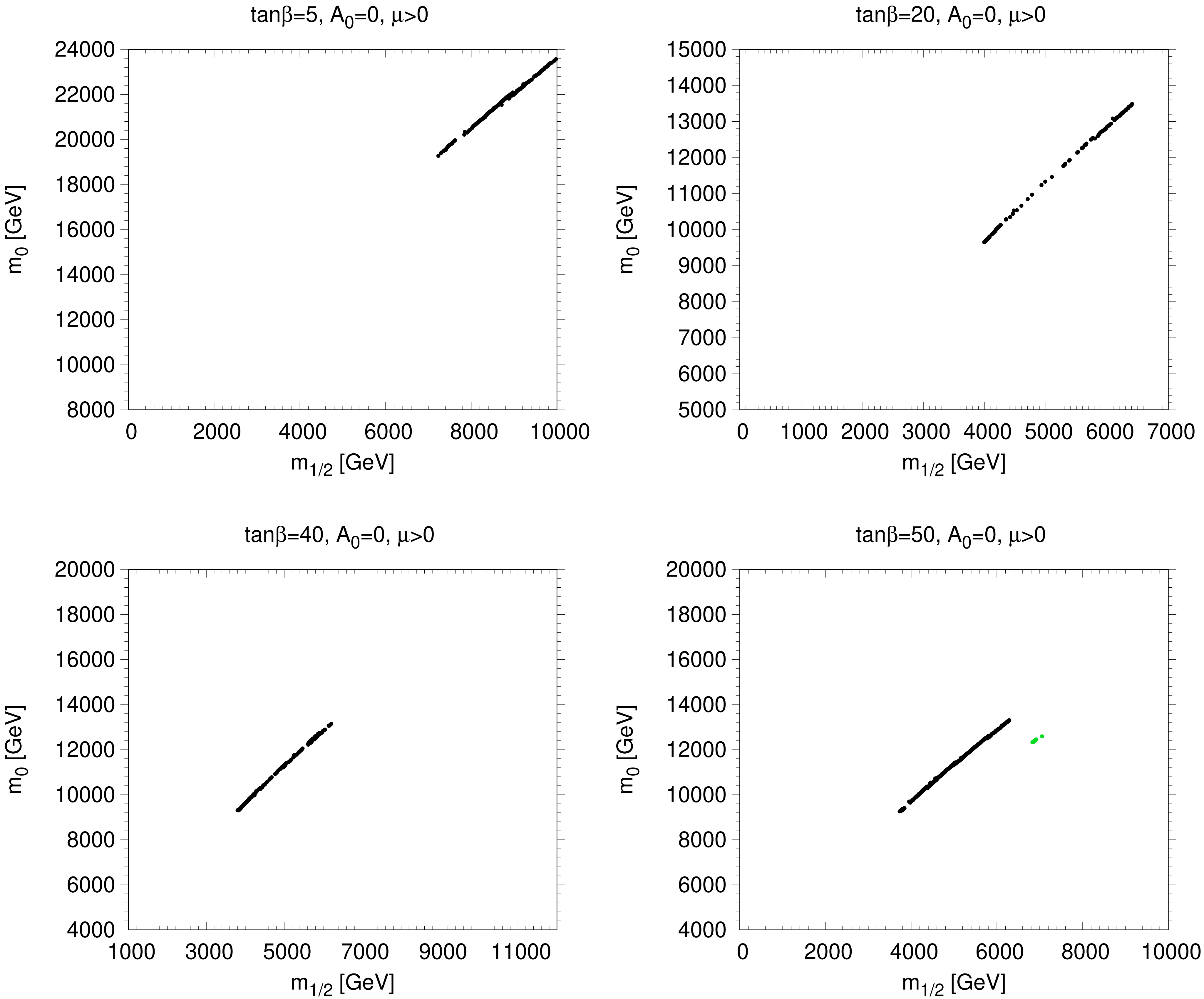
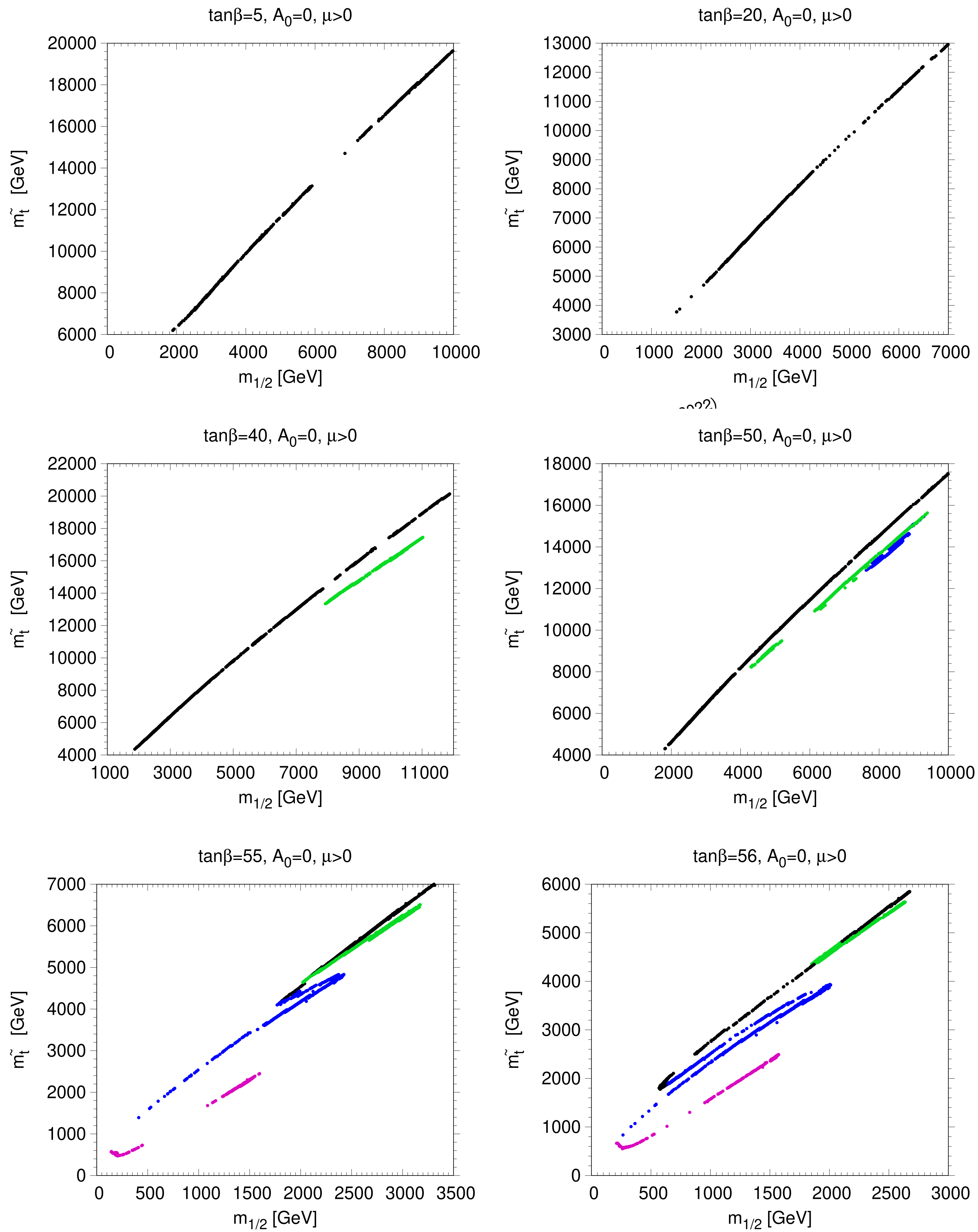
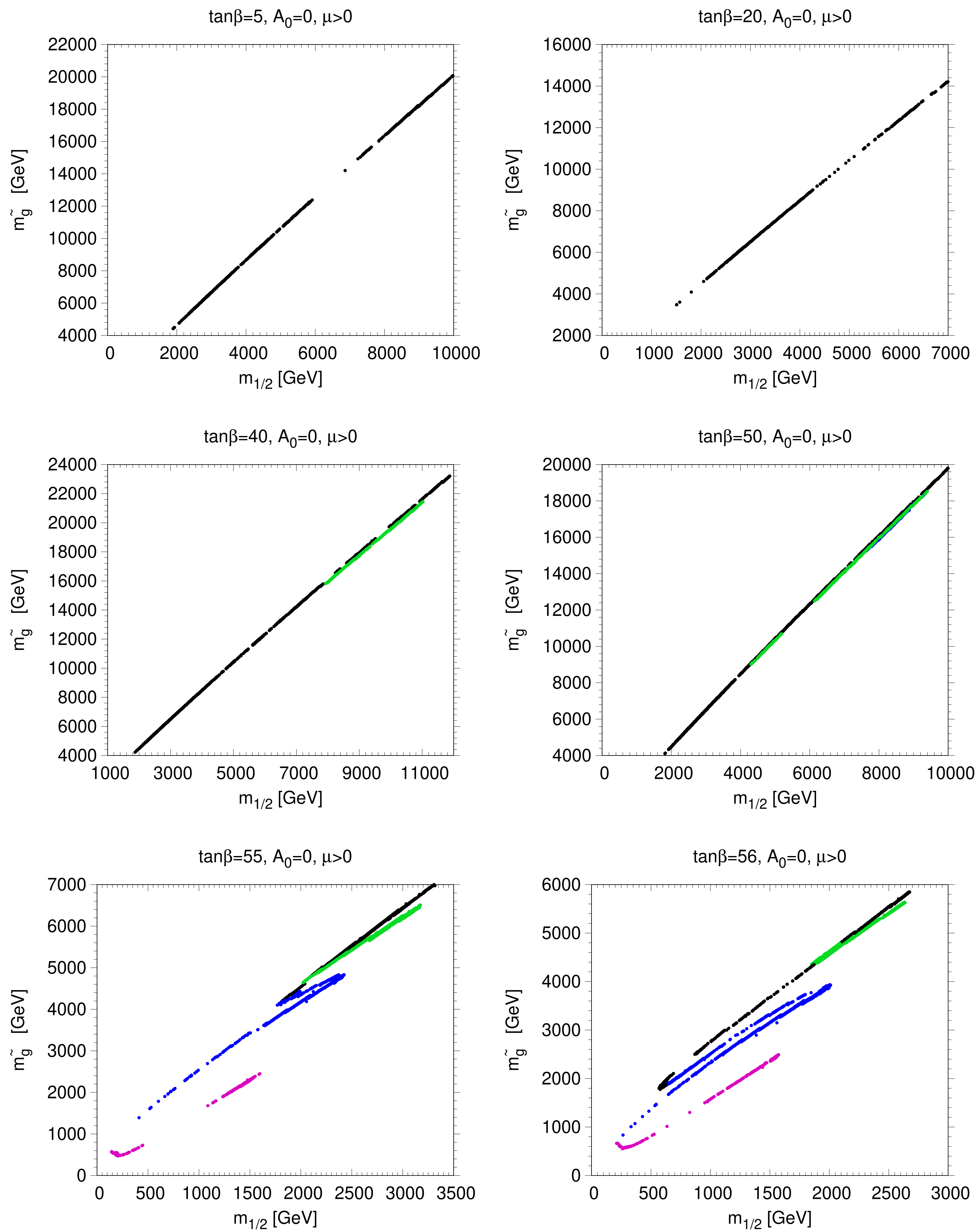
tanβ=20, A0=0, μ>0
tanβ=40, A0=0, μ>0
tanβ=50, A0=0, μ>0
Δχ2dSph
mχ [GeV]



