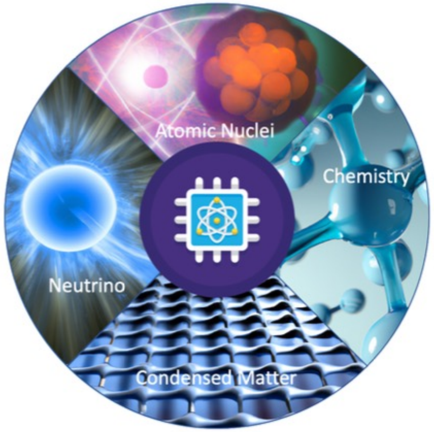
Doctoral Training Program - Orsay, 12-16 June 2023

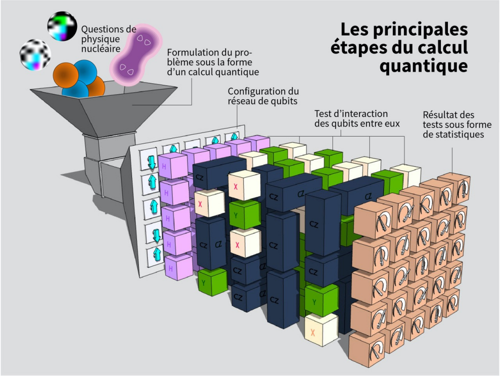
Quantum computing

Denis Lacroix (IJCLab, Paris-Saclay University)

General introduction on Quantum computing today

Current Challenges and applications





(First part)
General aspects of quantum computers and quantum advantage

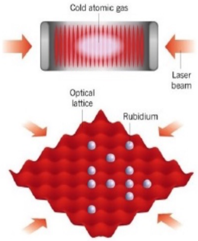

©Google

What means quantum devices today

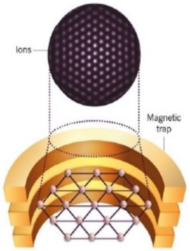
There are many types of quantum computers: ***analog versus digital quantum computers***

There are now many quantum objects one can manipulate

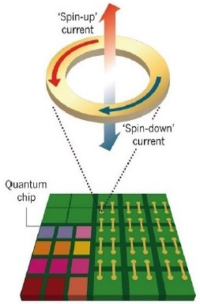
Cold atoms



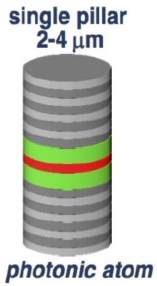
Trapped ions

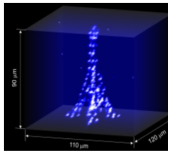


Superc. loops



Polaritons

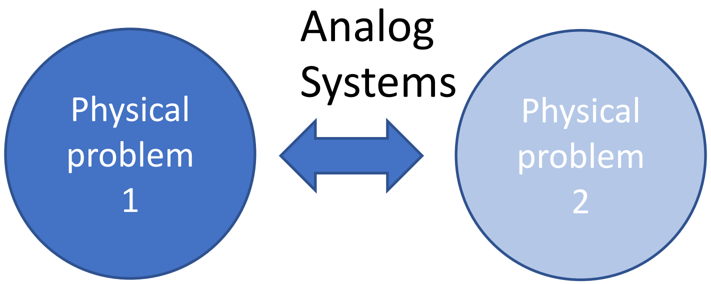




Atomes de
Rydberg dans
des pinces
optiques

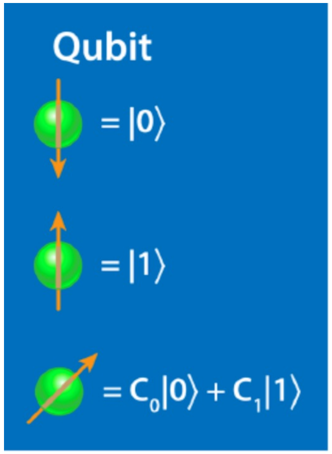
Analog quantum simulator

Digital quantum simulator



Complex problem that cannot or hardly be simulated on classical computers

Analog problem to 1 that could be tested in laboratory



➡ Non-universal

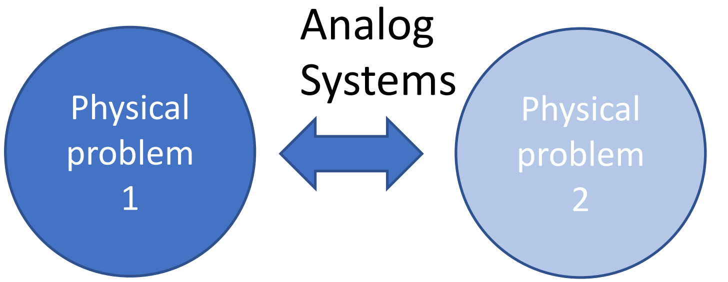
➡ Universal Quantum simulation

Quantum analogue simulation

There are many types of quantum computers: ***analog versus digital quantum computers***

A few examples

Analog quantum simulator



Physical problem 1

Analog Systems

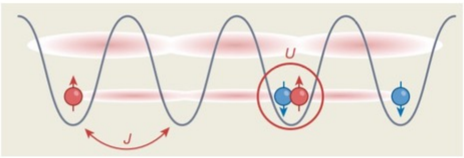
Physical problem 2

Complex problem that cannot or hardly be simulated on classical computers

Analog problem to 1 that could be tested in laboratory

- The mapping from one physical problem to another physical problem is a delicate issue
- It strongly depends on the problem itself (non-universality)

Analog systems on lattice (Fermi-Hubbard, Schwinger model, …)



Analog simulation of astrophysics/cosmology

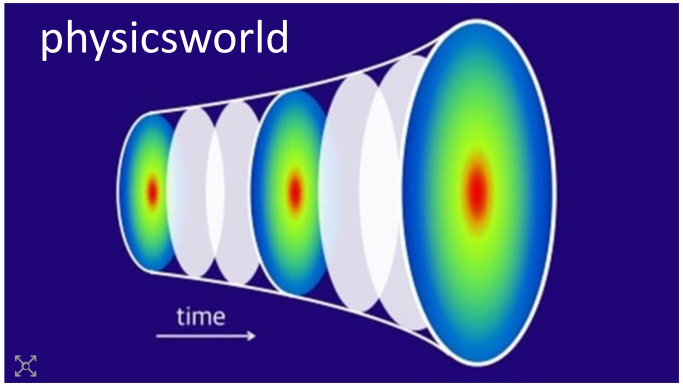
Ultracold Fermi gas

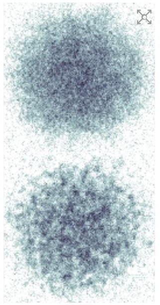
Neutron stars

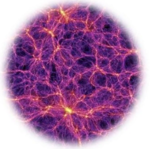
Viermann et al. Nature

An expanding universe is simulated in a quantum droplet

22 Mar 2023 Campbell McLauchlan





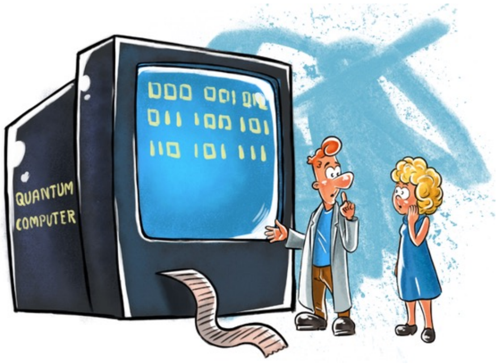


Digital quantum simulation with qubits

Digital quantum devices are supposed to be universal

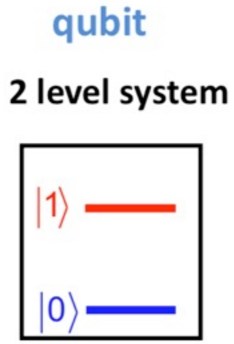
Classical computers
Works with bits

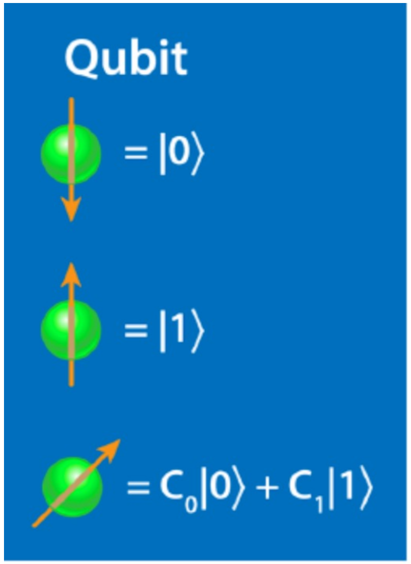
Quantum computers with
Quantum bits



Bits are only 0 or 1

Qubits can be seen
As two-level systems





Obvious advantage

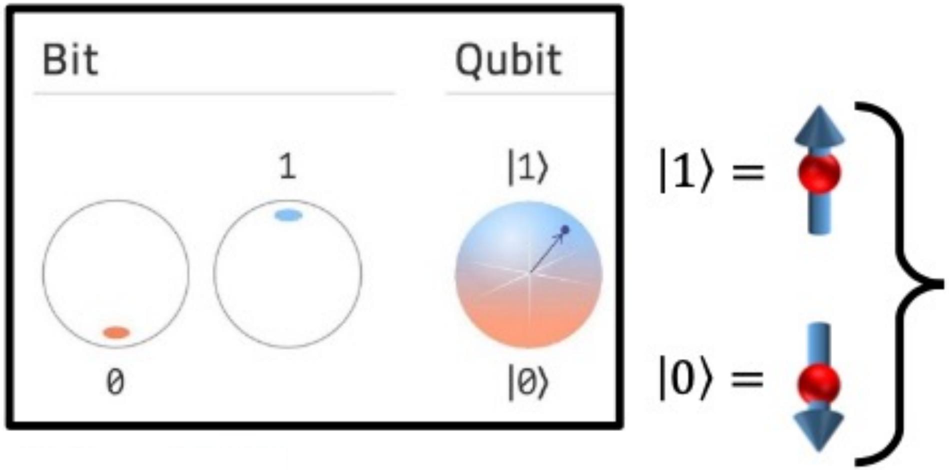
A single Qubit can be any superposition of 0 and 1

And with many
Qubits

New aspects can be used like quantum interference and entanglement

Where do we expect quantum advantage?

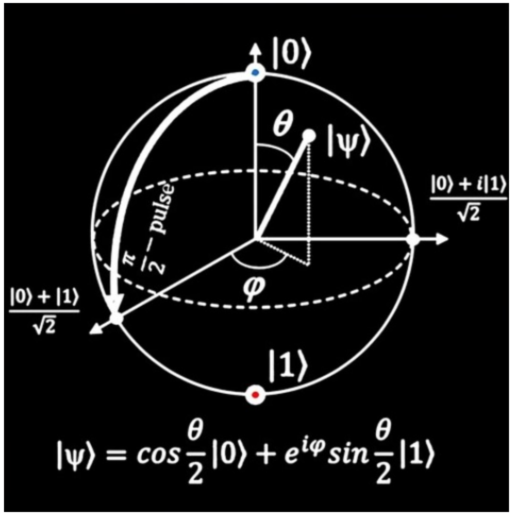
A few examples



Quantum RAM advantage

Storing with
-1 bit: 2 integers 0,1
-2 bits: 4 integers 0,1,2,3
…

Bloch sphere picture



Storing with one qubit

$$|\Phi\rangle = f_1(\theta)|0\rangle + f_2(\theta,\varphi)|1\rangle$$

Two-qubits

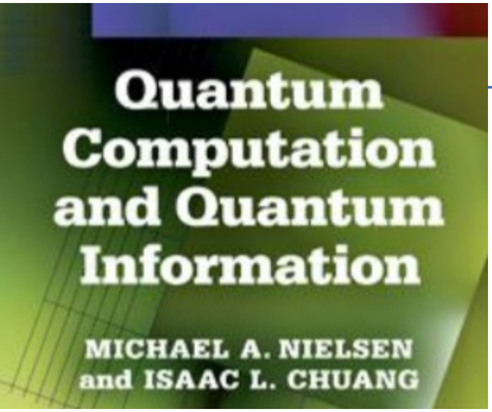
$$|\Phi\rangle = f_1(\theta_1,\theta_2)|00\rangle + \cdots$$

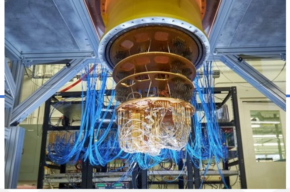
…

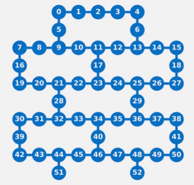
Direct multi-parameters function encoding
Continuous function programming

(+quantum speedup of runtime – see later)

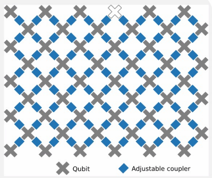
Why now?



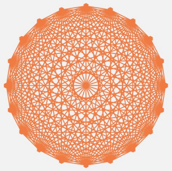




IBM
Rochester 53 qubits, october 2019
65 qubits, october 2020

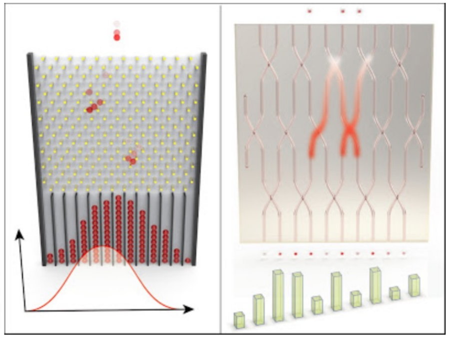


Google
Sycamore 53 qubits, october 2019

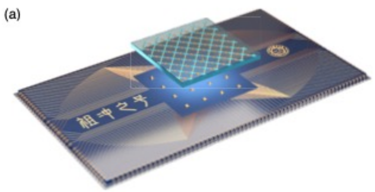


IonQ
11 qubits, 2018
32 qubits, 2020

Quantum computational advantage using photons, Science 370 (2020)

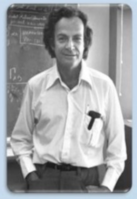


Zuchongzhi (66 qubits)
(PRL October 2021)

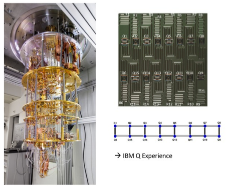


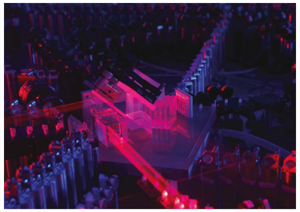
Simulating physics with computers-1982
Richard P. Feynman (Nobel Prize in Physics 1965)

> "Nature isn't classical, dammit, and if you want to make a simulation of nature, you'd better make it quantum mechanical, and by golly it's a wonderful problem, because it doesn't look so easy."

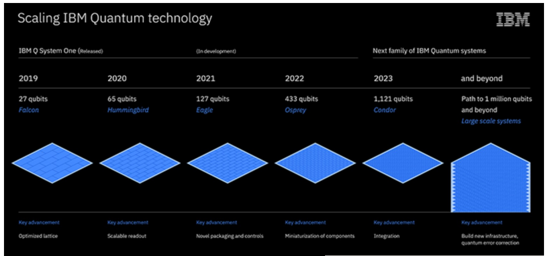


IBM QX5 (16 qubits)





This photonic computer did a task that would take a classical computer 2.5 billion years.



Scaling IBM Quantum technology

IBM Q System One (Released)		(In development)		Next family of IBM Quantum systems	
2019	2020	2021	2022	2023	and beyond
27 qubits *Falcon*	65 qubits *Hummingbird*	127 qubits *Eagle*	433 qubits *Osprey*	1,121 qubits *Condor*	Path to 1 million qubits and beyond *Large scale systems*
Key advancement Optimized lattice	Key advancement Scalable readout	Key advancement Novel packaging and controls	Key advancement Miniaturization of components	Key advancement Integration	Key advancement Build new infrastructure, quantum error correction

Quantum Theory	Quantum Computer	7 qubits Los Alamos	12 qubits MIT	128 qubits DWave	512 qubits DWave	1152 qubits DWave	17 qubits IBM	50 qubits IBM	2048 qubits DWave	72 qubits Google	128 qubits Rigetti
1927	1982	2000	2006	2011	2013	2015	2017	2017	2017	2018	2018

55 YEARS

18 YEARS

6 YEARS

1 YEAR

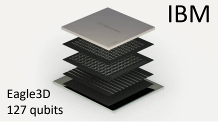
IBM Cloud

(2020)

(2021)

IonQ Gemini desk computer

?



IBM

Eagle3D
127 qubits

Quantum supremacy using a programmable superconducting processor

Nature | Vol 574 | 24 OCTOBER 2019 | **505**

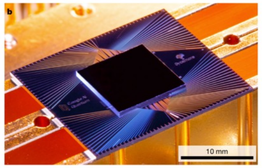
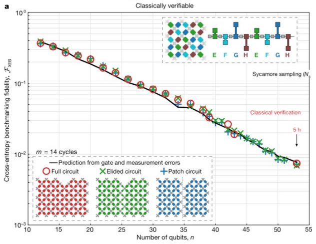


Fig. 1 | The Sycamore processor. a, Layout of processor, showing a rectangular array of 54 qubits (grey), each connected to its four nearest neighbours with couplers (blue). The inoperable qubit is outlined. **b**, Photograph of the Sycamore chip.

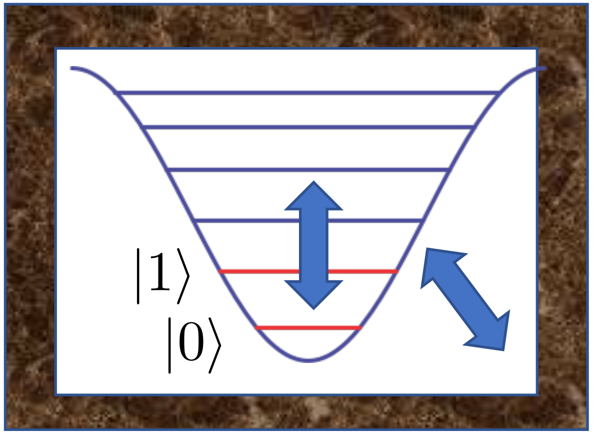


More on quantum computers and quantum hardwares

Quantum technologies – where do we stand ?

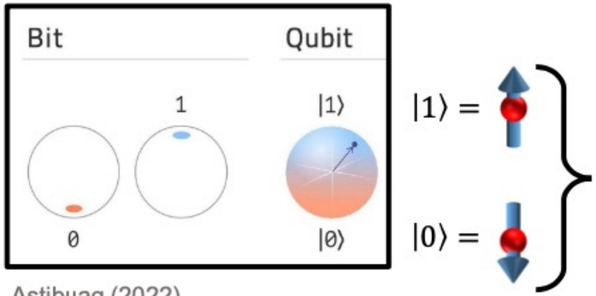
Quantum computing today is firstly a technological challenge

Simplified view of a general quantum system



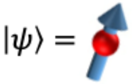
Challenges

- Be able to isolate and manipulate accurately a qubit



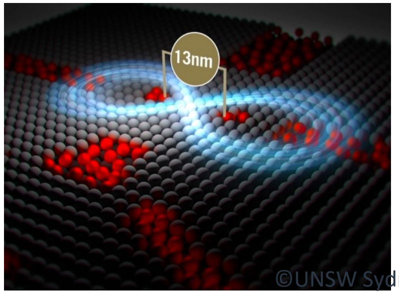
Astibuag (2022)

- Be able to manipulate accurately a qubit (fidelity)



- Be able to increase the number of “connected” qubits (scalability, connectivity)

Many technologies are explored



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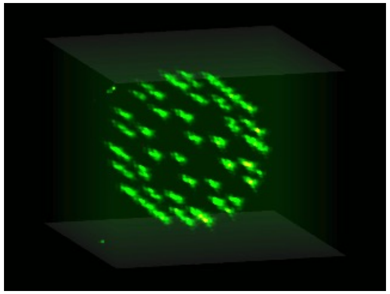
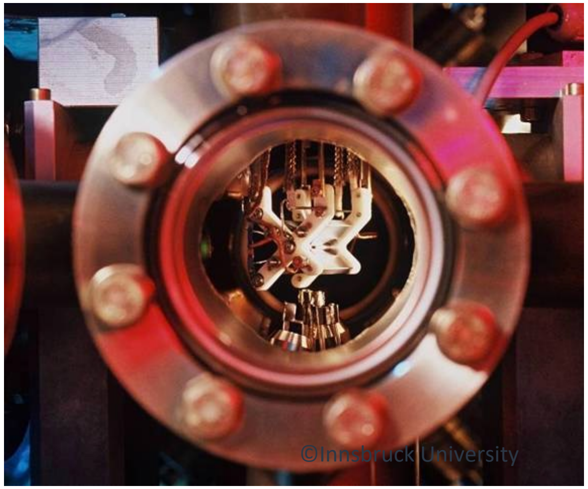
Silicon qubits



Neutral atoms



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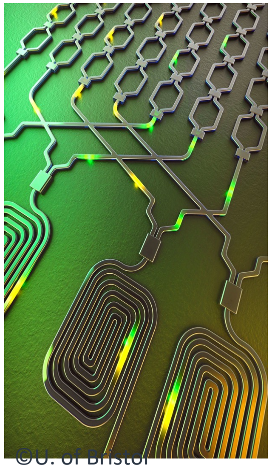
Trapped ions



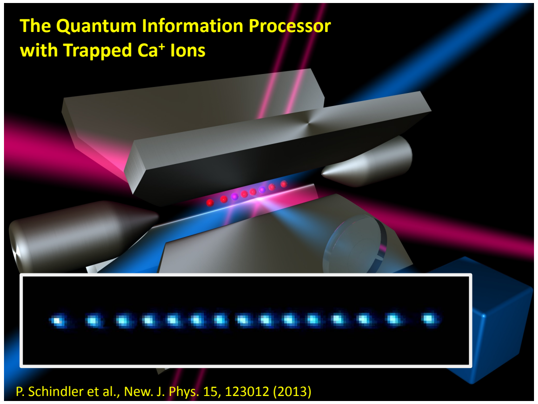
©U. of Bristol

Photons

©Google

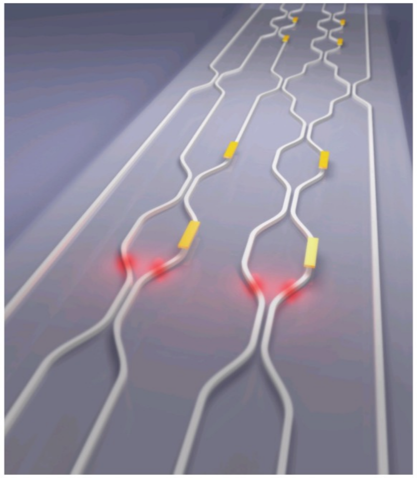
Superconducting qubits

NMR



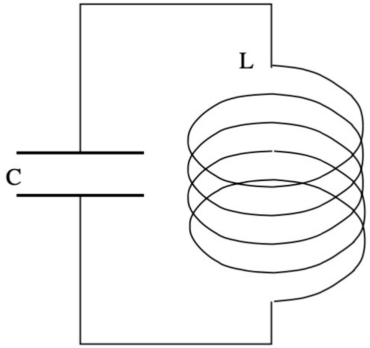
The Quantum Information Processor with Trapped Ca⁺ Ions

P. Schindler et al., New. J. Phys. 15, 123012 (2013)



Example 1: Superconducting qubits

Simple oscillator by LC circuit



$$H = \frac{1}{2L}\phi^2 + \frac{1}{2C}n^2$$

ϕ: flux in the inductor

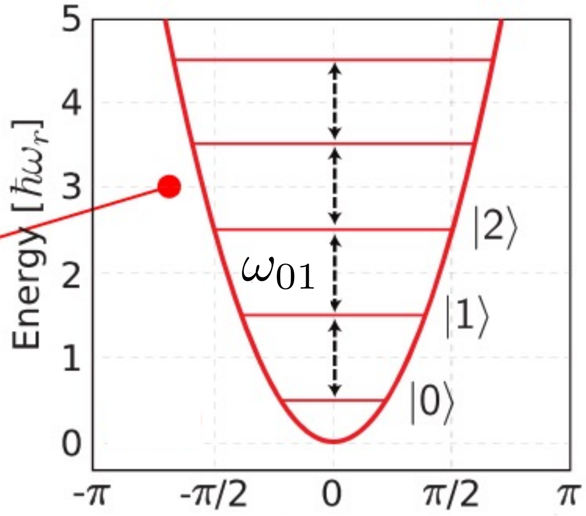
n : charge in the capacitor

Methodology

- We consider the limit where quantum effects are important

$$k_B T \ll \hbar\omega_{01}$$

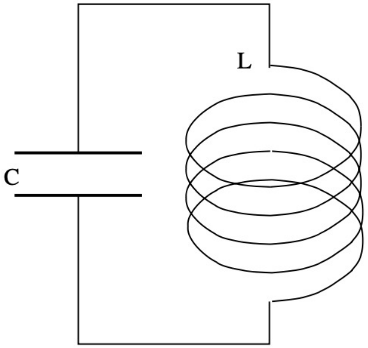
- n, ϕ becomes equivalent to canonical conjugated variables like *(x,p)* for the harmonic oscillator (HO)



Quantum
HO

Example : Superconducting qubits

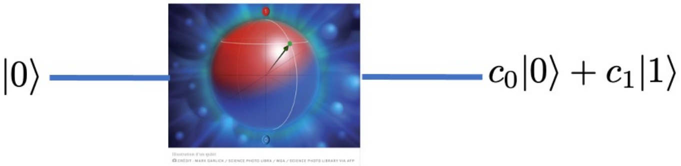
Simple oscillator by LC circuit



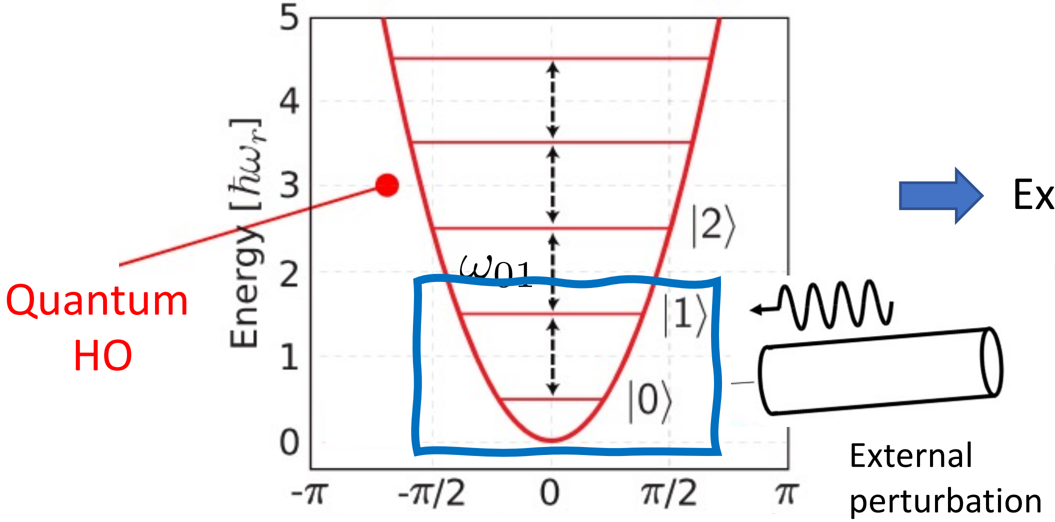
$$H = \frac{1}{2L}\phi^2 + \frac{1}{2C}n^2$$

ϕ: flux in the inductor
n : charge in the capacitor

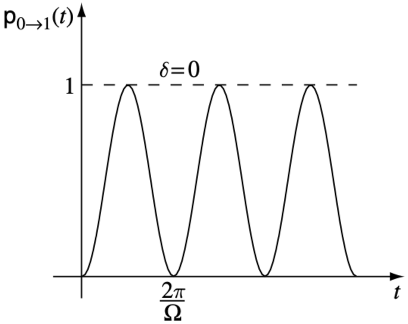
- We are interested in isolating $|0\rangle$ and $|1\rangle$
- We should prepare the system in any combination of them



Qubit identification and manipulation



- Example: induce Rabi-like oscillation



Two difficulties

Approximate operations

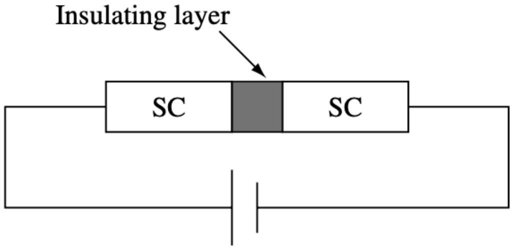
$$|0\rangle \longrightarrow |1\rangle$$

Unwanted transitions

$$|1\rangle \longrightarrow |2\rangle$$

Example: Superconducting qubits

The Josephson junction technology



ϕ: relative phase of the two superconductors

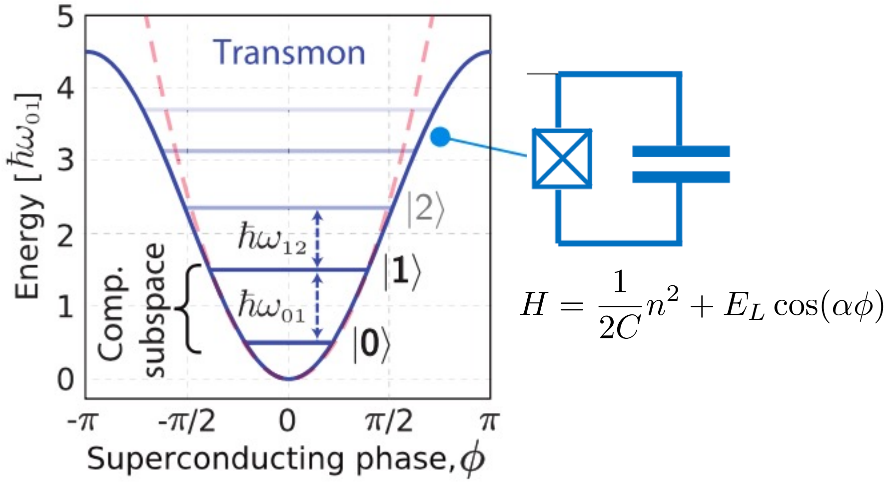
n : number of cooper pairs

$$H = \frac{1}{2C}n^2 + E_L\cos(\alpha\phi) + \frac{1}{2L}(\phi - \phi_{\text{ex}})^2 - I_{\text{ex}}\phi$$

Equivalent to LC but anharmonic

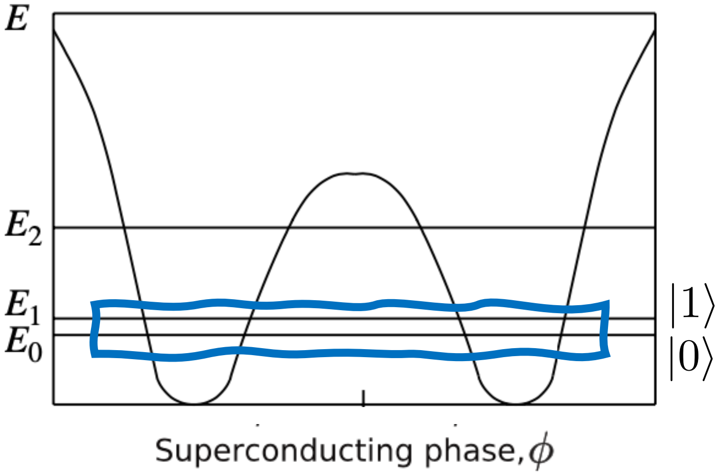
with external current

Transmon



$$H = \frac{1}{2C}n^2 + E_L\cos(\alpha\phi)$$

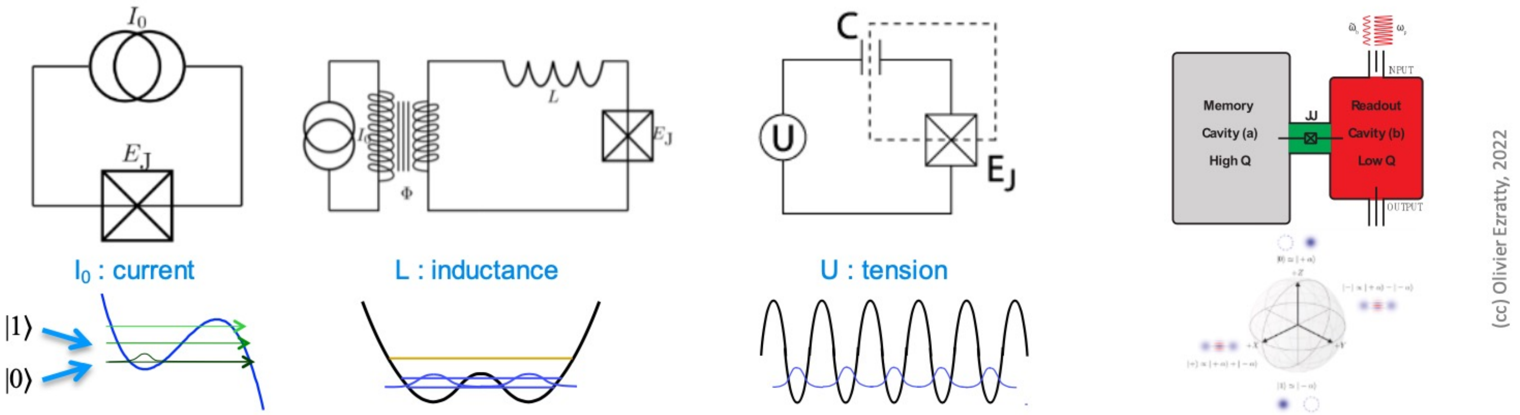
With an external current:



➡ Anharmonicity and gaps helps to reduce unwanted transitions

Example: Superconducting qubits

The Josephson junction technology



(cc) Olivier Ezratty, 2022

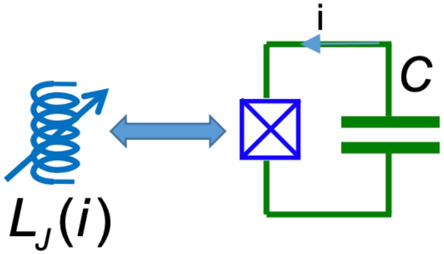
	phase qubit	flux qubit	charge qubit - transmon	cat-qubits
	Josephson junctions handle the qubit degree of liberty			Josephson junctions prepare, couple and correct the cat-qubits
|0⟩ and |1⟩ qubits	two energy levels in a potential well	two superconducting current directions	two levels of charge of Cooper pairs	pairs of entangled microwave photons in a cavity
quantum gates	micro-waves	magnetic field	micro-waves	micro-waves
qubits readout	resonator and micro-waves	magnetometer (SQUID)	resonator and micro-waves	resonator and micro-waves
commercial vendors	**abandonned**	D-Wave The Quantum Computing Company™ rigetti QILIMANJARO Alibaba bleximo ATLANTIC	IBM Google IQM OQC	qci amazon Nord Quantique ALICE & BOB

O. Ezratty book, website, blog, “Understand quantum computing” (in French)

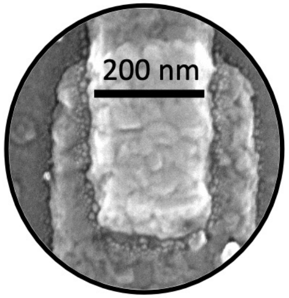
Example: Superconducting qubits

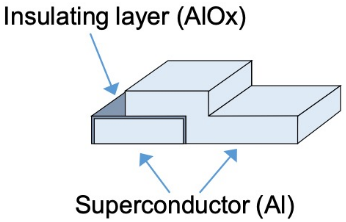
The Josephson junction technology

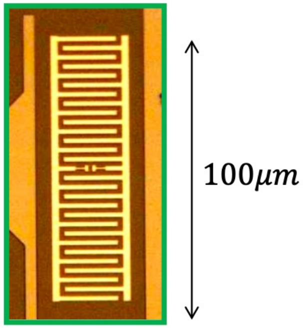
What it looks like



$$\nu_{01} \sim 5 - 10\text{GHz}$$
$$T = 10\text{mK}$$

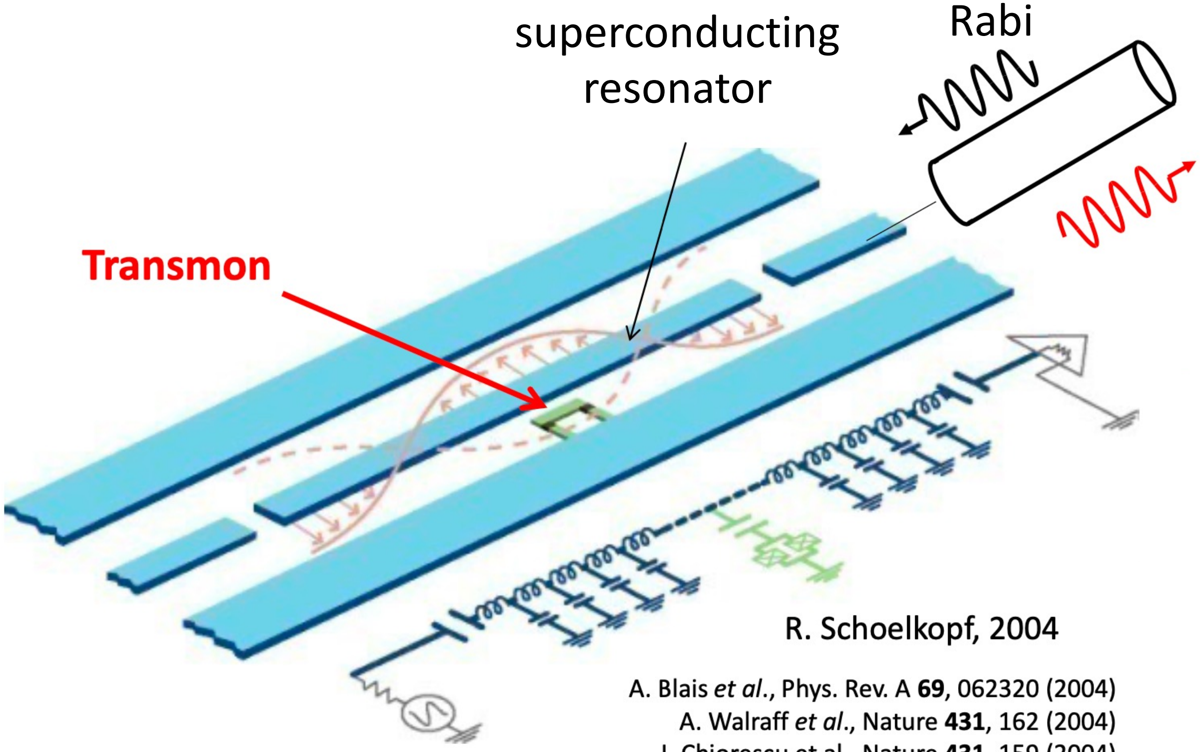






Example: Superconducting qubits

The Josephson junction technology



superconducting resonator

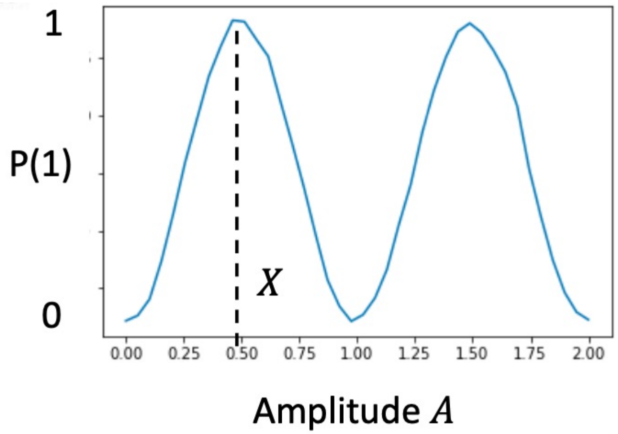
Rabi

Transmon

State read-out

R. Schoelkopf, 2004

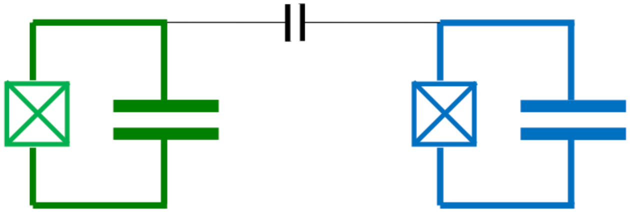
A. Blais *et al.*, Phys. Rev. A **69**, 062320 (2004)
A. Walraff *et al.*, Nature **431**, 162 (2004)
I. Chiorescu et al., Nature **431**, 159 (2004)



Example: Superconducting qubits

Next step: putting several qubits together

2 qubits can be coupled through electrostatic interactions



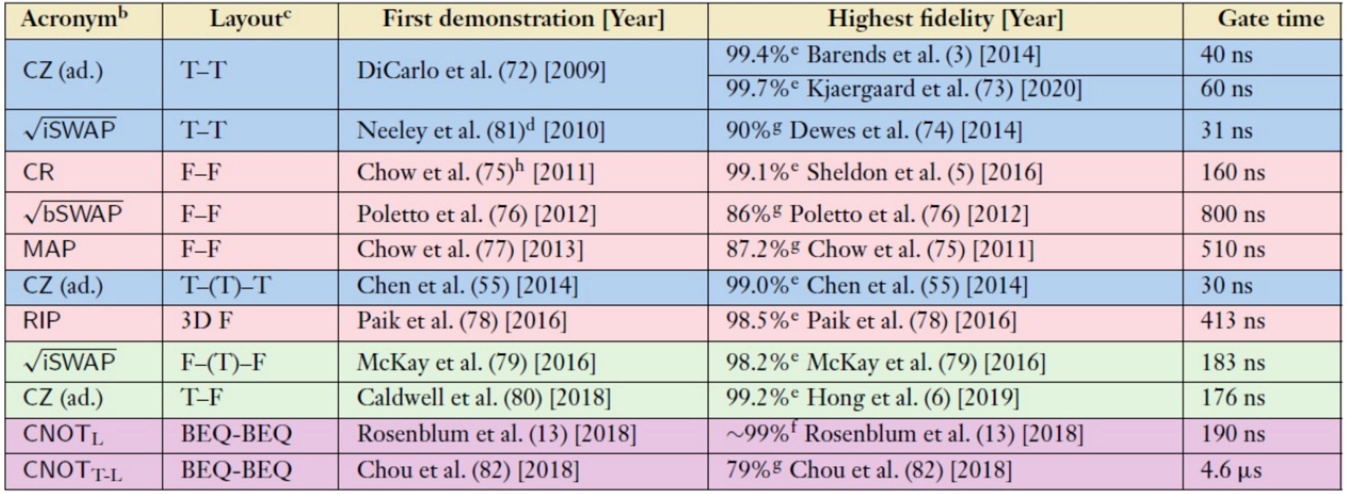
With this one can manipulate/entangle qubits

IBM Google rigetti IQM

Some specific Operations (see next lecture)

When

Fidelity



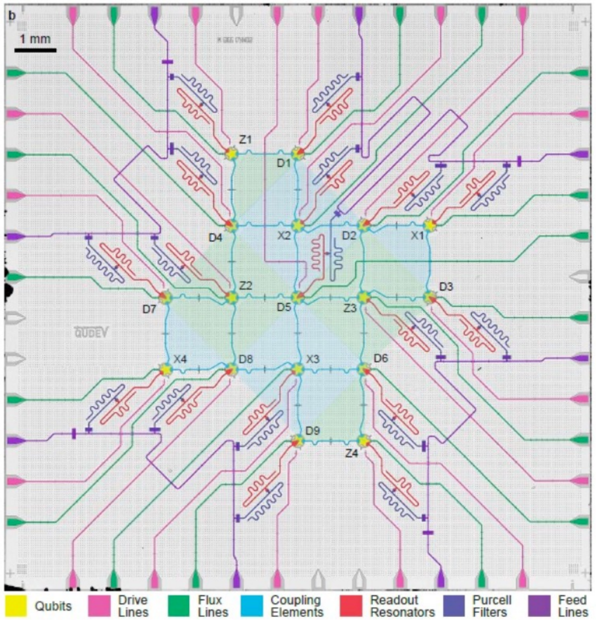
Acronym^{b}	Layout^{c}	First demonstration [Year]	Highest fidelity [Year]	Gate time
CZ (ad.)	T–T	DiCarlo et al. (72) [2009]	99.4%^{e} Barends et al. (3) [2014]	40 ns
			99.7%^{e} Kjaergaard et al. (73) [2020]	60 ns
$\sqrt{\text{iSWAP}}$	T–T	Neeley et al. (81)^{d} [2010]	90%^{g} Dewes et al. (74) [2014]	31 ns
CR	F–F	Chow et al. (75)^{h} [2011]	99.1%^{e} Sheldon et al. (5) [2016]	160 ns
$\sqrt{\text{bSWAP}}$	F–F	Poletto et al. (76) [2012]	86%^{g} Poletto et al. (76) [2012]	800 ns
MAP	F–F	Chow et al. (77) [2013]	87.2%^{g} Chow et al. (75) [2011]	510 ns
CZ (ad.)	T–(T)–T	Chen et al. (55) [2014]	99.0%^{e} Chen et al. (55) [2014]	30 ns
RIP	3D F	Paik et al. (78) [2016]	98.5%^{e} Paik et al. (78) [2016]	413 ns
$\sqrt{\text{iSWAP}}$	F–(T)–F	McKay et al. (79) [2016]	98.2%^{e} McKay et al. (79) [2016]	183 ns
CZ (ad.)	T–F	Caldwell et al. (80) [2018]	99.2%^{e} Hong et al. (6) [2019]	176 ns
CNOT_{L}	BEQ-BEQ	Rosenblum et al. (13) [2018]	~99%^{f} Rosenblum et al. (13) [2018]	190 ns
$\text{CNOT}_{\text{T-L}}$	BEQ-BEQ	Chou et al. (82) [2018]	79%^{g} Chou et al. (82) [2018]	4.6 µs

M. Kjaergaard, Annual Review of Cond. Matt (2020)

Example: Superconducting qubits

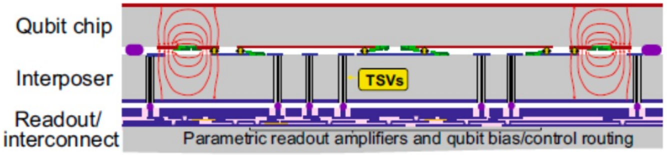
Where we are now ?

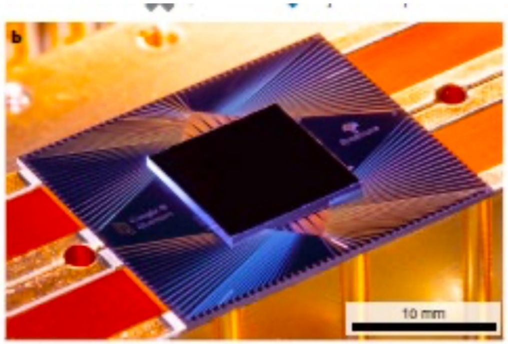
2D chip : 17 qubits



S. Krinner et al., arxiv (2021)
Wallraff group, ETHZ

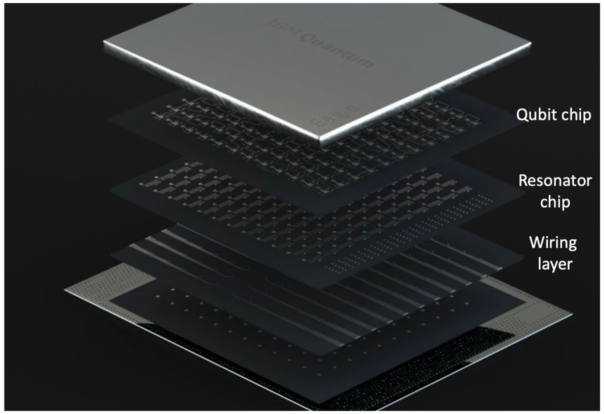
3D integration ~100 qubits demonstrated



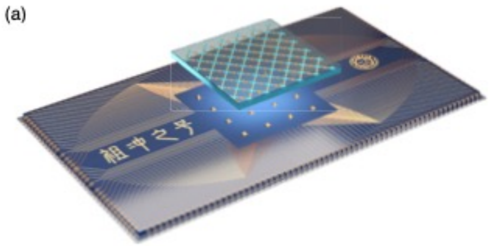


Rosenberg et al., NPJ (2017)
Arute et al., Nature (2019)

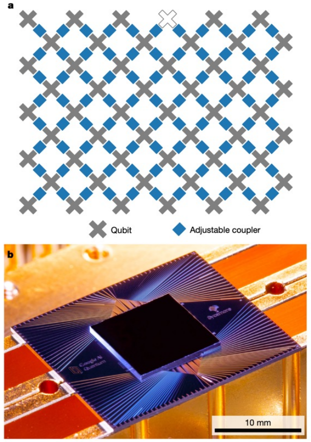
IBM Eagle
127 qubits



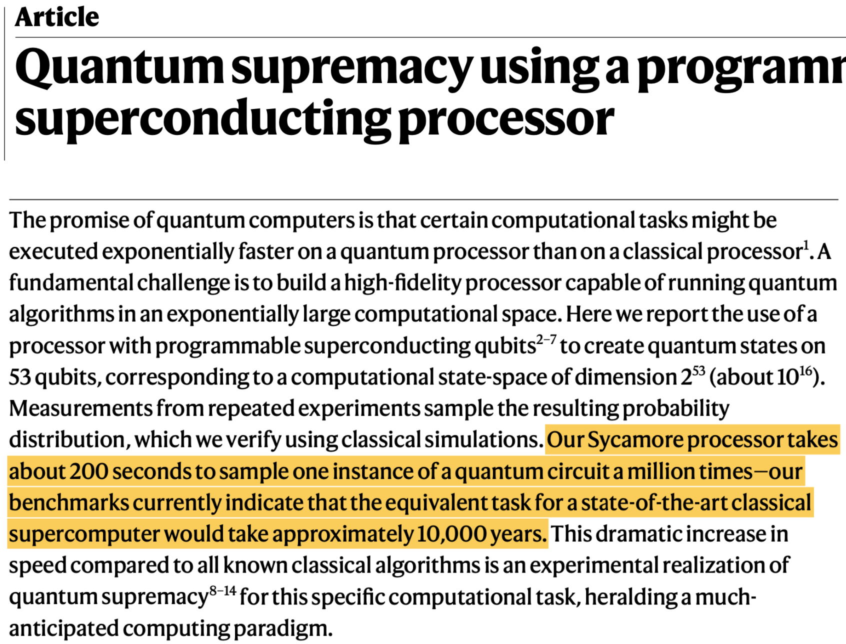
Zuchongzhi (66 qubits)
(PRL October 2021)



Example: Superconducting qubits



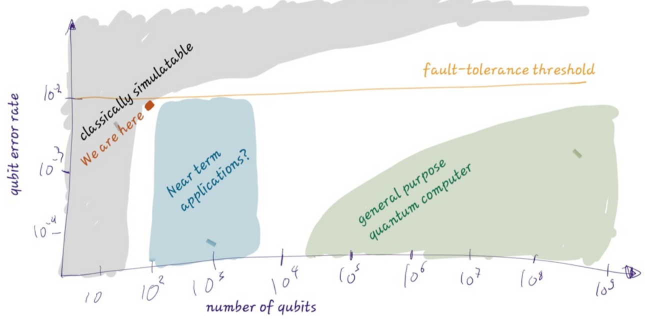
Quantum advantage



Article

Quantum supremacy using a programmable superconducting processor

The promise of quantum computers is that certain computational tasks might be executed exponentially faster on a quantum processor than on a classical processor[1]. A fundamental challenge is to build a high-fidelity processor capable of running quantum algorithms in an exponentially large computational space. Here we report the use of a processor with programmable superconducting qubits[2–7] to create quantum states on 53 qubits, corresponding to a computational state-space of dimension 2^{53} (about 10^{16}). Measurements from repeated experiments sample the resulting probability distribution, which we verify using classical simulations. Our Sycamore processor takes about 200 seconds to sample one instance of a quantum circuit a million times—our benchmarks currently indicate that the equivalent task for a state-of-the-art classical supercomputer would take approximately 10,000 years. This dramatic increase in speed compared to all known classical algorithms is an experimental realization of quantum supremacy[8–14] for this specific computational task, heralding a much-anticipated computing paradigm.

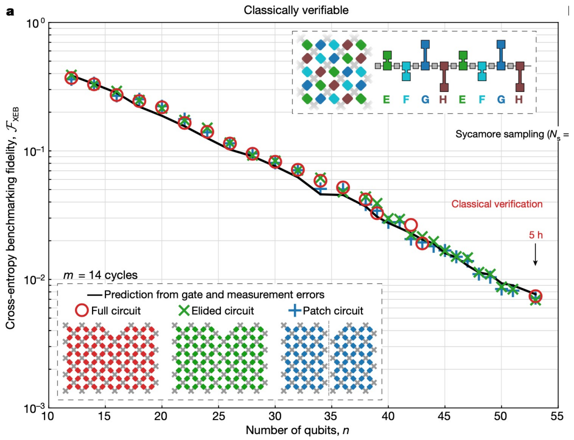


Task

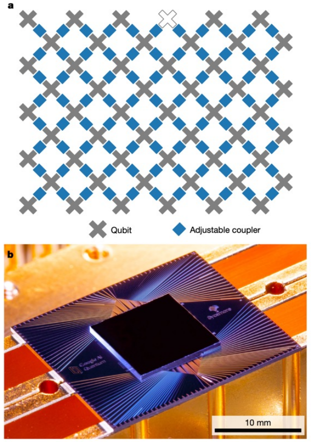
Compute output probabilities of a “random” circuit, i.e. a set of random operation.

⇨ Usefulness of the task ?

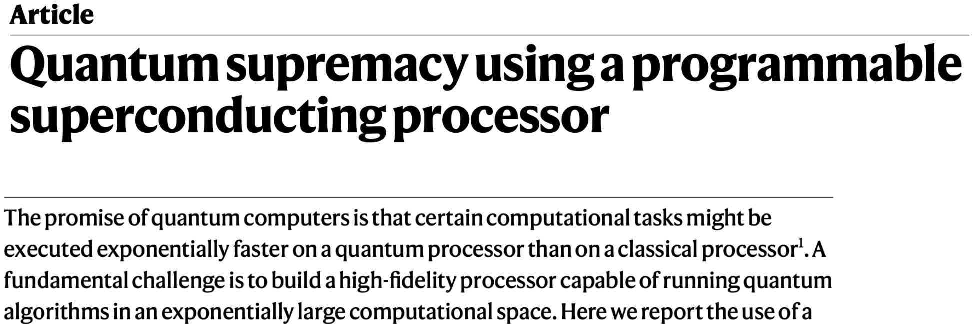
Proofs of fidelity



Example: Superconducting qubits



Quantum advantage



Article

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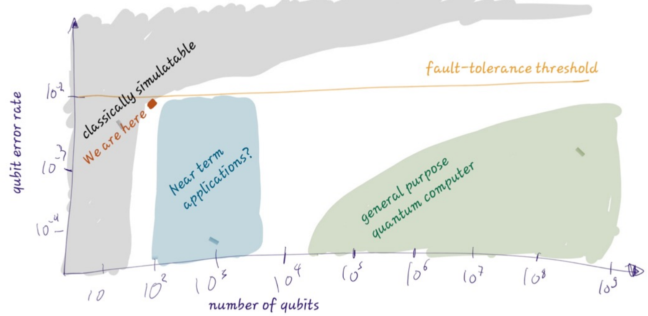

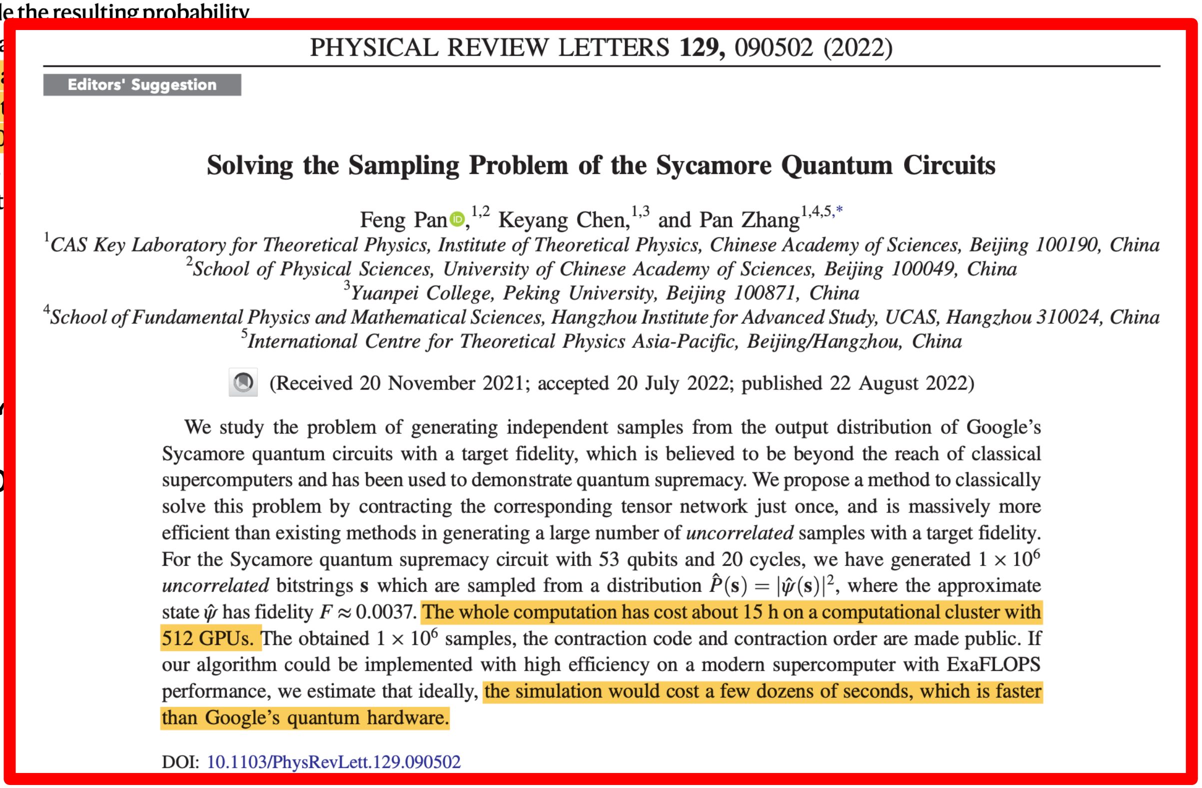

PHYSICAL REVIEW LETTERS **129,** 090502 (2022)

Editors' Suggestion

Solving the Sampling Problem of the Sycamore Quantum Circuits

Feng Pan,[1,2] Keyang Chen,[1,3] and Pan Zhang[1,4,5,*]
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[2] *School of Physical Sciences, University of Chinese Academy of Sciences, Beijing 100049, China*
[3] *Yuanpei College, Peking University, Beijing 100871, China*
[4] *School of Fundamental Physics and Mathematical Sciences, Hangzhou Institute for Advanced Study, UCAS, Hangzhou 310024, China*
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(Received 20 November 2021; accepted 20 July 2022; published 22 August 2022)

We study the problem of generating independent samples from the output distribution of Google's Sycamore quantum circuits with a target fidelity, which is believed to be beyond the reach of classical supercomputers and has been used to demonstrate quantum supremacy. We propose a method to classically solve this problem by contracting the corresponding tensor network just once, and is massively more efficient than existing methods in generating a large number of *uncorrelated* samples with a target fidelity. For the Sycamore quantum supremacy circuit with 53 qubits and 20 cycles, we have generated 1×10^6 *uncorrelated* bitstrings $\mathbf{s}$ which are sampled from a distribution $\hat{P}(\mathbf{s}) = |\hat{\psi}(\mathbf{s})|^2$, where the approximate state $\hat{\psi}$ has fidelity $F \approx 0.0037$. The whole computation has cost about 15 h on a computational cluster with 512 GPUs. The obtained 1×10^6 samples, the contraction code and contraction order are made public. If our algorithm could be implemented with high efficiency on a modern supercomputer with ExaFLOPS performance, we estimate that ideally, the simulation would cost a few dozens of seconds, which is faster than Google's quantum hardware.

