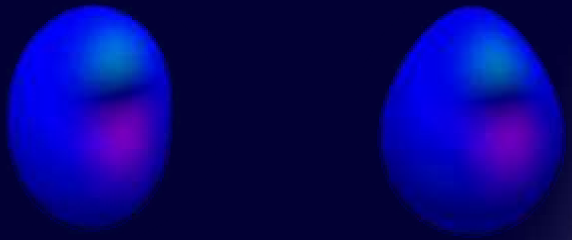
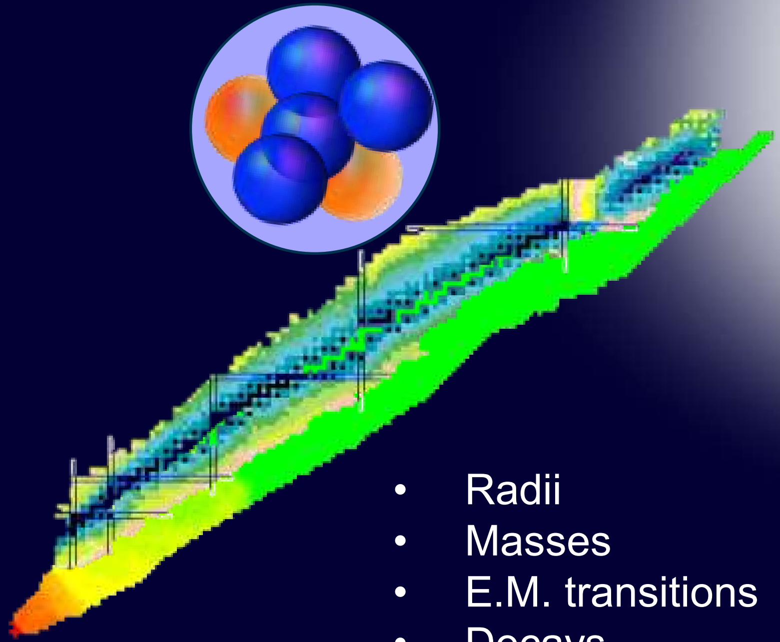
A journey into nuclear ~~structure~~ theory

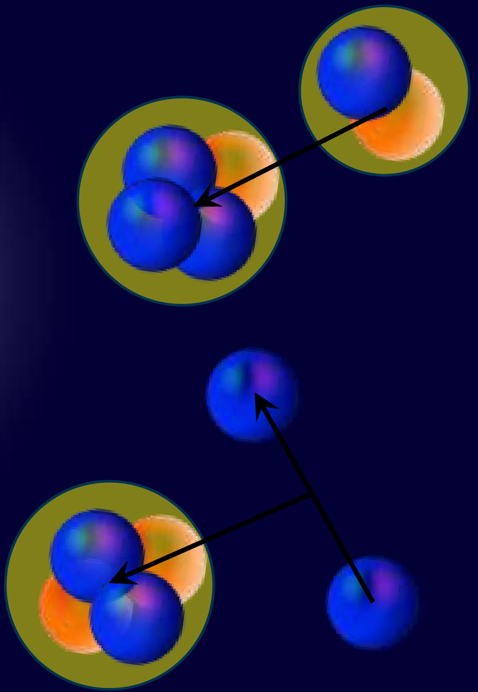
Nuclear structure from nuclear reactions





- Radii
- Masses
- E.M. transitions
- Decays

- Cross-sections
  - Nuclear
  - Photonuclear
  - Electronuclear
  - …



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Aknowledgement : D. Lacroix

A nuclear reaction in the laboratory

Typical experiment:

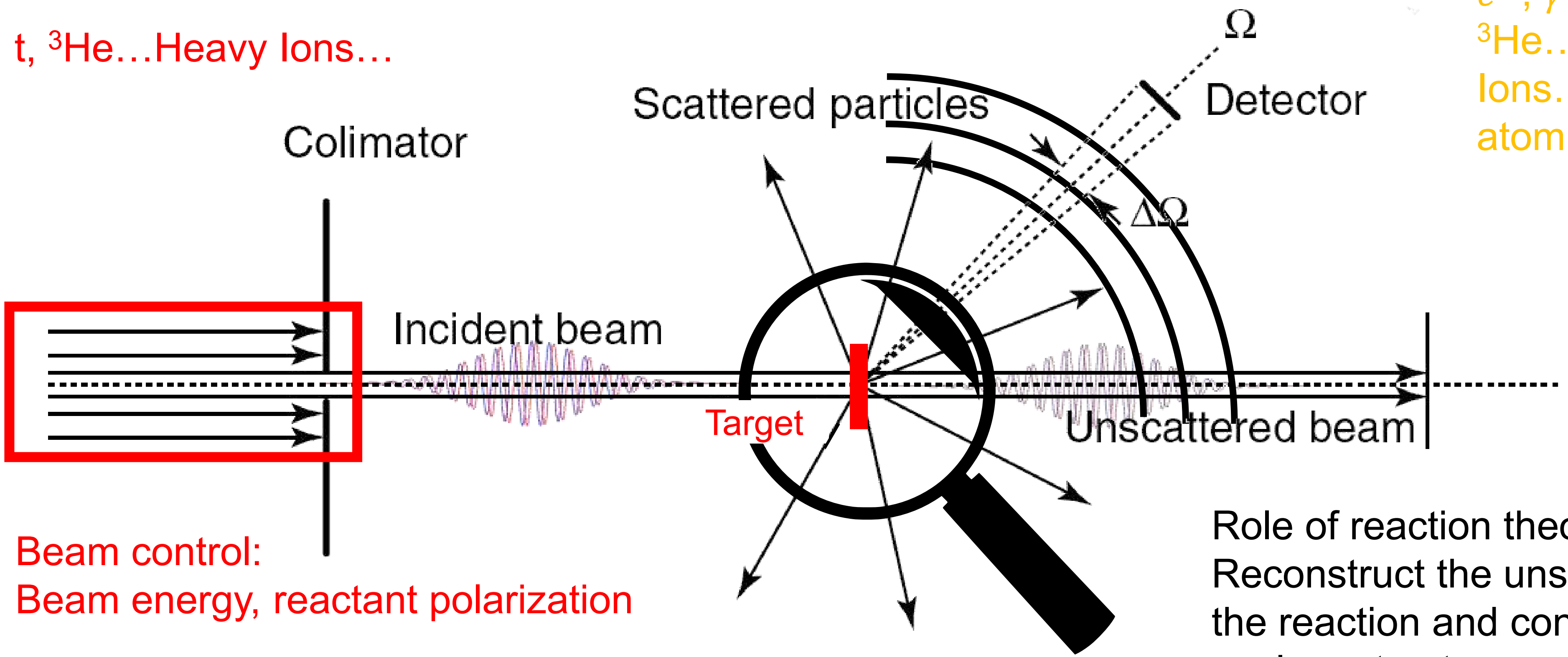
Expt. parameters

Theory

Detection

e^-, γ, p, n, t, ^{3}He…Heavy Ions…

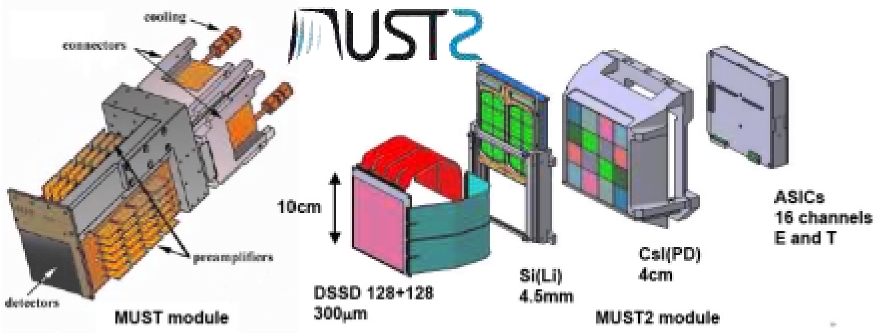
e^-, γ, p, n, t, ^{3}He…Heavy Ions…polarimeter, atomic transitions…



Beam control:
Beam energy, reactant polarization

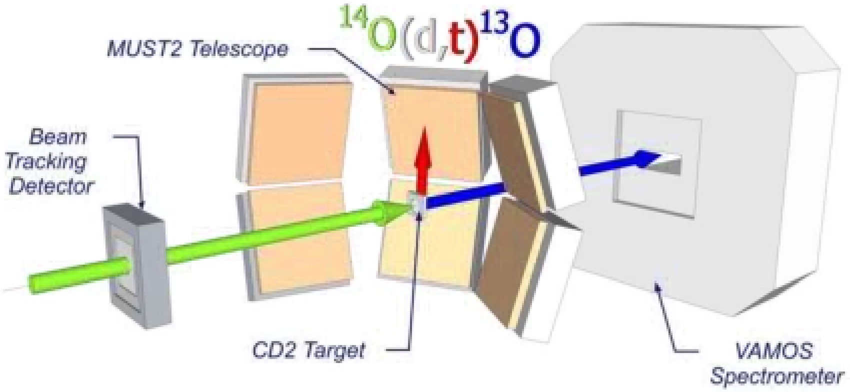
Role of reaction theory:
Reconstruct the unseen history of the reaction and connect to nuclear structure properties

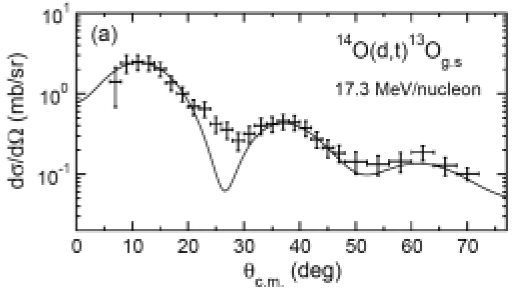
Today’s nuclear reactions in the laboratory

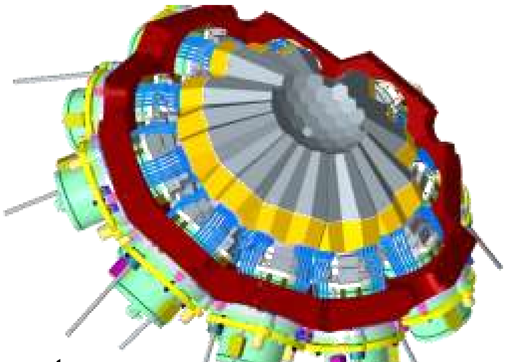


An example of experiment:

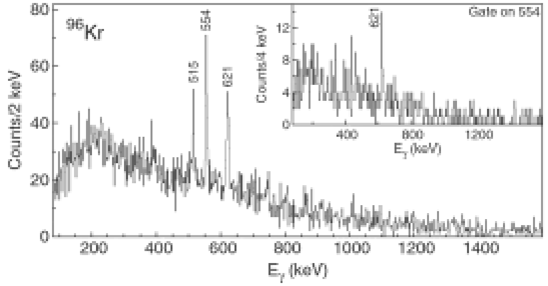
- Determination of the nature of the products (PID)
- Energy and angular resolution
- Momentum distributions
- γ measurements

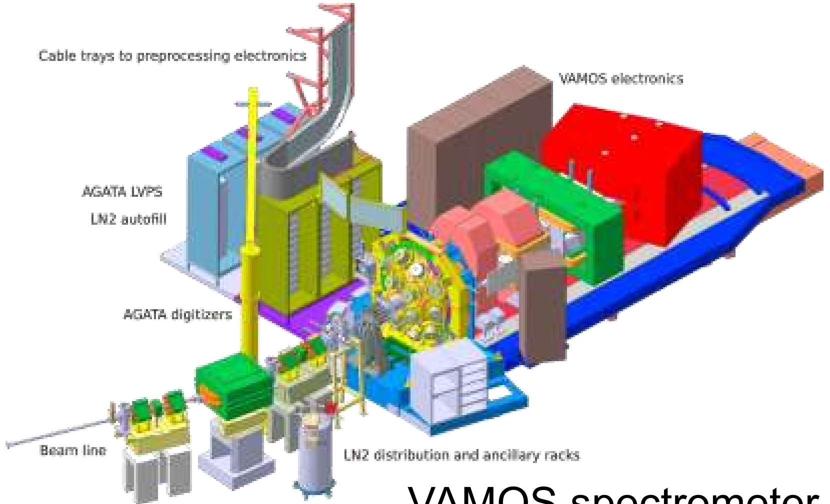






Agata:
4π gamma-ray detector
FWHM 6keV @ 1 MeV
ε~43%

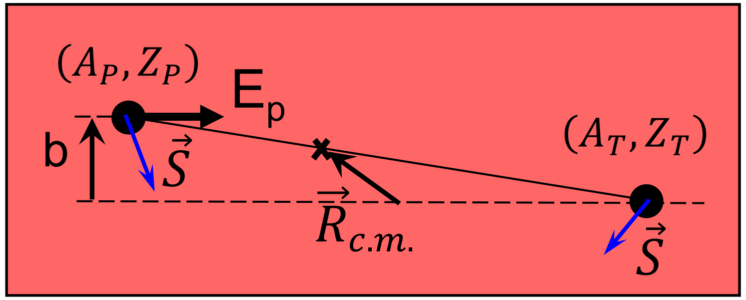




VAMOS spectrometer

Classification of reactions

Entrance channel



Exit channel

Inclusive/exclusive
Strongly detectors dependent

No energy transfer

$A + a \rightarrow A + a$ — Elastic scattering $A(a,a)A$

$A + \vec{a} \rightarrow \vec{A} + a$ — Spin transfer $A(\vec{a},a)\vec{A}$

Energy transfer

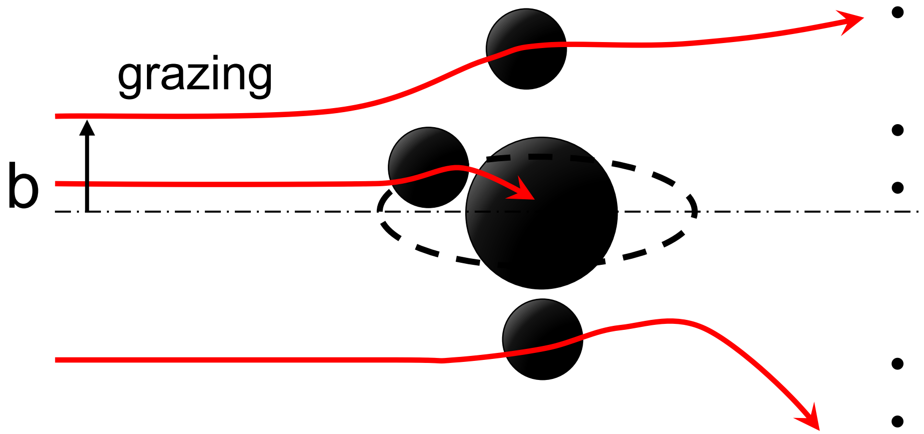
$A + a \rightarrow B + b$ — Transfer reaction $A(a,b)B$

$A + a \rightarrow A + a + \gamma$ — Nuclear Bremsstrahlung

$A + a \rightarrow B^* + b^* + \cdots$ — Inelastic scattering, Deeply inelastic…

$A + a \rightarrow C^* \rightarrow \text{decay}$ — Compound nucleus

Nuclear reactions by classifications

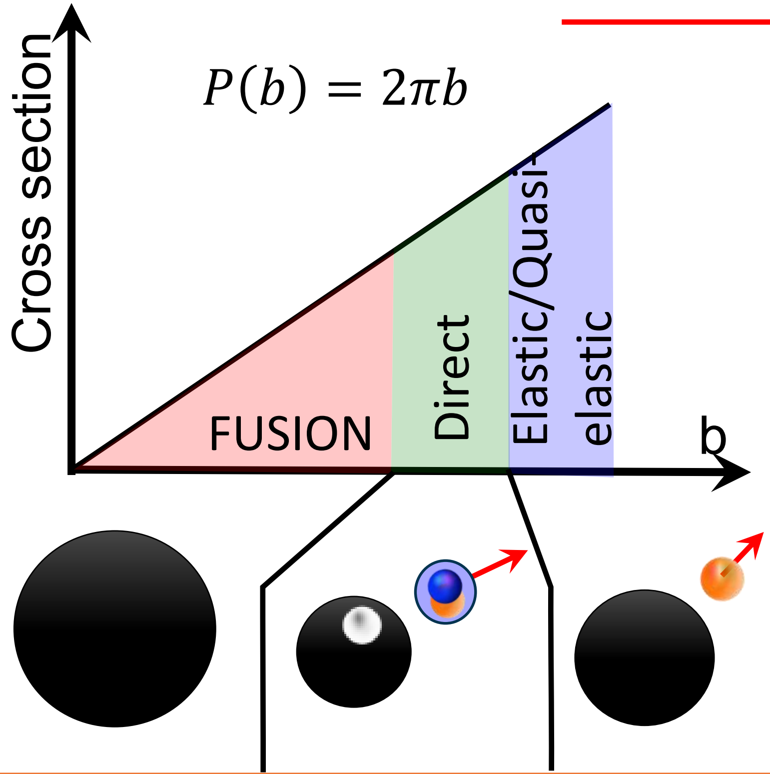


- Direct reaction
- Elastic and quasi-elastic

- Fusion
- Large amplitude Collective motion

- Multi-nucleon transfer
- Deep inelastic

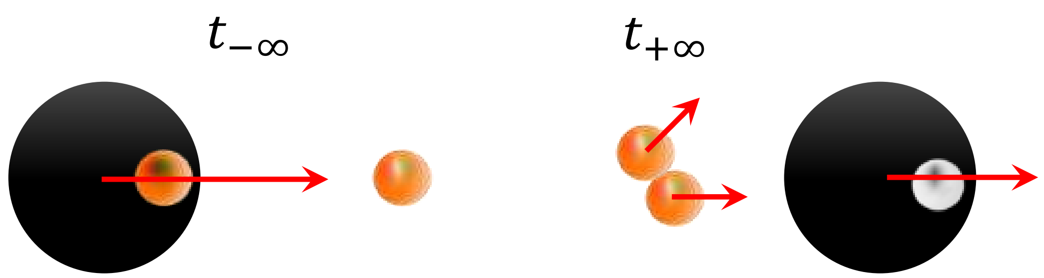
Adapted from : W. Nörenberg and H.A. Weidenmüller, “Introduction to Heavy-Ion theory”, Springer-Verlag 1981.



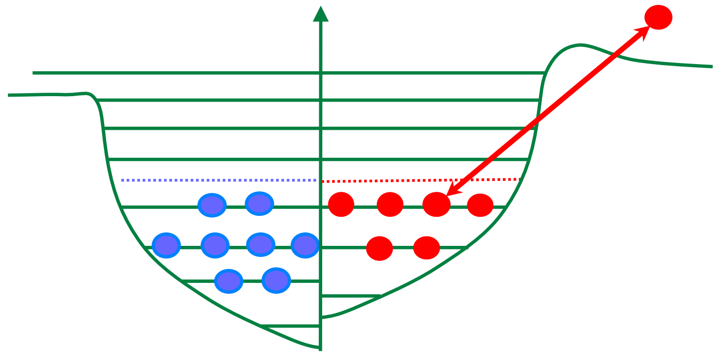
The more peripheral is the collision the highest is the associated cross-section

Nuclear structure

Knockout reactions (high energy)



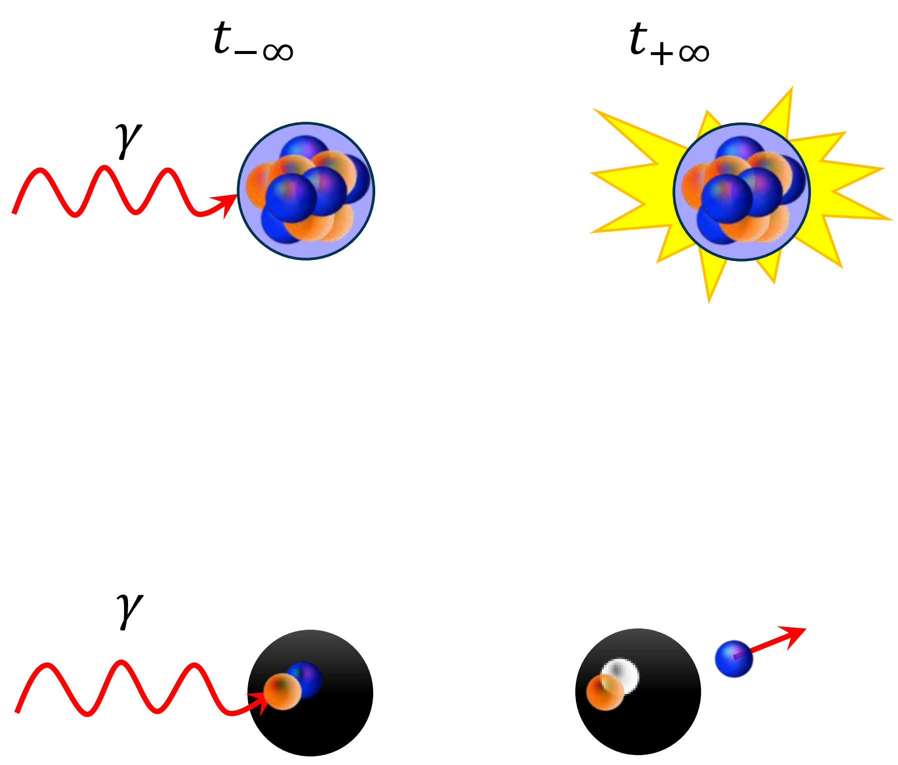
Spectroscopic information



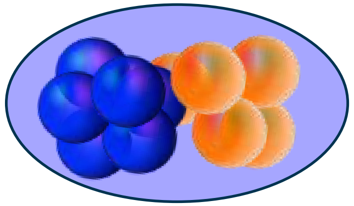
Transfer reactions (low energy)

Nuclear spectroscopy

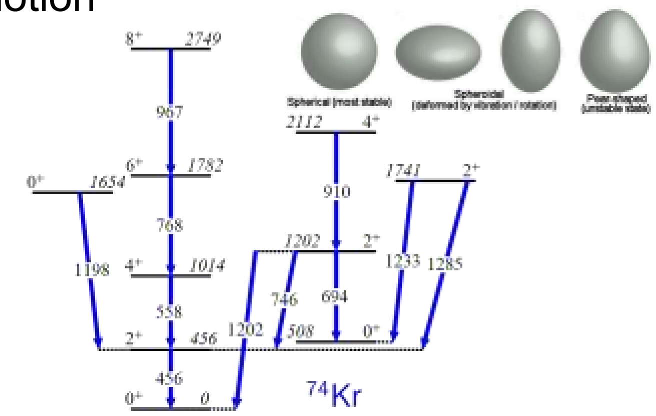
Photonuclear reactions



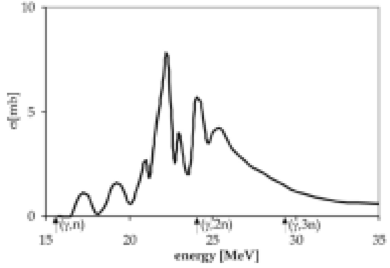
GDR excitation



Collective motion



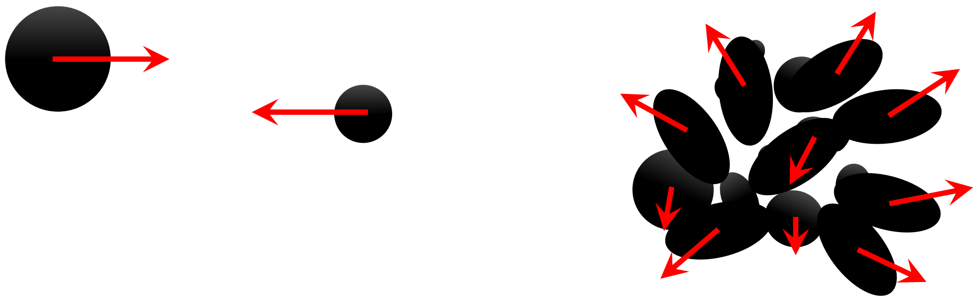
Bremsstrahlung, radiative capture.
Nuclear force imprints low-energy resonances.
Sensitive to off-shellness.

