# Brief Introduction to Nuclear Effective Field Theory

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# Outline

- Why EFT?
- What is EFT?
- What is EFT, *really*?
- QCD + ...
- Chiral EFT
- Pionless EFT
- Halo/Cluster EFT
- Conclusion

## To learn more

Lectures:

U. van Kolck, *Les Houches Lectures on Effective Field Theories for Nuclear and (some) Atomic Physics,* arXiv:1902.03141

Reviews:

U. van Kolck, *Naturalness in Nuclear Effective Field Theories,* arXiv:2003.09974

U. van Kolck, *The Problem of Renormalization of Chiral Nuclear Forces,* arXiv:2003.06721

S. König, H.-W. Hammer, and U. van Kolck, *Nuclear Effective Field Theory: Status and perspectives,* arXiv:1906.12122

# Why EFT?

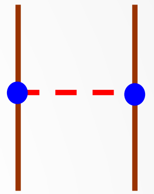
## Central goal of nuclear physics: How does nuclear structure emerge from the Standard Model?

R. Machleidt, arxiv:nucl-th/0609050

Table 1. Seven Decades of Struggle: The Theory of Nuclear Forces

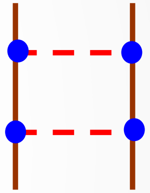
| | |
|---|---|
| **1935** | **Yukawa: Meson Theory** |
| **1950's** | *The "Pion Theories"*<br>One-Pion Exchange: o.k.<br>Multi-Pion Exchange: disaster |
| **1960's** | Many pions ≡ multi-pion resonances:<br>$\sigma, \rho, \omega, \ldots$<br>The One-Boson-Exchange Model |
| **1970's** | Refine meson theory:<br>Sophisticated $2\pi$ exchange models<br>(Stony Brook, Paris, Bonn) |
| **1980's** | Nuclear physicists discover<br>**QCD**<br>Quark Cluster Models |
| **1990's and beyond** | Nuclear physicists discover **EFT**<br>Weinberg, van Kolck<br>**Back to Meson Theory!**<br>*But, with Chiral Symmetry* |

Long-range physics
cf. photon exchange



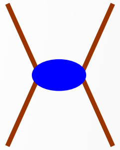
$R \sim \hbar / m_\pi c$

No renormalization!
cf. QED

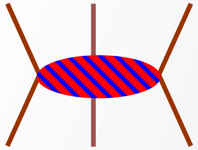


$+ \ \ldots \ \to \infty$

Phenomenological models
for short-range physics;
three-body forces?



$r \ll R$



$= ?$

Fail to account
for pion physics

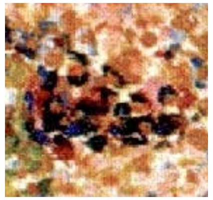
Next ~50 slides…

*And, much more!*

# What is EFT?

A paradigm to describe nature

- Relevant degrees of freedom



# What is EFT?

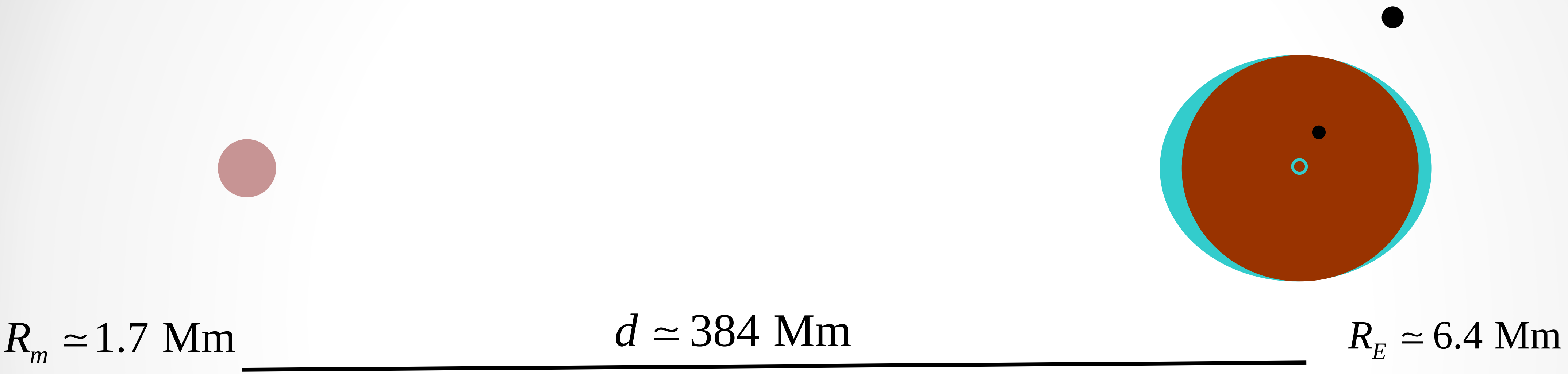
A paradigm to describe nature

- Relevant degrees of freedom

*choose the coordinates for the scales of the system*

- All possible interactions

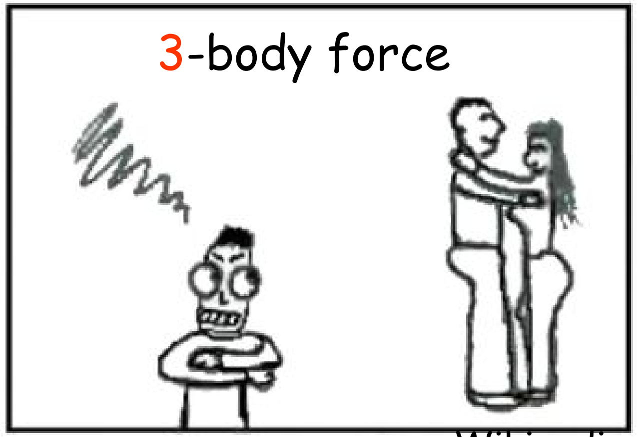
# Earth-moon-satellite system




2-body forces → 2+3-body forces

change in resolution

(RENORMALIZATION GROUP)




Wikipedia

# What is EFT?

A paradigm to describe nature

- Relevant degrees of freedom

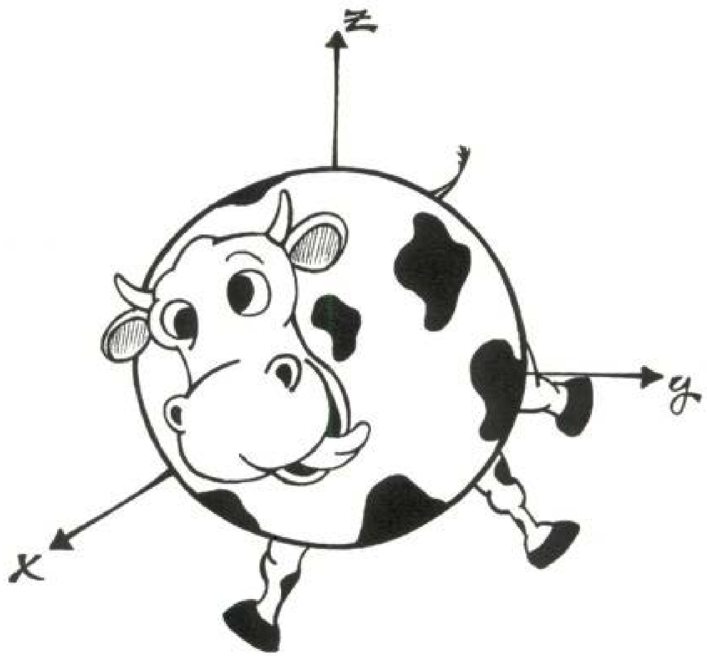
*choose the coordinates for the scales of the system*

- All possible interactions

*what is not forbidden is compulsory*

- Symmetries

A farmer is having trouble with a cow whose milk has gone sour.
He asks three scientists—a biologist, a chemist, and a physicist—to help him.
The biologist figures the cow must be sick or have some kind of infection,
but none of the antibiotics he gives the cow work.
Then, the chemist supposes that there must be a chemical imbalance affecting the production of milk,
but none of the solutions he proposes do any good either.
Finally, the physicist comes in and says, "First, we assume a spherical cow…"




$$\sum_{ij} \alpha_{ij} u_i v_j \to \vec{u} \cdot \vec{v} \qquad + \sum_{ij} \delta\alpha_{ij} u_i v_j$$

no, say, $u_1 v_2$

$$\left|\delta\alpha_{ij}\right| \ll 1$$

amenable to
perturbation theory

# What is EFT?

A paradigm to describe nature

- Relevant degrees of freedom

*choose the coordinates for the scales of the system*

- All possible interactions

*what is not forbidden is compulsory*

- Symmetries

*not everything is allowed*

- Naturalness

After scales have been identified,
the remaining, dimensionless parameters are

$$\mathcal{O}(1)$$

*unless suppressed by a symmetry*

't Hooft '79

*e.g.*, cow non-sphericity…

Occam's razor:
simplest assumption, to be revised if necessary

fine-tuning

Expansion in powers of $\frac{E}{E_{\rm hi}}$

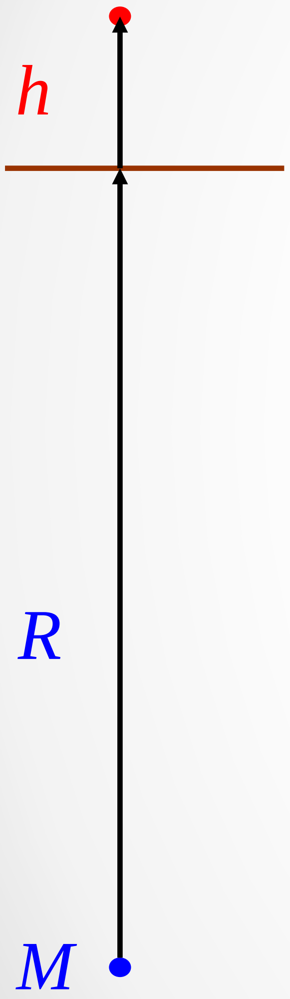
$E$: energy of process

$E_{\rm hi}$: energy scale of underlying theory

("POWER COUNTING")

# A classical example: the flat Earth

## light object near surface of a large body




$$E \sim mgh \ll E_{\rm hi} \equiv mgR$$

d.o.f.: point mass $m$

sym: $V_{\rm eff}(h,x,y) = V_{\rm eff}(h)$

$$V_{\rm eff}(h) = m\sum_{i=0}^{\infty} g_i h^i = {\rm const} + mgh\left\{1 + \frac{g_2}{g}h + \frac{g_3}{g}h^2 + \ldots\right\}$$

parameters

(neglecting quantum corrections…)

naturalness: $\dfrac{m g_i h^i}{m g_{i-1} h^{i-1}} = \dfrac{E}{E_{\rm hi}} \times \mathcal{O}(1) = \dfrac{h}{R} \times \mathcal{O}(1) \iff \dfrac{g_i}{g} = \mathcal{O}\left(\dfrac{1}{R^{i-1}}\right)$

😊

$$V_{\rm und}(h) = -GMm\frac{1}{R+h} = m\left(\frac{GM}{R^2}\right)\sum_{i=0}^{\infty}\left(\frac{-1}{R}\right)^{i-1} h^i \Rightarrow \frac{g_i}{g} = \frac{(-1)^{i-1}}{R^{i-1}}$$

$h \ll R$

$\frac{GM}{R^2} \equiv g$

itself the first term in a low-energy EFT of general relativity…

# What is EFT, *really*?

Euler + Heisenberg ’36
Weinberg ’67 ... ’79
Wilson, early 70s
...

d.o.f. → quantum field

“Folk Theorem”

Weinberg ‘79

“The quantum field theory generated by the most general Lagrangian with some assumed symmetries will produce the most general *S* matrix incorporating quantum mechanics, Lorentz invariance, unitarity, cluster decomposition and those symmetries, with no further physical content.”

