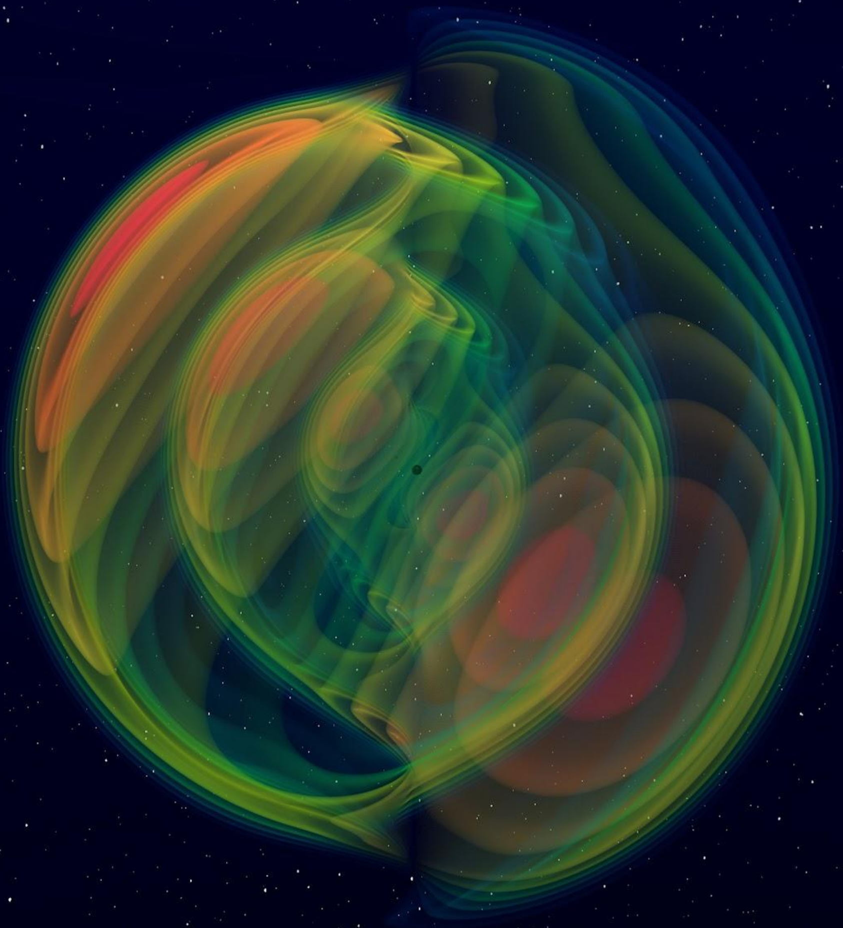
The Kerr BH in Einstein–Æther Gravity



Jacopo Mazza
IJCLab – Orsay, France

AstroParticle Symposium

November 9th, 2023

Based on

Franzin, Liberati, Mazza,

The Kerr Black Hole In Einstein–Æther Gravity

[arXiv: 2311:XXXXX]

[Nathan W. Pyle]