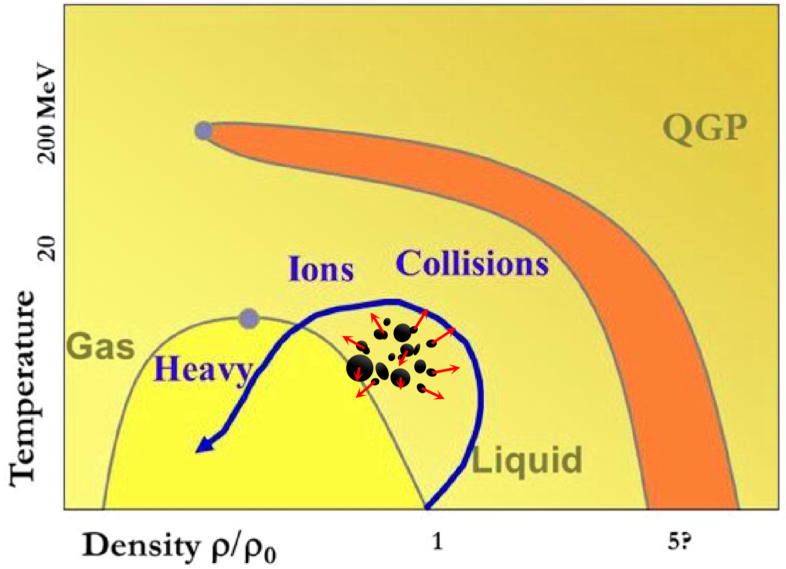


Out of equilibrium properties (away from nuclear density)

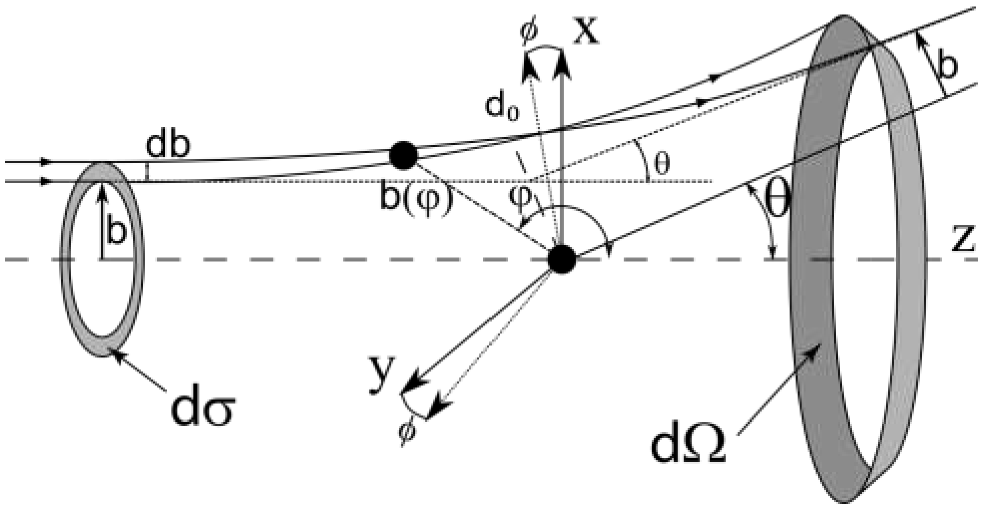
Reactions at Fermi energy

$t_{-\infty}$ $t_{+\infty}$





Definitions of cross-section and other quantities



Differential cross-section

$$\frac{d\sigma_{\text{channel}}(\theta,\varphi)}{d\Omega} = \frac{(\text{\# scattered particle.}s^{-1})(\theta,\varphi)}{(\text{Inc. flux})(\text{\# scatt. centers})(\text{unit of }\Omega)}$$

$$\xrightarrow{n=1} \frac{j_f(\theta,\varphi)}{j_i}$$

We define the outgoing solution

$$|\psi^+\rangle = |\phi_i\rangle + |\phi_f\rangle$$

$$\psi_k^+(\vec{r}) \xrightarrow{r\to\infty} A\left(e^{i\vec{k}.\vec{r}} + f(\Theta,\varphi)\frac{e^{ikr}}{r}\right)$$

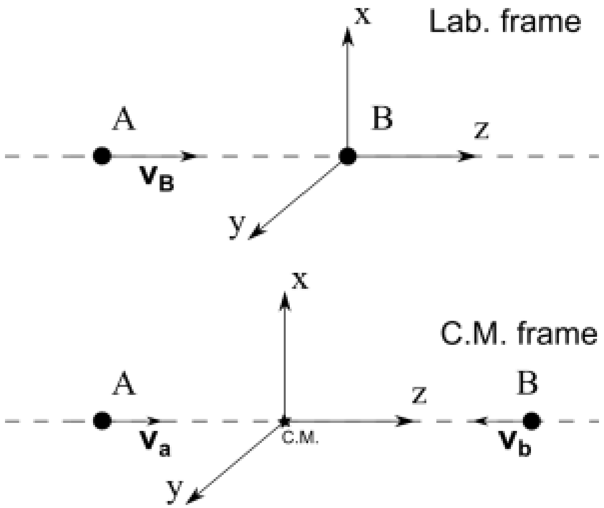
The flux is given by

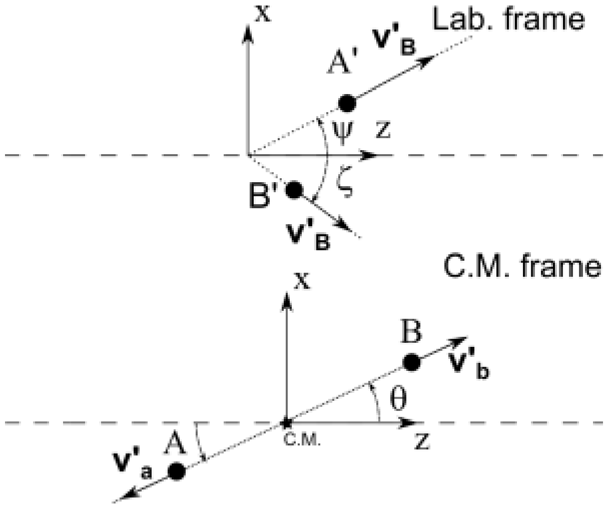
$$\vec{J} = \frac{\hbar}{2\mu i}\left((\psi_k^+)^*\nabla\psi_k^+ - \psi_k^+\nabla(\psi_k^+)^*\right)$$

We find the cross section to be:

$$\frac{d\sigma(\theta,\varphi)}{d\Omega} = |f(\Theta,\varphi)|^2$$

Translational invariance: C.M. frame





The Schrödinger equation

$$\left[-\frac{\hbar^2}{2m_A}\nabla_A^2-\frac{\hbar^2}{2m_B}\nabla_B^2+V(|\vec{r}_A-\vec{r}_B|)\right]\psi_k^+(\vec{r}_A,\vec{r}_B)=E\psi_k^+(\vec{r}_A,\vec{r}_B)$$

reduces in the c.m. frame

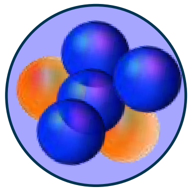
$$\left[-\frac{\hbar^2}{2\mu}\nabla^2+V(|\vec{r}|)\right]\psi_k^+(\vec{r}\,)=E_{\mathrm{C.M.}}\psi_k^+(\vec{r}\,)$$

phase velocity

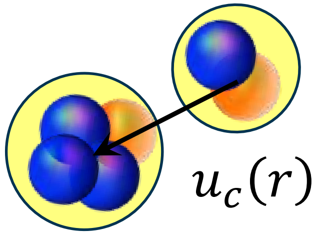
$$w=\frac{E}{\hbar}=\frac{\hbar k^2}{2\mu}=kv$$

Bound state E<0

$$\Psi_{A+a}^{J^\pi} = S_{a,A}^{A}\left[\Psi_a \Psi_A \chi_{k_\beta}^{f}(\vec{r})\right]^{J^\pi}$$



- $S_{a,A}^{A}$ spectroscopic amplitude
- $\left|S_{a,2}^{A}\right|^2$~ probability to form the configuration $(a + A)$ in the nucleus A



The Asymptotic Normalization Coefficient (ANC) $C_c = \frac{u_c(r)}{W_{-\eta_B, l+1/2}(2\kappa r)}$ pour $r > R$.

Internal region

$V = V_N + V_{\text{Coul}}$

$u_c(r) = \sum_n A_{cn} f_n(r)$

External region

$V = 0 \;(\text{or } V_{\text{Coul}})$

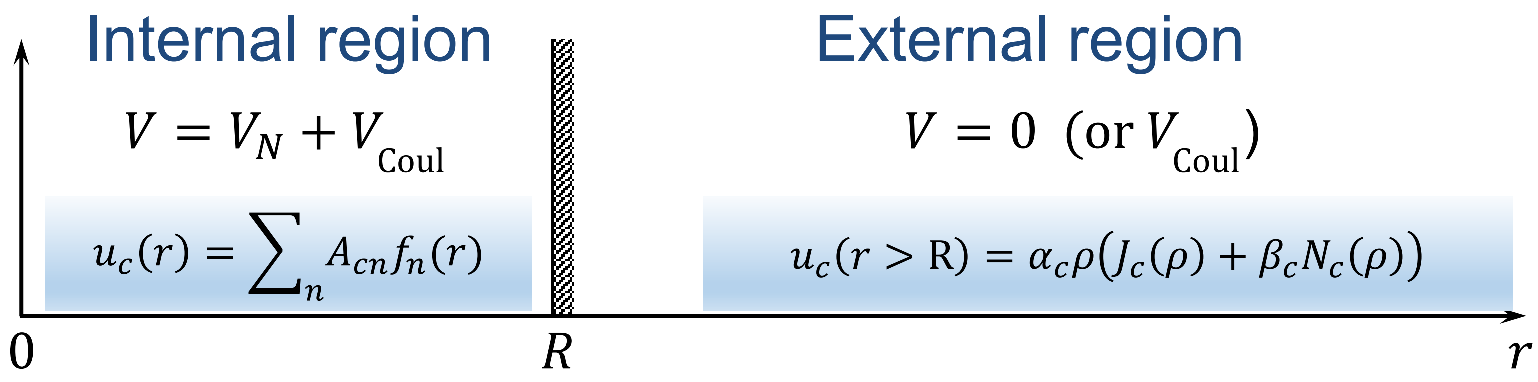
$u_c(r > R) = C_{c,l} W_{-\eta_B, l+1/2}(2\kappa r)$

0 R r

The tail of the wave function is determined by E_B and κ

$$u_c(r \to \infty) \sim x^{-\eta_B} e^{-\frac{2\kappa r}{2}}, \; \kappa = \sqrt{-\frac{2\mu E_B}{\hbar}}$$

Phase shifts, resonances E>0



Matching conditions on $u_{k,l\equiv c}(\rho)$ and $^{du_{k,l\equiv c}(\rho)}/_{dr}$ at $r = R$ between region I and II determines β_c.

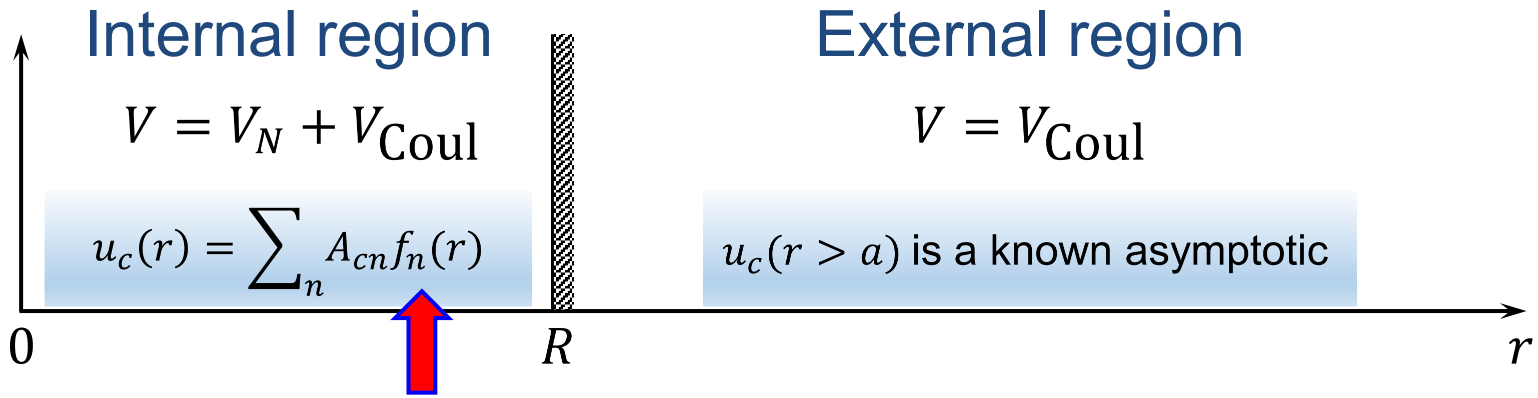
If we write $\beta_c = \tan \delta_c$, δ_c called the phase shift since:

$$u_c(r \to \infty) \sim \sin(\rho - l\,\pi/2 + \delta_c)$$

While the plane wave gives

$$\sin(\rho - l\,\pi/2)$$

Solution of the scattering problem with R-matrix method



Decomposition on a Lagrange mesh.

NCSMC can be cast as Bloch-Schrödinger equation:

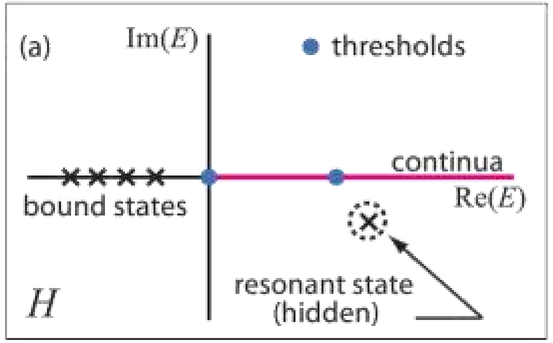
$$(C - EI)\vec{X} = Q(B)$$

And solved using R-matrix, which in the eigen basis of $C - EI$ reads:

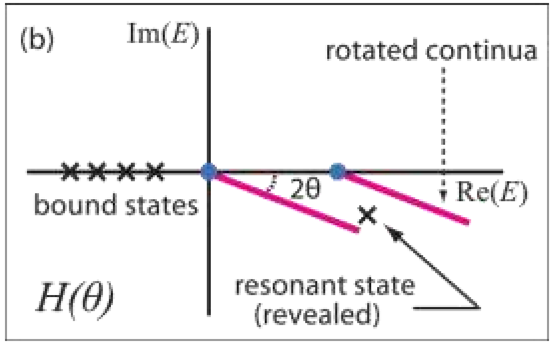
$$R_{cc'} = \sum_{\lambda} \frac{\gamma_{\lambda c}\gamma_{\lambda c'}}{E_\lambda - E}$$

Simple for binary reacting system, more involved for neutral ternary system and extremely challenging for charged breakup.

Asymptotically vanishing equivalent problem



Complex scaling



Kruppa *et al.* PRC**89** (2014)

The complex scaling and the resonance states

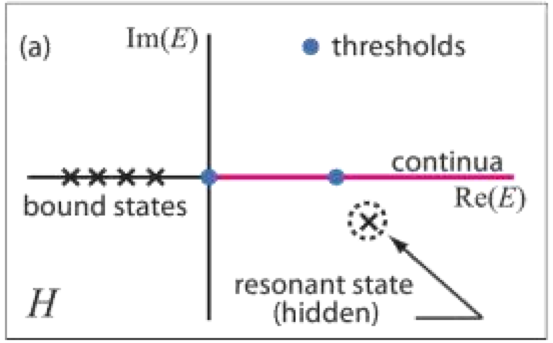
$$\hat{H}(r) = \hat{T} + \hat{V}(r) \quad \Rightarrow \quad \begin{aligned} \hat{H}(\theta) &= e^{-2i\theta}\hat{T} + \hat{V}\left(re^{i\theta}\right) \\ \hat{H}(r) &= \hat{U}(\theta)\hat{H}(r)\hat{U}^{-1}(\theta) \end{aligned}$$

Aguilar-Balslev-Combes theorem: the resonant states of the original Hamiltonian are invariant and the non-resonant scattering states are rotated and distributed on a 2θ ray that cuts the complex energy plane with a corresponding threshold being the rotation point. Math. Phys. **22**, 269 (1971)

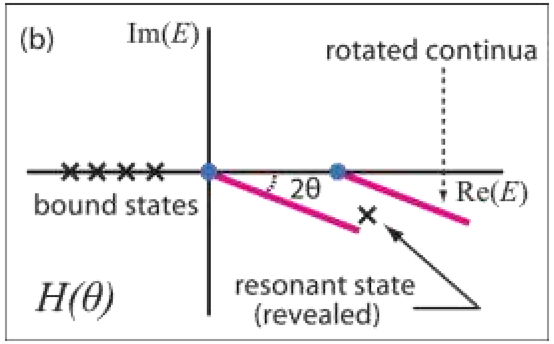
$$\hat{H}(r,\theta)\psi(r,\theta) = (E + i\Gamma)\psi(r,\theta)$$

Energy ↑ (E) ↑ ($i\Gamma$) Half-life

Asymptotically vanishing equivalent problem



Complex scaling



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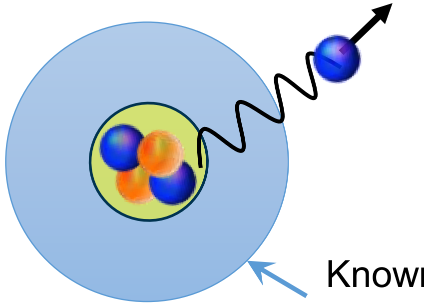
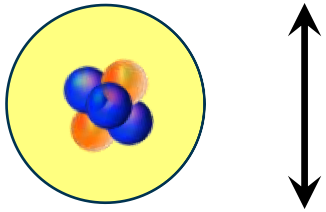
$$\hat{H}(r) = \hat{U}(\theta)\hat{H}(r)\hat{U}^{-1}(\theta)$$



Known asymptotic

$$U(\theta)H(r)U(\theta)^{-1}$$

$$\psi(r,\theta) \underset{\infty}{\sim} e^{-\boldsymbol{kr}\sin\theta}$$



Spatially extended but exponential fall off

Boundary limit problem

Bound state problem

From phase shifts to cross-sections

$$H_{A+B}|\psi_i^+\rangle = E_i|\psi_i^+\rangle$$

Solve

$$\left[-\frac{d^2}{dr^2} + \frac{l(l+1)}{r^2} + \frac{2\mu}{\hbar^2}V(r)\right]u_{k,l}(r) = k^2 u_{k,l}(r)$$

$$f_l(k) = \frac{e^{2i\delta_l(k)} - 1}{2ik}$$

$$\psi_k^+(\vec{r}) \longrightarrow \left(e^{i\vec{k}.\vec{r}} + f(\Theta,\varphi)\frac{e^{ikr}}{r}\right)$$

Match

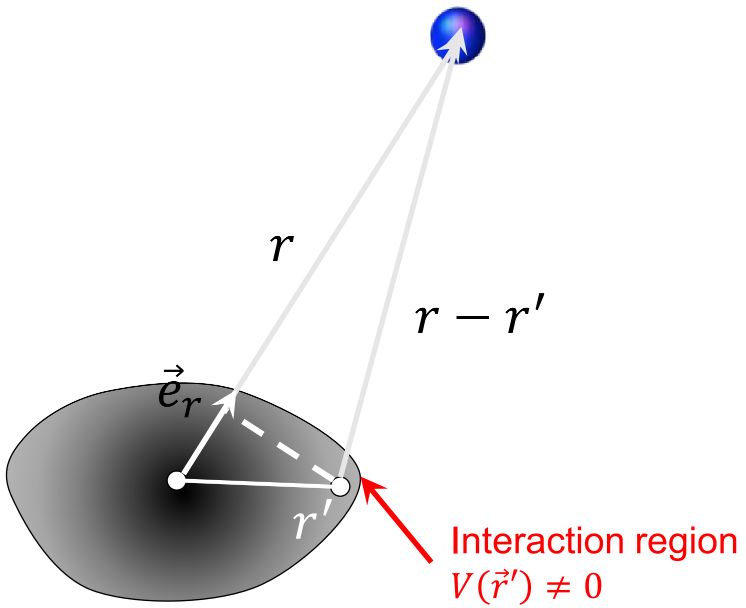
$$\frac{d\sigma(\theta,\varphi)}{d\Omega}, \sigma_{\text{tot}}$$