Issue:

QM virtual processes sensitive to high energies
– the "problem of infinities" in QFT

From here on

$\hbar = 1, c = 1$

$[m] = [E] = [p] = [r]^{-1} = [t]^{-1}$

1. Arbitrary regularization: points → blobs of size $\sim \Lambda^{-1}$

2. Renormalization: interaction strengths remove $\Lambda^{n \geq 0}$

   Residual regulator dependence no larger than next-order interactions for $\Lambda^{-1} \lesssim M_{\text{hi}}^{-1}$

3. Naturalness: magnitude of interactions strengths estimated by $\Lambda^{-1} \to M_{\text{hi}}^{-1}$

# A quantum example: contact interaction

$$V(\vec{r}) = C_0\, \delta^{(3)}(\vec{r})$$

Sch eq $$-\left(\frac{\nabla^2}{2\mu} + E\right)\psi(\vec{r}) = C_0\, \delta^{(3)}(\vec{r})\psi(0) \xrightarrow{\text{F.T.}} \left(\frac{p^2}{2\mu} - E\right)\tilde{\psi}(\vec{p}) = C_0\, \psi(0)$$

bound state

$$E = -B \equiv -\frac{\kappa^2}{2\mu} < 0$$

$$\Rightarrow \tilde{\psi}(\vec{p}) = \frac{2\mu C_0\, \psi(0)}{p^2 + \kappa^2} \Rightarrow \psi(\vec{r}) = 2\mu C_0\, \psi(0) \int \frac{d^3 p}{(2\pi)^3} \frac{\exp\left(-i\vec{p}\cdot\vec{r}\right)}{p^2 + \kappa^2}$$

consistency: $$C_0^{-1} = 2\mu \int \frac{d^3 p}{(2\pi)^3} \frac{1}{p^2 + \kappa^2} = \frac{\mu}{\pi^2} \int_0^{\infty} dp\, \frac{p^2}{p^2 + \kappa^2}$$ diverges!

regularization

$$V(\vec{r}) \to V(\vec{r},\Lambda) = C_0(\Lambda)\,\delta^{(3)}_{\Lambda^{-1}}(\vec{r})$$

smeared delta function

renormalization

$$C_0^{-1}(\Lambda) = \frac{\mu}{\pi^2}\int_0^{\Lambda} dp\,\frac{p^2}{p^2+\kappa^2} = \frac{\mu}{\pi^2}\left(\int_0^{\Lambda} dp - \kappa^2\int_0^{\Lambda} dp\,\frac{1}{p^2+\kappa^2}\right) = \frac{\mu}{2\pi}\left(\frac{2\Lambda}{\pi} - \kappa + \mathcal{O}\left(\frac{\kappa^2}{\Lambda}\right)\right)$$

$$\Rightarrow \quad \psi(\vec{r}) = N\,\frac{\exp(-\kappa r)}{r}\left(1+\mathcal{O}\left(\frac{\kappa}{\Lambda}\right)\right)$$

Repeat for scattering

$$E \equiv \frac{k^2}{2\mu} > 0$$

S matrix — phase shift

$$T_0(k) = \frac{S_0(k) - 1}{2i\mu k} = \frac{2\pi}{\mu k}\left(\cot\delta_0(k) - i\right)^{-1}$$

$$= \frac{2\pi}{\mu}\left(\frac{2\pi}{\mu}C_0^{-1}(\Lambda) - \frac{2\Lambda}{\pi} - ik + \mathcal{O}\left(\frac{k^2}{\Lambda}\right)\right)^{-1}$$

$$= \frac{2\pi}{\mu}\left(\underbrace{-\kappa + \mathcal{O}\left(\frac{\kappa^2}{\Lambda}\right)}_{-a_2^{-1}} - ik + \underbrace{\mathcal{O}\left(\frac{k^2}{\Lambda}\right)}_{r_2 k^2/2}\right)^{-1}$$

scattering length — effective range

No infinities,
no cutoff dependence
except for terms that can
be made arbitrarily small

effectively $C_0 \to -\dfrac{2\pi}{\mu\kappa}$

cutoff removed by $\Delta V(\vec{r}) = C_2 \nabla^2 \delta^{(3)}(\vec{r})$

effectively $\dfrac{C_2}{C_0^2} \to \dfrac{\mu r_2}{4\pi}$

Naturalness:

$$C_0 = \mathcal{O}\left(\frac{2\pi}{\mu M_{\rm hi}}\right) \qquad \frac{C_2}{C_0^2} = \mathcal{O}\left(\frac{\mu}{2\pi M_{\rm hi}}\right) \qquad etc.$$

- perturbation theory at all orders
- $\kappa = \mathcal{O}(M_{\rm hi})$ outside EFT

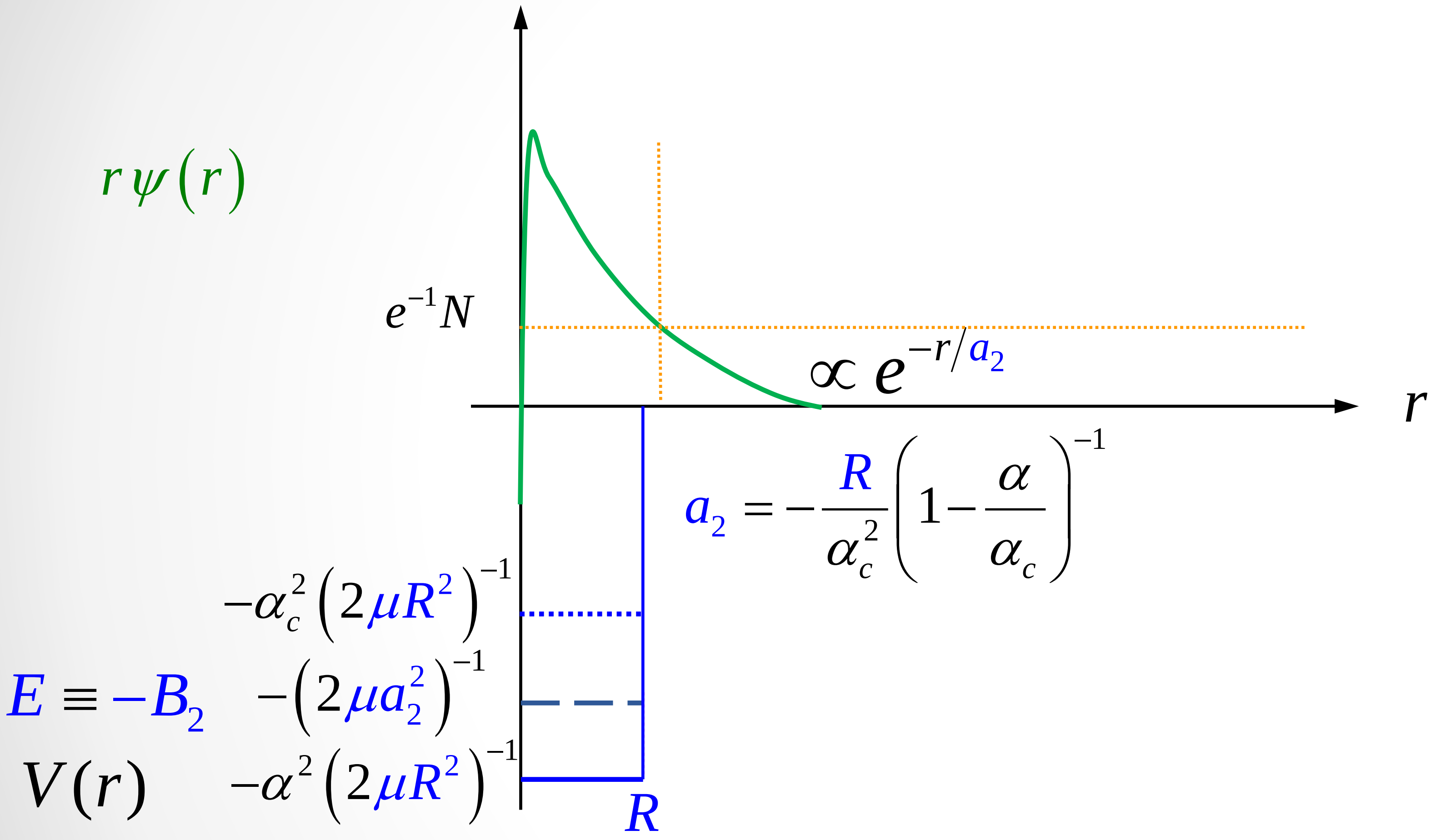
Fine tuning:

$$C_0 = \mathcal{O}\left(\frac{2\pi}{\mu M_{\rm lo}}\right) \qquad \frac{C_2}{C_0^2} = \mathcal{O}\left(\frac{\mu}{2\pi M_{\rm hi}}\right) \qquad etc.$$

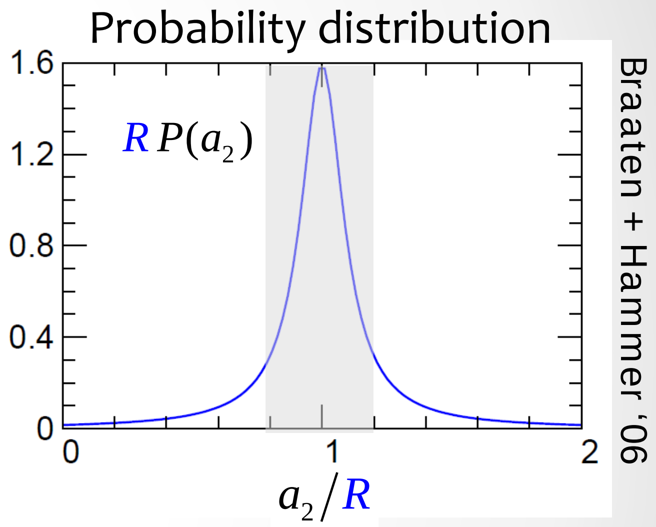
$$M_{\rm lo} \ll M_{\rm hi}$$

- amplitude nonperturbative at LO, distorted-wave perturbation at subLOs
- $\kappa = \mathcal{O}(M_{\rm lo})$ inside EFT

Toy model: $V(r) = -\frac{\alpha^2}{2\mu R^2}\,\theta\left(1 - \frac{r}{R}\right)$

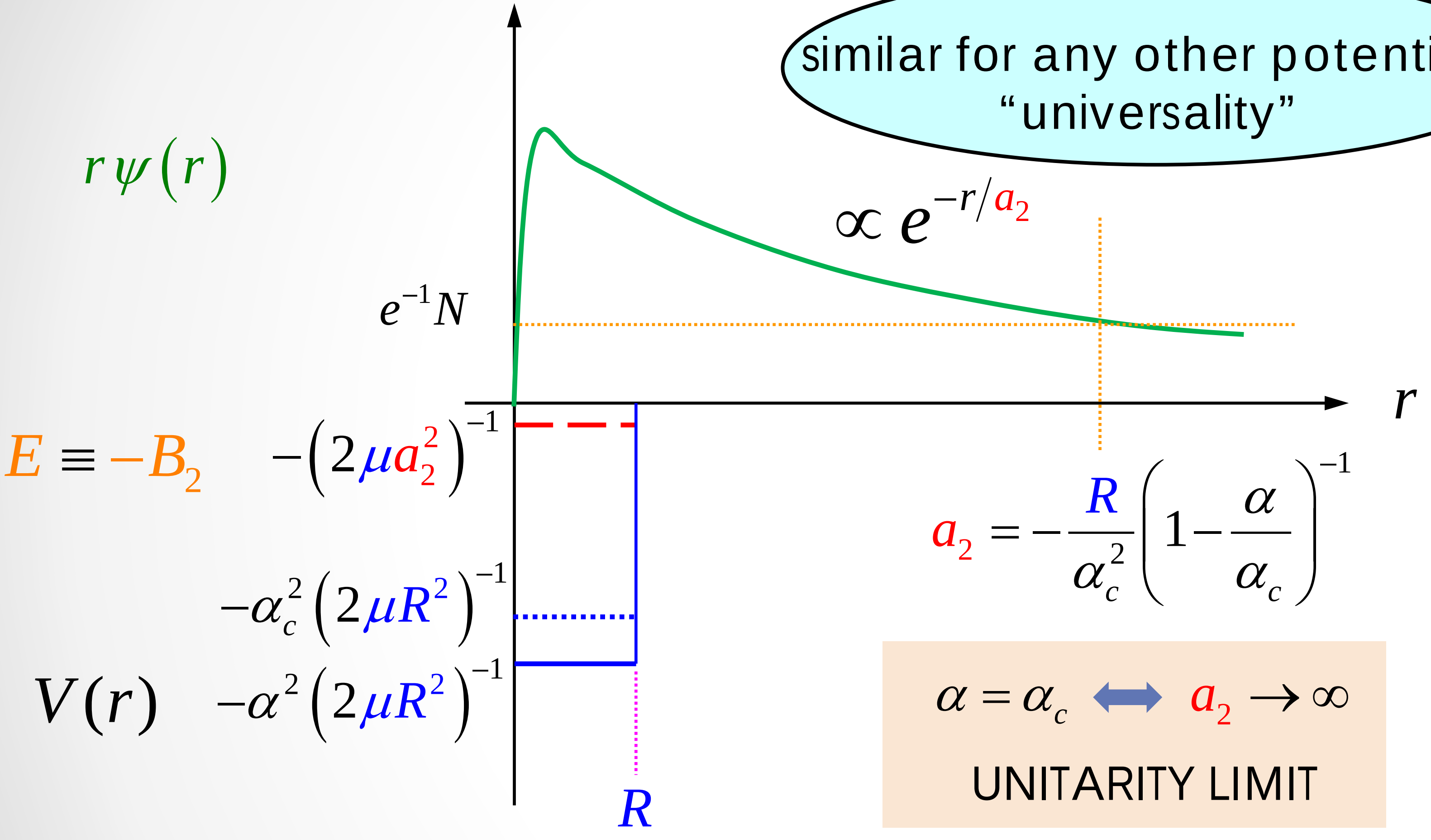



$\alpha_c \equiv (2n+1)\,\pi/2$



Toy model: $V(r) = -\frac{\alpha^2}{mR^2}\theta\left(1-\frac{r}{R}\right)$

similar for any other potential: "universality"