DOI: 10.1103/PhysRevLett.129.090502

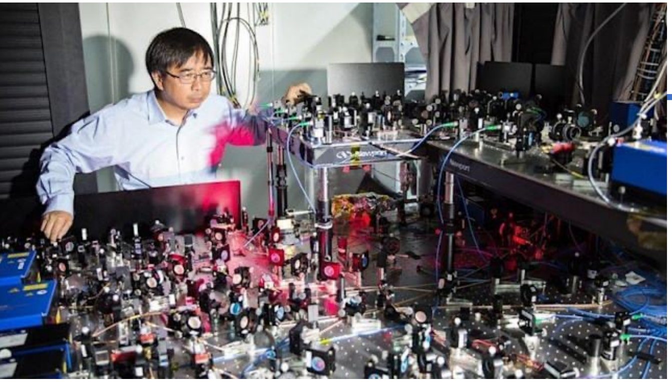
Task

Compute output probabilities of a “
circuit, i.e. a set of random operatio

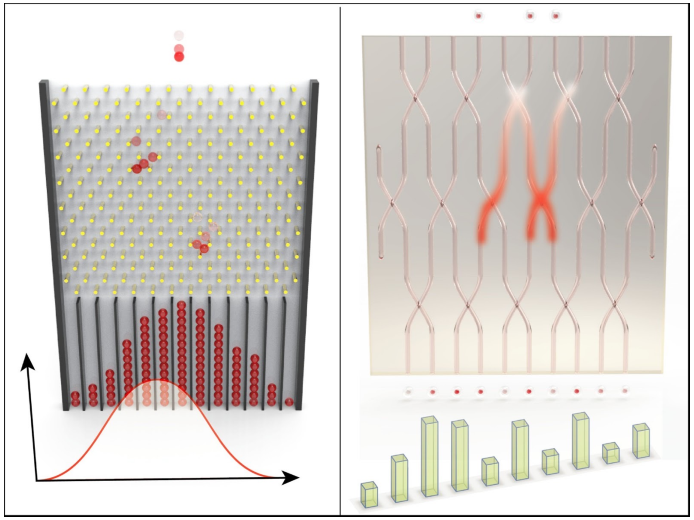
- Usefulness of the task ?
- Some controversial aspects

Example: photonic systems

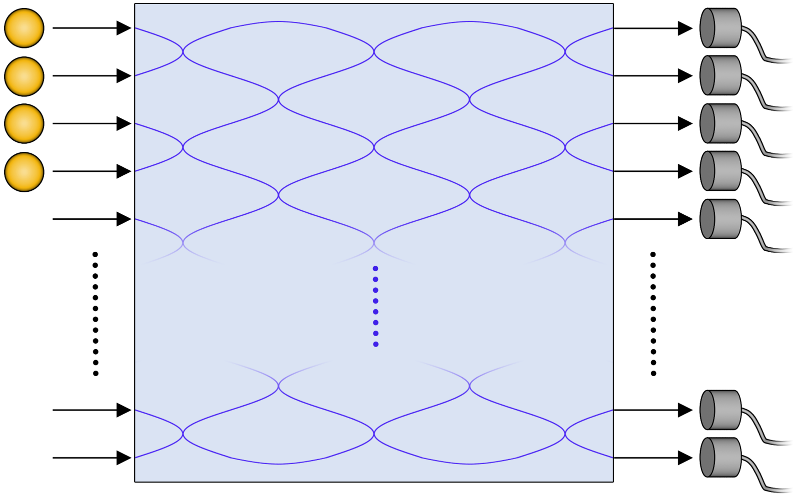
Quantum advantage



Boson sampling problem



Goal : predict the final distribution

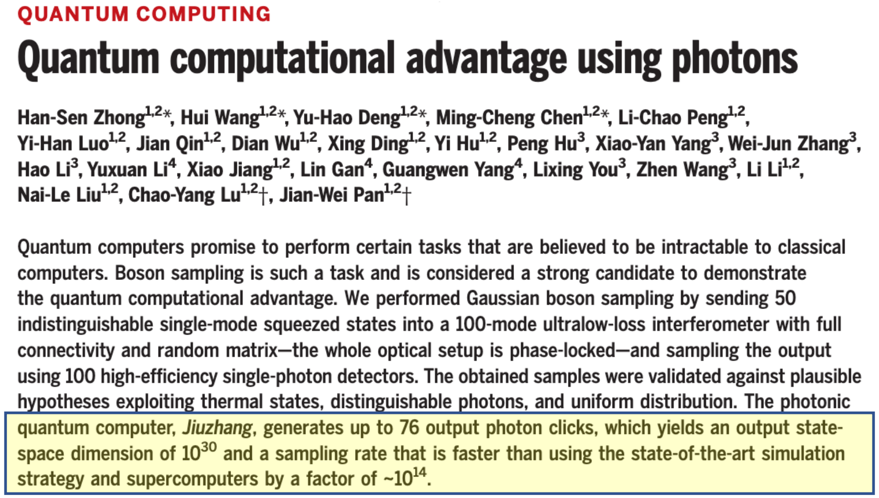


N single photons

M detectors

M layers of coupling gates

Example of operations: beam splitter, beam shifter



QUANTUM COMPUTING

Quantum computational advantage using photons

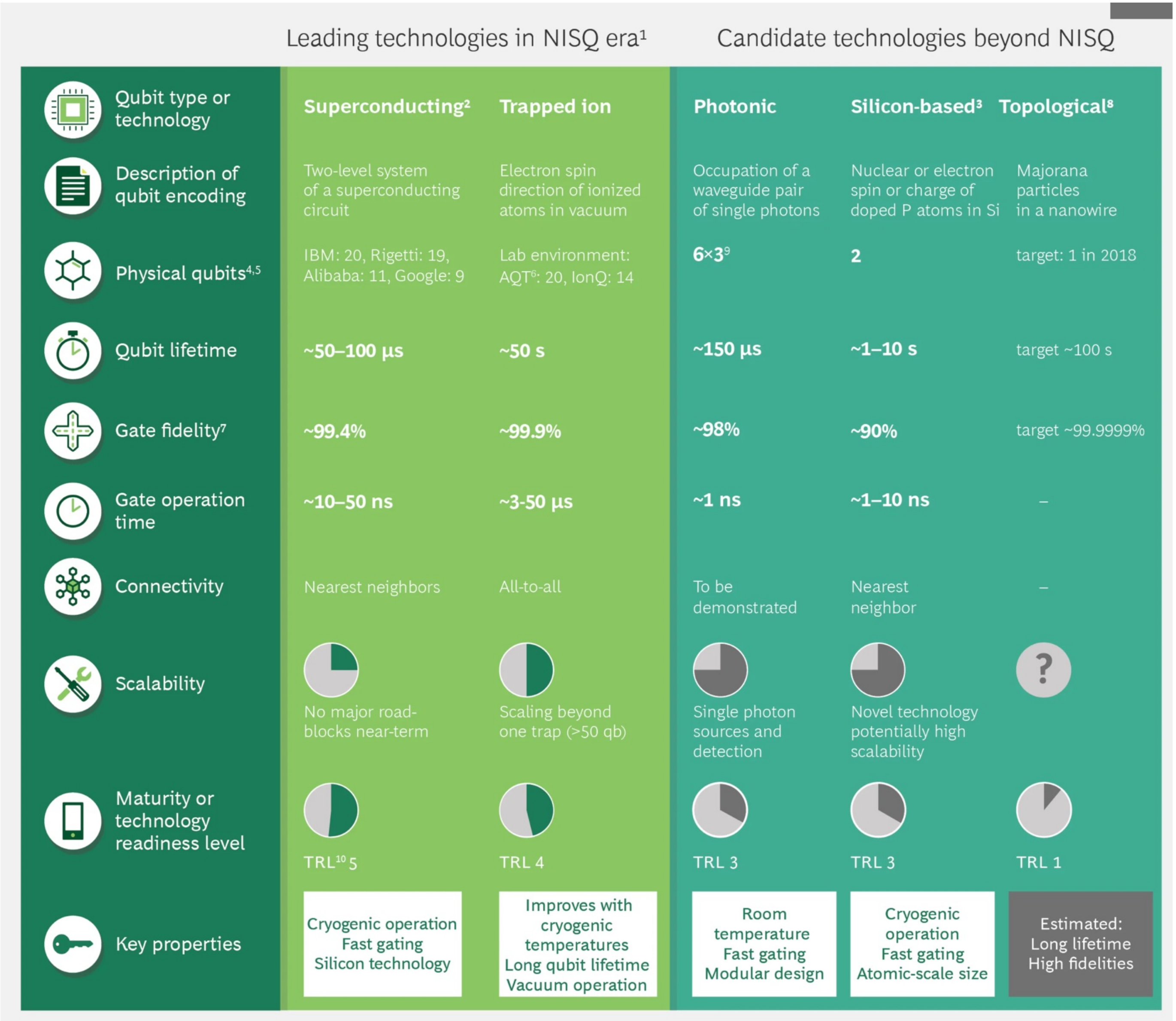
Han-Sen Zhong[1,2]*, Hui Wang[1,2]*, Yu-Hao Deng[1,2]*, Ming-Cheng Chen[1,2]*, Li-Chao Peng[1,2], Yi-Han Luo[1,2], Jian Qin[1,2], Dian Wu[1,2], Xing Ding[1,2], Yi Hu[1,2], Peng Hu[3], Xiao-Yan Yang[3], Wei-Jun Zhang[3], Hao Li[3], Yuxuan Li[4], Xiao Jiang[1,2], Lin Gan[4], Guangwen Yang[4], Lixing You[3], Zhen Wang[3], Li Li[1,2], Nai-Le Liu[1,2], Chao-Yang Lu[1,2]†, Jian-Wei Pan[1,2]†

Quantum computers promise to perform certain tasks that are believed to be intractable to classical computers. Boson sampling is such a task and is considered a strong candidate to demonstrate the quantum computational advantage. We performed Gaussian boson sampling by sending 50 indistinguishable single-mode squeezed states into a 100-mode ultralow-loss interferometer with full connectivity and random matrix—the whole optical setup is phase-locked—and sampling the output using 100 high-efficiency single-photon detectors. The obtained samples were validated against plausible hypotheses exploiting thermal states, distinguishable photons, and uniform distribution. The photonic quantum computer, *Jiuzhang*, generates up to 76 output photon clicks, which yields an output state-space dimension of 10^{30} and a sampling rate that is faster than using the state-of-the-art simulation strategy and supercomputers by a factor of ~10^{14}.

Science, 2020

(I) Quantum Hardware and the NISQ (Noisy Intermediate Scale quantum) period

Platforms comparison



	Leading technologies in NISQ era[1]		Candidate technologies beyond NISQ		
Qubit type or technology	**Superconducting[2]**	**Trapped ion**	**Photonic**	**Silicon-based[3]**	**Topological[8]**
Description of qubit encoding	Two-level system of a superconducting circuit	Electron spin direction of ionized atoms in vacuum	Occupation of a waveguide pair of single photons	Nuclear or electron spin or charge of doped P atoms in Si	Majorana particles in a nanowire
Physical qubits[4,5]	IBM: 20, Rigetti: 19, Alibaba: 11, Google: 9	Lab environment: AQT[6]: 20, IonQ: 14	**6×3[9]**	**2**	target: 1 in 2018
Qubit lifetime	**~50–100 µs**	**~50 s**	**~150 µs**	**~1–10 s**	target ~100 s
Gate fidelity[7]	**~99.4%**	**~99.9%**	**~98%**	**~90%**	target ~99.9999%
Gate operation time	**~10–50 ns**	**~3-50 µs**	**~1 ns**	**~1–10 ns**	–
Connectivity	Nearest neighbors	All-to-all	To be demonstrated	Nearest neighbor	–
Scalability	No major road-blocks near-term	Scaling beyond one trap (>50 qb)	Single photon sources and detection	Novel technology potentially high scalability	?
Maturity or technology readiness level	TRL[10] 5	TRL 4	TRL 3	TRL 3	TRL 1
Key properties	Cryogenic operation Fast gating Silicon technology	Improves with cryogenic temperatures Long qubit lifetime Vacuum operation	Room temperature Fast gating Modular design	Cryogenic operation Fast gating Atomic-scale size	Estimated: Long lifetime High fidelities

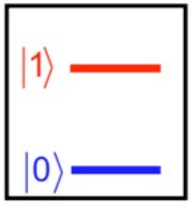
Boston Consulting Group – Nov 2018

NISQ (Noisy intermediate Scale Quantum) period

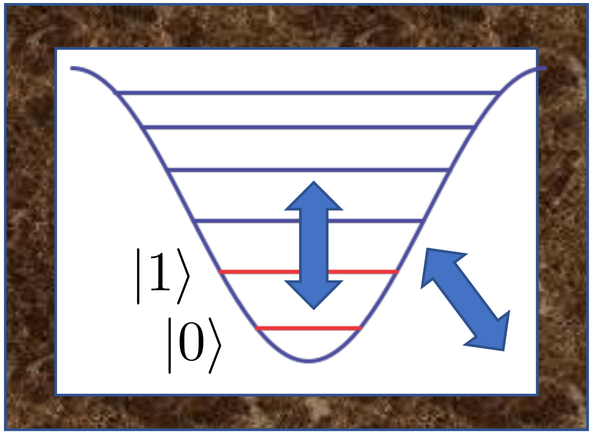
Many things tends to destroy the ideal qubits picture and the quantum coherence.

Ideal Qubits

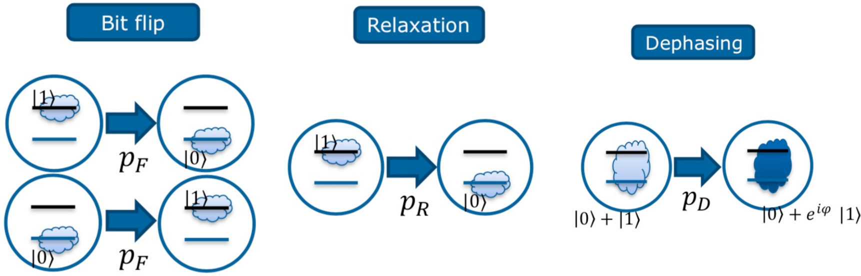
2 level system



External Exp. Setup



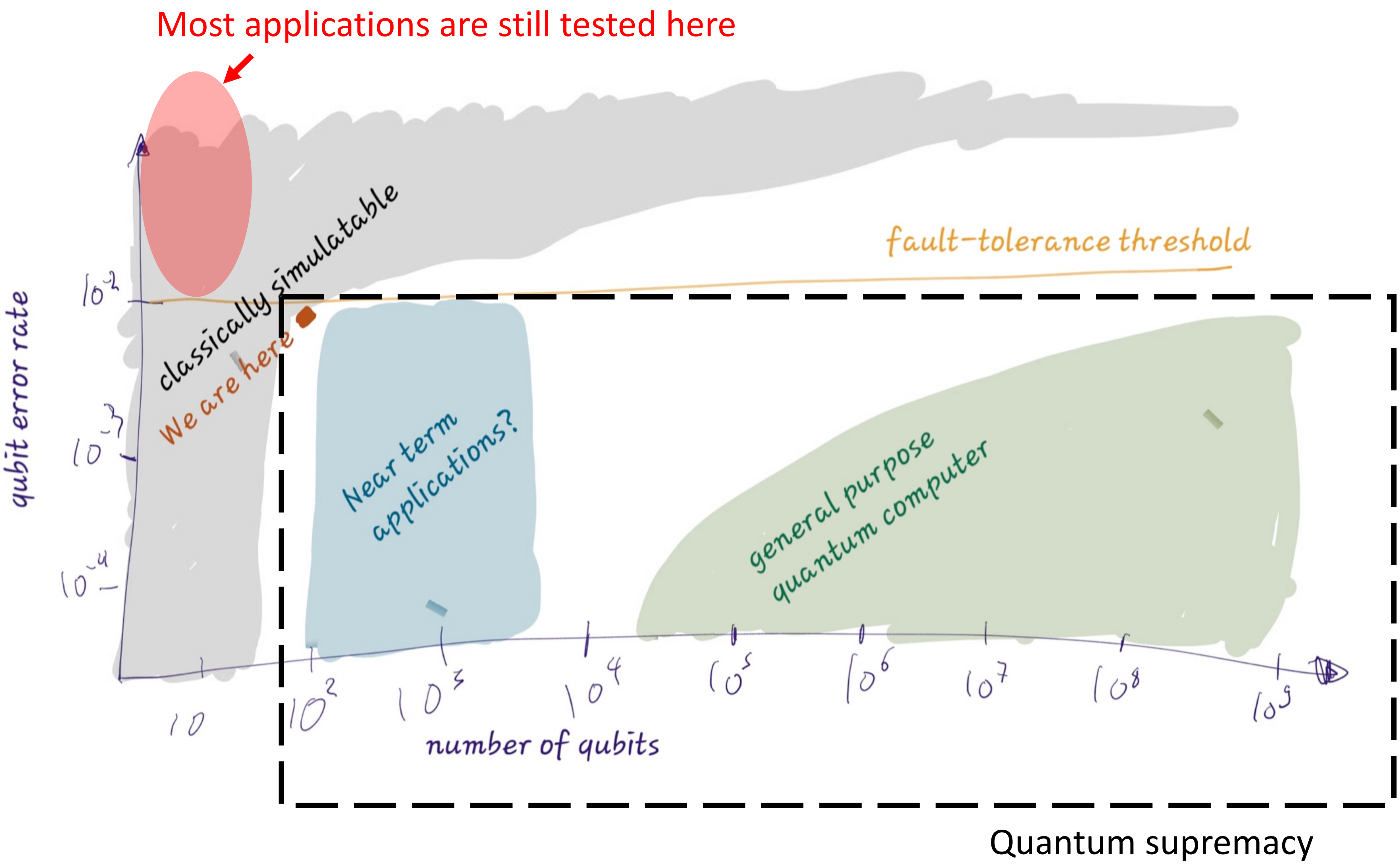
Leads to loss of information
And decoherence



For multi-qubits, also cross-talk (making operations on one qubit Impacts other qubits)

NISQ period: Working with quantum computers now means working in a noisy environment and "NISQ friendly" programs, i.e. only selected algorithm can be applied and error corrections should be made to get reasonable results.

The Google illustration



Most applications are still tested here

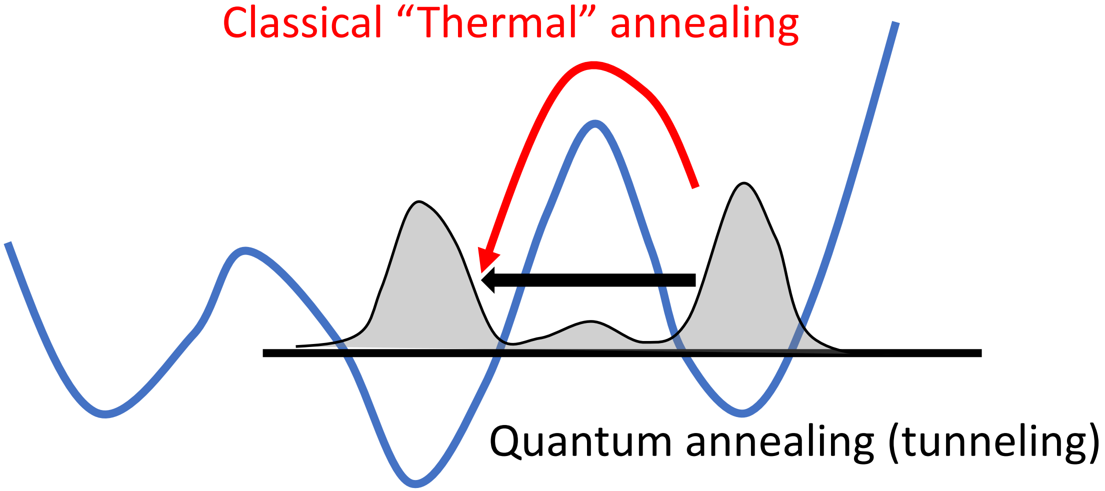
Quantum supremacy

Where ”quantum computing” is expected to be disruptive?

Short introduction to bit versus Qubits

Illustration of quantum advantages

Quantum Tunneling and quantum annealing



Classical “Thermal” annealing

Quantum annealing (tunneling)

Quantum entanglement

Assume two persons (Alice and Bob)  

The humor of A&B are encoded in the wave-function

$$|\Phi\rangle = \alpha|\overset{\overset{A}{\Downarrow}}{😀}\overset{\overset{B}{\Downarrow}}{😀}\rangle + \beta|\overset{\overset{A}{\Downarrow}}{😫}\overset{\overset{B}{\Downarrow}}{😫}\rangle$$

Suppose I measure Bob 😀 ➡ 😀

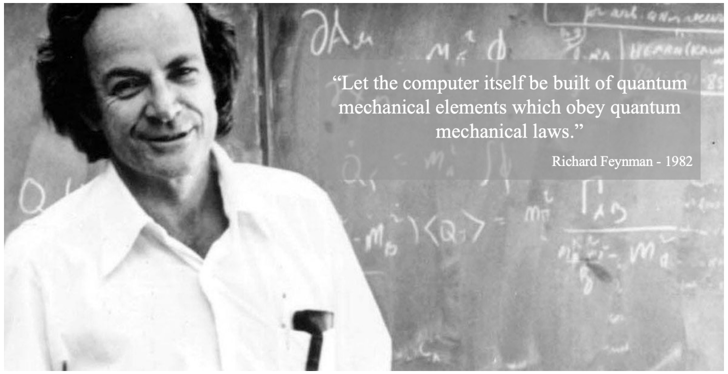
😫 ➡ 😫

I can measure partial info and get the full info

The info is destroyed after measurement

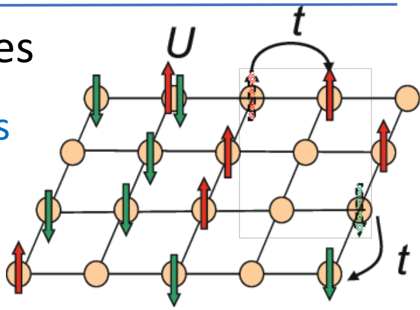
What are the anticipated applications ?

Simulation of Quantum complex systems



> "Let the computer itself be built of quantum mechanical elements which obey quantum mechanical laws."
>
> Richard Feynman - 1982

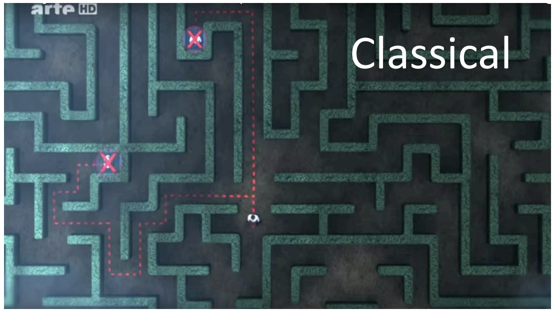
Ex: systems on lattices



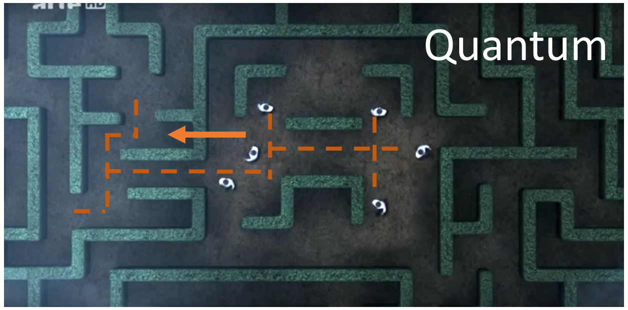
On classical computers
Can be solved exactly
For max 20 particles.

On quantum computers:
N sites means only N qubits

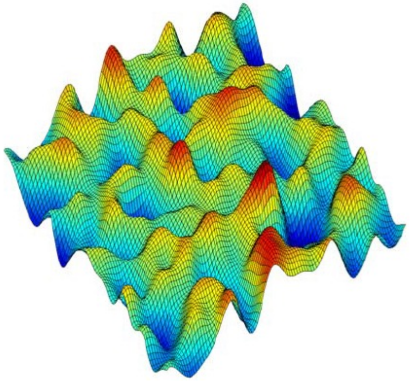
Quantum versus classical search



VS

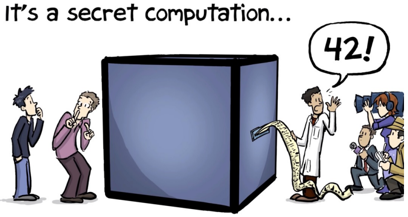


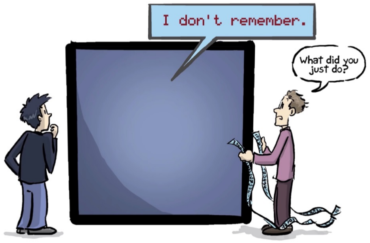
Credit: The Fabric of The Cosmos: Quantum Leap

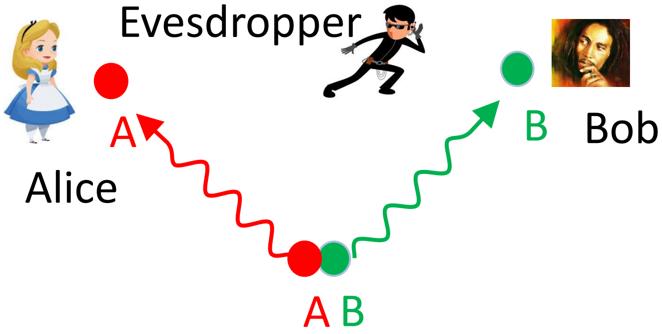


Exploring complex landscape:
molecules,
customers preferences (amazon), …

Quantum secrets (cryptography, quantum key, …)





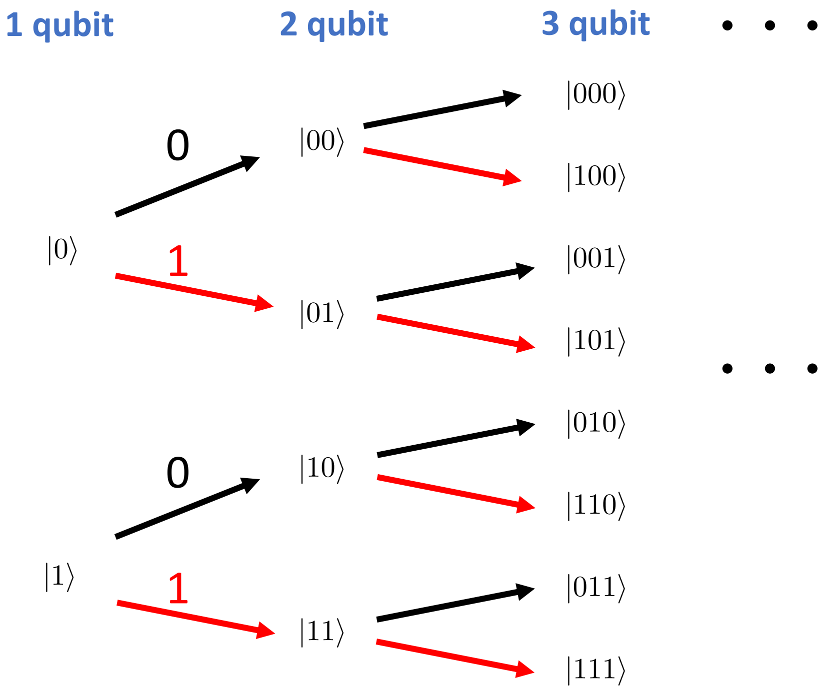


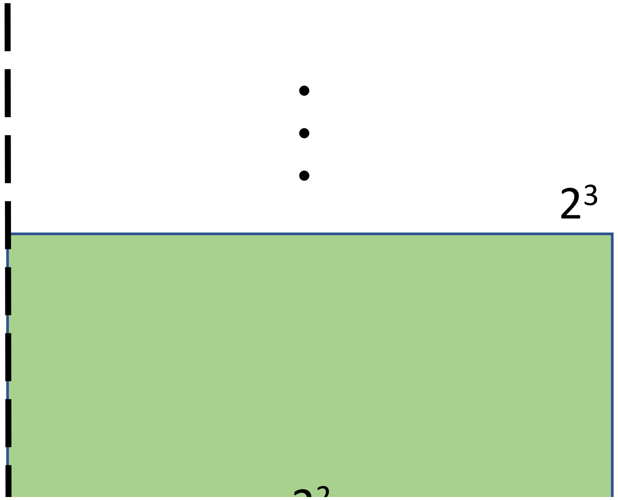
Hilbert Space dimention with qubits

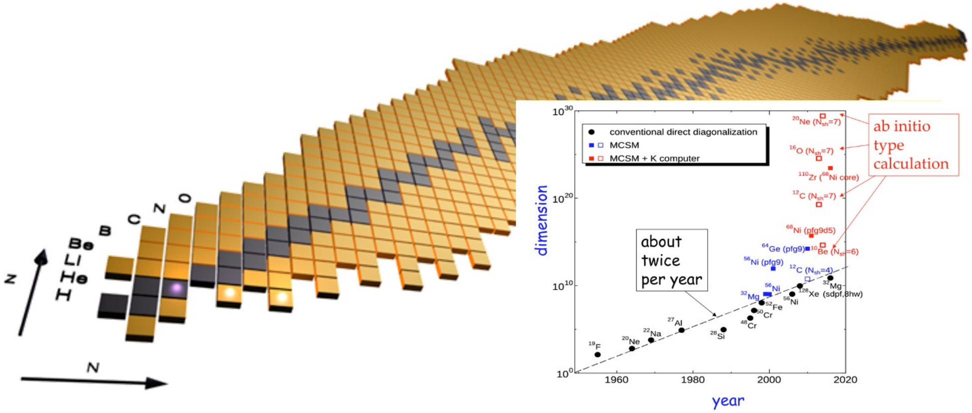
Illustration of quantum advantages

Systems described on qubits

$$|\Psi\rangle = \sum_{s_i=0,1} \Psi_{s_1,\cdots,s_N}|s_1,\cdots,s_n\rangle$$







Quantum supremacy

$\sim 2^{50}$

With 2^{300} (i.e. 300 qubits) the size is more than the number of particles In the universe. J. Preskill

A more precise view of problem complexity and quantum advantage

Complexity of a problem

A fundamental question is how much "computational time" it takes to solve problem – this is linked to the complexity of the problem (Church-Turing thesis, Problem complexity classification, …)

M.A. Nielsen and I. L. Chuang, "Quantum computation and quantum information", Cambridge university press.

➡ What could be computed is independent on computers – only speed will change with hardware, resources progresses

➡ Quantitatively, we are interested in the "speedup" to perform a calculation

Speedup can be:

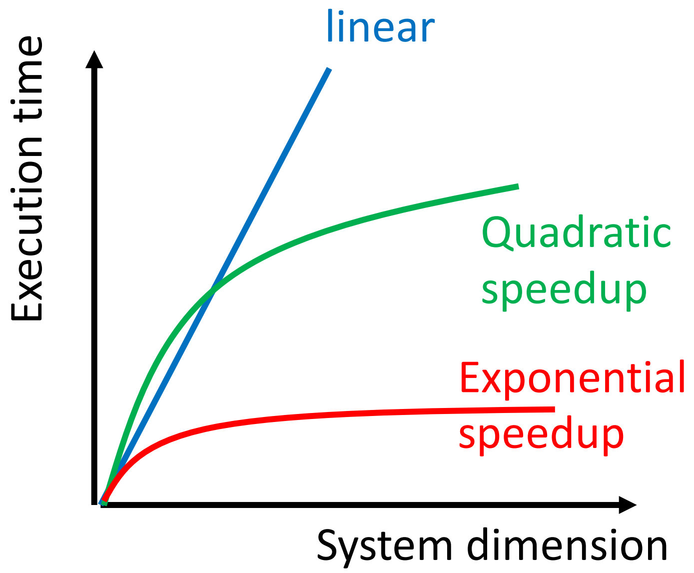
linear $t \rightarrow t/1000$

quadratic $t \rightarrow \sqrt{t}$ ➡ $t^2 \rightarrow t$

polynomial $t^x \rightarrow t$

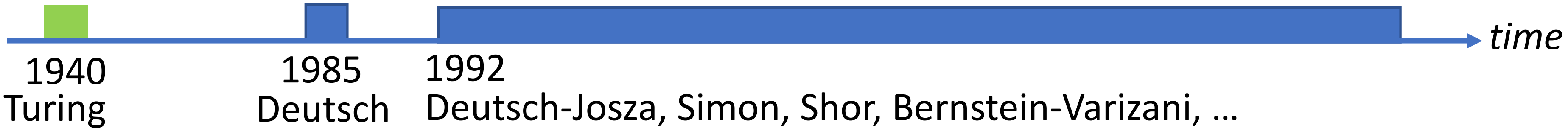
But the speed up could not be exponential ! $2^t \rightarrow t$

Quantum computers contradict this thesis- some algorithms Promises exponential speedup for specific calculations

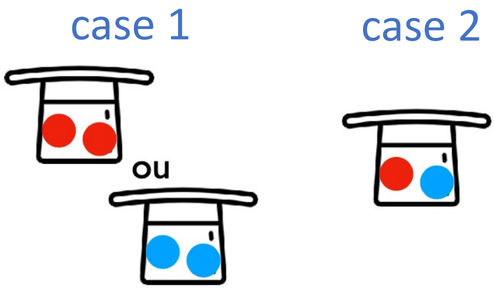


A calculation that's takes 1 year in "linear" case would takes 24 seconds with exponential speedups

Problem complexity and quantum advantage



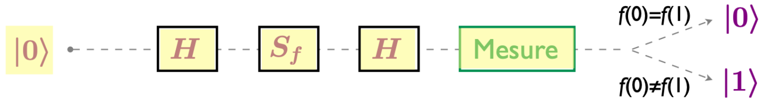
- Church Turing problem: analysis of a computational problem, complexity…



- The “simple” Deutsch problem: $f : \{0,1\} \rightarrow \{0,1\}$ (Oracle)

  Q: determine if $f(0)=f(1)$

  - Classically requires to have 2 answers $f(0)$? $f(1)$?
  - Quantum: one can directly ask $f(0)=f(1)$



- Deutsch-Josza problem (92)

  $f : \{0,1\}^n \rightarrow \{0,1\}$ Q: Is *f* constant or an equal mixing of *0* and *1* ?

  - Classically requires $1+2^{n-1}$ questions
  - Quantum: requires only *1* question but *n* qubits

Example *n=4*

$f(0000) \overset{?}{=} 0$
$f(0001) \overset{?}{=} 0$
$f(0010) \overset{?}{=} 0$
$f(0011) \overset{?}{=} 0$
$f(0100) \overset{?}{=} 0$
$f(0101) \overset{?}{=} 0$
$f(0110) \overset{?}{=} 0$
$f(0111) \overset{?}{=} 0$
$f(1000) \overset{?}{=} 0$
…

These problems are rather specific – usefulness?

Problem complexity and quantum advantage

Algorithmic – Some history

1940 Turing

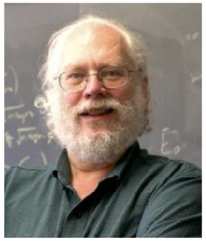
1985 Deutsch

1992 Deutsch-Josza, Simon, Shor, Bernstein-Varizani, Kitaev …

time

Tracking applications and progress

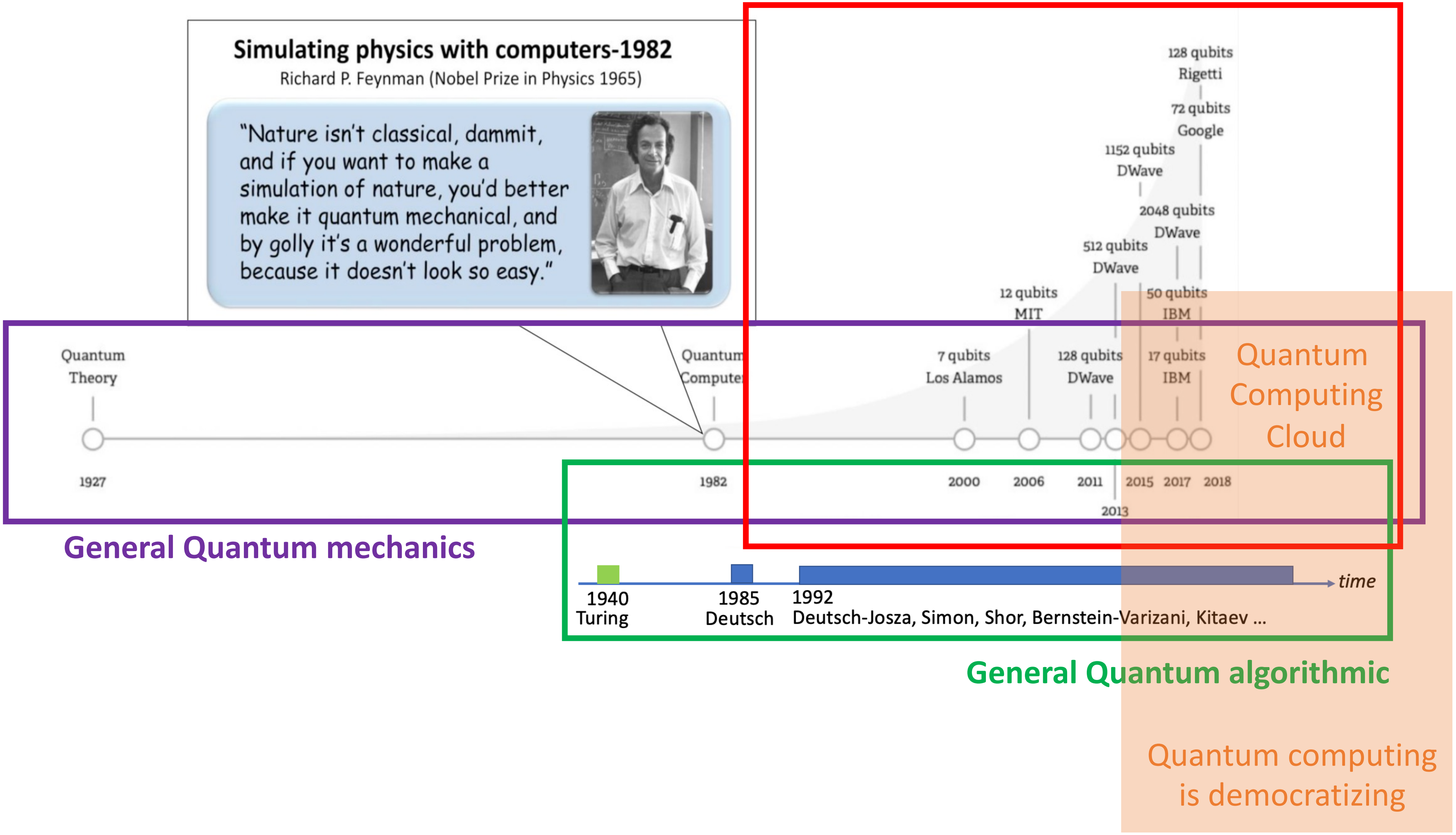
- Simon (1994): find the period of a function $f:\{0,1\}^n \rightarrow \{0,1\}^n$
  - Classically requires exponential number of questions.
  - Quantum: linear number of questions
  - Ingredient: Quantum Fourier transform (QFT) in qubit representation
- Shor (1994): find the period of a function $f:\mathbb{Z} \rightarrow \mathbb{Z}_N$
  - Ingredient: QFT, quantum phase estimation (QPE), no Oracle
  - Applications: factorization, discrete log (quantum algo can potentially break the RSA public key encryption technique)
  - Birth of the post-quantum cryptography



Peter Shor

- Kitaev 95: Circuits for QPE: eigenvalue problems, linear algebra, …
- Grover search algorithm (1997): unsupervised search algorithm, data mining, …

Why quantum computing is becoming mature now ?



Quantum technologies/devices

Simulating physics with computers-1982

Richard P. Feynman (Nobel Prize in Physics 1965)

> "Nature isn't classical, dammit, and if you want to make a simulation of nature, you'd better make it quantum mechanical, and by golly it's a wonderful problem, because it doesn't look so easy."

1927 Quantum Theory

1982 Quantum Computer

2000 7 qubits Los Alamos

2006 12 qubits MIT

2011 128 qubits DWave

2013 512 qubits DWave

2015 1152 qubits DWave

2017 17 qubits IBM; 50 qubits IBM; 2048 qubits DWave

2018 72 qubits Google; 128 qubits Rigetti

Quantum Computing Cloud

General Quantum mechanics

1940 Turing

1985 Deutsch

1992 Deutsch-Josza, Simon, Shor, Bernstein-Varizani, Kitaev ...

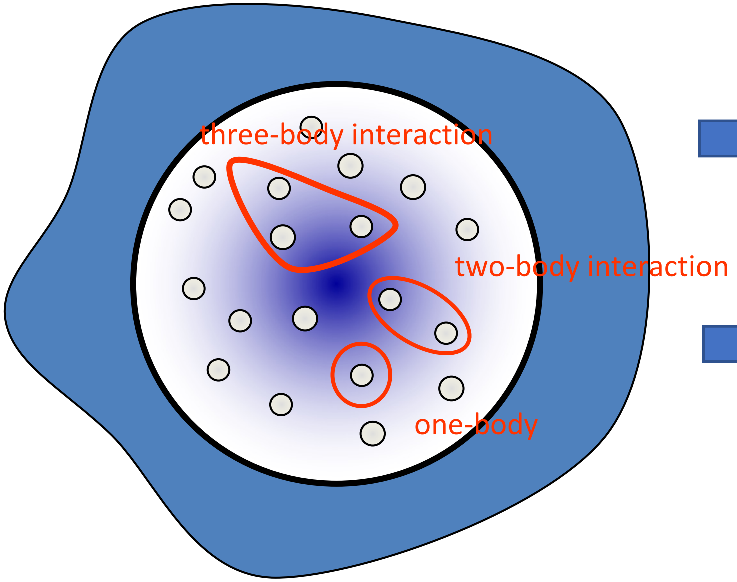
time

General Quantum algorithmic

Quantum computing is democratizing

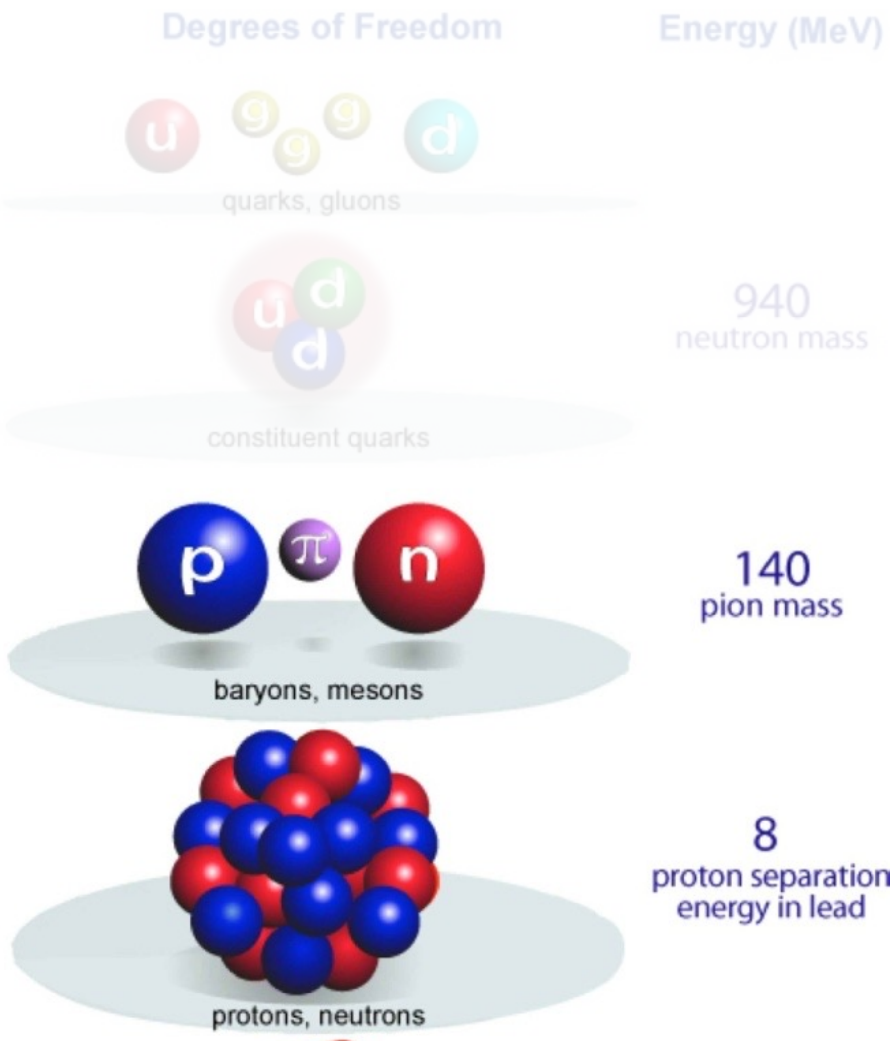
One example:

Simulation of complex quantum (interacting) systems



- If you have N one-body degrees of freedoms The Hilbert space has an exponential Scaling (~N!)
- Even today, only a limited area (small systems- few %) of the nuclear Chart can be calculated with most powerful Supercomputers.

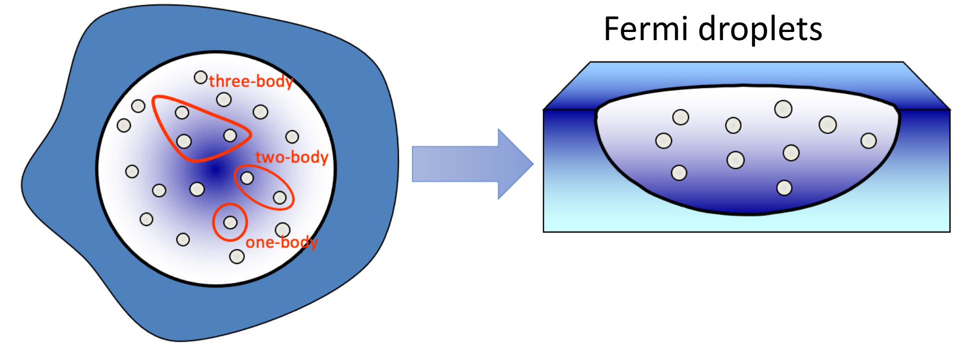
A short highlight of today's nuclear physics challenges



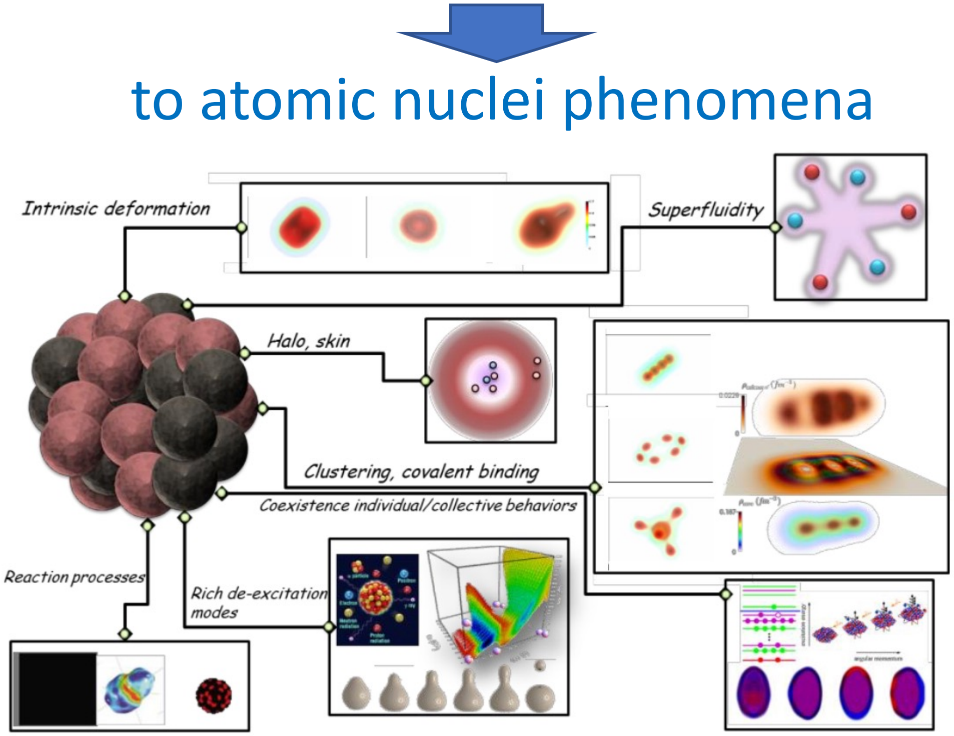
Some phenomenology

Strongly interacting fermions

Quantum self-bound Fermi droplets

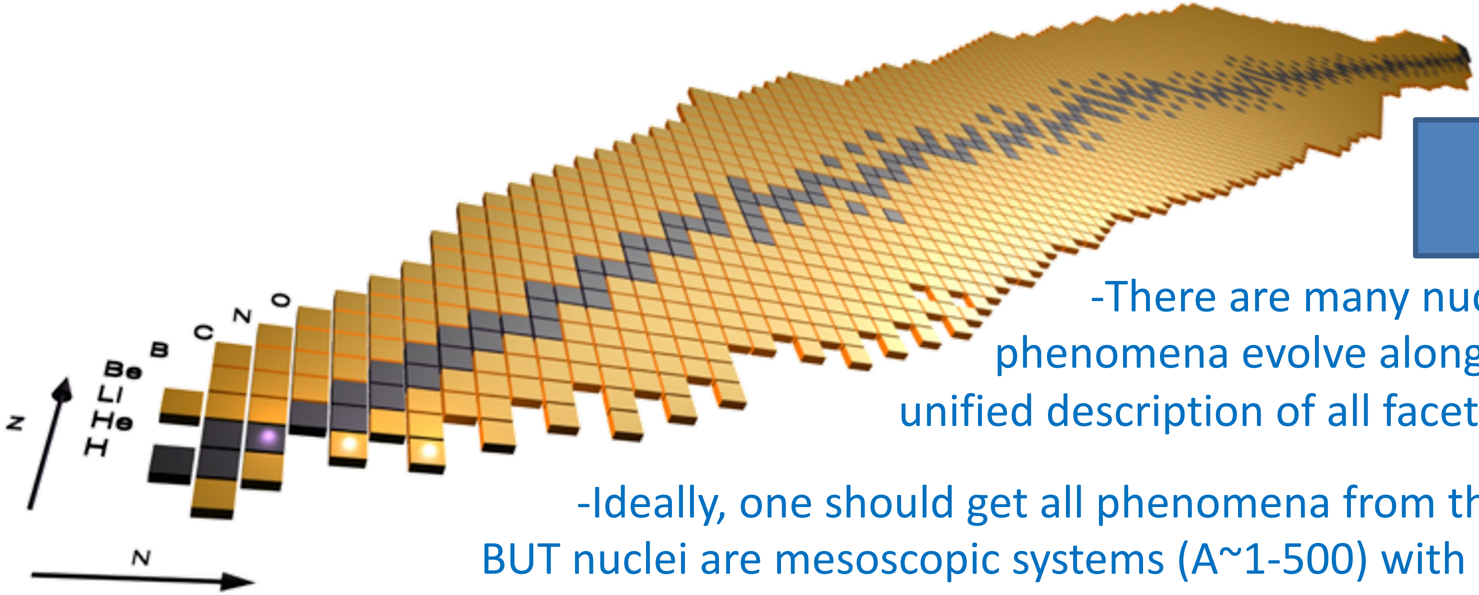


to atomic nuclei phenomena



(courtesy JP. Ebran)

Why quantum computing can help to describe nuclear physics ?

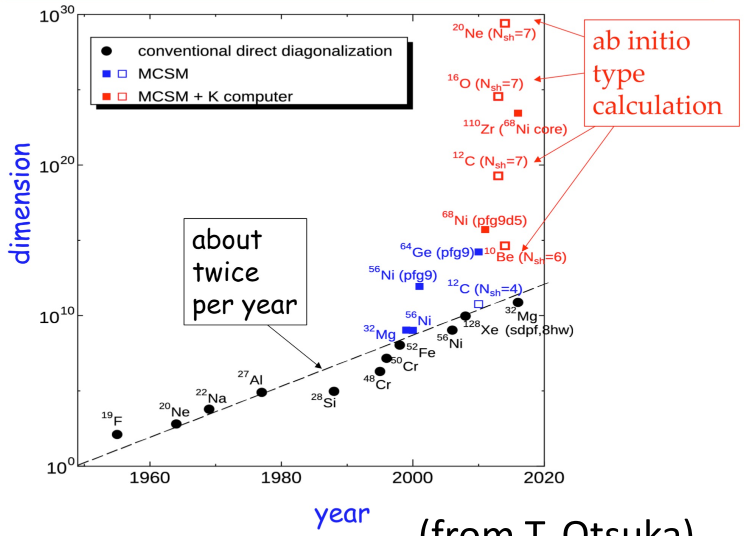


Some evident sources of complexity in nuclei

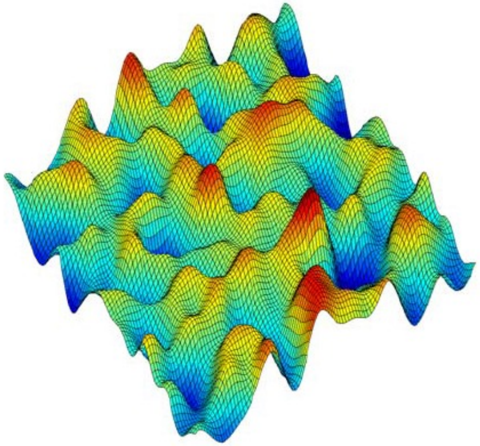
-There are many nuclei (>3000). Nuclear phenomena evolve along the nuclear chart. A unified description of all facets would be desirable.

-Ideally, one should get all phenomena from the bare interaction BUT nuclei are mesoscopic systems (A~1-500) with bad numerical scaling.

-Each nucleus is a complex problem per se.