Calculate

Differential cross-sections for central potential

$$\frac{d\sigma(\theta,\varphi)}{d\Omega} = \frac{1}{4k^2}\left|\sum(2l+1)\left(1 - \frac{e^{2i\delta_l(k)} - 1}{2ik}\right)P_l(\cos\Theta)\right|^2$$

Reaction theory as a series



The Lippmann-Schwinger equation gives

$$\psi_{\boldsymbol{k}}^{\pm}(\vec{r}) = \frac{e^{i\vec{k}\cdot\vec{r}}}{(2\pi)^{3/2}} - \frac{2\mu}{\hbar^2}\int d^3r' \frac{e^{\pm ik|r-r'|}}{4\pi|r-r'|} V(\vec{r}')\psi_{\boldsymbol{k}}^{\pm}(\vec{r}')$$

with $\vec{k}' = k\vec{e}_r$
$k' = k$

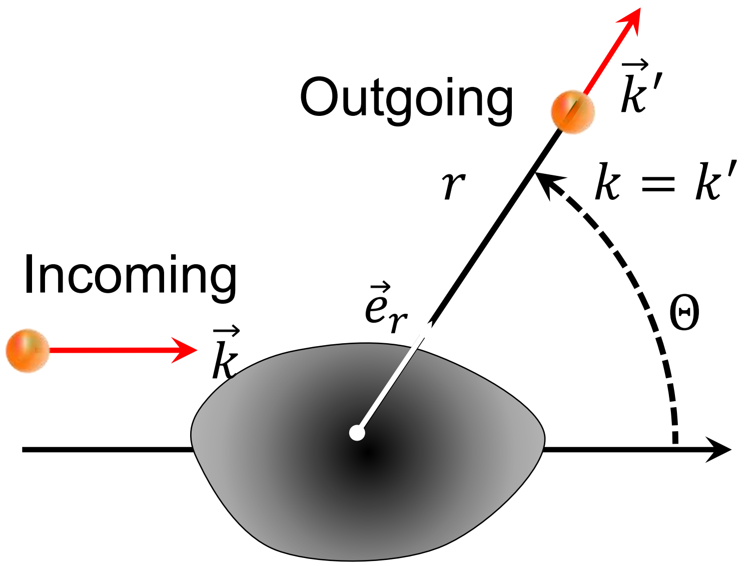
Looking for the solution following

$$\psi_k^{+}(\vec{r}) \longrightarrow \left(e^{i\vec{k}.\vec{r}} + f(\Theta,\varphi)\frac{e^{ikr}}{r}\right)$$

We obtain the scattering amplitude:

$$f(\theta,\varphi) = -2\pi^2\frac{2\mu}{\hbar^2}\int d^3r' \frac{e^{-i\vec{k}'\cdot\vec{r}'}}{(2\pi)^{3/2}} V(\vec{r}')\psi_{\boldsymbol{k}}^{+}(\vec{r}')$$

Reaction theory as a series



T-matrix is defined as

$$f(\theta,\varphi) = -2\pi^2 \frac{2\mu}{\hbar^2} \langle \varphi_{0,\boldsymbol{k}'} | V | \psi_{\boldsymbol{k}}^{+} \rangle = 2\pi^2 \frac{2\mu}{\hbar^2} |T_{\boldsymbol{k}',\boldsymbol{k}}|$$

We deduce the differential cross-section as:

$$\frac{d\sigma(\theta,\varphi)}{d\Omega} = \left(2\pi^2 \frac{2\mu}{\hbar^2}\right)^2 |T_{\boldsymbol{k}',\boldsymbol{k}}|^2$$

$T_{\boldsymbol{k}',\boldsymbol{k}}$ is the on-shell [$k = k'$] T-matrix element and relates to the S-matrix by

$$S_{\boldsymbol{k}',\boldsymbol{k}} = \delta(\vec{k} - \vec{k}') - 2\pi\delta(E_k - E_{k'})T_{\boldsymbol{k}',\boldsymbol{k}}$$

So $T_l(E) = -{}^1\!/_\pi \, e^{i\delta_l(E)} \sin\delta_l(E)$. Similarly $T_l(E) = \frac{2\mu}{\pi\hbar^2} \int dr\, rJ_l(kr)\, V(r) u_l(r)$

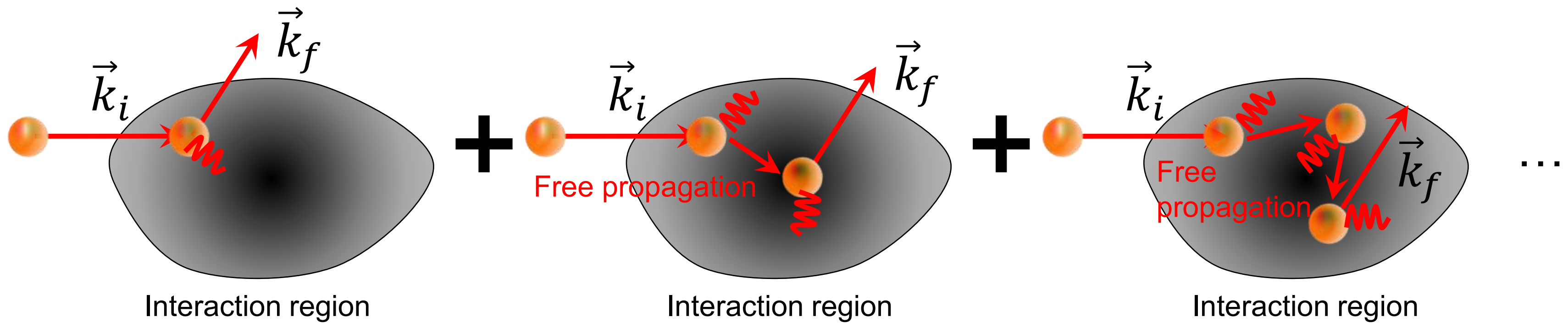
Reaction observables carry imprints of the nuclear force

Representation of the born series

From the Lippmann-Schwinger equation we define the Born series expansion

$$f(\Theta,\varphi) = -2\pi^2 \left\langle \varphi_{0,\boldsymbol{k}'} \middle| V \sum \left(\frac{2\mu}{\hbar^2} G_0 V \right)^n \middle| \varphi_{0,\boldsymbol{k}} \right\rangle$$

We can understand that the unperturbed plane wave undergoes a sequences of multiples scattering events from inside the potential region:

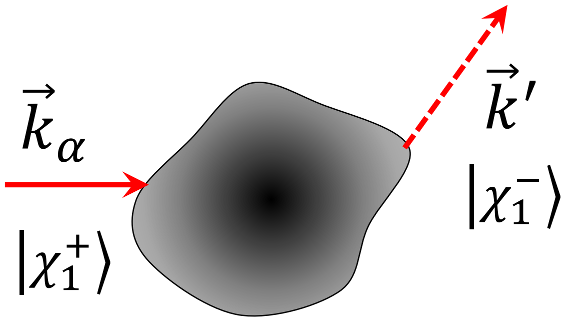


But the series may not converged until all terms are including if the potential is strong enough.

The Distorted Wave-Born Approximation (DWBA)

Born Approximation

$$\chi_k^+(\vec{r}) = \frac{e^{i\vec{k}_\alpha\cdot\vec{r}}}{(2\pi)^{3/2}} + \frac{2\mu}{\hbar^2}\int d^3r' \frac{e^{ik_\beta|r-r'|}}{4\pi|r-r'|} V(\vec{r}')\chi_k^+(\vec{r}')$$



Systematic constructive treatment

$$f_{\text{Born}} = -\frac{2\mu}{4\pi\hbar^2}\langle \boldsymbol{k}'|V|\mathbf{k}\rangle$$

In some cases, the free-wave approximation is rather poor starting point.

Suppose $V = V_{\text{MF}} + V_{\text{res}}$ and the solutions of $(\nabla^2 + k^2 - V_{\text{MF}})\chi_1(\boldsymbol{k},\boldsymbol{r}) = 0$ are known/computable

One can show $f = f_1 - \frac{2\mu}{4\pi\hbar^2}\int d^3r' \chi_1^-(\boldsymbol{k},\vec{r})V_{\text{res}}(\vec{r}')\chi_k^+(\vec{r}')$

The DWBA approximation consists in:

$\chi_k^+ \longrightarrow \chi_1^+(\boldsymbol{k},\boldsymbol{r})$ then $f = f_1 - \frac{2\mu}{4\pi\hbar^2}\langle\chi_1^-|V_{\text{res}}|\chi_1^+\rangle$

Multichannel reactions

$$f(\Theta,\varphi) = \sum_{\beta} f_\beta(\Theta,\varphi)$$

All energetically allowed opened channels β

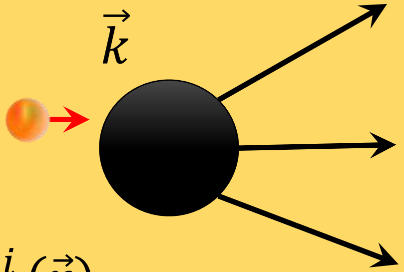
Note that, in a finite basis, closed channels also contribute but to reaction observables

$t = t_0$

Incoming wave

$|a + A\rangle$

$\vec{k}$



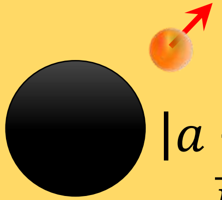
$\Psi_a \Psi_A \chi_k^i(\vec{r})$

$\chi_k^i(\vec{r}) \propto e^{i\vec{k}.\vec{r}}$

$H_a\Psi_a = E_0\Psi_a$
$H_A\Psi_A = E_0\Psi_A$

Ψ_a and Ψ_A quantum numbers defined conservation of total relative angular momentum

$t \gg t_0$



Elastic channel, always opened

$|a + A\rangle$ $\vec{k}_\beta$

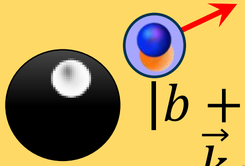
$\Psi_a \Psi_A \chi_{k_\beta}^f(\vec{r})$

$H_a\Psi_a = E_0\Psi_a$
$H_A\Psi_A = E_0\Psi_A$

$|C^*\rangle$ $\vec{k}_\beta$

Ψ_{C^*}

$H_C\Psi_{C^*} = E_n\Psi_{C^*}$



$|b + C\rangle$ $\vec{k}_\beta$

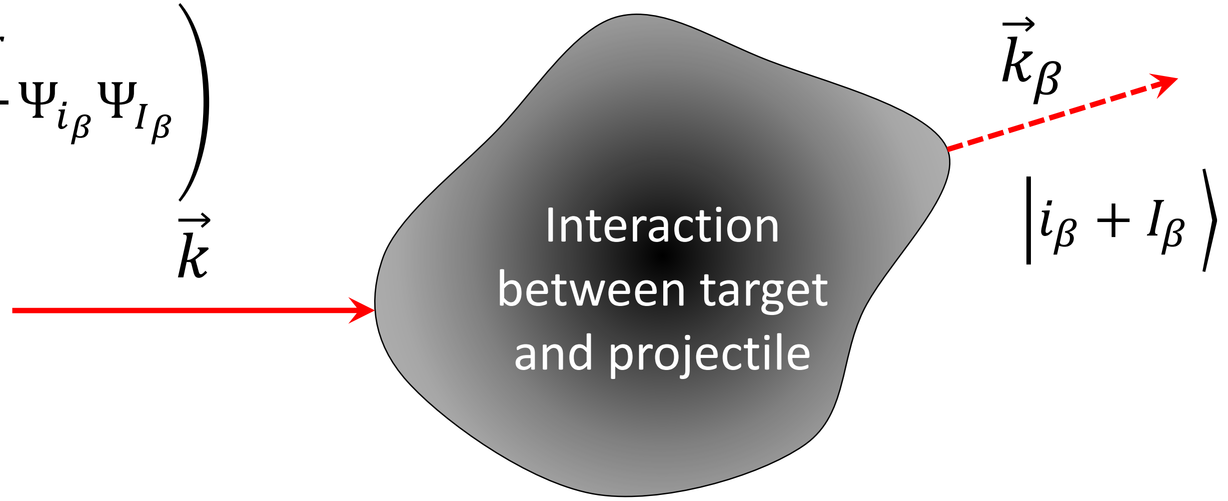
$\Psi_b \Psi_C \chi_{k_\beta}^f(\vec{r})$

$H_b\Psi_b = E_0\Psi_b$
$H_C\Psi_C = E_0\Psi_C$

channel Q-value

Scattering wave-function

$$\psi_k^+(\vec{r}) \longrightarrow \left(e^{i\vec{k}.\vec{r}}\Psi_a\Psi_A + \sum_\beta f_\beta(\Theta,\varphi)\frac{e^{ik_\beta r}}{r}\Psi_{i_\beta}\Psi_{I_\beta} \right)$$



Partial cross-section

$$\frac{d\sigma_\beta}{d\Omega} = \frac{\vec{J}_f \cdot d\vec{S}/_{r^2}}{\vec{J}_i \cdot \hat{k}}$$

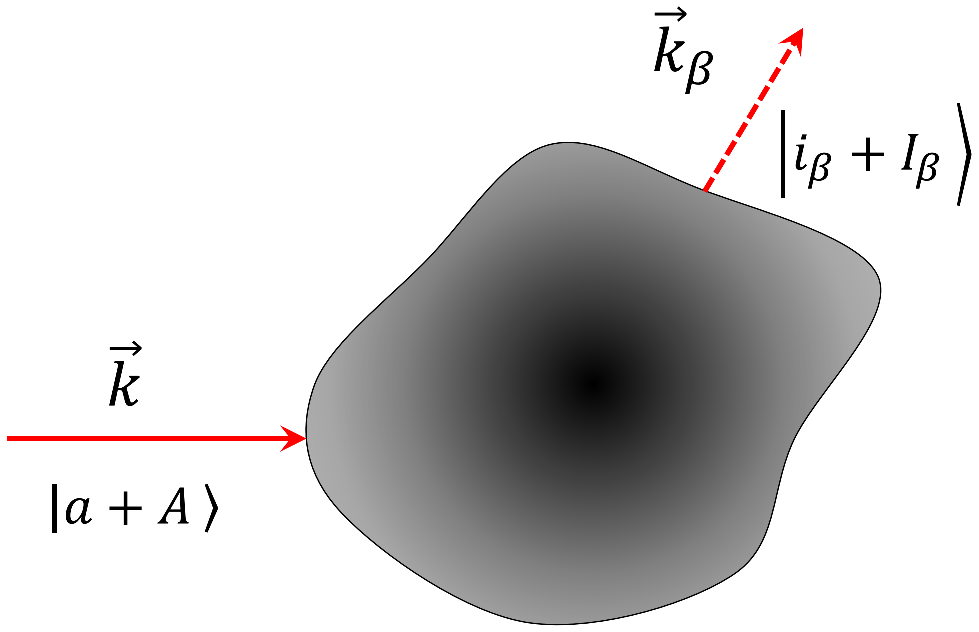
Since $\vec{J} = \rho\vec{v}$ with $\vec{v}$ the wave vector, we have that :

$$\frac{d\sigma_\beta}{d\Omega} = \frac{v_\beta}{v}\left|f_\beta(\Theta,\varphi)\right|^2$$

- Elastic scattering $v_\beta/_v = 1$
- Inelastic scattering $v_\beta/_v \neq 1$

Multichannel reactions

$$\frac{d\sigma_\beta}{d\Omega} = \frac{v_\beta}{v} \left|\breve{f}_\beta(\Theta, \varphi)\right|^2$$



Channels will all interfere…

$|a + A\rangle$

Elastic channel with $\frac{v_\beta}{v} = 1$, always opens

$|a^* + A^*\rangle$

Inelastic scattering $\frac{v_\beta}{v} \neq 1$,
Energetically opens if $E_{\mathrm{c.m.}}$ is greater than the reaction threshold

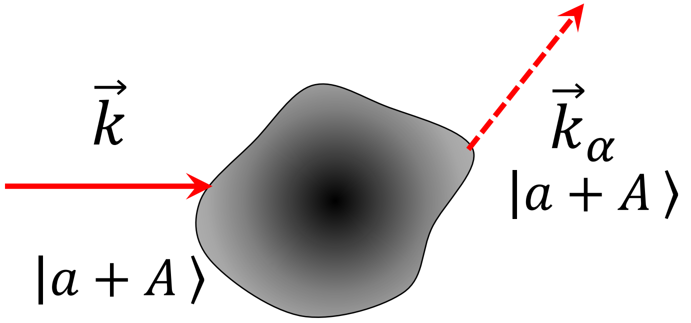
$|b + C\rangle$

$|b + d + C\rangle$

All other reaction channels energetically allowed ($\frac{v_\beta}{v} \neq 1$)

Inelastic channels

Influence of the non-elastic channels on cross-section



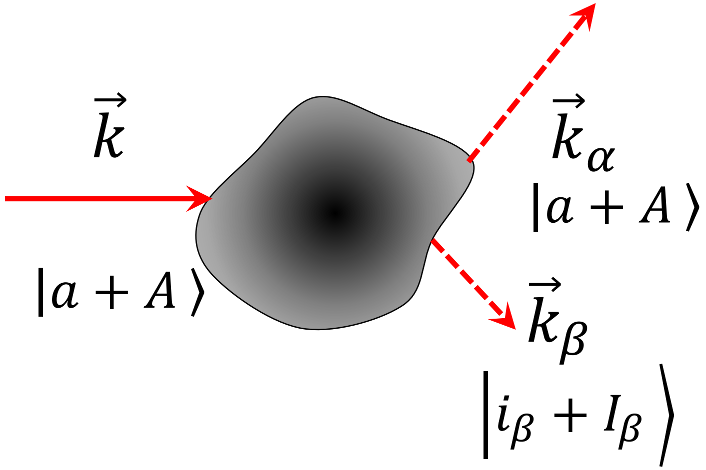
With only elastic channel

$$\psi_k^+(\vec{r}) \longrightarrow \left(e^{i\vec{k}.\vec{r}} + f(\Theta, \varphi)\frac{e^{ik_\alpha r}}{r} \right)$$

$$u_{\alpha,l}(r > \mathrm{R}) = A_{\alpha,l}\rho\left(H_l^-(\rho) - S_{\alpha,l}H_l^+(\rho)\right)$$

The conservation of the momentum leads to $k = k_\alpha$, the conservation of the flux [which implies the unitarity of the S-matrix i.e. $S_{\alpha,l}S_{\alpha,l}{}^* = 1$] means $S_{\alpha,l} = e^{2i\delta_l}, \delta \in \mathbb{R}$

Adding non-elastic channels



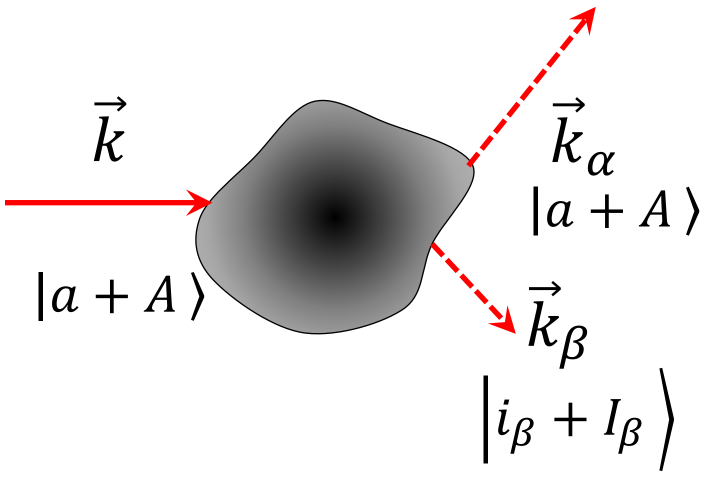
$$u_{\alpha,l}(r > \mathrm{R}) = A_{\alpha,l}\rho\left(H_l^-(\rho) - S_{\alpha,l}H_l^+(\rho)\right)$$

$$u_{\beta,l}(r > \mathrm{R}) = -A_{\beta,l}\rho S_{\beta,l}H_l^+(\rho)$$

Where $S_{\beta,l} = \sqrt{v/v_\beta}\,\tilde{S}_{\beta,l}$ and $\tilde{S}_{\beta,l} = e^{2i\delta_l}, \delta \in \mathbb{C}$. Total energy is conserved but $k_\beta \neq k$ due to energy consumed by the Q value. The flux is distributed among channels:

$$\left|S_{\alpha,l}(E)\right|^2 + \sum \left|S_{\beta,l}(E)\right|^2 = 1$$

Elastic, Reaction and Total cross section



Elastic channel:

$$\sigma_{\text{el}} = \frac{\pi}{k^2} \sum (2l+1) \left|1 - \tilde{S}_{\alpha,l}\right|^2$$

Inelastic channels:

$$\sigma_{\text{in}} = \frac{\pi}{k^2} \sum (2l+1) \left|\tilde{S}_{\beta,l}\right|^2$$

Sum of all inelastic channels (absorption cross-sec.):