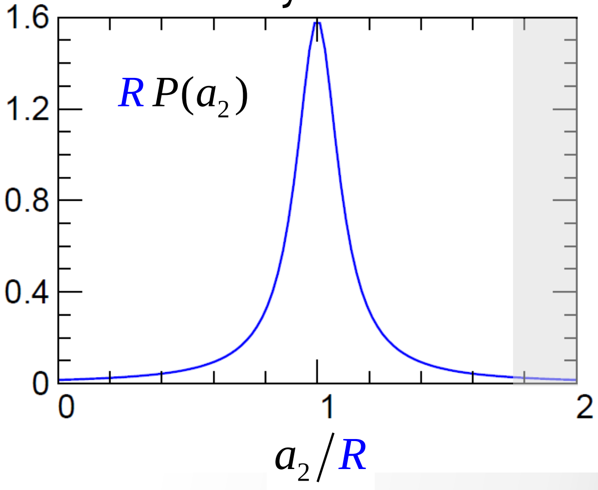

$$a_2 = -\frac{R}{\alpha_c^2}\left(1-\frac{\alpha}{\alpha_c}\right)^{-1}$$

$\alpha = \alpha_c \Leftrightarrow a_2 \to \infty$

UNITARITY LIMIT

$\alpha_c \equiv (2n+1)\pi/2$

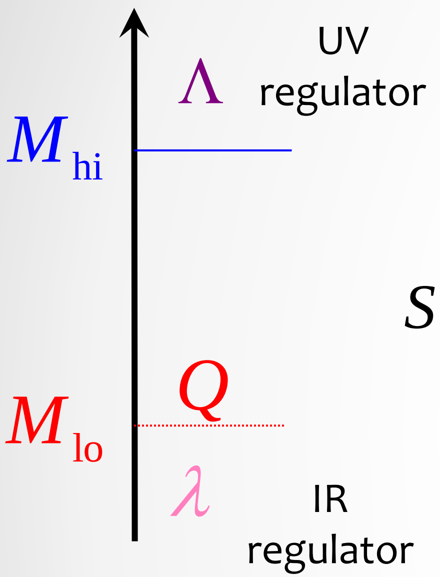
Probability distribution




Braaten + Hammer '06

In the quantum world, one **can** have a b.s. with size much larger than the range of the force *provided* there is fine-tuning *most* general $\mathcal{L} = \sum_{d,n,\ldots} \gamma_{d,n,\ldots} \partial^d \psi^n \ldots$

mass scales




$\nu = \nu(d, n, \ldots)$ "power counting"

order

expansion parameter

non-analytic functions, from dynamical eq

normalization

$$S^{(\bar\nu)}(Q \sim M_{lo} \ll M_{hi}) = 1 + \mathcal{N}(M_{hi}) \sum_{\nu=\nu_{min}}^{\bar\nu} \left[\frac{Q}{M_{hi}}\right]^{\nu} F_\nu\left(\frac{Q}{M_{lo}}, \frac{Q}{\Lambda}, \frac{\lambda}{Q}; \gamma_{\{\nu\}}\left(\frac{M_{lo}}{\Lambda}, \frac{\lambda}{M_{lo}}\right)\right)$$

$$\times \left\{1 + \mathcal{O}\left(\frac{Q^{\bar\nu+1}}{M_{hi}^{\bar\nu+1}}, \frac{Q^{\bar\nu+1}}{M_{hi}^{\bar\nu}\Lambda}, \frac{\lambda Q^{\bar\nu}}{M_{hi}^{\bar\nu+1}}\right)\right\}$$

"low-energy constants"

$N^{\bar\nu - \nu_{min}}LO$

CONTROLLED UNCERTAINTY

Want large "model space"

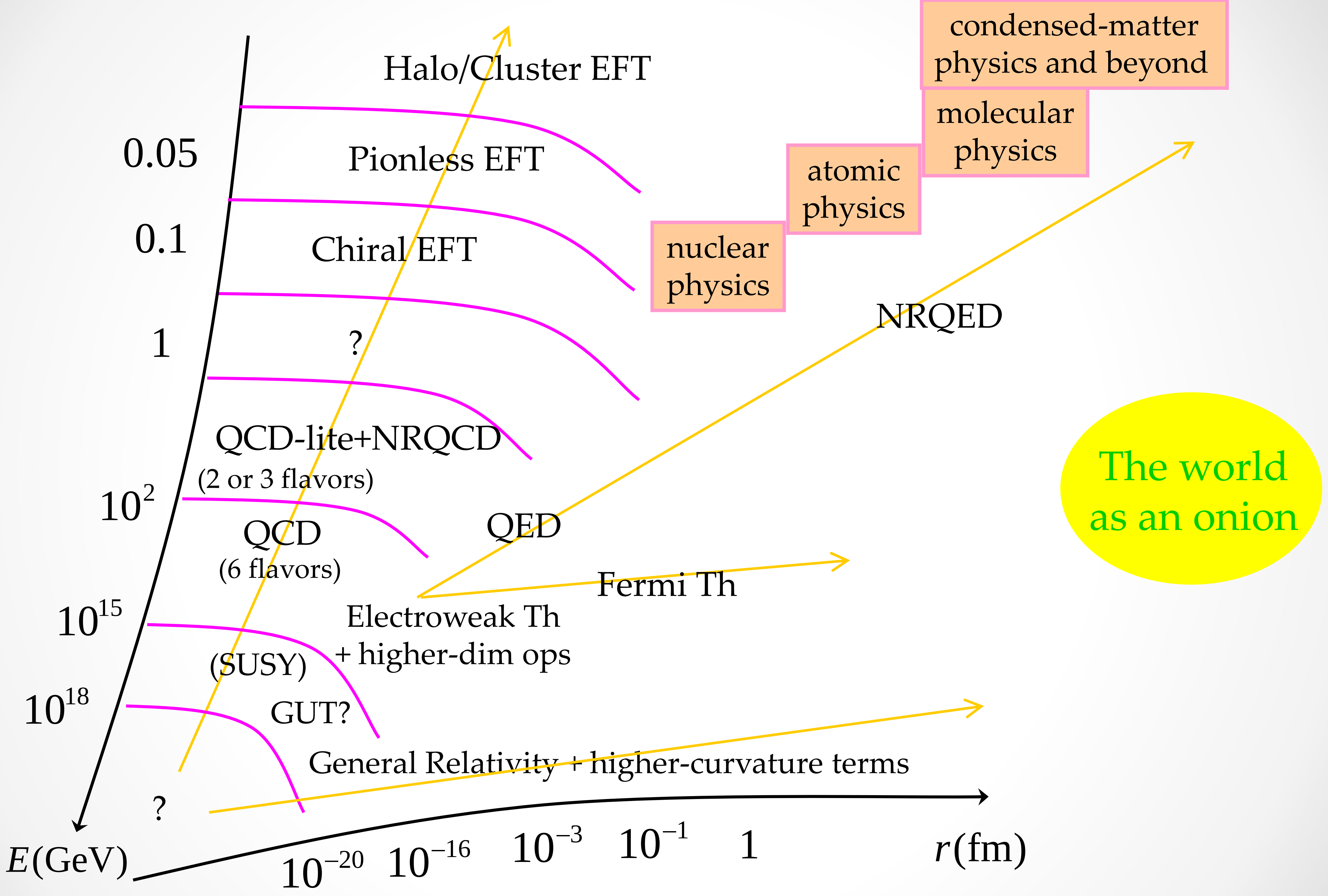
$\Lambda \gtrsim M_{hi}$

$\lambda \lesssim M_{lo}$

renormalization-group invariance

$$\left\{ \begin{array}{l} \dfrac{\Lambda}{S^{(\bar\nu)}} \dfrac{\partial S^{(\bar\nu)}}{\partial \Lambda} = \mathcal{O}\left(\dfrac{Q^{\bar\nu+1}}{M_{hi}^{\bar\nu}\Lambda}\right) \\ \dfrac{\lambda}{S^{(\bar\nu)}} \dfrac{\partial S^{(\bar\nu)}}{\partial \lambda} = \mathcal{O}\left(\dfrac{Q^{\bar\nu}\lambda}{M_{hi}^{\bar\nu+1}}\right) \end{array} \right.$$

MODEL INDEPENDENCE (insensitivity to high-mom details)



condensed-matter physics and beyond
molecular physics
atomic physics
nuclear physics
Halo/Cluster EFT
Pionless EFT
Chiral EFT
?
QCD-lite+NRQCD
(2 or 3 flavors)
QCD
(6 flavors)
QED
NRQED
Fermi Th
Electroweak Th
+ higher-dim ops
(SUSY)
GUT?
General Relativity + higher-curvature terms
?
0.05
0.1
1
$10^2$
$10^{15}$
$10^{18}$
$E$(GeV)
$10^{-20}$ $10^{-16}$ $10^{-3}$ $10^{-1}$ 1
$r$(fm)
The world as an onion

# QCD + …

EFT at a few GeV= underlying theory for nuclear physics

d.o.f.s leptons: $l_f = \begin{pmatrix} l^+ \\ \nu \end{pmatrix}_f$ quarks: $q = \begin{pmatrix} u \\ d \end{pmatrix}$ photon: $A_\mu$ gluons: $G_\mu^a$

symmetries: SO(3,1) global, $\mathrm{U_e}(1)$ gauge, $\mathrm{SU_c}(3)$ gauge

$$\mathcal{L}_{\text{und}} = \sum_{f=1}^{3} \bar{l}_f \left( i\partial\!\!\!/ + eQ_l A\!\!\!/ - m_f \right) l_f - \frac{1}{4} F_{\mu\nu} F^{\mu\nu}$$

$$+ \bar{q}\left( i\partial\!\!\!/ + eQ_q A\!\!\!/ + g_s G\!\!\!/ \right) q - \frac{1}{2} \mathrm{Tr}\left[ G_{\mu\nu} G^{\mu\nu} \right]$$

$$- \frac{1}{2}\left( m_u + m_d \right) \bar{q} q - \frac{1}{2}\left( m_u - m_d \right) \bar{q} \tau_3 q$$

QED + QCD-lite

$$Q_l = \begin{pmatrix} 1 & 0 \\ 0 & 0 \end{pmatrix}$$

$$Q_q = \begin{pmatrix} 2/3 & 0 \\ 0 & -1/3 \end{pmatrix} = \frac{1 + 3\tau_3}{6}$$

Pauli matrix

$$+ \frac{m_u m_d}{m_u + m_d} \bar{\theta}\, \bar{q} i \gamma_5 q + \ldots$$

higher-dimension interactions: suppressed by larger masses

e.g. $G_F \propto 1/M_{W,Z}^2$

unnaturally small T violation (strong CP problem)

$\bar{\theta} \lesssim 10^{-9}$

# Focus on strong-interacting sector: four parameters

1) $m_u = m_d = 0, \; e = 0, \; \bar{\theta} = 0$ "chiral limit"

single, *dimensionless* parameter

$$\int d^4x \, \mathcal{L}_{\rm QCD} = \int d^4x \left\{ \bar{q}\left(i\partial\!\!\!/ + g_s G\!\!\!/\right)q - \frac{1}{2}\mathrm{Tr}\, G^{\mu\nu}G_{\mu\nu} \right\}$$

invariant under scale transformations $\begin{cases} x \to \lambda^{-1}x \\ q \to \lambda^{3/2}q \\ G \to \lambda G \end{cases}$