Einstein-Æther

æ-theroy

(scalar-)vector-tensor theory of gravity

$$g_{\alpha\beta}$$

metric

$$u_\mu$$

æther

constraint

$$u_\mu u^\mu = -1$$

i.e. æther is a timelike congruence

æther never vanishes

provides a preferred time direction

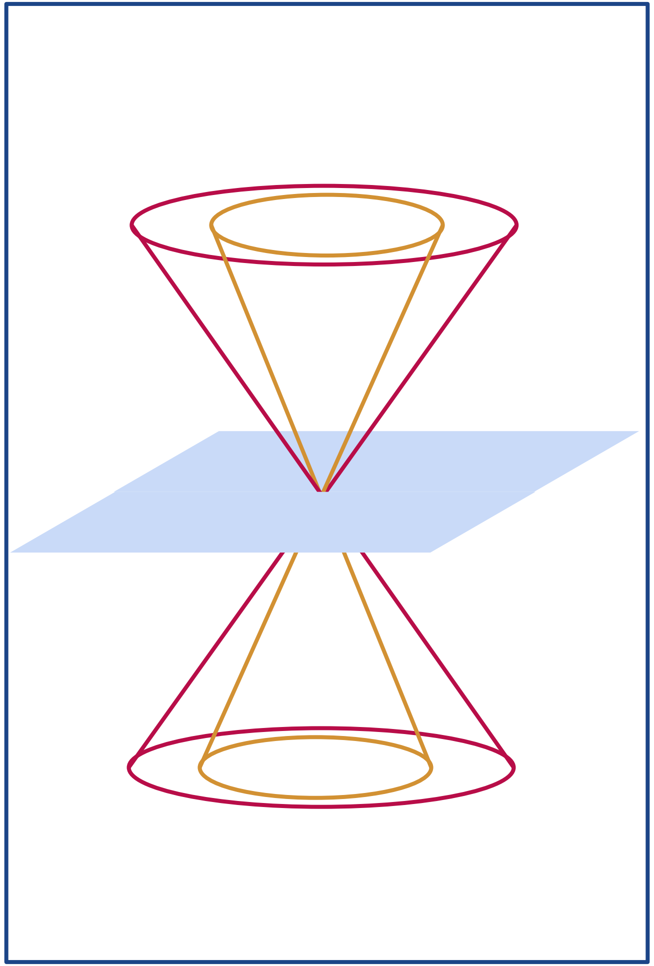
coupling with æther gives LIV

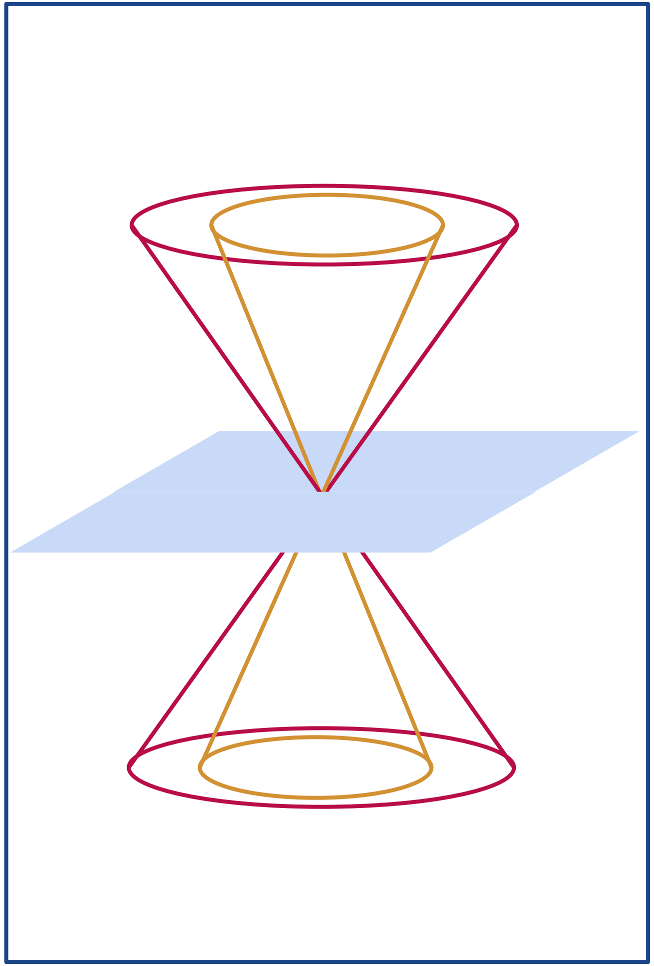
there can be superluminal causal signals

non-linear dispersion relations allowed:

$$\omega^2(k) = k^2 + \frac{k^4}{\Lambda^2} + \dots$$

æther determines causal structure





Why LIV Gravity?