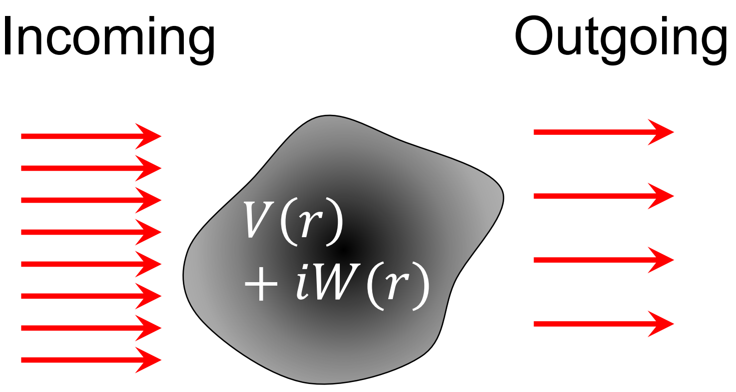
$$\sigma_{\text{abs}} = \frac{\pi}{k^2} \sum (2l+1)\left(1 - \left|\tilde{S}_{\alpha,l}\right|^2\right)$$

from $\left|S_{\alpha,l}\right|^2 + \sum \left|S_{\beta,l}\right|^2 = 1$, and total cross-section

$$\sigma_{\text{tot}} = \sigma_{\text{el}} + \sigma_{\text{abs}}$$

$$= \frac{2\pi}{k^2} \sum (2l+1)\left(1 - \text{Re}(\tilde{S}_{\alpha,l})\right)$$

Optical potential



Scattering equation with an imaginary potential

$$\left(\Delta + k^2 - \frac{2\mu}{\hbar^2}(V(\vec{r}) + iW(\vec{r})\right)\varphi(\vec{r}) = 0$$

With start with the conservation of matter

$$\frac{d\rho}{dt} = -\vec{\nabla}\cdot\vec{J}$$

Here we have $\hbar\,\vec{\nabla}\cdot\vec{J} = 2W(r)\rho(\mathrm{r})$ we immediately obtained the lost outgoing flux as $-\frac{2}{\hbar}\int d^3rW(r)\rho(\mathrm{r})$, the absorption cross-section reads

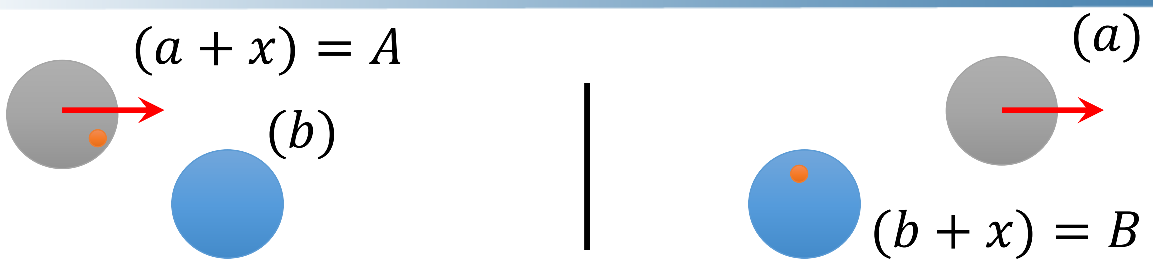
$$\sigma_{\mathrm{abs}} = -\frac{2}{\hbar v}\int d^3rW(r)\rho(\mathrm{r})$$

In practice, $W(r)$ is parametrized by various Wood-Saxon functions:

$$W(r,E) = -W^{\mathrm{vol}}(E,r) + 4a^{\mathrm{sur}}W^{\mathrm{sur}}(E)\frac{d}{dr}f(r) + W^{so}(r,E)$$

Nucleon transfer

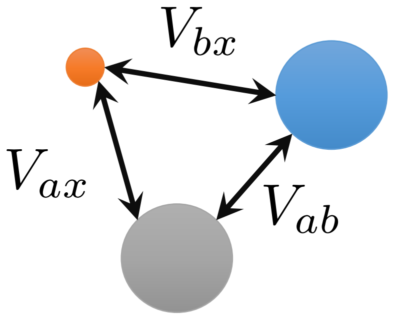
DWBA applied to nuclear transfer reaction



At high energies, we assume internal dof are frozen (spectator nucleons) only the transferred particle (n, p, d, …) ic considered explicitly:

$$A + b \rightarrow a + B \implies (a+x) + b \longrightarrow a + (b+x)$$

The interaction potentials should be between all constituents [3-body problem]:



Entrance

Exit

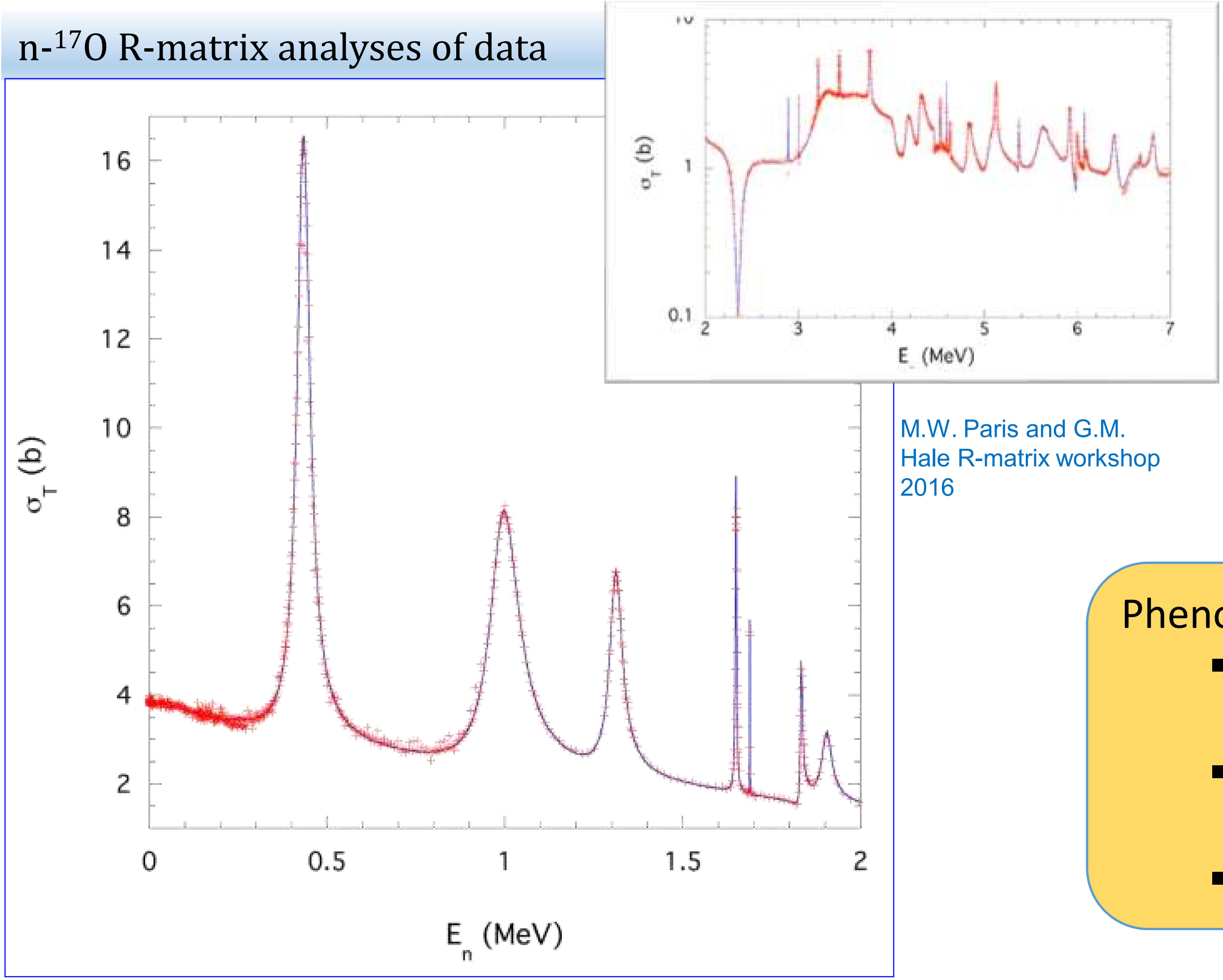
$$\Psi^{J^\pi}_{A=(a+x)} = S^A_{a,x}\left[\Psi_a \Psi_x \chi^f_{k_\beta}(\vec{r})\right]^{J^\pi} + \cdots \qquad \Psi^{J^\pi}_{B=(b+x)} = S^A_{b,x}\left[\Psi_b \Psi_x \chi^f_{k_{\beta'}}(\vec{r})\right]^{J^\pi} + \cdots$$

χ_1^- $\qquad$ χ_1^+

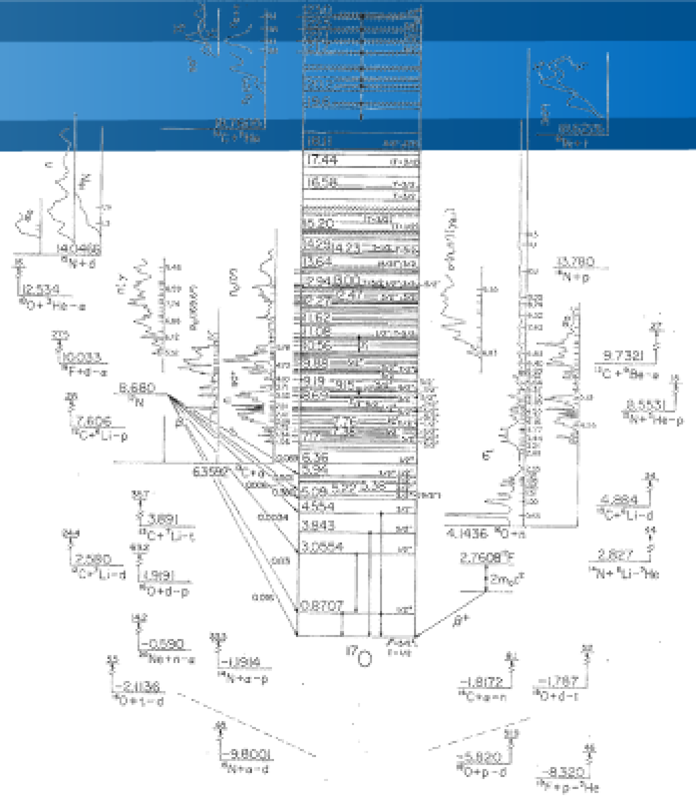
$$\langle \chi_1^- | V_{\text{res}} | \chi_1^+ \rangle \propto \left\langle \Psi_b \Psi_x \chi^f_{k_{\beta'}}(\vec{r}) \middle| V_{\text{res}} \middle| \Psi_a \Psi_x \chi^f_{k_\beta}(\vec{r}) \right\rangle$$

Extracting nuclear data for astrophysics

n-^{17}O R-matrix analyses of data



M.W. Paris and G.M. Hale R-matrix workshop 2016



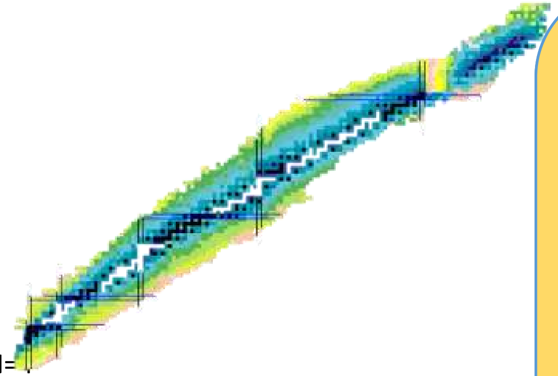
Phenomenological R-matrix is

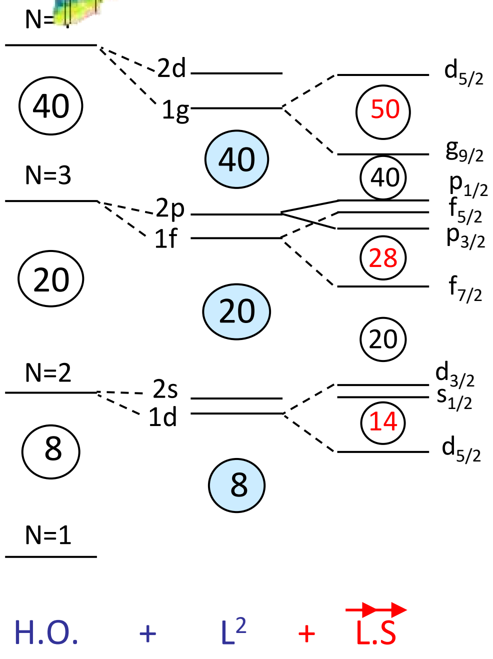
- Very efficient at extracting resonance properties and to evaluate data
- Well situated for nuclear astrophysics
- Not systematically improvable

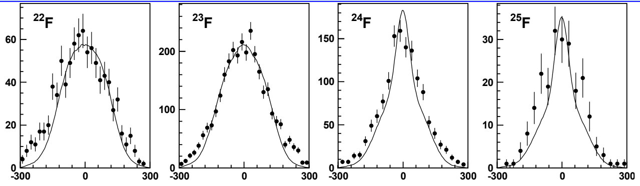
Evolution of nuclear structure and emergence of few-body prop.

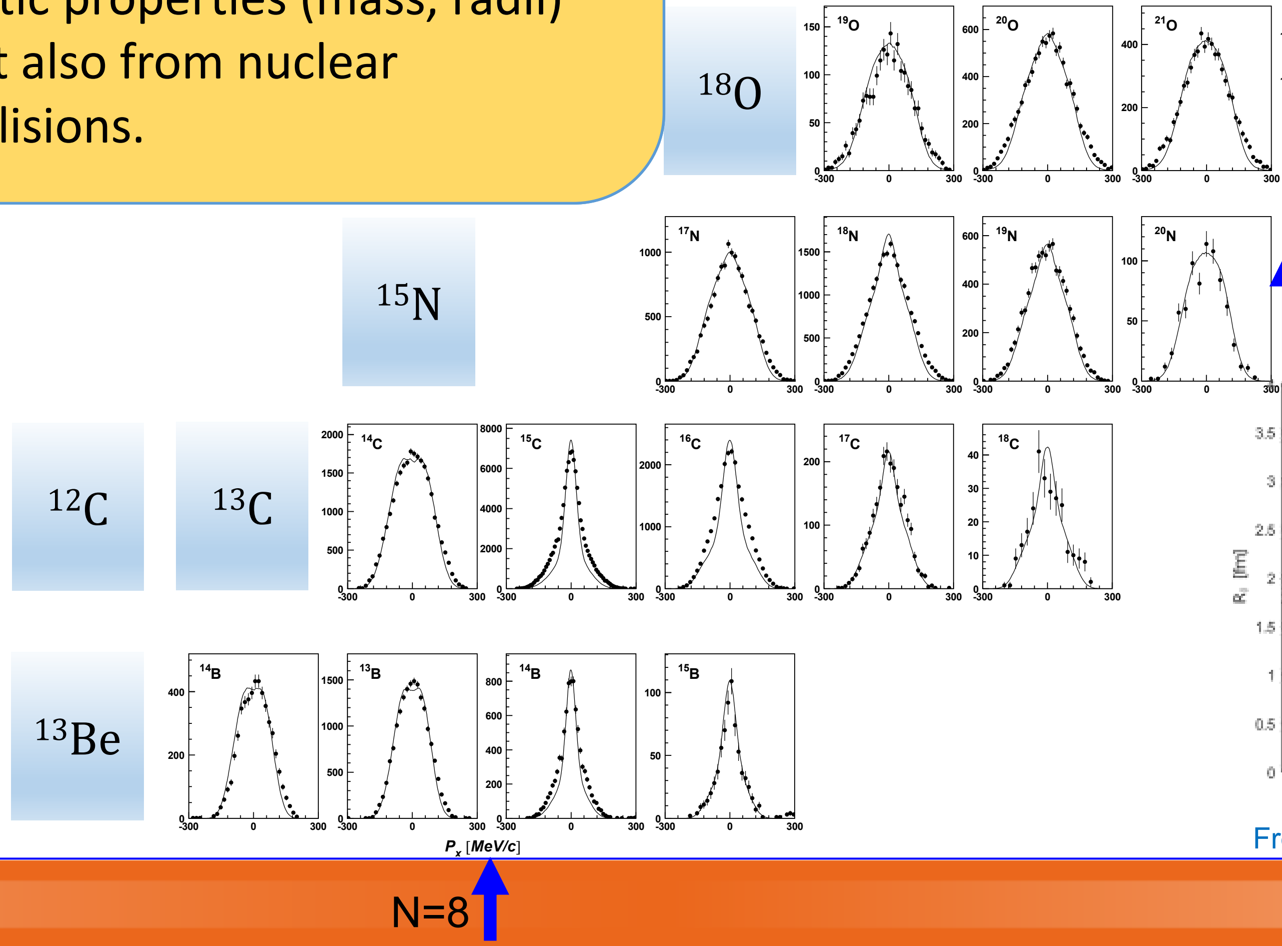
1. Parallel momentum distribution 1n knockout
2. Interaction cross-sections

To study the evolution of nuclear shells, it is more reliable to consider systematics of different observables, e.g. static properties (mass, radii) but also from nuclear collisions.

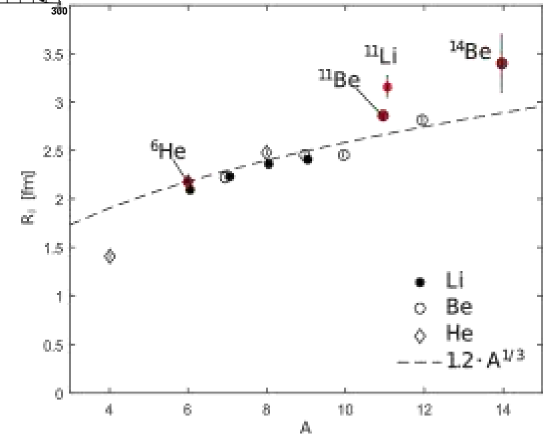






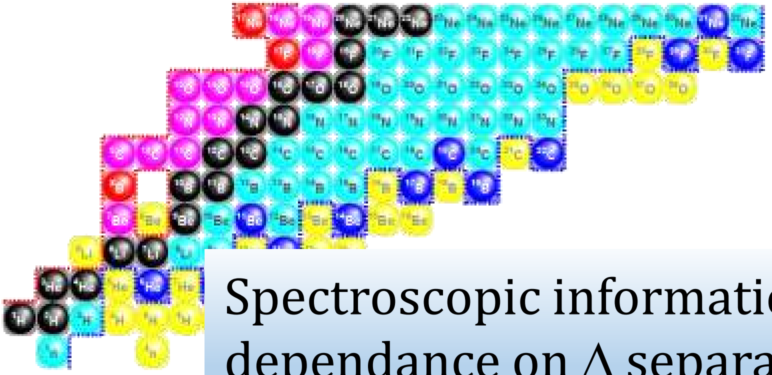


Adapted from : E. Sauvan *et al.* PRC**69** (2004)

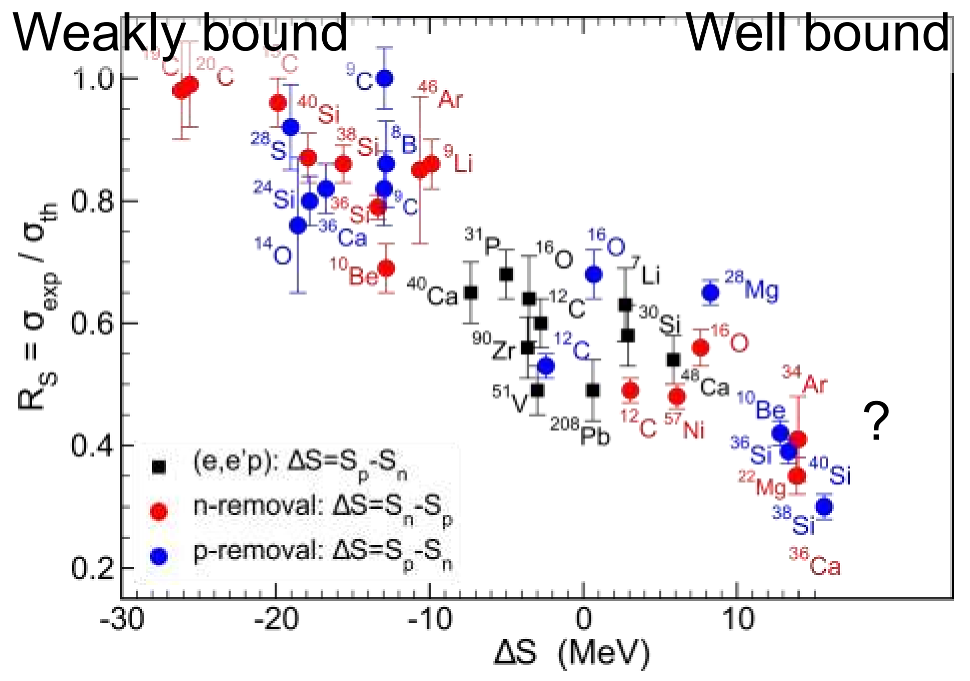


From : C. Hebborn PhD thesis (2020)

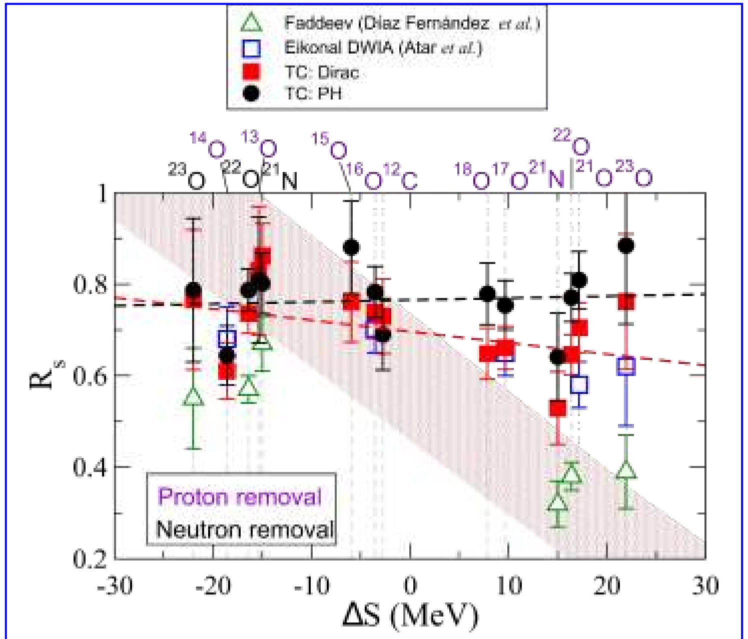
Extracting spectroscopic information



Spectroscopic information dependance on Δ separation energy



J. A. Tostevin and A. Gade PRC**90** (2014).



M.Gómez-Ramos and A.M.Moro PLB**785** (2018)

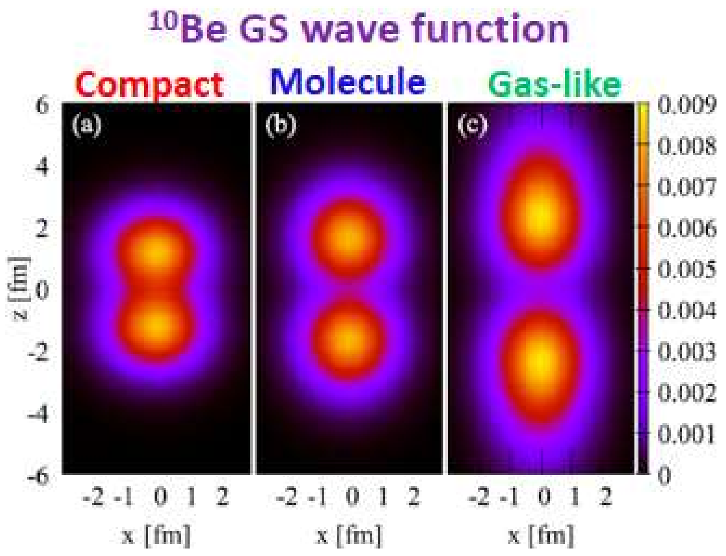
<u>Other models/reactions</u>:

- no apparent dependence on the Δ_S
- compatible with
  - (e,e'p) reactions on stable targets
  - predictions of *ab initio* calculations

Cluster structures using knockout

Observe clustering in nuclei, particularly <u>the type of clustering</u> (d, t, α, ^{6}He, ^{9}Li etc..).