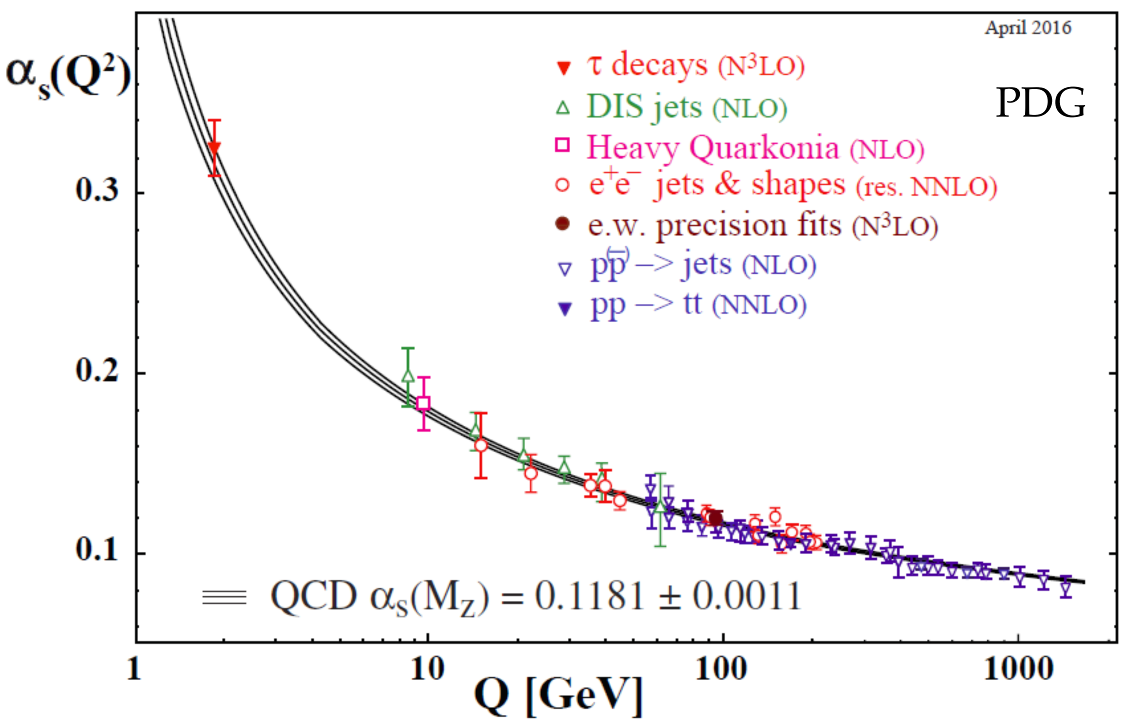
$$Z = \int DG \int D\bar{q} \int Dq \, \exp\left(i\int d^4x \, \mathcal{L}_{\rm QCD}\right)$$

scale invariance "anomalously broken"
by *dimensionful* regulator

⇨ coupling "runs"

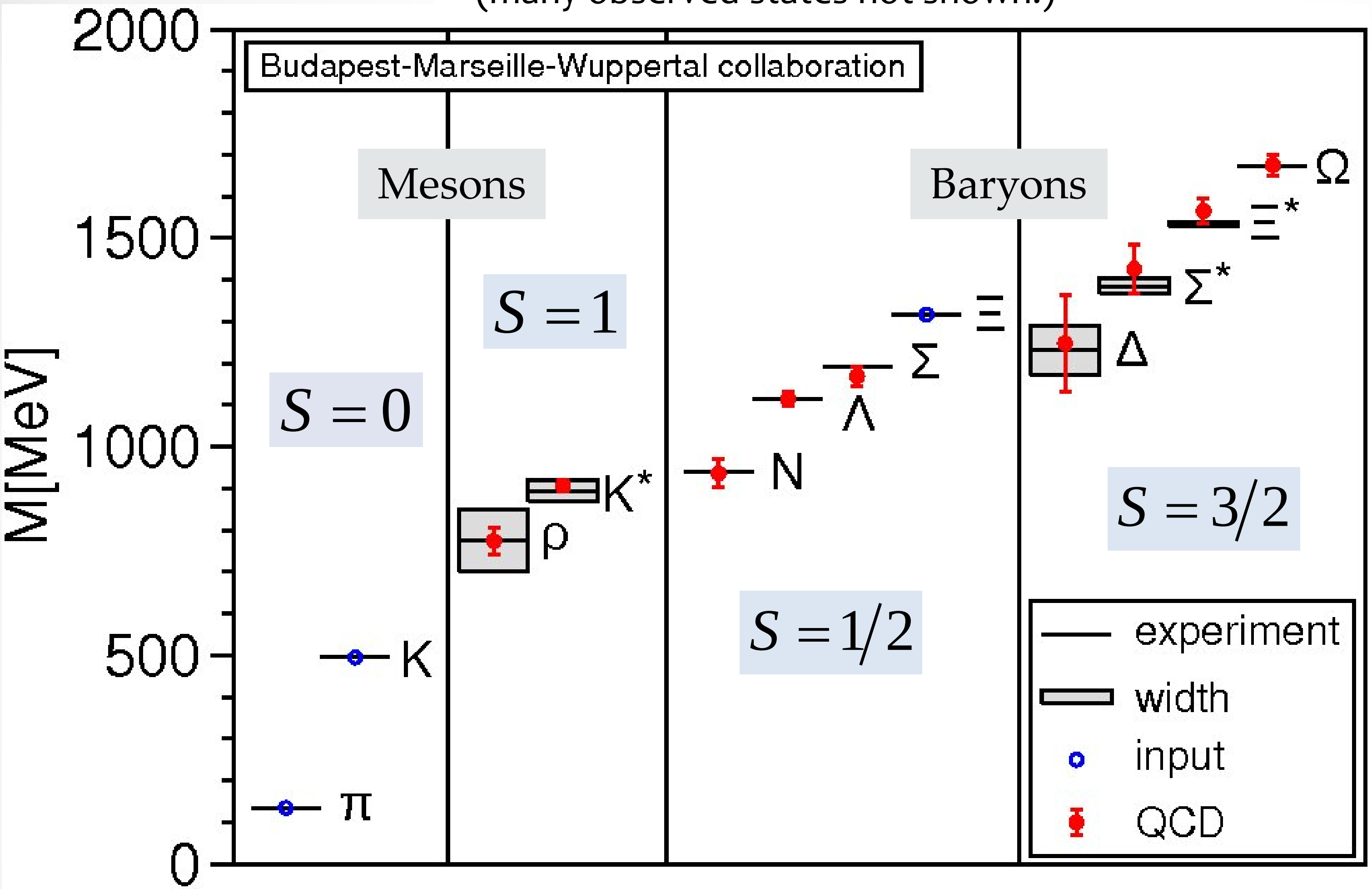
$$\alpha_s\left(Q \sim 1\,\mathrm{GeV}\right) \sim 1$$

("dimensional transmutation")



Strongly interacting particles (hadrons)

(many observed states not shown!)




naturalness

QCD scale

$M_{\rm QCD} \sim 1$ GeV

Exception: pion

$m_\pi \simeq 140$ MeV $\ll M_{\rm QCD}$

breakdown of naturalness? NO!

“spontaneous breaking” of chiral symmetry

# Why is the pion special?

$$\mathcal{L}_{QCD} = \bar{q}_L \left( i\not{\partial} + g_s \not{G} \right) q_L + \bar{q}_R \left( i\not{\partial} + g_s \not{G} \right) q_R - \frac{1}{2} \mathrm{Tr}\, G^{\mu\nu} G_{\mu\nu}$$

$$q = \begin{pmatrix} u \\ d \end{pmatrix} \qquad q_L = \frac{1-\gamma_5}{2} q \qquad q_R = \frac{1+\gamma_5}{2} q$$

invariant under

$$q_L \to \exp\left( i \boldsymbol{\alpha}_L \cdot \boldsymbol{\tau} \right) q_L$$
$$q_R \to \exp\left( i \boldsymbol{\alpha}_R \cdot \boldsymbol{\tau} \right) q_R$$

$$\mathrm{SU}(2)_L \times \mathrm{SU}(2)_R \sim \mathrm{SO}(4)$$

chiral symmetry

but

$$m_\sigma \gg m_\pi$$
$$m_{N_-} \gg m_{N_+}$$

***broken*** by vacuum down to

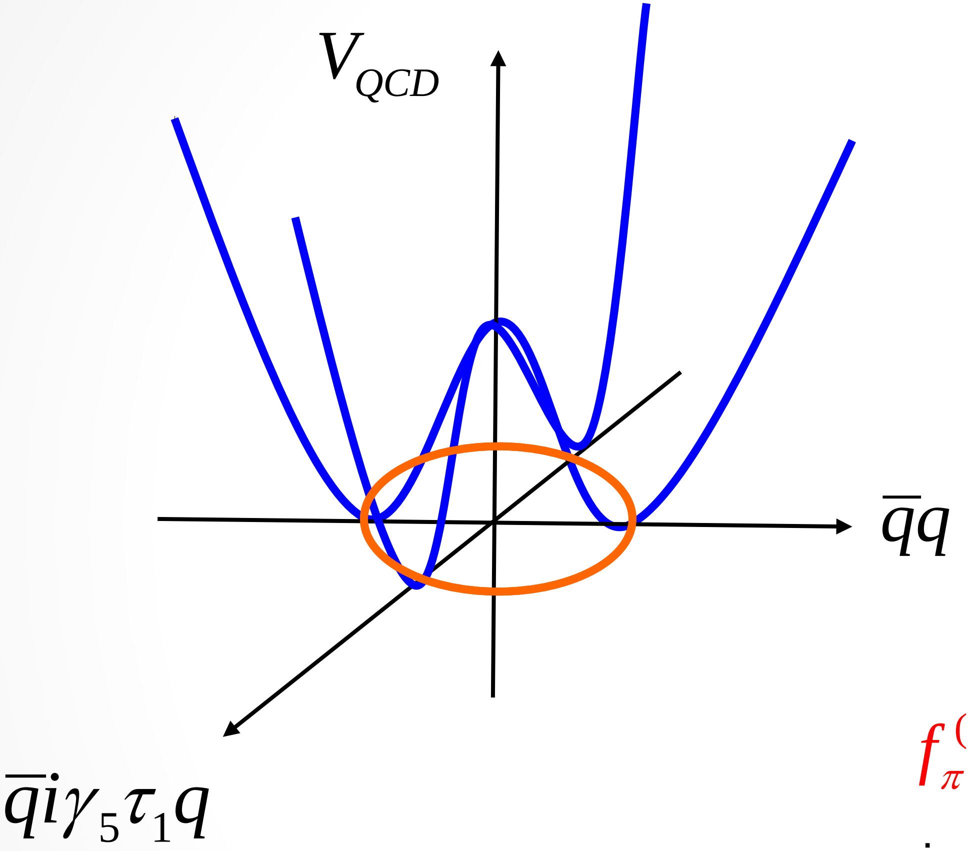
$$q \to \exp\left[ i \left( \boldsymbol{\alpha}_L + \boldsymbol{\alpha}_R \right) \cdot \boldsymbol{\tau} \right] q$$

$$\mathrm{SU}(2)_{L+R} \sim \mathrm{SO}(3)$$

isospin

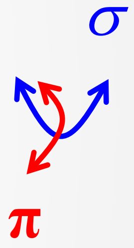
(axial transformations broken)

chiral limit




two isospin axis not shown

“chiral circle”




$$f_\pi^{(0)} = \mathcal{O}\left(M_{\text{QCD}}/4\pi\right)$$

pion decay constant (in chiral limit)

$$m_\sigma^2 = \mathcal{O}\left(M_{\text{QCD}}^2\right)$$

$$m_\pi^2 = 0$$

2) $m_u = m_d \equiv \overline{m} \neq 0,\ e = 0,\ \overline{\theta} = 0$

Weinberg '79

$$\mathcal{L}_{\text{QCD}} = \overline{q}\left(i\partial\!\!\!/ + g_s G\!\!\!/\right)q - \frac{1}{2}\text{Tr}\,G^{\mu\nu}G_{\mu\nu} + \overline{m}\,\underbrace{\overline{q}q} + \ldots$$

4th component of $S = \left(\overline{q}i\gamma_5\boldsymbol{\tau}q,\ \overline{q}q\right)$

$\text{SO}(4)$ vector!