? **Lorentz invariance** well tested in matter sector,
cornerstone of modern physics

- Several QG scenarios point to violations of LI in UV
- LIV can help build QG theories [Hořava]

Æ-theroy is EFT for generally covariant LIV gravity

Lagrangian

$$\sqrt{-g}\mathcal{L} = \sqrt{-g}\left\{R - \left[\frac{1}{3}c_\vartheta\vartheta^2 + c_\sigma\sigma^2 + c_\omega\omega^2 - c_a a^2\right] + \zeta\left(u^\mu u_\mu + 1\right)\right\}$$

$$\begin{cases} a_\mu &= u^\alpha\nabla_\alpha u_\mu \quad \text{(acceleration)} \\ \vartheta &= \nabla_\mu u^\mu \quad \text{(expansion)} \end{cases} \qquad \begin{cases} \sigma_{\mu\nu} &= \nabla_{(\mu}u_{\nu)} + u_{(\mu}a_{\nu)} - \frac{\theta}{3}p_{\mu\nu} \quad \text{(shear)} \\ \omega_{\mu\nu} &= \nabla_{[\mu}u_{\nu]} + u_{[\mu}a_{\nu]} \quad \text{(twist)} \end{cases}$$

hypersurface orthogonal u_μ $\qquad \omega_{\mu\nu} = 0$

≃ (IR) non-projectable **Hořava** gravity
a.k.a. khronometric theory

Compact Objects? BHs?

Spherical symmetry (static) → YES

- numerical
- analytical, for specific choices of couplings

Rotating (axial symmetry) → YES, but…

- slow rotation (numerical)
- some, arbitrary spin (numerical)

Adam, Figueras, Jacobson, Wiseman, Class. Quant. Grav. 39 (2022) 12, 125001 [2108.00005]

[no Killing Horizons]

? Can we find one ***analytical* rotating solution**? (possibly toy example)

Expectation

small couplings ⇒ GR solution
+ ‘painted’ u_μ

Idea

look for solution with
metric = Kerr

$$\sqrt{-g}\mathcal{L} = \sqrt{-g}\Big\{R - \Big[\tfrac{1}{3}c_\vartheta \vartheta^2 + \underbrace{c_\sigma \sigma^2 + c_\omega \omega^2 - c_a a^2}\Big]\Big\}$$

solve

$$\begin{cases} \nabla_\mu u^\mu = 0 \\ u_\mu u^\mu = -1 \end{cases}$$

$$c_\sigma = c_\omega = c_a = 0$$

“minimal æ-theory”

Metric

Kerr

$$ds^2 = -\left(1 - \frac{2Mr}{\Sigma}\right) \mathrm{d}t^2 + \frac{\Sigma}{\Delta}\mathrm{d}r^2 + \Sigma \mathrm{d}\theta^2 - \frac{4Mra\sin^2\theta}{\Sigma}\mathrm{d}t\mathrm{d}\phi + \frac{A\sin^2\theta}{\Sigma}\mathrm{d}\phi^2$$

$$\Sigma = r^2 + a^2\cos^2\theta$$
$$\Delta = r^2 - 2Mr + a^2$$
$$A = (r^2 + a^2)^2 - \Delta a^2 \sin^2\theta$$

Æther

Lie-dragged along Killing vectors

$$u_\mu(x^\alpha) = u_\mu(r,\theta)$$

vanishing expansion

$$\partial_r(u_r\Delta) + \frac{1}{\sin\theta}\partial_\theta(u_\theta \sin\theta) = 0$$

choose

$$u_\theta = 0$$

$$u_r = -\frac{M^2\Theta(\theta)}{\Delta}$$

unit norm

[...]