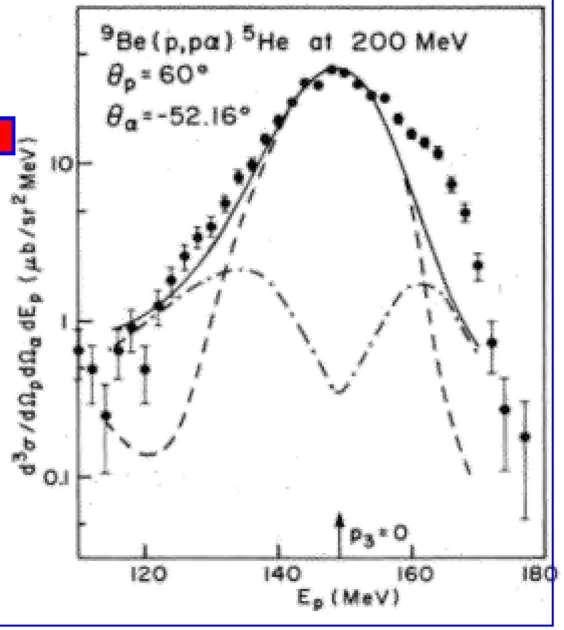
Alpha cluster spectroscopic factor S_α



From : D. Beaumel et J.P. Ebran GDR RESANET

S_α usually extracted from a normalization procedure DWIA cross-section (factorized form)

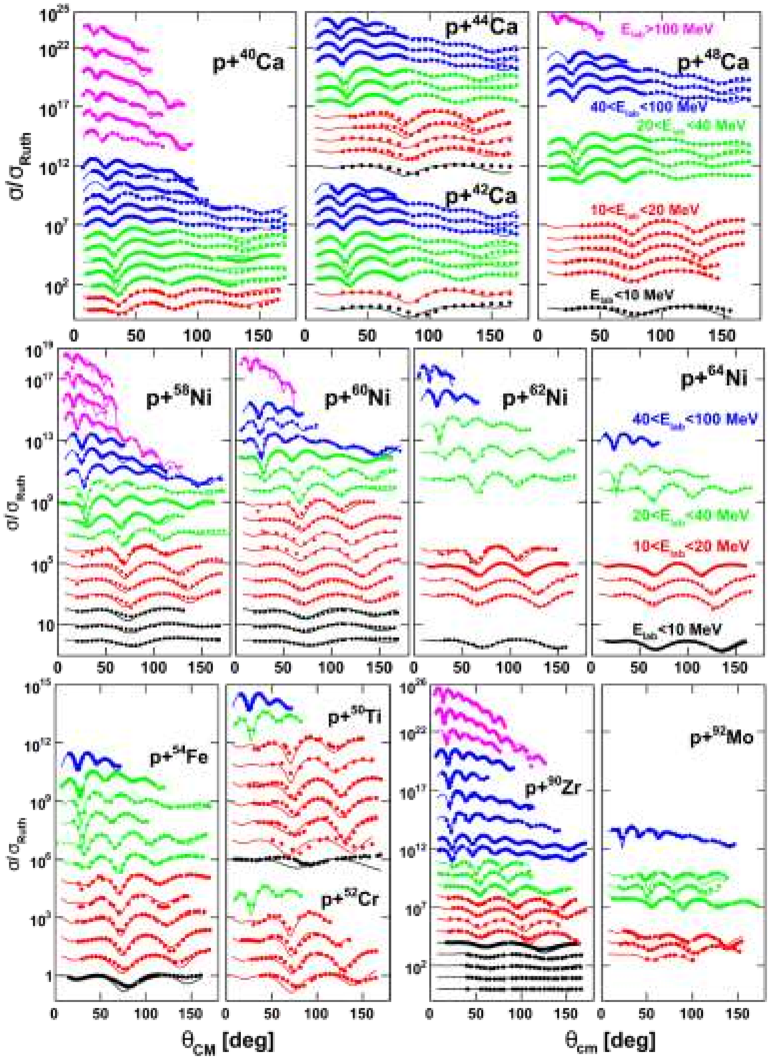
$$\frac{d^3\sigma}{d\Omega_p d\Omega_\alpha dE_p} = S_\alpha F_k \sum_c |T_{k,c}|^2 \frac{d\sigma}{d\Omega_{p-\alpha}}$$



Main uncertainties :

- Optical potentials (~ 25 %)
- Cluster wave functions

Determination of optical potential + applications



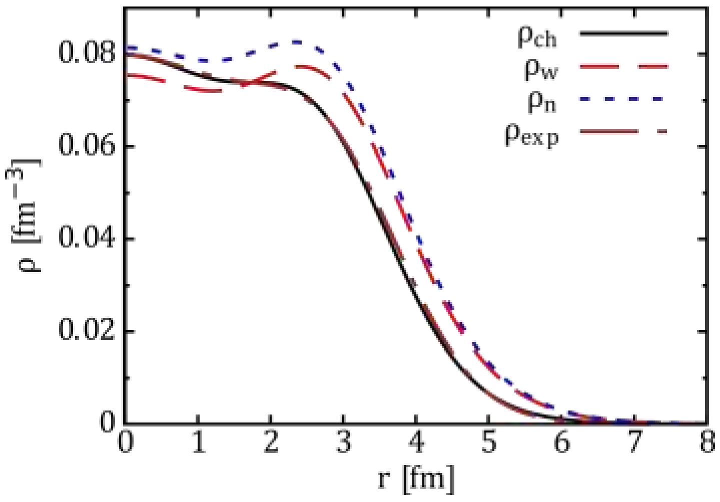
W.H. Dickhoff *et al.* JPG:NPP**44** 2017

Results of global fit to date of a dispersive optical model ~32 parameters

« Data driven »

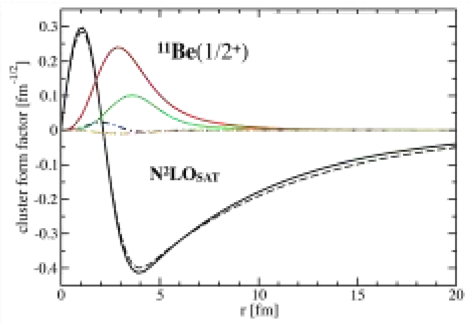
Optical potential are necessary for reaction theory to extract any information on nuclear structure.

For example:
Extraction of neutron and proton self energies to compute the neutron skin of ^{48}Ca 0.249 ± 0.023 fm

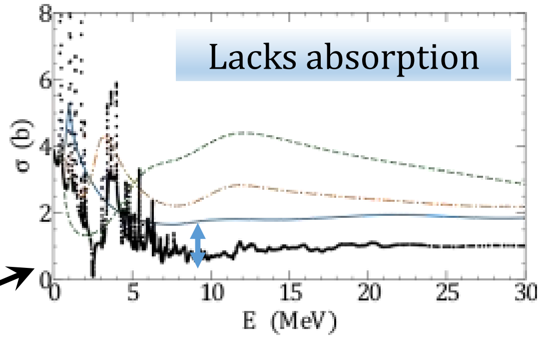


M.H. Mahzoon *et al.* PRL**119** 2017

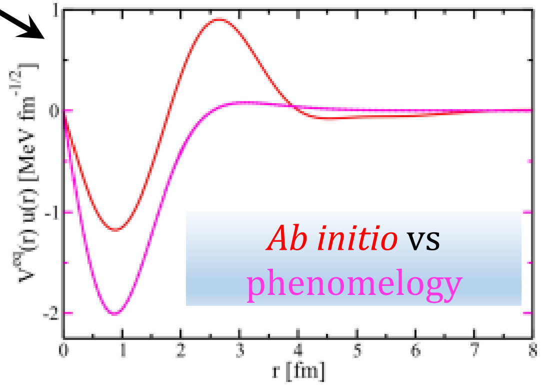
Ab initio inputs and optical potentials for reaction theory



- *Ab initio* methods can provide asymptotic properties of exotic nuclei w.f.
- Eventually optical potential



PRL**123** 092501 (2019)



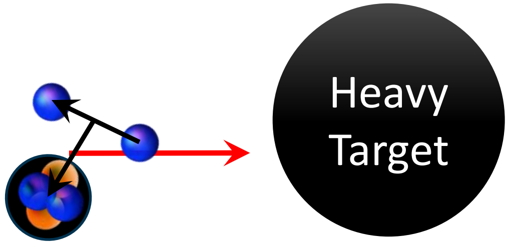
PRC**95** 024315 (2017)

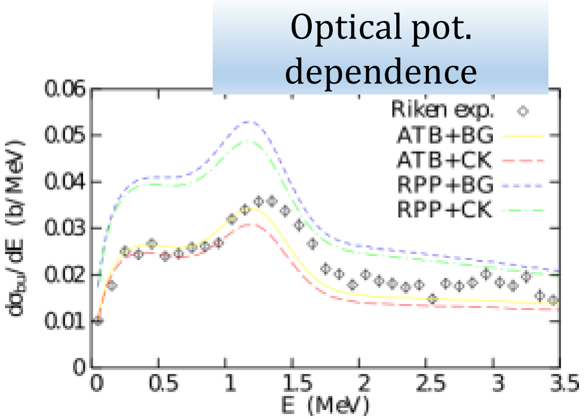
Needs reaction models:

- CDCC.
- Eikonal.
- Time-dependent etc…

Problems:

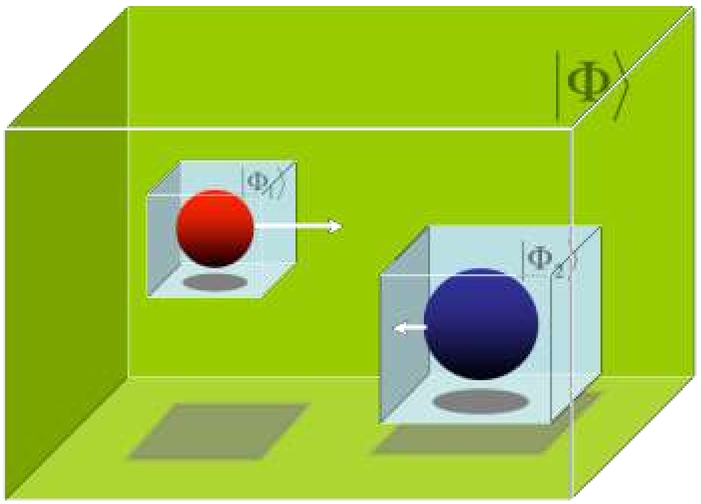
- Need inputs (E_b,E_r,τ_r), not always known.
- Dependent on optical pot.
- Need detail of structure.





AIPCP**791** 12 (2005)

Time-dependent approach: Hartree-Fock



Nuclear reaction with normal/superfluid nuclei on a 3D position mesh.
Hartree-Fock is a standard tool, the wave function $|\Psi_i\rangle$ is a Slater determinant and evolves by

$$i\hbar \frac{d\rho}{dt} = [h(\rho), \rho]$$

this generalizes to

$$i\hbar \frac{dR}{dt} = [\mathcal{H}(R), R],\ R = \begin{pmatrix} \rho & \kappa \\ -\kappa^* & 1-\rho \end{pmatrix}$$

quasiparticle evolution treats nuclear pairing.

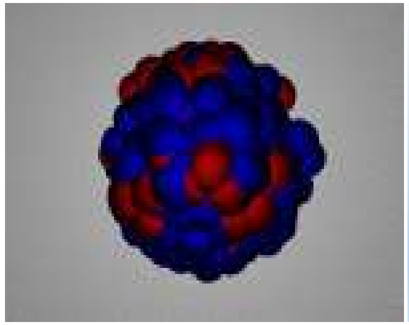
Simenel, Lacroix, Avez, arXiv:0806.2714v2

Yet TDHB is too complicated to be applied systematically → TDBCS limit of TDHFB

$$\Delta_{i,j} = \delta_{i,j} \rightarrow |\Psi(t)\rangle = \prod_{k>0} (u_k(t) + v_k(t) a_k^+(t) a_{\bar{k}}^+)\, |-\rangle$$

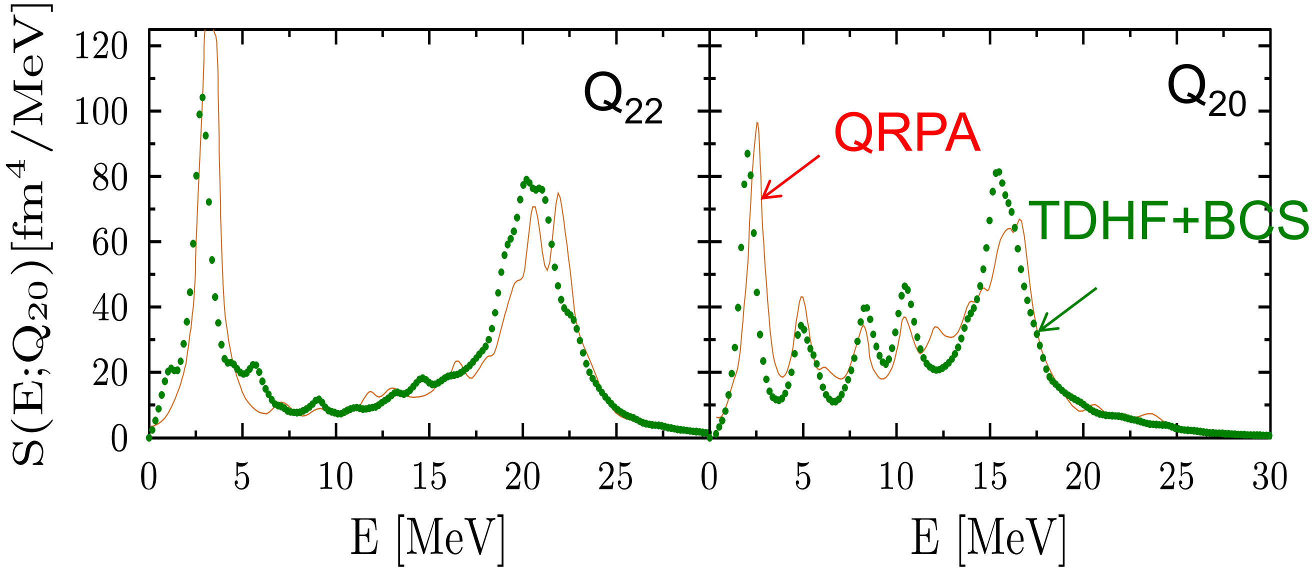
S. Ebata, T. Nakatsukasa *et al.* PRC**82** (2010)
G. Scamps and D. Lacroix, PRC**88** (2013).

Pairing effect on nuclear collective motion: GDR



- Inclusion of pairing provides realistic ground state deformation
- It allows the description of mid-shell nuclei
- It includes partially correlation effects

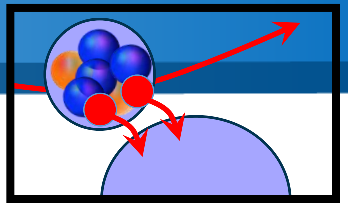
Strength distribution in deformed 34Mg

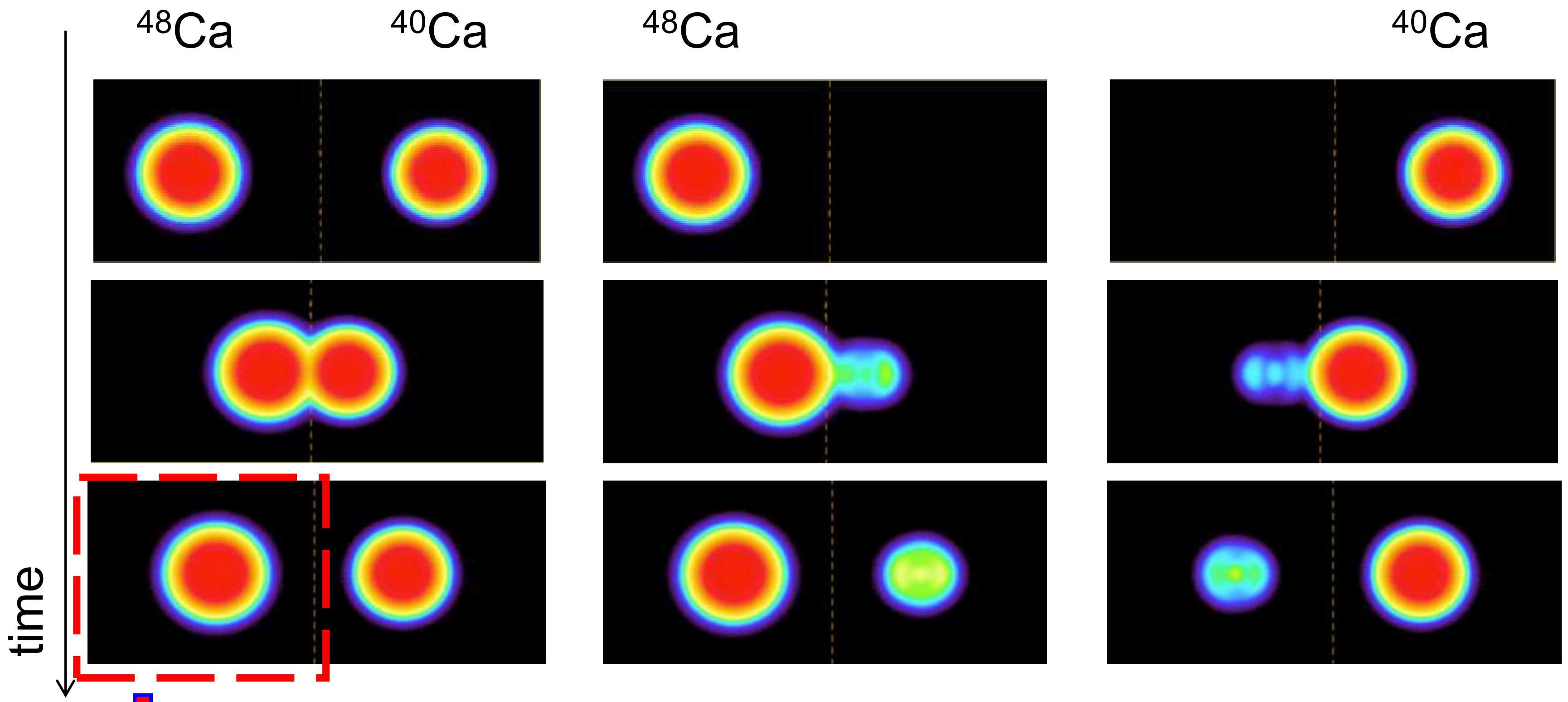


G. Scamps and D. Lacroix PoS**183**(2013)

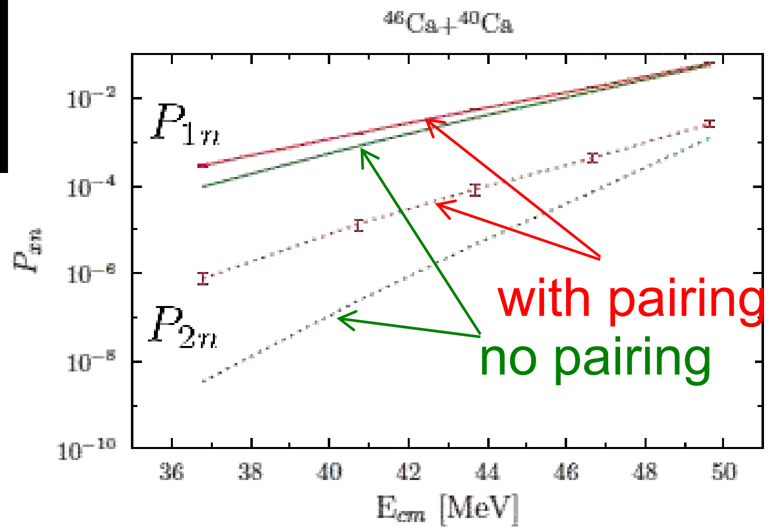
- Almost no difference between TDHF+BCS and TDHFB (QRPA)
- Main effect of pairing is to set the deformation

Transfer reactions – Nucleus-Nucleus





Extract one, two, … nucleons transfer probabilities



G. Scamps and D. Lacroix PRC**87** (2013).

No-core shell model: best for well bound states

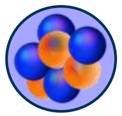
B.R. Barrett, P. Navrátil, J.P. Vary, J.P. Progr. Part. Nucl. Phys. 69 (2013)

- One way to solve the many-body problem

$$\Psi^{(A)}_{NCSM} = |A\lambda J^{\pi} T\rangle = \sum_{\alpha} c_{\alpha} \; |A\alpha j_z{}^{\pi} t_z\rangle$$

Mixing coefficients(unknown)

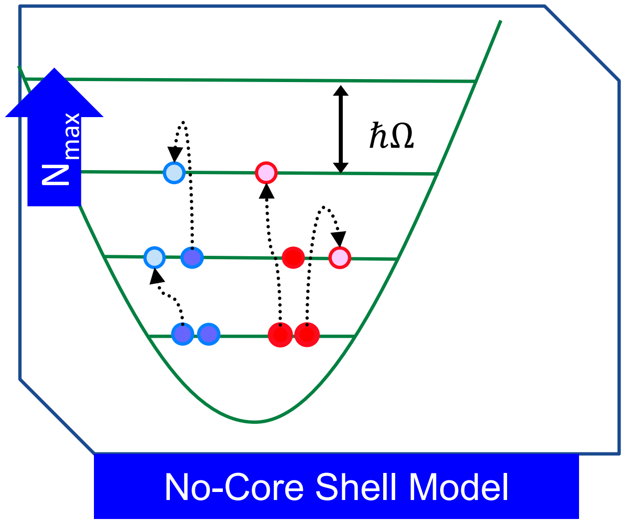
A-body harmonic oscillator states



$$|A\lambda J^{\pi} T\rangle_{SD} \phi_{00}(\vec{R}^{A}_{c.m.})$$

Second quantization

Can address bound and low-lying resonances (short range correlations)



Advantage of HO CI methods:

1. Center of mass is factorized.
2. Mathematically possible to derived s.p. to Jacobi coordinates transformation.
3. Fourier transform is trivial: NCSM, RGM with HO CI is equivalent in momentum or position space.