- [Ellis, Olive, VCS, Stamou (2023)]



tanβ=5, A0=0, μ>0
m~t [GeV]
m1/2 [GeV]
tanβ=20, A0=0, μ>0
m~t [GeV]
m1/2 [GeV]
tanβ=40, A0=0, μ>0
m~t [GeV]
m1/2 [GeV]
tanβ=50, A0=0, μ>0
m~t [GeV]
m1/2 [GeV]
tanβ=55, A0=0, μ>0
m~t [GeV]
m1/2 [GeV]
tanβ=56, A0=0, μ>0
m~t [GeV]
m1/2 [GeV]



tanβ=5, A0=0, μ>0
tanβ=20, A0=0, μ>0
tanβ=40, A0=0, μ>0
tanβ=50, A0=0, μ>0
tanβ=55, A0=0, μ>0
tanβ=56, A0=0, μ>0
m~g [GeV]
m1/2 [GeV]

Recap for neutralinos

- Neutralino DM is *well* studied in CMSSM
- The combination of Higgs mass bound and direct DM searches can be fulfilled in large parts of the parameter space, BUT well beyond the LHC reach
- The indirect searches gamma and neutrino fluxes do not constraint the parameter space

Gravitino as DM

- **Gravitino is the s=3/2 superpartner of graviton. Naturally is in the spectrum of any SUGRA model** [Ellis, Hagelin, Nanopoulos, Olive, Srednicki (1983), Khlopov, Linde (1984)]

- **The "classic" freeze-in DM candidate particle**

- **Naturally escapes all the direct and indirect DM searches**

- **Can be produced non-thermally: (i) inflaton decays** [Giudice, Riotto, Tkachev (1999); Kallosh, Kofman, Linde, Van Proeyen (2000); Nilles, Peloso, Sorbo (2001), Endo, Kawasaki, Takahashi, Yanagida (2006)] **(ii) decays from unstable particles, eg NLSP decays in GDM models** [Cyburt, Ellis, Field, Olive, VSC (2006); Kawasaki, Kohri, Moroi, Yotsuyanagi (2008)]

- **In the later case the BBN constraints should be applied** [Cyburt, Ellis, Field, Luo, Olive, VSC (2012)]

- **In any case the thermal gravitino production rate is vital to apply cosmological constraints**

Background of the calculation

- **Effective theory of light gravitinos, only 1/2 goldstino components**

  [Ellis, Kim, Nanopoulos (1984); Moroi, Murayama, Yamagushi, Kawasaki (1993,1994)]

- **Use of the Braaten, Pisarksi, Yan method including 3/2 components**

  [Ellis, Nanopoulos, Olive, Rey (1996), Bolz, Buchmuller, Plumacher, Brandenburger (1998,2001); Pedlar, Steffen (2007)]

- **1-loop calculation beyond the HTL approximation** [Rychkov, Strumia (2007)]

- **Our calculation: error corrections and proper parametrization of the result** [Eberl, Gialamas, VCS (2021)]

- **More improvements in the calculation will come** [Eberl, Gialamas, VCS (2022) to appear]

The setup of the calculation

The Braaten-Yuan prescription

[Braaten, Pisarski, Yuan (1990,1991)]

$$\gamma = \gamma|_{\rm hard}^{k^*<k} + \gamma|_{\rm soft}^{k^*>k}$$

where $gT \ll k^* \ll T$ assuming $g \ll 1$

Hard part is calculated from squared matrix elements

$$|\mathcal{M}(a\, b \to c\, \widetilde{G})|^2$$

Soft part is calculated from Imaginary part of the gravitino self-energy

$$\gamma|_{\rm hard}^{k^*<k} = A_{\rm hard} + B \ln\left(\frac{T}{k^*}\right)$$ and $$\gamma|_{\rm soft}^{k^*>k} = A_{\rm soft} + B' \ln\left(\frac{k^*}{m_{\rm thermal}}\right)$$

Thus

$$\gamma_{\rm BY} = \frac{3\zeta(3)}{16\pi^3}\,\frac{T^6}{M_{\rm P}^2}\sum_{N=1}^{3} c'_N\, g_N^2 \left(1+\frac{m_{\lambda_N}^2}{3m_{3/2}^2}\right)\ln\left(\frac{k'_N}{g_N}\right)$$

$c'_N = (11, 27, 72)$, $k'_N = (1.266, 1.312, 1.271)$ [Pradler, Steffen (2007)]

Analytical result, but valid only for $g \ll 1$

where $\gamma\big|_{\rm soft}$ is calculated in the Hard Thermal Loop (HTL) approx

The condition $g(T) \ll 1$ is not satisfied in the whole temperature range

especially if $g = g_3$

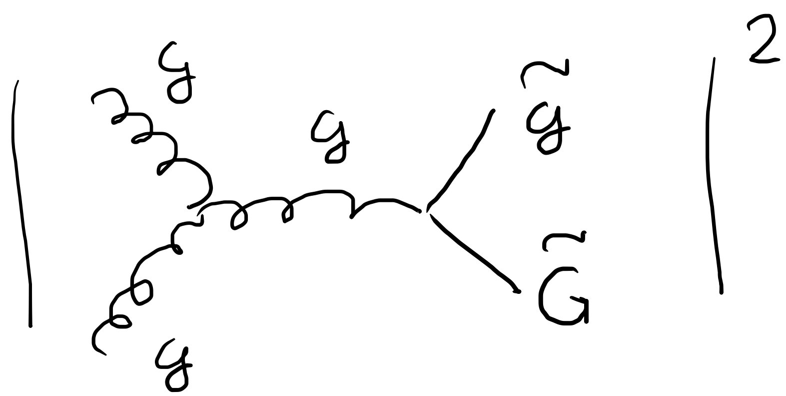
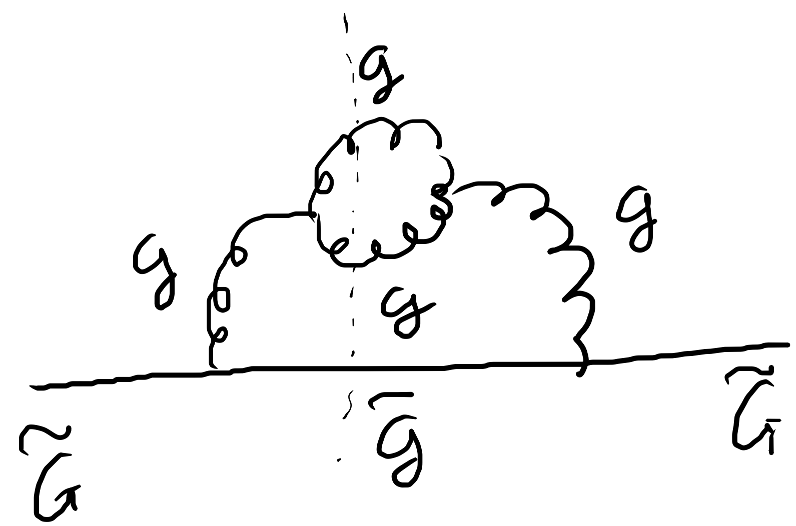
Beyond the HTL approx

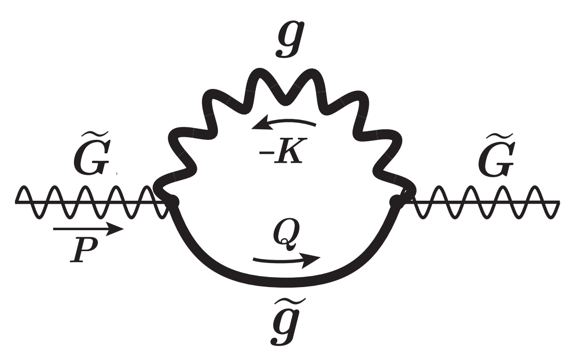
- **Calculate the full 1-loop gravitino self-energy beyond HTL approximation**

- **Calculate the so-called subtracted part of the $|\mathcal{M}|^2$** [Rychkov, Strumia (2007)]

The subtracted part of the squared amplitude is this that cannot be part of the gravitino self-energy

For example if *X:* $gg \to \tilde{g}\widetilde{G}$

$|\mathcal{M}_{X,s}|^2$ =  related to 

which is part of 

(D-graph)

where thick lines denote resumed thermal propagators

Thus $\gamma_{3/2} = \gamma_{\rm sub} + \gamma_{\rm D} + \gamma_{\rm top}$

X	process	$\lvert\mathcal{M}_{X,\mathrm{full}}\rvert^2$	$\lvert\mathcal{M}_{X,\mathrm{sub}}\rvert^2$
A	$gg \to \tilde{g}\widetilde{G}$	$4C_3(s+2t+2t^2/s)$	$-2sC_3$
B	$g\tilde{g} \to g\widetilde{G}$	$-4C_3(t+2s+2s^2/t)$	$2tC_3$
C	$\tilde{q}g \to q\widetilde{G}$	$2sC_3'$	0
D	$gq \to \tilde{q}\widetilde{G}$	$-2tC_3'$	0
E	$\tilde{q}q \to g\widetilde{G}$	$-2tC_3'$	0
F	$\tilde{g}\tilde{g} \to \tilde{g}\widetilde{G}$	$8C_3(s^2+t^2+u^2)^2/(stu)$	0
G	$q\tilde{g} \to q\widetilde{G}$	$-4C_3'(s+s^2/t)$	0
H	$\tilde{q}\tilde{g} \to \tilde{q}\widetilde{G}$	$-2C_3'(t+2s+2s^2/t)$	0
I	$q\tilde{q} \to \tilde{g}\widetilde{G}$	$-4C_3'(t+t^2/s)$	0
J	$\tilde{q}\tilde{q} \to \tilde{g}\widetilde{G}$	$2C_3'(s+2t+2t^2/s)$	0

Squared matrix elements for gravitino production in $SU(3)_c$ in terms of $g_3^2\, Y_3/M_{\mathrm{P}}^2$

$Y_3 = 1 + m_{\tilde{g}}^2/(3m_{3/2}^2)$, $C_3 = 24$ and $C_3' = 48$

$$|\mathcal{M}_{X,\mathrm{full}}|^2 = |\mathcal{M}_{X,s} + \mathcal{M}_{X,t} + \mathcal{M}_{X,u} + \mathcal{M}_{X,x}|^2$$

$$|\mathcal{M}_{X,D}|^2 = |\mathcal{M}_{X,s}|^2 + |\mathcal{M}_{X,t}|^2 + |\mathcal{M}_{X,u}|^2$$

$$|\mathcal{M}_{X,\mathrm{sub}}|^2 = |\mathcal{M}_{X,\mathrm{full}}|^2 - |\mathcal{M}_{X,D}|^2$$