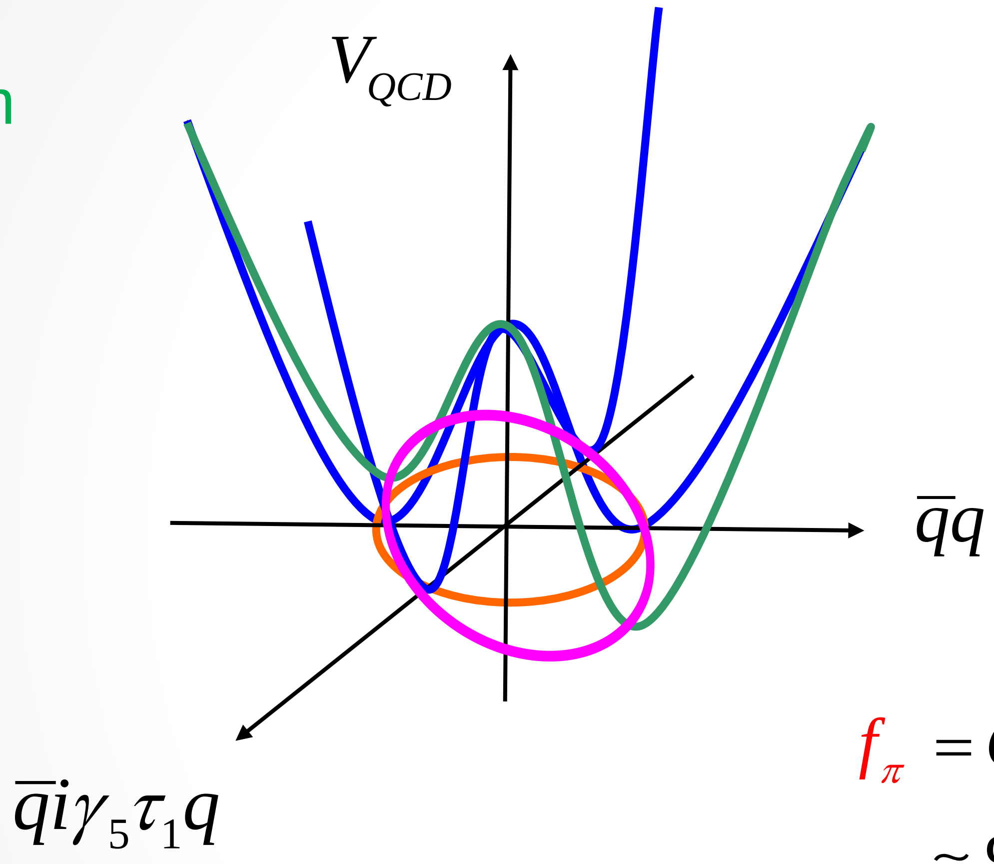
***breaks*** $\text{SO}(4) \rightarrow \text{SO}(3)$

*explicit* chiral-symmetry breaking

empirically $\overline{m} \sim 5\ \text{MeV}$

away from chiral limit

two isospin axis not shown




slightly-tilted chiral circle


$$f_\pi = \mathcal{O}\left(M_{\text{QCD}}/4\pi\right)$$
$$\simeq 93\ \text{MeV}$$

pion decay constant

$$m_\sigma^2 = \mathcal{O}\left(M_{\text{QCD}}^2\right)$$
$$m_\pi^2 = \mathcal{O}\left(\bar{m}\, M_{\text{QCD}}\right) \ll M_{\text{QCD}}^2$$

vK '93
Hockings, Mereghetti + vK '10

3) $m_d - m_u \equiv 2\varepsilon\bar{m} \neq 0,\ e \neq 0,\ \bar{\theta} \neq 0$

$$\mathcal{L}_{QCD} = \ldots \quad + \varepsilon\bar{m}\,\bar{q}\tau_3 q + e\,\bar{q}\not{A}Q_q q - \frac{1}{4}F_{\mu\nu}F^{\mu\nu} + 2(1-\varepsilon^2)\bar{m}\bar{\theta}\,\bar{q}i\gamma_5 q + \ldots$$

3rd component of

$$P = \left(\bar{q}\boldsymbol{\tau}q,\ \bar{q}i\gamma_5 q\right)$$

SO(4) vector

- "soft" photons – explicit d.o.f. in EFT
- "hard" photons – "integrated out" of EFT

34 component of

4th component of

$$P = \left(\bar{q}\boldsymbol{\tau}q,\ \bar{q}i\gamma_5 q\right)$$

*same* SO(4) vector!

SO(4) antisymmetric tensor $F_\mu = \begin{pmatrix} \varepsilon_{ijk}\bar{q}i\gamma_\mu\gamma_5\tau_k q & \bar{q}i\gamma_\mu\tau_j q \\ -\bar{q}i\gamma_\mu\tau_i q & 0 \end{pmatrix}$

***break*** SO(4) (and SO(3) in particular) $\rightarrow$ U(1)

isospin violation

empirically $\varepsilon \sim e \sim 1/3$

T

(linked to isospin violation)

empirically $\bar{\theta} \lesssim 10^{-9}$

4) higher-dimensional operators

*e.g.*

P-violating four-quark operators

$$\mathcal{O}\left(Q^2/M^2_{W,Z}\right)$$

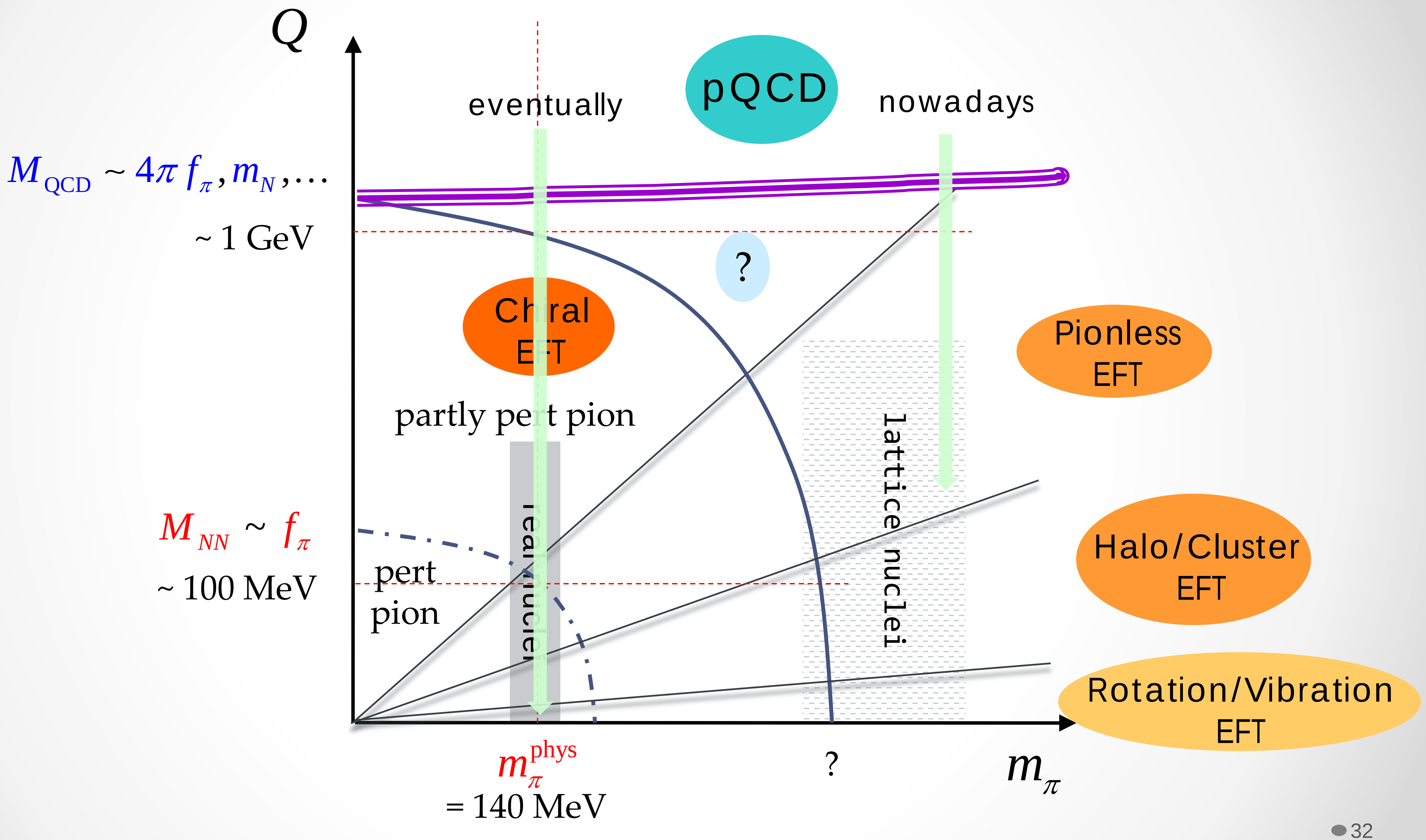
Kaplan + Savage ’96
Zhu, Maekawa, Holstein, Musolf + vK ’02

T-violating quark EDM and color-EDM

$$\mathcal{O}\left(Q^2/M^2_{\not T}\right)$$

De Vries, Mereghetti, Timmermans + vK ’12

# The Nuclear EFT Landscape



# Chiral EFT

$Q \sim M_{lo} \equiv m_\pi \ll M_{QCD} \equiv M_{hi}$

**d.o.f.s** nucleons, pions, photon + Deltas $(m_\Delta - m_N \sim 2m_\pi)$ + Roper? $(m_{N'} - m_N \sim 3m_\pi)$

$$N = \begin{pmatrix} p \\ n \end{pmatrix} \qquad \boldsymbol{\pi} = \begin{pmatrix} \pi_1 \\ \pi_2 \\ \pi_3 \end{pmatrix} = \begin{pmatrix} (\pi^+ + \pi^-)/\sqrt{2} \\ -i(\pi^+ - \pi^-)/\sqrt{2} \\ \pi^0 \end{pmatrix} \qquad A_\mu \qquad \Delta = \begin{pmatrix} \Delta^{++} \\ \Delta^+ \\ \Delta^0 \\ \Delta^- \end{pmatrix} \qquad N' = \begin{pmatrix} p' \\ n' \end{pmatrix}$$

**symmetries:** SO(3,1) global, $U_e(1)$ gauge, ~~$SU_L(2) \times SU_R(2)$~~ global

Non-linear realization of chiral symmetry

Weinberg '68
Callan, Coleman, Wess + Zumino '69

chiral invariants $\propto Q$ $\quad + S_4$'s, $P_3$'s, $F_{34}$'s, …

chiral covariant derivatives

pion $\quad \boldsymbol{D}_\mu \equiv \left(\frac{D_\mu \boldsymbol{\pi}}{f_\pi}\right)\left(1 - \frac{\boldsymbol{\pi}^2}{4 f_\pi^2} + \ldots\right)$

fermions $\quad \mathcal{D}_\mu \equiv D_\mu - \frac{i}{2}\boldsymbol{\tau} \cdot \left(\frac{\boldsymbol{\pi}}{f_\pi} \times \boldsymbol{D}_\mu\right)$

gauge covariant derivative

$D_\mu \equiv \partial_\mu - ieQA_\mu$

$\propto \bar{m} \sim m_\pi^2 / M_{QCD}$

$\propto \varepsilon \bar{m} \sim \varepsilon\, m_\pi^2 / M_{QCD}$

$\propto e^2 / 4\pi$

see for example Weinberg's Quantum Theory of Fields, vol 2

**CHIRAL SYMMETRY ⇨ WEAK PION INTERACTIONS**

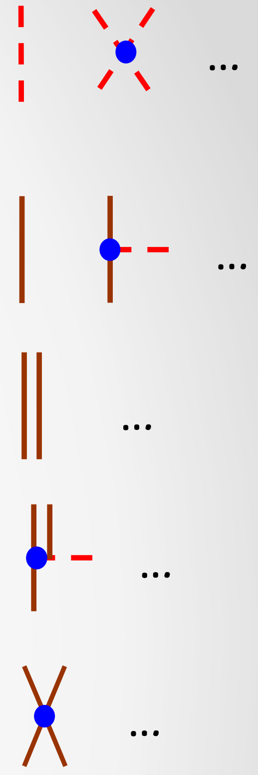
$$\mathcal{L}_{\chi\mathrm{EFT}} = \frac{1}{2}(\partial_\mu \boldsymbol{\pi})^2\left(1-\frac{\boldsymbol{\pi}^2}{2f_\pi^2}+\ldots\right) - \frac{1}{2}m_\pi^2\boldsymbol{\pi}^2\left(1-\frac{\boldsymbol{\pi}^2}{4f_\pi^2}+\ldots\right)$$

$$+N^+\left[i\partial_0 - \frac{1}{4f_\pi^2}\boldsymbol{\tau}\cdot(\boldsymbol{\pi}\times\partial_0\boldsymbol{\pi}) + \frac{\nabla^2}{2m_N}+\ldots+\frac{g_A}{2f_\pi}\boldsymbol{\tau}\vec{\sigma}\cdot\cdot(\vec{\nabla}\boldsymbol{\pi})\left(1-\frac{\boldsymbol{\pi}^2}{4f_\pi^2}+\ldots\right)+\ldots\right]N$$

$$+\Delta^+\left[i\partial_0-\left(m_\Delta-m_N\right)+\ldots\right]\Delta$$

$$+\Delta^+\left[\frac{h_A}{2f_\pi}\mathbf{T}\vec{S}\cdot\cdot(\vec{\nabla}\boldsymbol{\pi})\left(1+\ldots\right)+\ldots\right]N+\mathrm{H.c.}$$

$$-C_S\left(N^+N\right)^2-C_T\left(N^+\vec{\sigma}N\right)^2+\ldots$$




*Form* of pion interactions
determined by
chiral symmetry

low-energy constants
to be fitted to QCD
or experimental data

naturalness

$$f_\pi = \mathcal{O}\left(M_{\mathrm{QCD}}/4\pi\right) \qquad m_\pi^2=\mathcal{O}\left(\bar{m}M_{\mathrm{QCD}}\right)$$

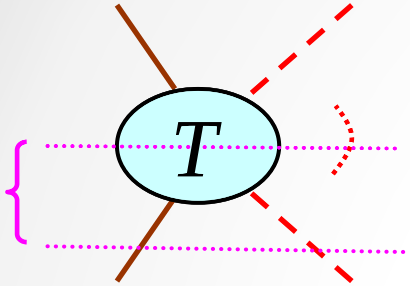
$$m_N = \mathcal{O}\left(M_{\mathrm{QCD}}\right) \qquad g_A, h_A = \mathcal{O}(1)$$

$$C_{S,T} = \mathcal{O}\left(4\pi/m_N f_\pi\right) \qquad \ldots$$

# $A$= 0, 1: Chiral Perturbation Theory

Weinberg '79
Gasser + Leutwyler '84
...
Gasser, Sainio + Švarč '87
Bernard, Kaiser + Meißner '90
Jenkins + Manohar '91
...