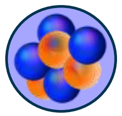
S. Quaglioni, P. Navrátil PRL101 (2008).

- One way to solve the many-body problem when two scales appear

$$\Psi^{(A)}_{NCSM} = |A\lambda J^\pi T\rangle = \sum_\alpha c_\alpha \; |A\alpha j_z{}^\pi t_z\rangle$$

Mixing coefficients(unknown)

A-body harmonic oscillator states



$|A\lambda J^\pi T\rangle_{SD}\phi_{00}(\vec{R}^A_{c.m.})$

Second quantization

Can address bound and low-lying resonances (short range correlations)

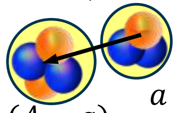
$$\Psi^{(A)}_{RGM} = \sum_\nu \int d\vec{r}\; g_\nu(\vec{r})\, \hat{A}_\nu \left|\Phi^{(A-a,a)}_{\nu\vec{r}}\right\rangle$$

Relative wave function (unknown)

Antisymmetrizer

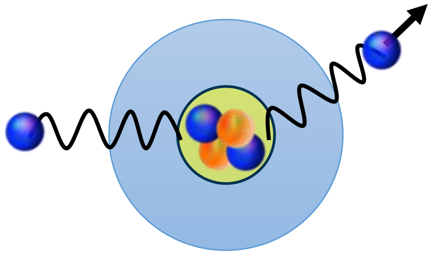
Channel basis

$\vec{r}_{A-a,a}$



$\psi^{(A-a)}_{\alpha_1}\psi^{(a)}_{\alpha_2}\,\delta(\vec{r}-\vec{r}_{A-a,a})$

Cluster expansion technique



NCSM/RGM
Cluster formalism for elastic/inelastic

Many-body basis is twice as large as Ψ_{NCSM}

- $\psi^{(A-a)}_{\alpha_1}$ spans $N_{\max}$
- $\psi^{(a)}_{\alpha_2}$ spans $N_{\max}$

Equal treatment of bound and resonant states: couple NCSM and NCSM/RGM (NCSMC)

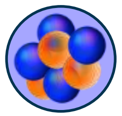
S. Baroni, P. Navrátil and S. Quaglioni PRL110 (2013); PRC93 (2013)

- One way to solve the many-body problem when two scales appear

$$\Psi^{(A)}_{NCSM} = |A\lambda J^\pi T\rangle = \sum_\alpha c_\alpha \; |A\alpha j_z{}^\pi t_z\rangle$$

Mixing coefficients (unknown)

A-body harmonic oscillator states

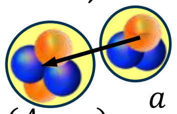


$$|A\lambda J^\pi T\rangle_{SD}\,\phi_{00}(\vec{R}^A_{c.m.})$$

Second quantization

Can address bound and low-lying resonances (short range correlations)

$$\vec{r}_{A-a,a}$$



$$\Psi^{(A)}_{RGM} = \sum_v \int d\vec{r}\, g_v(\vec{r})\, \hat{A}_v \left|\Phi^{(A-a,a)}_{v\vec{r}}\right\rangle$$

Relative wave function (unknown)

Antisymmetrizer

Channel basis

$$\psi^{(A-a)}_{\alpha_1}\psi^{(a)}_{\alpha_2}\;\delta(\vec{r}-\vec{r}_{A-a,a})$$

Cluster expansion technique

Design to account for scattering states (best for long range correlations)

- Our best ansatz combines both wave functions

$$\Psi^{(A)}_{NCSMC} = \sum_\lambda c_\lambda |A\lambda J^\pi T\rangle + \sum_v \int d\vec{r}\, g_v(\vec{r})\, \hat{A}_v \left|\Phi^{(A-a,a)}_{v\vec{r}}\right\rangle$$

NCSMC

Which $N_{\max}$ for $\Psi^{(A)}_{NCSMC}$?

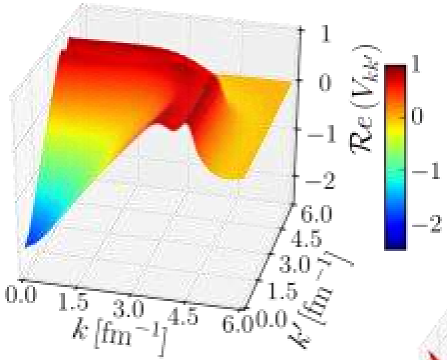
E. D. Jurgenson, P. Navrátil, R. J. Furnstahl PRL103 (2009); PRC83 (2011).

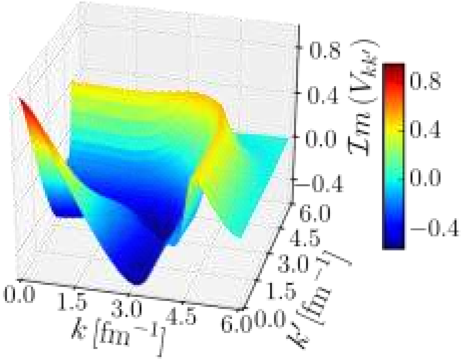
In configuration interaction methods we need to soften interaction to address the hard core. We can use the Similarity-Renormalization-Group (SRG) method

$$H_\lambda(\theta) = U_\lambda H(\theta) U_\lambda^T$$

Similarity Transformation

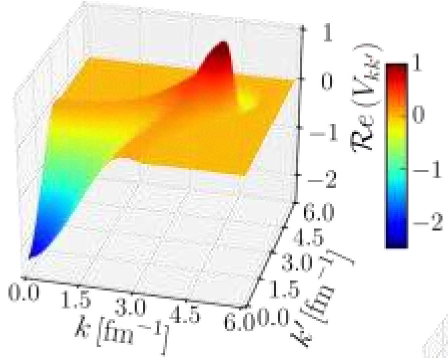
$$\begin{cases} \dfrac{dH_\lambda(\theta)}{d\lambda} = -\dfrac{4}{\lambda^5}[\eta(\lambda), H_\lambda(\theta)] \\ \eta(\lambda) = \dfrac{dU_\lambda}{d\lambda} U_\lambda^T \end{cases}$$

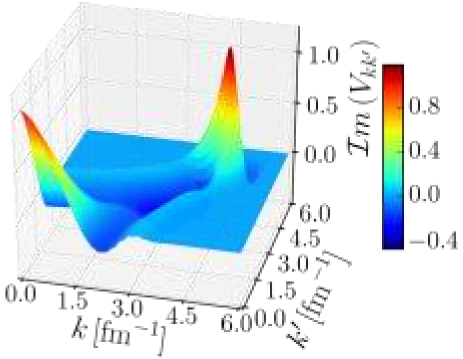




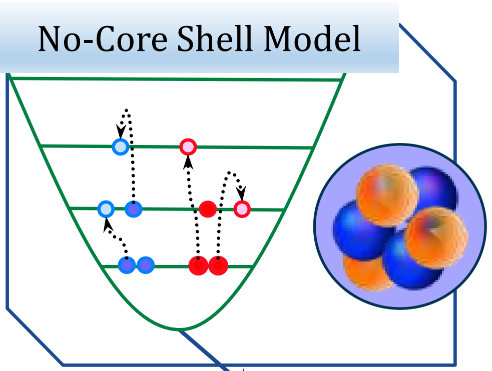
Evolution with flow parameter λ

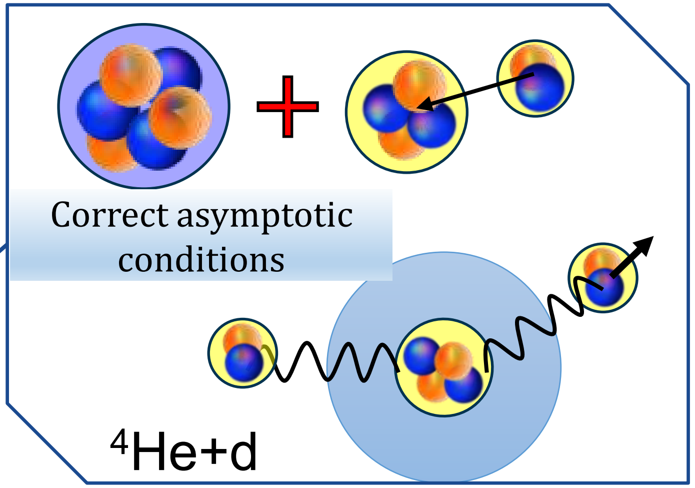
Consistent evolution of the imaginary part

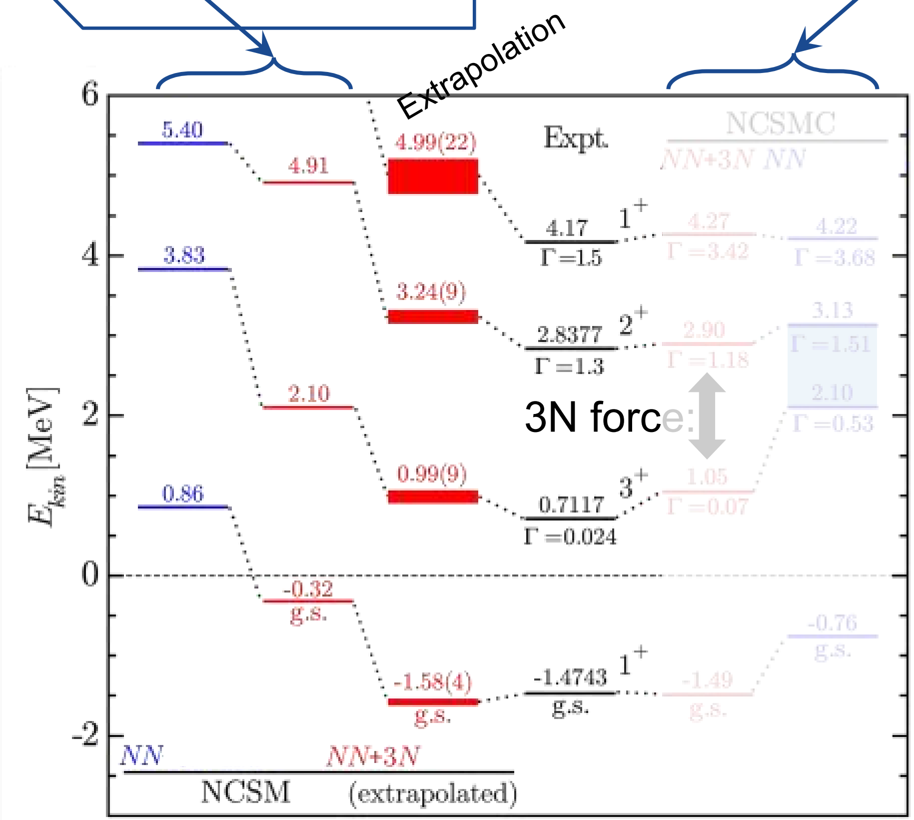




Example: Structure of ^{6}Li continuum





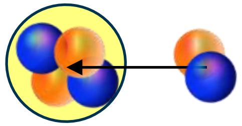


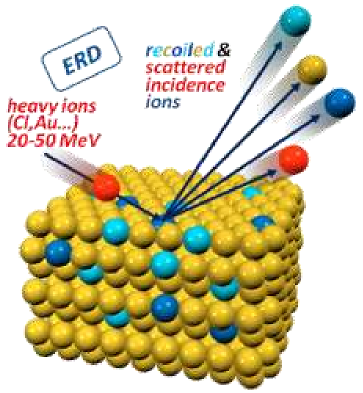
Allowing d+α clustering in the g.s. gives access to ANC.

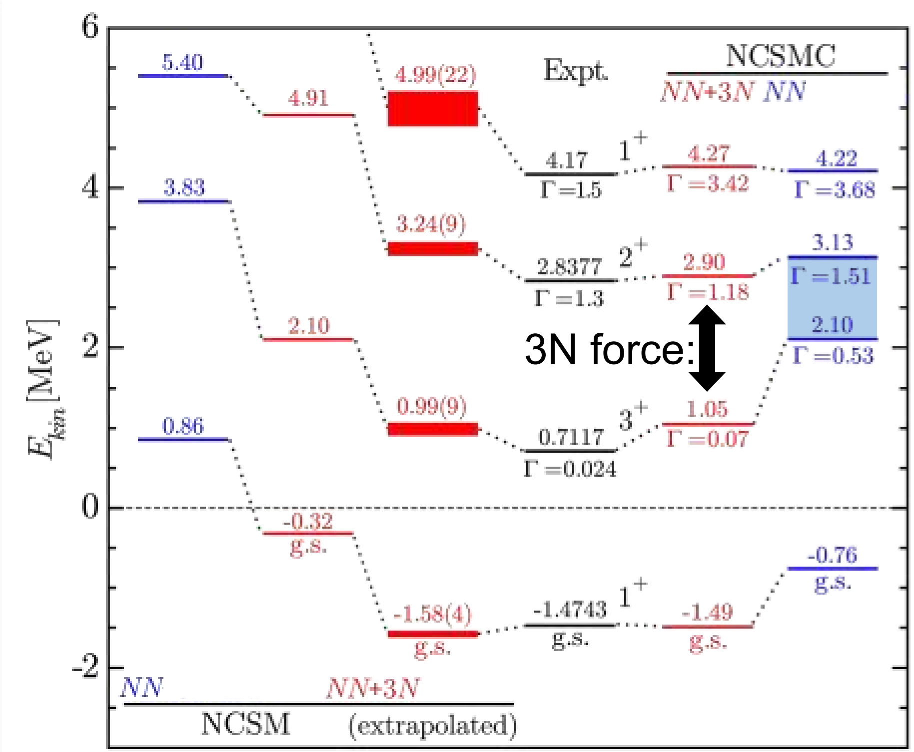
	NCSMC	Experiment[1]
C_0 $[\mathrm{fm}^{-1/2}]$	2.695	2.91 (9)
$C_2[\mathrm{fm}^{-1/2}]$	−0.074	−0,077 (18)
C_2/C_0	−0.027	0.025 (6)

[1]L. D. Blokhintsev *et al.* PRC**48** (1993).

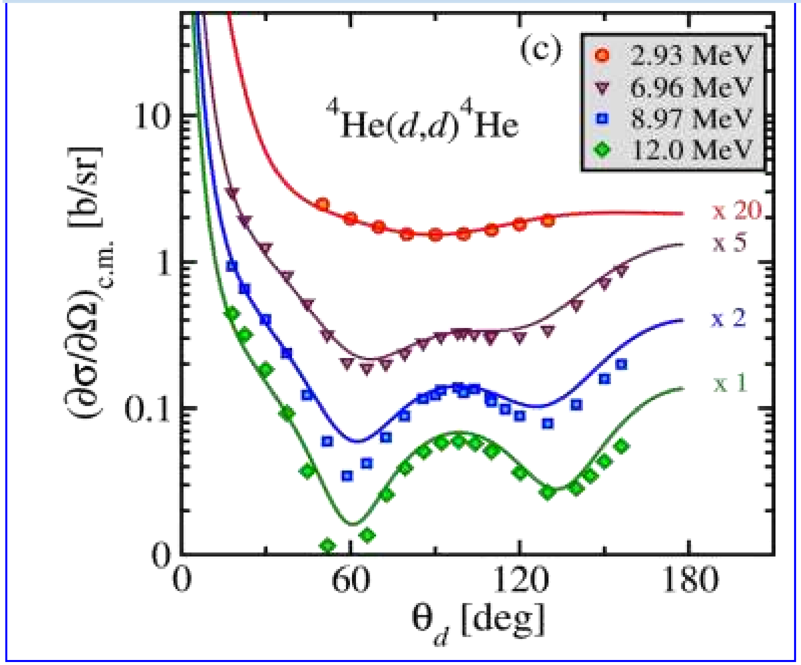
Example: Structure of ^{6}Li continuum



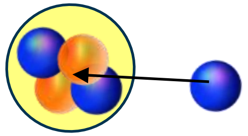




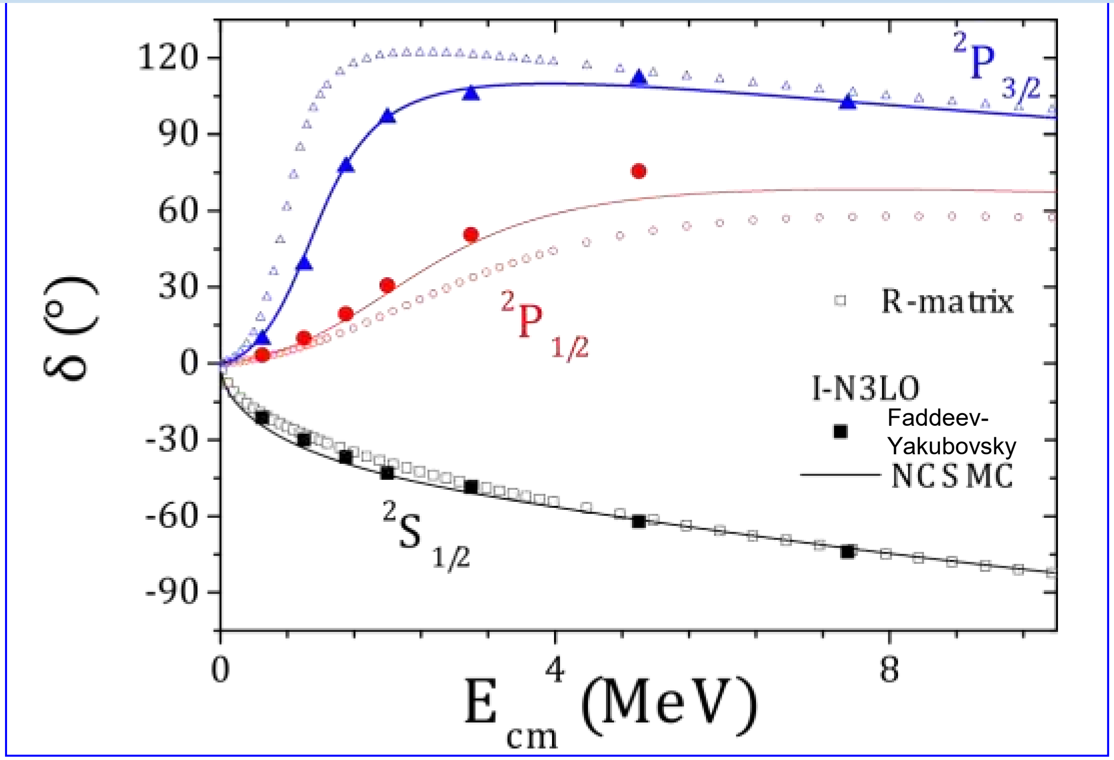
^{4}He(d,d) ^{4}He angular distribution



n-^{4}He scattering: NCSMC vs Faddeev-Yakubovsky

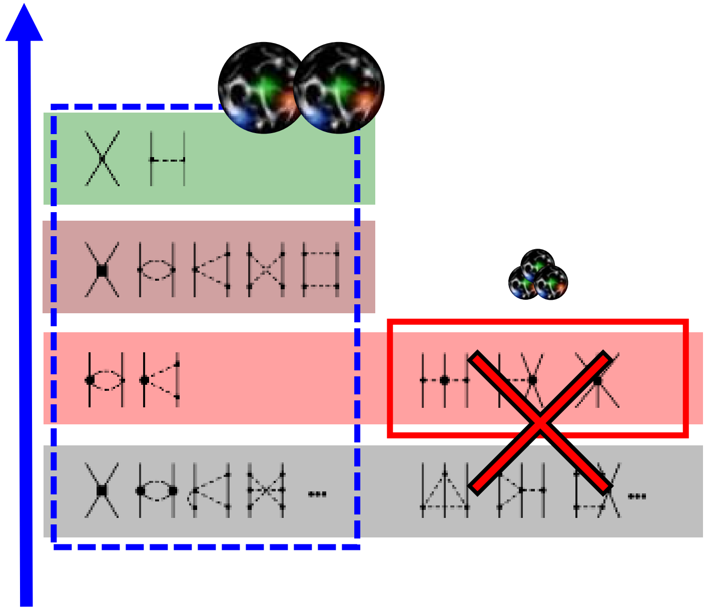


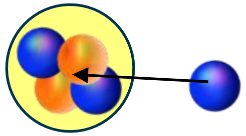
Benchmark: scattering phase shifts NCSMC/FY