$\gamma_{\rm sub}$

$$\gamma = \frac{1}{(2\pi)^8} \int \frac{\mathrm{d}^3\mathbf{p}_a}{2E_a} \frac{\mathrm{d}^3\mathbf{p}_b}{2E_b} \frac{\mathrm{d}^3\mathbf{p}_c}{2E_c} \frac{\mathrm{d}^3\mathbf{p}_{\widetilde{G}}}{2E_{\widetilde{G}}} \, |\mathcal{M}|^2 \, f_a \, f_b \, (1 \pm f_c) \times \delta^4(P_a + P_b - P_c - P_{\widetilde{G}})$$

$$f_{B|F} = \frac{1}{e^{\frac{E}{T}} \mp 1}$$

$$|\mathcal{M}_{A,\rm sub}|^2 + |\mathcal{M}_{B,\rm sub}|^2 = \frac{g_N^2}{M_{\rm P}^2} \left(1 + \frac{m_{\lambda_N}^2}{3m_{3/2}^2}\right) C_N(-s+2t)$$

as taken from the Table with the amplitudes

Performing numerical integration

$$\gamma_{\rm sub} = \frac{T^6}{M_{\rm P}^2} \sum_{N=1}^{3} g_N^2 \left(1 + \frac{m_{\lambda_N}^2}{3m_{3/2}^2}\right) C_N \left(-\mathcal{C}^s_{\rm BBF} + 2\,\mathcal{C}^t_{\rm BFB}\right)$$

$$\mathcal{C}^s_{\rm BBF} = 0.25957 \times 10^{-3}$$

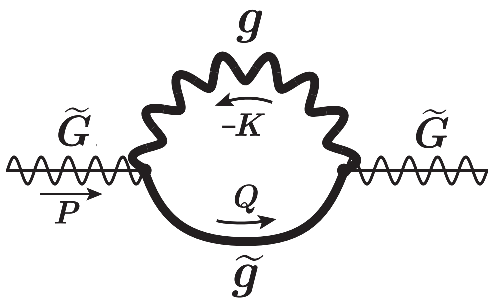
$$\mathcal{C}^t_{\rm BFB} = -0.13286 \times 10^{-3} \, .$$

γ_{D}

$$\Pi^{<}(P) = \frac{1}{16M_P^2}\sum_{N=1}^{3} n_N \left(1+\frac{m_{\lambda_N}^2}{3m_{3/2}^2}\right)\int \frac{\mathrm{d}^4K}{(2\pi)^4}\,\mathrm{Tr}\Big[\not{P}[\not{K},\gamma^\mu]\,{}^*S^{<}(Q)[\not{K},\gamma^\nu]\,{}^*D^{<}_{\mu\nu}(K)\Big]$$

$${}^*S^{<}(Q) = \frac{f_F(q_0)}{2}\left[(\gamma_0 - \boldsymbol{\gamma}\cdot\mathbf{q}/q)\,\rho_+(Q) + (\gamma_0 + \boldsymbol{\gamma}\cdot\mathbf{q}/q)\,\rho_-(Q)\right]$$

$${}^*D^{<}_{\mu\nu}(K) = f_B(k_0)\left[\Pi^T_{\mu\nu}\,\rho_T(K) + \Pi^L_{\mu\nu}\frac{k^2}{K^2}\,\rho_L(K) + \xi\frac{K_\mu K_\nu}{K^4}\right]$$



$$\gamma_D = \int \frac{\mathrm{d}^3\mathbf{p}}{2p_0(2\pi)^3}\,\Pi^{<}(p)$$

$$\gamma_{\mathrm{D}} = \frac{1}{4(2\pi)^5 M_{\mathrm{P}}^2}\sum_{N=1}^{3} n_N\left(1+\frac{m_{\lambda_N}^2}{3m_{3/2}^2}\right)\int_0^\infty \mathrm{d}p\int_{-\infty}^{\infty}\mathrm{d}k_0\int_0^\infty \mathrm{d}k\int_{|k-p|}^{k+p}\mathrm{d}q\; k\, f_B(k_0)\, f_F(q_0)$$
$$\times\Big[\rho_L(K)\,\rho_-(Q)\,(p-q)^2\big((p+q)^2-k^2\big) + \rho_L(K)\,\rho_+(Q)\,(p+q)^2\big(k^2-(p-q)^2\big)$$
$$+\,\rho_T(K)\,\rho_-(Q)\,\big(k^2-(p-q)^2\big)\Big(\big(1+k_0^2/k^2\big)\big(k^2+(p+q)^2\big)-4k_0(p+q)\Big)$$
$$+\,\rho_T(K)\,\rho_+(Q)\,\big((p+q)^2-k^2\big)\Big(\big(1+k_0^2/k^2\big)\big(k^2+(p-q)^2\big)-4k_0(p-q)\Big)\Big],$$

γ_{top}

$$\gamma_{\mathrm{top}} = \frac{T^6}{M_{\mathrm{P}}^2}\,72\,\mathcal{C}^s_{\mathrm{BBF}}\,\lambda_t^2\left(1+\frac{A_t^2}{3m_{3/2}^2}\right) \qquad \mathcal{C}^s_{\mathrm{BBF}} = 0.25957\times 10^{-3}$$

$$\rho_{L,T}(K) = 2\pi\Big\{Z_{L,T}(k)\Big[\delta(k_0 - \omega_{L,T}(k)) - \delta(k_0 + \omega_{L,T}(k))\Big] + \rho^{\rm cont}_{L,T}(K)\Big\}$$

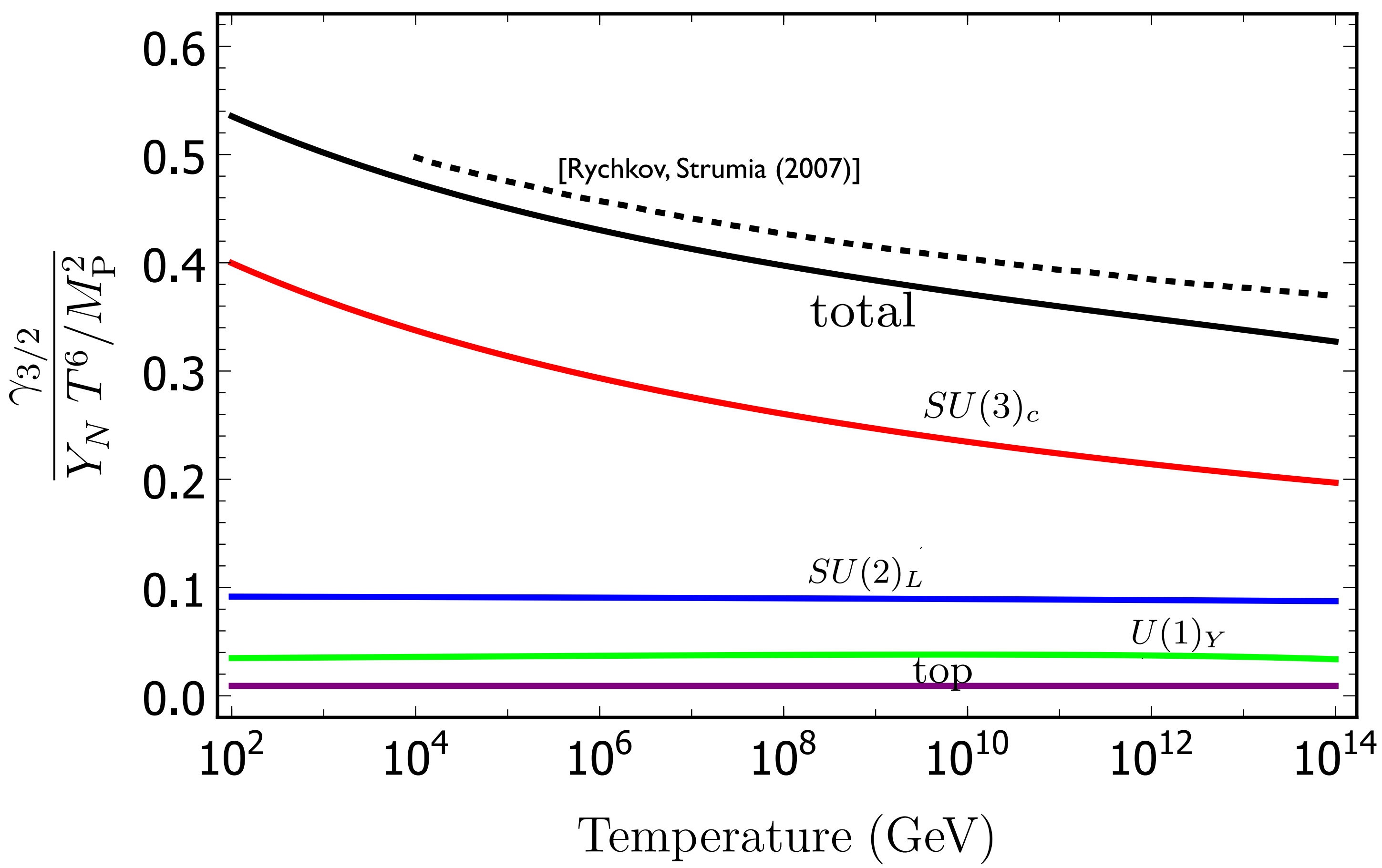
$$\rho_{\pm}(Q) = 2\pi\Big\{Z_{\pm}(q)\delta(q_0 - \omega \pm (q)) + Z_{\mp}(q)\delta(q_0 + \omega_{\mp}(q)) + \rho^{\rm cont}_{\pm}(Q)\Big\}\,.$$

$$Z_L(k) = \frac{\omega_L(k)(\omega_L^2(k) - k^2)}{k^2(k^2 + 2m^2 - \omega_L^2(k))}\ ,\ Z_T(k) = \frac{\omega_T(k)(\omega_T^2(k) - k^2)}{2m^2\omega_T^2(k) - (\omega_T^2(k) - k^2)^2}\ ,\ Z_{\pm}(q) = \frac{\omega_{\pm}(q)^2 - q^2}{2m_f^2}$$

Result and cosmological consequences

$$\gamma_{\rm sub}+\gamma_{\rm D} = \frac{3\zeta(3)}{16\pi^3}\,\frac{T^6}{M_{\rm P}^2}\sum_{N=1}^{3} c_N\, g_N^2 \left(1+\frac{m_{\lambda_N}^2}{3m_{3/2}^2}\right)\ln\left(\frac{k_N}{g_N}\right)$$