$$\frac{1}{\Delta E} \sim \frac{1}{Q}$$




$$\sim \sum_{\nu} c_\nu \left(\frac{Q}{M_{\rm QCD}}\right)^{\nu} F_\nu\left(\frac{Q}{m_\pi}\right)$$

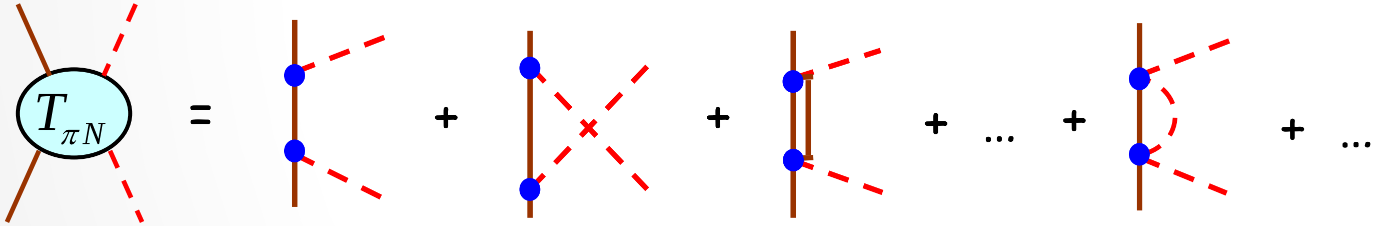
$$\nu = 2 - A + 2L + \sum_i \left(d_i + \frac{f_i}{2} - 2\right) \geq \nu_{\rm min} = 2 - A$$

\# loops

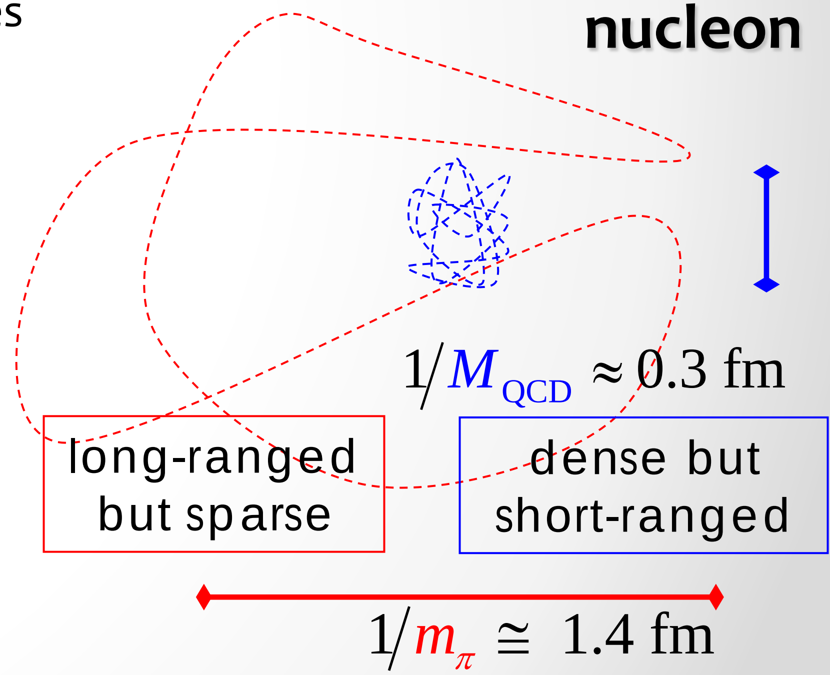
sum over vertices

\# derivatives/pion masses

\# fermion fields

e.g. 


**slow-moving nucleon**



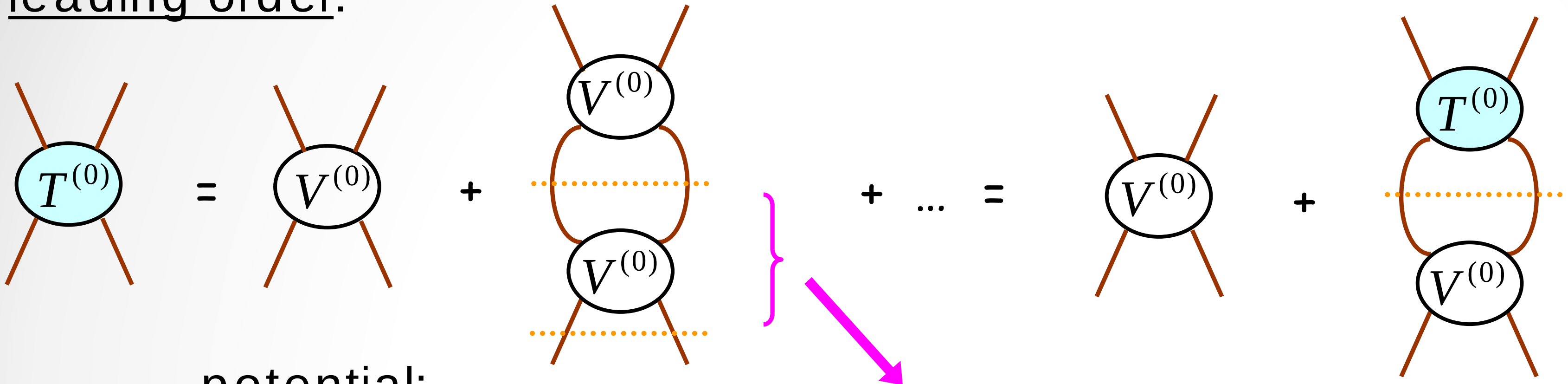

expansion in

$$\frac{Q}{M_{\rm QCD}} \sim \begin{cases} Q/m_N & \text{non-relativistic} \\ Q/m_\rho, \ldots & \text{multipole} \\ Q/4\pi f_\pi & \text{pion loop} \end{cases}$$

# A $\geq$ 2: resummed Chiral Perturbation Theory

Weinberg ‘90 ‘91

leading order:




Lippmann-Schwinger eq.
→ Schrödinger eq.

potential:
A-nucleon irreducible

A-nucleon reducible

$$\frac{1}{\Delta E} \sim \frac{m_N}{Q^2}$$

infrared enhancement!

$$\sim \frac{4\pi}{m_N f_\pi}\left\{1+\mathcal{O}\left(\frac{4\pi}{m_N f_\pi}\frac{Q^3}{4\pi}\frac{m_N}{Q^2}\right)+\ldots\right\} \sim \frac{4\pi}{m_N f_\pi}\frac{1}{1-\mathcal{O}\left(\frac{Q}{f_\pi}\right)}$$

$$= \frac{m_N Q}{4\pi}$$ reducible loop

bound state or resonance at

$$Q \sim f_\pi \Rightarrow |E| \sim \frac{Q^2}{m_N} \sim \frac{f_\pi^2}{M_{\rm QCD}}$$

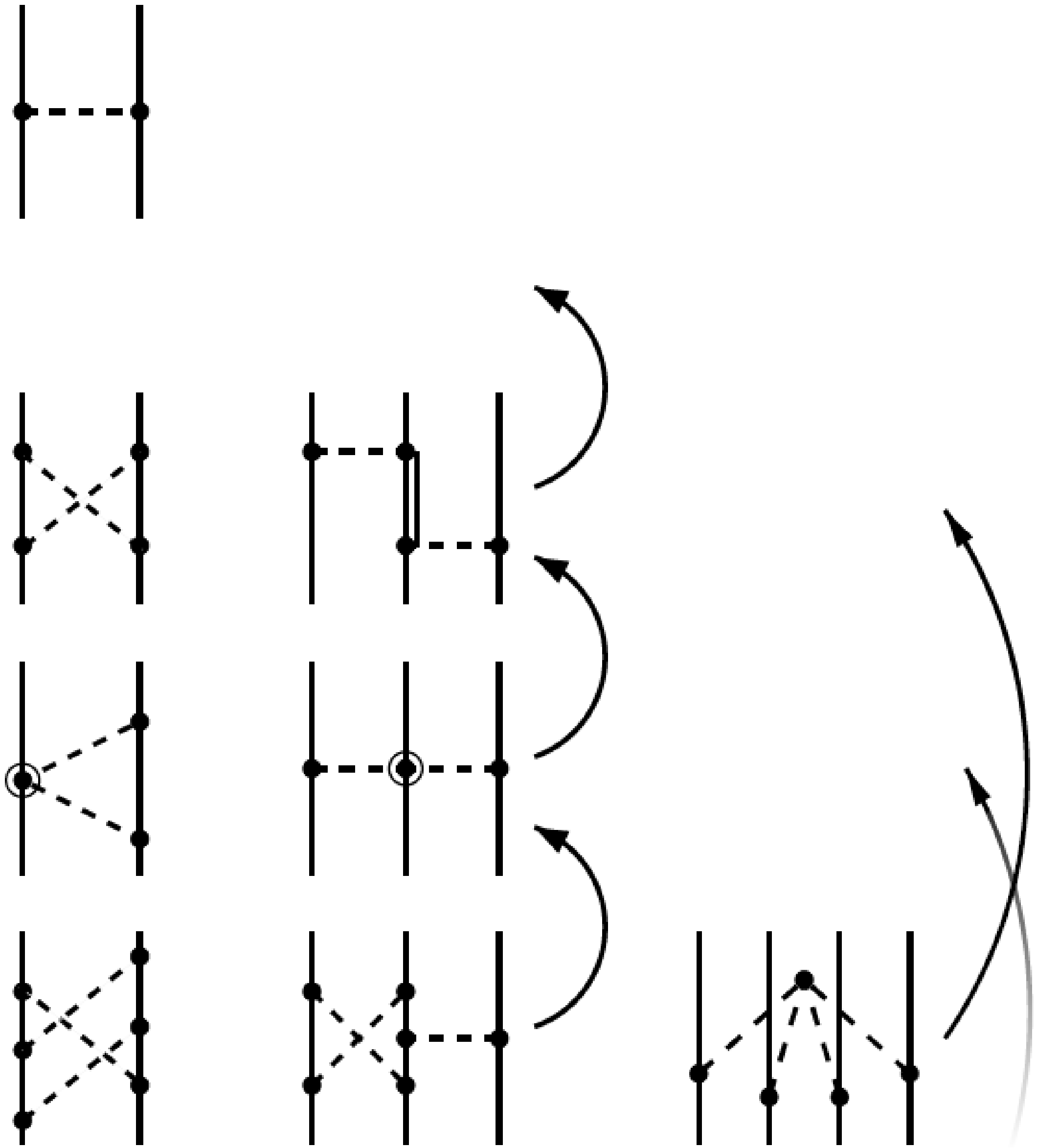
$$\approx 10 \text{ MeV}$$

subleading orders: expansion in $Q/M_{\rm QCD}$

$M_{hi} \sim M_{QCD}$

long-range potential

| | | 2N | 3N | 4N | … |
|---|---|---|---|---|---|
| LO | $O(1)$ | | | | |
| NLO | $O(Q/M_{hi})$ | | | | |
| NNLO | $O(Q^2/M_{hi}^2)$ | | | | |
| NNNLO | $O(Q^3/M_{hi}^3)$ | | | | |
| NNNNLO | $O(Q^4/M_{hi}^4)$ | | | | |
| in popular use | … | | | | |