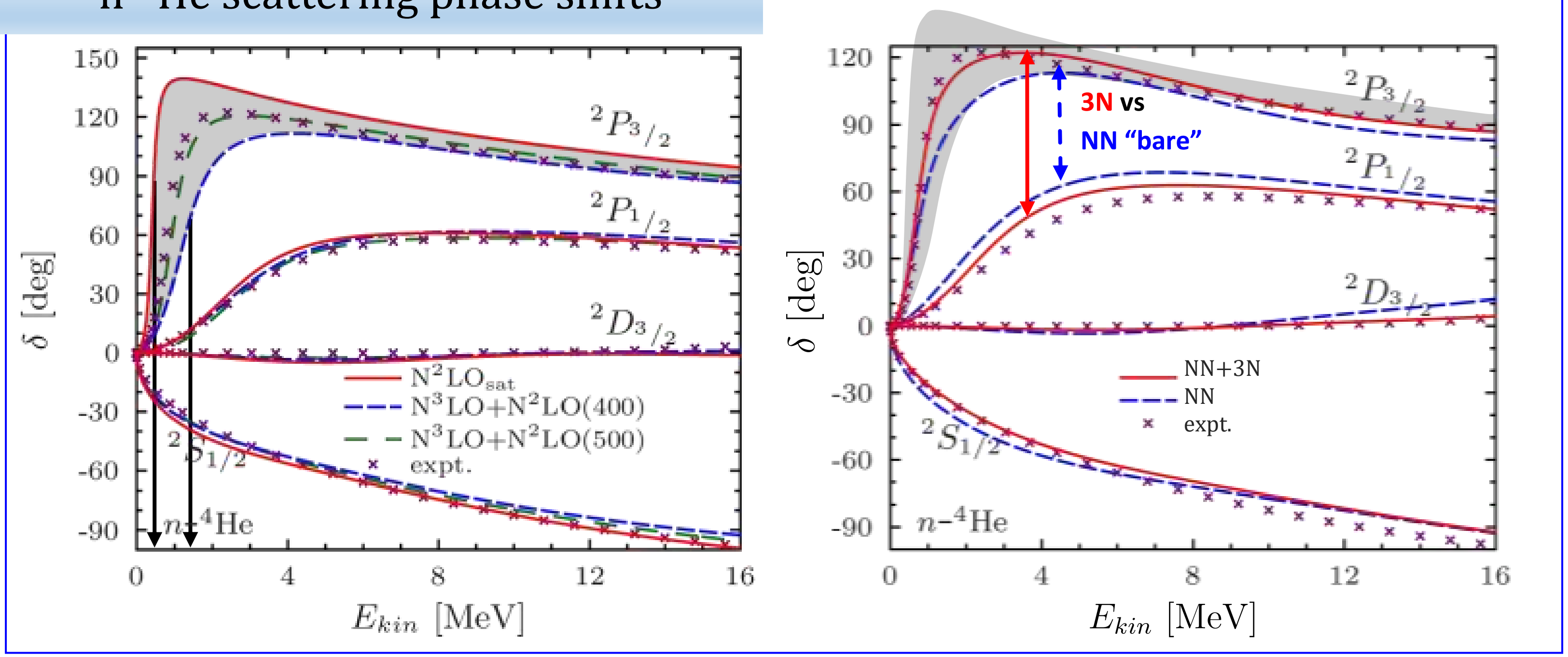
R. Lazauskas, PRC **97** (2018).

Good agreement between the two methods.





n-^{4}He scattering phase shifts



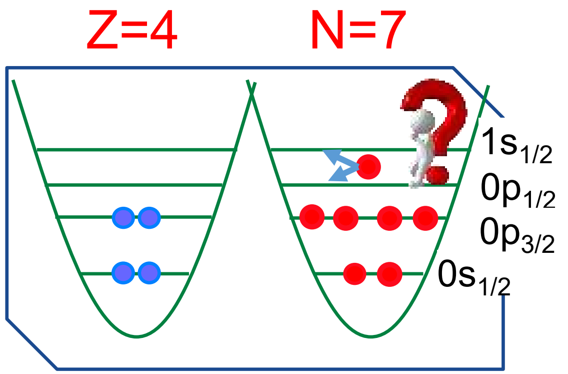
R-matrix results from G. Hale

Some of the shortcomings of the nuclear interaction can already be **probed** in *p*-shell nuclei **through reactions**.
[known since the work of K. Nollett]

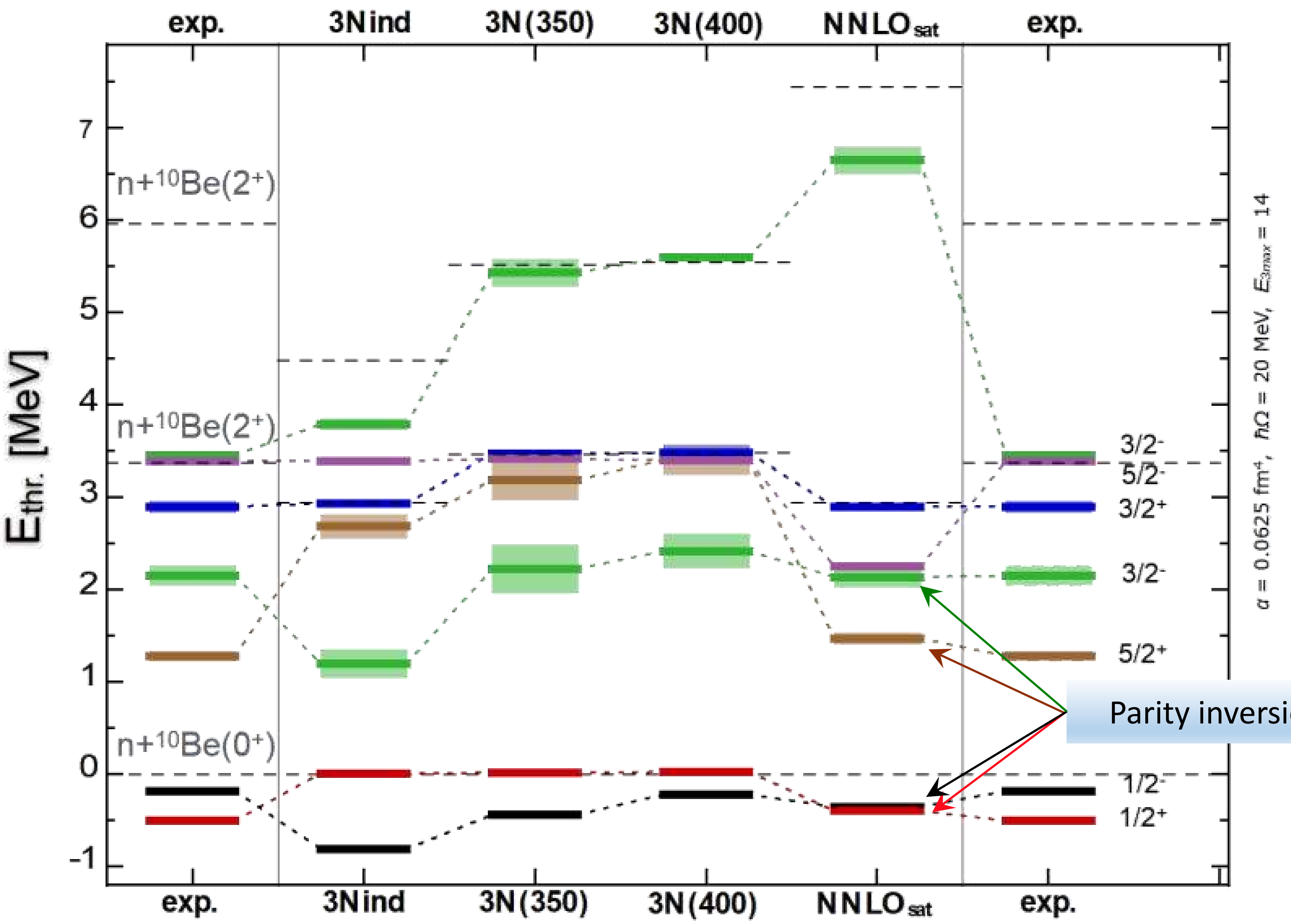
- The 3N interactions **influence** mostly the ***P*-waves**.
- Conservative estimate of EFT accuracy is in the range of 3N force effects.

^{11}Be within NCSMC: Discrimination among chiral nuclear forces

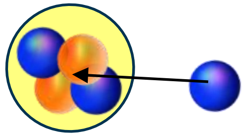
A. Calci, P. Navratil, G. Hupin, S. Quaglioni, R. Roth *et al.* with IRIS collaboration, in preparation



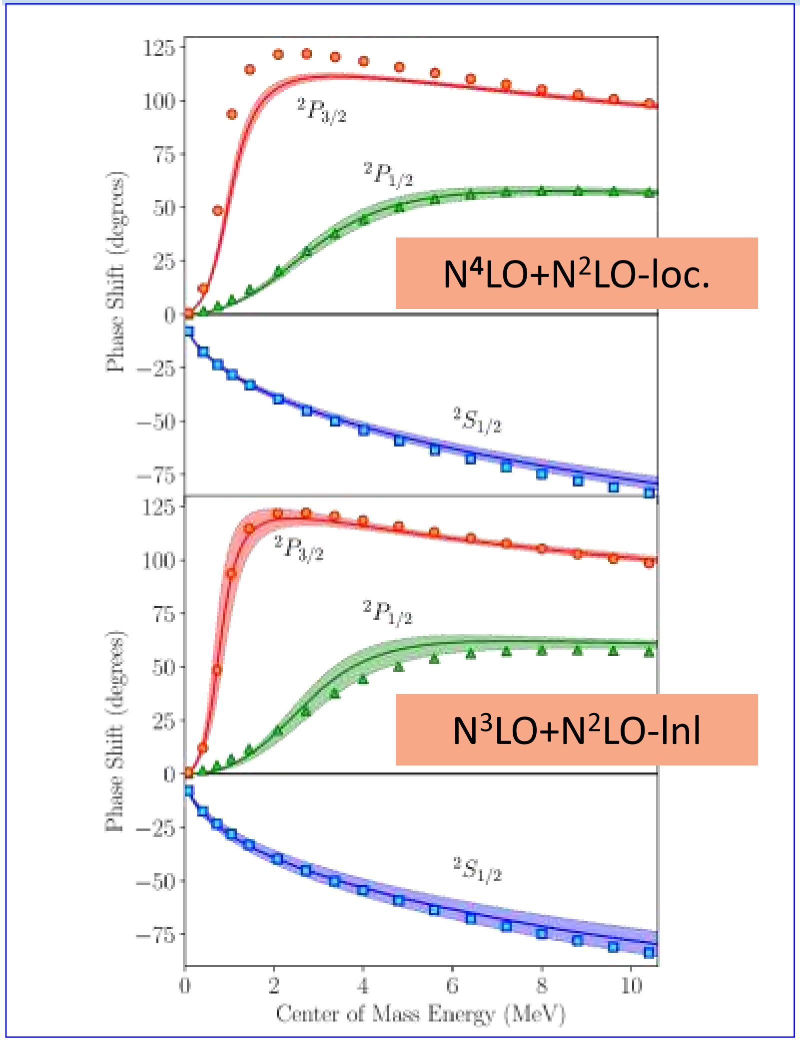
Single particle interpretation



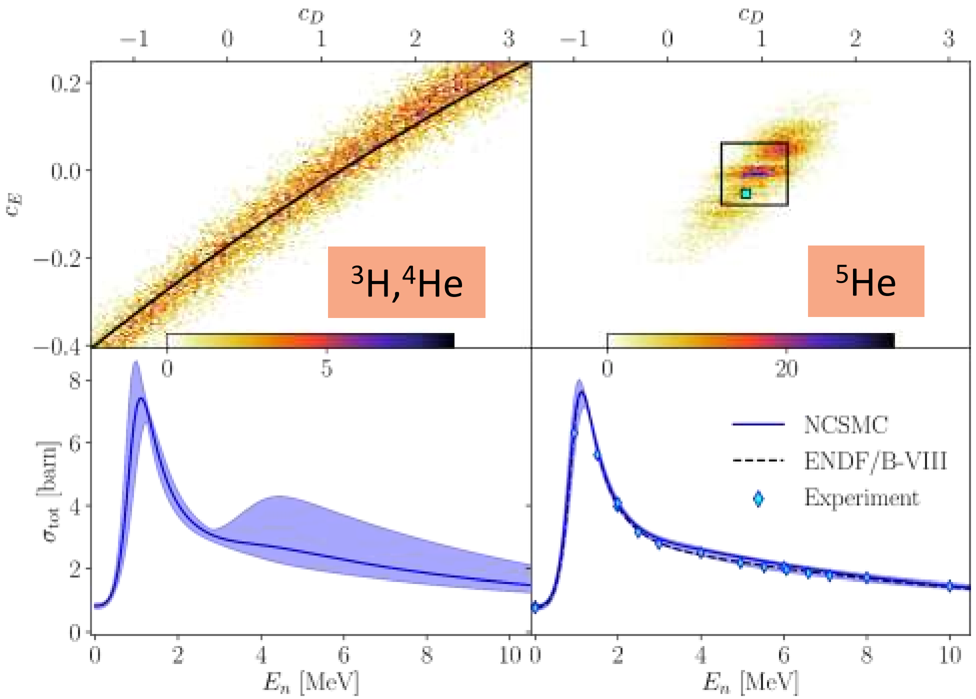
Parity inversion



Sensitivity to c_D and c_E

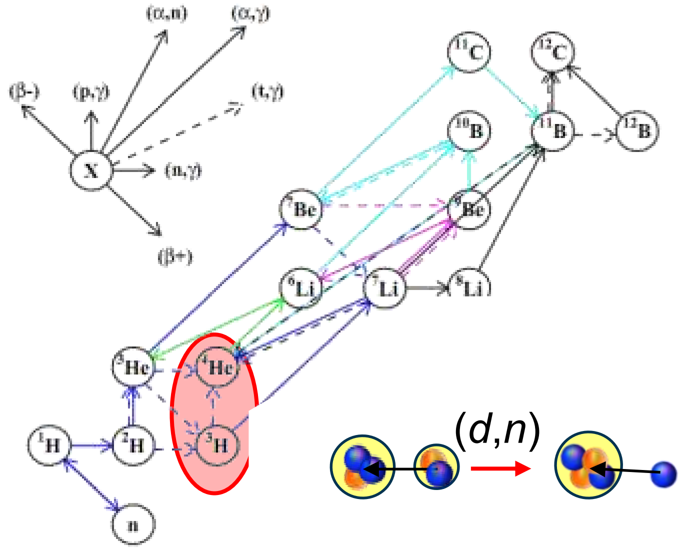


- NN N^4LO + 3N N^2LO cannot reproduce the *p*-wave splitting.
- **Tighter posterior distribution** if the properties of the ^5He are included in the fit.

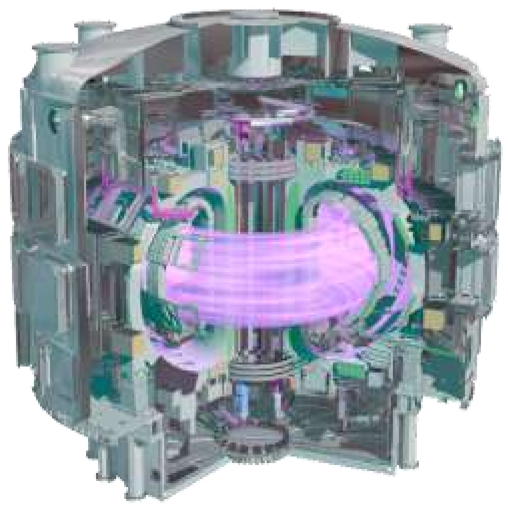


Low-energy Transfer reactions (*d,N*)

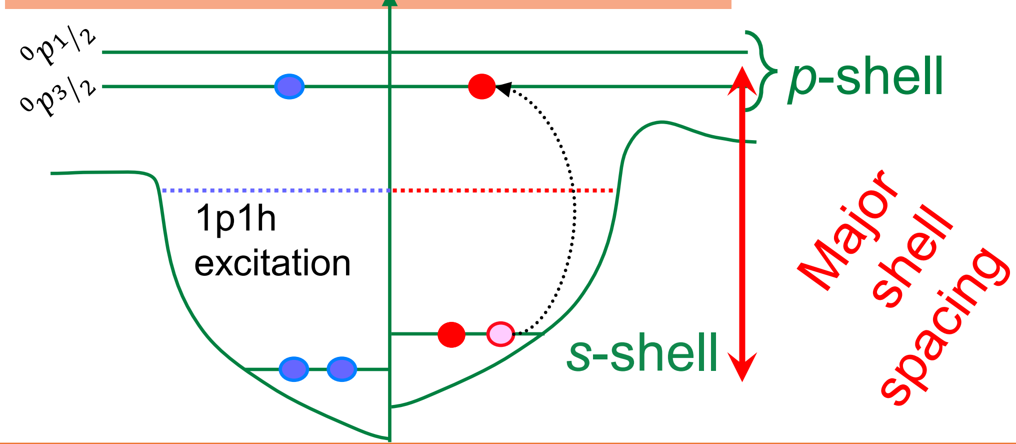
Primordial Nucleosynthesis **(blue)**



ITER design (Cadarache, France)



Structure of the ^{5}He $^{3}/_{2}{}^{+}$ resonance

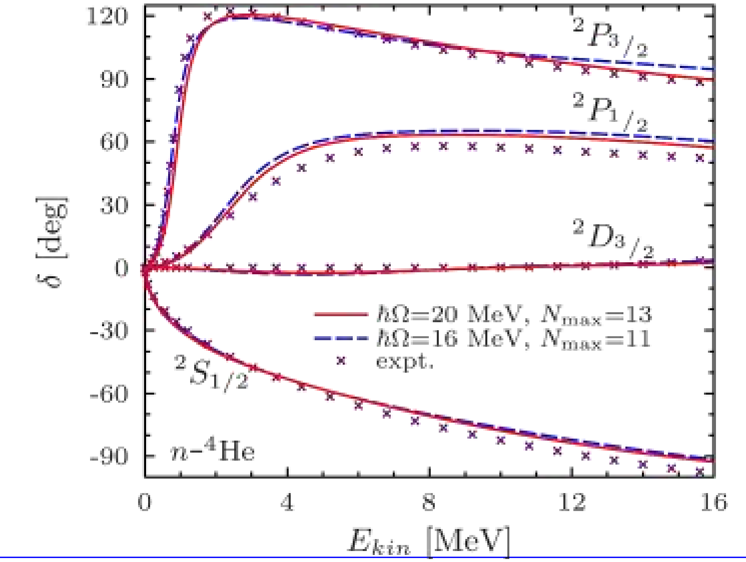


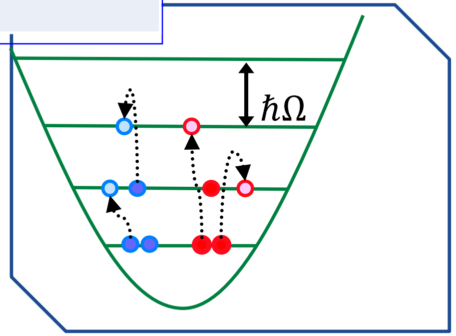
$^3H(d,n)^4He$ fusion reaction: Model convergence

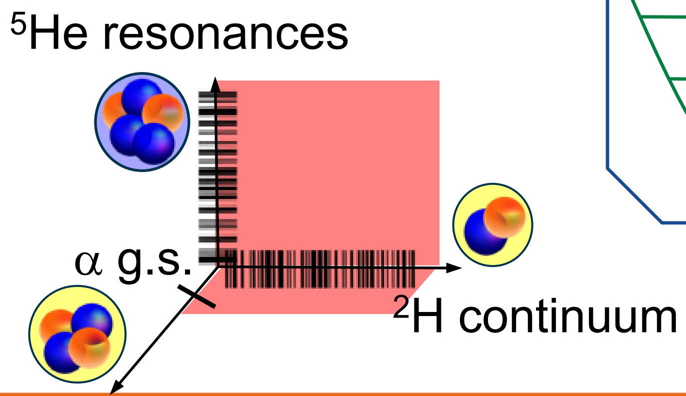
Convergence of $^3/_2{}^+$ resonance

N_{max}	$\hbar\omega$=20 MeV Λ_{SRG}=2.0 fm^{-1}	$\hbar\omega$=16 MeV Λ_{SRG}=1.7 fm^{-1}
7	78.70%	42.29%
9	45.04%	18.85%
11	25.68%	8.41%
13	13.78%	-

n-^{4}He phase shifts

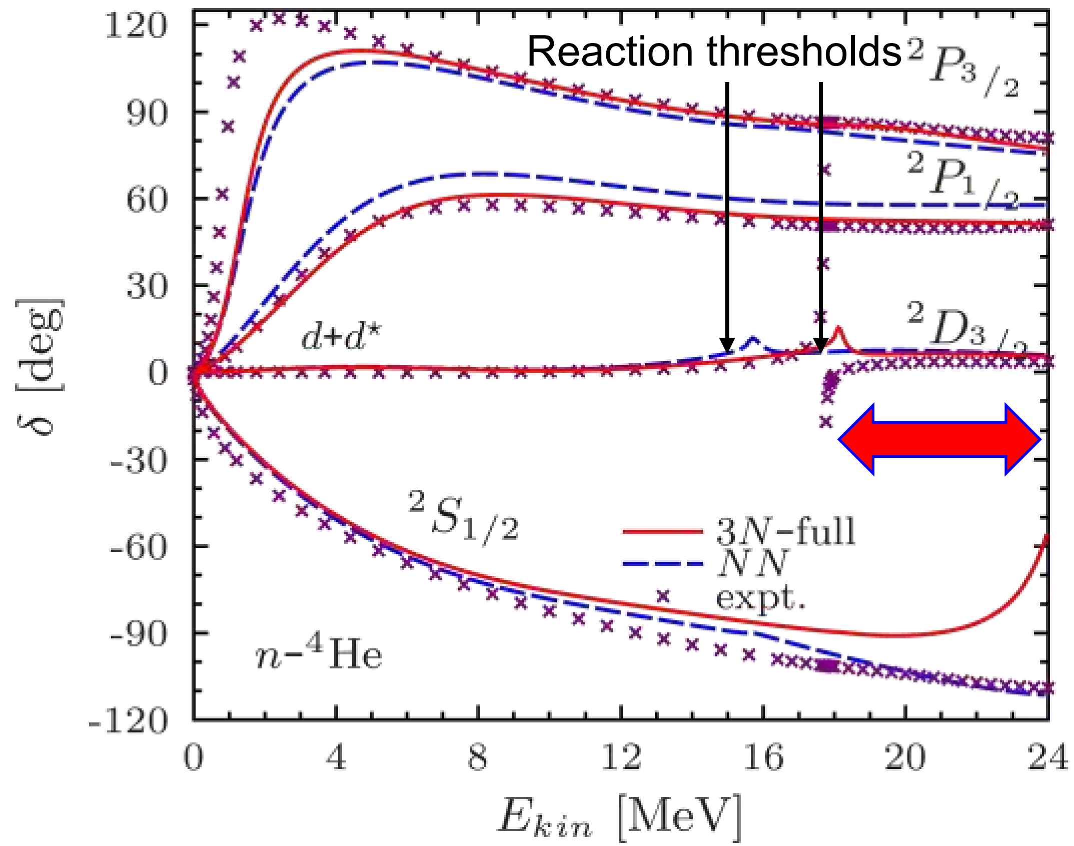


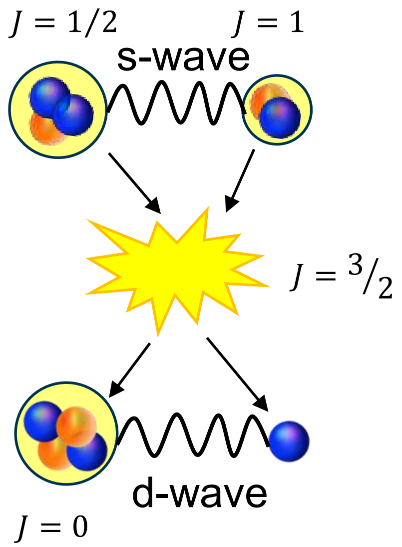




- $^3/_2{}^+$ resonance converges the **fastest** with $\hbar\omega = 16$ MeV, understood from **major shell splitting**.
- n-^{4}He elastic scattering independent of HO frequency and SRG flow.