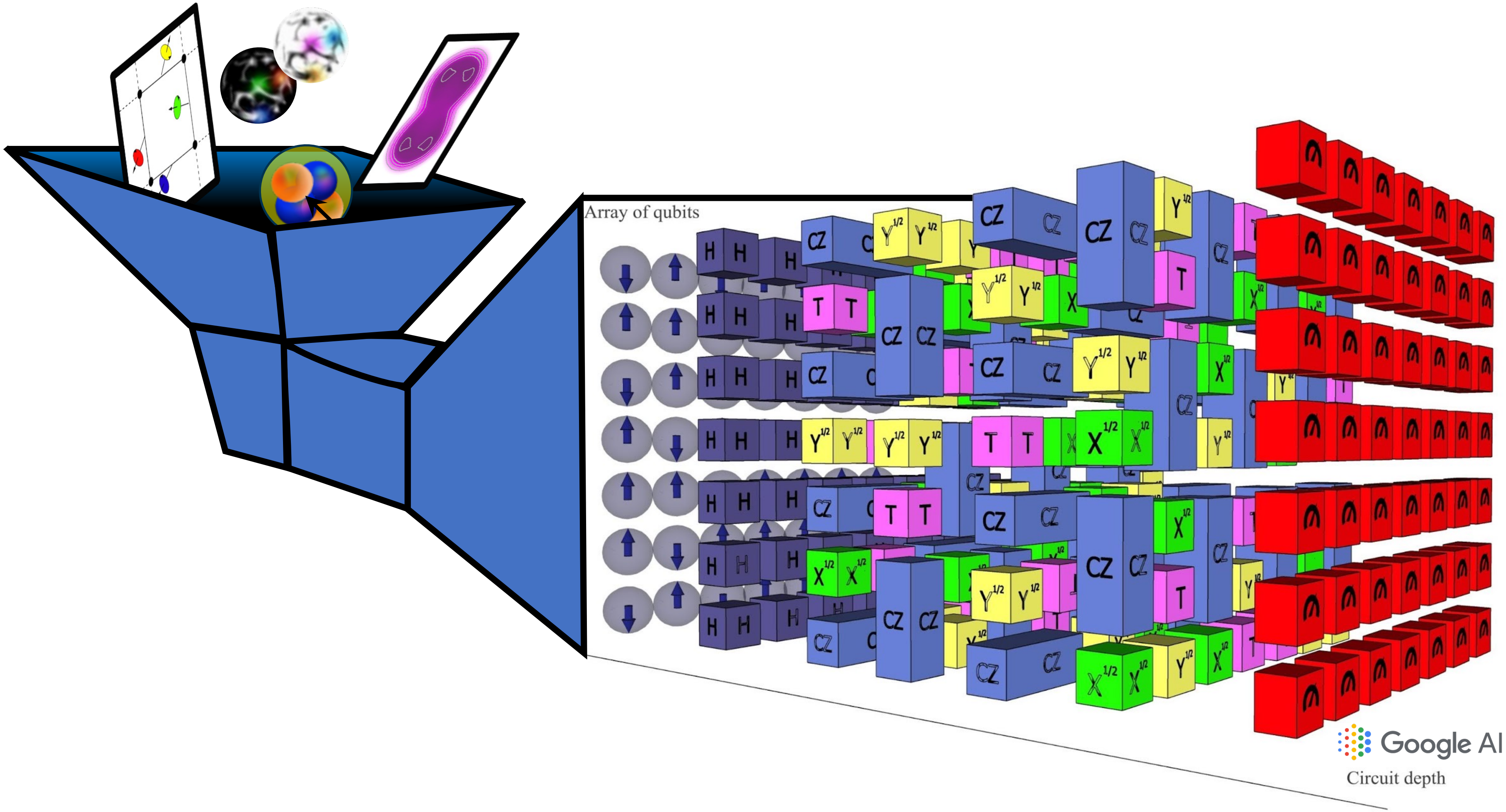
(from T. Otsuka)



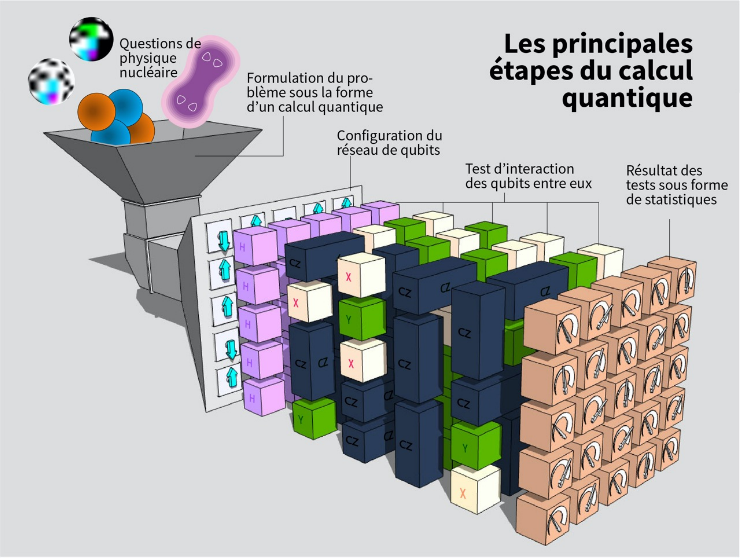
(Energy landscape of a molecule)

This motivates the search of disruptive techniques (high risk/high potential benefits)

Ongoing projects that are starting now



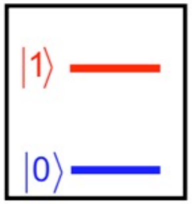
(Second part)
“PLAYING” WITH QUBITS



Introduction to digital quantum computing

qubit

2 level system



Manipulate the Qubits (Make rotations)

$|0\rangle$ Initial state

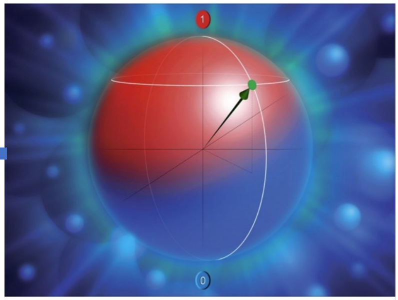


Illustration d'un qubit
CRÉDIT : MARK GARLICK / SCIENCE PHOTO LIBRA / MGA / SCIENCE PHOTO LIBRARY VIA AFP

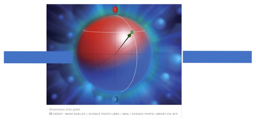
Final state $c_0|0\rangle + c_1|1\rangle$

Measure the state

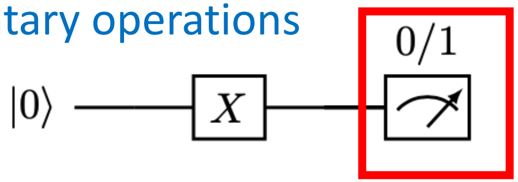
$|c_0|^2, \ |c_1|^2$

("destroy" the state)

Example of simple unary operations (gates)

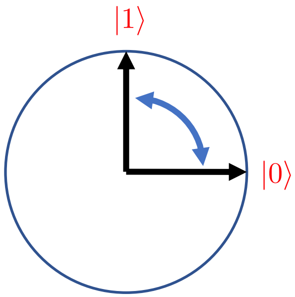


Elementary operations

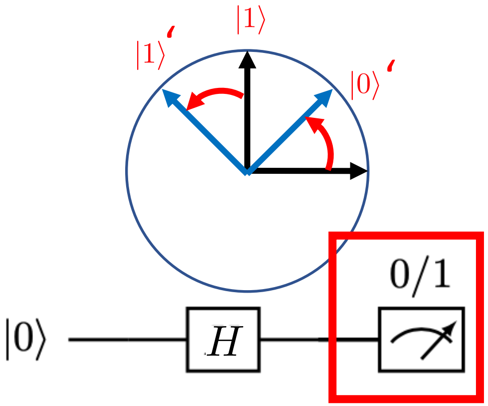


Measure

Here all events gives 1



Hadamard



Measure: 0,0, 1, 0, 1, 1, …

Pauli matrices

Pauli-X (X)	X	⊕	$\begin{array}{cc\|l} \|0\rangle & \|1\rangle & \\ 0 & 1 & \|0\rangle \\ 1 & 0 & \|1\rangle \end{array}$
Pauli-Y (Y)	Y		$\begin{bmatrix} 0 & -i \\ i & 0 \end{bmatrix}$
Pauli-Z (Z)	Z		$\begin{bmatrix} 1 & 0 \\ 0 & -1 \end{bmatrix}$

NOT operation

Other useful unary operations

Hadamard (H)	H	$\frac{1}{\sqrt{2}}\begin{bmatrix} 1 & 1 \\ 1 & -1 \end{bmatrix}$
Phase (S, P)	S	$\begin{bmatrix} 1 & 0 \\ 0 & i \end{bmatrix}$
$\pi/8$ (T)	T	$\begin{bmatrix} 1 & 0 \\ 0 & e^{i\pi/4} \end{bmatrix}$

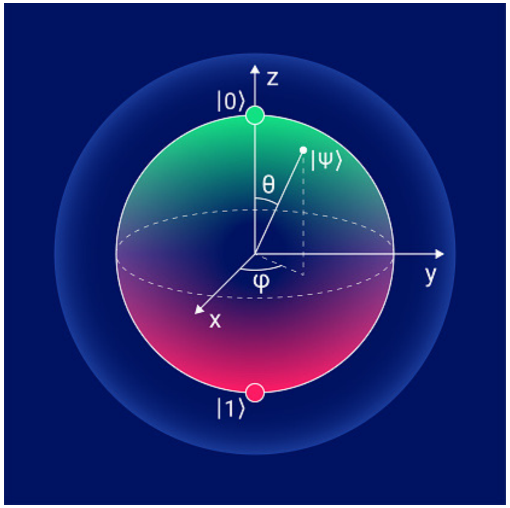
Another useful representation of operation (more familiar to physicists)

The Bloch sphere

From qubits to spins

$$\begin{aligned}\sigma_x &= X = |1\rangle\langle 0| + |0\rangle\langle 1| \\ \sigma_y &= Y = i|1\rangle\langle 0| - i|0\rangle\langle 1| \\ \sigma_z &= Z = |0\rangle\langle 0| - |1\rangle\langle 1|\end{aligned}$$

➡ Obey standard spin algebra



Rotations corresponds to gates

$$\boxed{R_X(\varphi) = e^{-i\varphi X/2}}$$

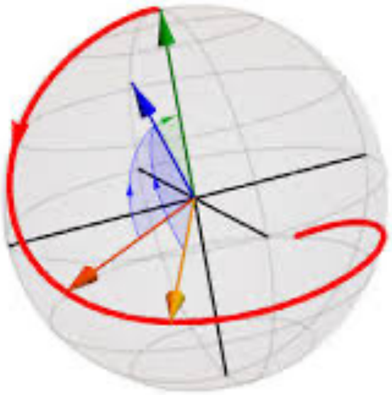
Any state can be written as

$$|\Psi\rangle = \cos(\theta)|0\rangle + e^{i\varphi}\sin(\theta)|1\rangle$$

Such state can always be obtained from 3 rotations

$$|\Psi\rangle = R_Y(\theta_1)R_Z(\theta_2)R_Y(\theta_3)|0\rangle$$

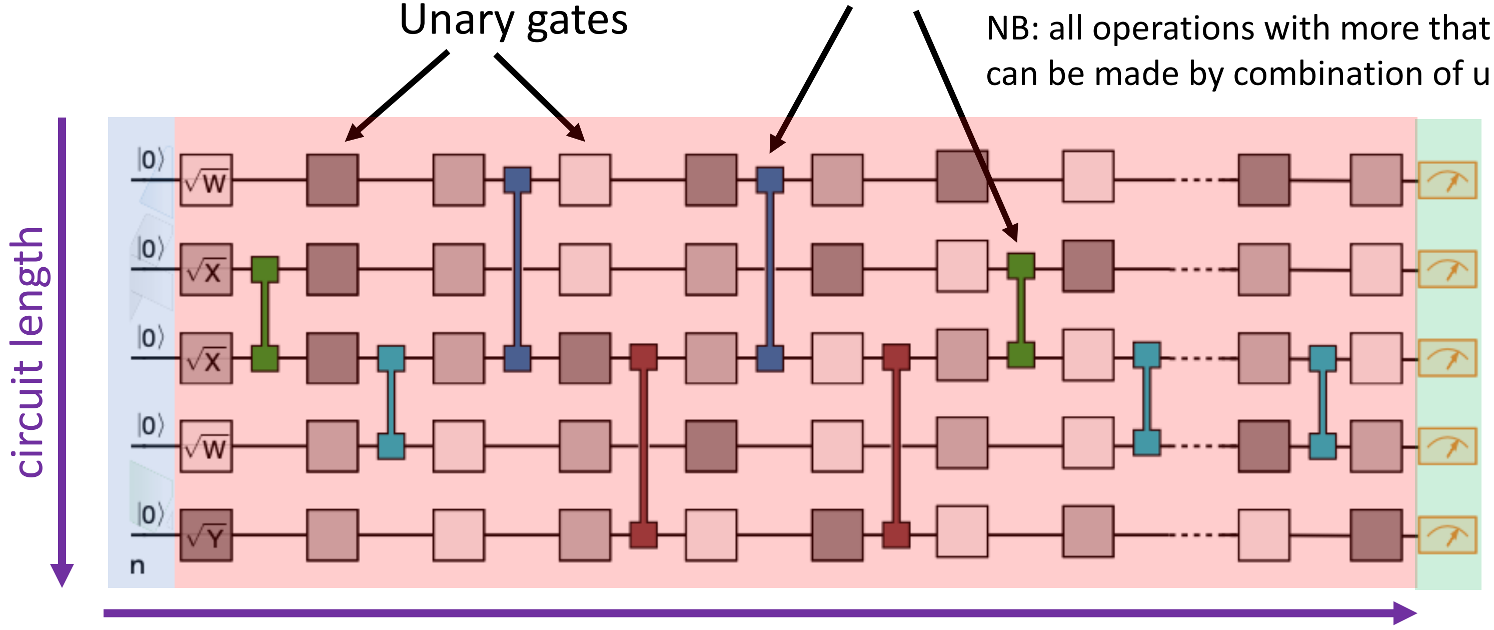
Unary operation can be represented as a path in the Bloch sphere



Quantum computing with more than one qubit

Some terminology – general aspects

Quantum circuits with more than one qubit



Unary gates

Binary gates

NB: all operations with more that two qubits can be made by combination of unary and binary gates

circuit length

circuit depth

Quantum register:
Define the qubit computational basis: $|0,0,\cdots,0,0\rangle$

$|0,0,\cdots,1,1\rangle$

$|0,0,\cdots,0,1\rangle$

$\cdots$

Hilbert space size 2^n

Quantum circuits:

Constraint: the circuit makes Unitary transformation, i.e. no loss of information

Advantage: one can imagine to do e^{itH}

Quantum measurement of qubits – leads to a set of events In the form of bitstring

$$\sum_{i_k=0,1} a_{i_1 i_2 i_3 i_4 \dots i_{2N}} \left| i_1, i_2, i_3 \dots i_{2^N} \right\rangle$$

⬇

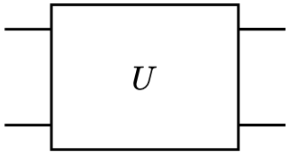
Gives the |a|^2

Quantum computing with more than one qubit

Some terminology – general aspects

Binary gates

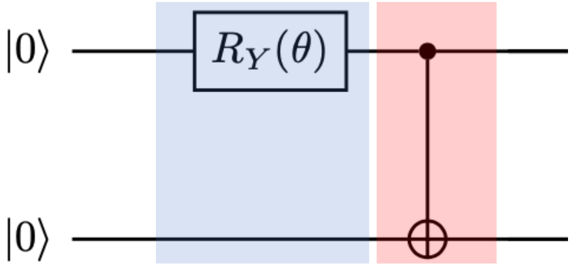
Binary gates are operations acting simultaneously on two qubits and induced entanglement



Remember the example:

$|\Phi\rangle = \alpha|$😀😀$\rangle + \beta|$😫😫$\rangle$ [equivalent to Bell states]

Quantum circuit



Rotation

Control-X gate
Also known as CNOT

Step 1

$$|00\rangle \rightarrow \cos(\theta/2)|00\rangle + \sin(\theta/2)|01\rangle$$

Step 2

$$\rightarrow \cos(\theta/2)|00\rangle + \sin(\theta/2)|11\rangle$$

Indirect measurement and Born rule for measurement

If I measure 0 in first qubit ⇒ After the measurement the state collapse to $|00\rangle$

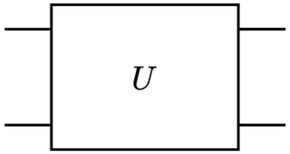
If I measure 1 in first qubit ⇒ After the measurement the state collapse to $|11\rangle$

Quantum computing with more than one qubit

Some terminology – general aspects

Binary gates

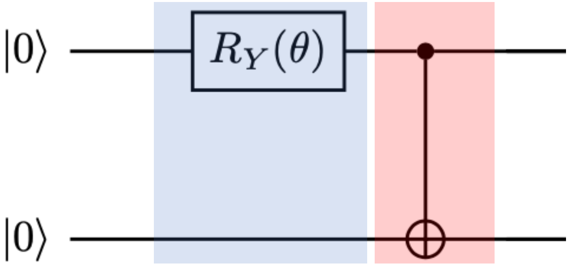
Binary gates are operations acting simultaneously on two qubits and induced entanglement



Remember the example:

$$|\Phi\rangle = \alpha|\text{😀😀}\rangle + \beta|\text{😫😫}\rangle$$ [equivalent to Bell states]

Quantum circuit



Rotation

Control-X gate
Also known as CNOT

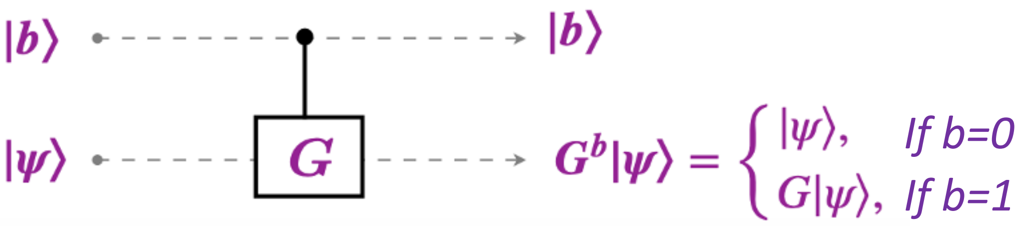
Step 1

$$|00\rangle \rightarrow \cos(\theta/2)|00\rangle + \sin(\theta/2)|01\rangle$$

Step 2

$$\rightarrow \cos(\theta/2)|00\rangle + \sin(\theta/2)|11\rangle$$

Controlled gates (IF-THEN-ELSE)

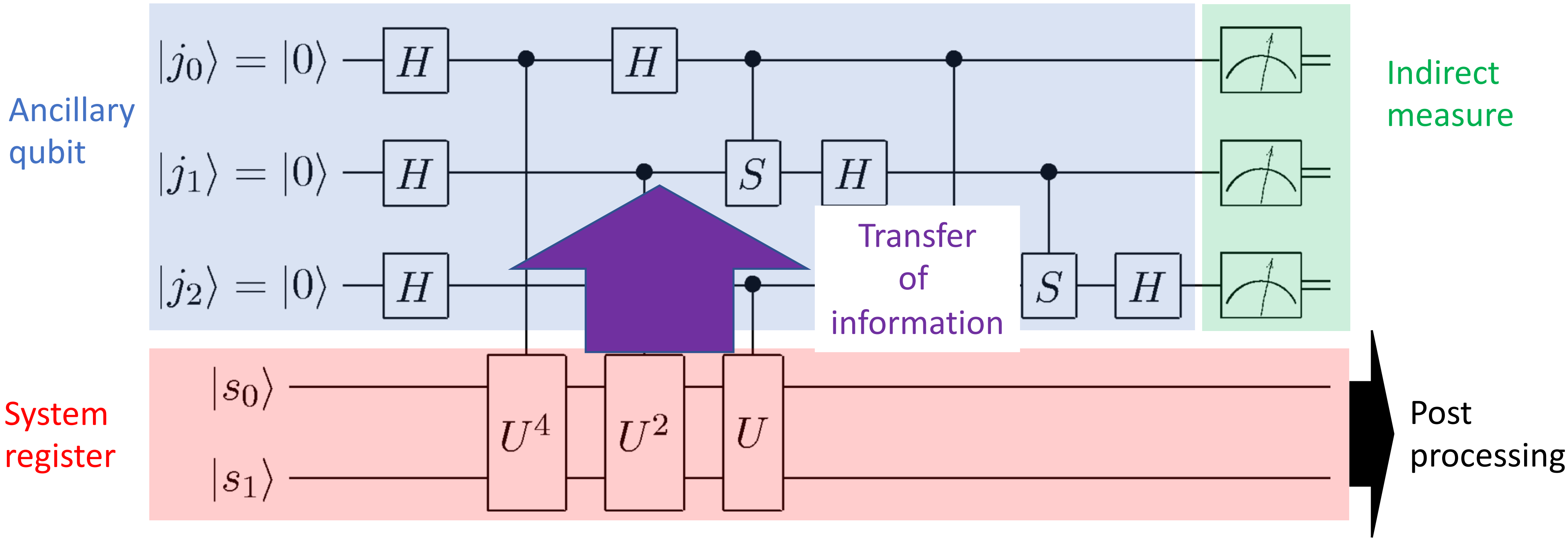


$$G^b|\psi\rangle = \begin{cases} |\psi\rangle, & \text{If } b=0 \\ G|\psi\rangle, & \text{If } b=1 \end{cases}$$

Quantum computing with more than one qubit

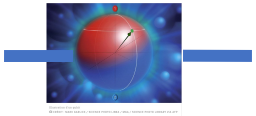
Some terminology – general aspects

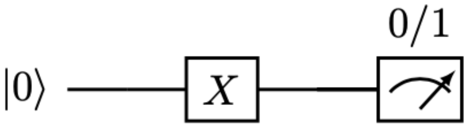
A more complete view of quantum computing circuits



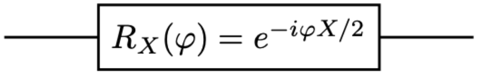
The quantum computing toolkit

Unary operations



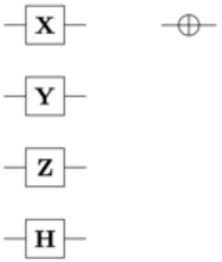


Rotations



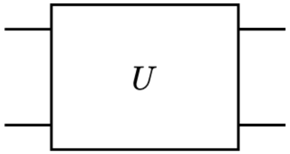
$$R_X(\varphi) = e^{-i\varphi X/2}$$

Standard examples

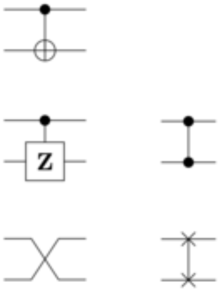


Pauli-X (X)	$\begin{array}{cc} \begin{matrix}\vert 0\rangle & \vert 1\rangle\end{matrix} & \\ \begin{bmatrix}0 & 1\\ 1 & 0\end{bmatrix} & \begin{matrix}\vert 0\rangle\\ \vert 1\rangle\end{matrix}\end{array}$
Pauli-Y (Y)	$\begin{bmatrix}0 & -i\\ i & 0\end{bmatrix}$
Pauli-Z (Z)	$\begin{bmatrix}1 & 0\\ 0 & -1\end{bmatrix}$
Hadamard (H)	$\frac{1}{\sqrt{2}}\begin{bmatrix}1 & 1\\ 1 & -1\end{bmatrix}$

Binary operations

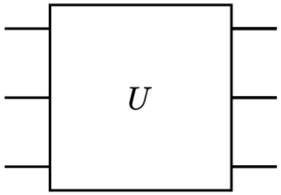


Standard examples



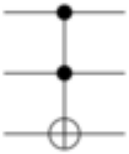
Controlled Not (CNOT, CX)	$\begin{bmatrix}1&0&0&0\\0&1&0&0\\0&0&0&1\\0&0&1&0\end{bmatrix}$	$\vert 11\rangle \leftrightarrow \vert 10\rangle$
Controlled Z (CZ)	$\begin{bmatrix}1&0&0&0\\0&1&0&0\\0&0&1&0\\0&0&0&-1\end{bmatrix}$	$\vert 11\rangle \leftrightarrow -\vert 11\rangle$
SWAP	$\begin{array}{cc}\begin{bmatrix}1&0&0&0\\0&0&1&0\\0&1&0&0\\0&0&0&1\end{bmatrix} & \begin{matrix}\vert 00\rangle\\ \vert 01\rangle\\ \vert 10\rangle\\ \vert 11\rangle\end{matrix}\end{array}$	$\vert 01\rangle \leftrightarrow \vert 10\rangle$

Ternary operations



Standard example

Toffoli (CCNOT, CCX, TOFF)

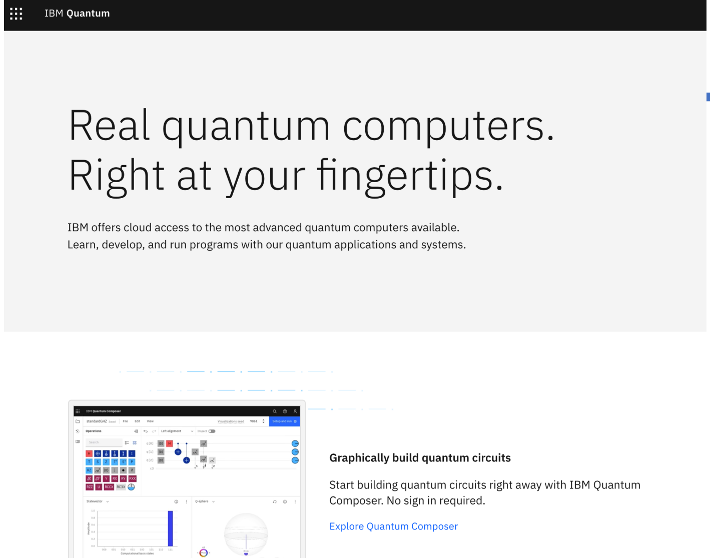


$$\begin{bmatrix}1&0&0&0&0&0&0&0\\0&1&0&0&0&0&0&0\\0&0&1&0&0&0&0&0\\0&0&0&1&0&0&0&0\\0&0&0&0&1&0&0&0\\0&0&0&0&0&1&0&0\\0&0&0&0&0&0&0&1\\0&0&0&0&0&0&1&0\end{bmatrix}$$

$$|110\rangle \leftrightarrow |111\rangle$$

If you want to play with qubits

Try the “IBM Quantum experience” online



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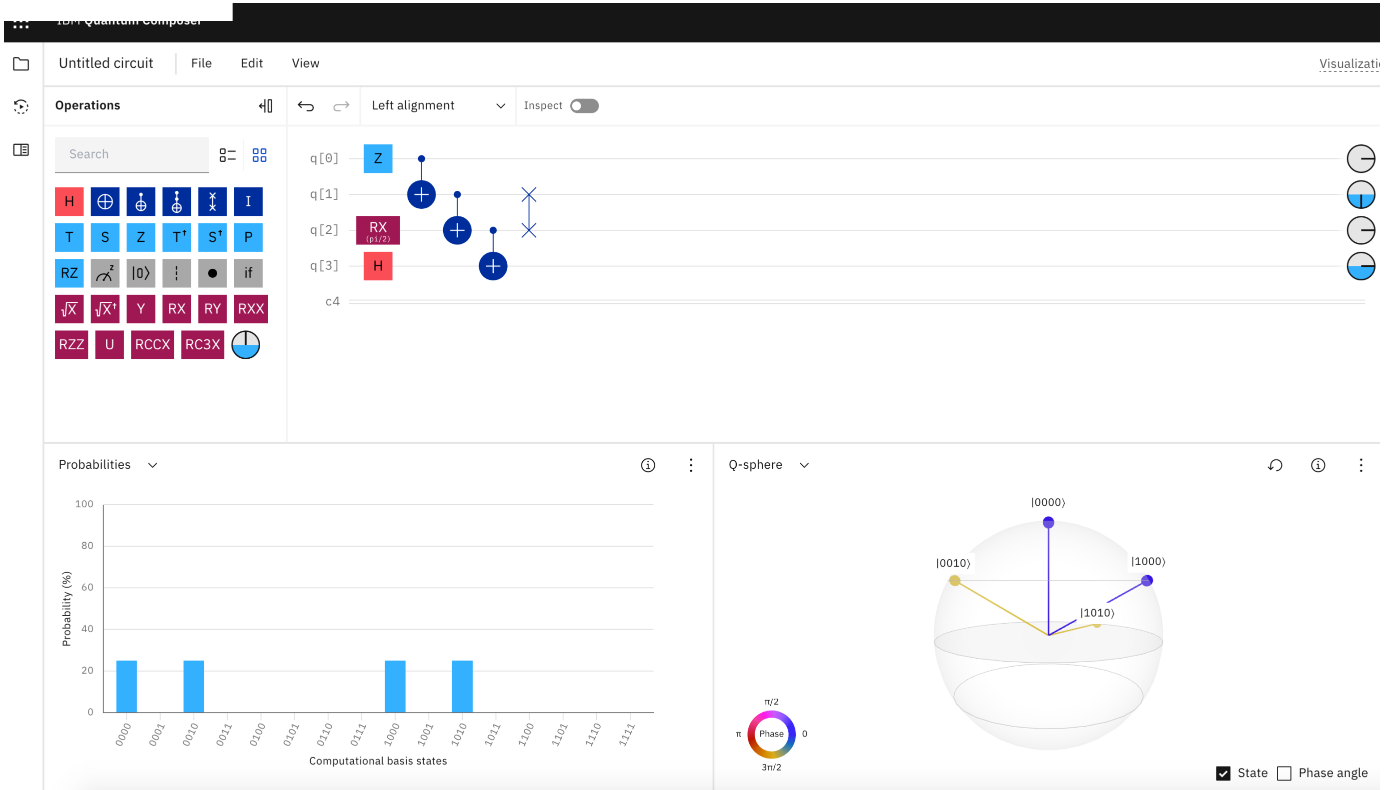
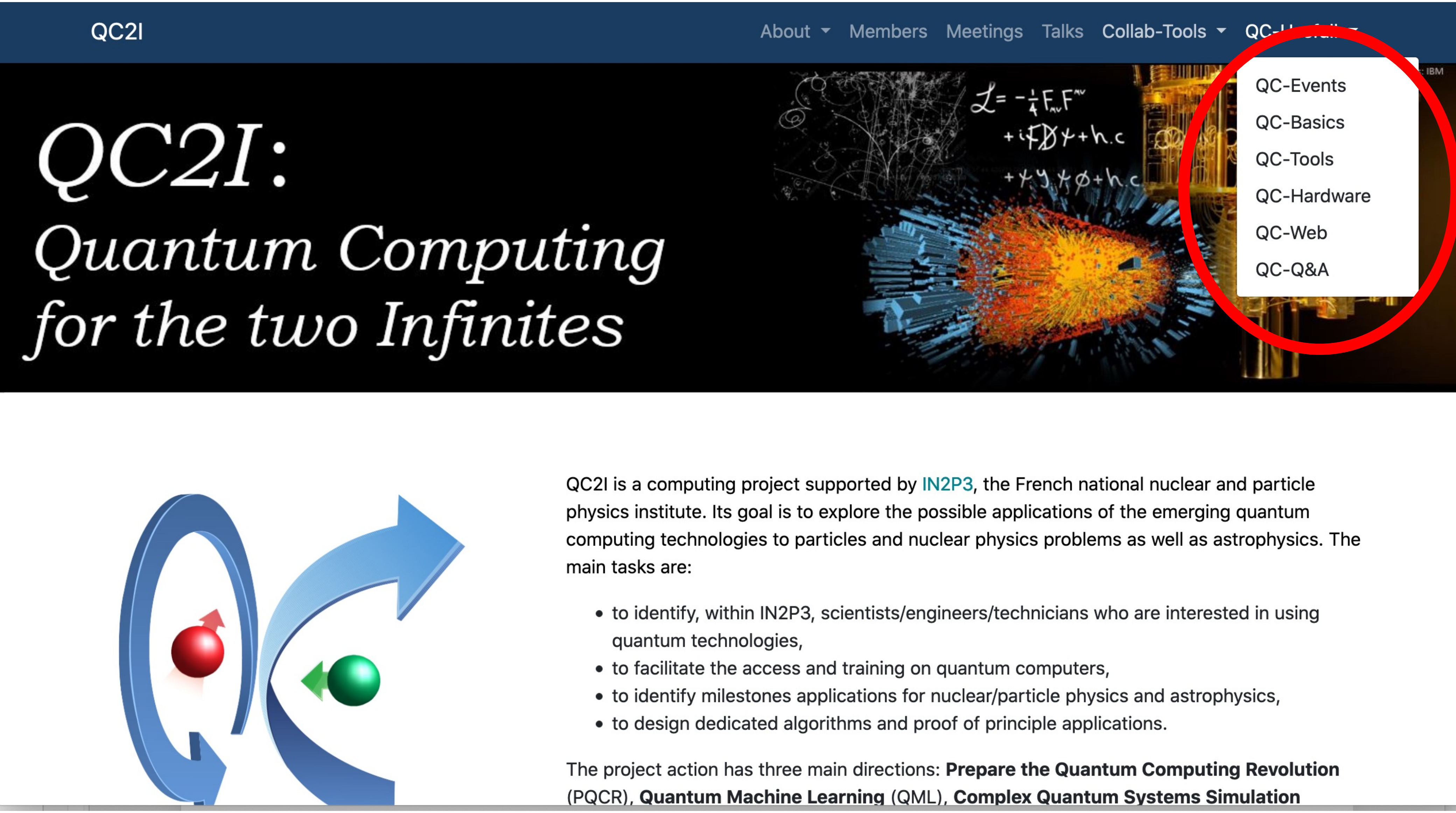


☑ State ☐ Phase angle

Some advertisement – for some ressources

If you want to start QC programming

https://qc.pages.in2p3.fr/web/



QC2I:

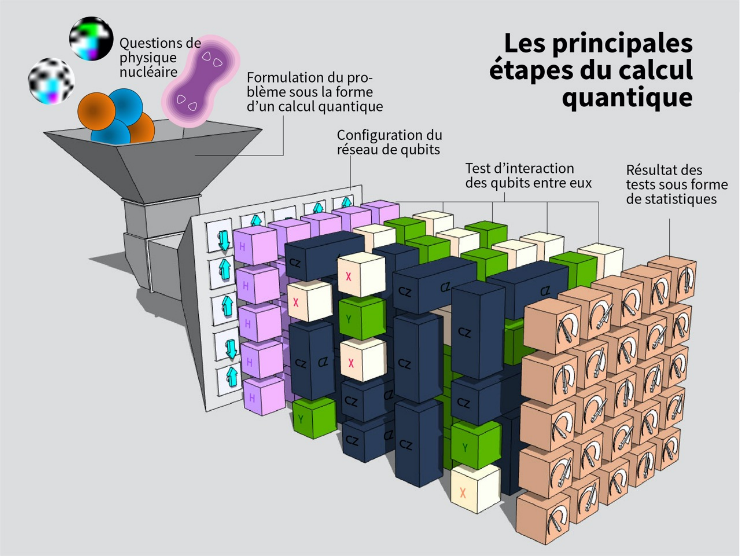
Quantum Computing for the two Infinites

QC2I is a computing project supported by IN2P3, the French national nuclear and particle physics institute. Its goal is to explore the possible applications of the emerging quantum computing technologies to particles and nuclear physics problems as well as astrophysics. The main tasks are:

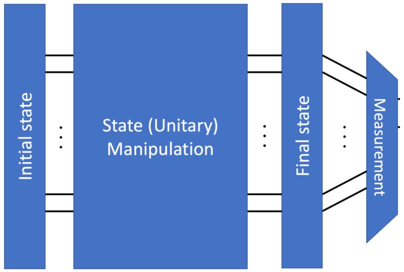
- to identify, within IN2P3, scientists/engineers/technicians who are interested in using quantum technologies,
- to facilitate the access and training on quantum computers,
- to identify milestones applications for nuclear/particle physics and astrophysics,
- to design dedicated algorithms and proof of principle applications.

The project action has three main directions: **Prepare the Quantum Computing Revolution** (PQCR), **Quantum Machine Learning** (QML), **Complex Quantum Systems Simulation**

Some illustrations of applications



General strategy for quantum computing



Take a problem
(classical or quantum)

Encode the problem
Onto a set of qubits

There are many ways to encode a classical problem (phase, amplitude, time, …)

Schuld and Petruccione, Supervised learning with quantum computers (2018)

Make Quantum Computing
Program to solve the problem

For instance
Qiskit, cirq,
myQLM, …

Test on a
QC emulator

Test on a true
QC

IBM Quantum

Amazon Braket

…

First Example:

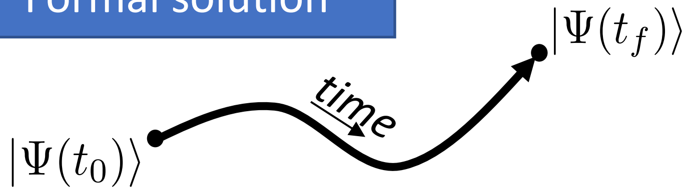
Solution of a 1D
Time-Dependent Schroedinger Eq.

Example 1: solving a 1D Schrödinger Equation on a quantum computer

I want to solve $$i\hbar\frac{d}{dt}\Psi(x) = \left\{\frac{-\hbar^2}{2m}\Delta + V(x)\right\}\Psi(x)$$

Discretization on a mesh

Formal solution



Formal solution

$$|\Psi(t)\rangle = e^{\frac{1}{i\hbar}(t-t_0)H}|\Psi(t_0)\rangle$$

H is usually a big matrix

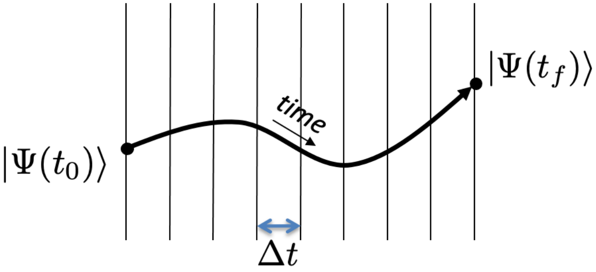
Practical solution

$$i\hbar\dot{\mathbf{F}}(t) = \mathbf{H}\times\mathbf{F}(t)$$

$$\mathbf{F}(t+\Delta t) = \exp\left(\frac{\Delta t}{i\hbar}\mathbf{H}\right)\times\mathbf{F}(t)$$

Time discretization

time: $\{t_i\}$ time-step: Δt



Non unitary

Direct

$$\exp\left(-\frac{\Delta t}{i\hbar}\mathbf{H}\right) \simeq 1 + \frac{\Delta t}{i\hbar}\mathbf{H} + \frac{1}{2!}\left(\frac{\Delta t}{i\hbar}\mathbf{H}\right)^2 + \cdots$$

(Δt)ⁿ, non-unitary, any dim.

Requires to inverse a matrix

Crank-Nicholson

$$\mathbf{F}(t+\Delta t) = \frac{1-\frac{\Delta t}{2i\hbar}\mathbf{H}}{1+\frac{\Delta t}{2i\hbar}\mathbf{H}}\mathbf{F}(t)$$

$(\Delta t)^2$, unitary, 1D only

Might be used

Split-Operator

$$\mathbf{F}(t+\Delta t) \simeq e^{-i\Delta t\frac{\mathbf{P}^2}{4\hbar m}}e^{-\frac{i}{\hbar}\Delta t\mathbf{V}}e^{-i\Delta t\frac{\mathbf{P}^2}{4\hbar m}}\times\mathbf{F}(t)$$

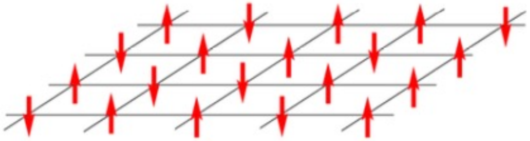
$(\Delta t)^2$, unitary, any dim.

Solving a 1D Schrödinger Equation on a *quantum computer*

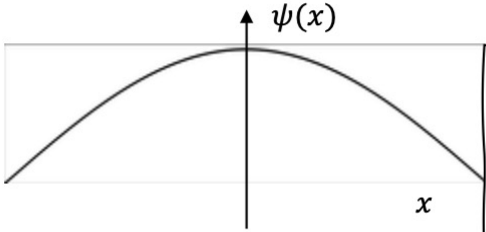
I want to solve $$i\hbar\frac{d}{dt}\Psi(x) = \left\{\frac{-\hbar^2}{2m}\Delta + V(x)\right\}\Psi(x)$$ → Discretization on a mesh

$x \to i\Delta x$

Jing Zhang
PhD IJCLab



- **Discretization**



$$T = -\frac{\hbar^2}{2m}\frac{\mathrm{d}^2}{\mathrm{d}x^2}$$

$$V = V(x)$$

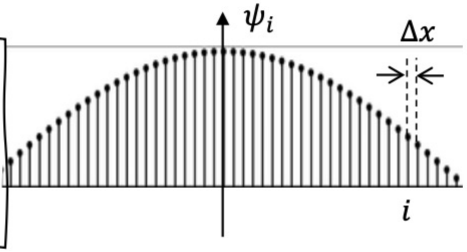
Discretize

Discrete space basis

$$\delta(x - x_i) = |i\rangle$$

$$\Delta x\ \psi(x_i) = \langle i|\psi\rangle = \psi_i$$

$\hbar = 1$
$\Delta x = 1$
$2m = 1$



$$T = \sum_i |i\rangle\langle i+1| - 2\,|i\rangle\langle i| + |i\rangle\langle i-1|$$

$$V = \sum_i V_i\,|i\rangle\langle i|$$

$$\text{e.g.,}\ \langle\psi|T|\psi\rangle = -\frac{\hbar^2}{2m}\int \psi^*\frac{\mathrm{d}^2}{\mathrm{d}x^2}\psi\mathrm{d}x$$

$$\approx -\frac{\hbar^2}{2m}\sum_i \psi_i^*\frac{\psi_{i+1} - 2\psi_i + \psi_{i-1}}{(\Delta x)^2}$$

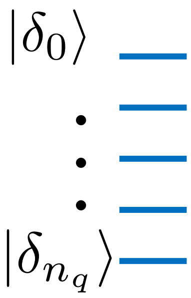
$$= \sum_i \langle\psi|i\rangle\,\frac{\langle i+1|\psi\rangle - 2\,\langle i|\psi\rangle + \langle i-1|\psi\rangle}{(\Delta x)^2}$$

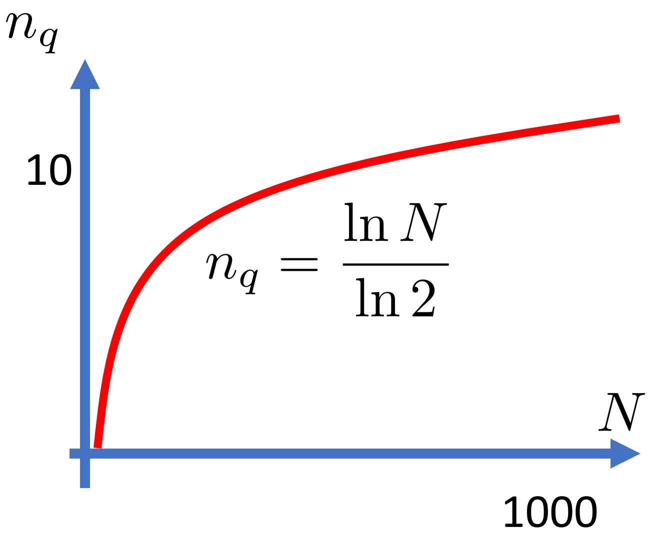
Solving a 1D Schrödinger Equation on a *quantum computer*

Encoding into qubits

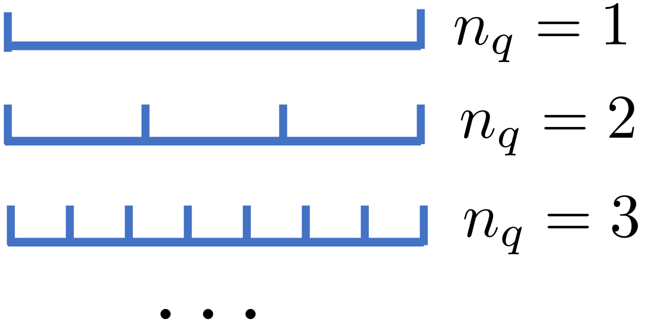
We have a discrete basis $|i = 0 \cdots N-1\rangle$

Natural encoding – Use the binary representation $|i\rangle = |\delta_{n_q} \cdots \delta_0\rangle$ *with* $i = \sum_k \delta_k 2^k$





Constraint $N \leq 2^{n_q}$



$\|i\rangle$	Standard Binary (SB) Encoding	Gray code(GC) Encoding
$\|0\rangle$	$\|000\rangle$	$\|000\rangle$
$\|1\rangle$	$\|001\rangle$	$\|001\rangle$
$\|2\rangle$	$\|010\rangle$	$\|011\rangle$
$\|3\rangle$	$\|011\rangle$	$\|010\rangle$
$\|4\rangle$	$\|100\rangle$	$\|110\rangle$
$\|5\rangle$	$\|101\rangle$	$\|111\rangle$
$\|6\rangle$	$\|110\rangle$	$\|101\rangle$
$\|7\rangle$	$\|111\rangle$	$\|100\rangle$

Solving a 1D Schrödinger Equation on a *quantum computer*

Encoding into qubits with Gray code

Gray code encoding of Hamiltonian

$$H = \sum_k \left(|k\rangle\langle k+1| - 2\,|k\rangle\langle k| + |k\rangle\langle k-1|\right)$$

➡ Goal : transform the Hamiltonian into gates products of *(X,Y,Z)*

How this works:

$$X = |1\rangle\langle 0| + |0\rangle\langle 1|$$
$$Y = i|1\rangle\langle 0| - i|0\rangle\langle 1|$$
$$Z = |0\rangle\langle 0| - |1\rangle\langle 1|$$

We add the identity $I = |0\rangle\langle 0| + |1\rangle\langle 1|$

➡ $$\begin{cases} |1\rangle\langle 1| \rightarrow \frac{1}{2}(I - Z) \\ |0\rangle\langle 0| \rightarrow \frac{1}{2}(I + Z) \end{cases}$$

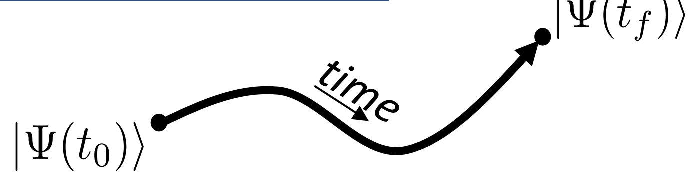
e.g., $$\begin{aligned} |2\rangle\langle 3| + |3\rangle\langle 2| &= |011\rangle\langle 010| + |010\rangle\langle 011| \\ &= |0\rangle\langle 0| \otimes |1\rangle\langle 1| \otimes (|1\rangle\langle 0| + |0\rangle\langle 1|) \\ &= \frac{1}{2}(I + Z_0) \otimes \frac{1}{2}(I - Z_1) \otimes X_2 \end{aligned}$$

This can be made systematically

$$H = \text{fct}(X_\alpha, Y_\alpha, Z_\alpha)$$

Solving a 1D Schrödinger Equation on a *quantum computer*

Time-evolution



Formal solution

This can be a priori computed
Directly on a QC

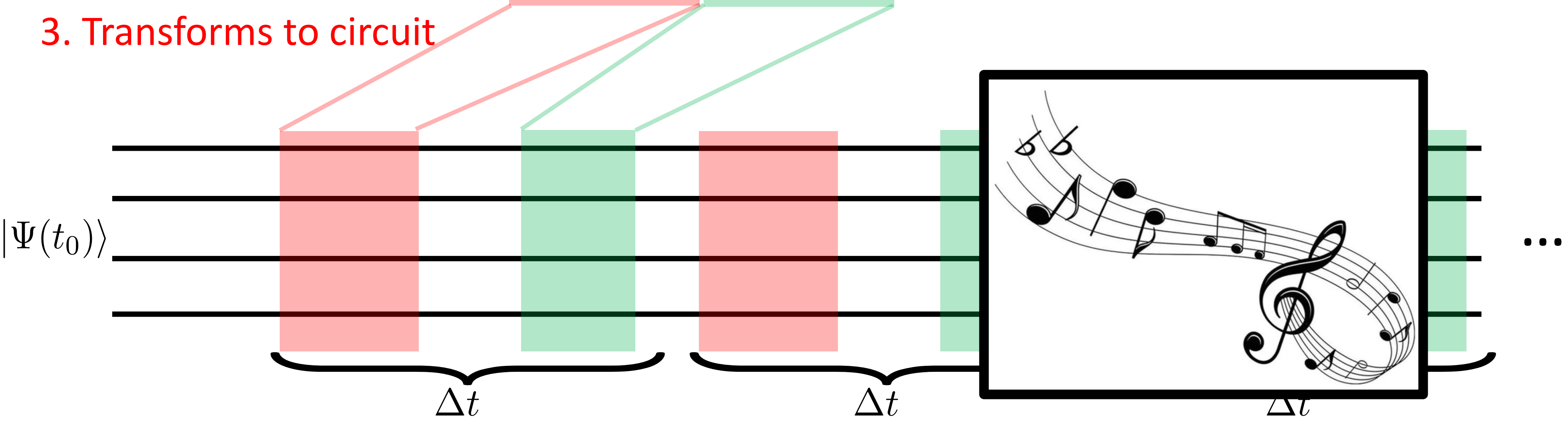
$$|\Psi(t)\rangle = e^{\frac{1}{i\hbar}(t-t_0)H}|\Psi(t_0)\rangle$$

1. Decomposition of H into elementary blocks $H = \sum_l H_l$

2. Use a transformation (Trotter-Suzuki) $e^{-ix(A+B)} = \left(e^{-iAx/N}e^{-iBx/N}\right)^N + \mathcal{O}(t^2/N)$

Example : $e^{i\Delta t H_1/\hbar} = e^{-i\Delta t H_1/\hbar} e^{-i\Delta t H_2/\hbar}$

3. Transforms to circuit



Solving a 1D Schrödinger Equation on a *quantum computer*

Deeper in technical details

The case of free propagation: $H = \frac{p^2}{2m}$

position $\xrightarrow[\text{tranform}]{\text{Fourier}}$ momentum

Hamiltonian decomposition

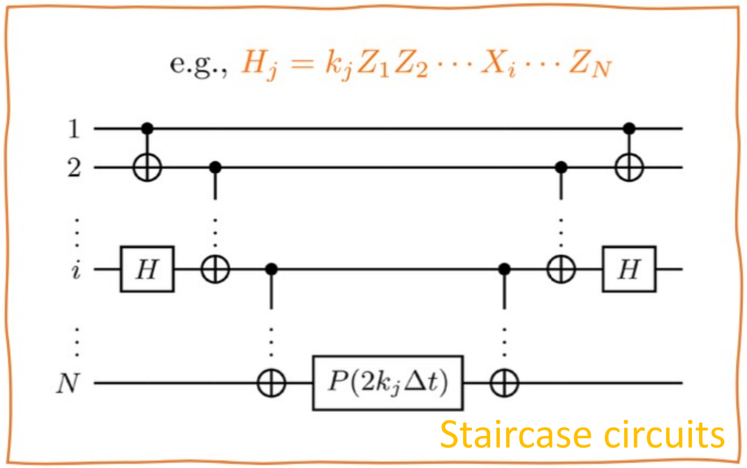
Time evolution:

$$\exp\left(-i\hat{H}t\right) = \exp\left(-i\textstyle\sum_j \hat{H}_j t\right)$$

$$\approx \left[\prod_j \boxed{\exp\left(-i\hat{H}_j\Delta t\right)}\right]^{t/\Delta t}$$

Trotter decomposition

$$e^{i(A+B)t} = \lim_{\Delta t\to 0}\left(e^{iA\Delta t}e^{iB\Delta t}\right)^{t/\Delta t}$$



e.g., $H_j = k_j Z_1 Z_2 \cdots X_i \cdots Z_N$

Staircase circuits

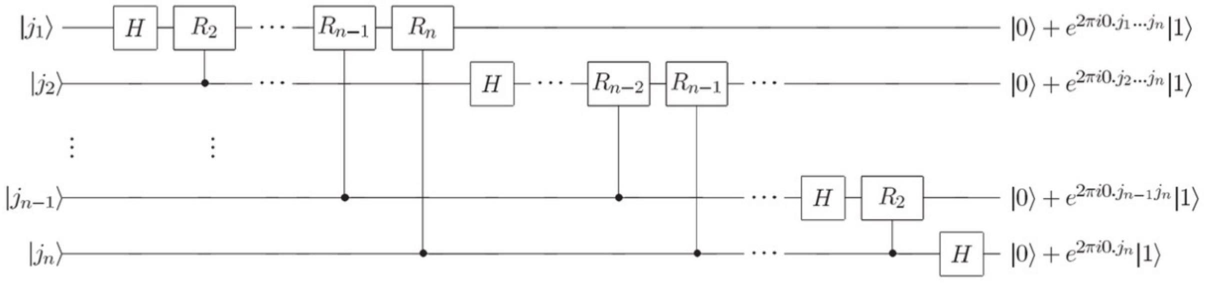
Digital or Quantum Fourier transformation

$$QFT: \quad |j\rangle \to \frac{1}{2^{N/2}}\sum_{k=0}^{2^N-1} e^{2\pi i j(k-2^{N-1})/2^N}\,|k\rangle$$

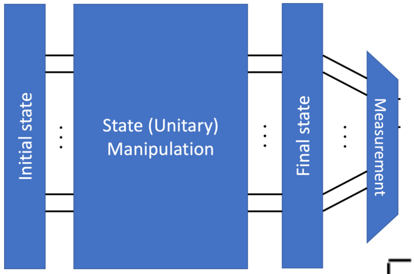
$$= \frac{e^{-\pi i j}}{2^{N/2}}\left(|0\rangle + e^{2\pi i 0.j_N}\,|1\rangle\right)\otimes\left(|0\rangle + e^{2\pi i 0.j_{N-1}j_N}\,|1\rangle\right)$$

$$\otimes\cdots\otimes\left(|0\rangle + e^{2\pi i 0.j_1 j_2\cdots j_N}\,|1\rangle\right).$$

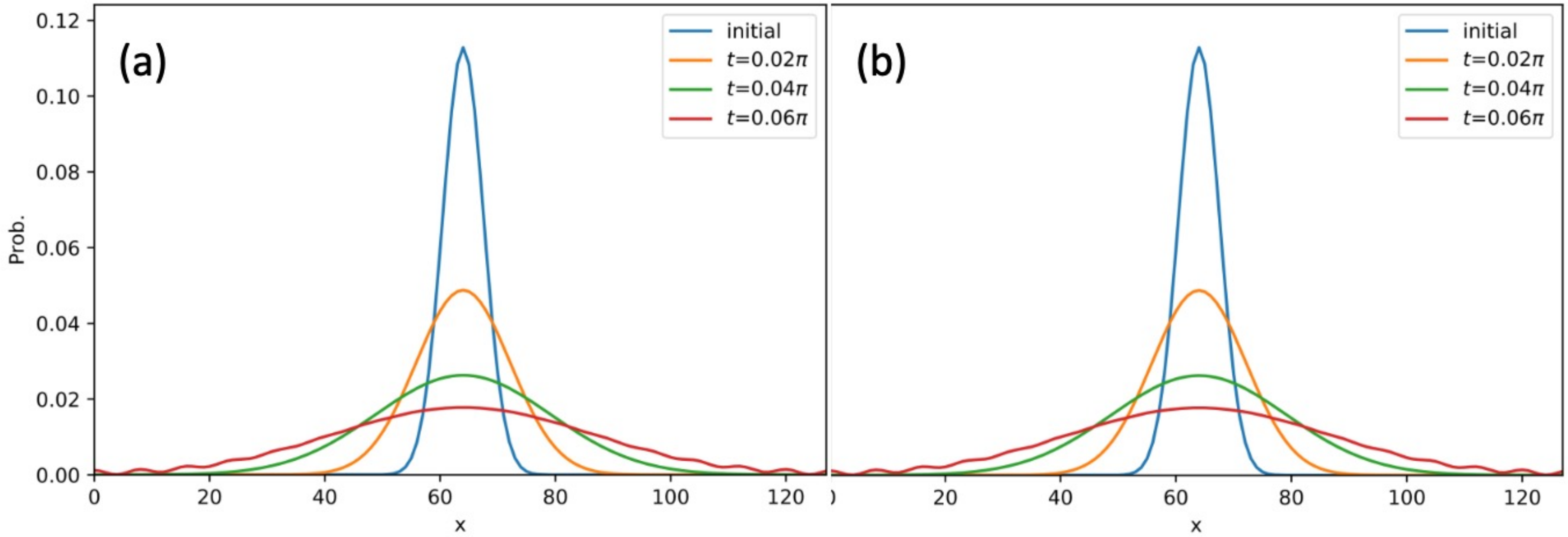
QFT circuit



Solving a 1D Schrödinger Equation on a *quantum computer*



Initial state preparation → QFT → Time propagation → QFT^{-1} → Final state

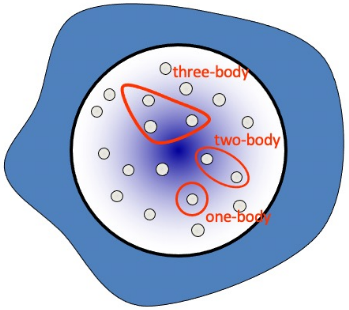


Simulation on classical computer

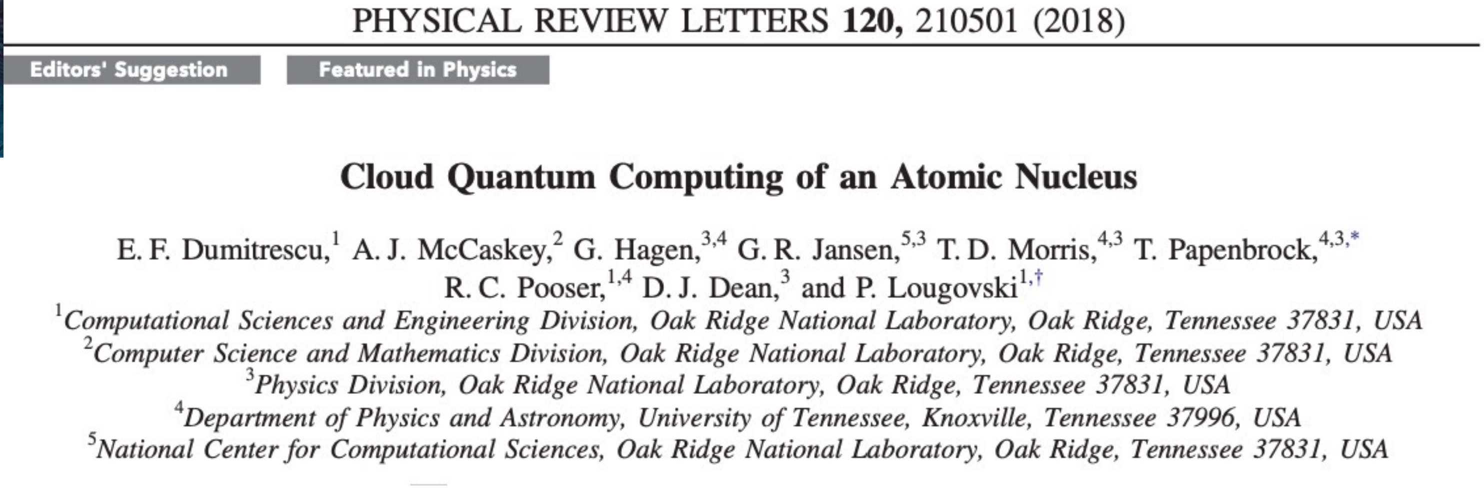
Simulation on quantum computer (emulator)

Some shortcomings:

- The number of qubits to get sufficient accuracy is already quite large
- Absorbing waves at the boundary?



Second Example:
The deuteron problem in a schematic model



PHYSICAL REVIEW LETTERS **120,** 210501 (2018)

Editors' Suggestion **Featured in Physics**

Cloud Quantum Computing of an Atomic Nucleus

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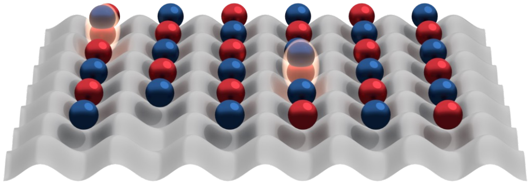
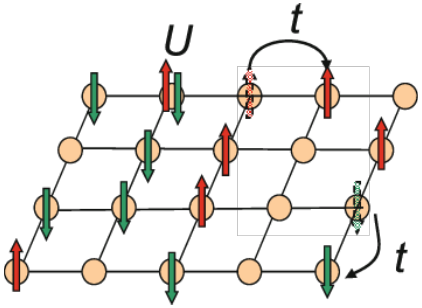
Solution of simple many-body problem with QC: a brief history

Jordan-Wigner (1928)

Fermions on [1D] lattice



Spin systems



Quantum chemistry
Condensed matter

1997- Abrams and Loyds

A quantum algorithm
For eigenvalue problems

2001- Bravyi-Kitaev
Mapping fermions-Qubits

-2011-2012-Whitfield et al
-Seeley et al
The H_2 Hamiltonian

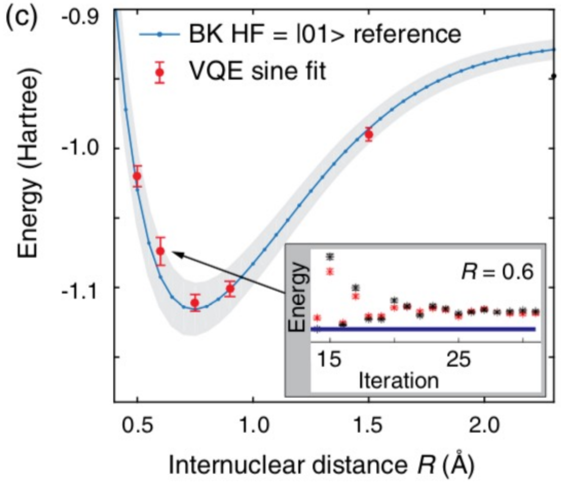
-2014 – Peruzzo et al,
The VQE algorithm
For classical-quantum calc.