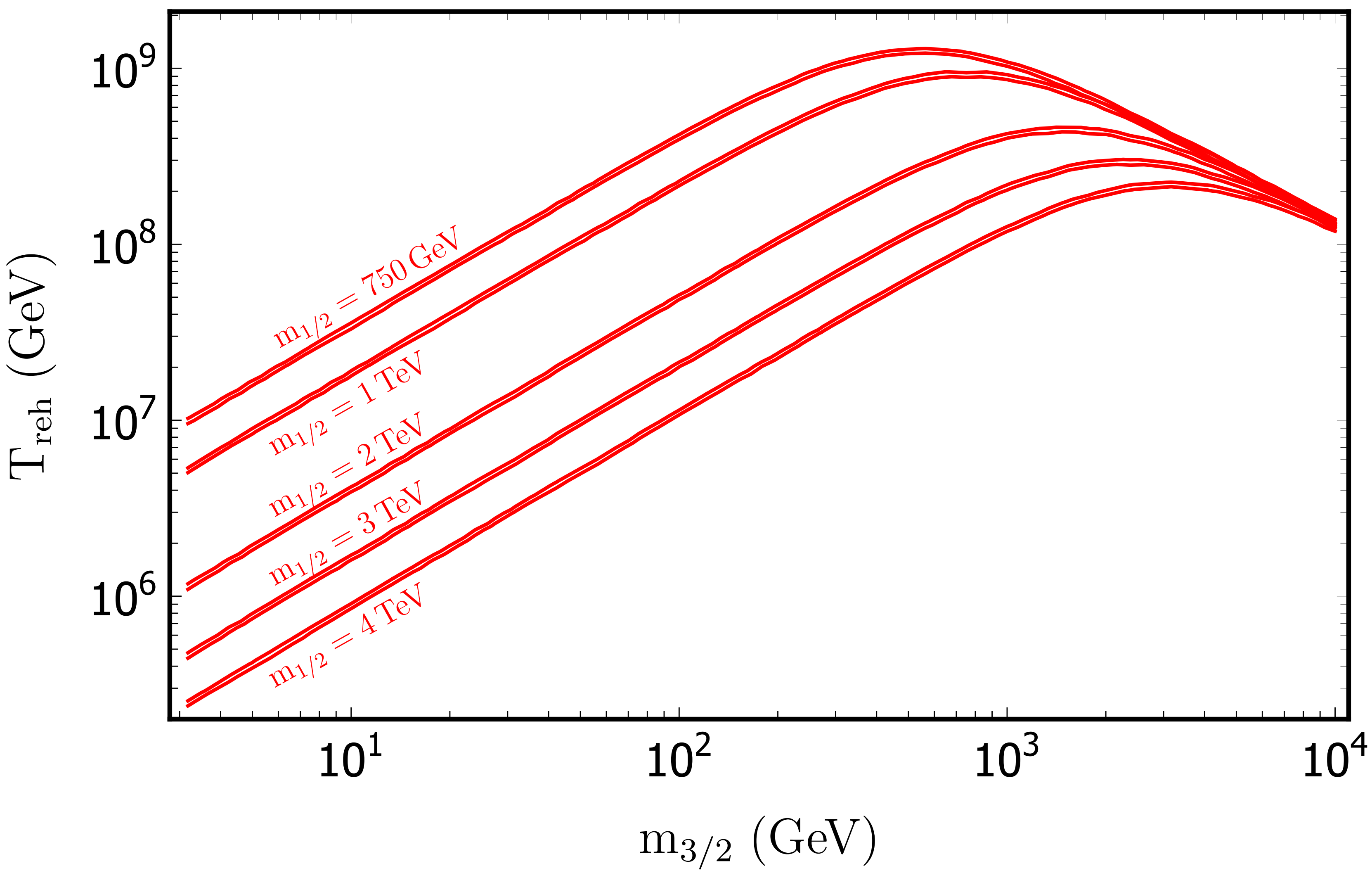
Gauge group	c_N	k_N
$U(1)_Y$	41.937	0.824
$SU(2)_L$	68.228	1.008
$SU(3)_c$	21.067	6.878



Gravitino abundance

$$Y_{3/2}(T) \simeq \frac{\gamma_{3/2}(T_{\rm reh})}{H(T_{\rm reh})\; n_{\rm rad}(T_{\rm reh})} \; \frac{g_{*s}(T)}{g_{*s}(T_{\rm reh})}$$

$$\Omega_{\rm DM} h^2 = \frac{\rho_{3/2}(t_0)\, h^2}{\rho_{\rm cr}} = \frac{m_{3/2}\, Y_{3/2}(T_0)\, n_{\rm rad}(T_0)\; h^2}{\rho_{\rm cr}} \simeq 1.33 \times 10^{24}\, \frac{m_{3/2}\;\gamma_{3/2}(T_{\rm reh})}{T_{\rm reh}^5}$$



Recap for gravitinos

- Gravitino is a natural DM candidate in SUGRA
- Thermally produced (Freeze-in mechanism) details explained. Improvements for this calculation are possible.
- No-thermal production (e.g. through inflaton decays) requires a particular inflation model.
- Assuming $m_{1/2}$> 750 GeV (~LHC bound) for T_{reh}~ 10^9 GeV, we get $m_{3/2}$=550 GeV. For T_{reh}~10^8 GeV for the same $m_{3/2}$, $m_{1/2}$~3,4 TeV.

PBH from SUGRA models

- Using as basis the no-scale SUGRA models one can show that adding modifications either to Kaehler potential or to superpotential can create features in the scalar potential, i.e. an inflection point, that can produce a significant enhancement in the power spectrum

- Around the inflection point the slow-roll approximation is not working, thus the numerical solution of the Mukhanov-Sasaki equation

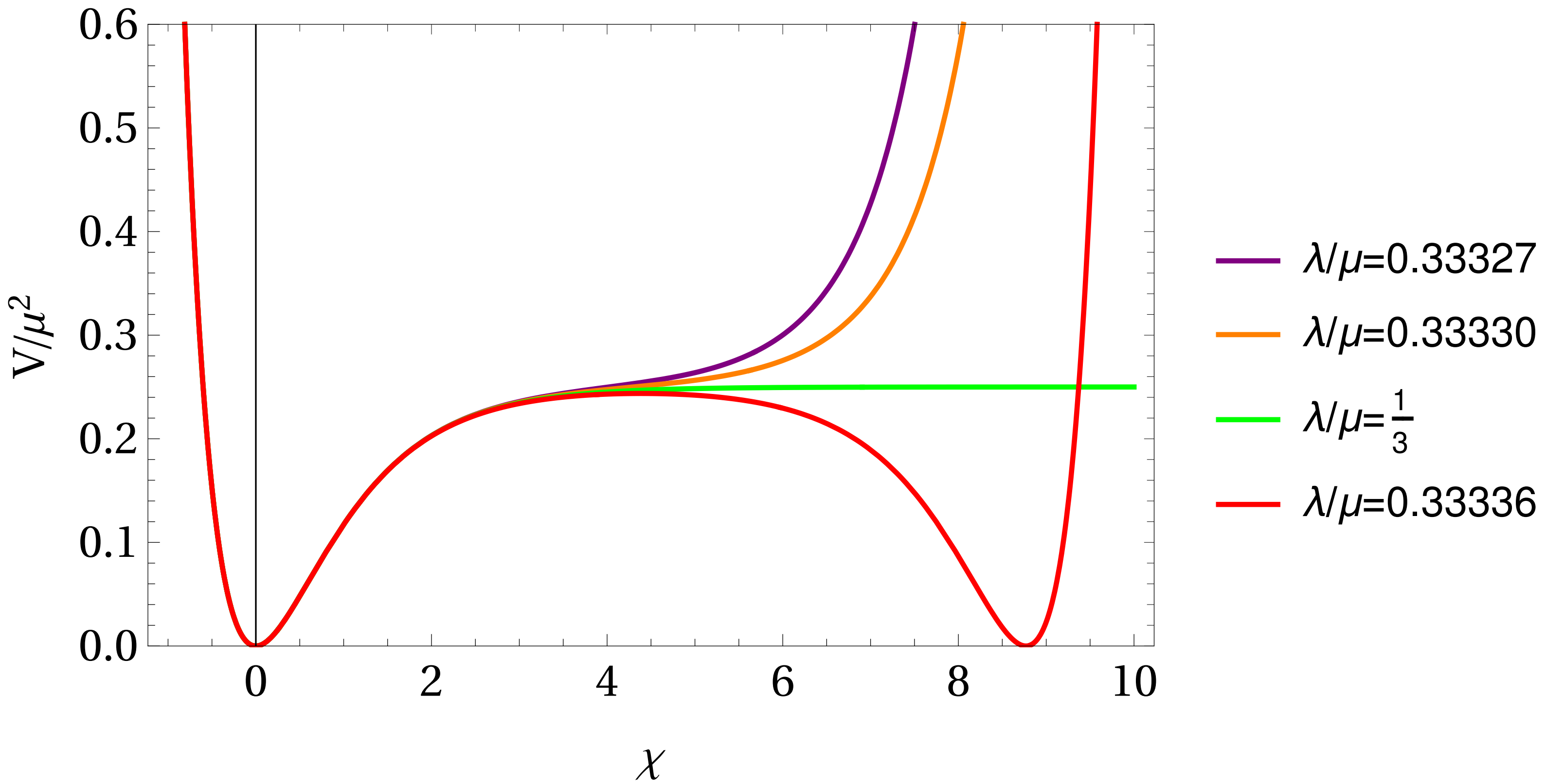
- In each case the models satisfy the Planck constraints for inflation, produce significant amount of DM in form of PBH and GW detectable at LISA, NANOGrav etc.