^{3}H(*d*,*n*)^{4}He fusion reaction: impact of 3N force



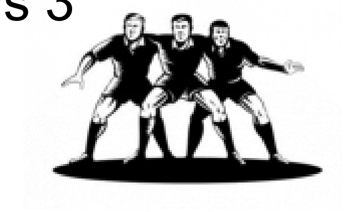


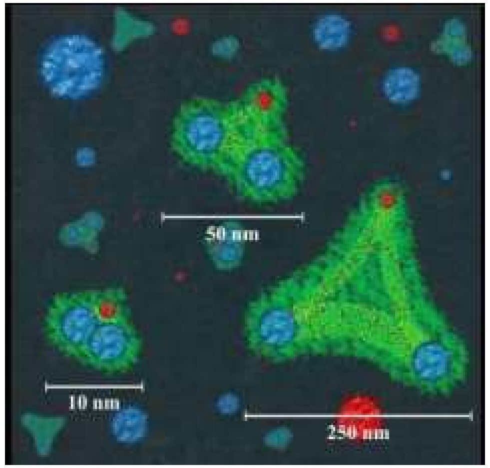
- 3N force impacts:
  - ➢ The threshold position (i.e. reproduction of nuclear masses).
  - ➢ The **positions and splitting** between the $^3/_2{}^+$ and $^1/_2{}^+$ resonances.
- **Tensor force** is essential to model the ^{3}H(*d*,*n*)^{4}He transfer reaction.

Few-body physics get a boost at the dripline

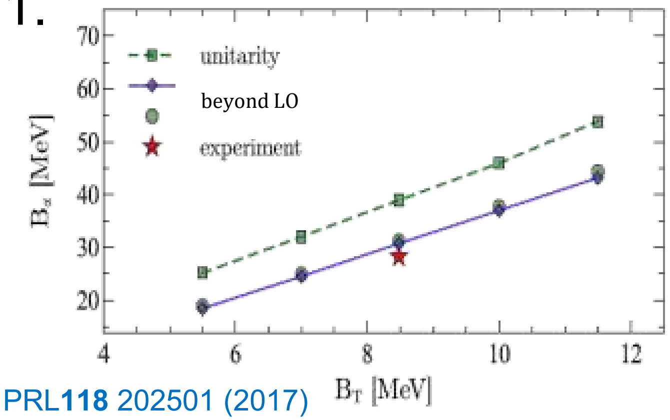
Residual short force + three-body leads to strange universal laws!

2 vs 3





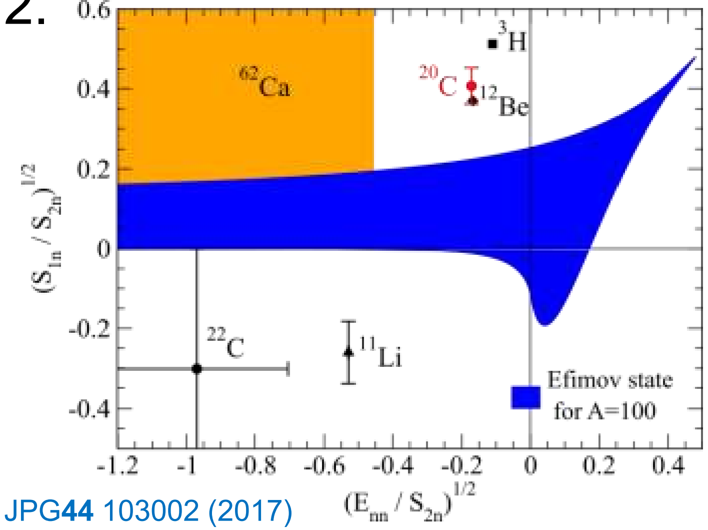
1.



PRL**118** 202501 (2017)

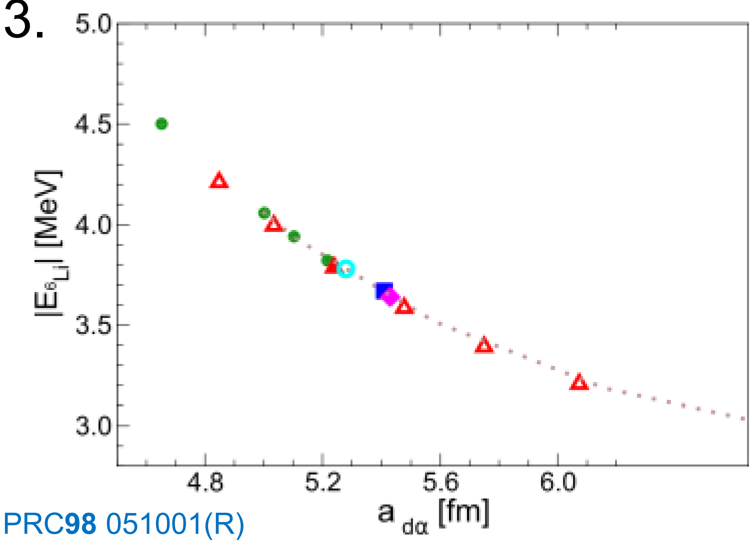
Universal correlations between observables due to the few-body nature of the system

2.



JPG**44** 103002 (2017)

3.



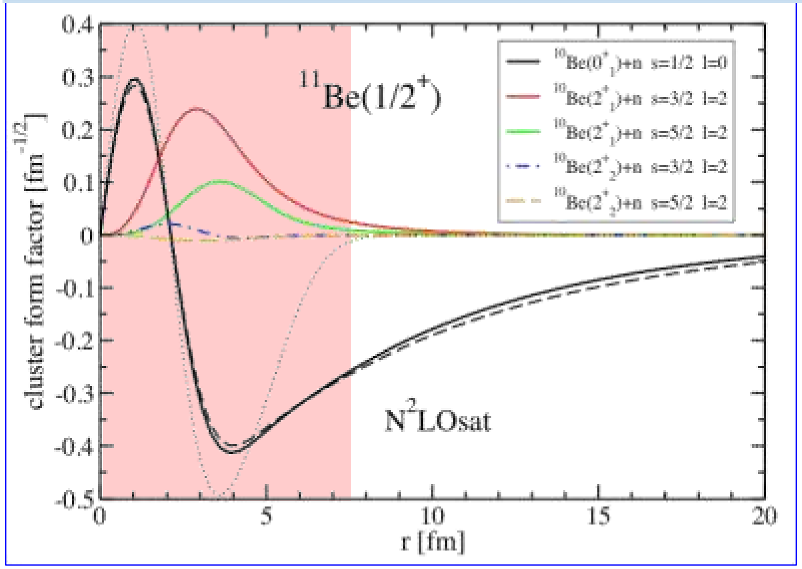
PRC**98** 051001(R)

1. Light nuclei are close to unitarity.
2. Exotic nuclei g.s. in the vicinity of Efimov states.
3. Metastable ^{6}Li g.s. shows universal behavior.

^{11}Be 1n Halo nuclei : EM probes

A. Calci, *et al.* PRL117 (2016)

Halo structure

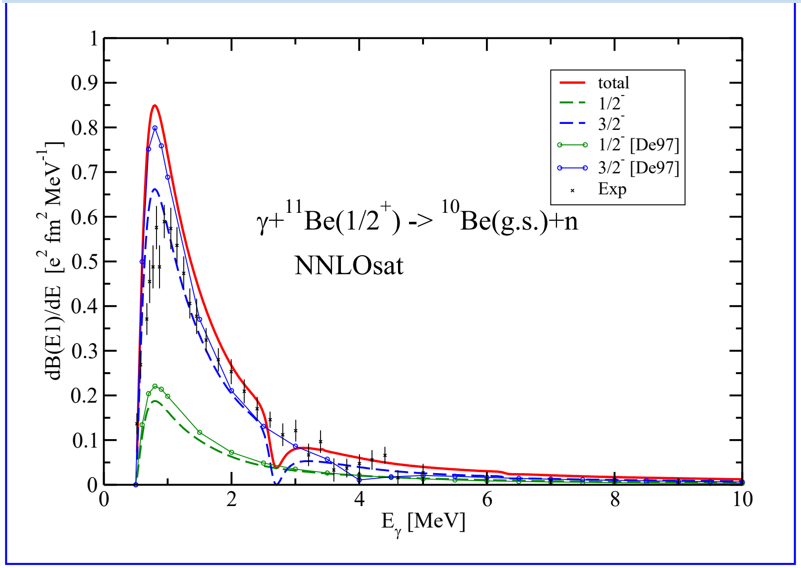


N_{max}=10 model space i.e. $r \approx 7$ fm HO spatial box

E1 transition bound to bound

	NCSM	NCSM-pheno	Expt.
$B\left(E1; \frac{1}{2}^+ \rightarrow \frac{1}{2}^-\right)$ $[e^2\ \mathrm{fm}^2]$	5.10^{-6}	0.118	0.102 (2)

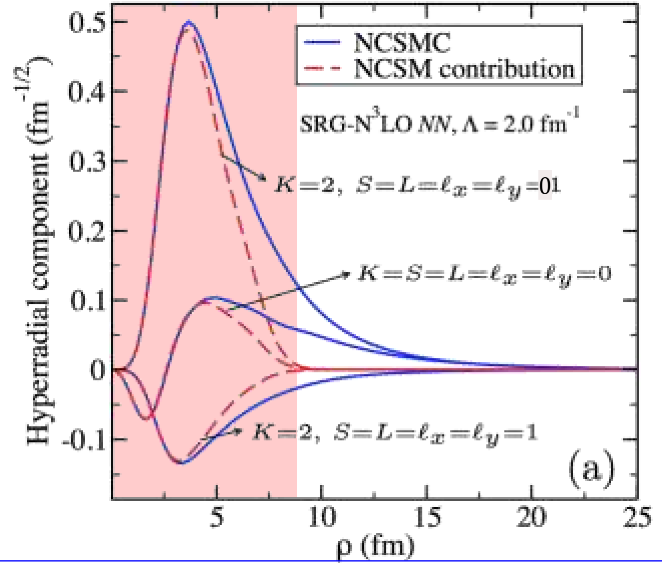
E1 transition bound to continuum



^{6}He 2n Halo nuclei

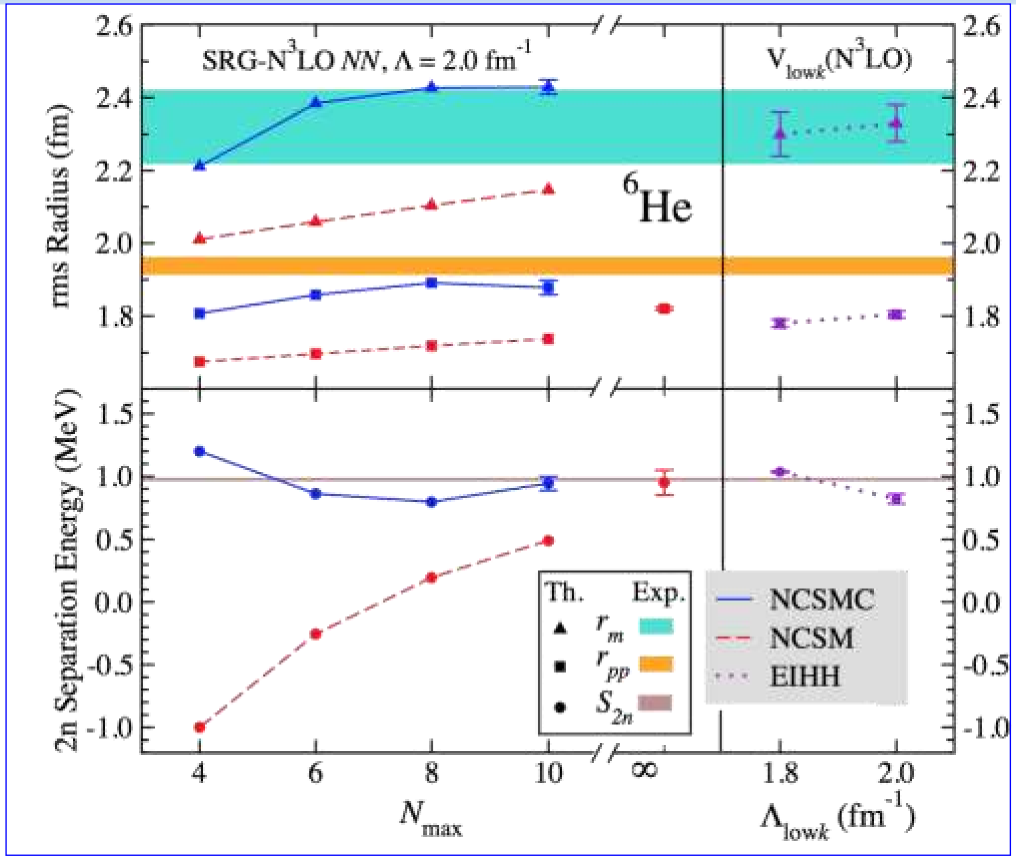
C. Romero-Redondo, S. Quaglioni *et al.* PRL117 (2016)

Halo structure



N_{max}=12 model space i.e. $r \approx 8$ fm HO spatial box

^{6}He g.s. energy, proton and matter radii



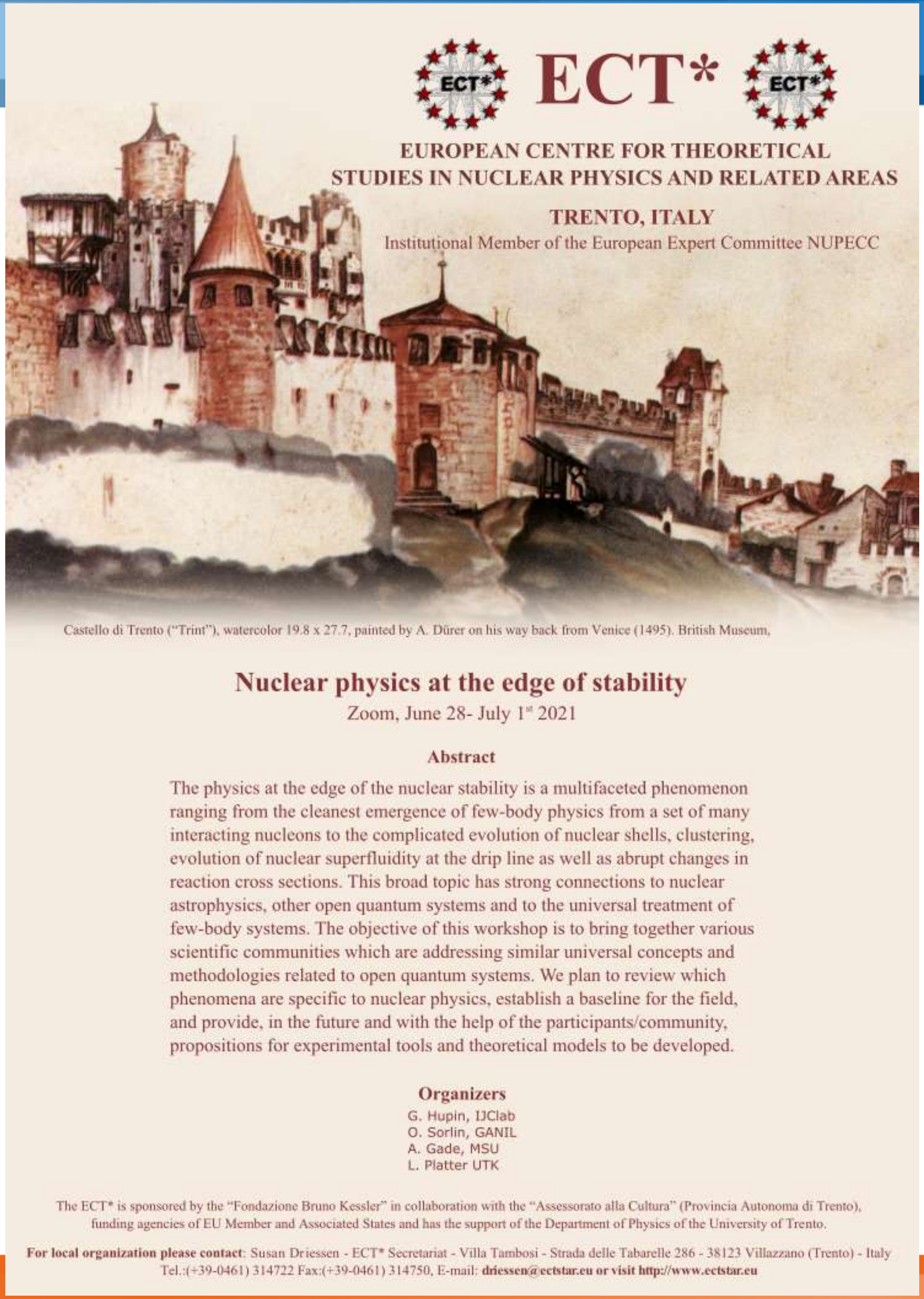
Long-distance components are mandatory to model ^{6}He.

g.s. energy, proton and matter radii from chiral NN force are close to data.

Propaganda slide

ECT*

1. To be a center of **frontline research** in **theoretical nuclear physics**
2. To promote active contacts between **theory** and **experiments**, and to **related areas** of research
3. To further the **training** of **young researchers**



ECT*

EUROPEAN CENTRE FOR THEORETICAL STUDIES IN NUCLEAR PHYSICS AND RELATED AREAS

TRENTO, ITALY

Institutional Member of the European Expert Committee NUPECC

Castello di Trento ("Trint"), watercolor 19.8 x 27.7, painted by A. Dürer on his way back from Venice (1495). British Museum,

Nuclear physics at the edge of stability

Zoom, June 28- July 1st 2021

Abstract

The physics at the edge of the nuclear stability is a multifaceted phenomenon ranging from the cleanest emergence of few-body physics from a set of many interacting nucleons to the complicated evolution of nuclear shells, clustering, evolution of nuclear superfluidity at the drip line as well as abrupt changes in reaction cross sections. This broad topic has strong connections to nuclear astrophysics, other open quantum systems and to the universal treatment of few-body systems. The objective of this workshop is to bring together various scientific communities which are addressing similar universal concepts and methodologies related to open quantum systems. We plan to review which phenomena are specific to nuclear physics, establish a baseline for the field, and provide, in the future and with the help of the participants/community, propositions for experimental tools and theoretical models to be developed.

Organizers

G. Hupin, IJClab
O. Sorlin, GANIL
A. Gade, MSU
L. Platter UTK

The ECT* is sponsored by the "Fondazione Bruno Kessler" in collaboration with the "Assessorato alla Cultura" (Provincia Autonoma di Trento), funding agencies of EU Member and Associated States and has the support of the Department of Physics of the University of Trento.

For local organization please contact: Susan Driessen - ECT* Secretariat - Villa Tambosi - Strada delle Tabarelle 286 - 38123 Villazzano (Trento) - Italy Tel.:(+39-0461) 314722 Fax:(+39-0461) 314750, E-mail: **driessen@ectstar.eu or visit http://www.ectstar.eu**

Thank you !