choose

$$u_\phi = 0$$

$$(u_t)^2 = \frac{\Delta\Sigma + M^4\Theta^2}{A}$$

Æther solution

One simple solution is

$$u_\mu = \left\{ \mp\sqrt{\frac{\Sigma\Delta + M^4\Theta^2}{A}}, -\frac{M^2\Theta}{\Delta}, 0, 0 \right\}_\mu$$

$$u^\mu = \left\{ \pm\frac{A}{\Delta\Sigma}\sqrt{\frac{\Sigma\Delta + M^4\Theta^2}{A}}, -\frac{M^2\Theta}{\Sigma}, 0, \pm\frac{2Mar}{\Delta\Sigma}\sqrt{\frac{\Sigma\Delta + M^4\Theta^2}{A}} \right\}^\mu$$

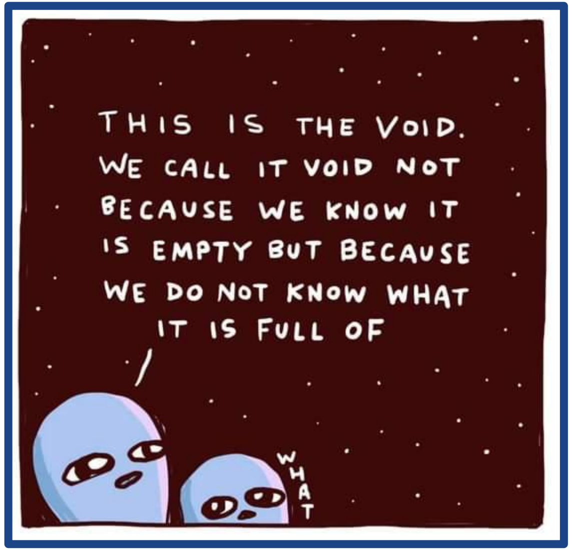
$\Theta(\theta)$ — free function of angle

angular velocity of frame dragging — $\frac{u^\phi}{u^t} = \frac{g^{t\phi}}{g^{tt}}$

$$\omega_{\mu\nu} \neq 0,\ \sigma_{\mu\nu} \neq 0,\ a_\mu \neq 0$$

Is that it?

Not so quickly…



[Nathan W. Pyle]

Remark

One step back…

square

$$(u_t)^2 = \frac{\Delta\Sigma + M^4\Theta^2}{A}$$

can become negative

$r > 0$

DEN > 0
NUM ??

NUM can be < 0 where $\Delta < 0$
i.e. between KHs

must choose
$\Theta(\theta)$
$M^4\Theta^2 \geq -\min(\Delta\Sigma)$

$r < 0$

NUM > 0
DEN ??

$A < 0$ somewhere
(associated to CTCs)

must exclude

Can we fix $\boldsymbol{\Theta}$?

Static and spherically symmetric solution
known
in this corner of parameter space