Starobinsky no-scale SUGRA model

$$K = -3\ln\left(T + \bar{T} - \frac{\varphi\bar{\varphi}}{3}\right) \qquad W = \frac{\hat{\mu}}{2}\varphi^2 - \frac{\lambda}{3}\varphi^3$$

$$T = \bar{T} = \frac{c}{2}\,, \quad \mathrm{Im}\varphi = 0 \qquad \varphi = \sqrt{3\,c}\,\tanh\left(\frac{\chi}{\sqrt{3}}\right)$$

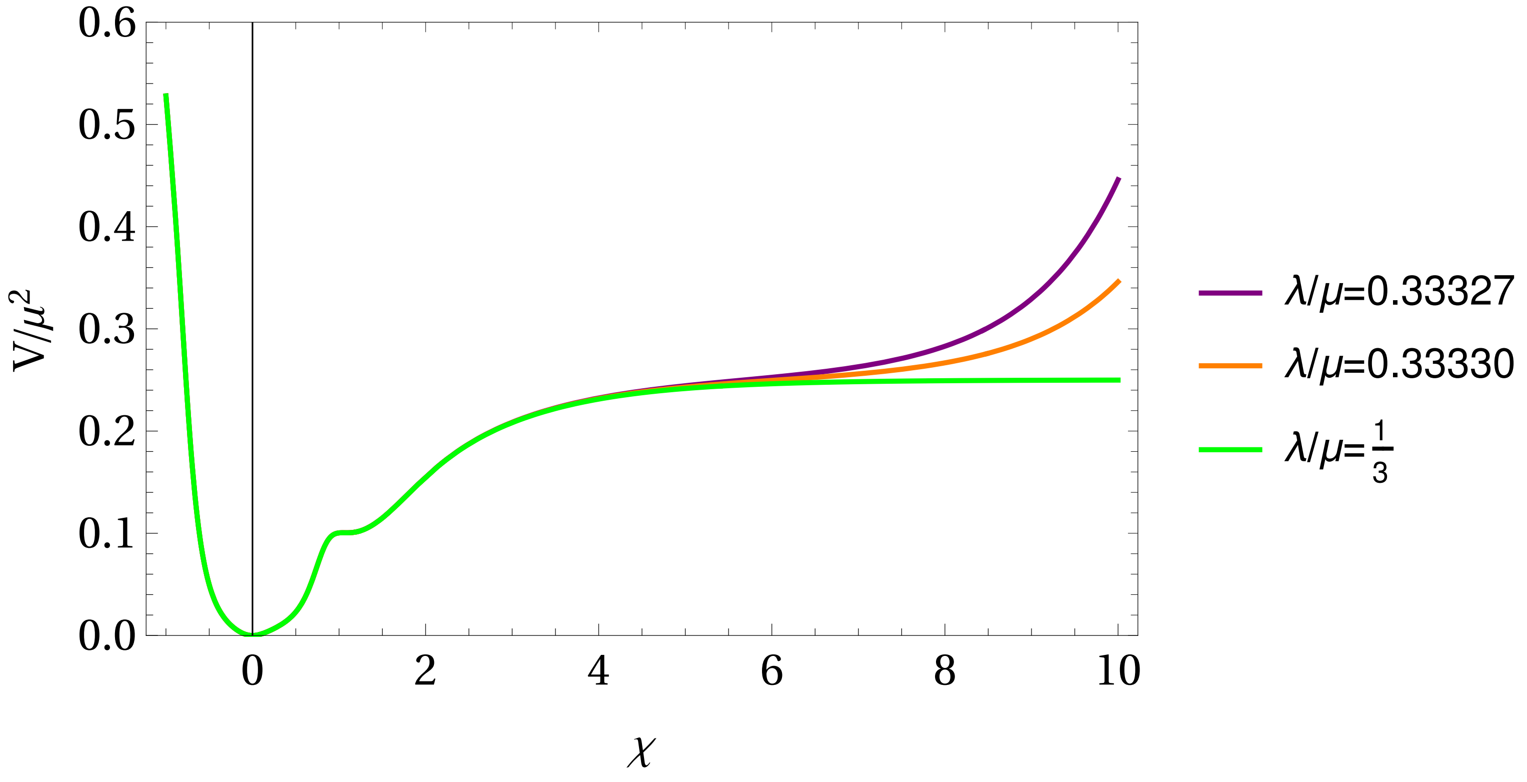
$$V(\chi) = \frac{\mu^2}{4}\left(1 - e^{-\sqrt{\frac{2}{3}}\chi}\right)^2$$



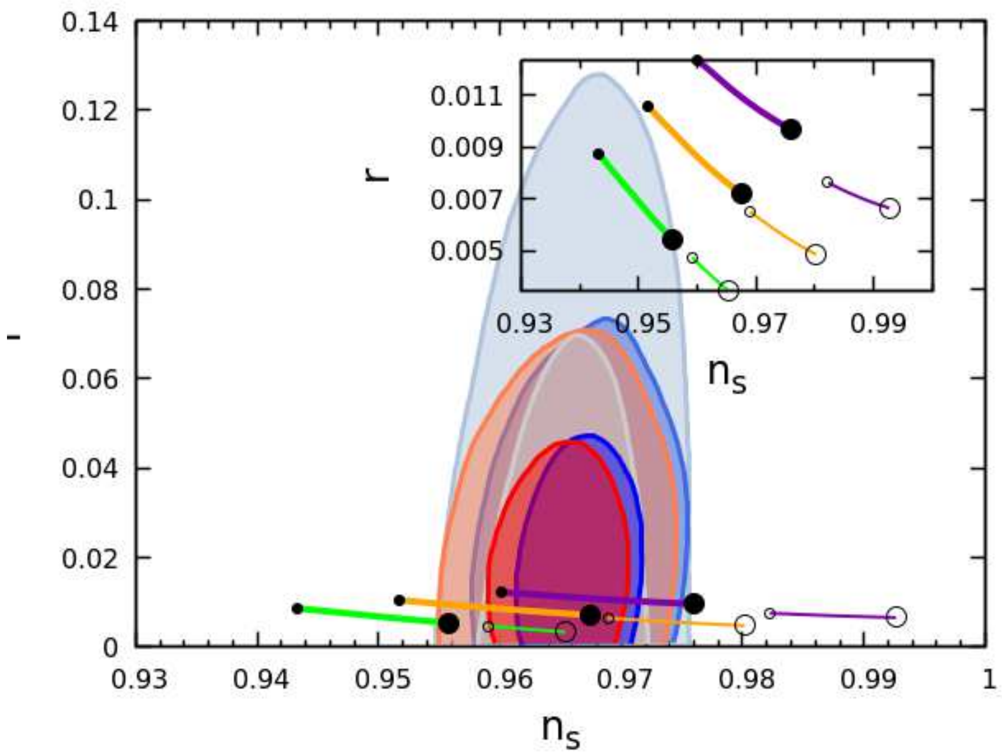
Modifying the Kaehler potential

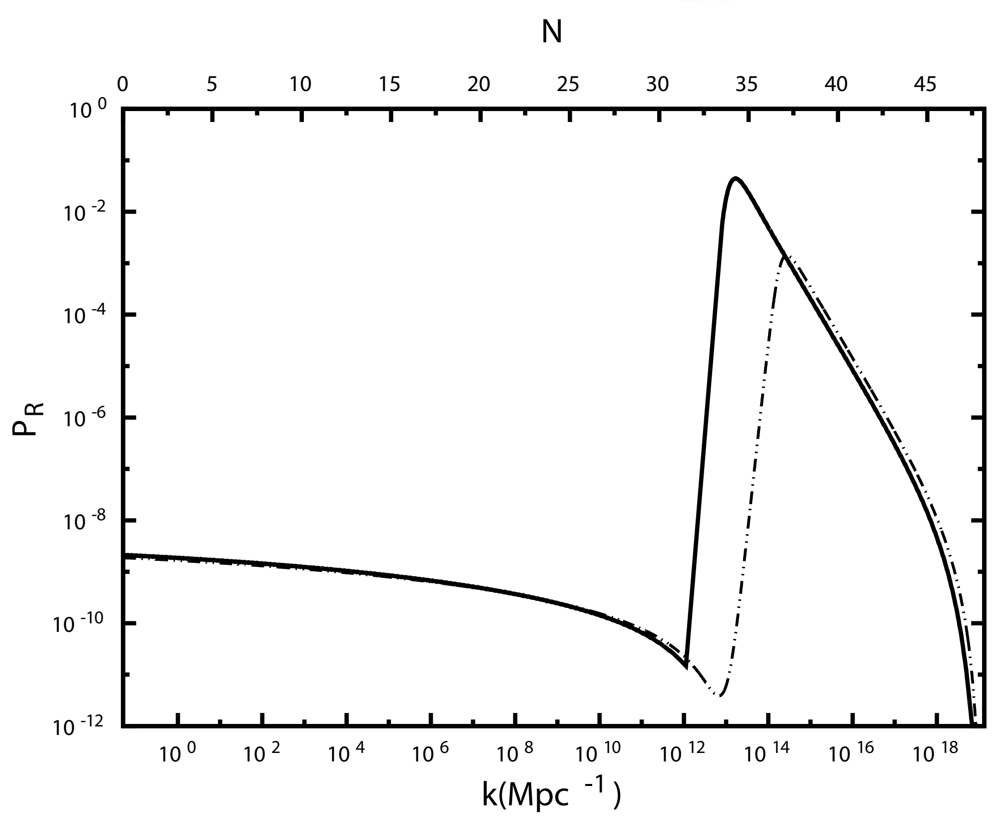
$$K = -3\ln\left[T + \bar{T} - \frac{\varphi\bar{\varphi}}{3} + a\, e^{-b(\varphi+\bar{\varphi})^2}(\varphi+\bar{\varphi})^4\right]$$

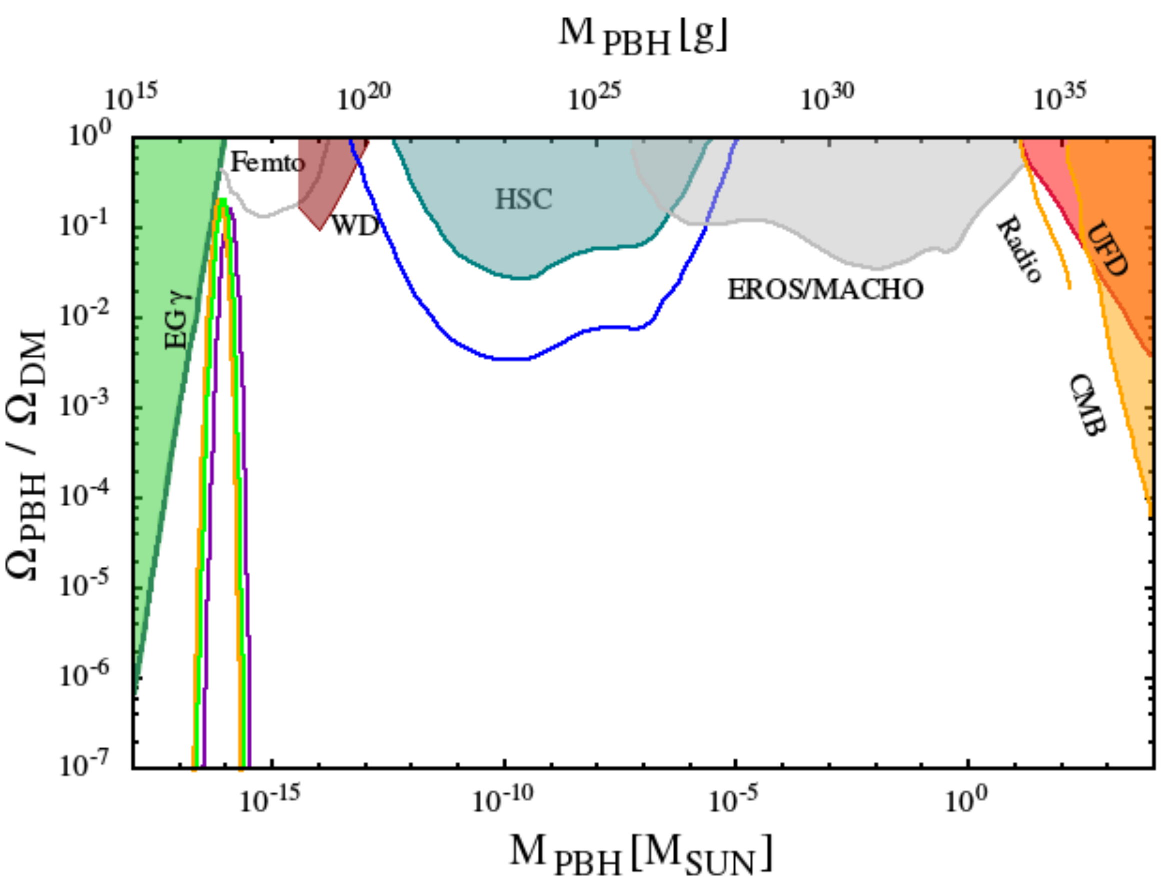
	λ/μ	b
1.	0.33327	87.379427
2.	0.33330	87.390563
3.	1/3	87.402941