-2016 - O'Malley et al
First "real" calculations H_2

-2017 - Kandala et al
Calculations for H_2, LiH, HBeH

-2018 - Hempel et al
…

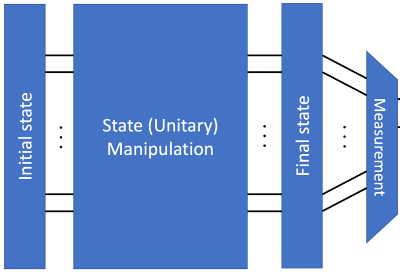
Nuclear Physics



Creation of a US coll. To Prepare QC

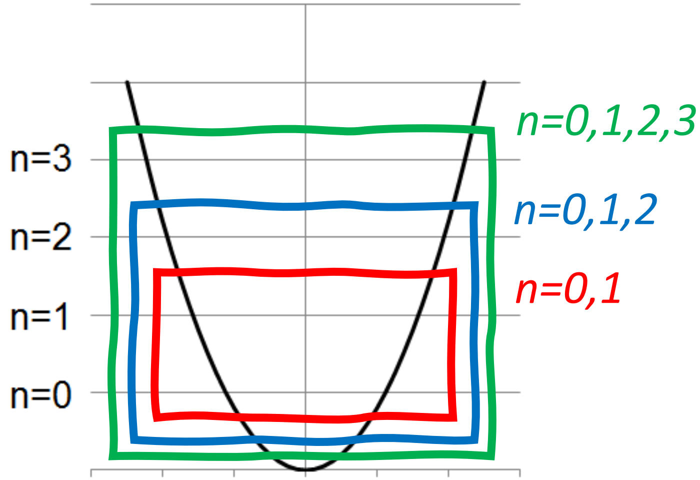
-2018 - Dimitrescu et al
First "real" calculations
For deuteron

-2019 - Lu et al
^3H, ^3He, α



Solving the deuteron problem on a quantum computer

Goal : map the deuteron problem into a simple Hamiltonian
Usable in current quantum devices



Schematic Hamiltonian in a truncated HO basis

$$H_N = \sum_{n,n'=0}^{N-1} \langle n'|(T+V)|n\rangle a_{n'}^{\dagger} a_n$$

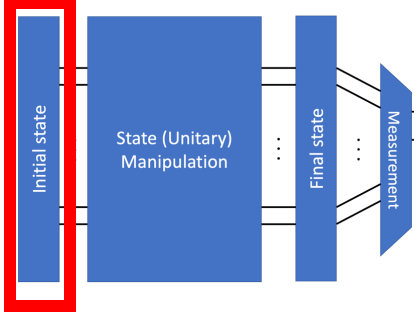
$$\langle n'|T|n\rangle = \frac{\hbar\omega}{2}\Big[(2n+3/2)\delta_n^{n'} - \sqrt{n(n+1/2)}\,\delta_n^{n'+1} - \sqrt{(n+1)(n+3/2)}\,\delta_n^{n'-1}\Big],$$

$$\langle n'|V|n\rangle = V_0 \delta_n^0 \delta_n^{n'}.$$

The parameters are adjusted to reproduce the ground state and excited deuteron energy

$$V_0 = -5.68658111\,\text{MeV}, \quad \hbar\omega = 7\ \text{MeV}$$

- This problem is a simple tri-diagonal problem easy to solve on a classical computer with a direct matrix diagonalization
- Still it addresses the problem of a system of fermions and applications on real QC but can be considered as a proof of principle



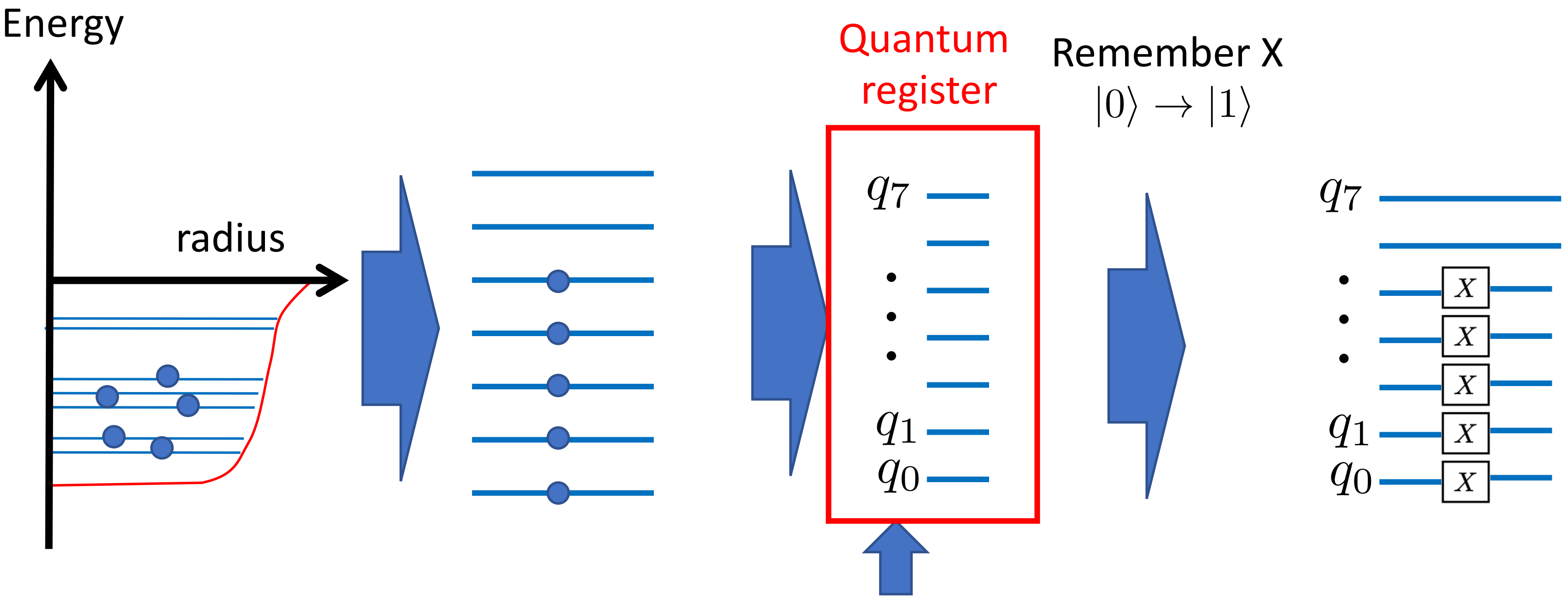
“Nuclear” Many-Body problem: initial state state preparation

The nuclear “toy” shell in configuration interaction

Direct mapping from Fock to qubit space:

Assign one qubit to each single-particle state

$|0\rangle$ State is occupied
$|1\rangle$ State is unoccupied



Remember X
$|0\rangle \rightarrow |1\rangle$

Initially all set to zero

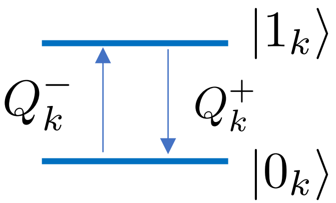
One-difficulty – the antisymmetric nature of fermions

The Jordan-Wigner transformation solution

Mapping the Fock space into Qubits

$$|-\rangle = |0\cdots0\rangle$$

For qubits



For fermions

$$a_k^\dagger|-\rangle = |0\cdots0\ 1_k\ 0\cdots0\rangle \quad\Longleftrightarrow\quad Q_k^+ = \frac{1}{2}(X_k - iY_k) = \begin{bmatrix} 0 & 0 \\ 1 & 0 \end{bmatrix}$$

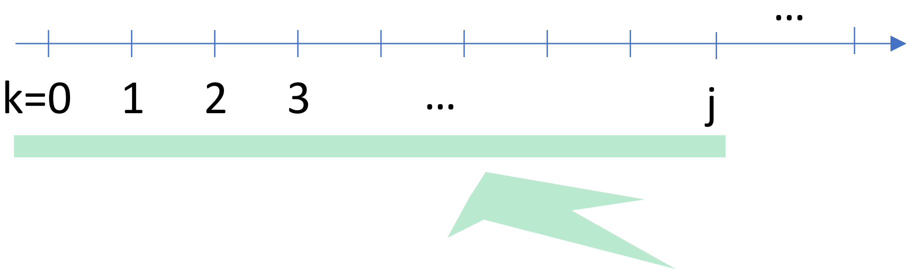
$$\{a_k, a_k^\dagger\} = 1 \qquad\qquad \{Q_k^-, Q_k^+\} = 1$$

Problem $\{a_k, a_l^\dagger\} = 0$ **while** $[Q_k^-, Q_l^+] = 0$

One possible solution (Jordan-Wigner transformation -1928)

① Order the index like in a lattice

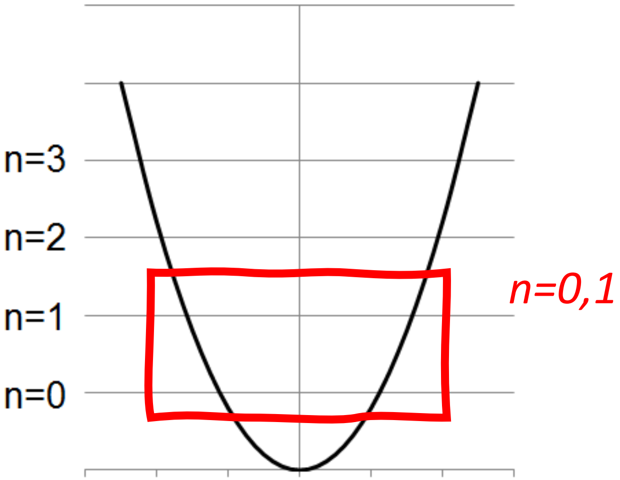
② Define new mapping



$$a_k^\dagger \to \prod_{k<j}(-\sigma_z^j)\sigma_j^+$$

The JWT method

The JWT method is systematic, how this works:



$$H_N = \sum_{n,n'=0}^{N-1} \langle n'|(T+V)|n\rangle a^{\dagger}_{n'} a_n$$

$$H_1 = \varepsilon_0 a_0^{\dagger} a_0 + \varepsilon_1 a_1^{\dagger} a_1 + v_{01}(a_1^{\dagger} a_0 + a_0^{\dagger} a_1)$$

JWT mapping

$$a_0^{\dagger} \to Q_0^{+}$$
$$a_1^{\dagger} \to -Z_0 Q_1^{+}$$

$$(Q_k^{+} Z_k = -Q_k^{+})$$

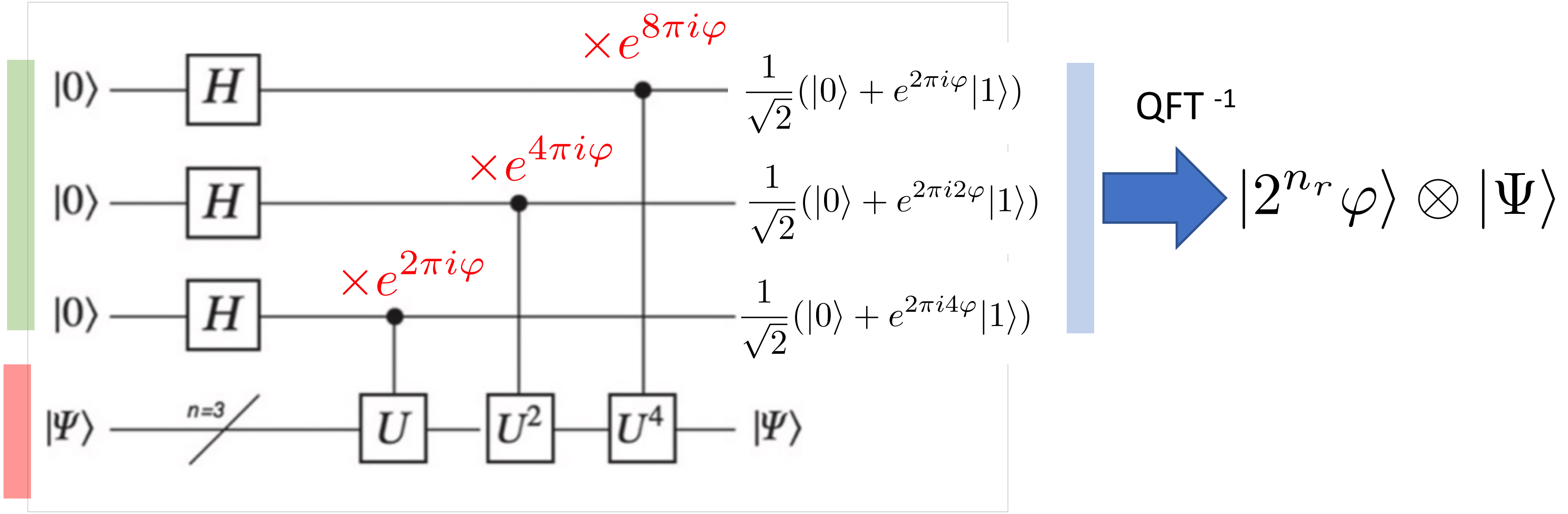
$$H_1 = \frac{\varepsilon_0}{2}(I - Z_0) + \varepsilon_1 (I - Z_1) + v_{01}(X_0 X_1 + Y_0 Y_1)$$

The Quantum-killer application for eigenvalue problem

the quantum-Phase estimation (QPE) algorithm

Assume a unitary operator U

Assume an eigenstate $|\Psi\rangle$ Such that $U|\Psi\rangle = e^{2\pi i\varphi}|\Psi\rangle$ $0 \leq \varphi < 1$



nr Register qubits

Working qubits

The quantum-Phase estimation (QPE) algorithm

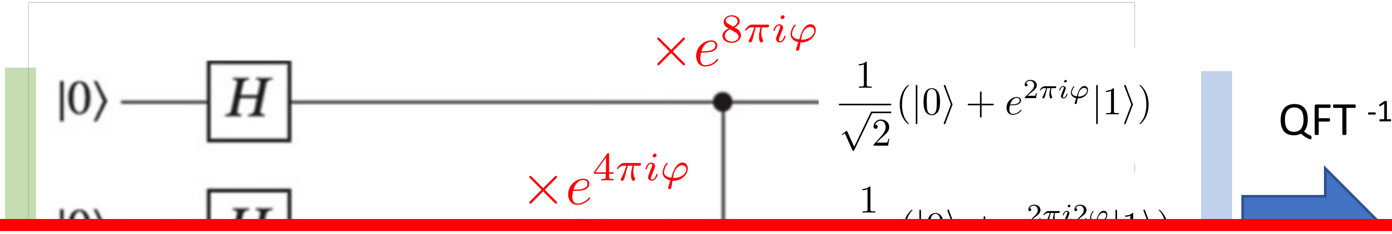
For known eigenvalue problems

Assume a unitary operator U

Assume an eigenstate $|\Psi\rangle$ Such that $U|\Psi\rangle = e^{2\pi i\varphi}|\Psi\rangle$ $0 \leq \varphi < 1$

nr Register

q



$|2^{n_r}\varphi\rangle \otimes |\Psi\rangle$

V

c

What is the meaning of $|2^{n_r}\varphi\rangle$?

$\varphi < 1$ Can be written as a binary fraction

$$\varphi = 0.j_1 \cdots j_n = \frac{j_1}{2} + \frac{j_2}{2^2} + \cdots + \frac{j_n}{2^n} \cdots$$

Example $0.1011 \rightarrow 1 \times 0.5 + 0 \times 0.25 + 1 \times 0.125 + 1 \times 0.0625$
$\rightarrow 0.6875$

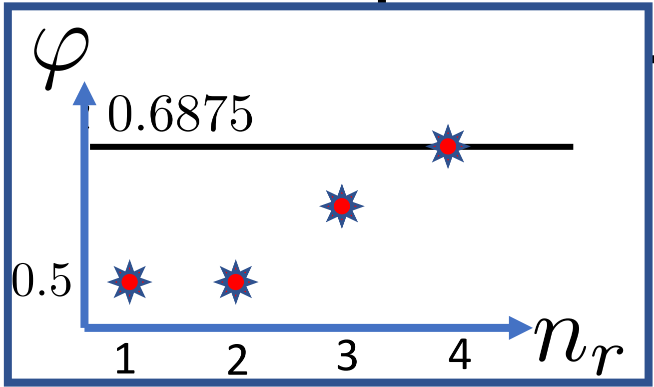
2^{-1}	$=$	0.5
2^{-2}	$=$	0.25
2^{-3}	$=$	0.125
2^{-4}	$=$	0.0625
2^{-5}	$=$	0.03125
2^{-6}	$=$	0.015625
2^{-7}	$=$	0.0078125

$n_r = 1$ ➡ $|2^{n_r}\varphi\rangle = |1\rangle$ ➡ $\varphi \simeq 0.5$

$n_r = 2$ ➡ $|2^{n_r}\varphi\rangle = |10\rangle$ ➡ $\varphi \simeq 0.5$

$n_r = 3$ ➡ $|2^{n_r}\varphi\rangle = |101\rangle$ ➡ $\varphi \simeq 0.625$

$n_r = 4$ ➡ $|2^{n_r}\varphi\rangle = |1011\rangle$ ➡ $\varphi \simeq 0.6875$



The quantum-Phase estimation (QPE) algorithm

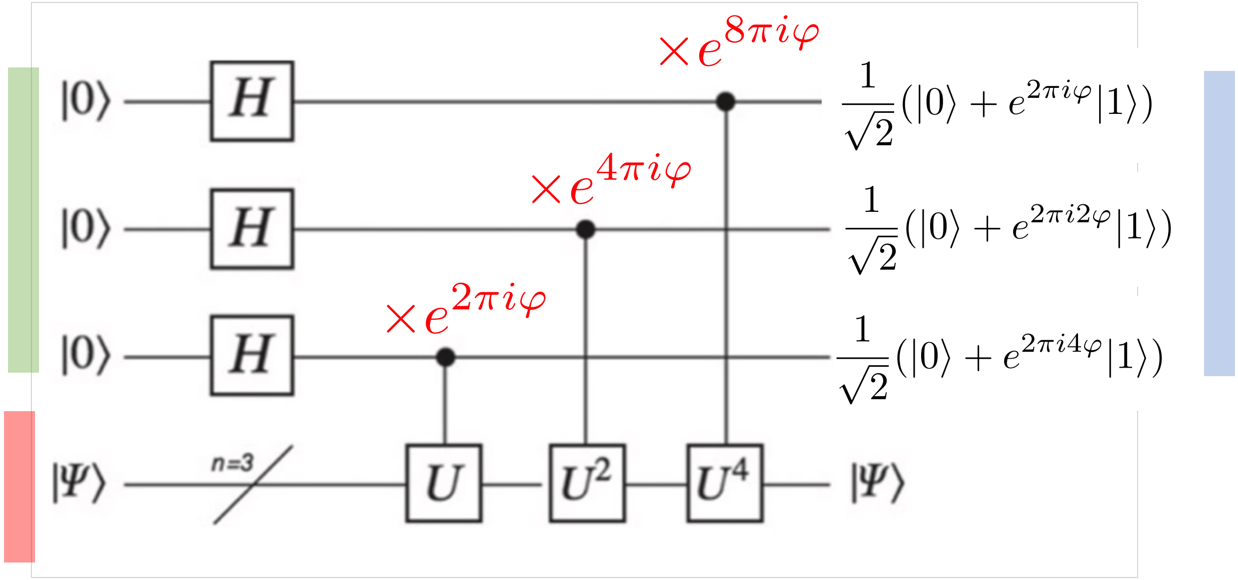
For known eigenvalue problems

Assume a unitary operator U

Assume an eigenstate $|\Psi\rangle$ Such that $U|\Psi\rangle = e^{2\pi i\varphi}|\Psi\rangle$ $\quad 0 \leq \varphi < 1$

nr Register qubits

Working qubits



QFT $^{-1}$

$$\Rightarrow |2^{n_r}\varphi\rangle \otimes |\Psi\rangle$$

General Case

QPE

$$|\Psi\rangle = \sum_k \alpha_k|\phi_k\rangle \Rightarrow \sum_k \alpha_k|\theta_k 2^{n_r}\rangle \otimes |\phi_k\rangle$$

🟩 register 🟥 eigenstate

For an Hamiltonian problem

One can simply use

$$U = e^{i\alpha(H-E_0)}$$

and get any eigenvalues with arbitrary precisions

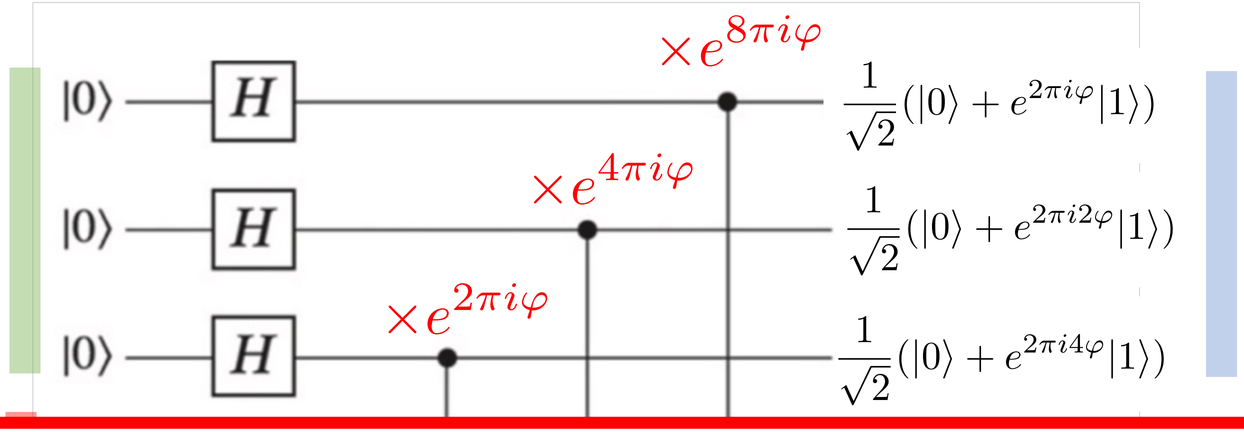
The quantum-Phase estimation (QPE) algorithm

For known eigenvalue problems

Assume a unitary operator U

Assume an eigenstate $|\Psi\rangle$ Such that $U|\Psi\rangle = e^{2\pi i\varphi}|\Psi\rangle$ $0 \leq \varphi < 1$

nr Register qubits



QFT $^{-1}$

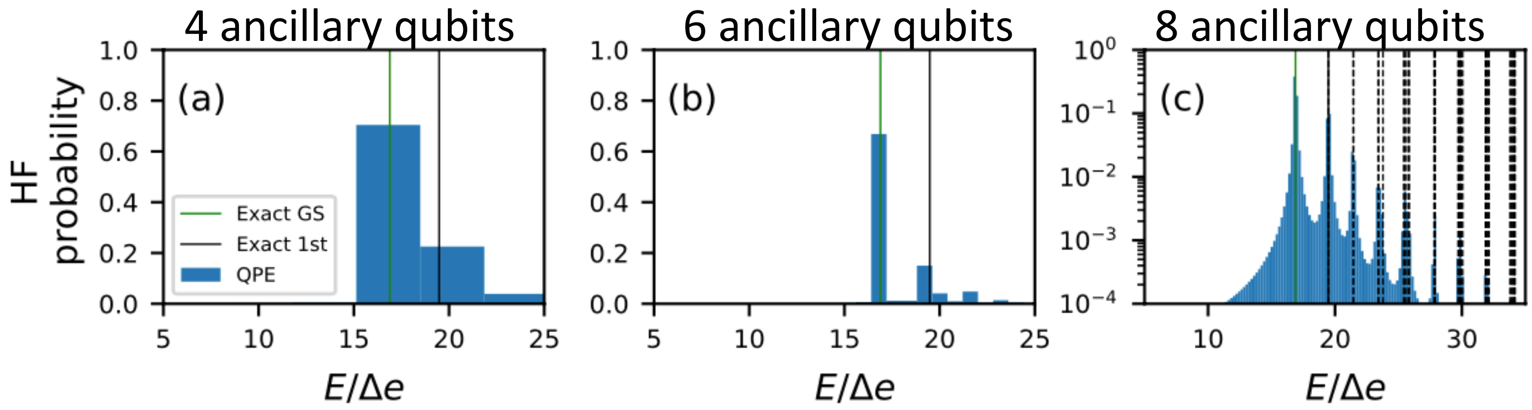
$$|2^{n_r}\varphi\rangle \otimes |\Psi\rangle$$

Wo

qu

$|\Psi\rangle$ =

And this works



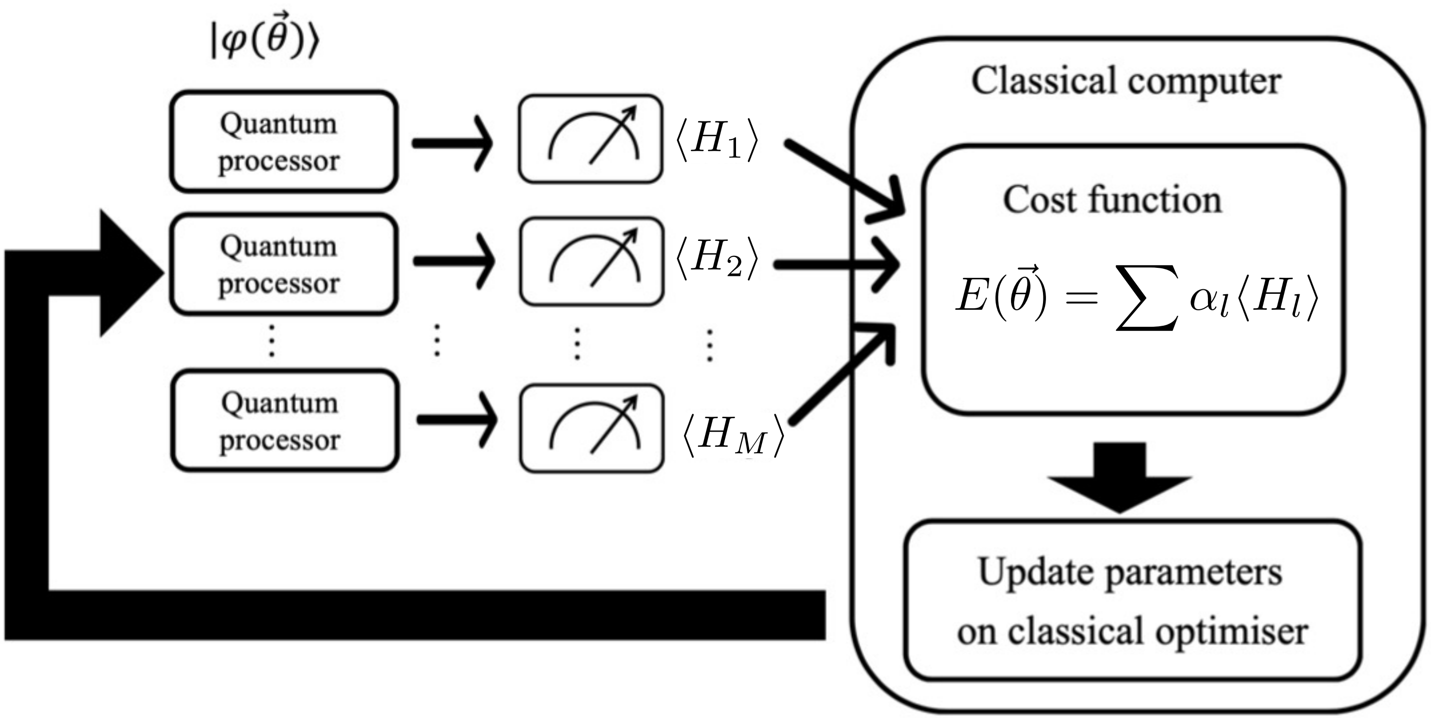
Unfortunately this is beyond the NISQ period

trary

The Variational Quantum Eigensolver (VQE) method

Main Idea of VQE

- Wave-functions can be better controlled by parametrizing them
- One can reduce the QC effort by computing expectation values of simpler operators $H = \sum \alpha_l H_l$
- ☐ The optimization task is left to a classical computer

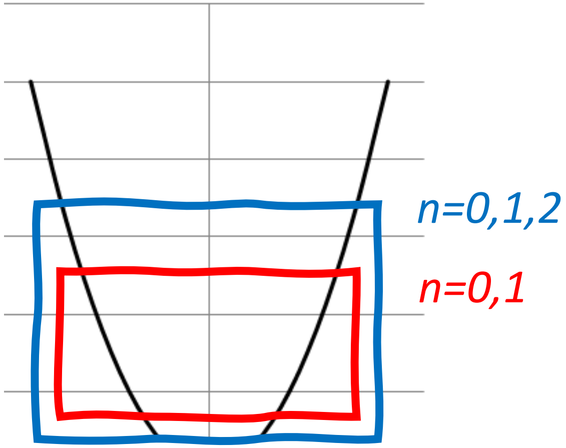


Adapted from S. Endo et al, arxiv:2011.01382
(see also K. Bharti et al., Rev. Mod. Phys. 94, 015004 (2022).

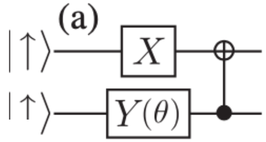
Energy minimization

Wave-function as parametric ansatz $|\Psi(\theta)\rangle = U(\theta)\underbrace{X_0 \cdots X_k|0\cdots 0\rangle}$

Hartree-Fock state with k occupied levels

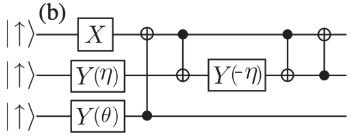


n=0,1 $$U(\theta) \equiv e^{\theta(a_0^\dagger a_1 - a_1^\dagger a_0)} = e^{i(\theta/2)(X_0Y_1 - X_1Y_0)}$$



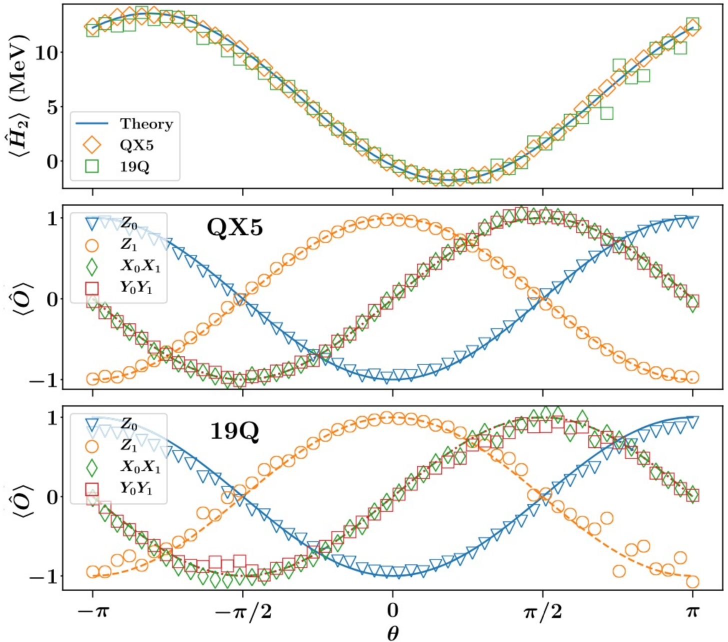
n=0,1,2 $$U(\eta,\theta) \equiv e^{\eta(a_0^\dagger a_1 - a_1^\dagger a_0) + \theta(a_0^\dagger a_2 - a_2^\dagger a_0)}$$

$$\approx e^{i(\eta/2)(X_0Y_1 - X_1Y_0)} e^{i(\theta/2)(X_0Z_1Y_2 - X_2Z_1Y_0)}$$



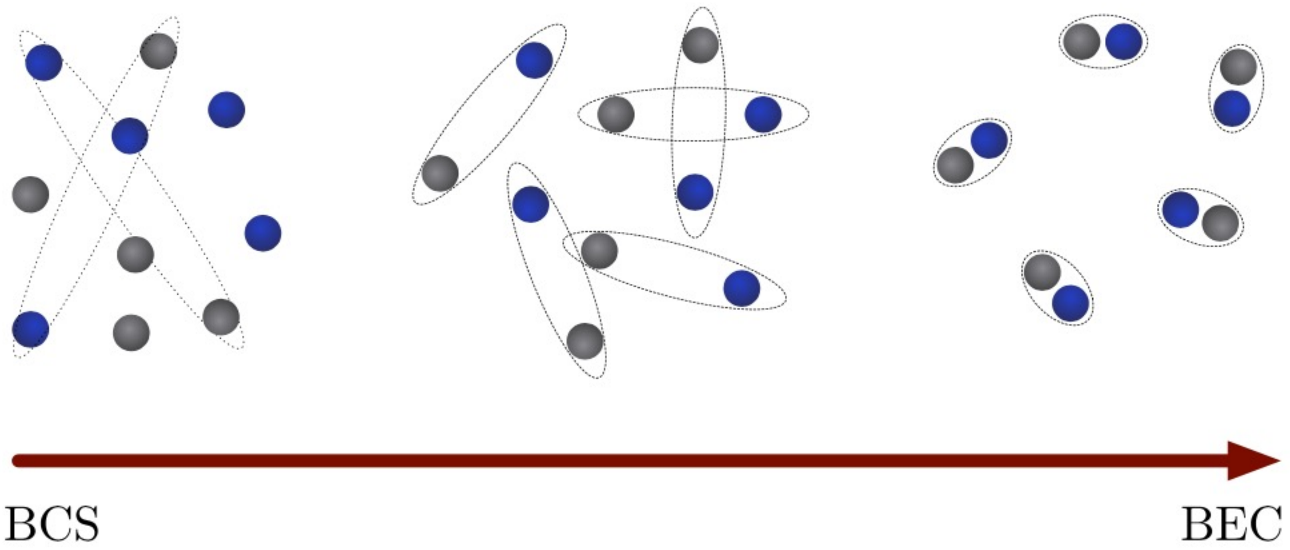
Note that this is a Unitary Coupled Cluster (UCC)

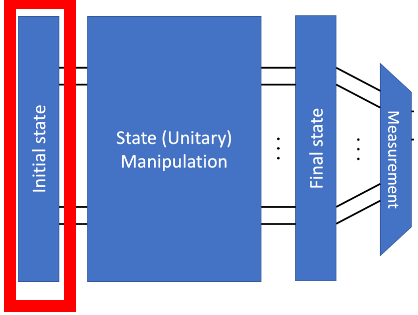
$$H_1 = \frac{\varepsilon_0}{2}(I - Z_0) + \varepsilon_1(I - Z_1) + v_{01}(X_0X_1 + Y_0Y_1)$$



Dumitrescu et al, Phys. Rev. Lett. 120 (2018)

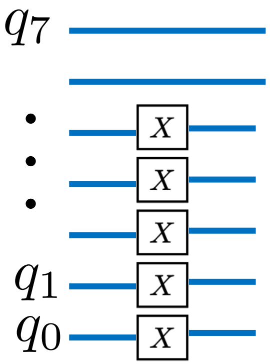
Third Example:
Treatment of superconductivity



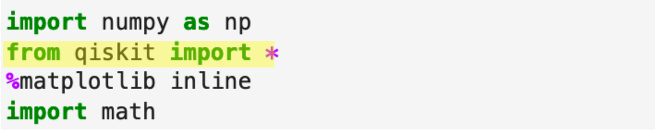


"Nuclear" Many-Body problem: initial state preparation

Starting point : Slater determinants

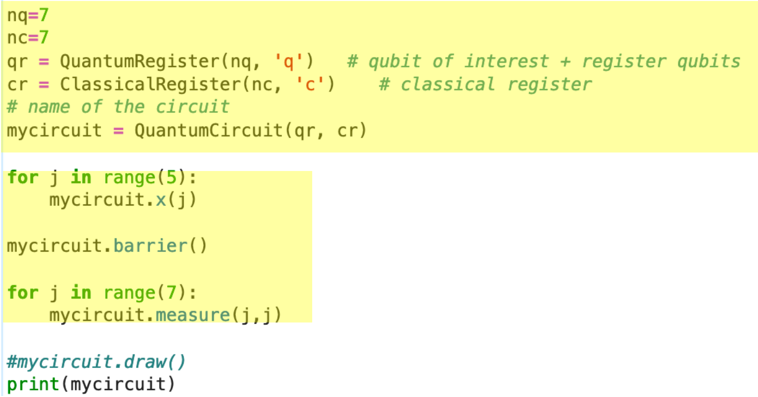


Demystifying QC Illustration with qiskit-IBM

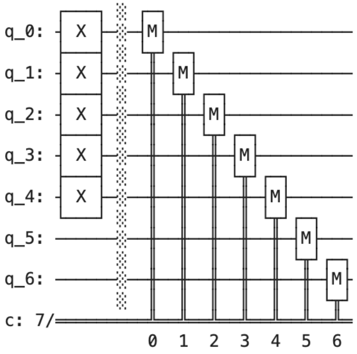


```
import numpy as np
from qiskit import *
%matplotlib inline
import math
```

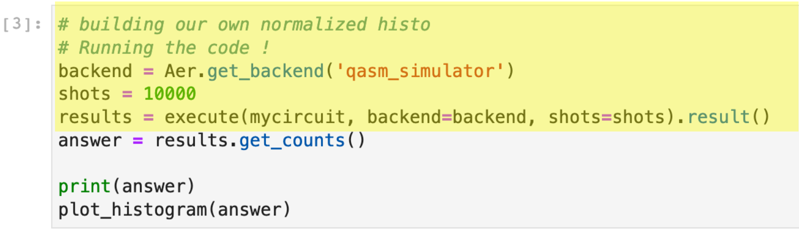
Creation of the circuit



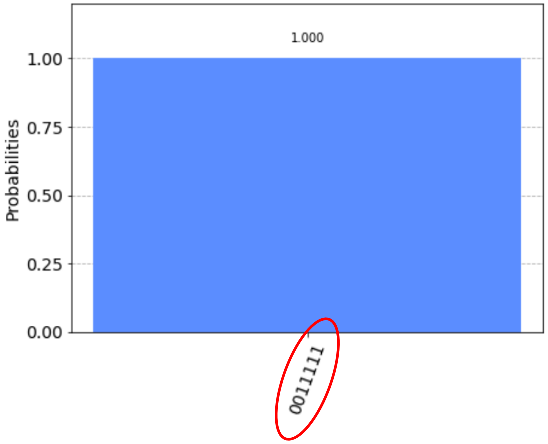
```
nq=7
nc=7
qr = QuantumRegister(nq, 'q')   # qubit of interest + register qubits
cr = ClassicalRegister(nc, 'c')    # classical register
# name of the circuit
mycircuit = QuantumCircuit(qr, cr)

for j in range(5):
    mycircuit.x(j)

mycircuit.barrier()

for j in range(7):
    mycircuit.measure(j,j)

#mycircuit.draw()
print(mycircuit)
```



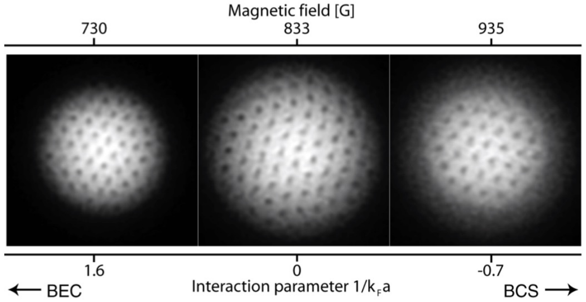
Running the circuit



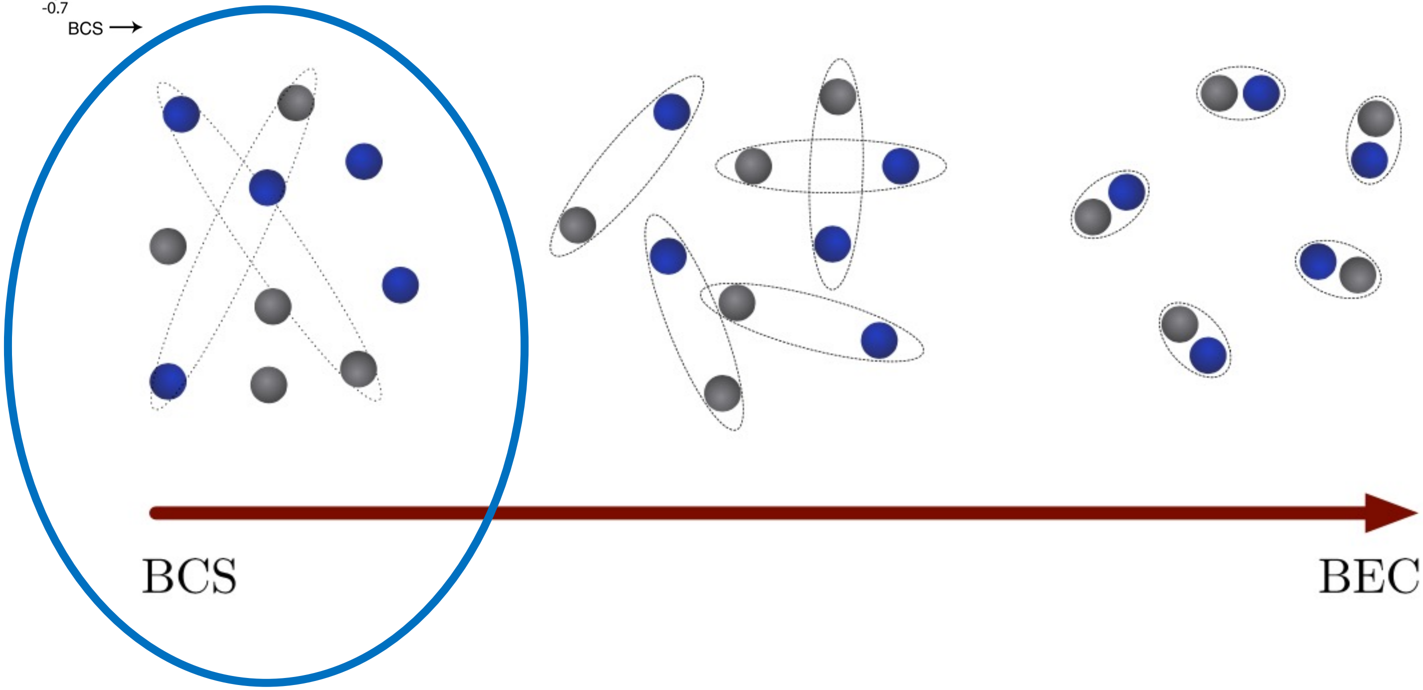
```
[3]: # building our own normalized histo
     # Running the code !
     backend = Aer.get_backend('qasm_simulator')
     shots = 10000
     results = execute(mycircuit, backend=backend, shots=shots).result()
     answer = results.get_counts()

     print(answer)
     plot_histogram(answer)
```



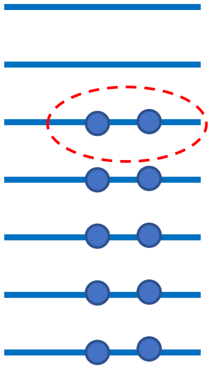
Preparing superfluid systems



Cooper pairs and superfluidity are rather universal phenomena: (condensed matter, Atomic physics, Nuclear physics, …)



This problem is an archetype of spontaneous symmetry breaking.
A "easy" way to describe it is to break the particle number symmetry, i.e. consider wave-function that mixes different particle number



Example

$$|\Phi_0\rangle = \Pi_i (u_i + v_i a_i^\dagger a_{\bar{i}}^\dagger)|-\rangle$$

➡ Mixes states with 0, 2, 4, … particles

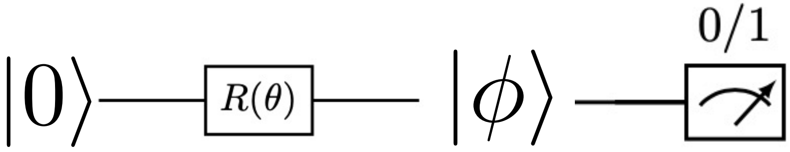
We say that a symmetry (particle number) is broken

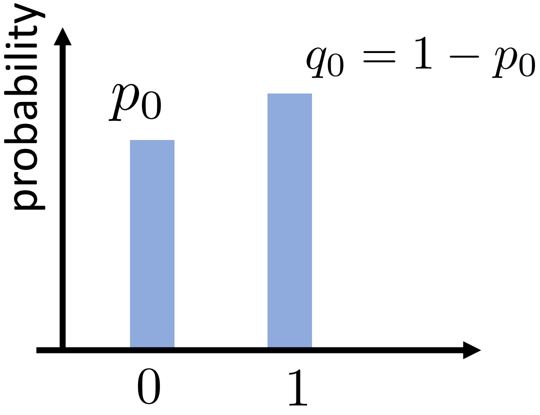
Preparing state that mixes particle number

I assign a qubit to each qubit

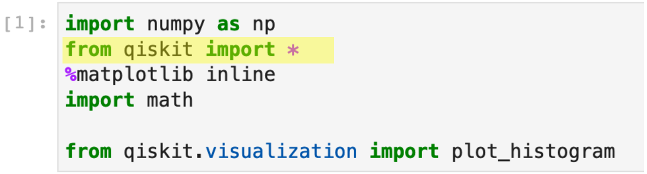
$$|\phi\rangle = \sqrt{p_0}|0\rangle + \sqrt{1-p_0}|1\rangle$$

Measuring the qubit gives the probability



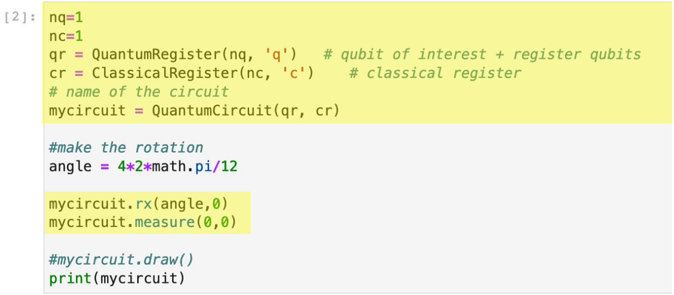


On the Qiskit toolkit

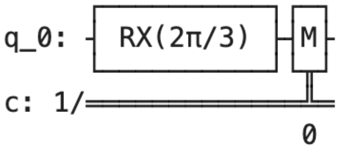


```
[1]: import numpy as np
     from qiskit import *
     %matplotlib inline
     import math

     from qiskit.visualization import plot_histogram
```

Creation of the circuit



```
[2]: nq=1
     nc=1
     qr = QuantumRegister(nq, 'q')   # qubit of interest + register qubits
     cr = ClassicalRegister(nc, 'c')    # classical register
     # name of the circuit
     mycircuit = QuantumCircuit(qr, cr)

     #make the rotation
     angle = 4*2*math.pi/12

     mycircuit.rx(angle,0)
     mycircuit.measure(0,0)

     #mycircuit.draw()
     print(mycircuit)
```

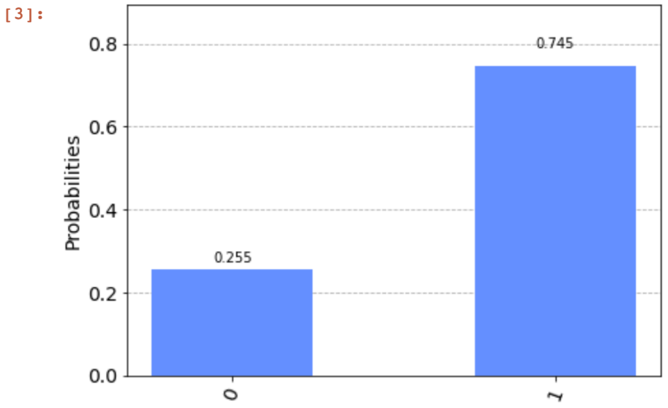


```
q_0: ┤ RX(2π/3) ├┤M├
                   ║
c: 1/══════════════╩═
                   0
```

Running the circuit

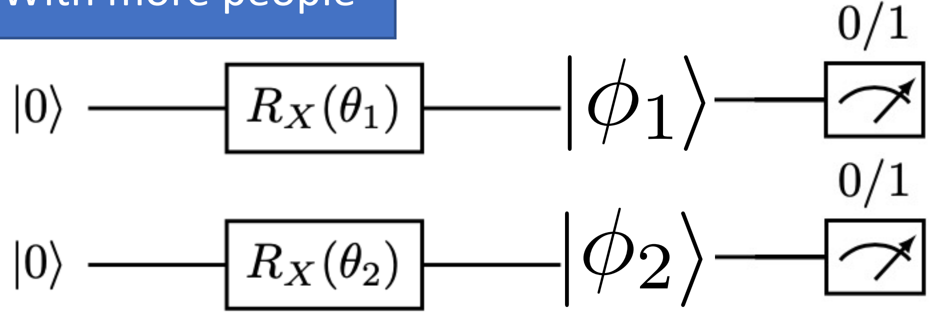
```
[3]: # building our own normalized histo
     # Running the code !
     backend = Aer.get_backend('qasm_simulator')
     shots = 10000
     results = execute(mycircuit, backend=backend, shots=shots).result()
     answer = results.get_counts()

     print(answer)
     plot_histogram(answer)
```

```
{'0': 2546, '1': 7454}
```



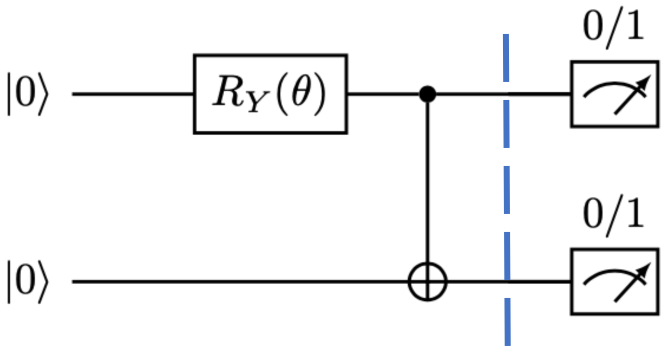
Preparing state that mixes particle number

With more people



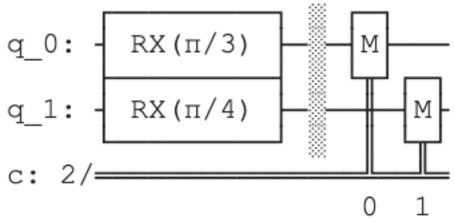
$$|\phi_i\rangle = \alpha_i|0\rangle + \beta_i|1\rangle$$

Qubit can be entangled



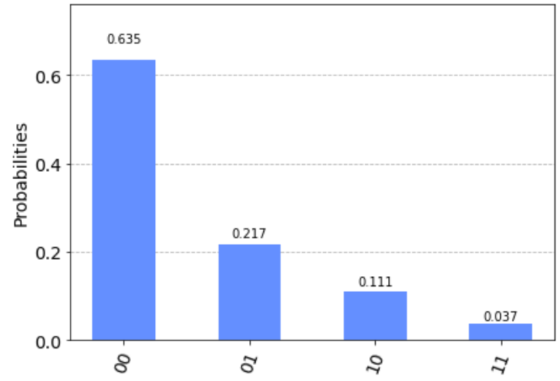
$$|\phi_{12}\rangle = \alpha|00\rangle + \beta_i|11\rangle$$

Here I created a Bell state

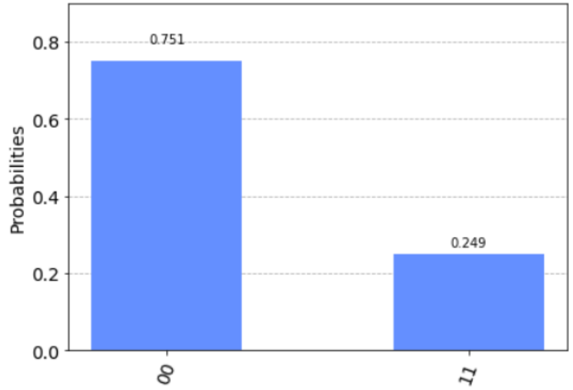


```
{'00': 6351, '01': 2169, '10': 1112, '11': 368}
```

[15]:



```
{'00': 7513, '11': 2487}
```



Application to the N-body pairing problem

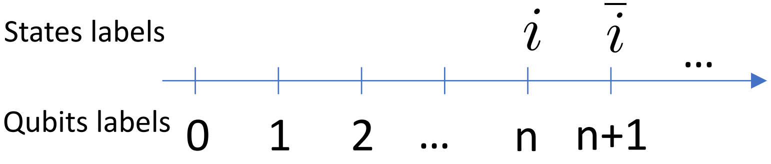
Hamiltonian and initial state

Pairing Hamiltonian

$$H_{\mathrm{P}} = \sum_{i>0} \varepsilon_i (a_i^\dagger a_i + a_{\bar{i}}^\dagger a_{\bar{i}}) - g \sum_{i,j>0} a_i^\dagger a_{\bar{i}}^\dagger a_{\bar{j}} a_j$$

Jordan-Wigner trans: $\frac{1}{2}(I_i - Z_i)$

State ordering is important !


States labels i $\bar{i}$ ...

Qubits labels 0 1 2 ... n n+1

$$a_i^\dagger a_{\bar{i}}^\dagger \Rightarrow Q_n^+ Q_{n+1}^+$$

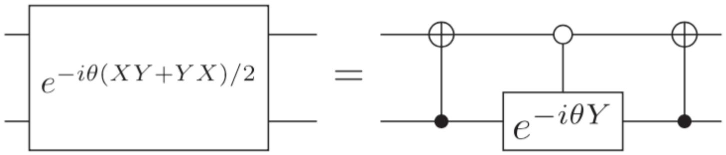
Initial (symmetry breaking) state preparation

$$|\Psi\rangle = \exp\left\{ -\sum_{i>0} \varphi_i \left(a_i^\dagger a_{\bar{i}}^\dagger - a_{\bar{i}} a_i \right) \right\} |0\rangle \xrightarrow{\varphi_i = \varphi} |\Psi\rangle = \prod_{n>0} e^{i\varphi(X_n Y_{n+1} + Y_n X_{n+1})/2} |-\rangle$$

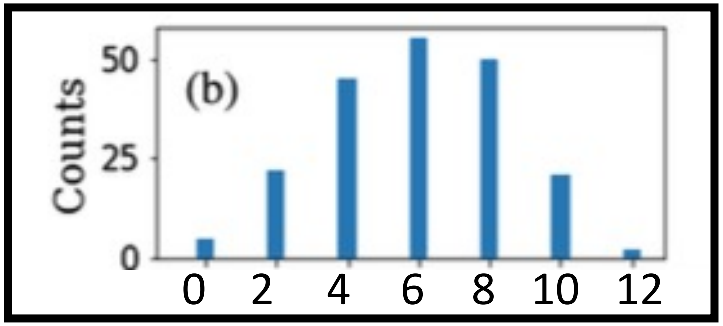
Simplified circuit (generalized Bell state)

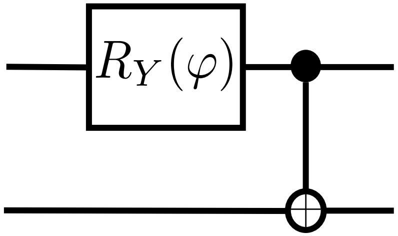
$$|\Psi\rangle = \prod_n \left[\cos\left(\frac{\varphi}{2}\right) I_n \otimes I_{n+1} + \sin\left(\frac{\varphi}{2}\right) Q_n^+ Q_{n+1}^+ \right] |-\rangle$$

Equivalent universal gate on pairs

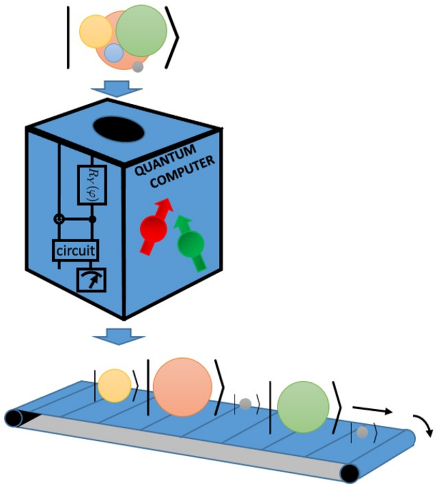


Zhang Jiang et al,
Phys. Rev. Applied 9, 044036 (2018).

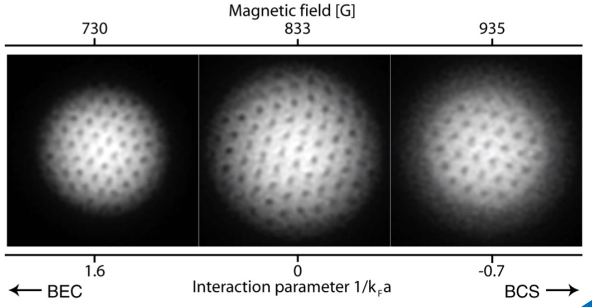




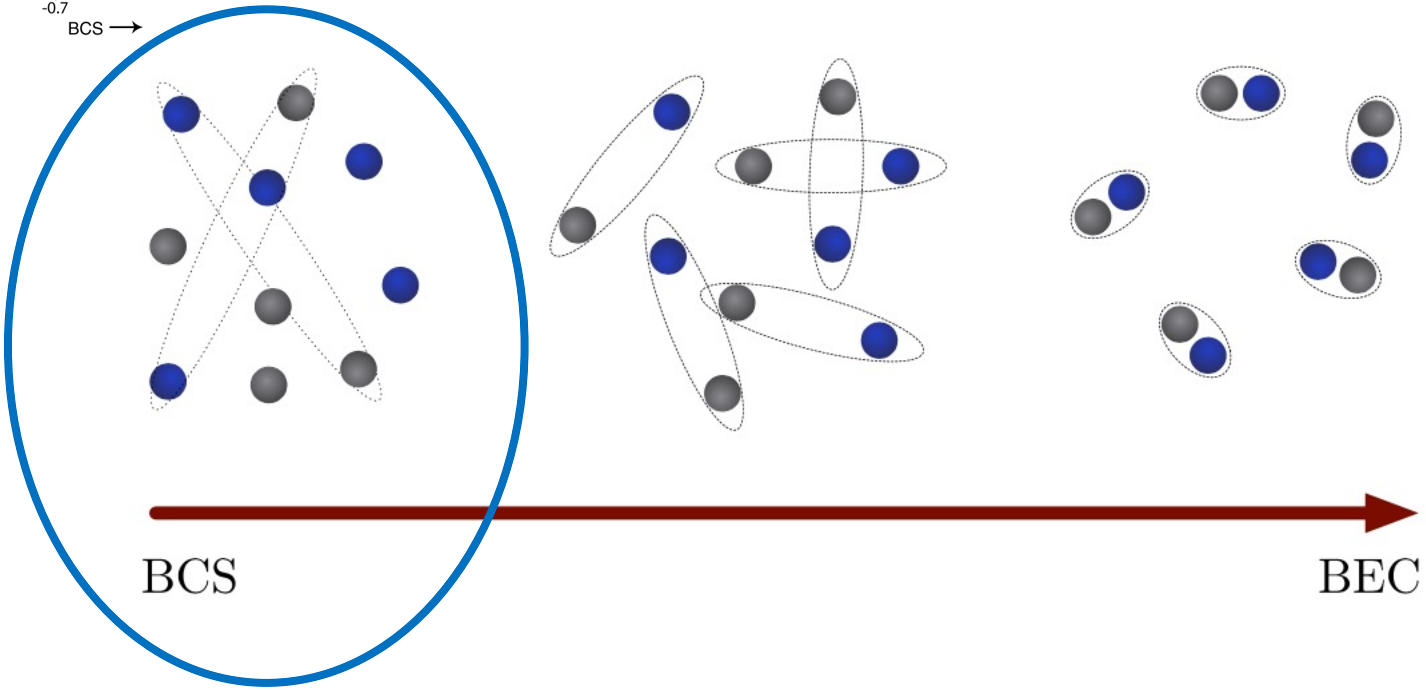
Fourth Example: restoration of broken symmetries



But what is the connection with interacting systems ???