make sure that
limit $a \to 0$ mathces

$\boldsymbol{\Theta}$ must become constant
at $a = 0$

$$\lim_{a\to 0} M^4 \boldsymbol{\Theta}^2 = \boldsymbol{r}^2_{\text{ae}}$$

integration
constant

$$\boldsymbol{r}^2_{\text{ae}} \geq \sqrt{\frac{27}{16}} M^2$$

also has to satisfy a bound
to ensure u_μ real-valued

Fixing $\boldsymbol{\Theta}$

Option 1

forget angular dependence

$$M^4\Theta^2 = r^4_{\mathrm{ae}}$$

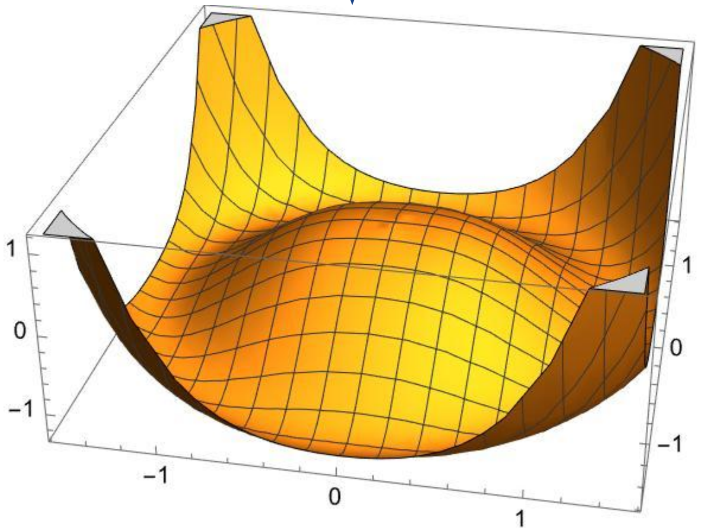
choice is ok,
not very interesting

Fixing Θ - Cont.

Option 2

More complicated but more intersting

$$(u_t)^2 = \frac{\Delta\Sigma + M^4\Theta^2}{A}$$



$$r_{\text{QUH}} : \quad \partial_r(\Delta\Sigma)\big|_{r=r_{\text{QUH}}} = 0$$

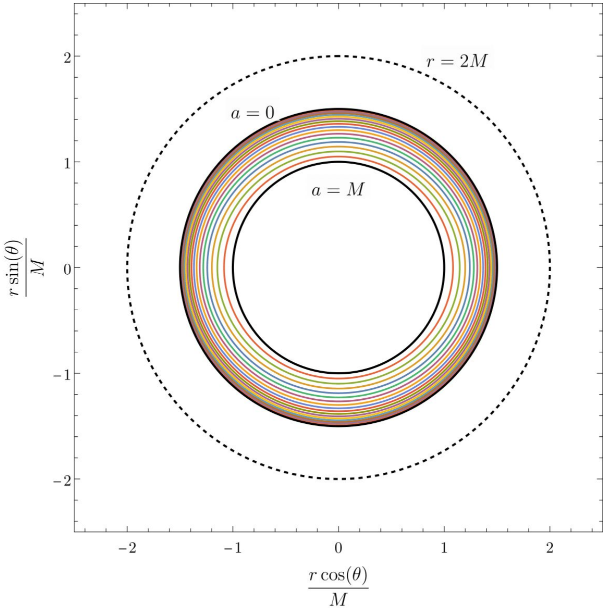
(has multiple sol's, take minimum)

$$M^4\Theta^2 = -\Delta\Sigma\big|_{r=r_{\text{QUH}}}$$

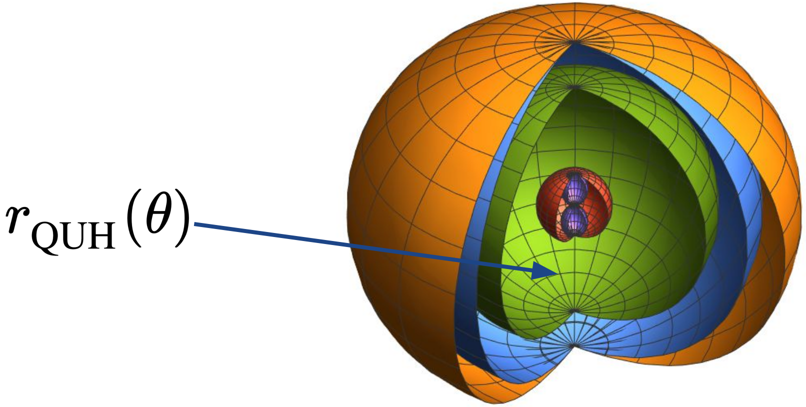
$u_t(r,\theta)$ is zero on $r = r_{\text{QUH}}(\theta)$

Properties of $r_{QUH}(\theta)$

- spacelike surface
- not constant r, but mild θ-dependence



- sandwiched between KHs



- not orthogonal to æther

$$n_\mu = -(a_\alpha \chi^\alpha) u_\mu + 2\omega_{\mu\alpha}\chi^\alpha$$

Why Option 2?