PBH production

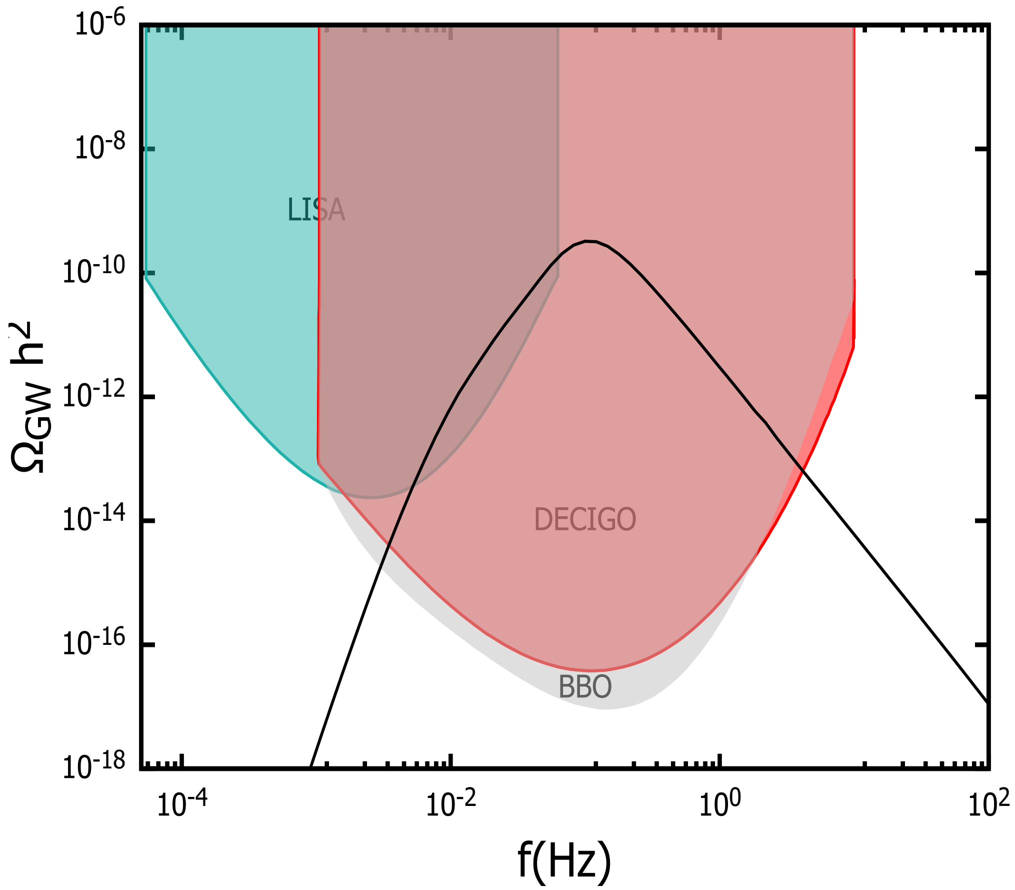


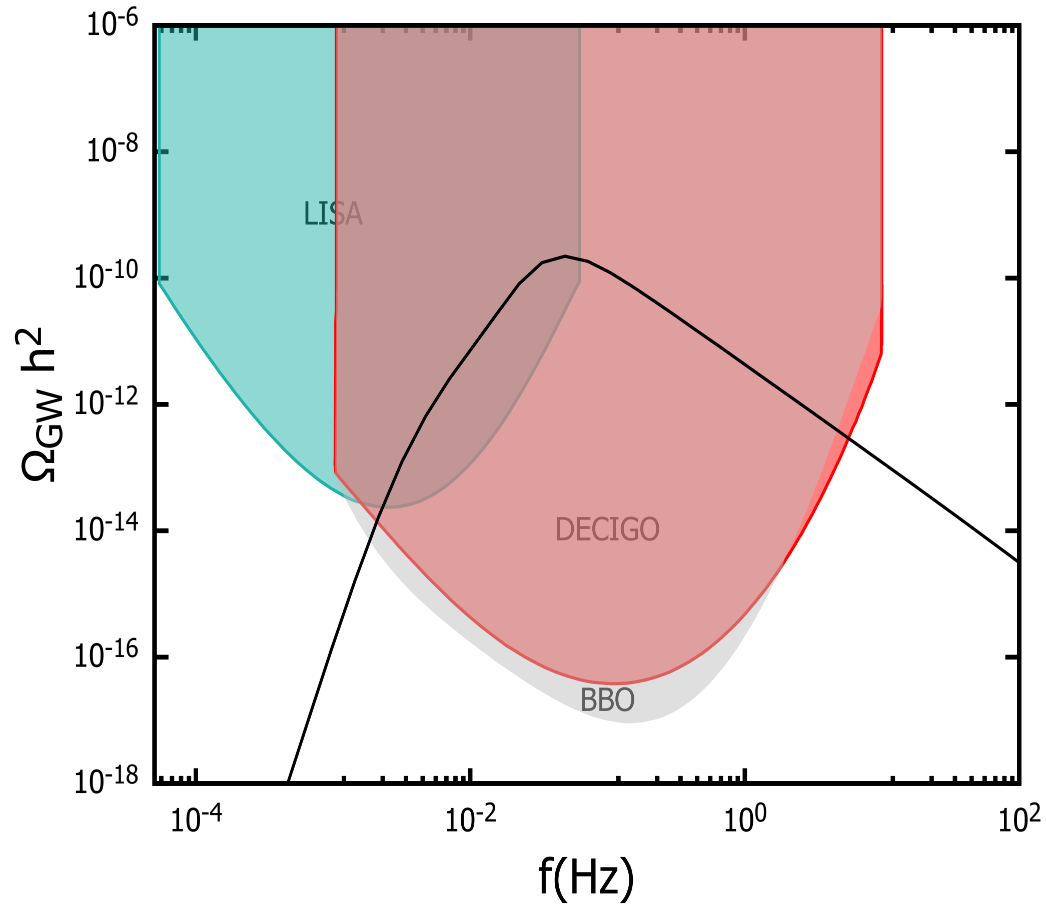




[Nanopoulos, VCS, Stamou (2020)]

GW production





[Stamou (2021); VCS, Stamou (2022)]

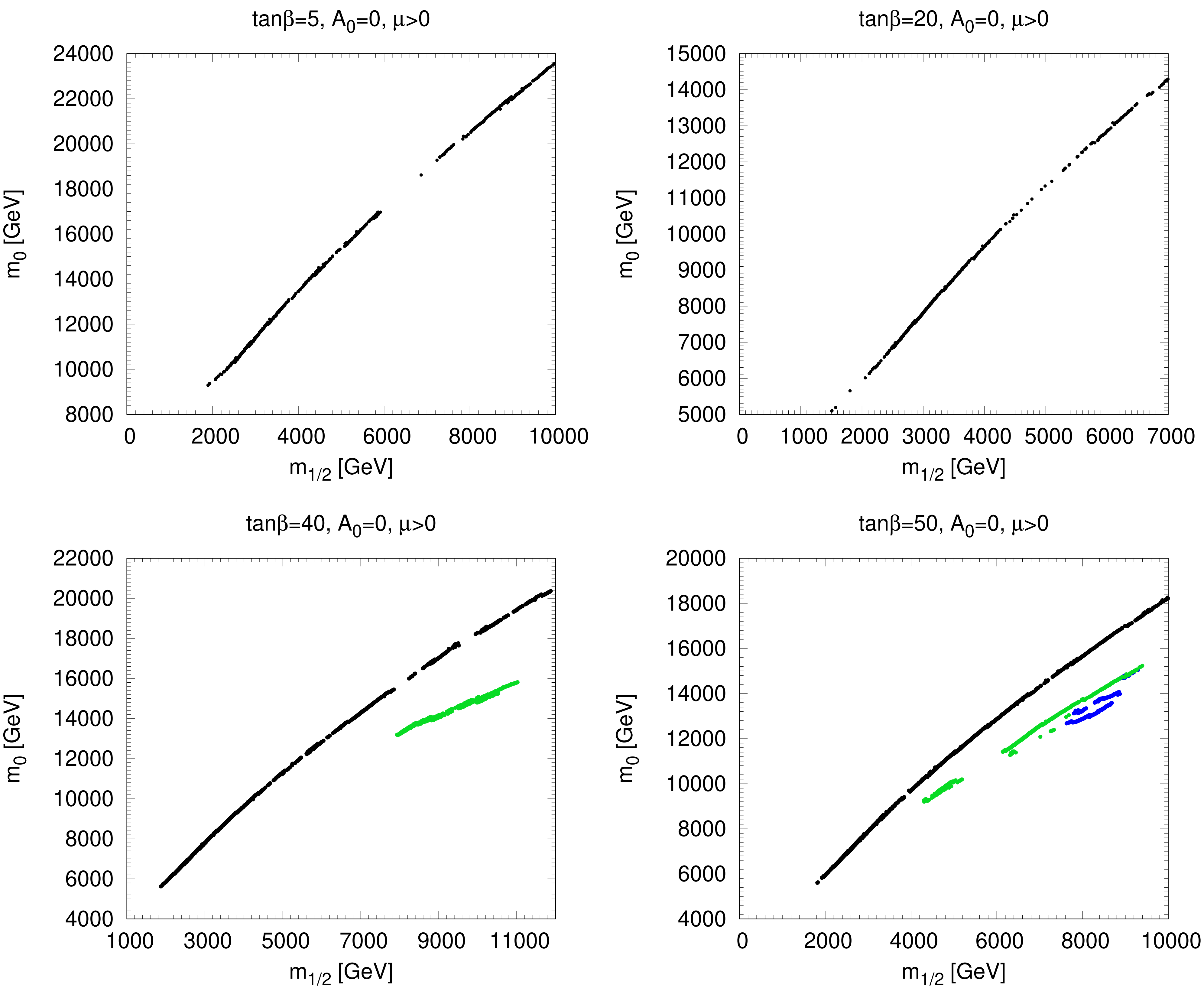
Recap for PBH

- By modifying the Kaehler potential or the superpotential PBH and GW are produced
- Tuning the parameters of the models inflationary constraints are satisfied and significant amount of DM in the form of PBH is produced, up to 90% or even higher
- In the most of the case sizeable fine tuning is required in order to achieve these
- GW that are produced can be detected in current and future interferometers, like NANOGrav, LISA, Decigo etc