Cooper pairs and superfluidity are rather universal phenomena: (condensed matter, Atomic physics, Nuclear physics, …)



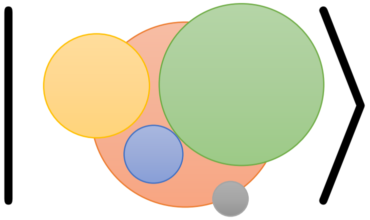
This problem is an archetype of spontaneous symmetry breaking.
A “easy” way to describe it is to break the particle number symmetry, i.e. consider wave-function that mixes different particle number

Example

$$|\Phi_0\rangle = \Pi_i(u_i + v_i a_i^\dagger a_{\bar{i}}^\dagger)|-\rangle$$

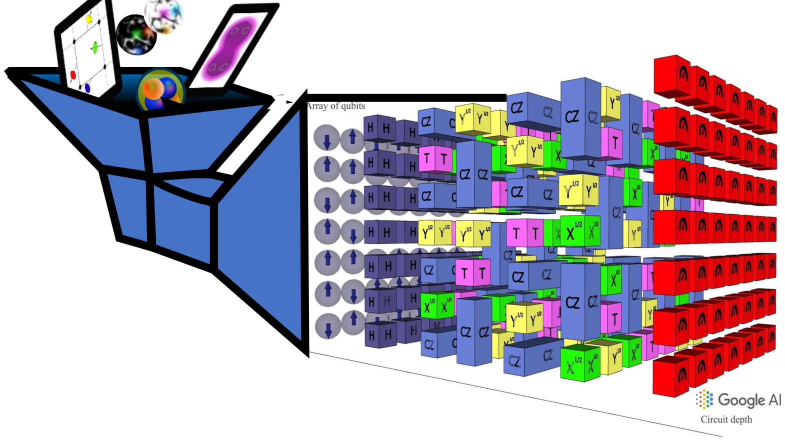
→ Mixes states with 0, 2, 4, … particles

We say that a symmetry (particle number) is broken

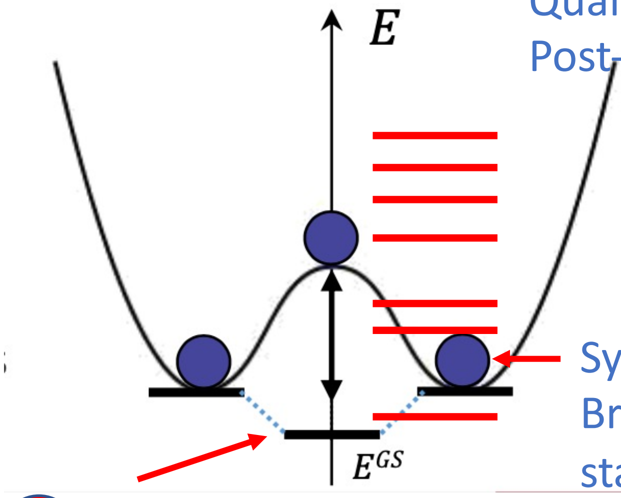


But ultimately number of Particle should be restored !

What we have started to do

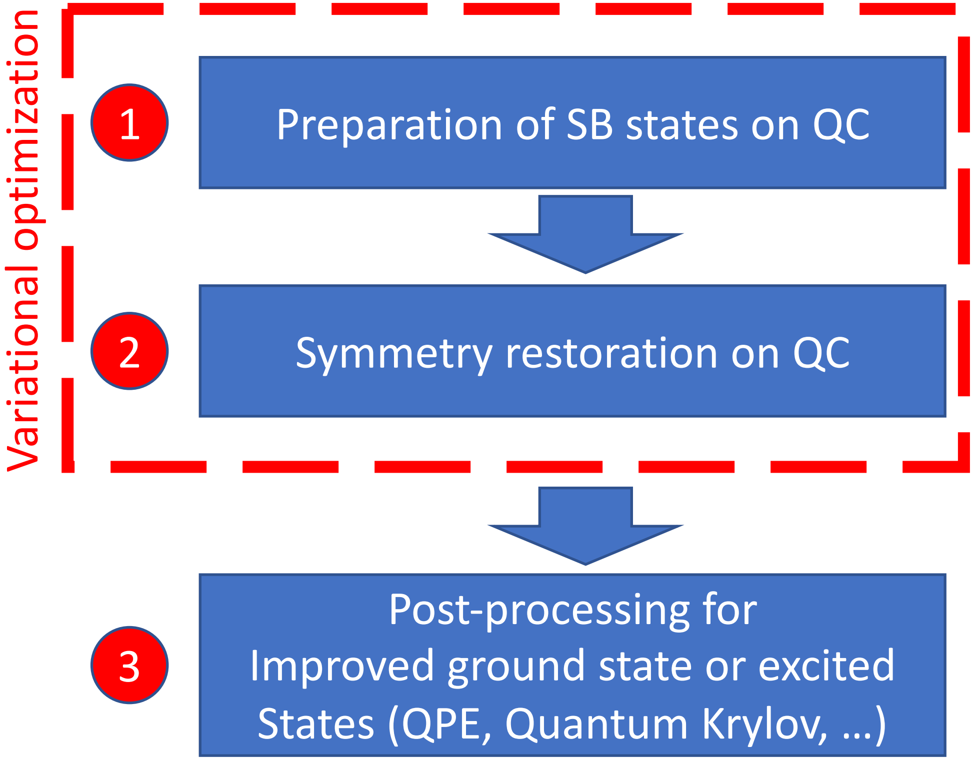


③ Further Quantum or hybrid Quantum-Classical Post-processing



① Symmetry Breaking (SB) state

② Symmetry Restored (SR) state (multi-reference)



① Preparation of SB states on QC

② Symmetry restoration on QC

③ Post-processing for Improved ground state or excited States (QPE, Quantum Krylov, …)

Illustration with small superconductors

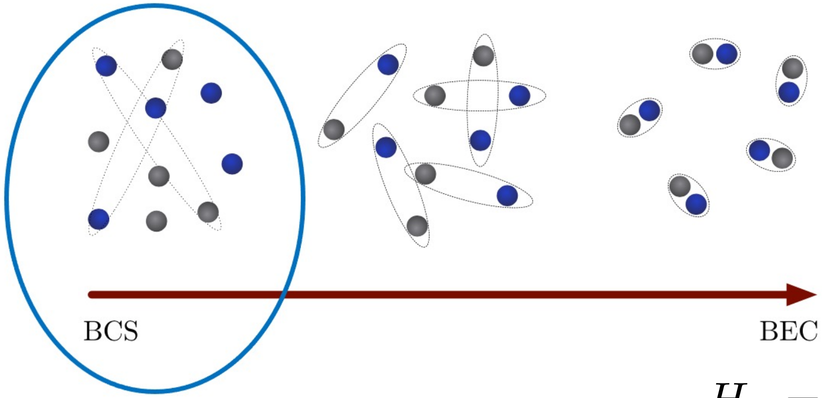
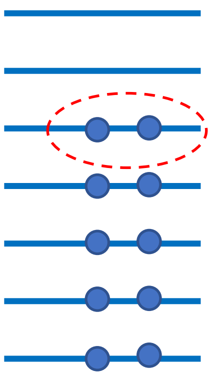


Illustration with the Richardson Hamiltonian

$$H_{\rm P} = \sum_{i>0} \varepsilon_i (a_i^\dagger a_i + a_{\bar{i}}^\dagger a_{\bar{i}}) - g \sum_{i,j>0} a_i^\dagger a_{\bar{i}}^\dagger a_{\bar{j}} a_j$$



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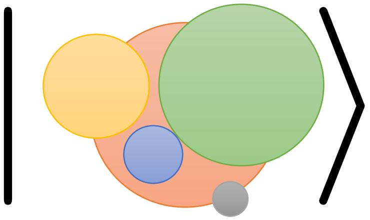
An “easy” way to describe it is to break the particle number symmetry, i.e. consider wave-function that mixes different particle number

Example

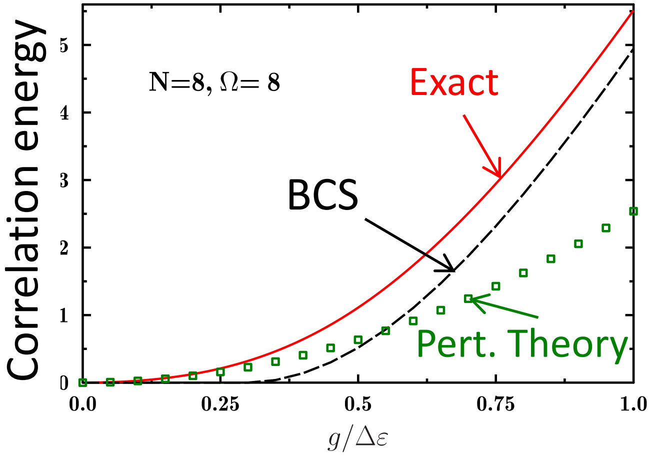
$$|\Phi_0\rangle = \Pi_i (u_i + v_i a_i^\dagger a_{\bar{i}}^\dagger)|-\rangle$$

⇨ Mixes states with 0, 2, 4, … particles

The particle number - U(1) symmetry)
is broken



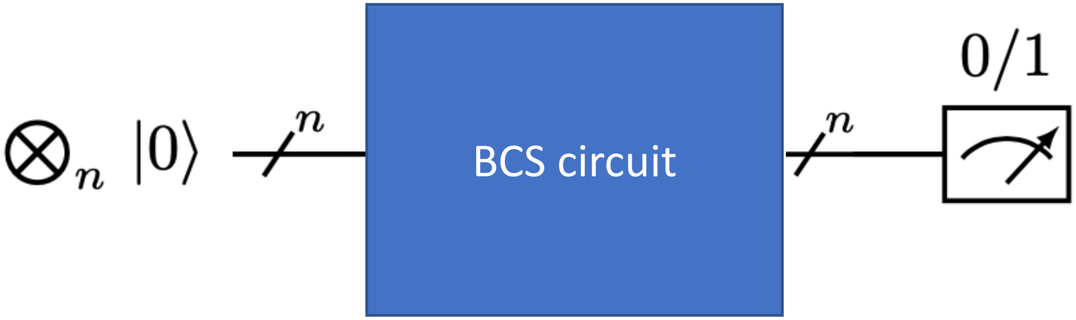
But ultimately number of
Particle should be restored !



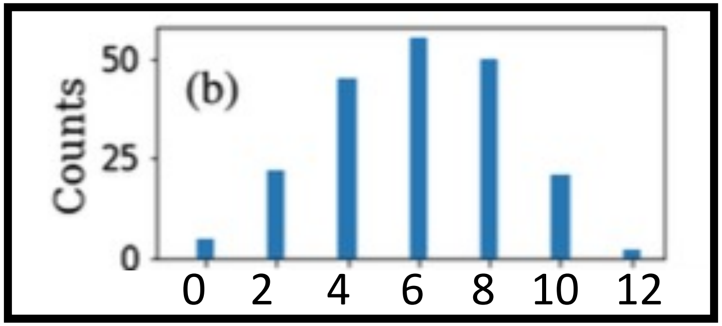
Restoration of particle number symmetry

The counting statistics problem

Direct estimate of Counting statistics

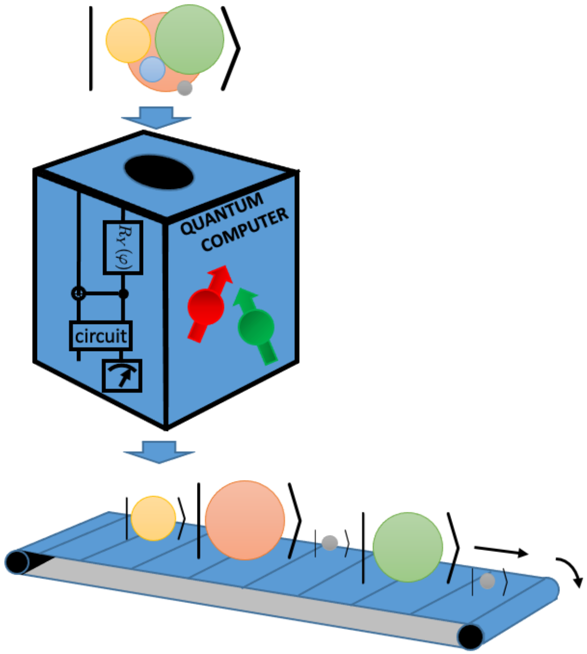


Example of mixing for 12 qubits (with qiskit)



Projection on particle number

$$|\Psi\rangle = \sum_N c_N |N\rangle \Rightarrow |N\rangle$$



For 2 qubits

$$|\Psi\rangle = \alpha|00\rangle + \beta|01\rangle + \gamma|10\rangle + \delta|11\rangle$$

$$|N=0\rangle \qquad \propto |N=1\rangle \qquad |N=2\rangle$$

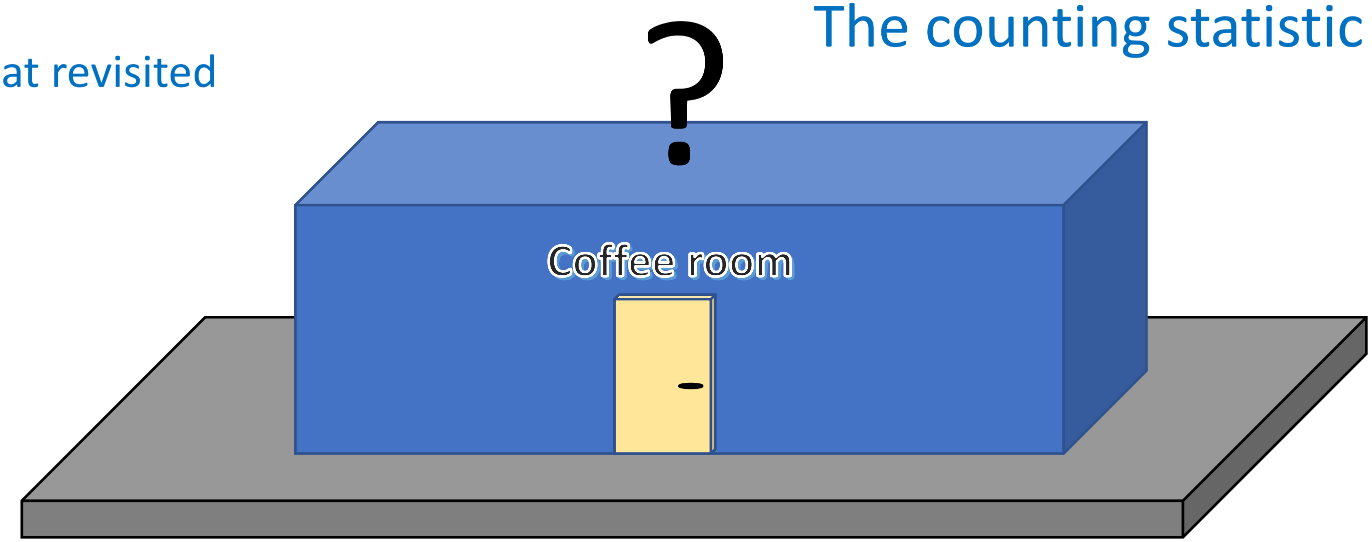
➡ A possible way to perform the projection is to use
The Quantum-Phase-Estimation method with *N* itself

D. Lacroix, "*Symmetry-Assisted Preparation of Entangled Many-Body States on a Quantum Computer*", PRL 125, 230502 (2020).

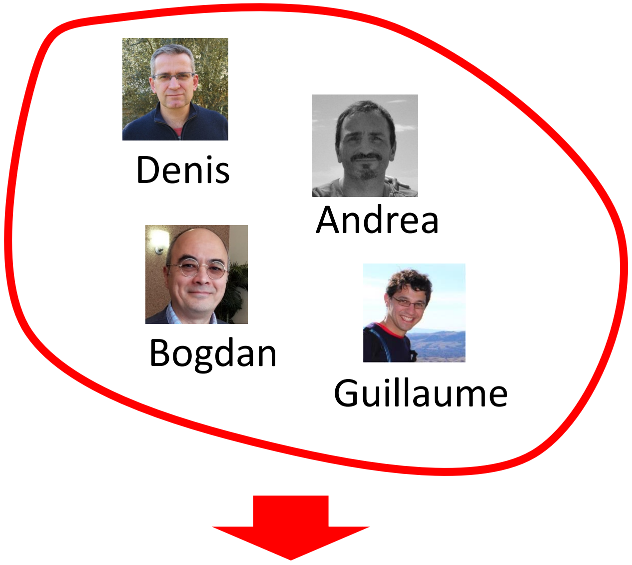
Broken symmetry/restoration

The counting statistic problem

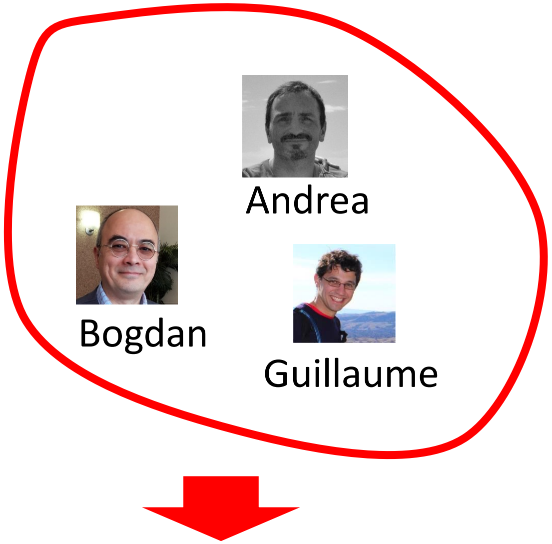
The schroedinger cat revisited



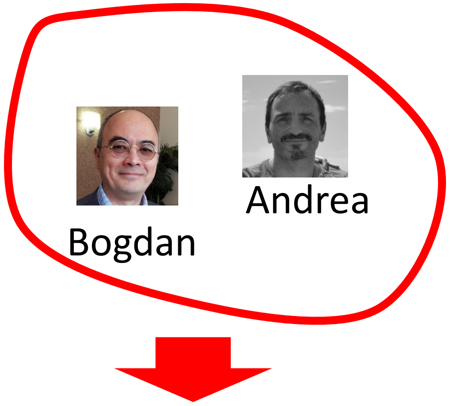
A more specific example



4 persons



3 persons



2 persons

With many events we can do
Probabilities, statistical analysis

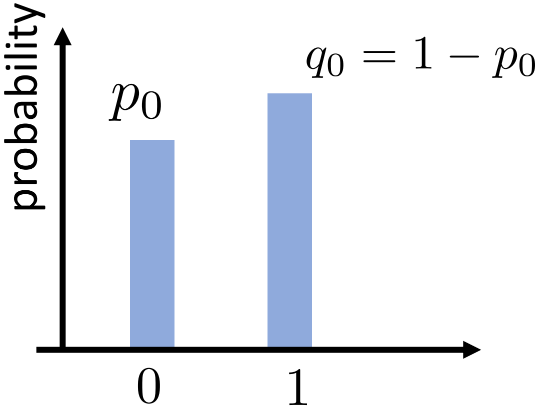
The counting statistic problem

In quantum systems

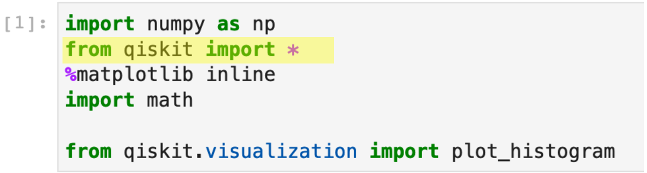
I assign a qubit to each person

$$\left|\quad\right\rangle = \sqrt{p_0}|0\rangle + \sqrt{1-p_0}|1\rangle$$

Measuring the qubit gives the probability

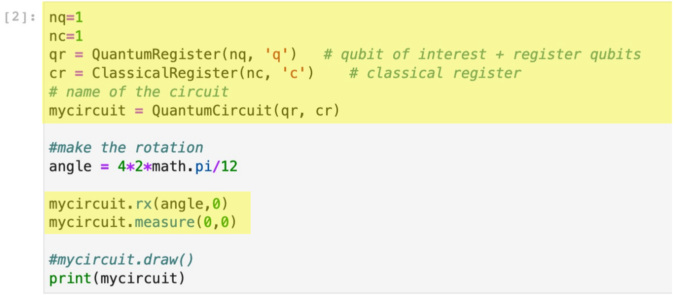


Demystifying QC Illustration with qiskit

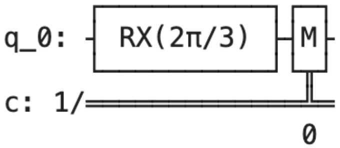


```
[1]: import numpy as np
     from qiskit import *
     %matplotlib inline
     import math

     from qiskit.visualization import plot_histogram
```

Creation of the circuit

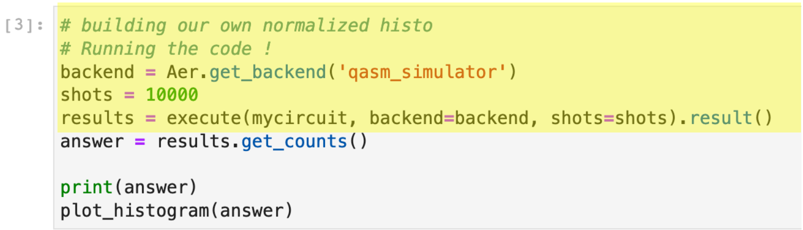


```
[2]: nq=1
     nc=1
     qr = QuantumRegister(nq, 'q')   # qubit of interest + register qubits
     cr = ClassicalRegister(nc, 'c')    # classical register
     # name of the circuit
     mycircuit = QuantumCircuit(qr, cr)

     #make the rotation
     angle = 4*2*math.pi/12

     mycircuit.rx(angle,0)
     mycircuit.measure(0,0)

     #mycircuit.draw()
     print(mycircuit)
```



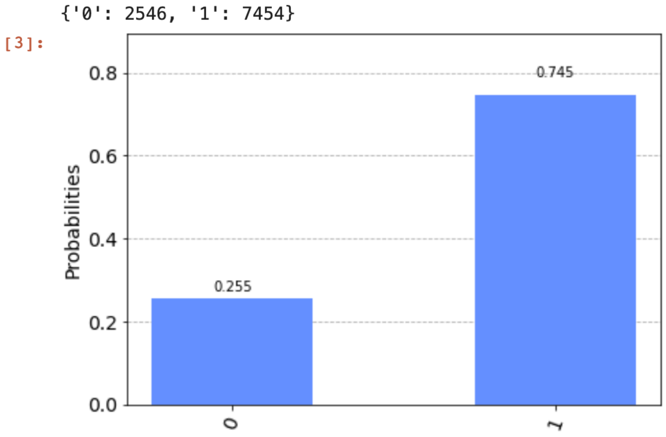
```
     ┌───────────┐┌─┐
q_0: ┤ RX(2π/3) ├┤M├
     └───────────┘└╥┘
c: 1/══════════════╩═
                   0
```

Running the circuit



```
[3]: # building our own normalized histo
     # Running the code !
     backend = Aer.get_backend('qasm_simulator')
     shots = 10000
     results = execute(mycircuit, backend=backend, shots=shots).result()
     answer = results.get_counts()

     print(answer)
     plot_histogram(answer)
```

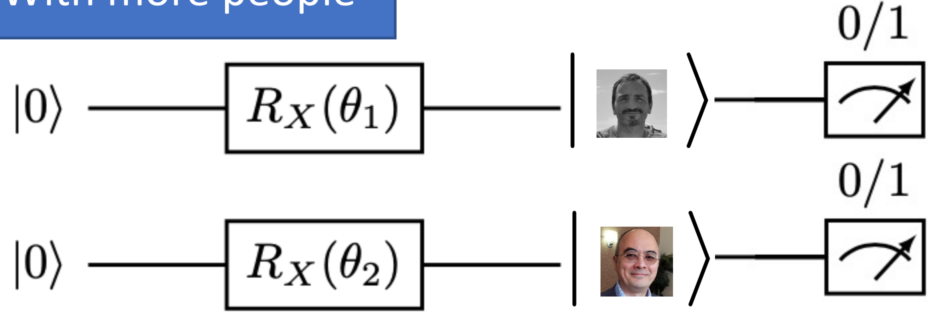
```
{'0': 2546, '1': 7454}
```



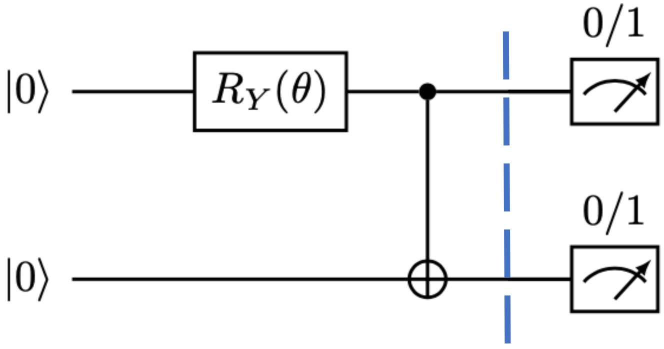
The counting statistic problem

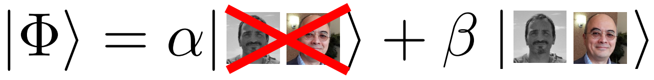
In quantum systems

With more people

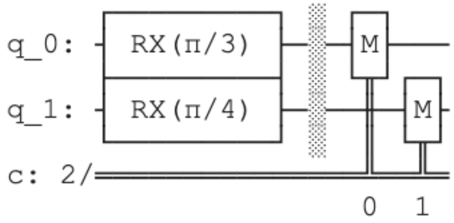


People can be entangled



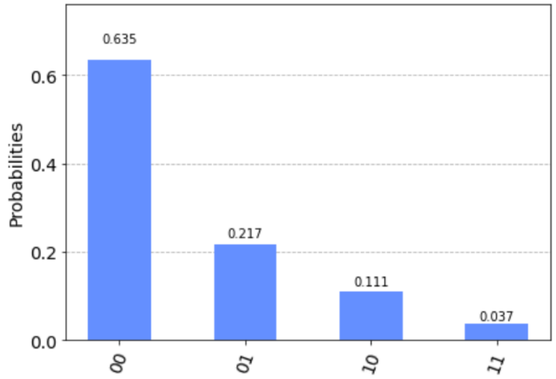


Here I created a Bell state

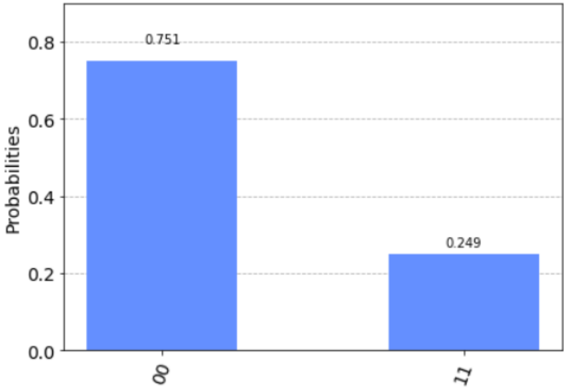


```
{'00': 6351, '01': 2169, '10': 1112, '11': 368}
```

[15]:



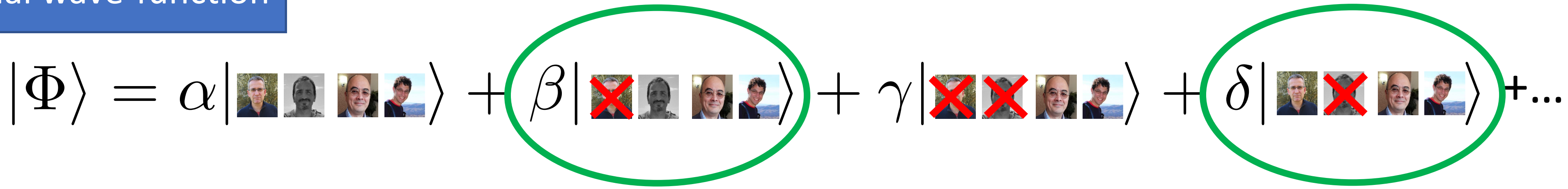
```
{'00': 7513, '11': 2487}
```



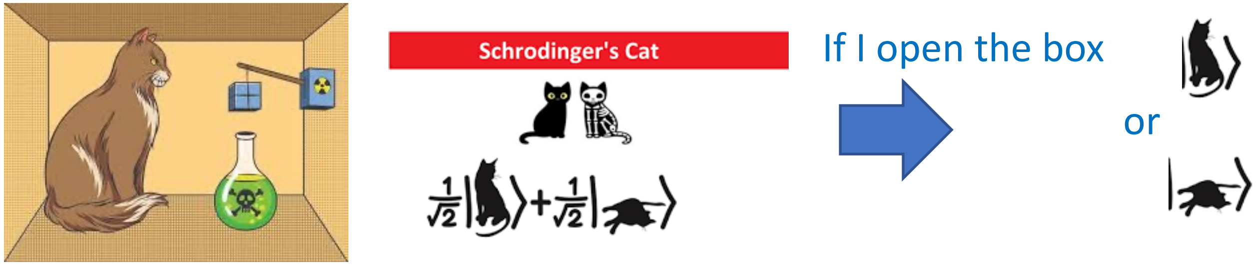
The counting statistic problem

without destroying the wave-function

Initial wave-function

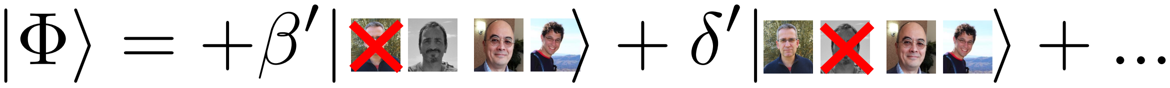


After the measurement the wave-function collapse to one of the state

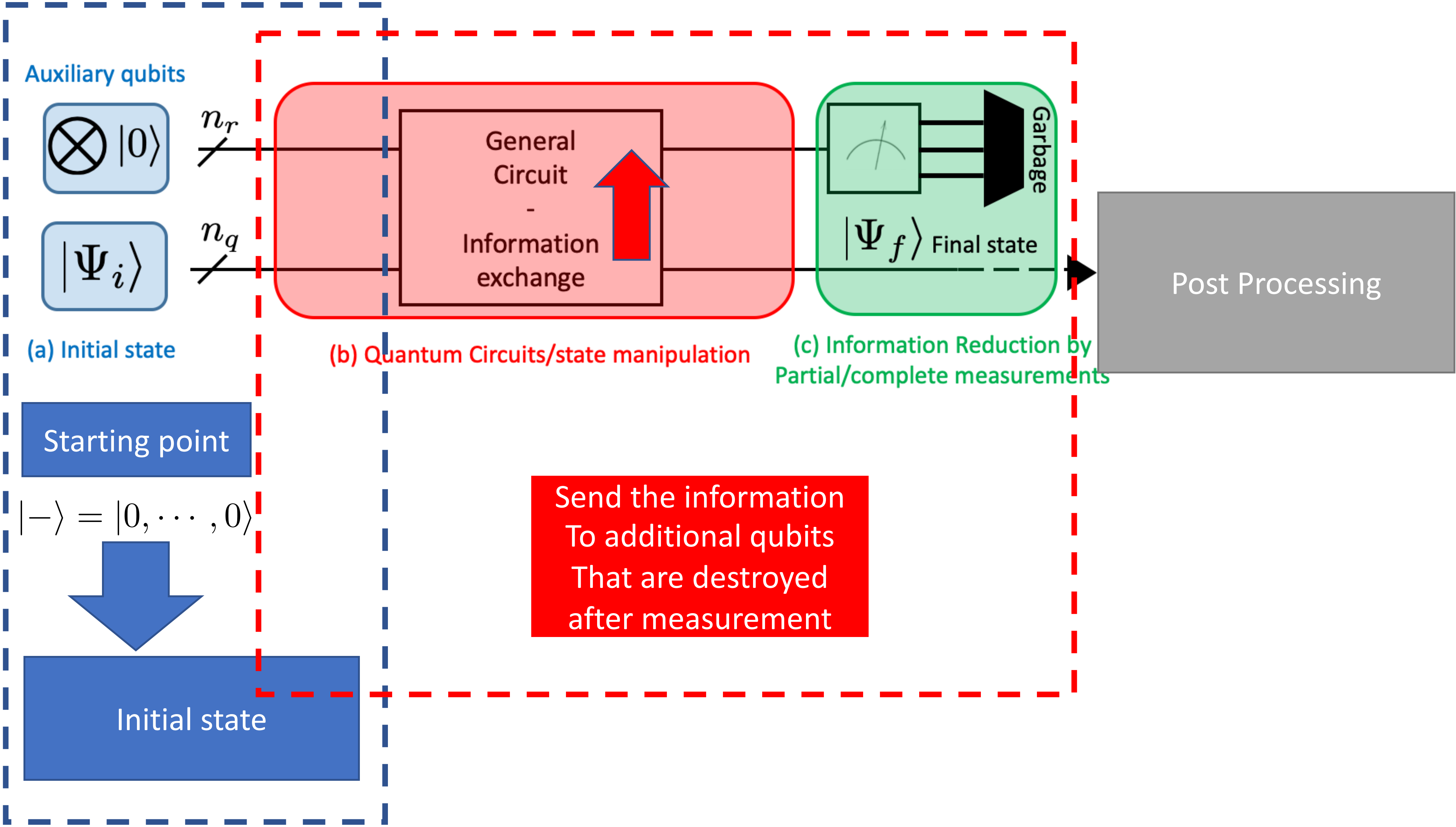


A more difficult problem

I want to select the component with 3 persons without completely destroying it



Non-destructive counting on a quantum computer



Auxiliary qubits

(a) Initial state

(b) Quantum Circuits/state manipulation

(c) Information Reduction by Partial/complete measurements

Post Processing

Starting point

$|-\rangle = |0, \cdots, 0\rangle$

Initial state

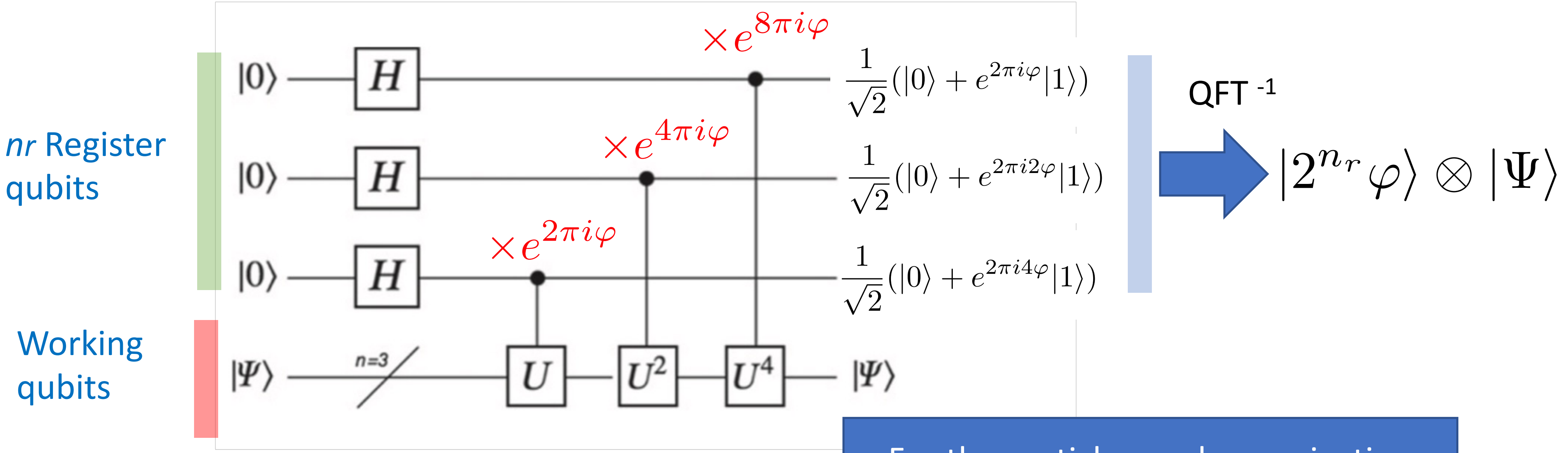
Send the information
To additional qubits
That are destroyed
after measurement

The quantum-Phase estimation (QPE) algorithm

For known eigenvalue problems

Assume a unitary operator U

Assume an eigenstate $|\Psi\rangle$ Such that $U|\Psi\rangle = e^{2\pi i\varphi}|\Psi\rangle$ $\quad 0 \leq \varphi < 1$



General Case

$$|\Psi\rangle = \sum_k \alpha_k|\phi_k\rangle \xrightarrow{\text{QPE}} \sum_k \alpha_k \underbrace{|\theta_k 2^{n_r}\rangle}_{\text{register}} \otimes \underbrace{|\phi_k\rangle}_{\text{eigenstate}}$$

For the particle number projection

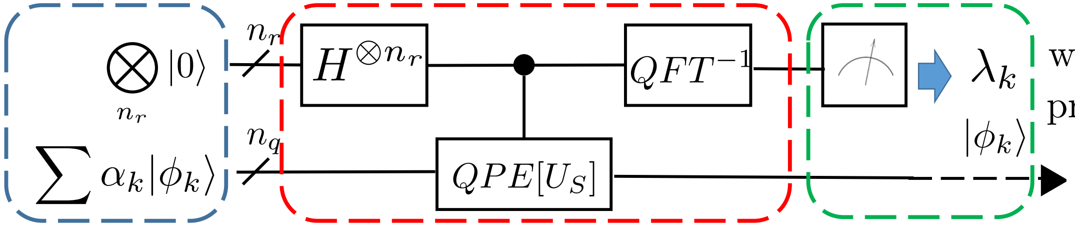
$U = U_N = e^{2\pi i \frac{N}{2^{n_r}}}$ with $N = \frac{1}{2}\sum_i (I_i - Z_i)$

Assume eigenvalues $\{0, 1, \cdots, A\}$

Constraint: $0 \leq \frac{A}{2^{n_r}} < 1$ then $\frac{A}{2^{n_r}} = 0.\underline{a_1 \cdots a_{n_r-1}}$

If I measure given binary number in the ancillary qubit. After measurement, I have the projection on the associated particle number component

Eigenvalues-Ground state and excited states



with prob. $|\alpha_k|^2$

Initial configuration with symmetry breaking state

Symmetry operator eigenvalues encoding
In the register qubits

Symmetry restoration through measurement

Measurement

Example of an event:

$$\underbrace{|011\cdots010\rangle}_{=A^{(\lambda)}}{}^{(\lambda)} \otimes |\phi_{A^{(\lambda)}}\rangle$$

BCS/HFB state

$$|\Psi\rangle = \prod_n \left[\cos\left(\frac{\varphi}{2}\right) I_n \otimes I_{n+1} + \sin\left(\frac{\varphi}{2}\right) Q_n^+ Q_{n+1}^+\right] |-\rangle$$

Measurement

Projected BCS or with varying number Of particles

Degenerate case

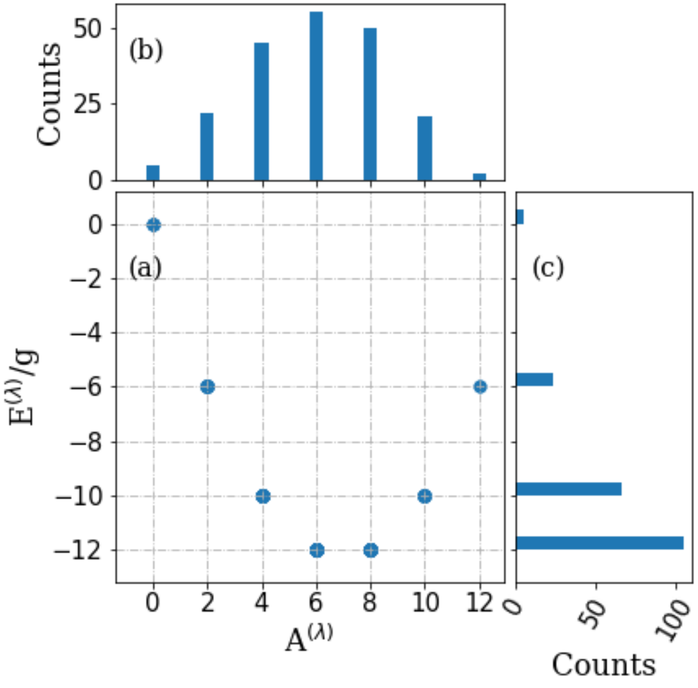
$$H_{\mathrm{P}} = -g \sum_{i,j>0} a_i^\dagger a_{\bar{i}}^\dagger a_{\bar{j}} a_j$$

$$\langle \phi_{A^{(\lambda)}} | H | \phi_{A^{(\lambda)}} \rangle$$

H was encoded on the full Fock space with $A < n_q$

For the degenerate case, this should give the exact solution

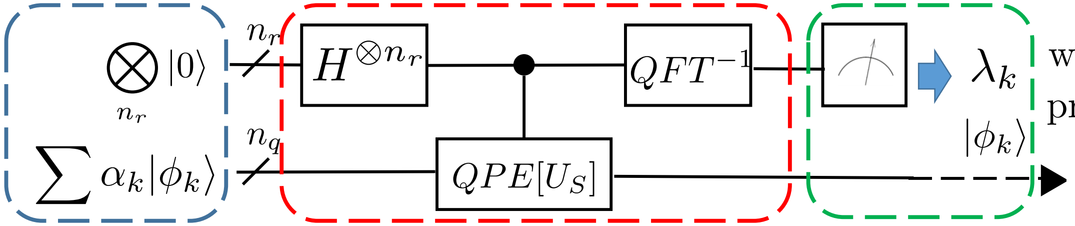
6 pairs



Exact solution

$$E/g = -\frac{1}{4}(A-\nu)(2n_q - A - \nu - 2).$$

Eigenvalues-Ground state and excited states



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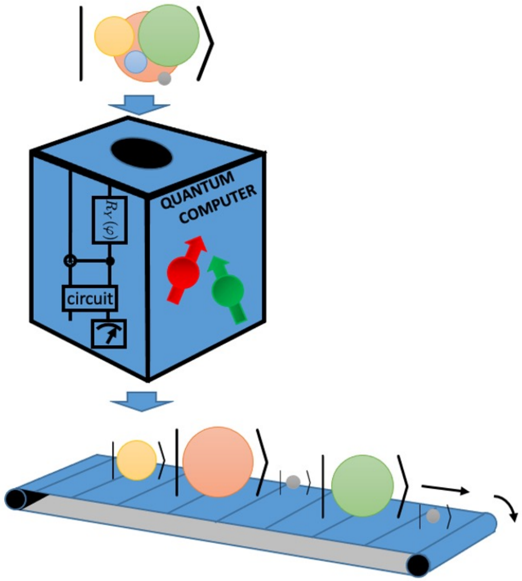
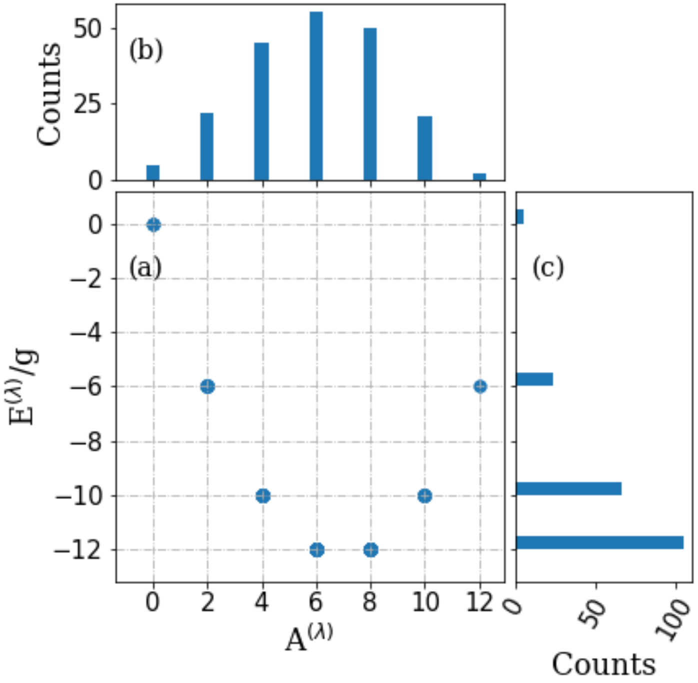


6 pairs

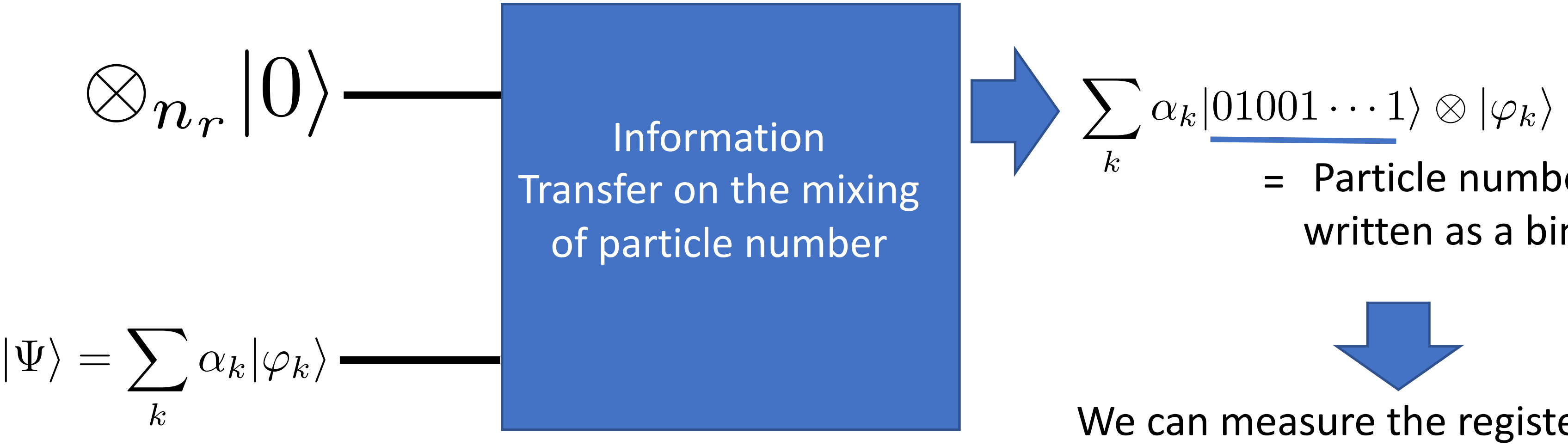


Exact solution

$$E/g = -\frac{1}{4}(A-\nu)(2n_q - A - \nu - 2).$$

A schematic view

Making projection on particle number



$\bigotimes_{n_r}|0\rangle$

Information Transfer on the mixing of particle number

$|\Psi\rangle = \sum_k \alpha_k |\varphi_k\rangle$

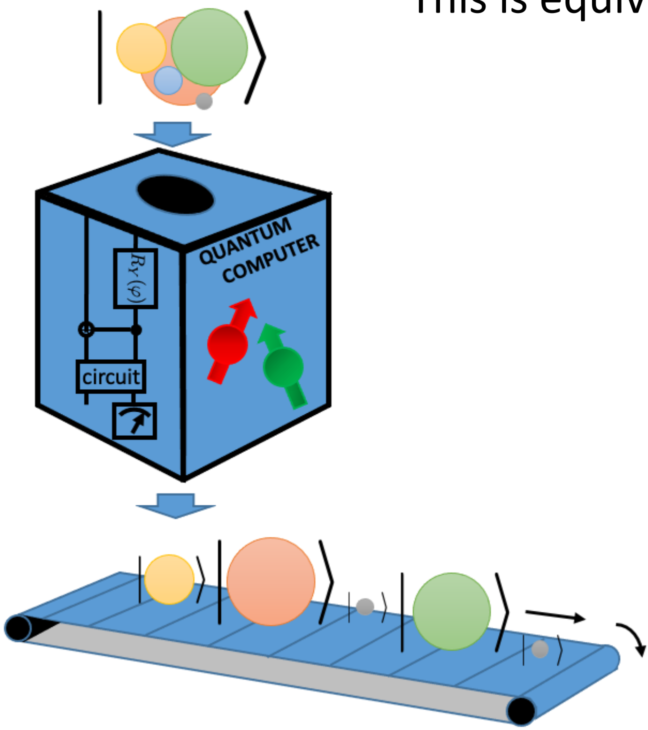
$\sum_k \alpha_k \underline{|01001\cdots 1\rangle} \otimes |\varphi_k\rangle$

= Particle number written as a binary number

We can measure the register qubit

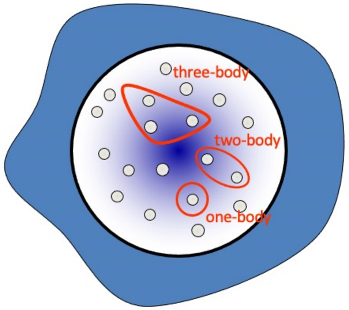
This is equivalent to project on $|\varphi_k\rangle$

An even more schematic view



Then I can use this Wave-function for post-processing

D. Lacroix, "*Symmetry-Assisted Preparation of Entangled Many-Body States on a Quantum Computer*", PRL 125, 230502 (2020).



Combining restoration of broken symmetries and variational method: The Quantum Variation After rojection (Q-VAP)

Coming back to our superconducting problem

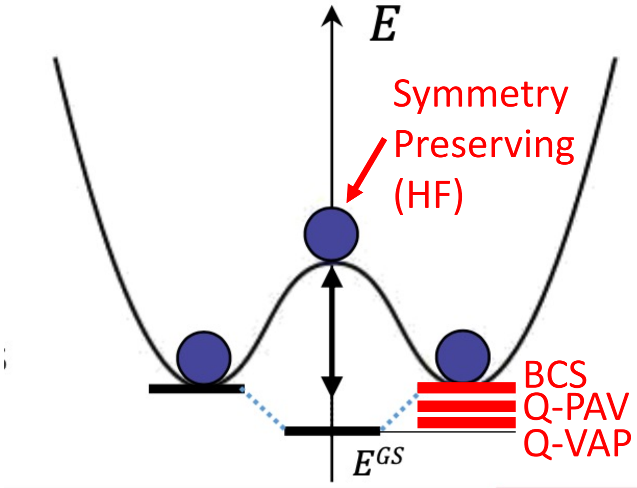
Combining projection with variational method

Possible optimization schemes

Pair occupation are now encoded

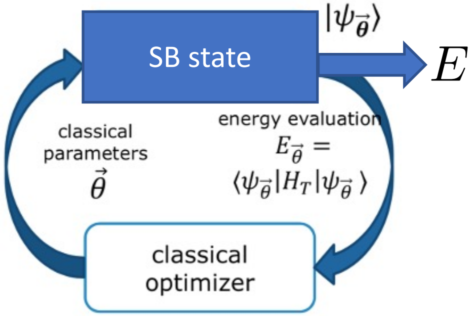
Variational
Symmetry-Breaking ansatz

$$|\Psi(\{\theta_p\})\rangle = \bigotimes_{p=1}^{N-1} [\sin(\theta_p)|0_p\rangle + \cos(\theta_p)|1_p\rangle]$$

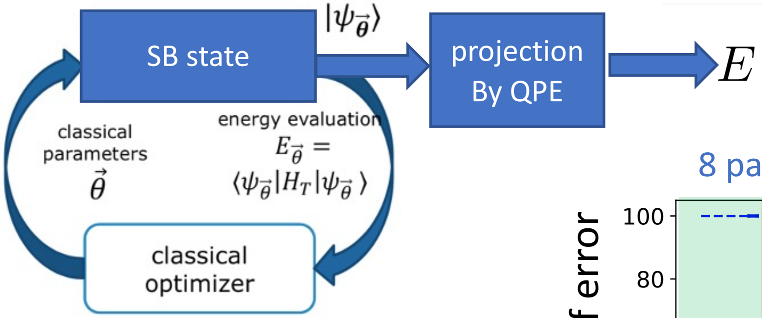


Quantum-Classical optimizers

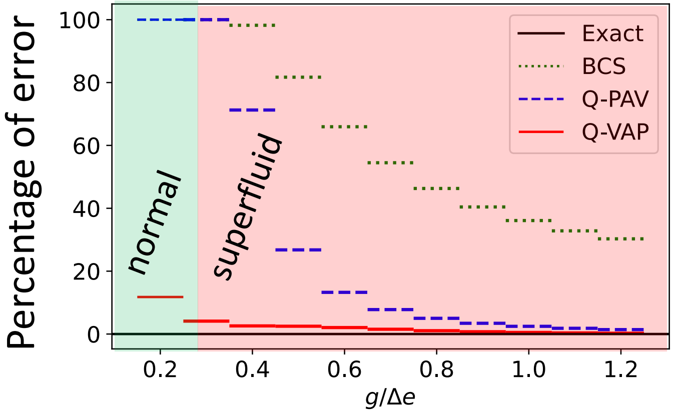
- Standard BCS theory



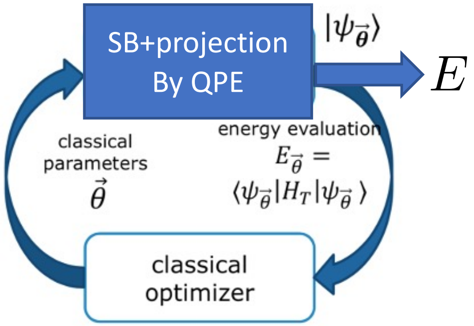
- Project after optimization
Q-PAV: Quantum Projection
After Variation



8 particles on 8 equidistant levels



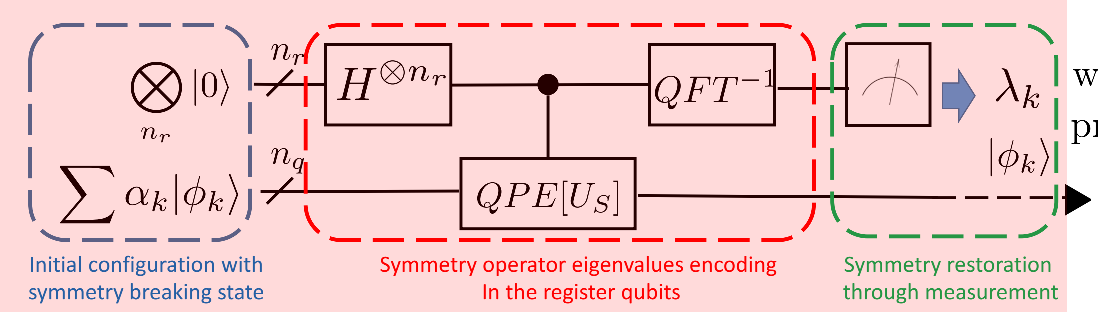
- The optimization is made on the
Symmetry restored state.
Q-VAP: Quantum Variation
After Projection



Post-processing of projected states

Complete strategy

E. A. Ruiz Guzman



Initial configuration with symmetry breaking state

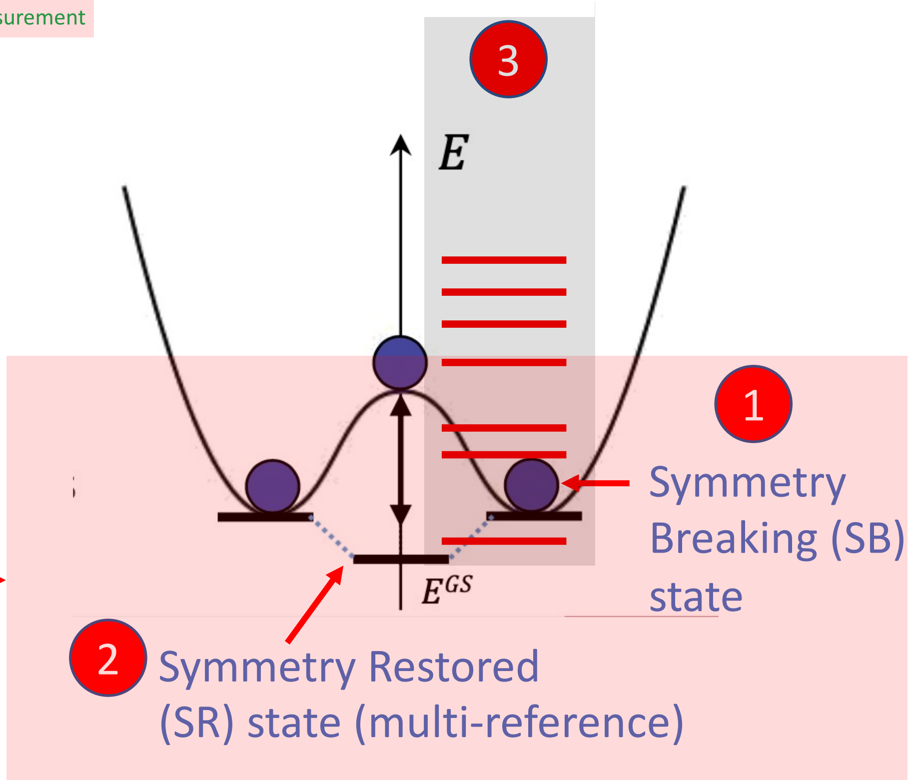
Symmetry operator eigenvalues encoding In the register qubits

Symmetry restoration through measurement

with prob. $|\alpha_k|^2$

Initial state preparation (HF, BCS, P-VAP, Q-VAP)