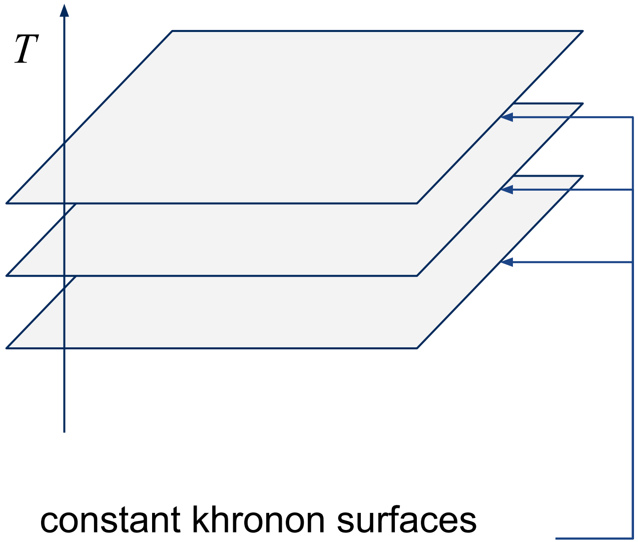
Analogy with

a) spherically symmetric case

b) hypersurface-orthogonal case
[Hořava]

$$u_\mu = N\,\nabla_\mu T$$

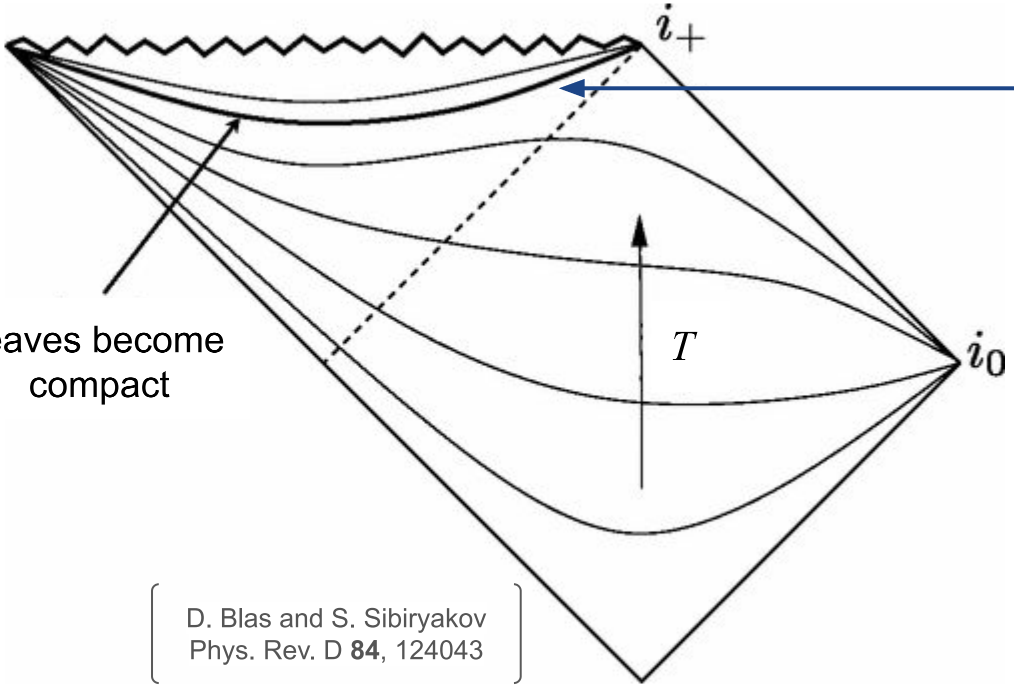
T called ‘khronon’



constant khronon surfaces
provide (preferred) foliation

Universal Horizons

Sometimes, the leaves can change topology



leaves become compact

universal horizon (UH)
traps modes of any speed

need them to 'save' BHs in LIV gravity

otherwise, they are low-energy phenomenon only

D. Blas and S. Sibiryakov
Phys. Rev. D **84**, 124043

UHs

properly characterised in
stationary (foliated) spacetimes

locally identifid by

$$u_\mu \chi^\mu = 0$$

[χ Killing vector]

(note: UHs not metric concept)

similar to GR horizons in
many ways

- mechanics
- Hawking radiation

characterisation only makes sense in *foliated* manifolds,
but
they are ‘needed’ even with non-hypersurface-orthogonal æther

? Can we find a UH
in our solution?