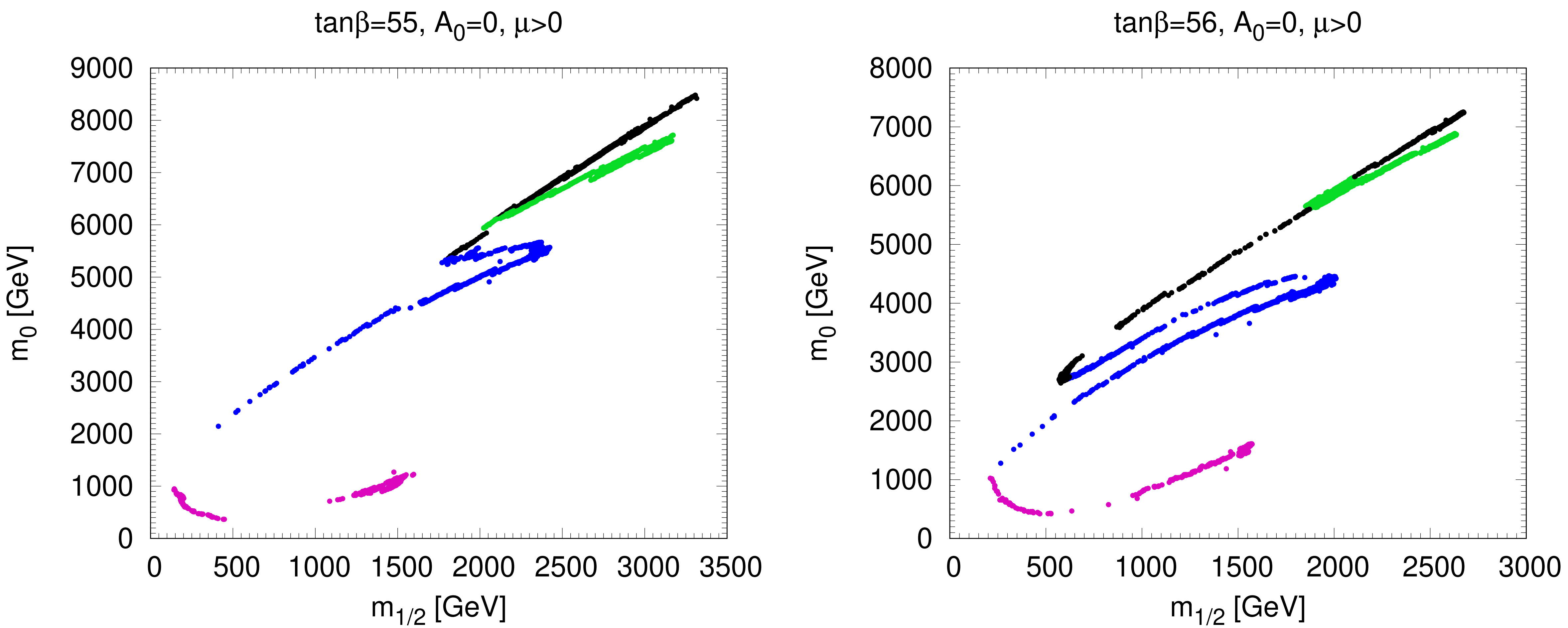
Summary

- We presented results for neutralino, gravitino and PBH DM in the context of various SUGRA models
- Gravitino DM scenario not susceptible either to direct or indirect searches, but other constraints, e.g. from BBN should apply
- Even in PBH scenarios based on SUGRA gravitino contribution cannot be avoided