➡ Post-processing to get excited states: Quantum Phase Estimation

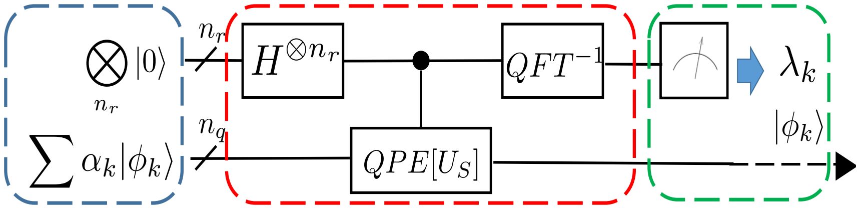


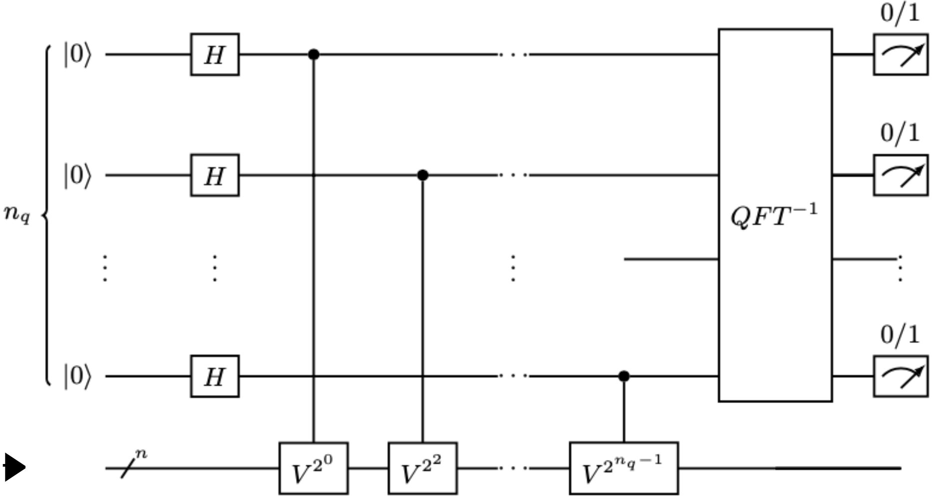
Symmetry Breaking (SB) state

Symmetry Restored (SR) state (multi-reference)

Illustration of the QPE method with projected state

Initial state preparation





Some technical details

$$V = \exp\left\{-2\pi i\left(\frac{H - E_{\min}}{E_{\max} - E_{\min}}\right)\right\}$$

➡ For the propagator, we used the Trotter-Suzuki method

$$U(\tau) = e^{-i\tau H}$$

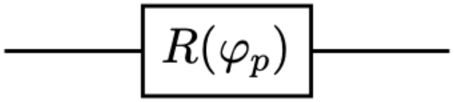
$$U(\tau) = \prod U(\Delta\tau) \longrightarrow \prod U_\varepsilon(\Delta\tau) U_g(\Delta\tau)$$

$$H_{\mathrm{P}} = \sum_{i>0} \varepsilon_i (a_i^\dagger a_i + a_{\bar{i}}^\dagger a_{\bar{i}}) - g \sum_{i,j>0} a_i^\dagger a_{\bar{i}}^\dagger a_{\bar{j}} a_j$$

$$\prod_p \begin{pmatrix} 1 & 0 \\ 0 & \exp\left(-2i\tilde{\varepsilon}_p \Delta t\right) \end{pmatrix} \quad \prod_{p>q} \begin{pmatrix} 1 & 0 & 0 & 0 \\ 0 & \cos(\lambda_{pq}) & i\sin(\lambda_{pq}) & 0 \\ 0 & i\sin(\lambda_{pq}) & \cos(\lambda_{pq}) & 0 \\ 0 & 0 & 0 & 1 \end{pmatrix}$$

with

$$\lambda_{pq} = g\Delta t$$



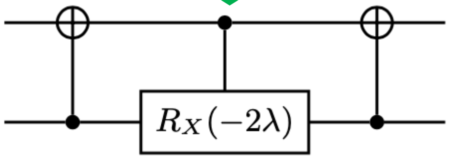
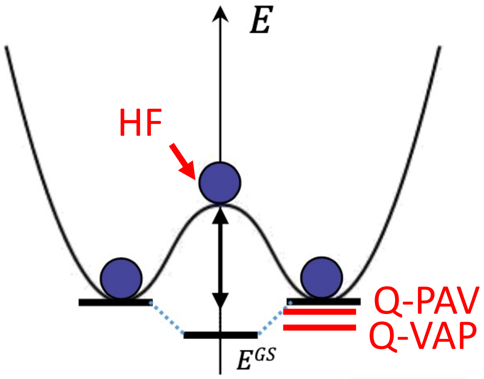
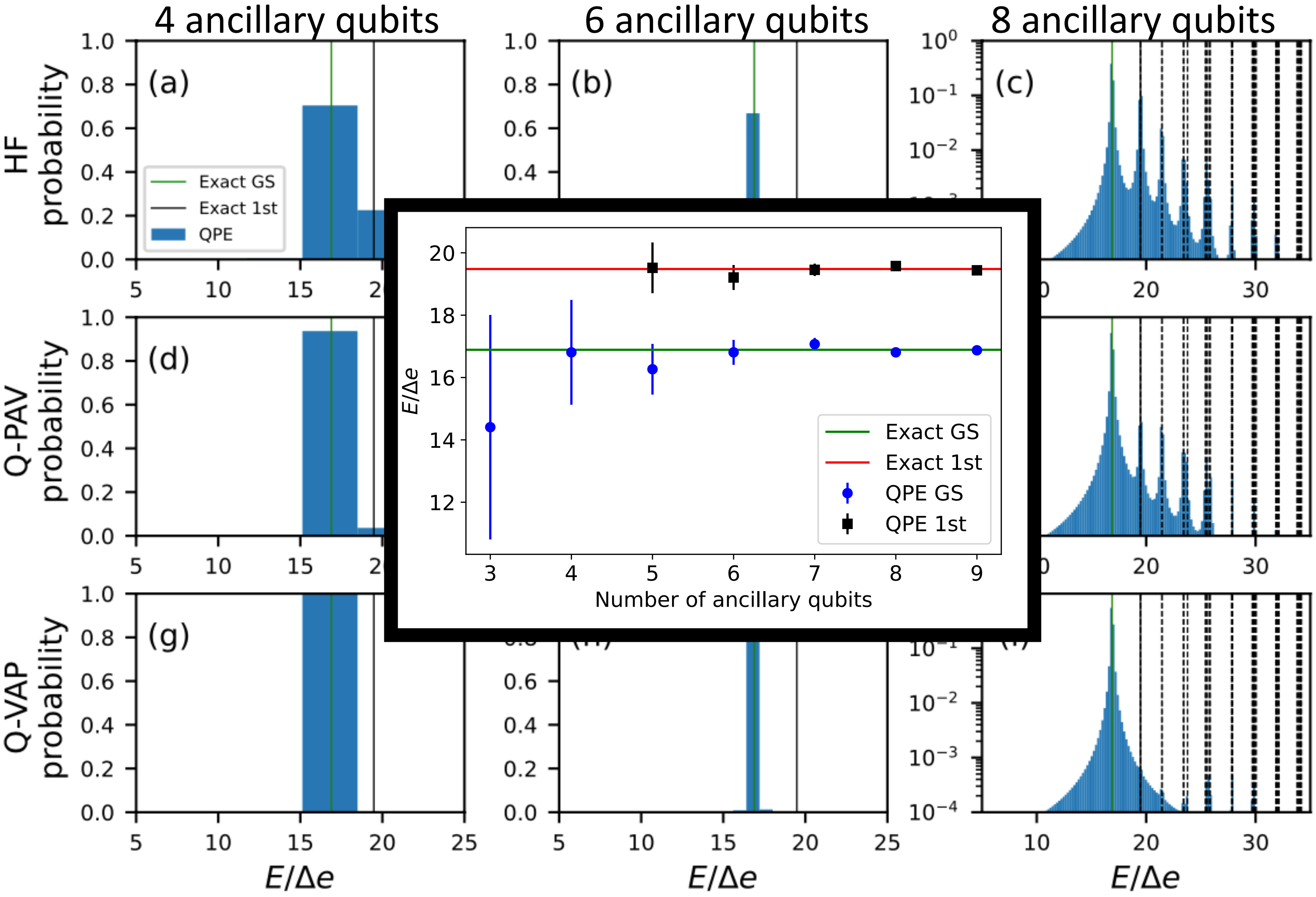


Illustration of the QPE method with projected state

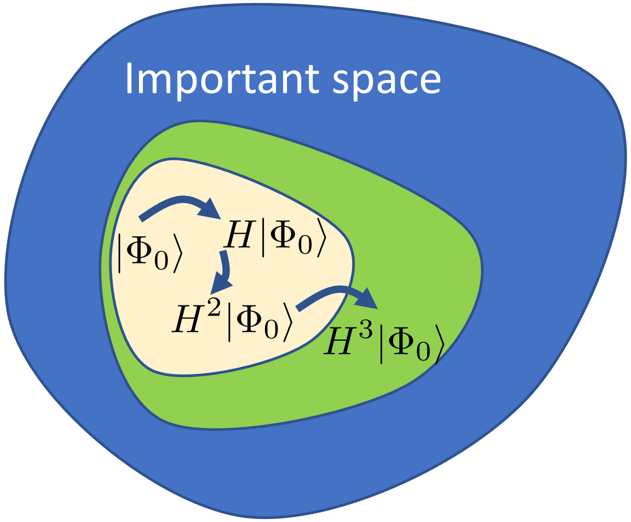




E. A. Ruiz Guzman and DL, Phys. Rev. C 105, 024324 (2022)

Approximate method : Krylov Based methods

Hilbert space



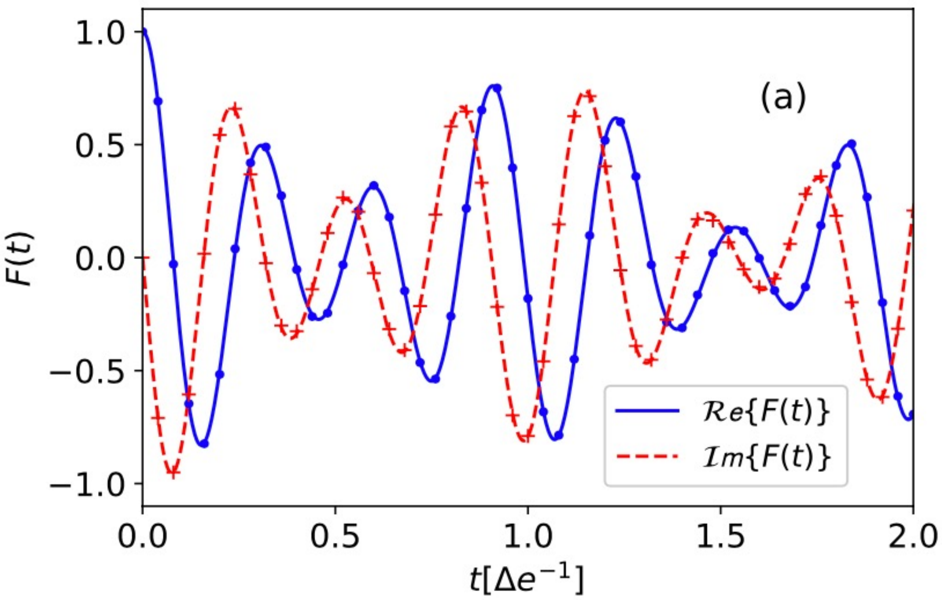
$$\{|\Psi\rangle,\ H|\Psi\rangle, \cdots,\ H^{M-1}|\Psi\rangle\} \equiv \{|\Phi_i\rangle\}_{i=0,M-1}$$

Diagonalize in the non-orthogonal subspace

$$O_{ij} = \langle \Phi_i | \Phi_j \rangle \qquad H_{ij} = \langle \Phi_i | H | \Phi_j \rangle$$

Generalized eigenvalue problem

$$\sum_n c_n(\alpha) O_{in}$$



Our strategy

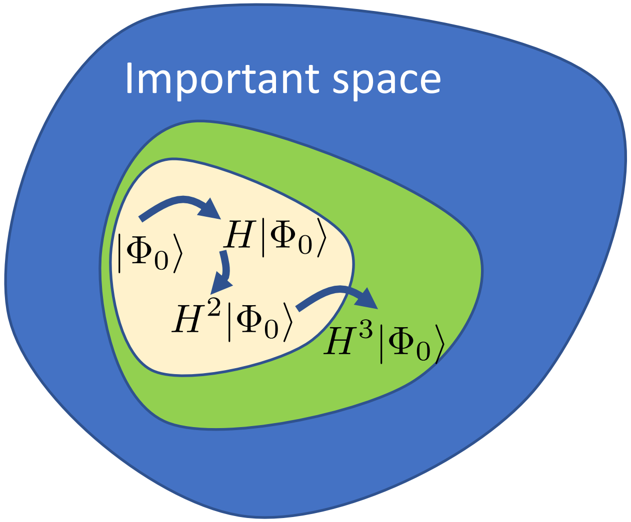
Compute overlap and Hamiltonian matrix elements on the quantum computer

Solve the eigenvalue problem on the classical computer

E. A. Ruiz-Guzman and DL, arXiv:2104.08181

Approximate method : Krylov Based methods

Highly Truncated Hilbert space

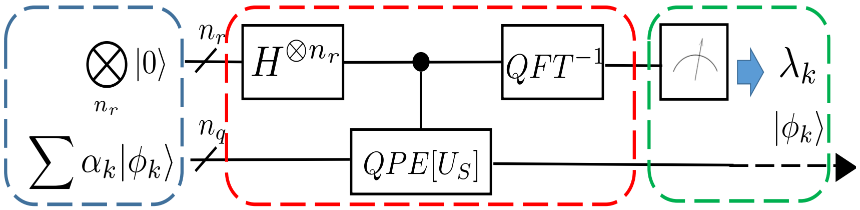


$$\{|\Psi\rangle,\ H|\Psi\rangle, \cdots ,\ H^{M-1}|\Psi\rangle\} \equiv \{|\Phi_i\rangle\}_{i=0,M-1}$$

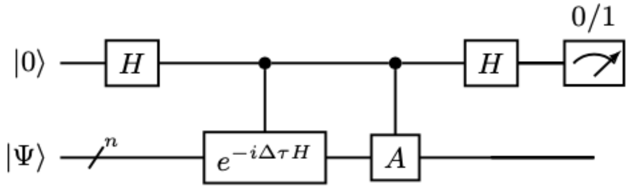
$$\{|\Psi\rangle,\ e^{-i\tau_1 H}|\Psi\rangle, \cdots ,\ e^{-i\tau_{M-1} H}|\Psi\rangle\}$$

$$O_{ij} = \langle\Phi_i|\Phi_j\rangle = \langle\Psi|e^{-i(\tau_j-\tau_i)H}|\Psi\rangle \qquad H_{ij} = \langle\Psi|He^{-i(\tau_j-\tau_i)H}|\Psi\rangle$$

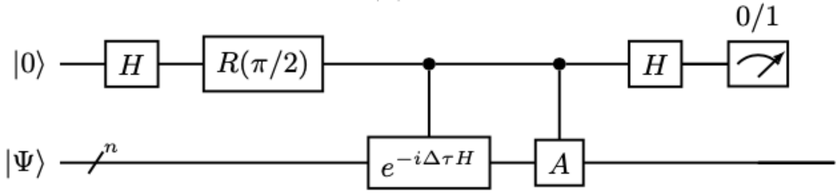
Initial state preparation



Hadamard test for the real part of *O* and *H*

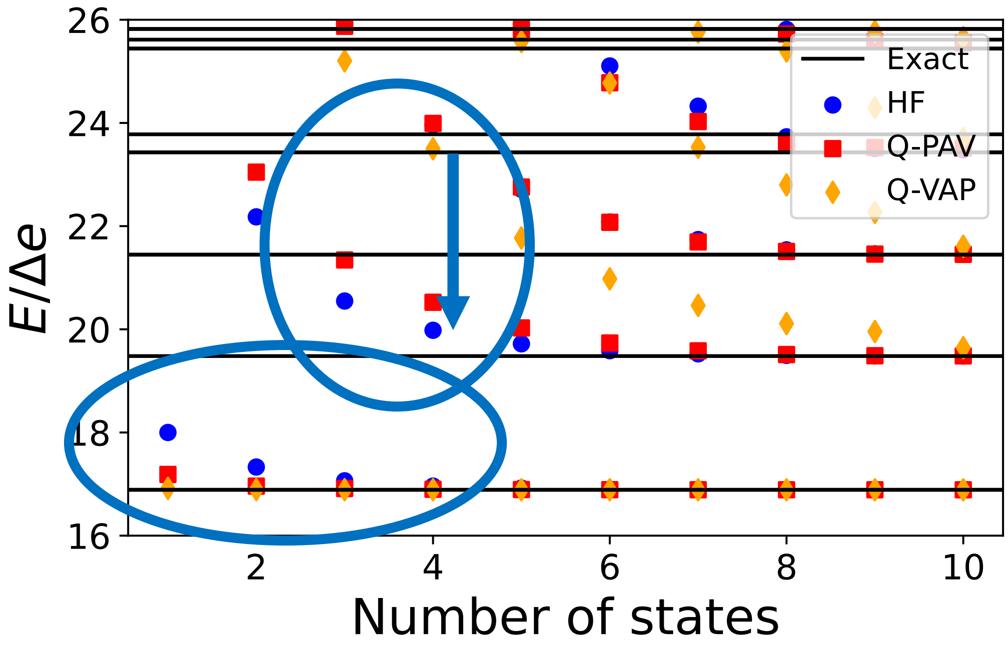


Modified Hadamard test for the imaginary part



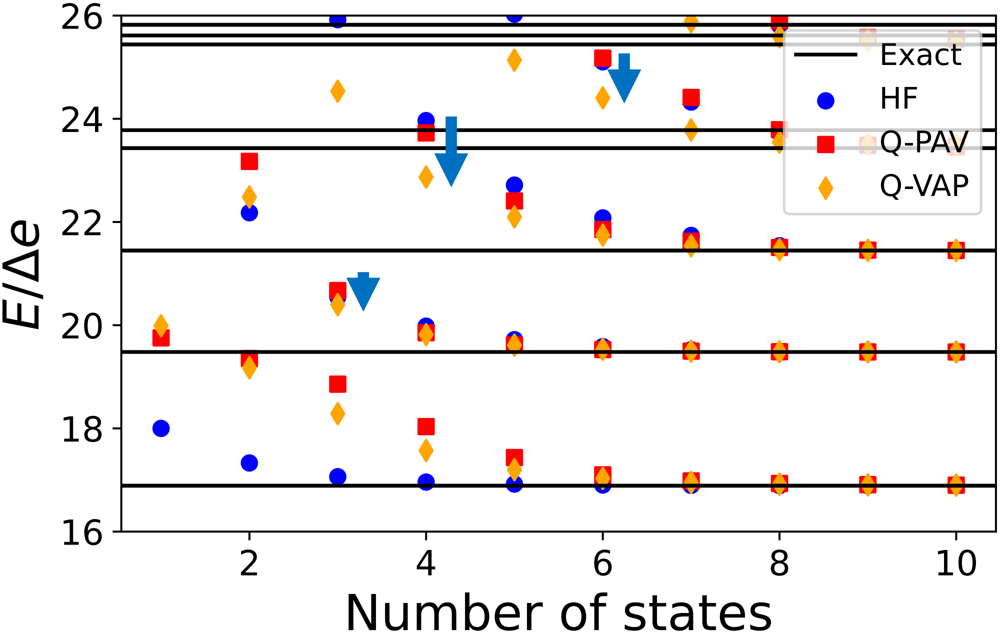
Diagonalization on a classical computer

Comparison QPE vs Quantum Kr



- The combination of Q-VAP + Quantum Krylov
  Is very good for the Ground state

- But Q-VAP + Quantum Krylov
  is worth than others for excited states



A possible solution

$$|\Psi\rangle = \bigotimes_p [\sin(\theta_p)|0_p\rangle + \cos(\theta_p)|1_p\rangle]$$

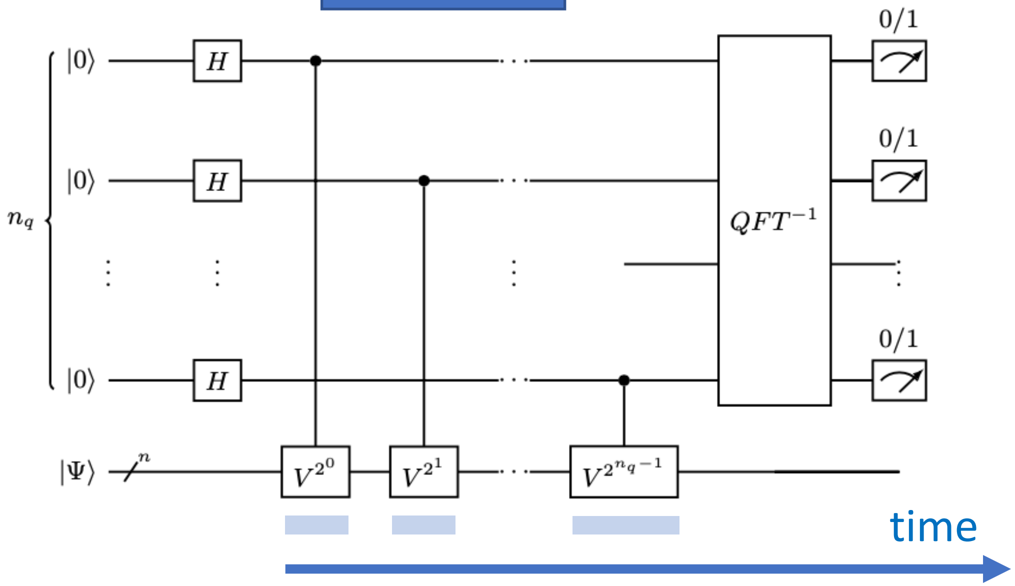
$$\Downarrow$$

$$|\Psi'\rangle = [-\cos(\theta_i)|0_i\rangle + \sin(\theta_i)|1_i\rangle] \bigotimes_{p\neq i} [\sin(\theta_p)|0_p\rangle + \cos(\theta_p)|1_p\rangle]$$

$$\langle\Psi'|\Psi\rangle = 0$$

Comparison QPE vs Quantum Krylov after Q-VAP

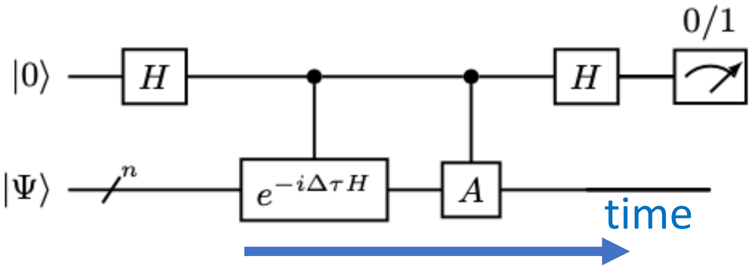
QPE



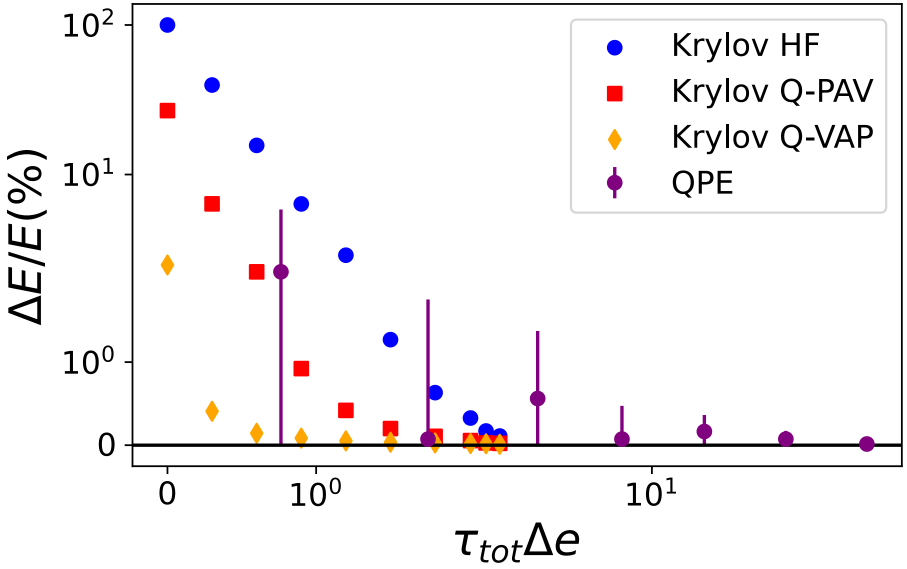
$$\tau_{\rm QPE} + 2\tau_{\rm QPE} + \cdots + 2^{n_q-1}\tau_{\rm QPE}$$

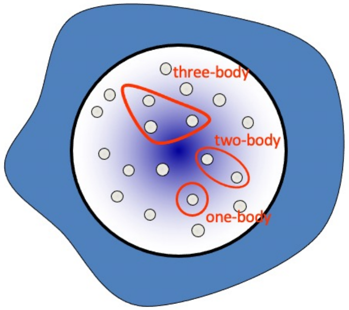
$$\tau_{\rm tot}^{\rm QPE}(n_q) = (2^{n_q}-1)\,\tau_{\rm QPE}$$

Quantum Krylov



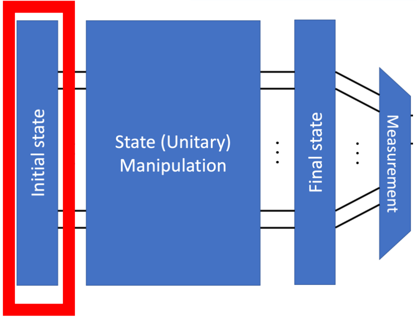
$$\tau_{\rm tot}^{QK}(M)$$





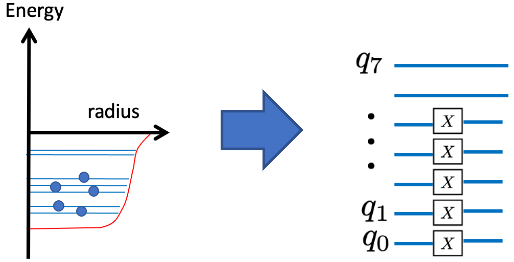
More on many-body ansatz in quantum computing

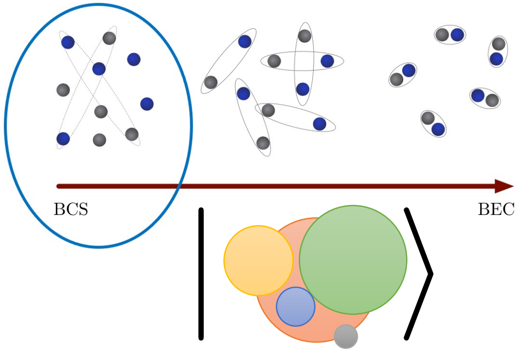
Some flash of what is happening now: nuclear and other many-body systems

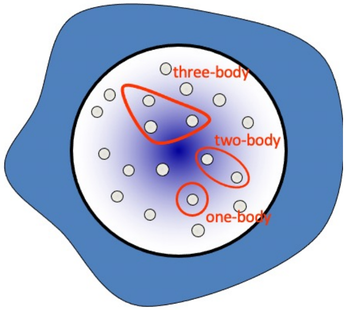


The many ways to prepare a system

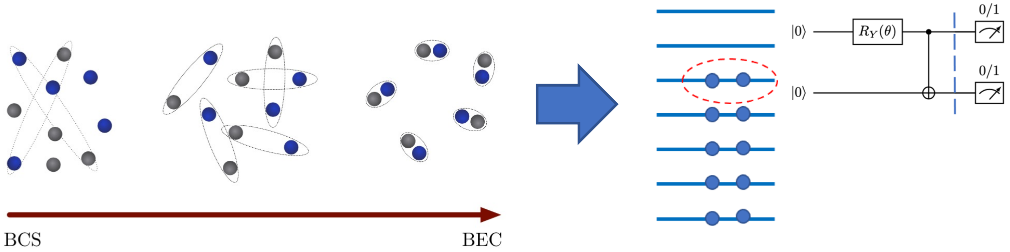
Independent particles



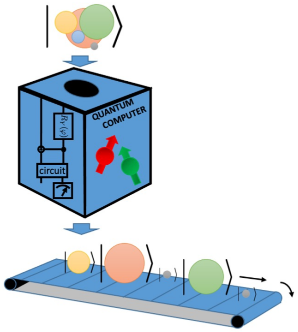




Superfluid systems



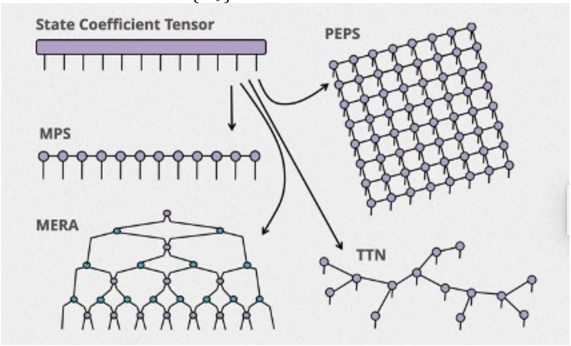
Requires symmetry breaking and restauration tools



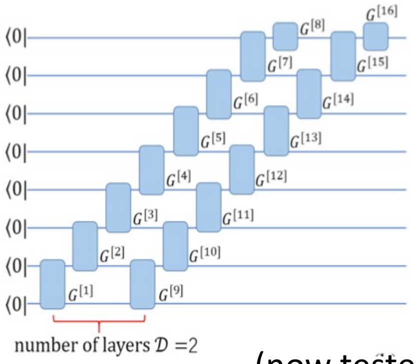
D. Lacroix, PRL 125, 230502 (2020).

Parametrizing general states: Tensor network

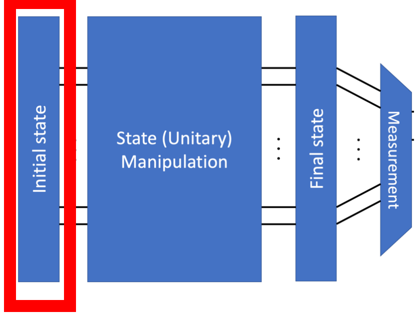
$$\Psi^{s_1,s_2,\cdots s_{n-1},s_n} = \sum_{\{\alpha_i\}} G^{s_1}_{\alpha_1} G^{s_2}_{\alpha_1\alpha_2} \cdots G^{s_{n-1}}_{\alpha_{n-2}\alpha_{n-1}} G^{s_n}_{\alpha_{n-1}}$$



Examples:



(now tested with A. Ruiz Guzman [PhD-IJCLab])



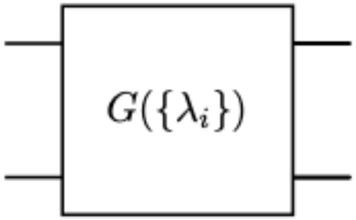
Many-Body problem

Other ways to prepare correlated systems

What is the most general way for 2 qubits ?

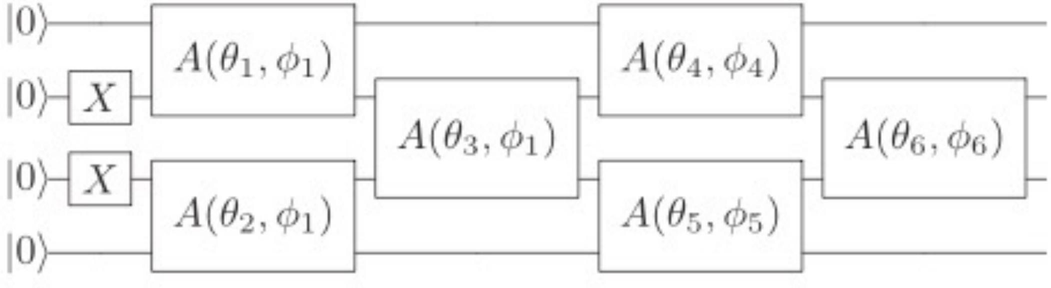
In total, this gives a priori $4 + 6 \times 2 - 1 = 15$ parameters

$$\alpha|00\rangle + \beta|01\rangle + \gamma|10\rangle + \delta|11\rangle$$

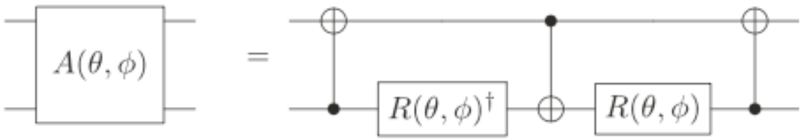


$$\alpha'|00\rangle + \beta'|01\rangle + \gamma'|10\rangle + \delta'|10\rangle$$

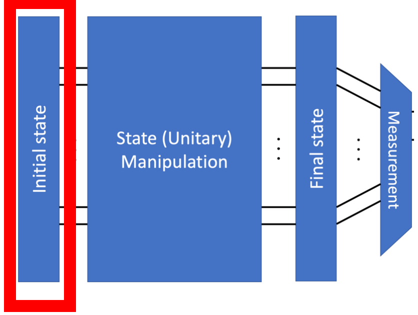
An example of structure that do not mixes particle number



with



Gard et al, Quant Inf (2020)



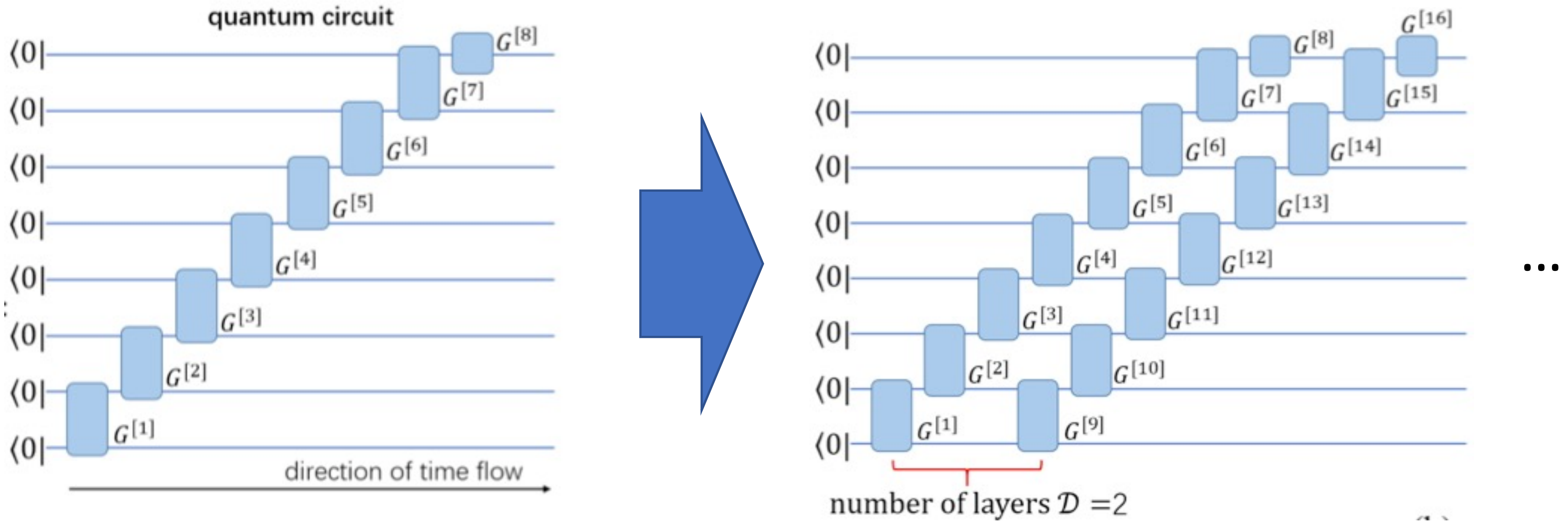
Many-Body problem

Other ways to prepare correlated systems

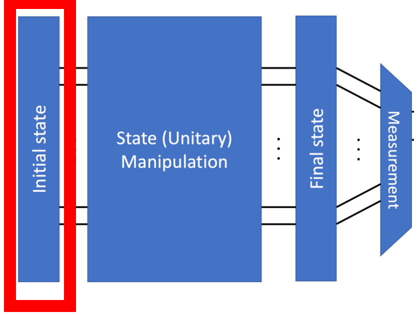
From Two to many qubits

We might use a “product like” form

$$|\Psi\rangle = \sum_{s_i=0,1} \Psi_{s_1,\cdots,s_N}|s_1,\cdots,s_n\rangle \Rightarrow \Psi^{s_1,s_2,\cdots s_{n-1},s_n} = \sum_{\{\alpha_i\}} G^{s_1}_{\alpha_1} G^{s_2}_{\alpha_1\alpha_2} \cdots G^{s_{n-1}}_{\alpha_{n-2}\alpha_{n-1}} G^{s_n}_{\alpha_{n-1}}$$



Ran, Phys. Rev. A 101 (2020)



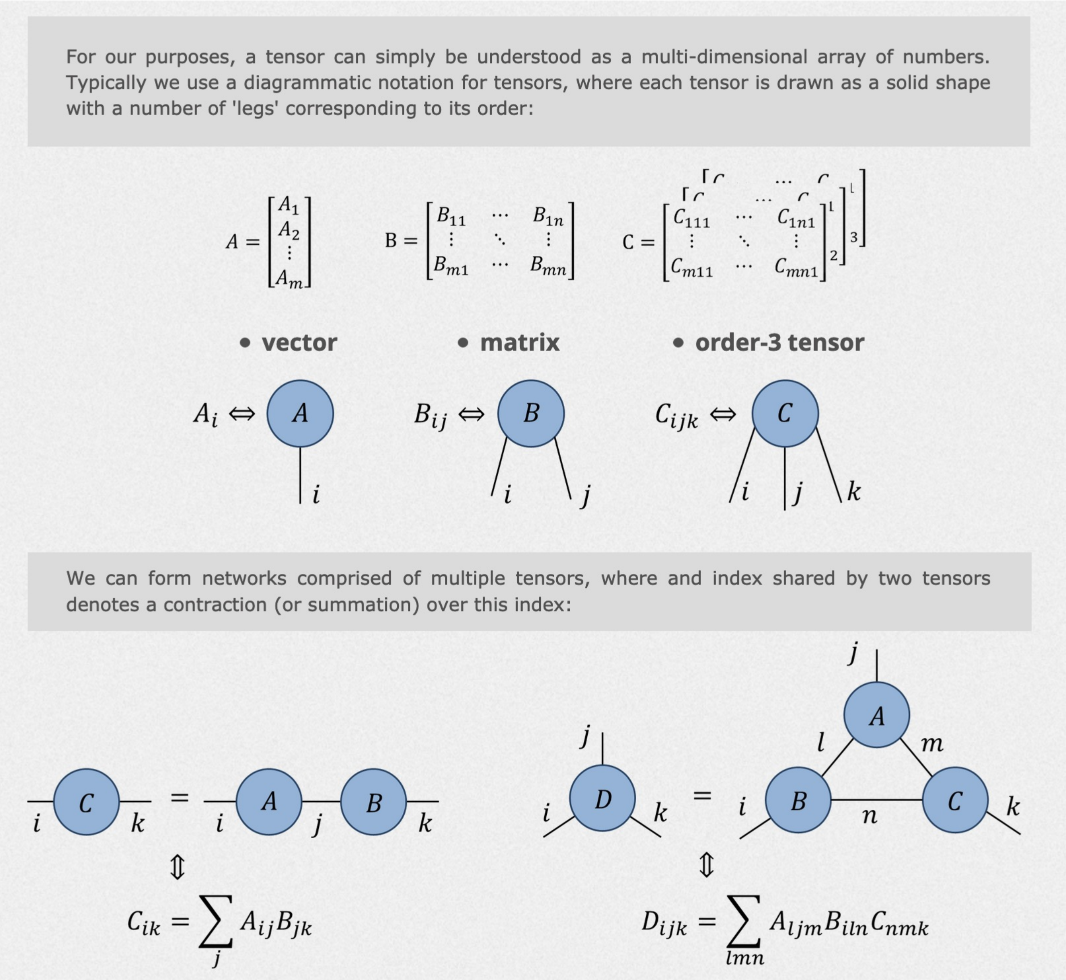
Many-Body problem

Other ways to prepare correlated systems

$$\Psi^{s_1,s_2,\cdots s_{n-1},s_n} = \sum_{\{\alpha_i\}} G^{s_1}_{\alpha_1} G^{s_2}_{\alpha_1\alpha_2} \cdots G^{s_{n-1}}_{\alpha_{n-2}\alpha_{n-1}} G^{s_n}_{\alpha_{n-1}}$$

A few words on tensor network

`https://www.tensors.net/`
`https://tensornetwork.org/`



For our purposes, a tensor can simply be understood as a multi-dimensional array of numbers. Typically we use a diagrammatic notation for tensors, where each tensor is drawn as a solid shape with a number of 'legs' corresponding to its order:

$$A = \begin{bmatrix} A_1 \\ A_2 \\ \vdots \\ A_m \end{bmatrix} \qquad B = \begin{bmatrix} B_{11} & \cdots & B_{1n} \\ \vdots & \ddots & \vdots \\ B_{m1} & \cdots & B_{mn} \end{bmatrix} \qquad C = \begin{bmatrix} C_{111} & \cdots & C_{1n1} \\ \vdots & \ddots & \vdots \\ C_{m11} & \cdots & C_{mn1} \end{bmatrix}$$

- **vector**
- **matrix**
- **order-3 tensor**

$A_i \Leftrightarrow$ $\quad$ $B_{ij} \Leftrightarrow$ $\quad$ $C_{ijk} \Leftrightarrow$

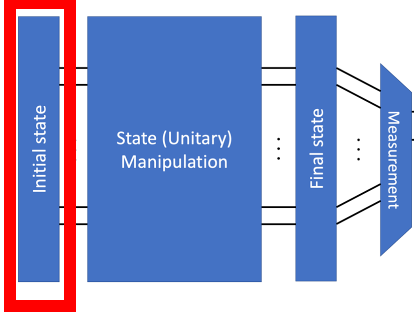
We can form networks comprised of multiple tensors, where and index shared by two tensors denotes a contraction (or summation) over this index:

$$\Updownarrow$$

$$C_{ik} = \sum_j A_{ij} B_{jk}$$

$$\Updownarrow$$

$$D_{ijk} = \sum_{lmn} A_{ljm} B_{iln} C_{nmk}$$



Many-Body problem

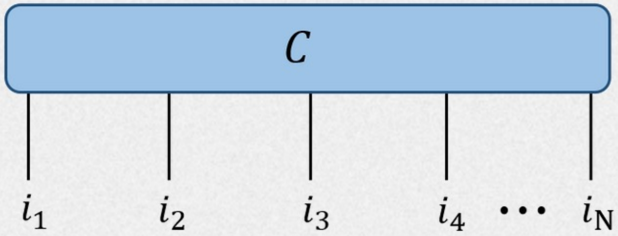
Other ways to prepare correlated systems

$$\Psi^{s_1,s_2,\cdots s_{n-1},s_n} = \sum_{\{\alpha_i\}} G^{s_1}_{\alpha_1} G^{s_2}_{\alpha_1\alpha_2} \cdots G^{s_{n-1}}_{\alpha_{n-2}\alpha_{n-1}} G^{s_n}_{\alpha_{n-1}}$$

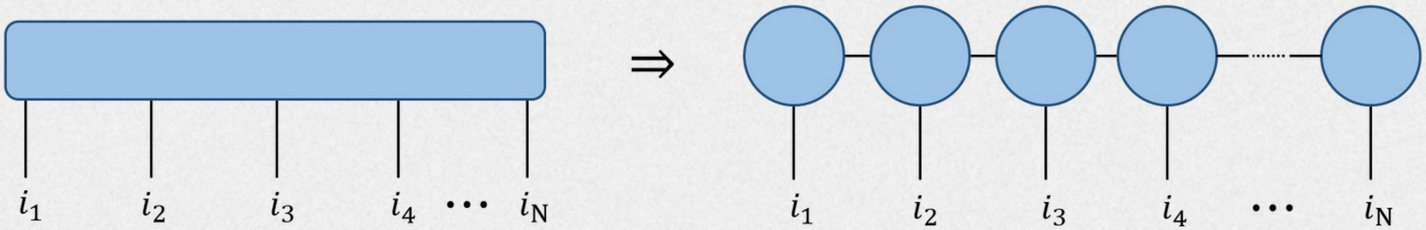
A few words on tensor network

`https://www.tensors.net/`
`https://tensornetwork.org/`

$$|\psi\rangle = \sum_{i_1,i_2,\ldots,i_N} C_{i_1,i_2,\ldots,i_N} |i_1\rangle|i_2\rangle \cdots |i_N\rangle$$

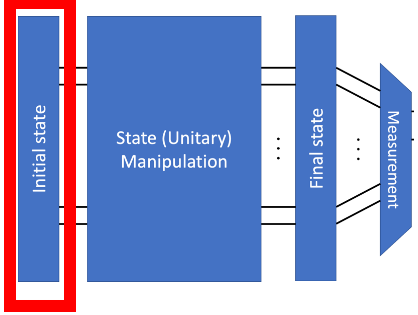


Matrix Product State



Order-N tensor:
contains ~*exp(N)* parameters

Network of low-order tensors:
contains ~*poly(N)* parameters



Many-Body problem

Other ways to prepare correlated systems

$$\Psi^{s_1,s_2,\cdots s_{n-1},s_n} = \sum_{\{\alpha_i\}} G^{s_1}_{\alpha_1} G^{s_2}_{\alpha_1\alpha_2} \cdots G^{s_{n-1}}_{\alpha_{n-2}\alpha_{n-1}} G^{s_n}_{\alpha_{n-1}}$$

A few words on tensor network

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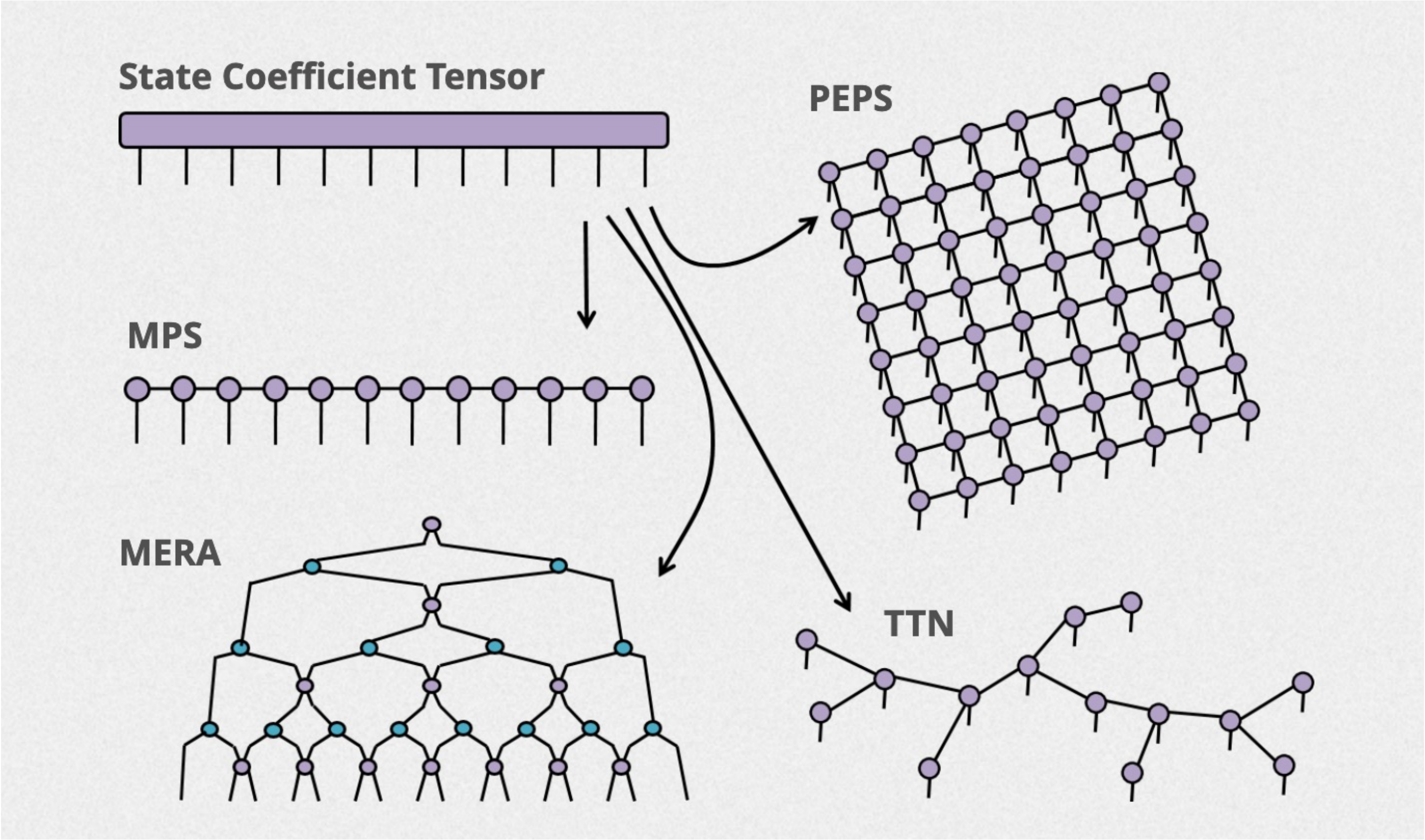
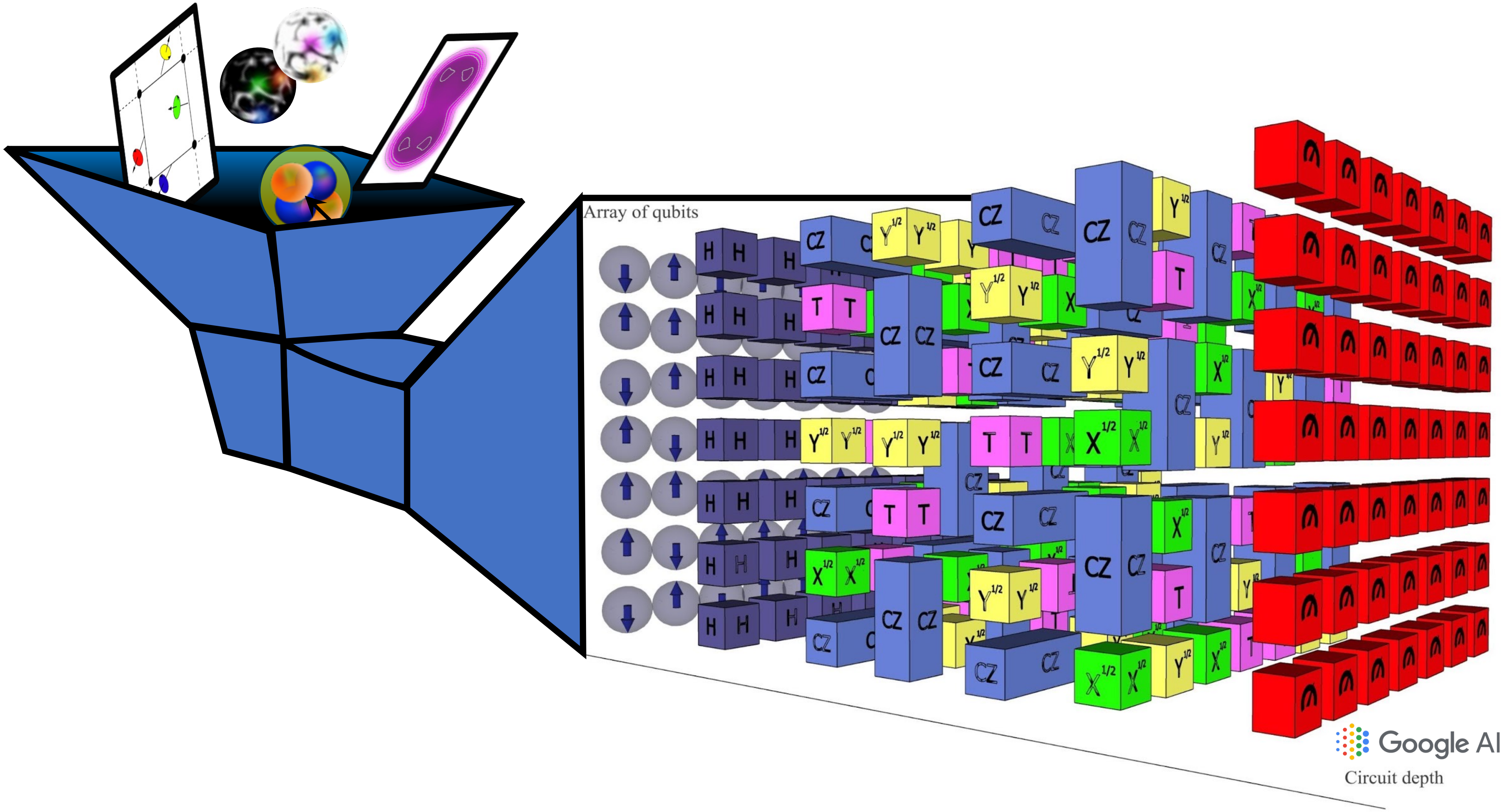
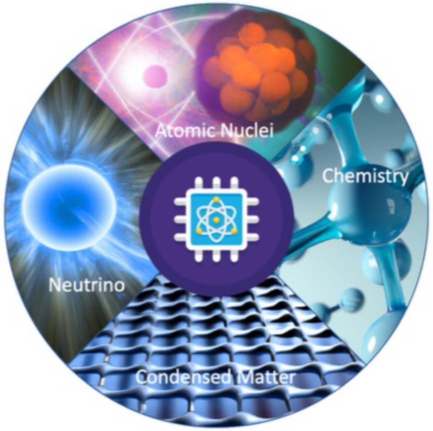