Is QUH a UH?

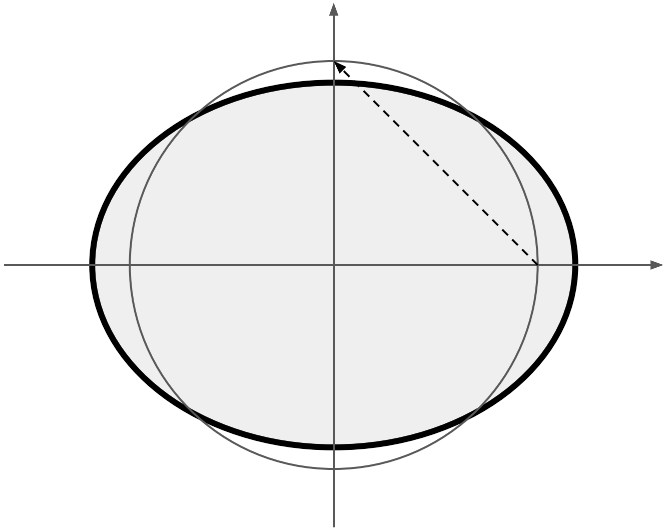
$r_{QUH}(\theta)$ seems perfect candidate,
because in these coordinates

$$u_\mu \chi^\mu = u_t$$

? Is QUH a UH

No.

QUH not orthogonal to u_μ
so its seems possible to exit following a causal curve



However…

… it really looks like one:

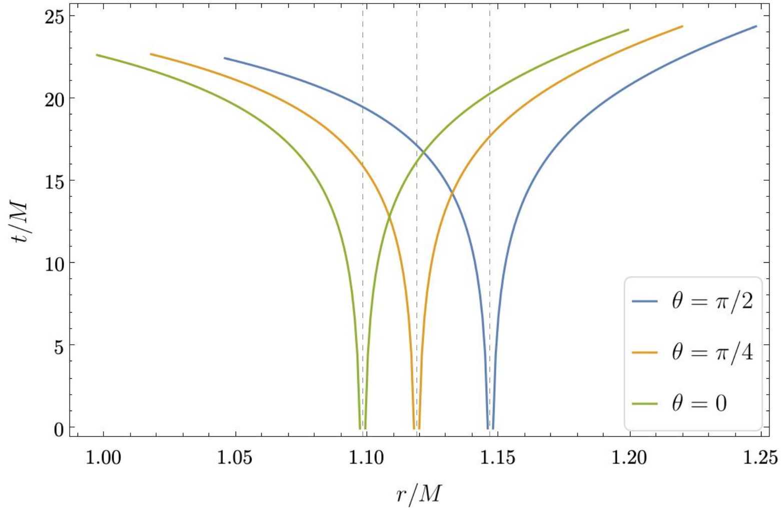
- at low spin, it's
  'UH ($a = 0$) + $O(a^2)$'
- at $a = M$ it seems an extremal UH
- it exhibits *some* peeling properties…