Weinberg '90'91'92
Ordóñez + vK '92
vK '94
…

Friar '97

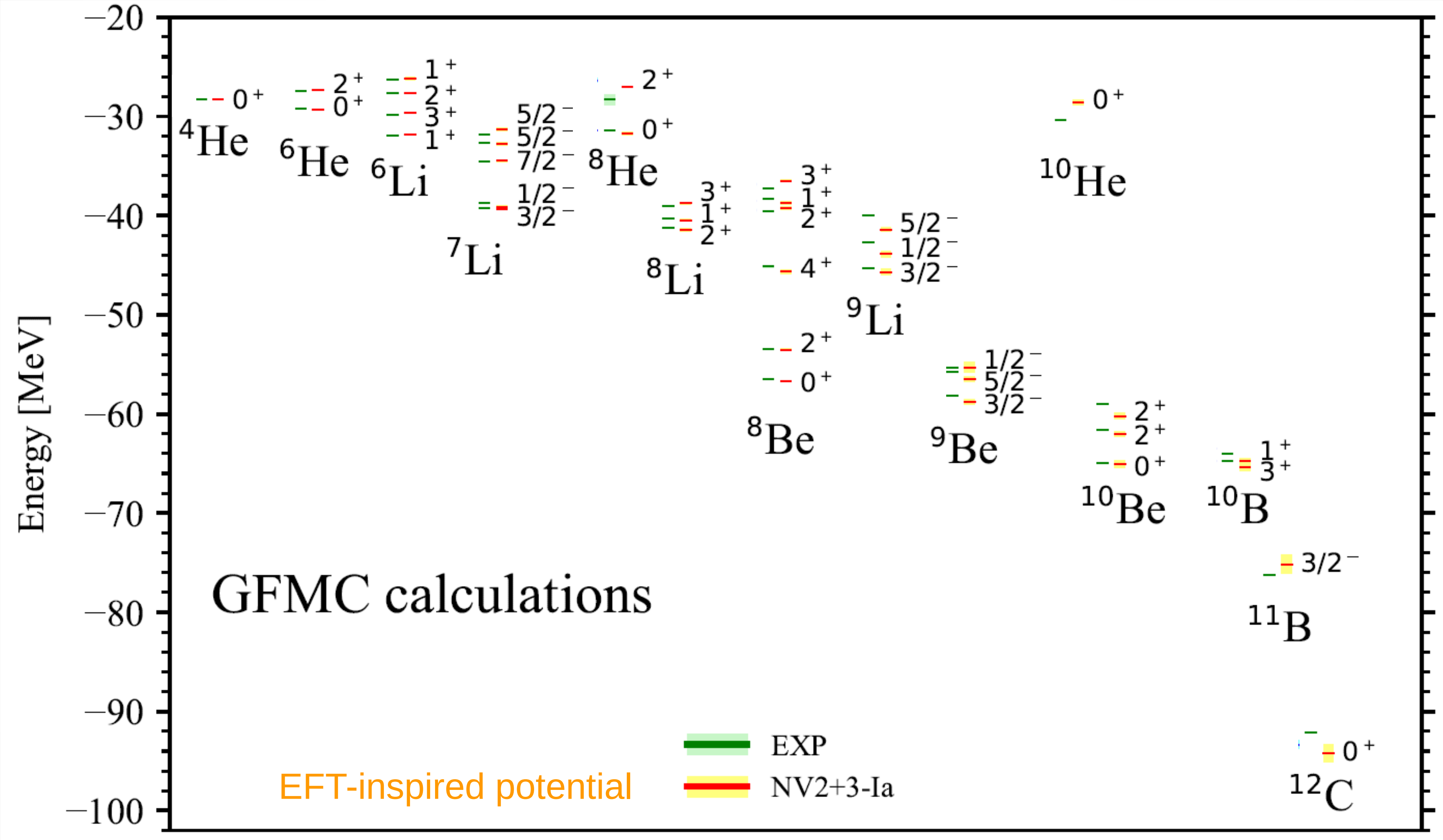
# Weinberg’s prescription

adopted almost exclusively by nuclear physicists;
successful phenomenologically

- Naturalness under the assumption that reducible loops do not require further renormalization

- Schrödinger eq. solved exactly for truncated potential even at subLO

# Example




M. Piarulli *et al.*, PRL 2018

# Problems with Weinberg's prescription

adopted almost exclusively by nuclear physicists only
mostly successful phenomenologically only at high orders

- Naturalness under the assumption that reducible loops do not require further renormalization

  Reducible loops <u>do</u> require further renormalization
  → enhanced contact interactions

- Schrödinger eq. solved exactly for truncated potential even at subLO

  Loss of renormalizability
  → perturbation in subleading interactions necessary

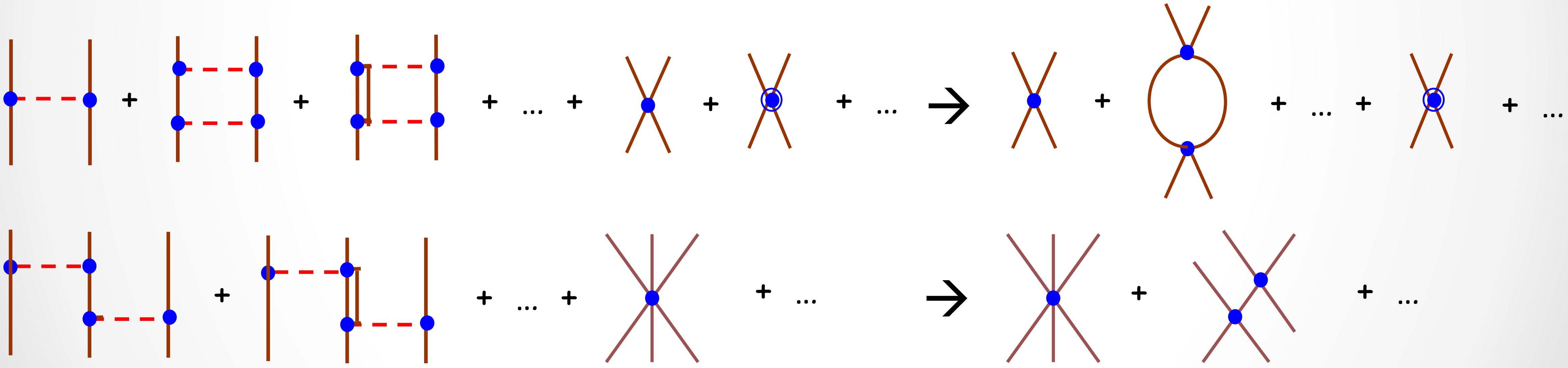
Oscar Edmund Berninghaus
A wagon train...

THE FRONTIER

# Pionless EFT

$$Q \sim M_{\text{lo}} \equiv \sqrt{m_N B_3} \ll m_\pi \equiv M_{\text{hi}}$$

- Integrate out pions + Deltas (+ Roper?) from Chiral EFT
- Chiral symmetry plays no role




*etc.*

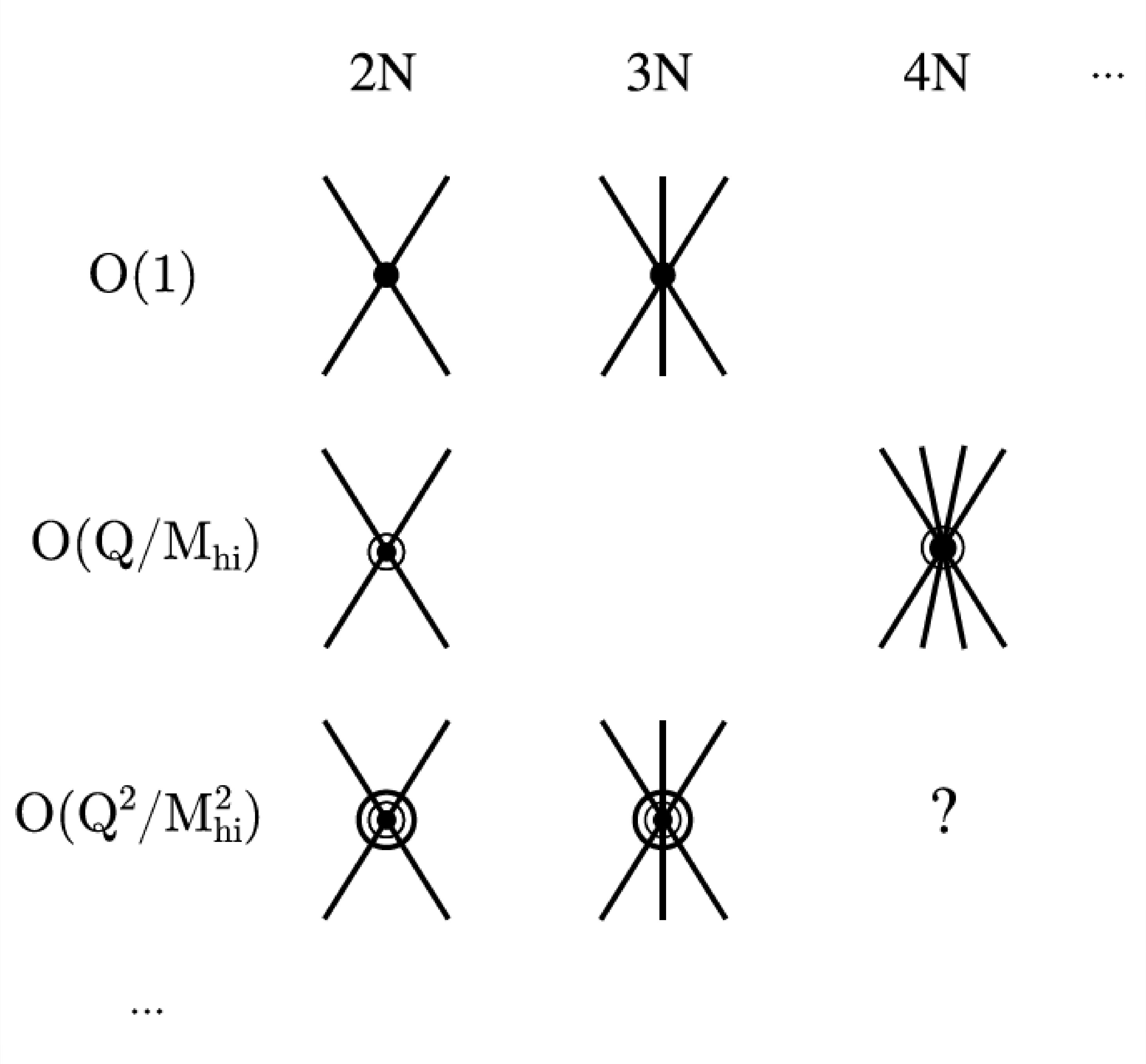
$M_{\text{hi}} \sim m_\pi$

potential

LO

NLO

NNLO



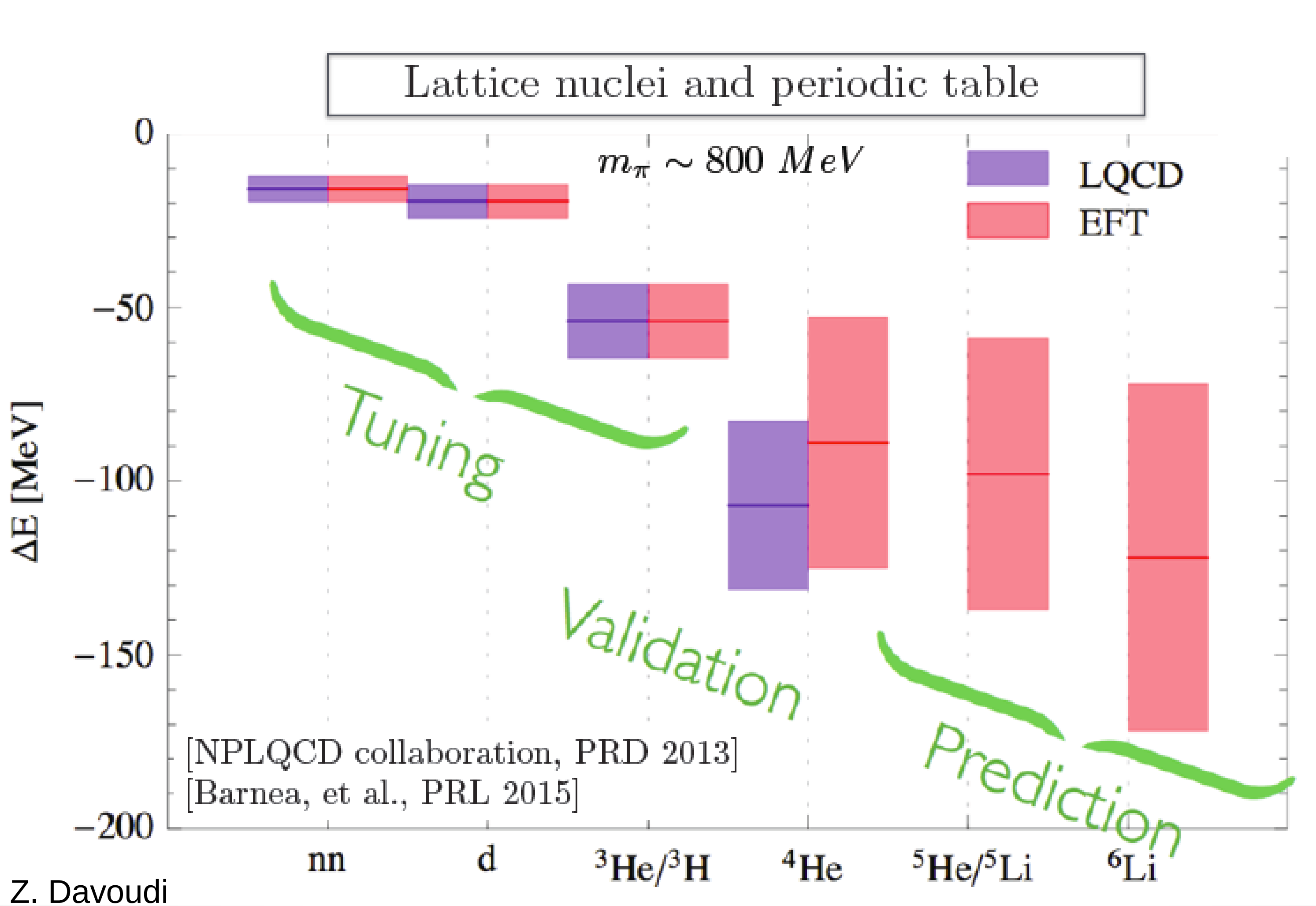

vK '97
Bedaque, Hammer + vK '98
Bazak, Kirscher, König, Pavón Valderrama, Barnea + vK '19