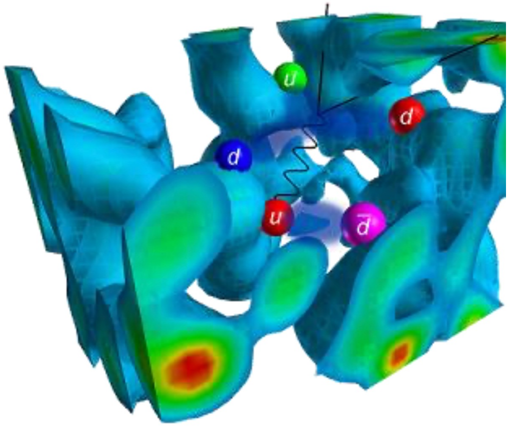
Summary



Quantum computers are often themselves many-body interacting systems



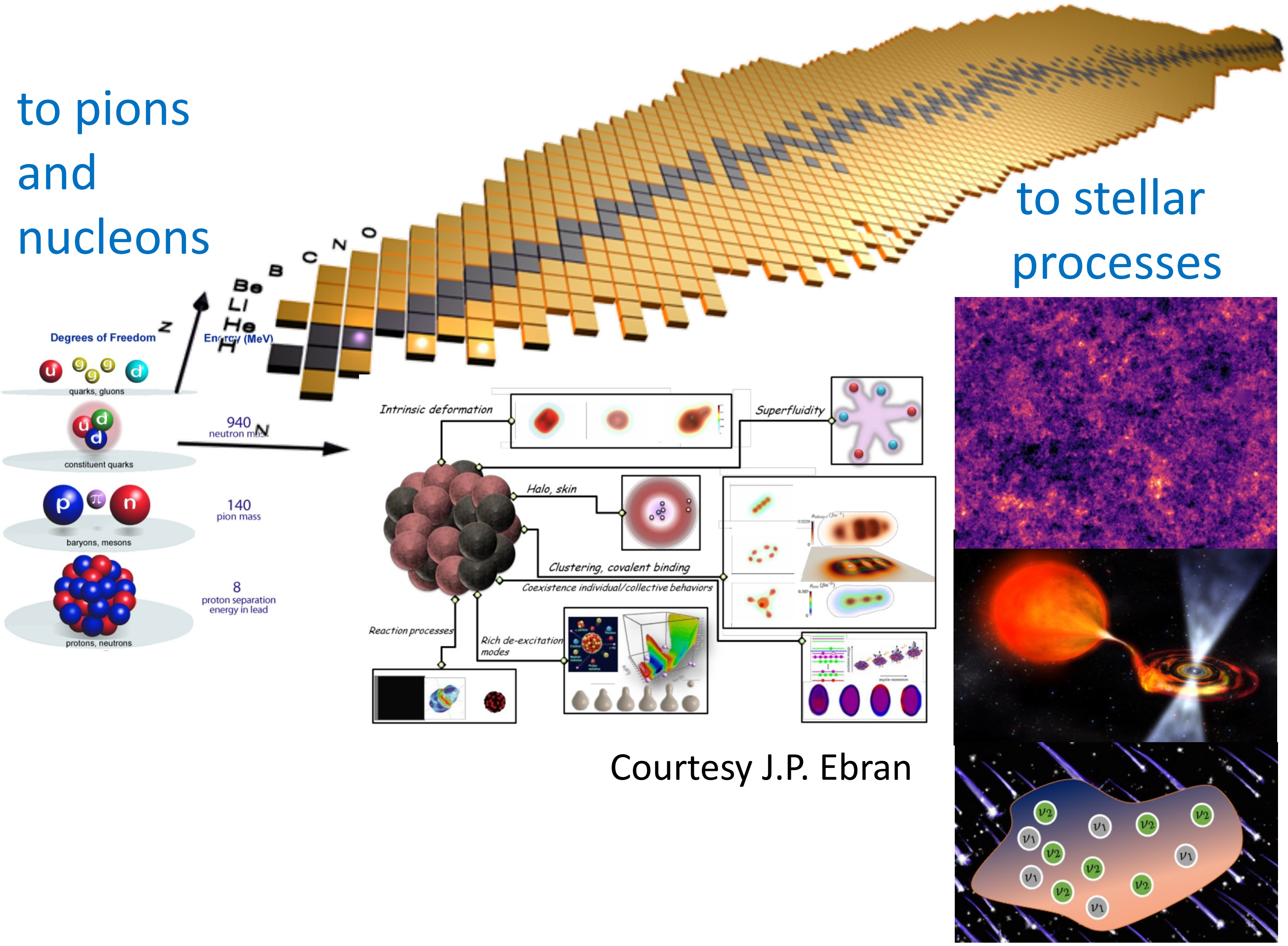
from quark



to pions
and
nucleons

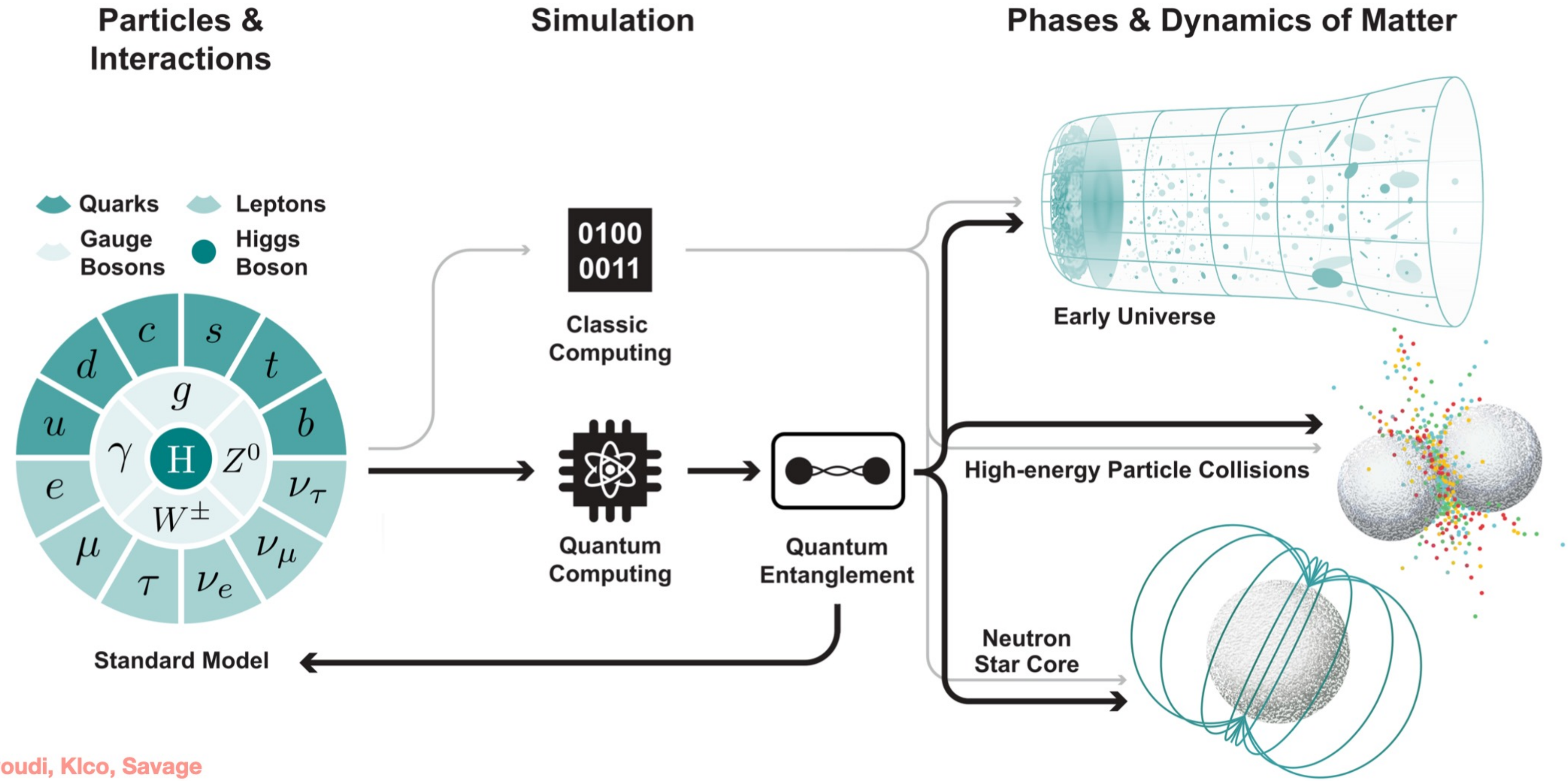
to atomic nuclei

to stellar
processes



Courtesy J.P. Ebran

Quarks and Gluons – Effective Field theories

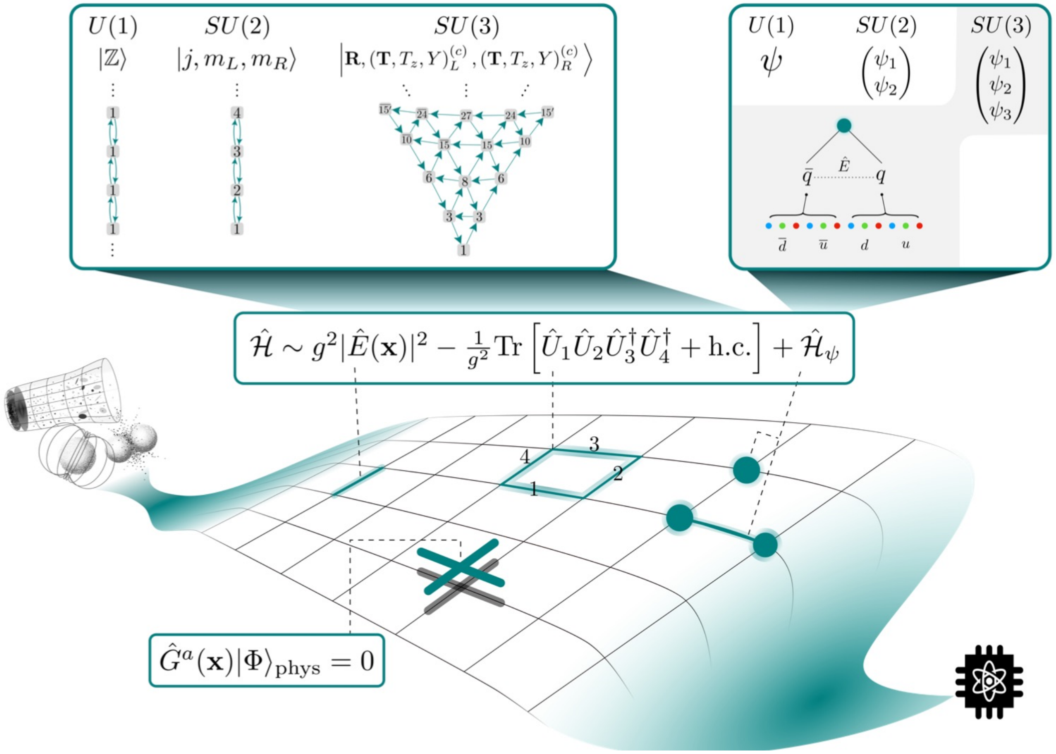


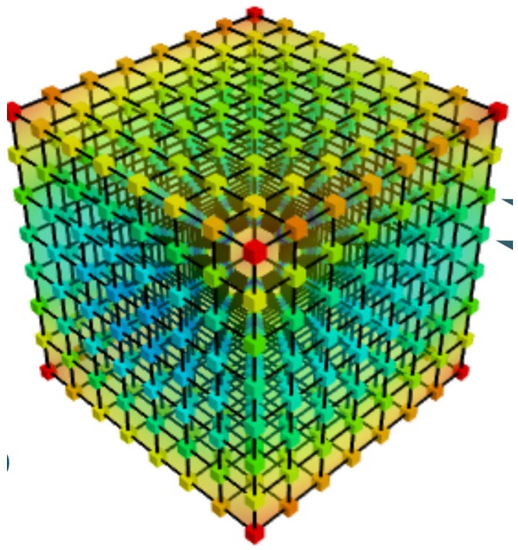
Bauer, Davoudi, Klco, Savage

Courtesy M. Savage

Quarks and Gluons – Effective Field theories

Digital Quantum Chromodynamics

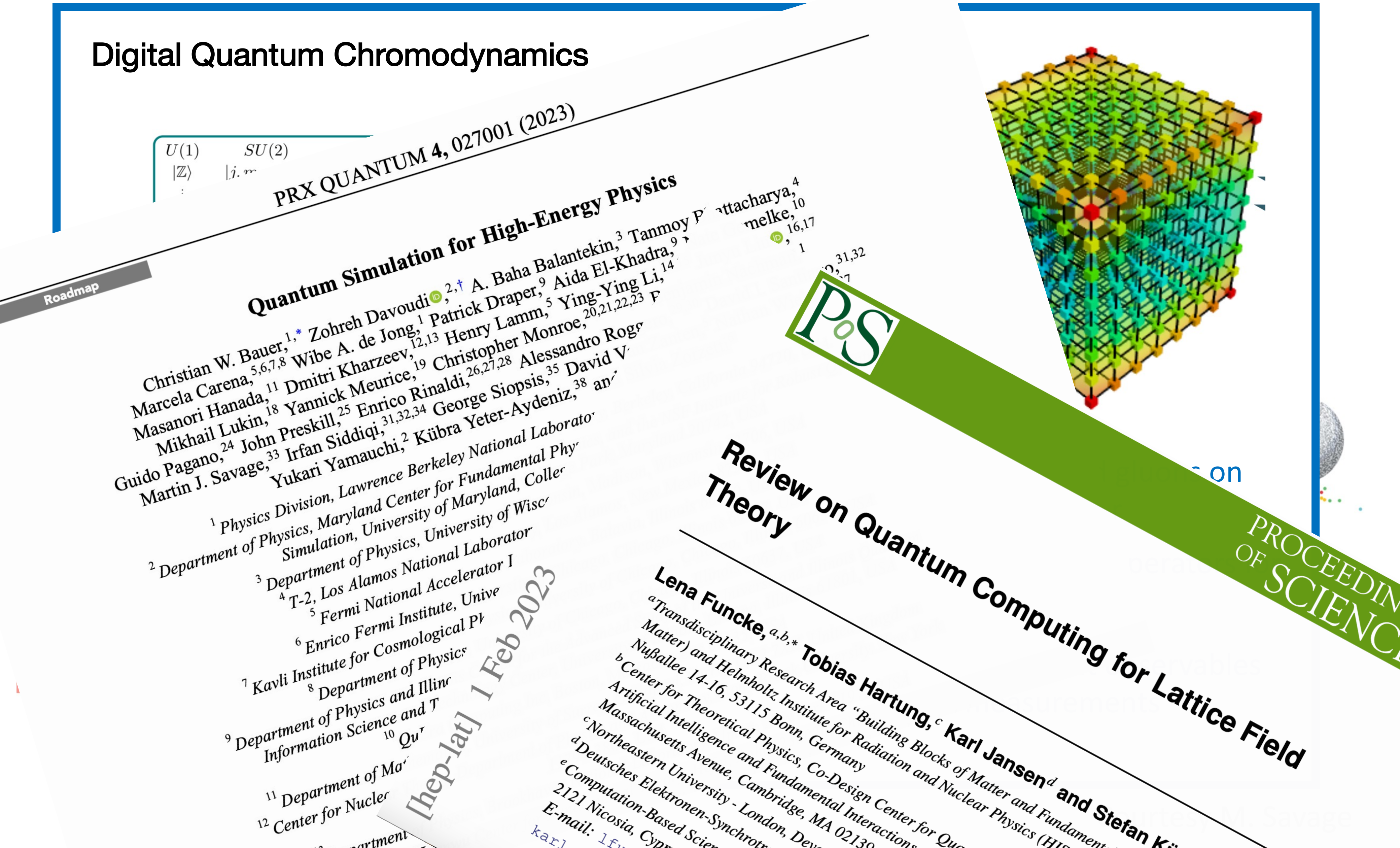




- Map quarks and gluons on quantum register
- Develop unitary operators for their evolution
- Obtain relevant observables from measurements

Courtesy M. Savage

Quarks and Gluons – Effective Field theories



Digital Quantum Chromodynamics

$U(1)$ $SU(2)$

$|\mathbb{Z}\rangle$ $|j, m$

s on

PRX QUANTUM **4**, 027001 (2023)

Roadmap

Quantum Simulation for High-Energy Physics

Christian W. Bauer,[1,*] Zohreh Davoudi,[2,†] A. Baha Balantekin,[3] Tanmoy P ttacharya,[4]
Marcela Carena,[5,6,7,8] Wibe A. de Jong,[1] Patrick Draper,[9] Aida El-Khadra,[9] melke,[10]
Masanori Hanada,[11] Dmitri Kharzeev,[12,13] Henry Lamm,[5] Ying-Ying Li,[14] [16,17]
Mikhail Lukin,[18] Yannick Meurice,[19] Christopher Monroe,[20,21,22,23] F [1]
Guido Pagano,[24] John Preskill,[25] Enrico Rinaldi,[26,27,28] Alessandro Rogg [31,32]
Martin J. Savage,[33] Irfan Siddiqi,[31,32,34] George Siopsis,[35] David V
Yukari Yamauchi,[2] Kübra Yeter-Aydeniz,[38] an

[1] *Physics Division, Lawrence Berkeley National Laborato*
[2] *Department of Physics, Maryland Center for Fundamental Phy*
Simulation, University of Maryland, Colle
[3] *Department of Physics, University of Wisc*
[4] *T-2, Los Alamos National Laborator*
[5] *Fermi National Accelerator I*
[6] *Enrico Fermi Institute, Unive*
[7] *Kavli Institute for Cosmological Ph*
[8] *Department of Physics*
[9] *Department of Physics and Illin*
Information Science and T
[10] *Qu*
[11] *Department of Ma*
[12] *Center for Nucle*

[hep-lat] 1 Feb 2023

PoS

PROCEEDIN
OF SCIENC

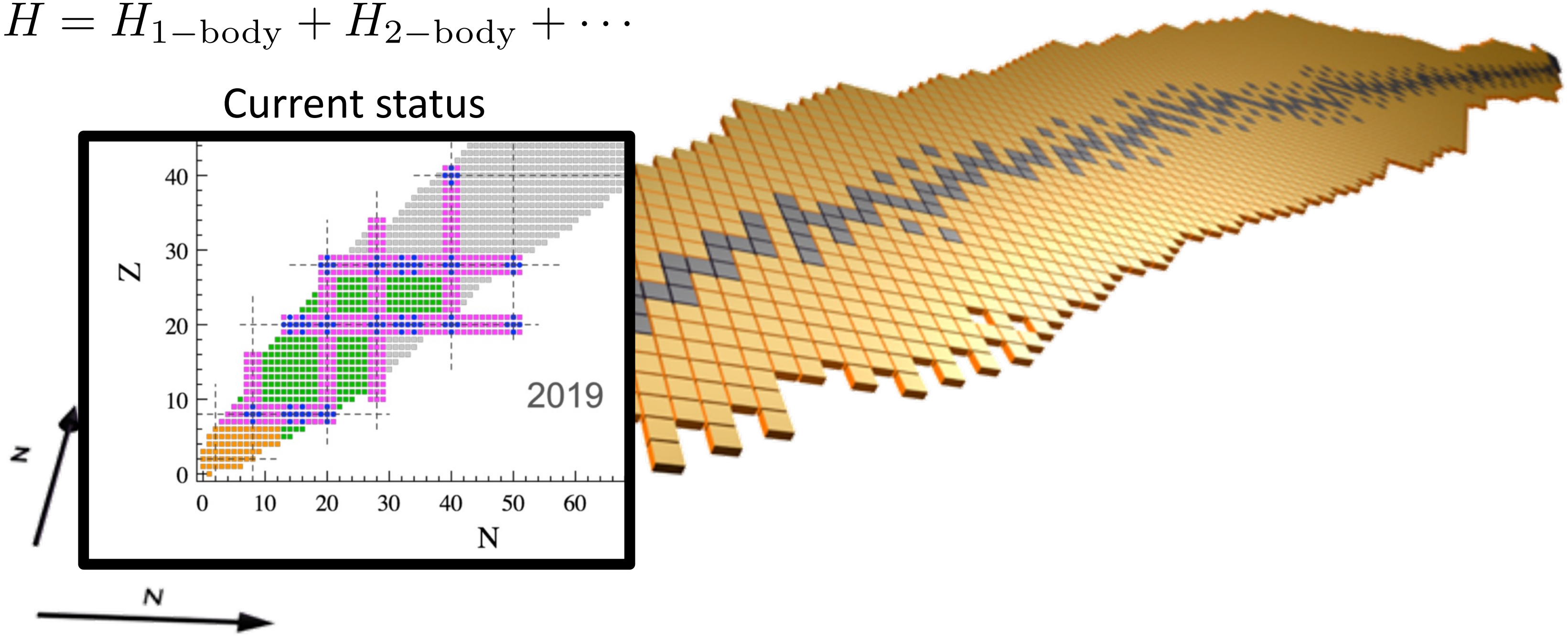
Review on Quantum Computing for Lattice Field Theory

Lena Funcke,[a,b,*] **Tobias Hartung,**[c] **Karl Jansen**[d] **and Stefan K**

[a] *Transdisciplinary Research Area "Building Blocks of Matter and Fundament*
Matter) and Helmholtz Institute for Radiation and Nuclear Physics (HIS
Nußallee 14-16, 53115 Bonn, Germany
[b] *Center for Theoretical Physics, Co-Design Center for Qu*
Artificial Intelligence and Fundamental Interactions
Massachusetts Avenue, Cambridge, MA 0213
[c] *Northeastern University - London, Dev*
[d] *Deutsches Elektronen-Synchrot*
[e] *Computation-Based Scien*
2121 Nicosia, Cypr
E-mail: karl lfu

Actual tendency : Towards Full configuration-Interaction description ?

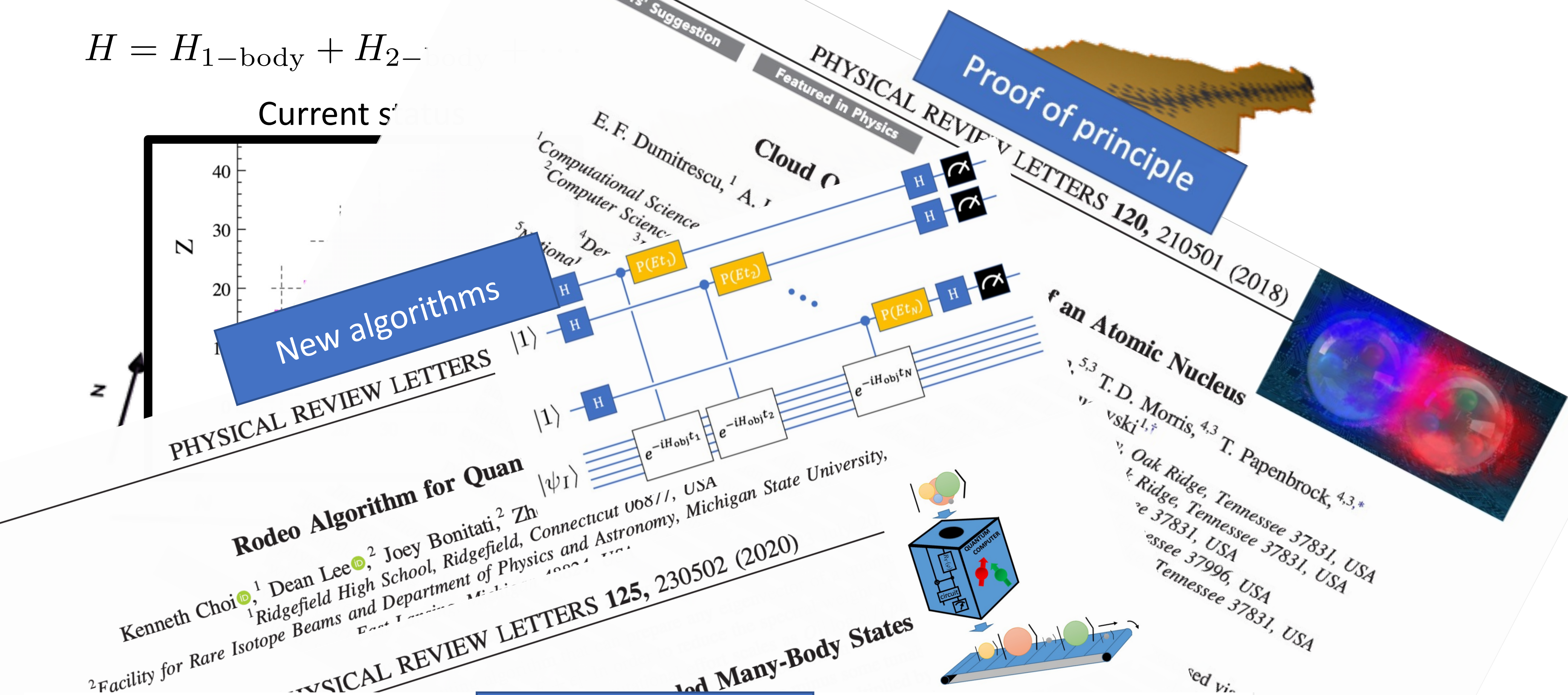
$$H = H_{1-\mathrm{body}} + H_{2-\mathrm{body}} + \cdots$$



Actual tendency : Towards Full configuration-Interaction description ?

$$H = H_{1-\text{body}} + H_{2-\text{body}} + \cdots$$

Current status



Editors' Suggestion

Featured in Physics

PHYSICAL REVIEW LETTERS 120, 210501 (2018)

Cloud Q

f an Atomic Nucleus

E. F. Dumitrescu,1 A. J

5,3 T. D. Morris,4,3 T. Papenbrock,4,3,*

wski1,†

1Computational Science

2Computer Scienc

Oak Ridge, Tennessee 37831, USA

Ridge, Tennessee 37831, USA

e 37831, USA

essee 37996, USA

Tennessee 37831, USA

PHYSICAL REVIEW LETTERS

Rodeo Algorithm for Quan

Kenneth Choi,1 Dean Lee,2 Joey Bonitati,2 Zh

1Ridgefield High School, Ridgefield, Connecticut 06877, USA

2Facility for Rare Isotope Beams and Department of Physics and Astronomy, Michigan State University,

YSICAL REVIEW LETTERS 125, 230502 (2020)

led Many-Body States

Systematic strategy

Nuclear shell-model simulation in digital quantum computers

A. Pérez-Obiol*

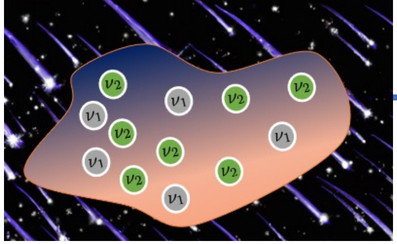
Barcelona Supercomputing Center, 08034 Barcelona, Spain

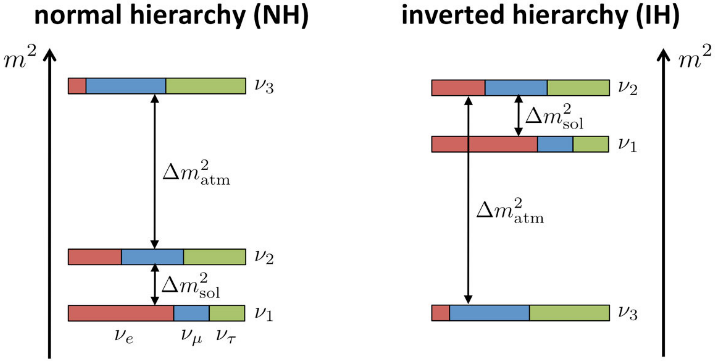
A. M. Romero,† J. Menéndez,‡ and A. Rios§

Departament de Física Quàntica i Astrofísica (FQA),

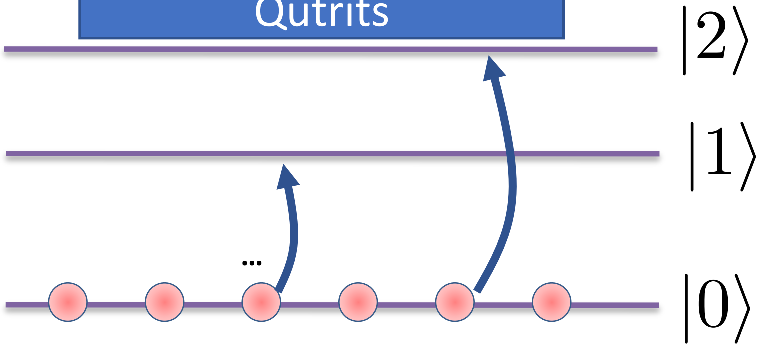
Universitat de Barcelona (UB), c. Martí i Franquès, 1, 08028, Barcelona, Spain

Neutrino oscillations

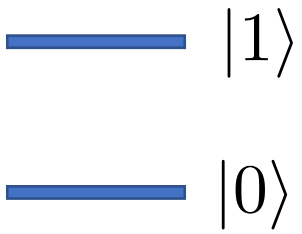




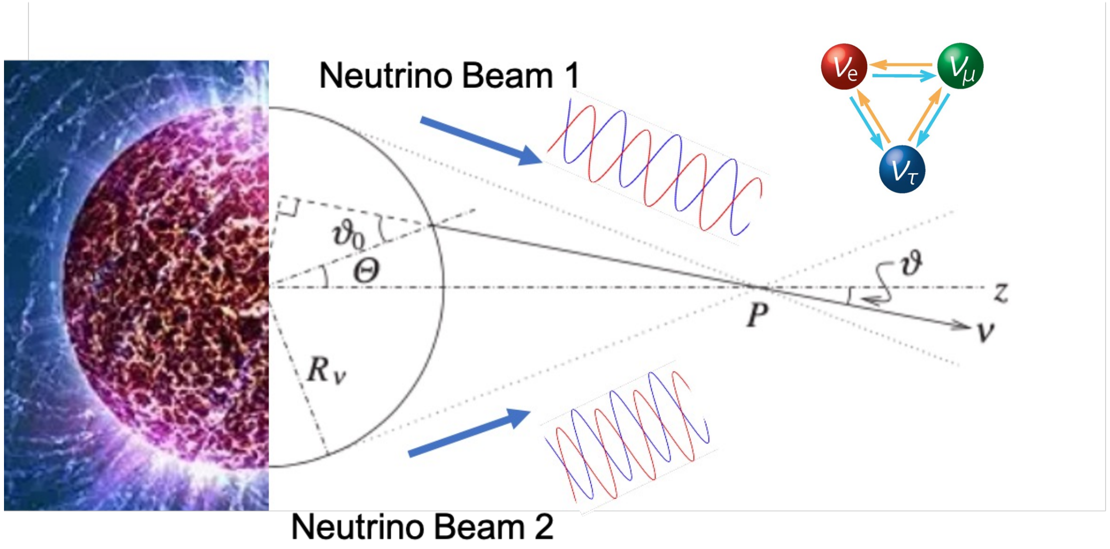
Natural treatment as Qutrits



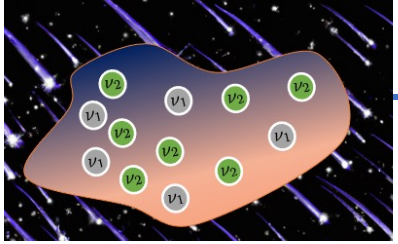
2-flavor approx. directly treated as Qubit



Neutrino oscillations in beams



Neutrino oscillations

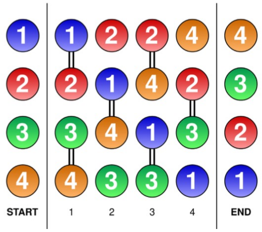


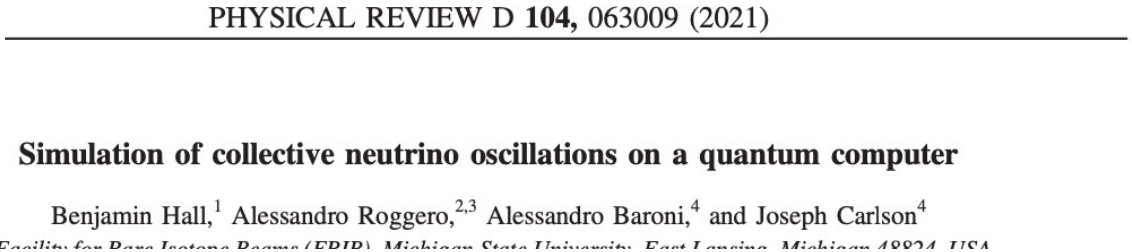
Nowadays: Increasing number of applications

2-flavor approx. directly treated as

- $|1\rangle$
- $|0\rangle$

Quantum Information And entanglement

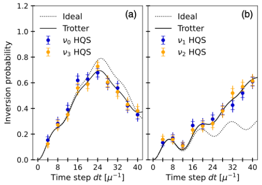


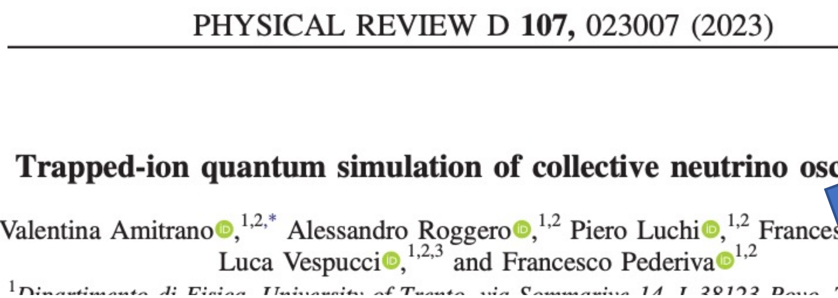

PHYSICAL REVIEW D **104,** 063009 (2021)

Simulation of collective neutrino oscillations on a quantum computer

Benjamin Hall,[1] Alessandro Roggero,[2,3] Alessandro Baroni,[4] and Joseph Carlson[4]

[1]Facility for Rare Isotope Beams (FRIB), Michigan State University, East Lansing, Michigan 48824, USA

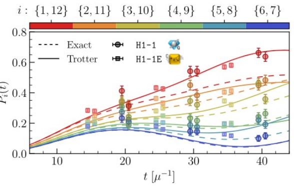


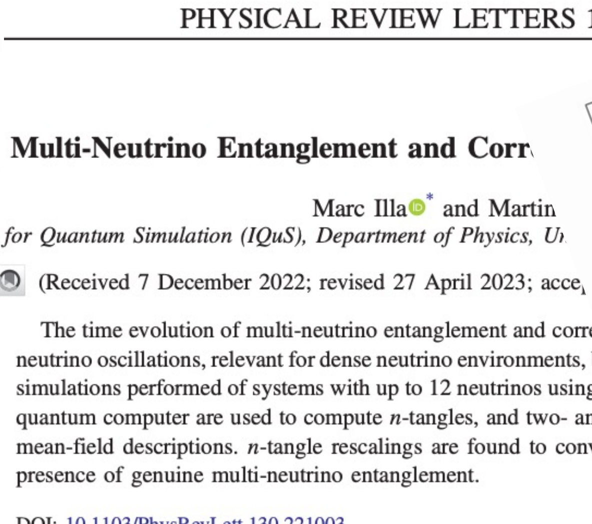

PHYSICAL REVIEW D **107,** 023007 (2023)

Trapped-ion quantum simulation of collective neutrino oscill

Valentina Amitrano,[1,2,*] Alessandro Roggero,[1,2] Piero Luchi,[1,2] Frances
Luca Vespucci,[1,2,3] and Francesco Pederiva[1,2]

[1]Dipartimento di Fisica, Università di Trento, via Sommarive 14, I-38123 Povo, Tr




PHYSICAL REVIEW LETTERS **130,** 221003

Multi-Neutrino Entanglement and Corr

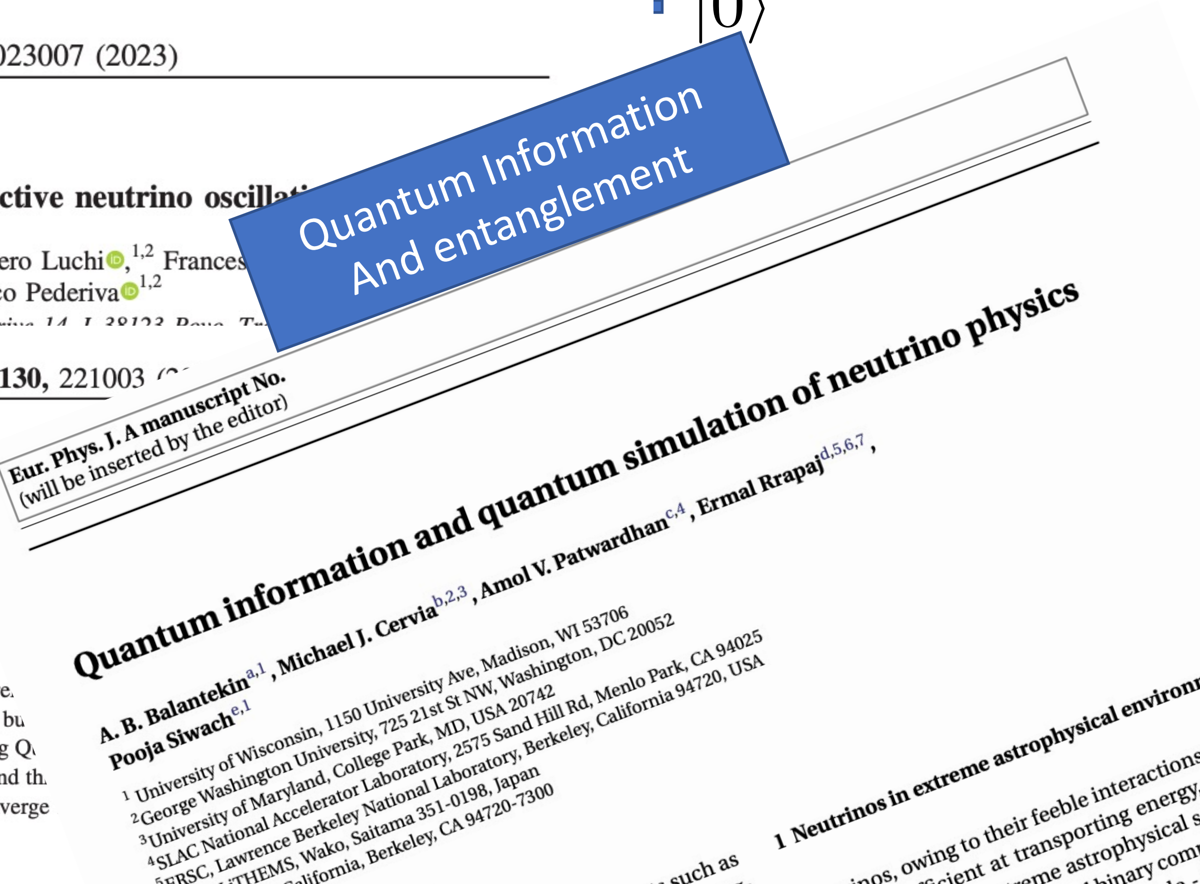
Marc Illa[*] and Martin

InQubator for Quantum Simulation (IQuS), Department of Physics, U

(Received 7 December 2022; revised 27 April 2023; acce

The time evolution of multi-neutrino entanglement and corre
neutrino oscillations, relevant for dense neutrino environments, bu
simulations performed of systems with up to 12 neutrinos using Q
quantum computer are used to compute n-tangles, and two- and th
mean-field descriptions. n-tangle rescalings are found to converge
presence of genuine multi-neutrino entanglement.

DOI: 10.1103/PhysRevLett.130.221003


Eur. Phys. J. A manuscript No.
(will be inserted by the editor)

Quantum information and quantum simulation of neutrino physics

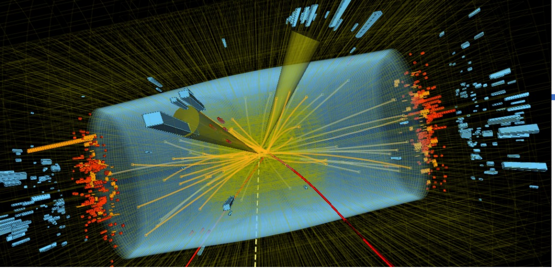
A. B. Balantekin[a,1], Michael J. Cervia[b,2,3], Amol V. Patwardhan[c,4], Ermal Rrapaj[d,5,6,7], Pooja Siwach[e,1]

[1] University of Wisconsin, 1150 University Ave, Madison, WI 53706
[2] George Washington University, 725 21st St NW, Washington, DC 20052
[3] University of Maryland, College Park, MD, USA 20742
[4] SLAC National Accelerator Laboratory, 2575 Sand Hill Rd, Menlo Park, CA 94025
[5] ERSC, Lawrence Berkeley National Laboratory, Berkeley, California 94720, USA
[6] iTHEMS, Wako, Saitama 351-0198, Japan
[7] California, Berkeley, CA 94720-7300

1 Neutrinos in extreme astrophysical environ

inos, owing to their feeble interactions
fficient at transporting energy
treme astrophysical s
binary com

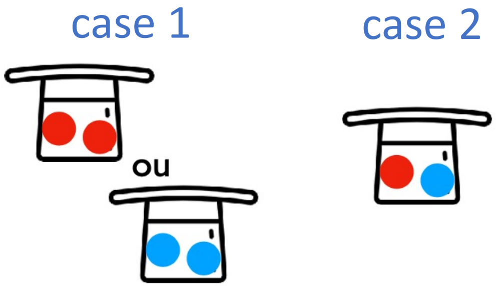
such as



Quantum Machine Learning
and event classification

Some quantum historical algorithms are very fast for pattern recognition.

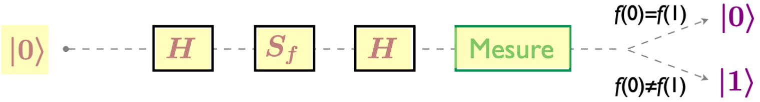
Deutsch (1985), Simon (1994), …

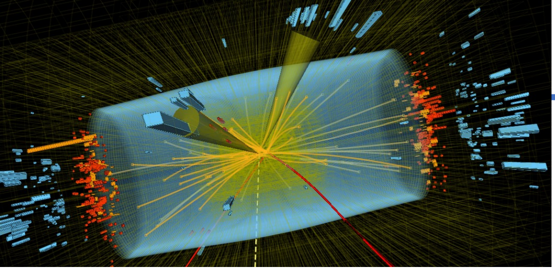


The “simple” Deutsch problem: $f:\{0,1\}\rightarrow\{0,1\}$ (Oracle)

Q: determine if $f(0)=f(1)$

- Classically requires to have 2 answers $f(0)$? $f(1)$?
- Quantum: one can directly ask $f(0)=f(1)$





Quantum Machine Learning
and event classification

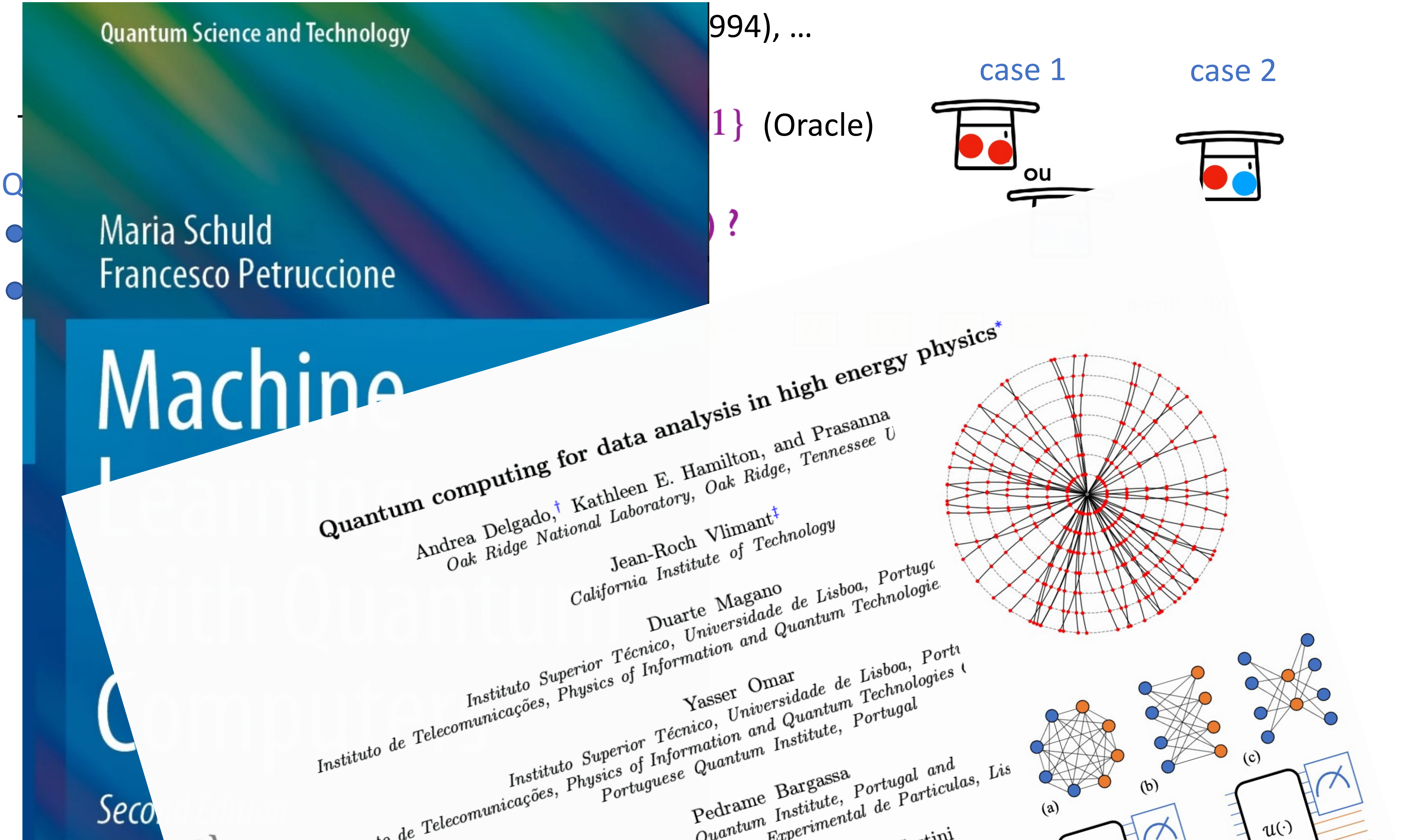
Some quantum historical algorithms are very fast for pattern recognition.

994), …

1 } (Oracle)

) ?

Q



Quantum Science and Technology

Maria Schuld
Francesco Petruccione

Machine
Learning
with Quantum
C

Seco

Quantum computing for data analysis in high energy physics*[*]

Andrea Delgado,[†] Kathleen E. Hamilton, and Prasanna
Oak Ridge National Laboratory, Oak Ridge, Tennessee U

Jean-Roch Vlimant[‡]
California Institute of Technology

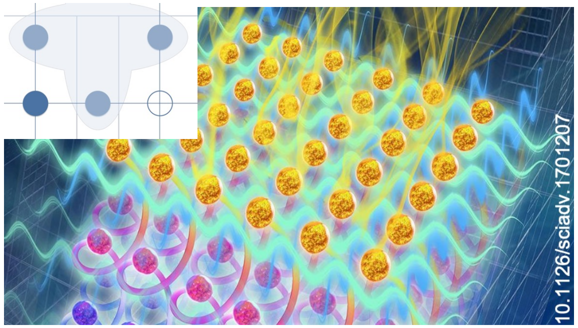
Duarte Magano
Instituto Superior Técnico, Universidade de Lisboa, Portug
Instituto de Telecomunicações, Physics of Information and Quantum Technologie

Yasser Omar
Instituto Superior Técnico, Universidade de Lisboa, Port
Instituto de Telecomunicações, Physics of Information and Quantum Technologies (
Portuguese Quantum Institute, Portugal

Pedrame Bargassa
Quantum Institute, Portugal and
Experimental de Particulas, Lis

Few initiated applications

Lattice gauge theories



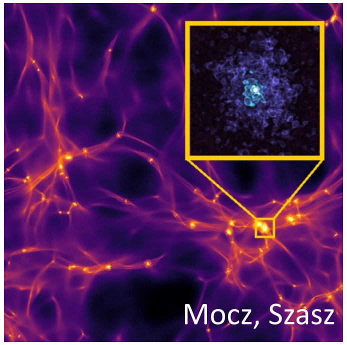
Zohar, Kolck, Savage, …

N-body problem

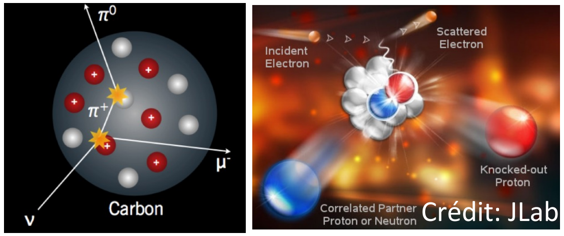
N-body nuclear systems

Dumitrescu, Hagen, Carlson, Papenbrock…

Dark matter



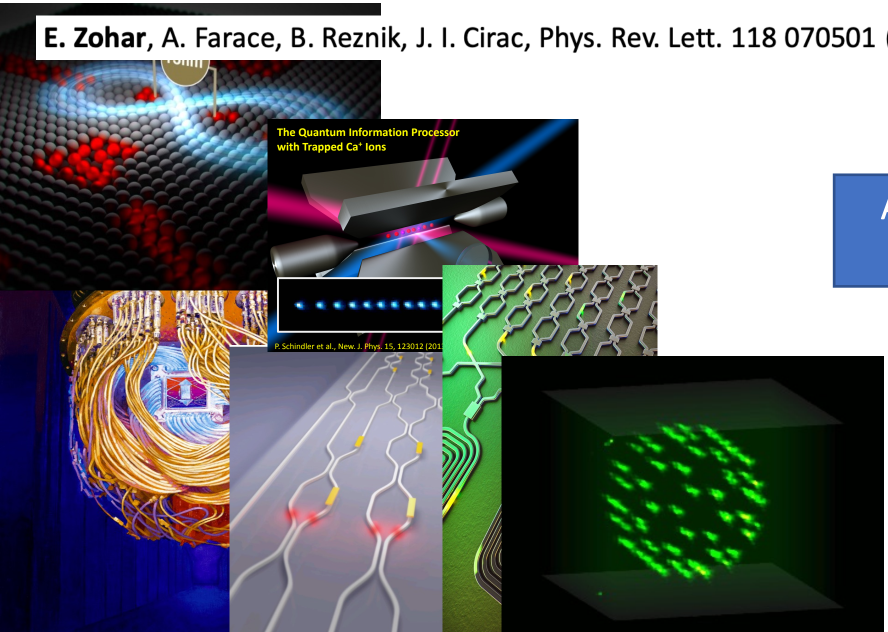
Dynamics: e, ν scattering



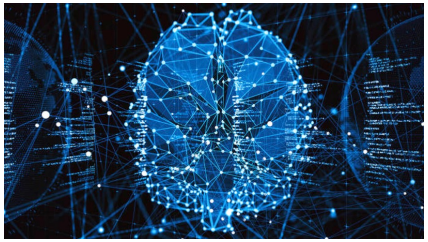
Roggero, Carlson, …

E. Zohar, J. I. Cirac, B. Reznik, Phys. Rev. Lett. **110**, 125304 (2013)
E. Zohar, J. I. Cirac, B. Reznik, Phys. Rev. A **88** 023617 (2013)
E. Zohar, J. I. Cirac, B. Reznik, Rep. Prog. Phys. **79**, 014401 (2016)
D. González Cuadra, **E. Zohar**, J. I. Cirac, New J. Phys. **19** 063038 (2017)

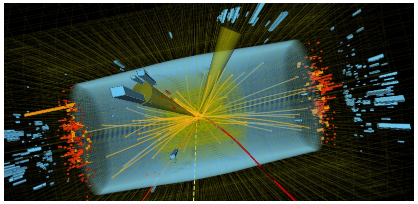
E. Zohar, A. Farace, B. Reznik, J. I. Cirac, Phys. Rev. Lett. 118 070501 (2017)



Applications to data mining (event classification)



CMS-detector



Summary

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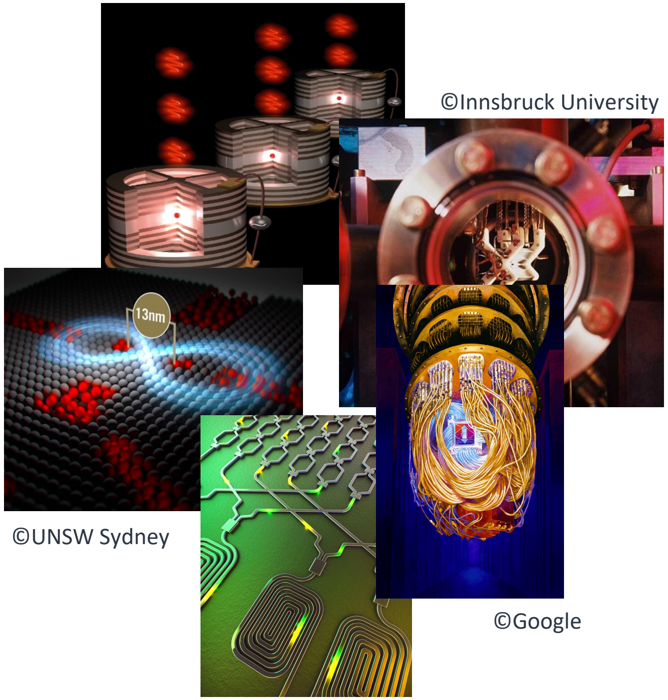

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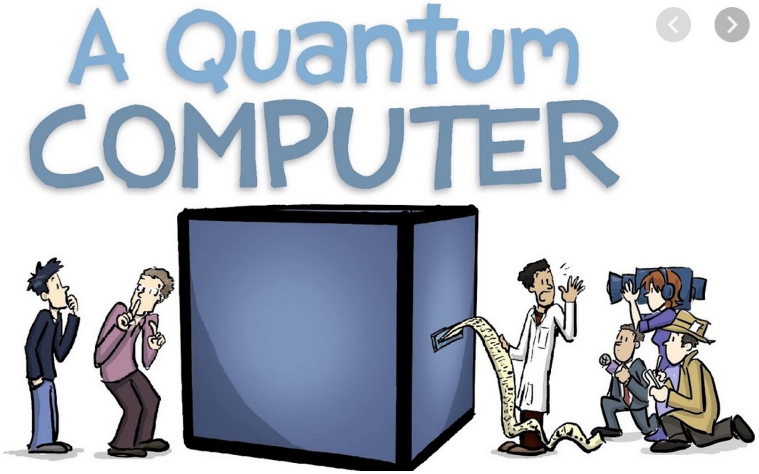
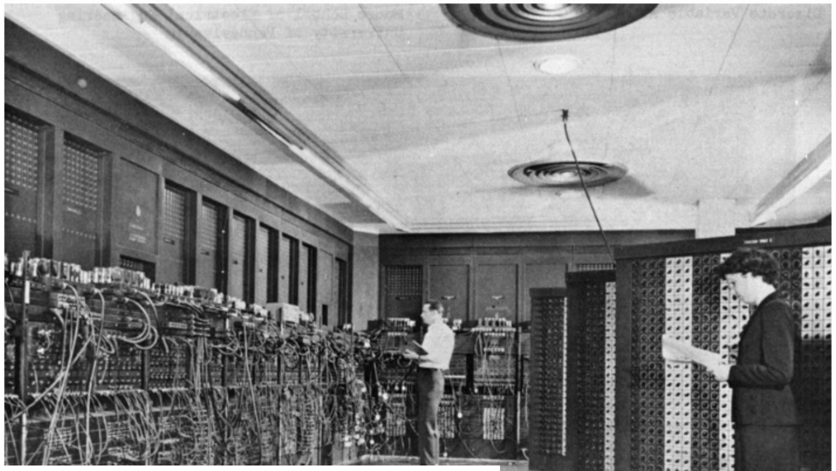
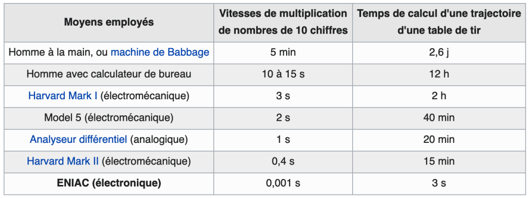
- Quantum computing is a high risk/high benefit interdisciplinary field
- It might lead to unprecedented boost in theory (or more generally in complex problems)
- It leads to natural link between public research and private companies (IBM, Google, …)
- Emerging QC programs in France



Eniac ~1950





Moyens employés	Vitesses de multiplication de nombres de 10 chiffres	Temps de calcul d'une trajectoire d'une table de tir
Homme à la main, ou machine de Babbage	5 min	2,6 j
Homme avec calculateur de bureau	10 à 15 s	12 h
Harvard Mark I (électromécanique)	3 s	2 h
Model 5 (électromécanique)	2 s	40 min
Analyseur différentiel (analogique)	1 s	20 min
Harvard Mark II (électromécanique)	0,4 s	15 min
ENIAC (électronique)	0,001 s	3 s

IBM ~2020

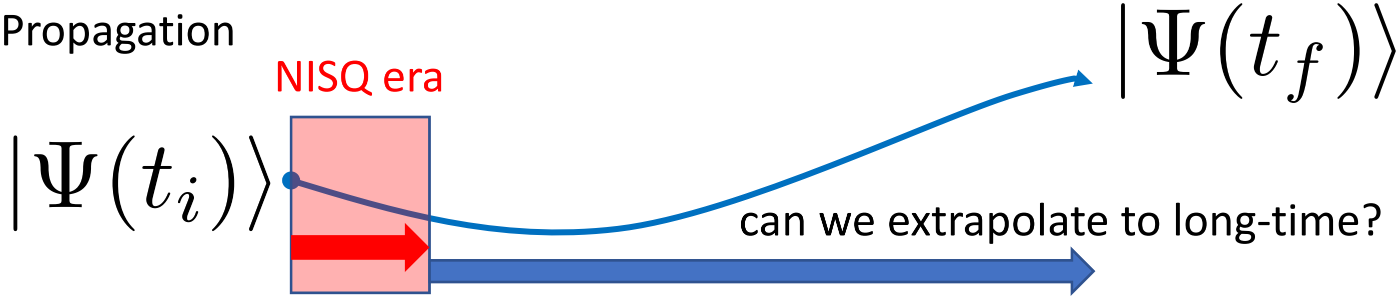
?

Thank you

Additional material

Predicting long time dynamics from short-time evolution

- Prediction of long time evolution from short-time Propagation



NISQ era

$|\Psi(t_i)\rangle$

$|\Psi(t_f)\rangle$

can we extrapolate to long-time?

What is the physical content of short-time evolution ?