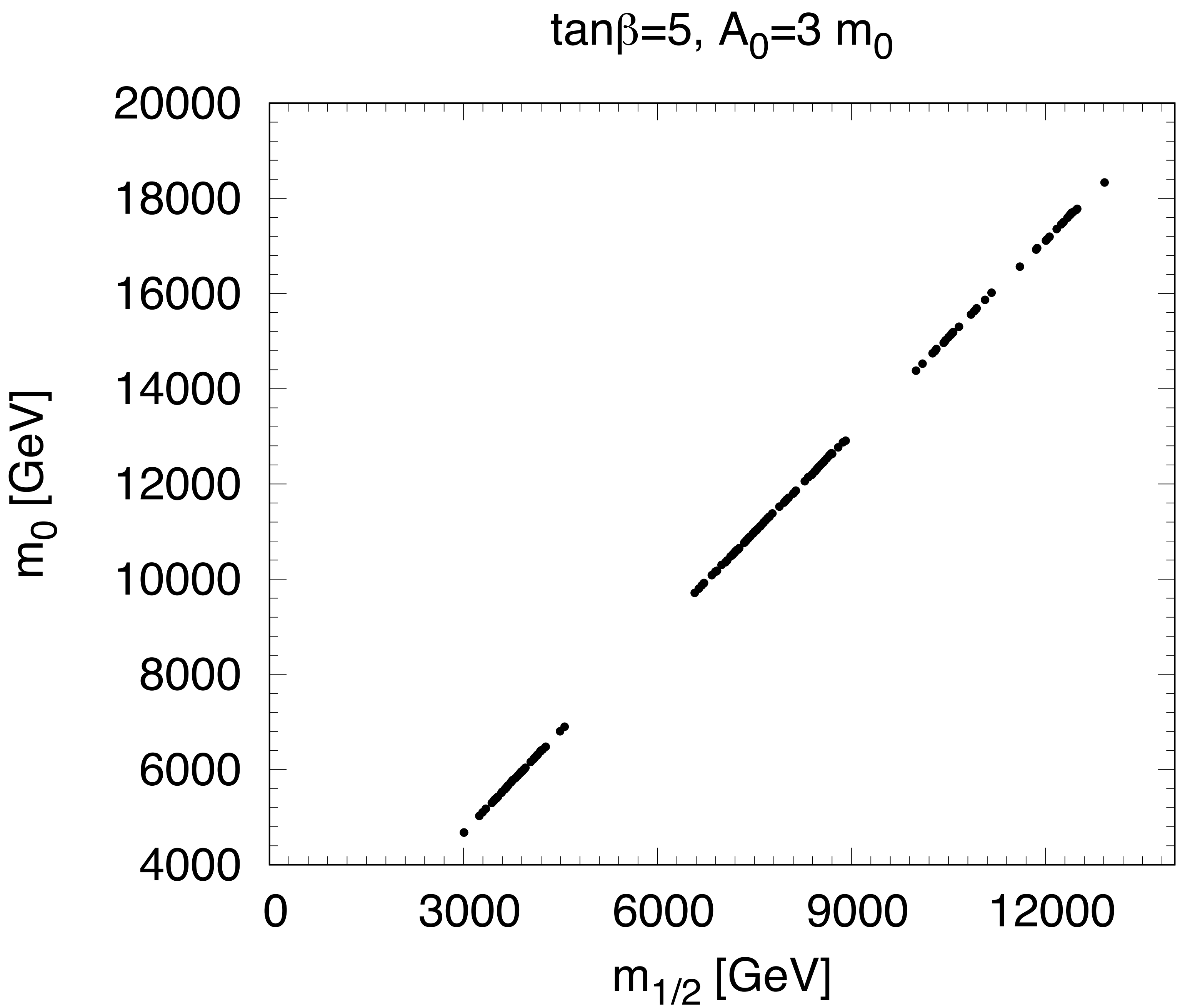
Backup slides

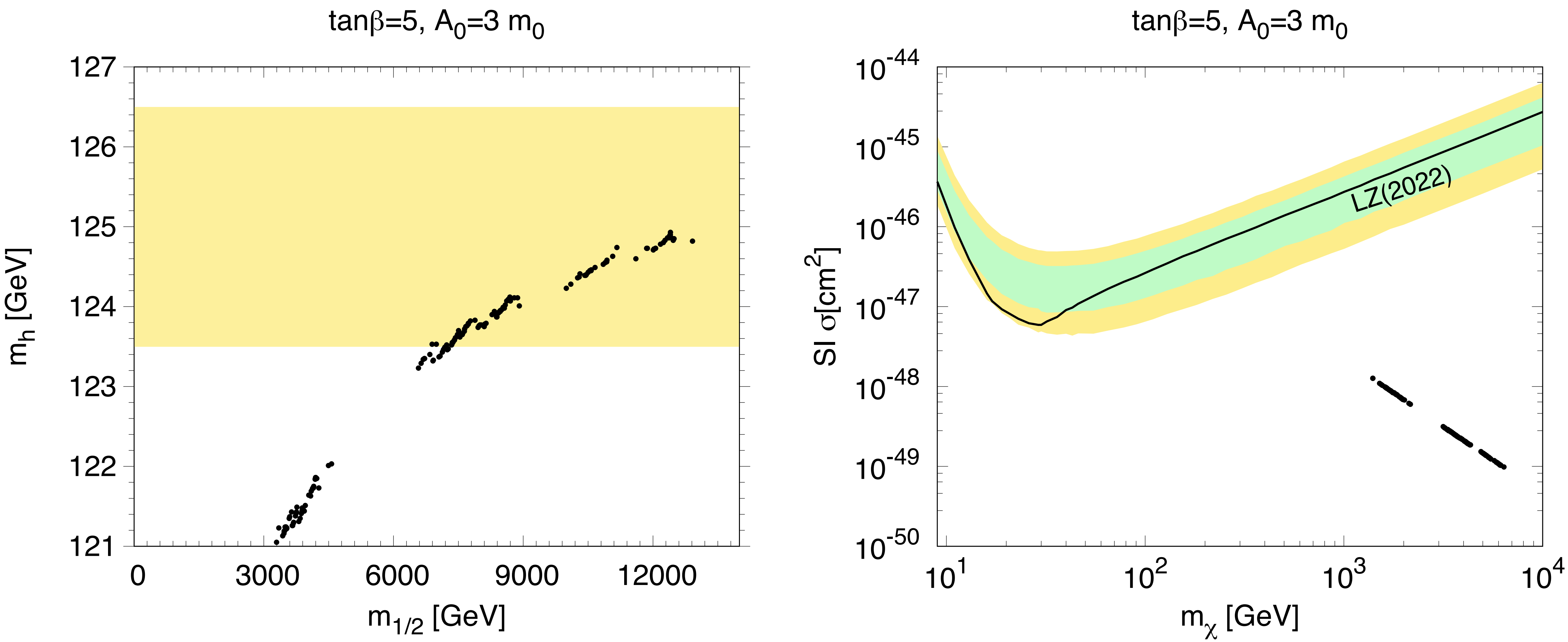


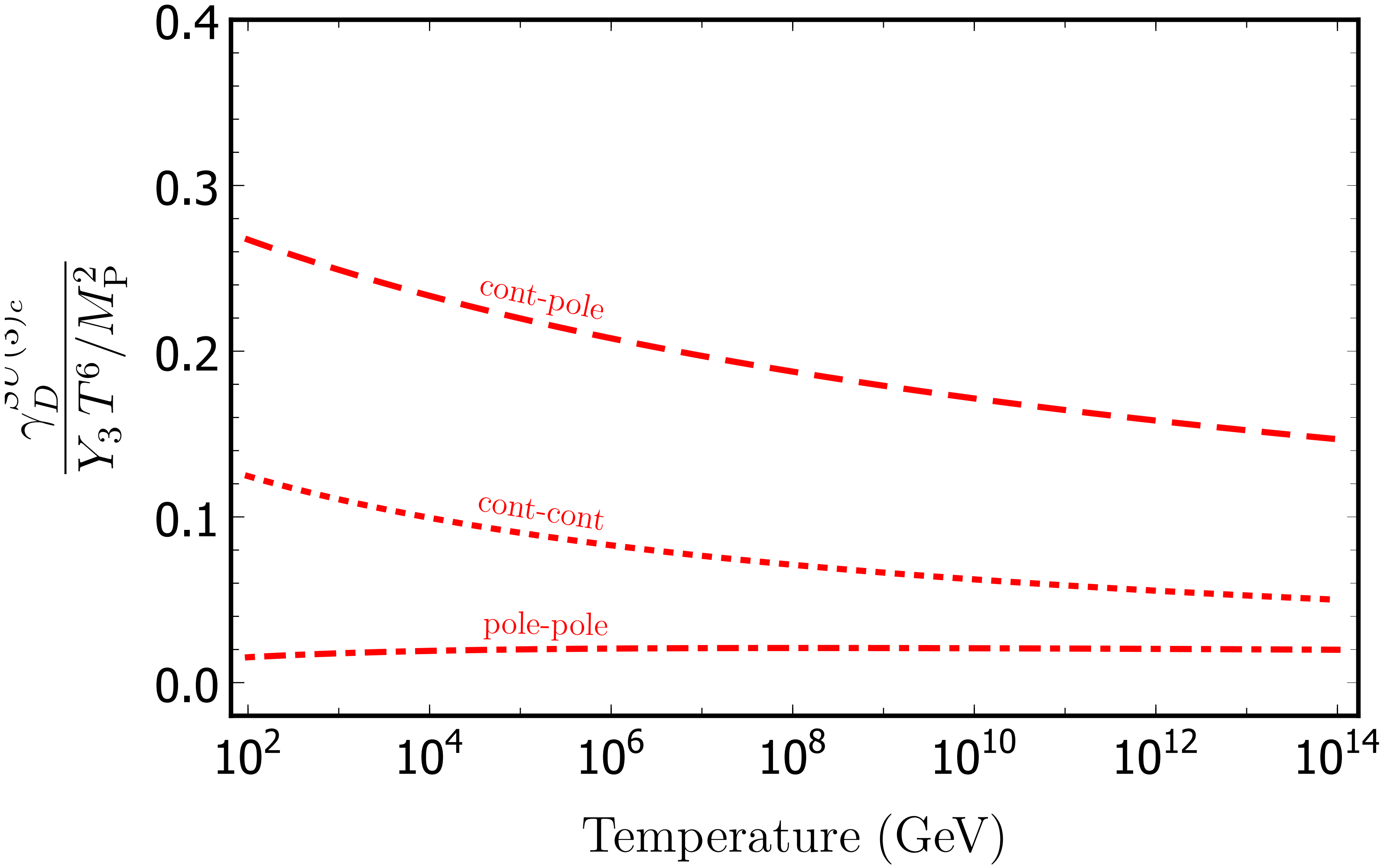
tanβ=5, A0=0, μ>0
tanβ=20, A0=0, μ>0
tanβ=40, A0=0, μ>0
tanβ=50, A0=0, μ>0
m0 [GeV]
m1/2 [GeV]



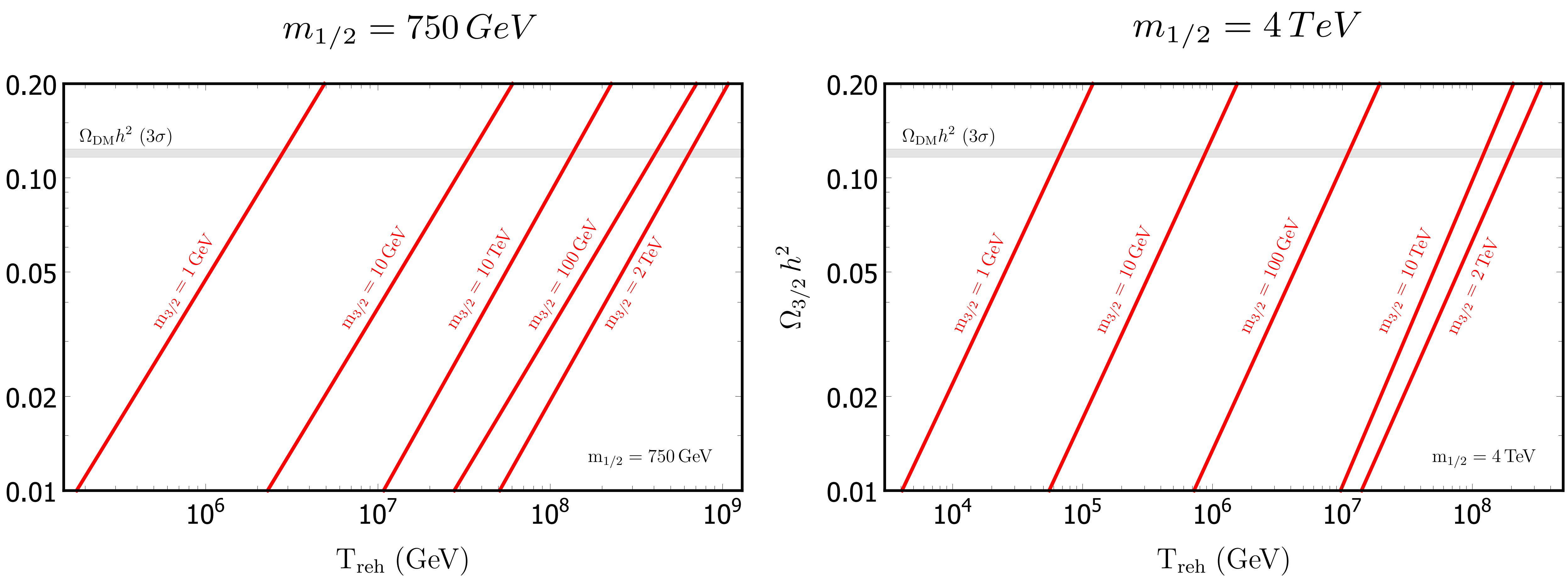
tanβ=55, A_0=0, μ>0
9000
8000
7000
6000
5000
4000
3000
2000
1000
0
0
500
1000
1500
2000
2500
3000
3500
m_0 [GeV]
$m_{1/2}$ [GeV]
tanβ=56, A_0=0, μ>0
8000
7000
6000
5000
4000
3000
2000
1000
0
0
500
1000
1500
2000
2500
3000
m_0 [GeV]
$m_{1/2}$ [GeV]


tanβ=5, A0=3 m0
20000
18000
16000
14000
12000
10000
8000
6000
4000
m0 [GeV]
0
3000
6000
9000
12000
m1/2 [GeV]


tanβ=5, A0=3 m0
127
126
125
124
123
122
121
mh [GeV]
0
3000
6000
9000
12000
m1/2 [GeV]
tanβ=5, A0=3 m0
10-44
10-45
10-46
10-47
10-48
10-49
10-50
SI σ[cm2]
LZ(2022)
101
102
103
104
mχ [GeV]



0.4
0.3
0.2
0.1
0.0
cont-pole
cont-cont
pole-pole
10^2
10^4
10^6
10^8
10^{10}
10^{12}
10^{14}
Temperature (GeV)



$m_{1/2} = 750\,GeV$
$m_{1/2} = 4\,TeV$
0.20
0.10
0.05
0.02
0.01
$\Omega_{DM}h^2$ (3σ)
$m_{3/2} = 1$ GeV
$m_{3/2} = 10$ GeV
$m_{3/2} = 10$ TeV
$m_{3/2} = 100$ GeV
$m_{3/2} = 2$ TeV
$m_{1/2} = 750$ GeV
$m_{1/2} = 4$ TeV
10^4
10^5
10^6
10^7
10^8
10^9
T_{reh} (GeV)
$\Omega_{3/2}\,h^2$

$$\Omega_{GW}(k) = \frac{c_g\,\Omega_r}{36}\int_0^{\frac{1}{\sqrt{3}}} \mathrm{d}d \int_{\frac{1}{\sqrt{3}}}^{\infty} \mathrm{d}s \left[\frac{(s^2-1/3)(d^2-1/3)}{s^2+d^2}\right]^2 P_R(kx)P_R(ky)(I_c^2+I_s^2)$$

$$x = \frac{\sqrt{3}}{2}(s+d), \quad y = \frac{\sqrt{3}}{2}(s-d).$$

$$I_c = -36\,\pi\,\frac{(s^2+d^2-2)^2}{(s^2-d^2)^3}\,\Theta(s-1)$$

$$I_s = -36\,\frac{(s^2+d^2-2)^2}{(s^2-d^2)^2}\left[\frac{(s^2+d^2-2)}{(s^2-d^2)}\,\ln\left|\frac{d^2-1}{s^2-1}\right| + 2\right]$$

$$\frac{\Omega_{PBH}}{\Omega_{DM}}(M_{\rm PBH}) = \frac{\beta(M_{\rm PBH})}{8\times 10^{-16}}\left(\frac{\gamma}{0.2}\right)^{3/2}\left(\frac{g_*(T_f)}{106.75}\right)^{-1/4}\left(\frac{M_{\rm PBH}}{10^{-18}\ \rm grams}\right)^{-1/2}$$

$$f_{PBH} = \int \frac{dM_{\rm PBH}}{M_{\rm PBH}}\,\frac{\Omega_{PBH}}{\Omega_{DM}}$$

$$M_{\rm PBH}(k) = 10^{18}\left(\frac{\gamma}{0.2}\right)\left(\frac{g_*(T_f)}{106.75}\right)^{-1/6}\left(\frac{k}{7\times 10^{13}\,\rm Mpc^{-1}}\right)^{-2}\ \text{in grams}$$

$$\beta(M_{\rm PBH}) = \frac{1}{\sqrt{2\pi\sigma^2(M_{\rm PBH})}}\int_{\delta_c}^{\infty} d\delta\ \exp\left(-\frac{\delta^2}{2\sigma^2(M_{\rm PBH})}\right)$$

$$\sigma^2\left(M_{\rm PBH}(k)\right) = \frac{16}{81}\int \frac{dk'}{k'}\left(\frac{k'}{k}\right)^4 P_R(k')\tilde{W}\left(\frac{k'}{k}\right)$$