- ✓ Fully renormalized with existing power counting
- ✓ Convergent within range of applicability
- ✓ Equivalent to effective-range expansion in two-body system
- ✓ Approximate discrete scale invariance in more-body systems

  -- “Efimov physics”
- ✓ Can be matched to lattice QCD at unphysically large quark masses
- ✓ Applicable to any short-range interaction, *e.g.* atomic systems

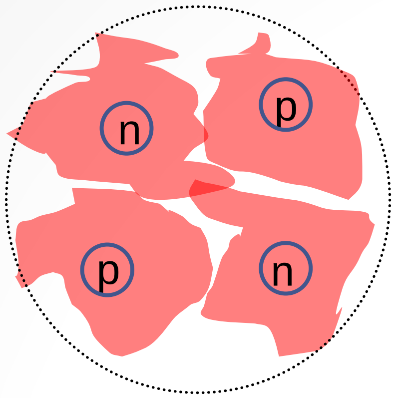
  -- “universality”

# Example




Z. Davoudi

*typical nucleus*




$M_{QCD}^{-1}$

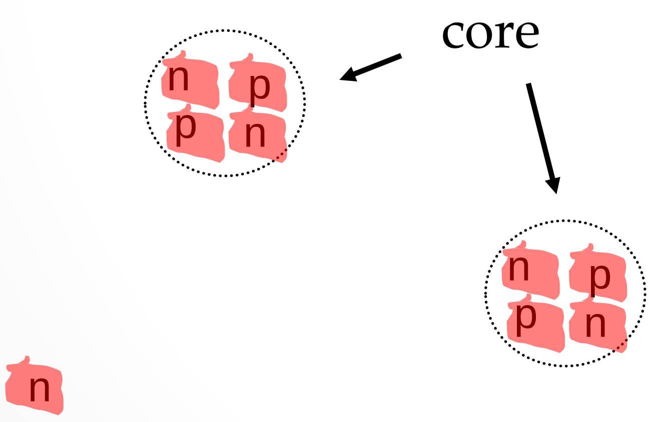
$m_\pi^{-1} \gg M_{QCD}^{-1}$

$M_{QCD} \sim m_N, m_\rho, 4\pi f_\pi, \ldots \sim 1\ \mathrm{GeV}$

$m_\pi \sim \sqrt{\overline{m} M_{QCD}} \simeq 140\ \mathrm{MeV}$

$R_c \sim A_c^{1/3} m_\pi^{-1}$ ⇐ Chiral EFT, Pionless EFT

*halo/cluster states*




p valence nucleon

$R_c \sim A_c^{1/3} m_\pi^{-1}$

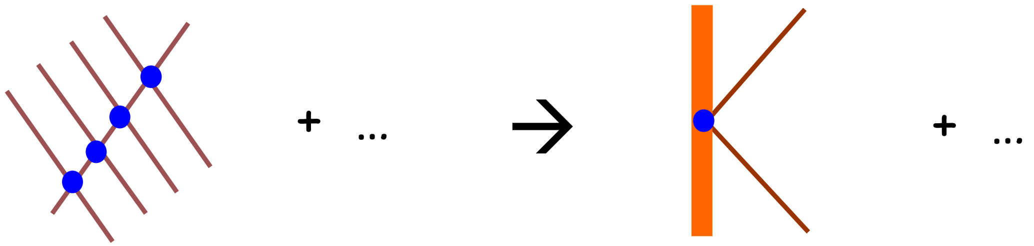
*e.g.*
$R_{^4\mathrm{He}} \approx 1.5\ \mathrm{fm}$
$R_{^6\mathrm{He}} \approx 2.4\ \mathrm{fm}$

$R_h \gg R_c > m_\pi^{-1}$ ⇐ Halo/Cluster EFT

# Halo/Cluster EFT

$$Q \sim M_{lo} \equiv R_h^{-1} \ll R_c^{-1} \equiv M_{hi} \sim A_c^{-1/3} m_\pi$$

cannot resolve details of the core → core as a point d.o.f.




*etc.*

Power counting
- bound state: same as Pionless EFT
- resonance: two parameters at LO, unitarity term perturbative for shallow resonance

+ addition expansion in inverse number of core nucleons for heavy halos

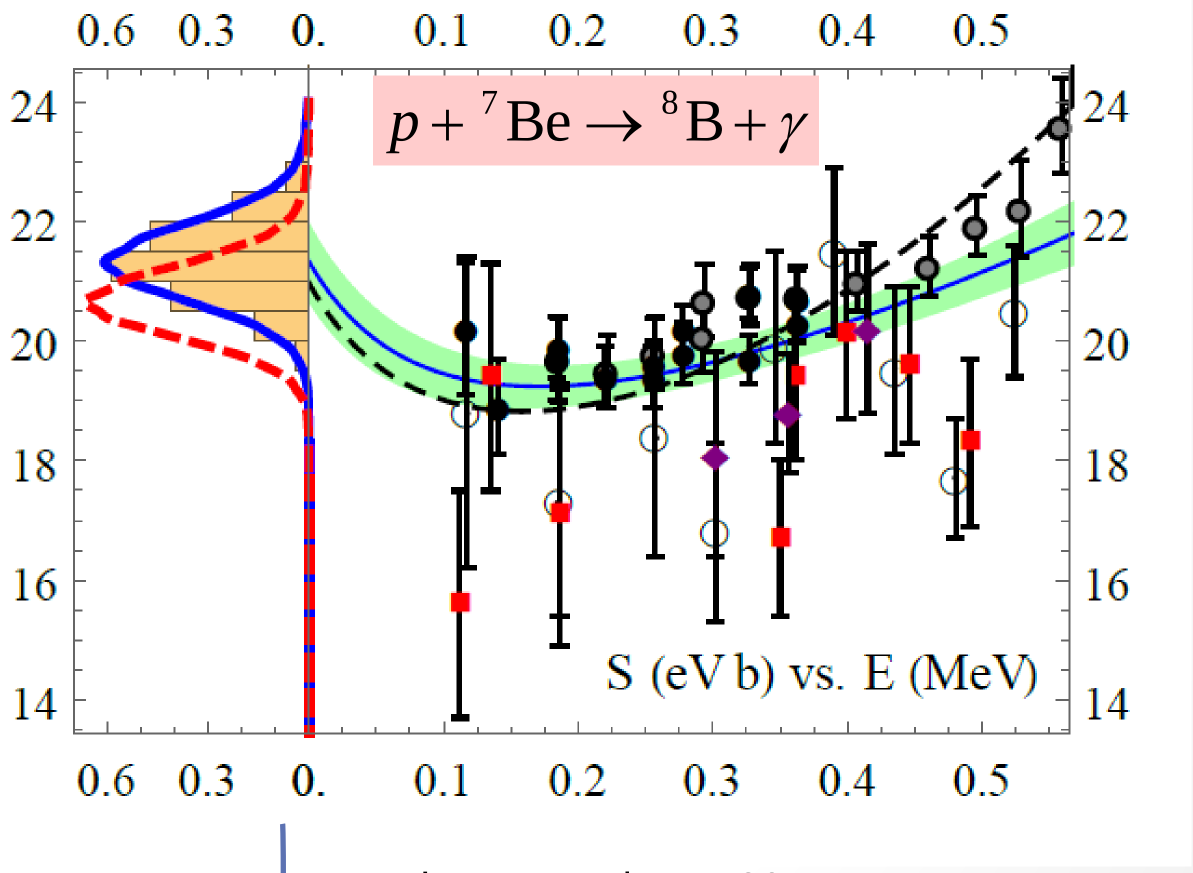
Bertulani, Hammer + v.K. '02
Bedaque, Hammer + v.K. '03

Forssén, Phillips, Ryberg + v.K. '19

# Example

astrophysical
S-factor

$$\sigma(E) \equiv \exp\left(-\frac{Z_1 Z_2 e^2}{2}\sqrt{\frac{\mu}{E}}\right)\frac{S(E)}{E}$$

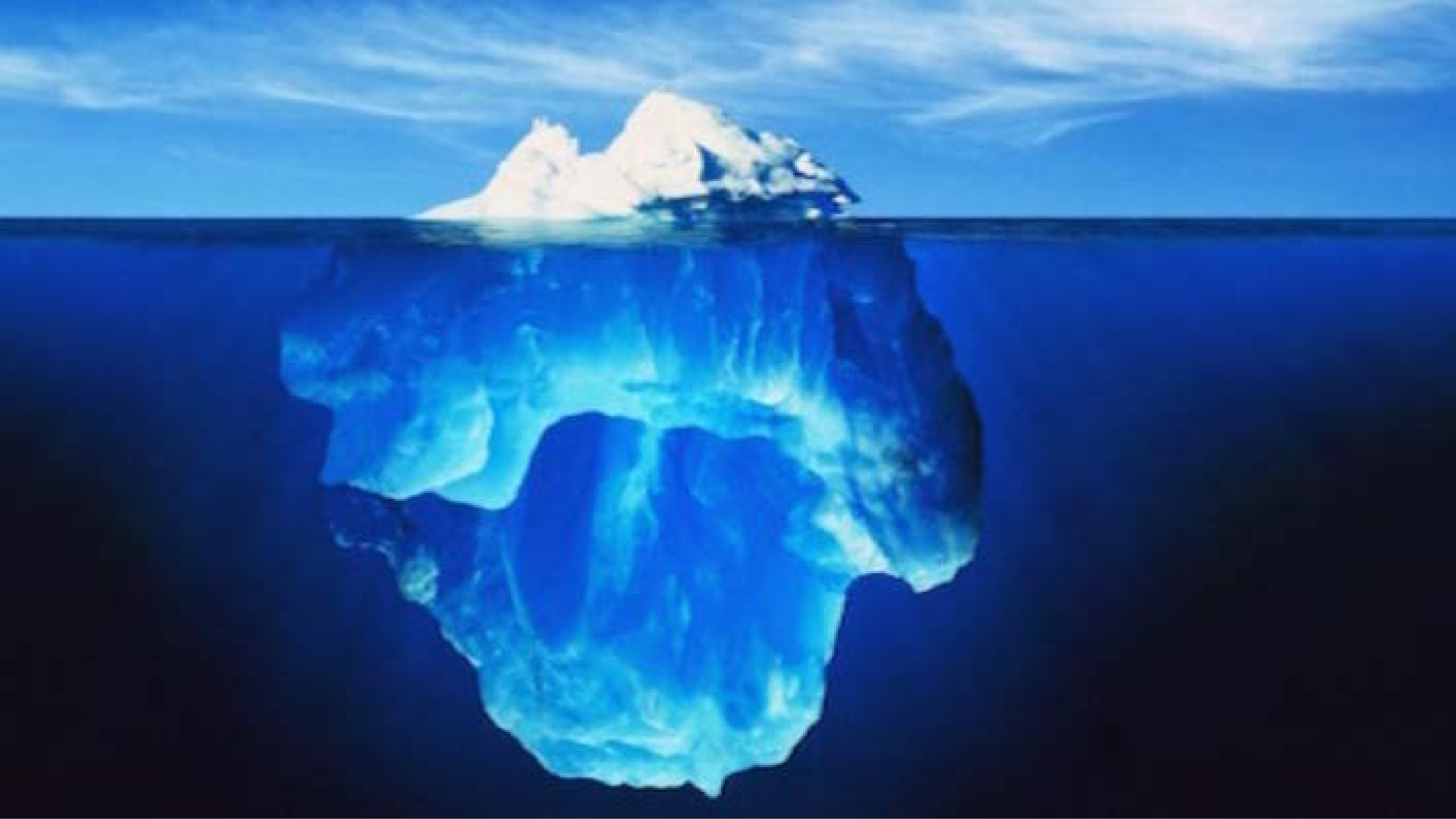



LO

NLO

$S\ (E = 0\,\mathrm{MeV}) = 21.3 \pm 0.7$ eV b.

Zhang et al. PLB 2015



# Conclusion

EFT

a general framework

for theory construction

- ✓ same method across scales
- ✓ model independent
- ✓ controlled expansion

a paradigm

in nuclear physics

- ✓ encodes QCD (more generally, B/SM)
- ✓ incorporates hadronic physics
- ✓ generates nuclear structure

the frontier:

many bodies & lattice QCD

- ❑ interplay with *ab initio* methods
- ❑ new EFTs