Consider

$$s_\mu = \{s_t, s_r, 0, 0\}_\mu$$
$$s_\mu s^\mu = +1 \quad s_\mu u^\mu = 0$$

('radially outgoing infinite-speed rays')

it's integral curves peel off the QUH



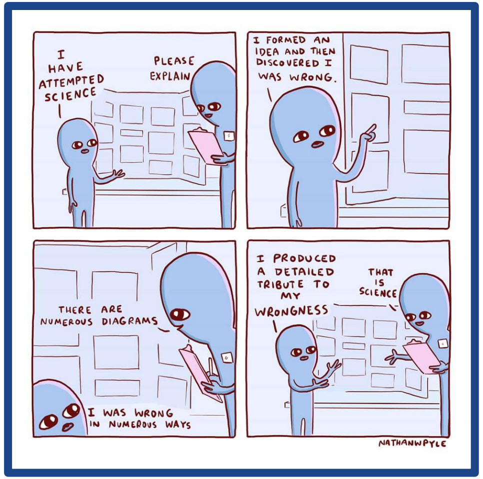
Quasi
Universal
Horison

Conclusions

Upshot

Found an analytical rotating (stationary & axysymmetric) solution of (minimal) Einstein–æther theory

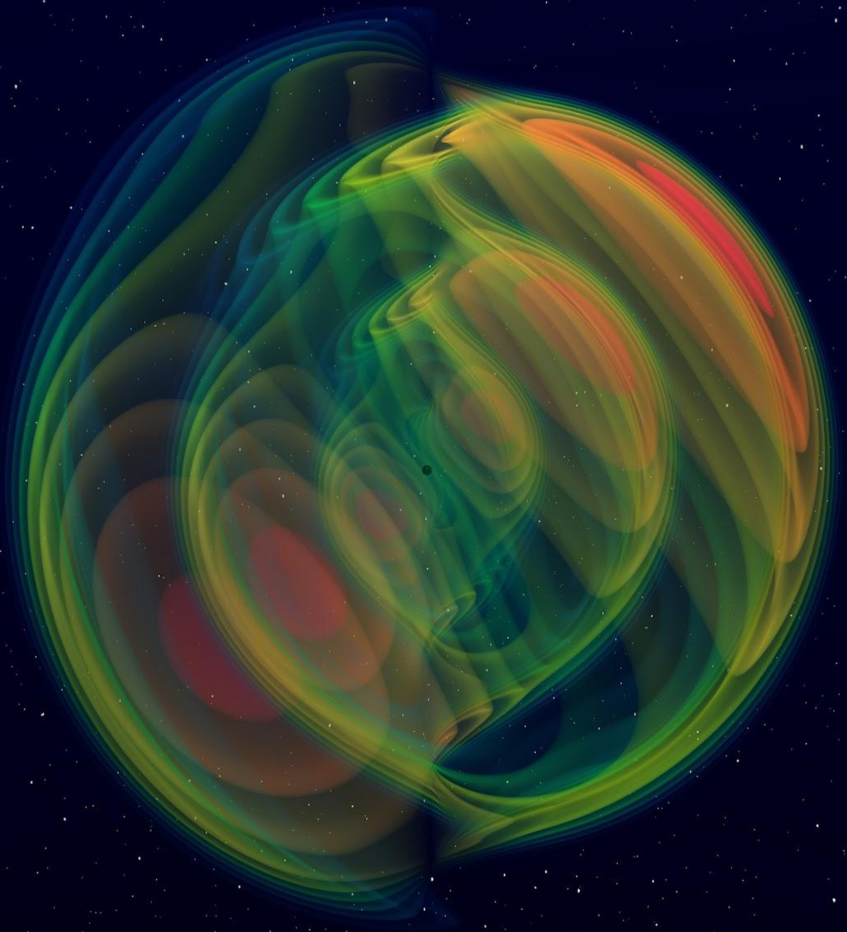
greatly restricted coupling space,
many simplifying assumptions



Opens new questions:

Q: do UHs exist in non-foliated manifolds?

Q: if so, how are they characterised?



Thanks!

N. Fischer, H. Pfeiffer, A. Buonanno (Max Planck Institute for Gravitational Physics), Simulating eXtreme Spacetimes (SXS) Collaboration.