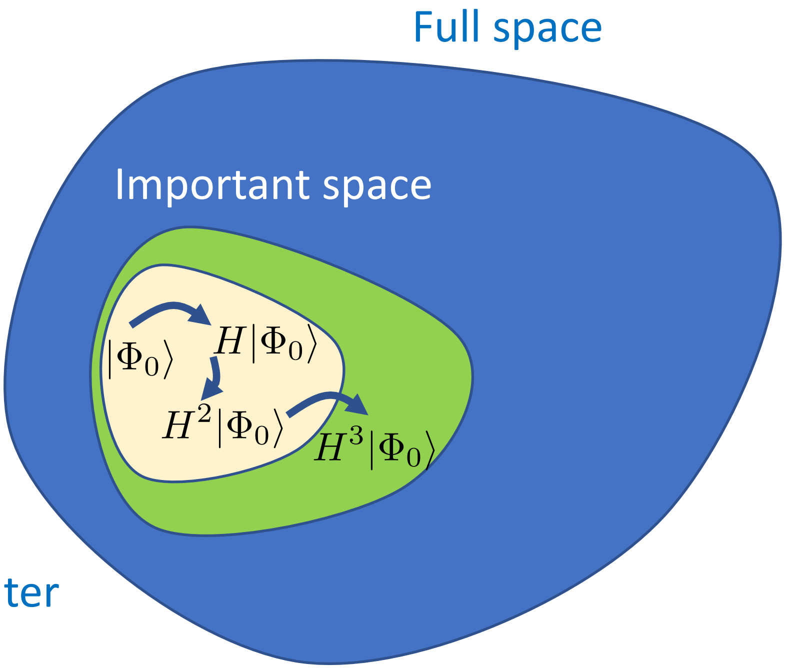
$$|\Phi(t)\rangle = \left(1 - itH + \frac{(-it)^2}{2!}H^2 + \cdots\right)|\Phi(0)\rangle$$

- $H^K|\Phi(0)\rangle$

Are the so-called Krylov states

But they cannot be computed easily on a quantum computer

- We propose instead to compute $\langle H^K\rangle_0$



Full space

Important space

$|\Phi_0\rangle$

$H|\Phi_0\rangle$

$H^2|\Phi_0\rangle$

$H^3|\Phi_0\rangle$

Hamiltonian moments calculation on a quantum computer

With minimal qubits number

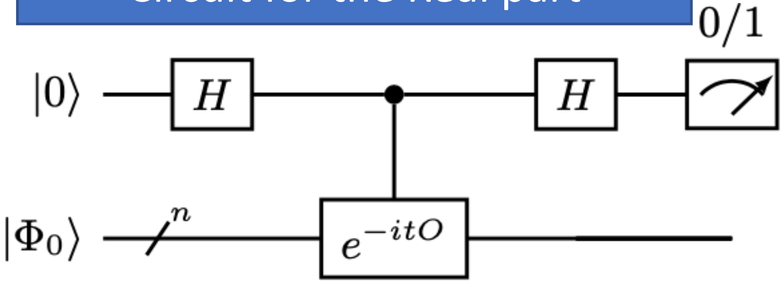
Generating function concept

$$F(t) = \langle \Phi_0 | e^{-itH} | \Phi_0 \rangle$$

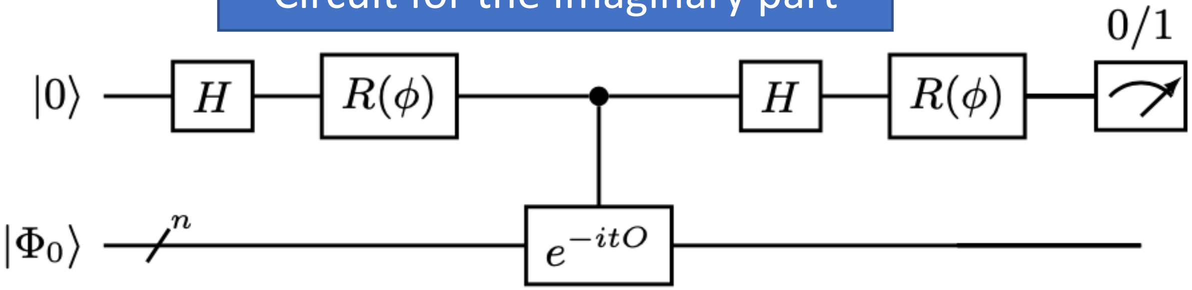
$$F(t) = 1 - it\langle H\rangle_0 + \frac{(-it)^2}{2}\langle H^2\rangle_0 + \cdots \Rightarrow \langle H^K\rangle_0 = i^K \left.\frac{d^K F(t)}{dt^K}\right|_{t=0}$$

Practical method to get *F(t)*

Circuit for the Real part



Circuit for the Imaginary part



Hamiltonian moments calculation on a quantum computer

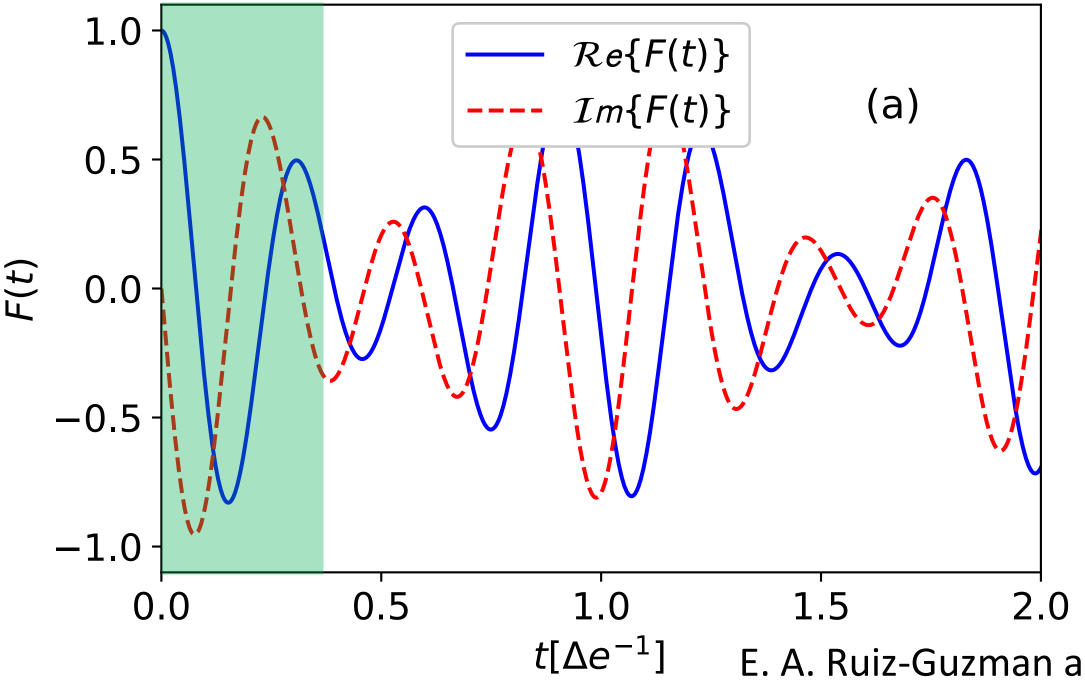
With minimal qubits number

Generating function concept

$$F(t) = \langle \Phi_0 | e^{-itH} | \Phi_0 \rangle$$

$$F(t) = 1 - it\langle H \rangle_0 + \frac{(-it)^2}{2}\langle H^2 \rangle_0 + \cdots \Rightarrow \langle H^K \rangle_0 = i^K \left. \frac{d^K F(t)}{dt^K} \right|_{t=0}$$

Illustration for the cooper pair problem



finite difference made on a classical computer

$\Rightarrow \langle H^K \rangle_0$

E. A. Ruiz-Guzman and DL, in preparation

Next use the moments for post-processing

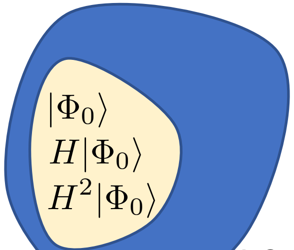
$\langle H^K \rangle_0$

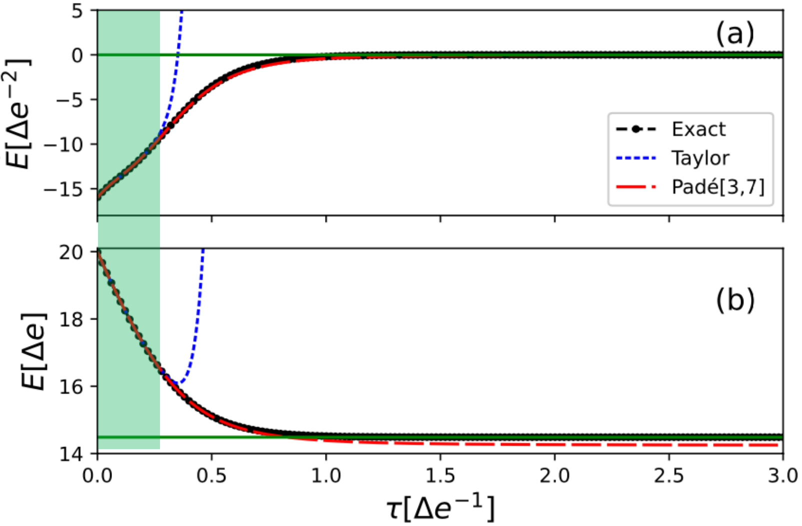
Ground state property (imaginary time evolution)

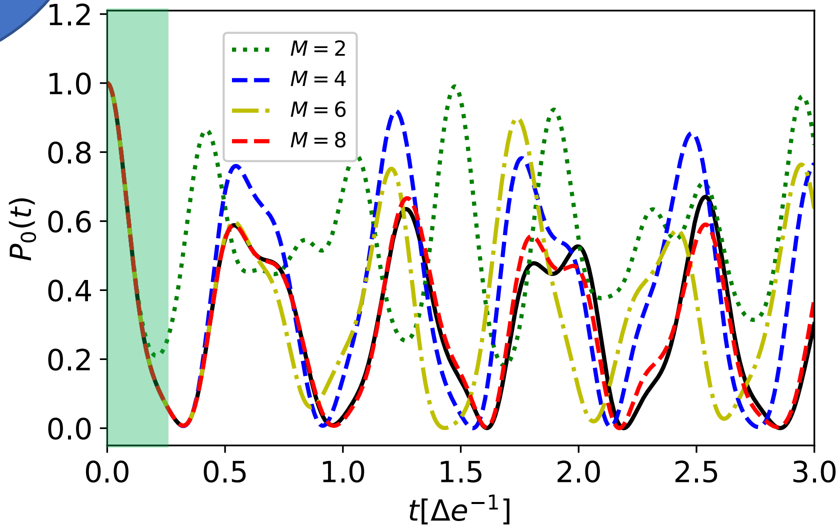
Evolution: Krylov without Krylov states

$$E(\tau) = \frac{\langle He^{-\tau H}\rangle}{\langle e^{-\tau H}\rangle}$$

$$\frac{d}{d\tau}E(\tau) \simeq -\sum_{K=0}^{L-2}\frac{(-\tau)^K}{K!}\kappa_{K+2}$$







E. A. Ruiz-Guzman and DL, in preparation

Illustration of the specific effort to make with qubits

Playing with binary numbers

Integer representation

Binary representation

$$j \xrightarrow{j = j_1 2^{n-1} + j_2 j^{n-2} \cdots j_n 2^0} \{j_1, \cdots, j_n\}$$

Binary fraction

$$\frac{j}{2^n} \longrightarrow 0.j_1 \cdots j_n = \frac{j_1}{2} + \frac{j_2}{2^2} + \cdots + \frac{j_n}{2^n}$$

Note that this is just the equivalent to our base 10 writing

$$0.j_1 \cdots j_n = \frac{j_1}{10} + \frac{j_2}{10^2} + \cdots + \frac{j_n}{10^n}$$

Example

$$\begin{aligned} 0.1011 &\rightarrow 1 \times 0.5 + 0 \times 0.25 + 1 \times 0.125 + 1 \times 0.0625 \\ &\rightarrow 0.6875 \end{aligned}$$

Approximation of 0.6875 can be obtained by removing some terms

$$\begin{aligned} 0.101 &\rightarrow 0.625 \\ 0.1 &\rightarrow 0.5 \end{aligned}$$

$$\begin{aligned} 2^{-1} &= 0.5 \\ 2^{-2} &= 0.25 \\ 2^{-3} &= 0.125 \\ 2^{-4} &= 0.0625 \\ 2^{-5} &= 0.03125 \\ 2^{-6} &= 0.015625 \\ 2^{-7} &= 0.0078125 \\ &\cdots \end{aligned}$$

Illustration of the specific effort to make with qubits

A seminal example: the Quantum Fourier Transform (QFT)

Reminder: classical FT

Assume a set of values $\{x_0, \cdots, x_{N-1}\}$, we can define a set of new numbers $\{f_0, \cdots, f_{N-1}\}$ such that:

$$f_k = \frac{1}{\sqrt{N}} \sum_{j=0}^{N-1} x_k e^{2i\pi k(j/N)}$$

Then the inverse Fourier transform is given by^1:

$$x_j = \frac{1}{\sqrt{N}} \sum_{k=0}^{N-1} f_k e^{-2i\pi j(k/N)}$$

Quantum FT

$$|j\rangle \rightarrow |\tilde{j}\rangle \equiv \frac{1}{\sqrt{N}} \sum_{k=0}^{N-1} e^{2i\pi\alpha(k/N)}|k\rangle$$

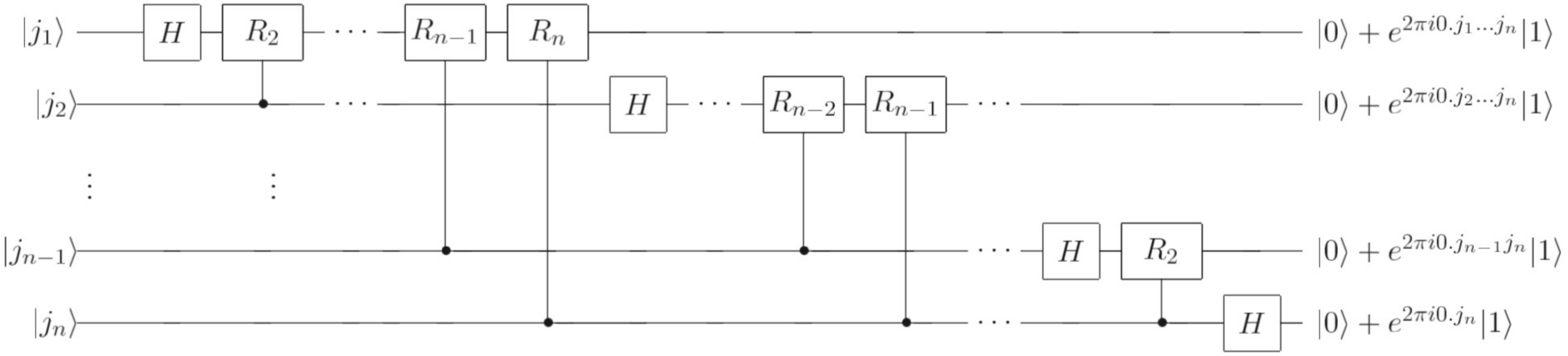
Qubit FT

$$\begin{aligned} |j\rangle = |j_{n-1}\cdots j_0\rangle \;\rightarrow\; |\tilde{j}\rangle &= \frac{1}{2^{n/2}} \sum_{k=0}^{n-1} e^{2\pi i j k/2^n}|k\rangle \\ &= \frac{1}{2^{n/2}} \bigotimes_{l=1}^{n} \left[|0\rangle + e^{2\pi i j 2^{-l}}|1\rangle\right] \\ &= \frac{1}{2^{n/2}} \left(|0\rangle + e^{2\pi i 0.j_0}|1\rangle\right)\left(|0\rangle + e^{2\pi i 0.j_1 j_0}|1\rangle\right)\cdots\left(|0\rangle + e^{2\pi i 0.j_{n-1}\cdots j_1 j_0}|1\rangle\right) \end{aligned}$$

Quantum circuits for Qubit FT

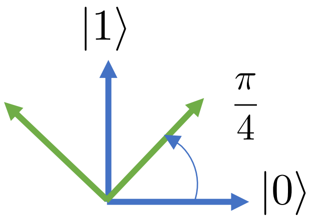
$$|j\rangle \rightarrow \frac{1}{2^n}\bigotimes_{l=1}^{n}\left[|0\rangle + e^{2\pi i j 2^{-l}}|1\rangle\right]$$

Quantum circuits



H-gate (unary gate)

Controlled –U gate (binary gate)



$$H = \frac{1}{\sqrt{2}}\begin{bmatrix} 1 & 1 \\ 1 & -1 \end{bmatrix}$$

$$Controlled-U = \begin{array}{cc} \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & U_{11} & U_{12} \\ 0 & 0 & U_{21} & U_{22} \end{bmatrix} & \begin{array}{l} |00\rangle \\ |01\rangle \\ |10\rangle \\ |11\rangle \end{array} \end{array}$$

$$R_k = \begin{bmatrix} 1 & 0 \\ 0 & e^{2\pi i/2^k} \end{bmatrix}$$

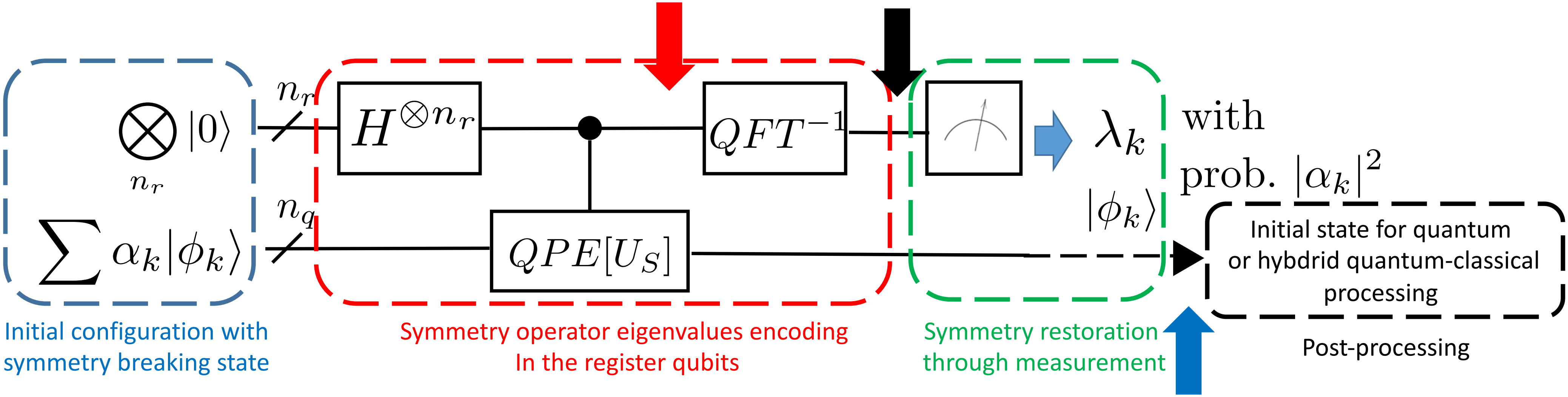
The quantum-Phase estimate (QPE) algorithm

For eigenvalue problems

Full protocol

here give access
to the generating function
of the moments

Here give access to the probability
Distribution *P(S)*



Initial configuration with
symmetry breaking state

Symmetry operator eigenvalues encoding
In the register qubits

Symmetry restoration
through measurement

Post-processing

After measurement
The state is projected
on eigenstates or a set of
Eigenstates if degenerated

Important remarks

In practice, for each measurement we obtain a binary string

$$\{0110\cdots\}_{n_r} \Rightarrow \theta_k \Rightarrow \left|2^{n_r}\theta_k\right\rangle \otimes \left|\phi_k\right\rangle \Rightarrow \left|\phi_k\right\rangle$$ for post-processing

Main idea of the paper arxiv:2006.06491

Use the QPE approach for operators with known eigenvalues to obtain entangled states

Hypothesis:

- Assume a hermitian operator *S* acting on *nq* qubits
- Assume that *S* has discrete eigenvalues $\{\lambda_0 \leq \cdots \leq \lambda_M\}$ with $\lambda_k = a m_k$
  $a = \text{cst}$
- Define the operator

$$U_S = \exp\left\{2\pi i \left[\frac{S - \gamma_0}{a 2^{n_0}}\right]\right\}$$

- Eigenvalues of U_S are given by $e^{2\pi i \theta_k}$ with $\theta_k = (m_k - m_0)/2^{n_0}$

If $(m_k - m_0) < 2^{n_0}$ ➡ $\theta_k < 1$

and θ_k is exactly written as a binary fraction

Examples

- parity
- Part. number
- $J_z = \hbar m$
- $J^2 = \hbar^2 j(j+1)$

➡ It is then optimal for the QPE use.
An optimal choice for the number of register qubits is $n_r = n_0$

and $n_r - 1 \leq \ln(m_k - m_0)/\ln 2 < n_r.$

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Examples

- parity
- Part. number
- $J_z = \hbar m$
- $J^2 = \hbar^2 j(j+1)$

Illustration of the Jordan-Wigner use

Quantum Computing notations

$$\{\sigma_x, \sigma_y, \sigma_z\} \Rightarrow \{X, Y, Z\}$$

Reminder

$$\begin{aligned} X &= \begin{bmatrix} 0 & 1 \\ 1 & 0 \end{bmatrix} = |1\rangle\langle 0| + |0\rangle\langle 1|, \\ Y &= i\begin{bmatrix} 0 & -1 \\ 1 & 0 \end{bmatrix} = i|1\rangle\langle 0| - i|0\rangle\langle 1| \\ Z &= \begin{bmatrix} 1 & 0 \\ 0 & -1 \end{bmatrix} = |0\rangle\langle 0| - |1\rangle\langle 1| \end{aligned}$$

Convention:

$$Q_j^+ = \frac{1}{2}(X_j - iY_j) \Rightarrow Q_j^+|0_j\rangle = |1_j\rangle$$

Example: particle number

$$\hat{N} = \sum_j a_j^\dagger a_j \xrightarrow[Z_j^2]{a_k^\dagger \to \prod_{k<j}(-Z_k)Q_j^+} N = \sum_j Q_j^+ Q_j = \frac{1}{2}\sum_j (I_j - Z_j) \Rightarrow \begin{aligned} Q_j^+ Q_j|0_j\rangle &= 0 \\ Q_j^+ Q_j|1_j\rangle &= |1_j\rangle \end{aligned}$$

N counts the number n of 1 in qubits

Particle number projection with the QPE

For n_q qubits, $n = 0, \cdots, n_q$

$$U_N = \exp\{2\pi i N/2^{n_r}\}$$

With the Condition $n_q <\sim 2^{n_r}$

$n_q = 1 \rightarrow n_r = 1$
$2 \leq n_q < 3 \rightarrow 2$
$4 \leq n_q < 7 \rightarrow 3$
$8 \leq n_q < 15 \rightarrow 4$
$16 \leq n_q < 31 \rightarrow 5$
$32 \leq n_q < 63 \rightarrow 6$
$64 \leq n_q < 127 \rightarrow 7$
$128 \leq n_q < 255 \rightarrow 8$
$255 \leq n_q < 511 \rightarrow 9$
$512 \leq n_q < 1023 \rightarrow 10$

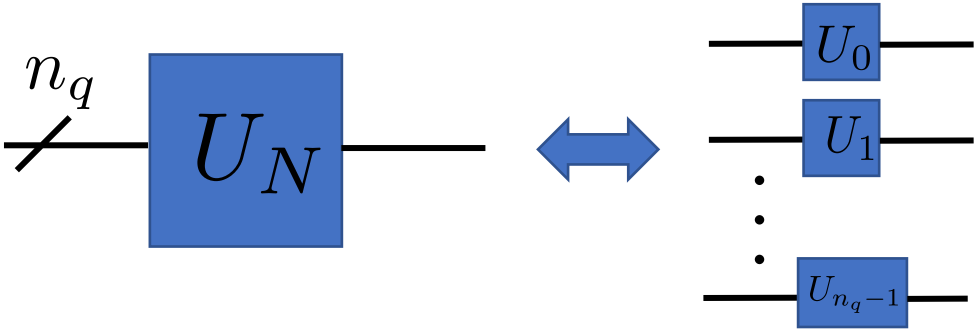
QPE applied to particle number projection

Practical details

$$U_N = \prod_j U_j$$

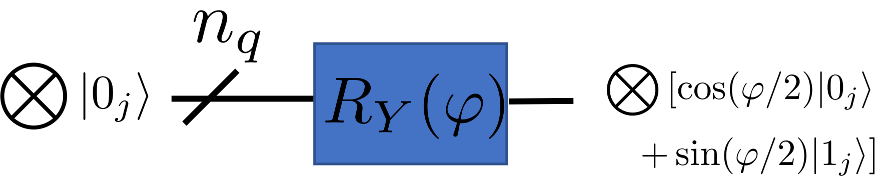
$$U_i = |0_i\rangle\langle 0_i| + \exp(i\pi/2^{n_0-1})|1_i\rangle\langle 1_i|$$

$$U_i = \begin{bmatrix} 1 & 0 \\ 0 & e^{i\pi/2^{n_0-1}} \end{bmatrix}$$



Example: Qubit counting statistics

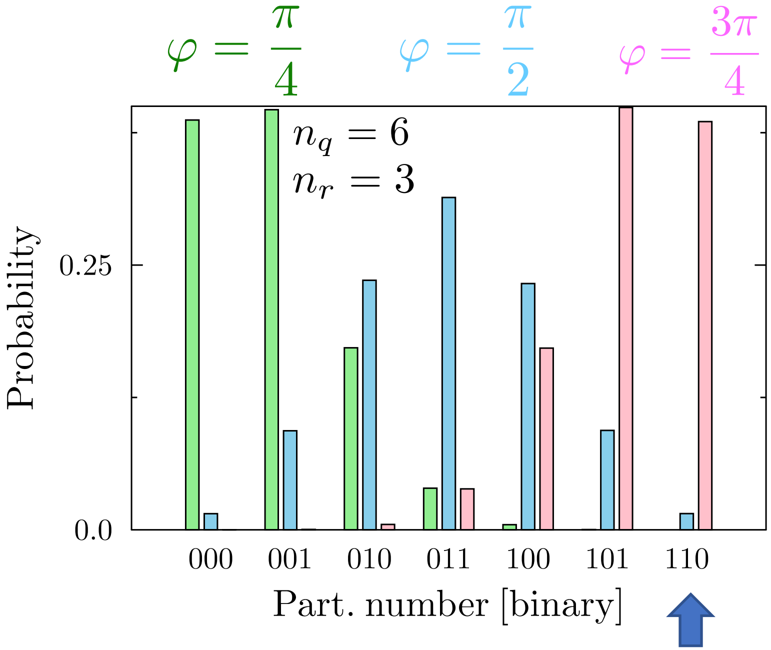
Initial state



$$P(A) = C_{n_q}^{A} p^A (1-p)^{n_q-A}$$

$$p = \sin^2(\varphi/2)$$

Calculation made with the IBM Qiskit python package



$$6/2^4 = 1/2 + 1/4 + 0/8 \equiv [110]$$

Applying the strategy to the pairing problem

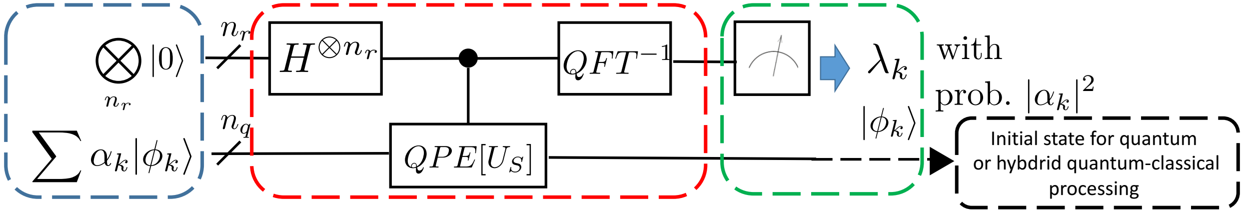
$\bigotimes |0_j\rangle$

Symmetry-breaking state

QPE+QFT^{-1}

Register Qubit Measurement

Compute the Energy on a classical comp.



Initial state for quantum or hybdrid quantum-classical processing

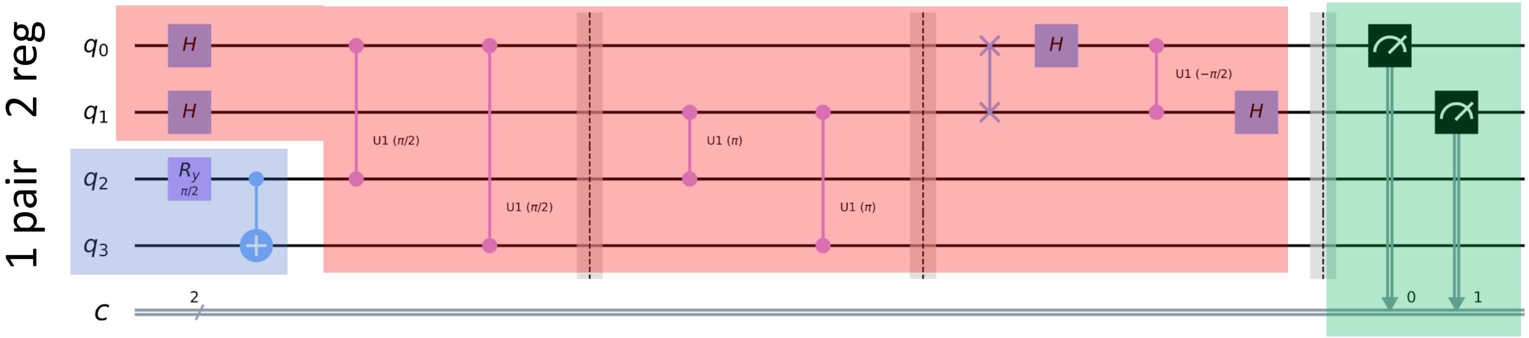
Initial configuration with symmetry breaking state

Symmetry operator eigenvalues encoding In the register qubits

Symmetry restoration through measurement

Post-processing

Qiskit circuit for a single pair



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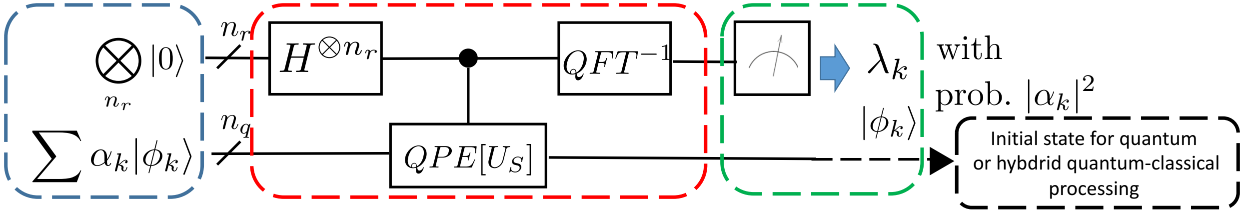


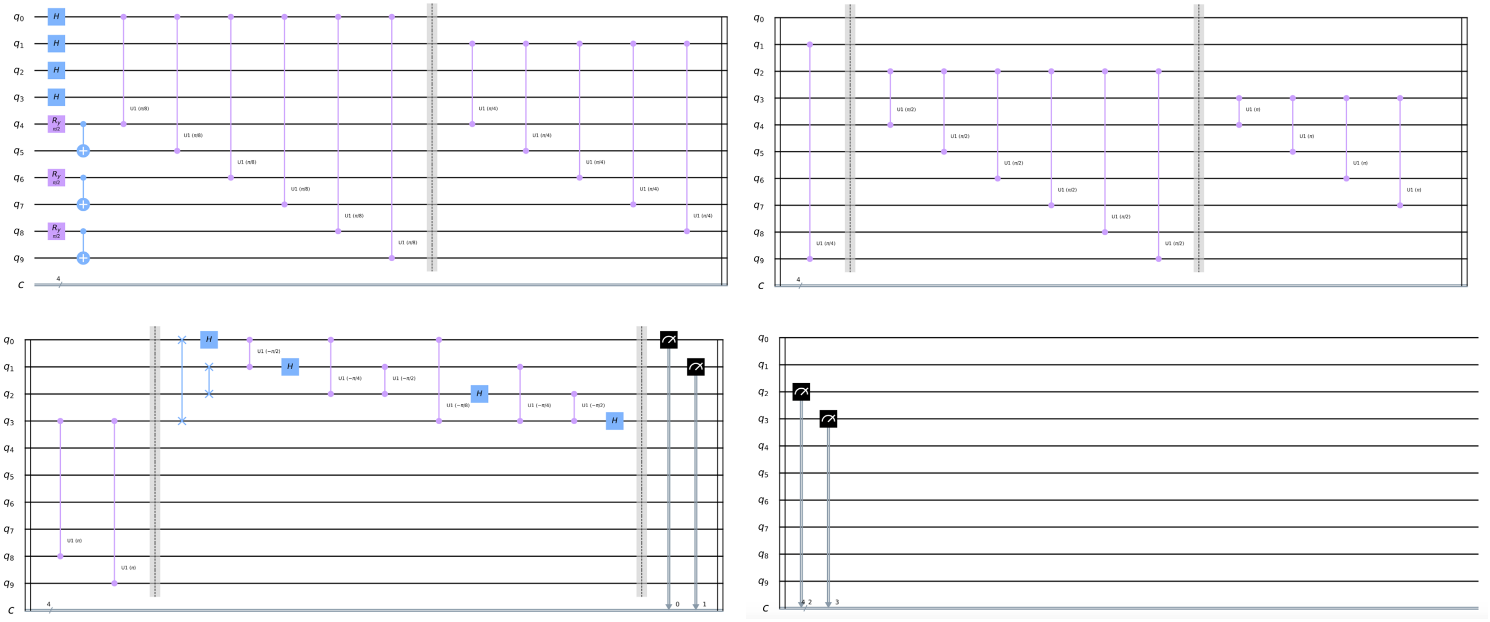
Initial configuration with symmetry breaking state

Symmetry operator eigenvalues encoding In the register qubits

Symmetry restoration through measurement

Post-processing

3 pairs, 4 register



Applying the strategy to the pairing problem

$\bigotimes |0_j\rangle$

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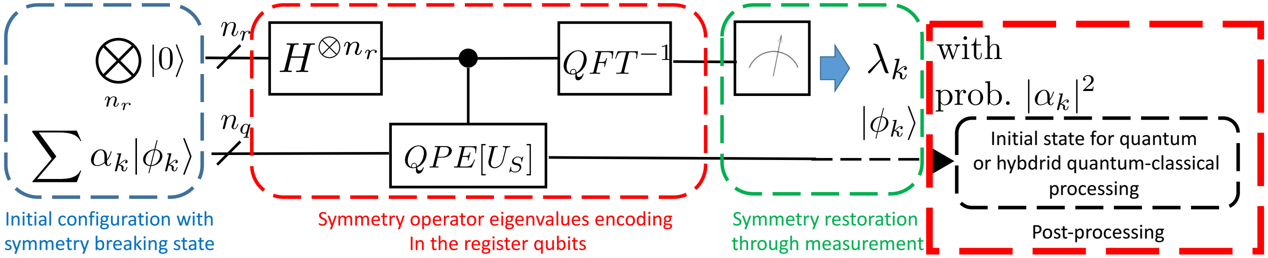
QPE+QFT^{-1}

Register Qubit Measurement

Compute the Energy on a classical comp.



Initial configuration with symmetry breaking state

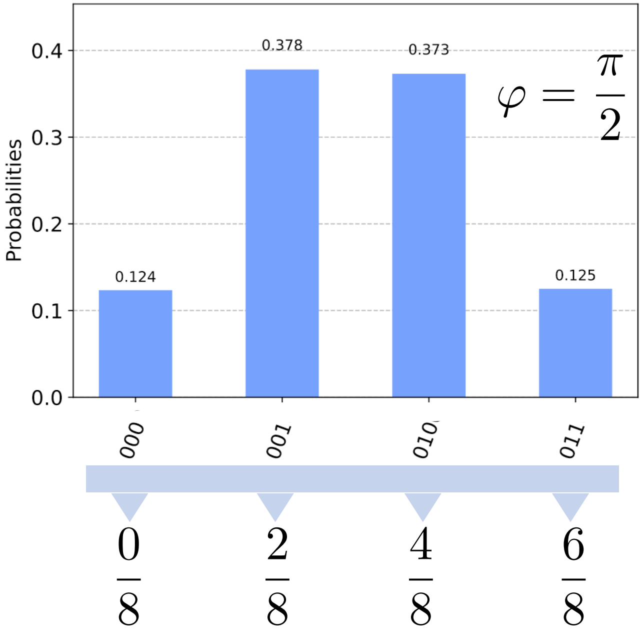
Symmetry operator eigenvalues encoding In the register qubits

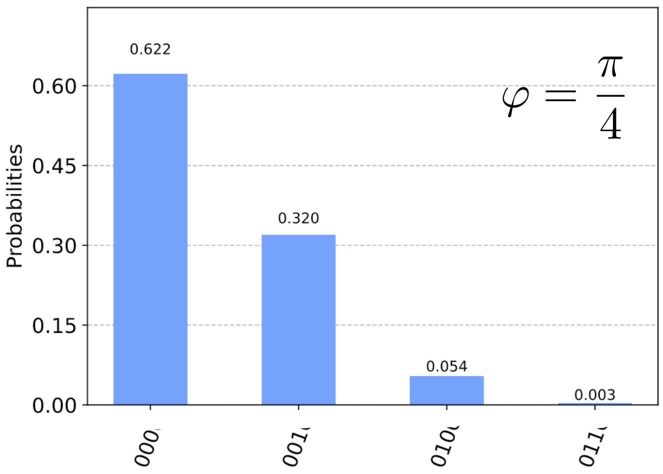
Symmetry restoration through measurement

Qubit counting statistics

Initial state $$|\Psi\rangle = \prod_n \left[\cos\left(\frac{\varphi}{2}\right) I_n \otimes I_{n+1} + \sin\left(\frac{\varphi}{2}\right) Q_n^+ Q_{n+1}^+\right] |-\rangle$$

Example: 3 pairs, 3 registers



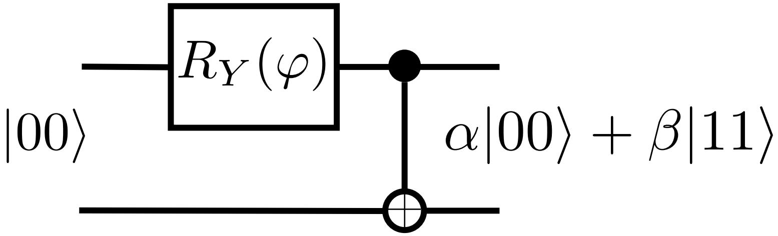


Exploring the possibilities of QC

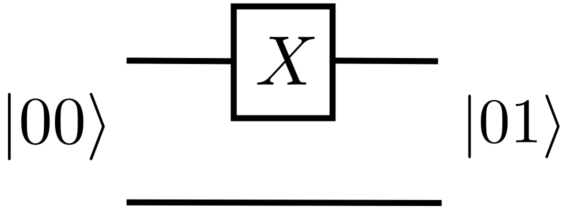
Using the Broken pair approximation

BCS/HFB state

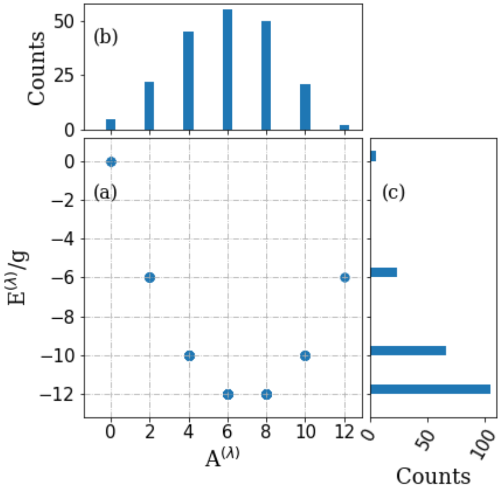
All pairs= generalized Bell state



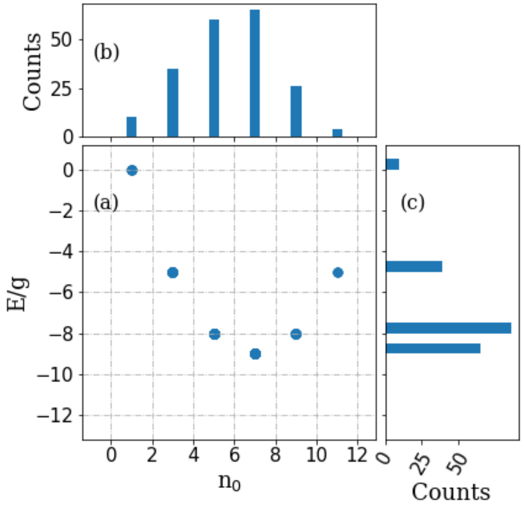
BCS/HFB state + some broken pairs



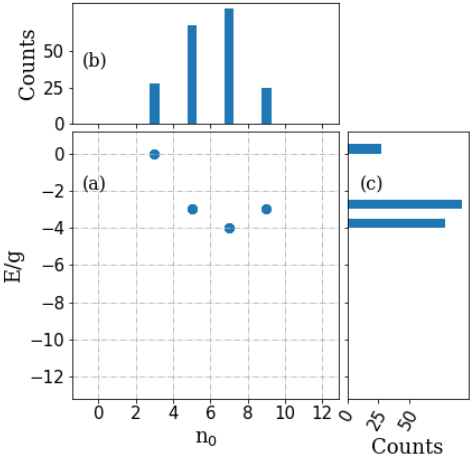
No broken pairs



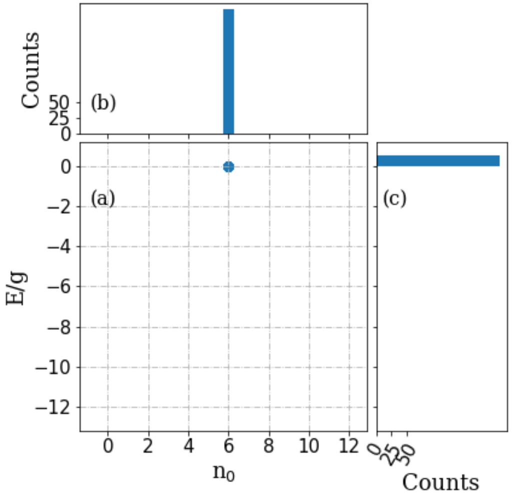
1 broken pairs



3 broken pairs



6 broken pairs

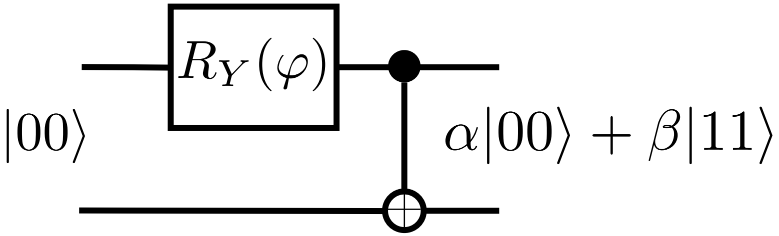


Exploring the possibilities of QC

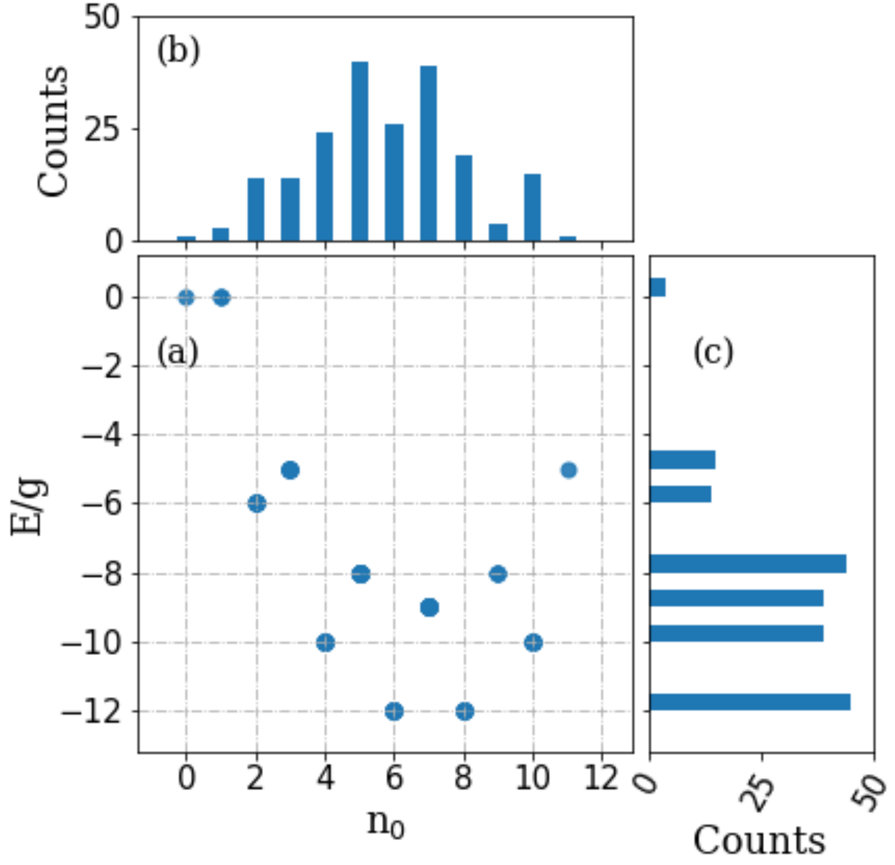
Some additional remarks

BCS/HFB state

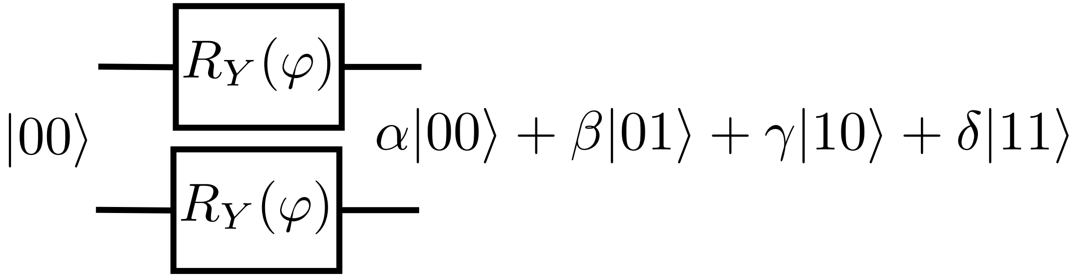
All pairs= generalized Bell state



Can give both odd and even simultaneously



Alternative circuits

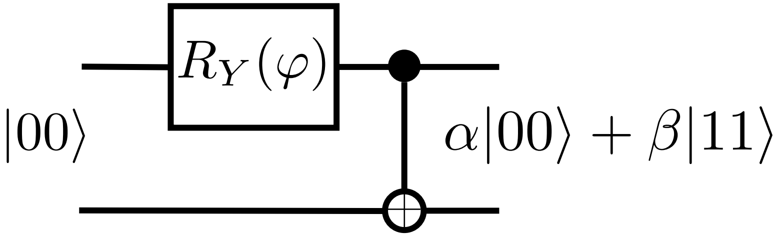


Exploring the possibilities of QC

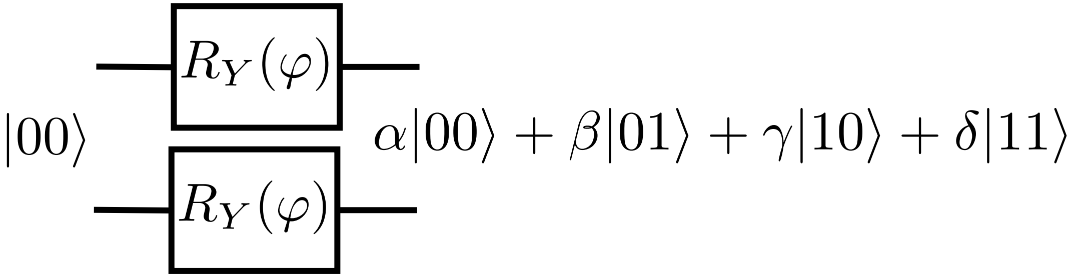
Some additional remarks

BCS/HFB state

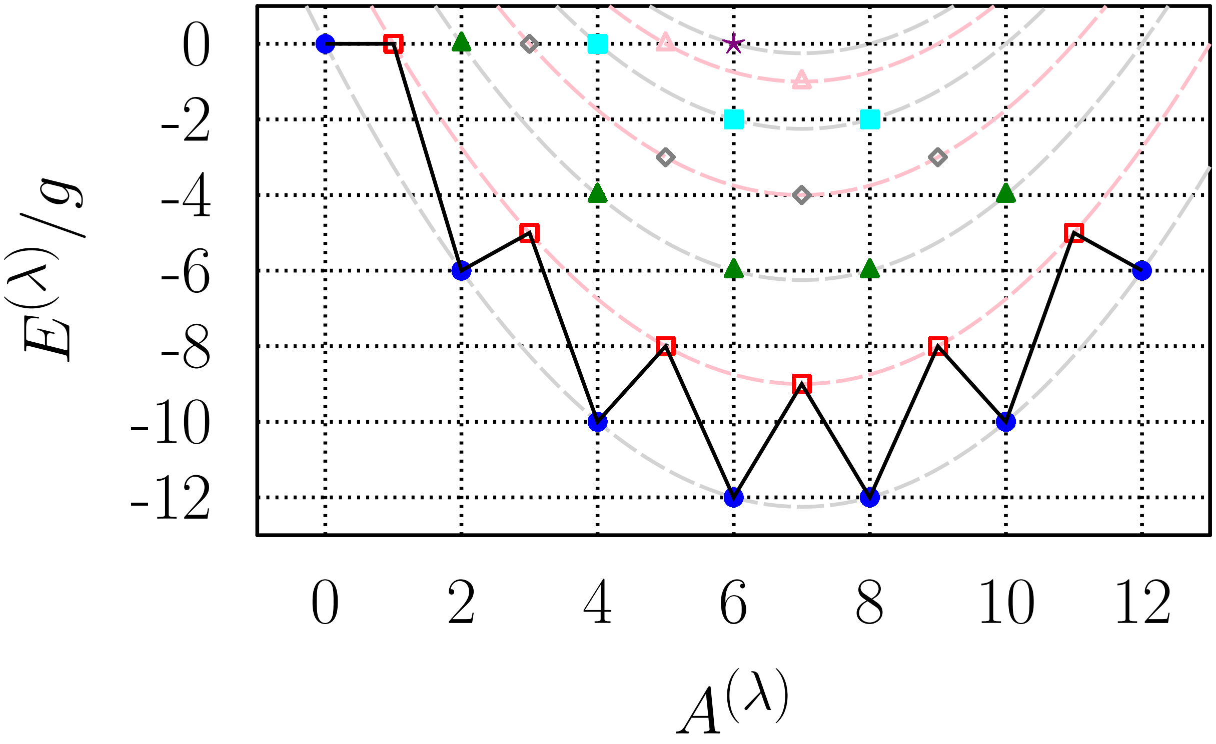
All pairs= generalized Bell state



Alternative circuits



Altogether

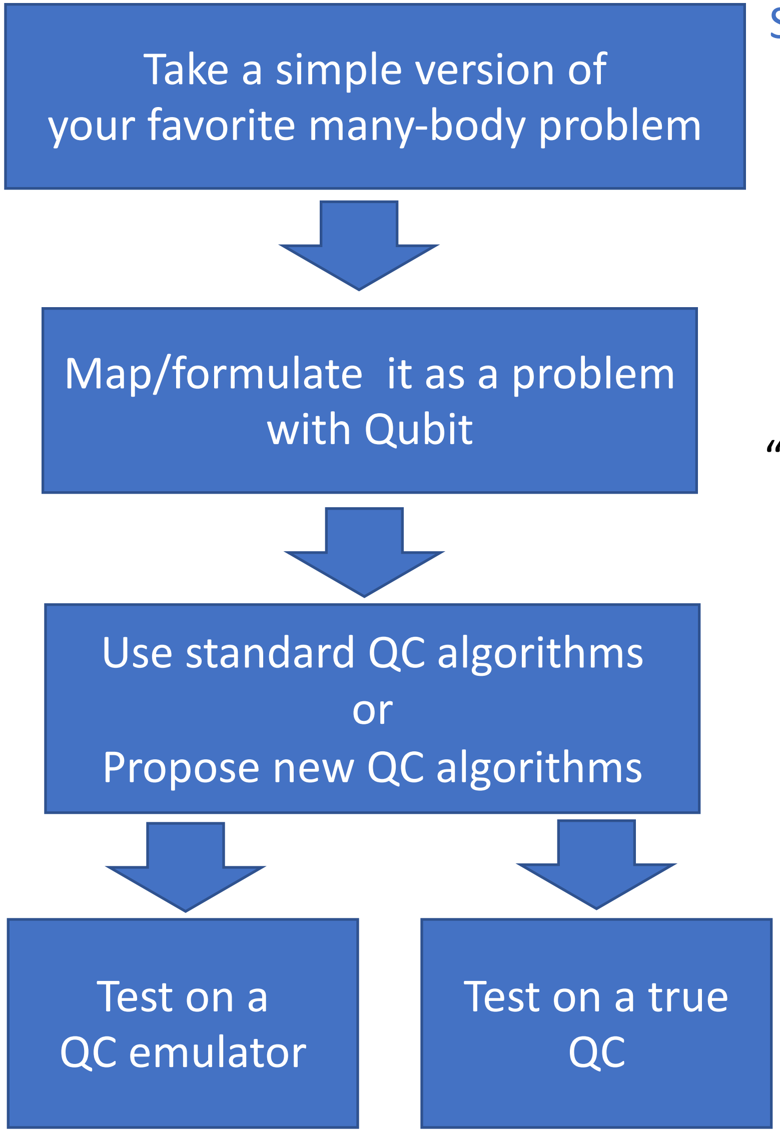


Exact solution (lines)

$$E/g = -\frac{1}{4}(A-\nu)(2n_q - A - \nu - 2).$$

Illustration II

Strategy



Take a simple version of your favorite many-body problem

Map/formulate it as a problem with Qubit

Use standard QC algorithms
or
Propose new QC algorithms

Test on a QC emulator

Test on a true QC

PHYSICAL REVIEW LETTERS **123,** 090501 (2019)

σ Models on Quantum Computers

Andrei Alexandru,[1,2,*] Paulo F. Bedaque,[2,†] Henry Lamm,[2,‡] and Scott Lawrence[2,§]

(NuQS Collaboration)

Start with the discretized σ model

$$\mathcal{H} = \sum_r \left(\frac{g^2}{2}\boldsymbol{\pi}(r)^2 + \frac{1}{2g^2\Delta x^2}[\mathbf{n}(r+1)-\mathbf{n}(r)]^2\right)$$

Map it to a Spin algebra (fuzzy sphere)

$$-\frac{g^2}{2}\nabla^2\psi \to H^0\Psi = \kappa\frac{g^2}{2}\sum_{k=1}^{3}[\mathbb{J}_k,[\mathbb{J}_k,\Psi]],$$

J_k are generators of the SU(2) algebra

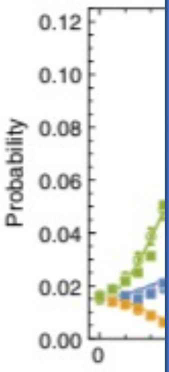
“only” j=1/2 was considered

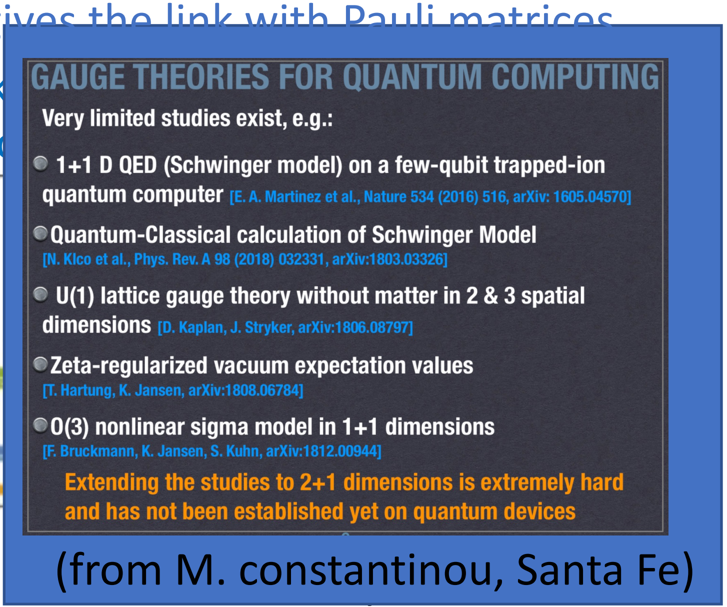
$$j_1 = \mathbb{1}\otimes\sigma_2/\sqrt{3}, \quad j_2 = \sigma_2\otimes\sigma_3/\sqrt{3}, \quad j_3 = \sigma_2\otimes\sigma_1/\sqrt{3},$$

This gives the link with Pauli matrices

Use the Suzuk

+ specific QC





GAUGE THEORIES FOR QUANTUM COMPUTING

Very limited studies exist, e.g.:

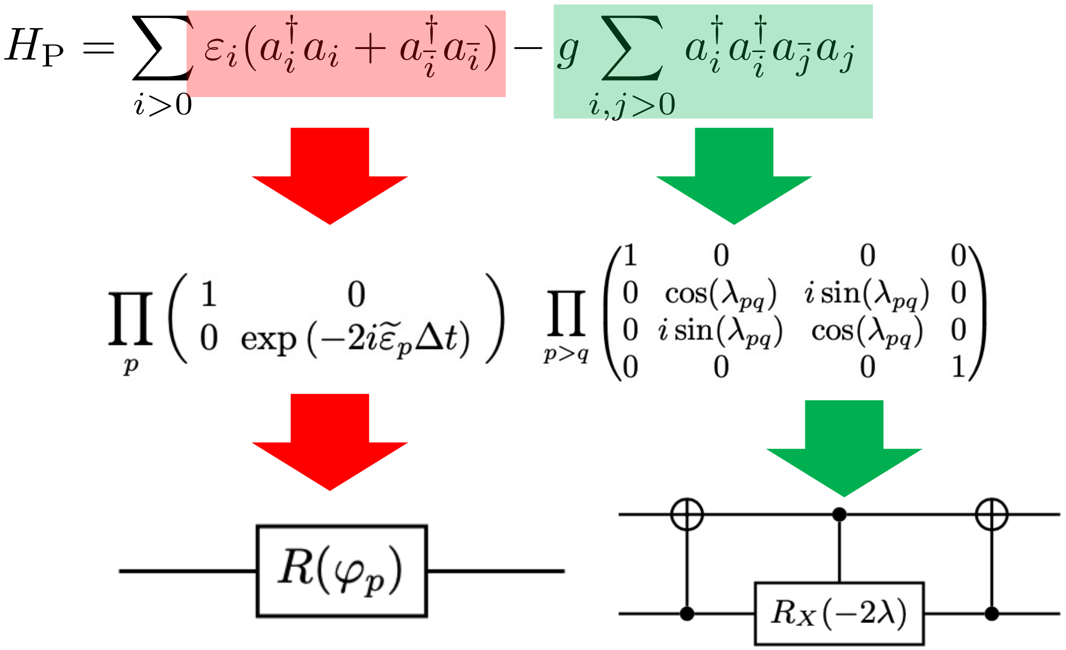
- 1+1 D QED (Schwinger model) on a few-qubit trapped-ion quantum computer [E. A. Martinez et al., Nature 534 (2016) 516, arXiv: 1605.04570]
- Quantum-Classical calculation of Schwinger Model [N. Klco et al., Phys. Rev. A 98 (2018) 032331, arXiv:1803.03326]
- U(1) lattice gauge theory without matter in 2 & 3 spatial dimensions [D. Kaplan, J. Stryker, arXiv:1806.08797]
- Zeta-regularized vacuum expectation values [T. Hartung, K. Jansen, arXiv:1808.06784]
- O(3) nonlinear sigma model in 1+1 dimensions [F. Bruckmann, K. Jansen, S. Kuhn, arXiv:1812.00944]

Extending the studies to 2+1 dimensions is extremely hard and has not been established yet on quantum devices

(from M. constantinou, Santa Fe)

Illustration with the cooper pair problem

Pairing Hamiltonian



$$H_{\mathrm{P}} = \sum_{i>0} \varepsilon_i (a_i^\dagger a_i + a_{\bar{i}}^\dagger a_{\bar{i}}) - g \sum_{i,j>0} a_i^\dagger a_{\bar{i}}^\dagger a_{\bar{j}} a_j$$

$$\prod_p \begin{pmatrix} 1 & 0 \\ 0 & \exp\left(-2i\widetilde{\varepsilon}_p \Delta t\right) \end{pmatrix} \quad \prod_{p>q} \begin{pmatrix} 1 & 0 & 0 & 0 \\ 0 & \cos(\lambda_{pq}) & i\sin(\lambda_{pq}) & 0 \\ 0 & i\sin(\lambda_{pq}) & \cos(\lambda_{pq}) & 0 \\ 0 & 0 & 0 & 1 \end{pmatrix}$$

The problem is that we can nowadays perform only few operations and with a limited *fidelity*

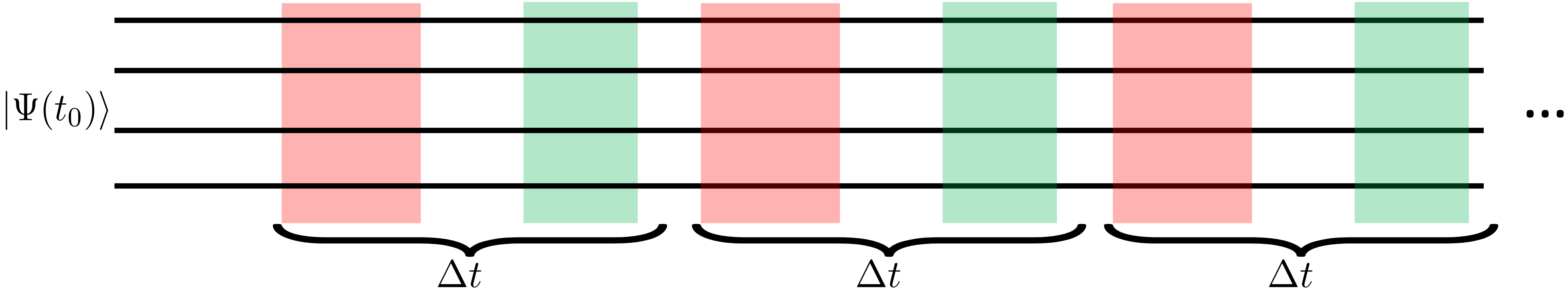
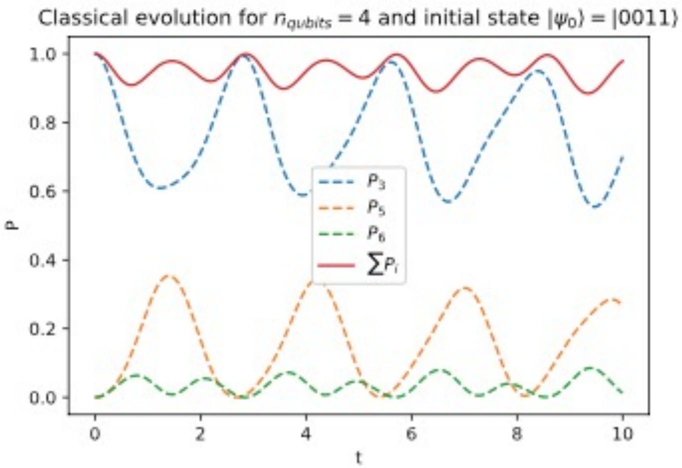
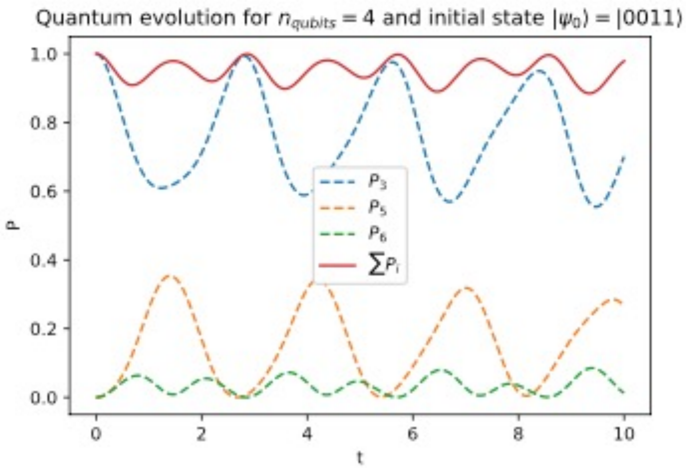
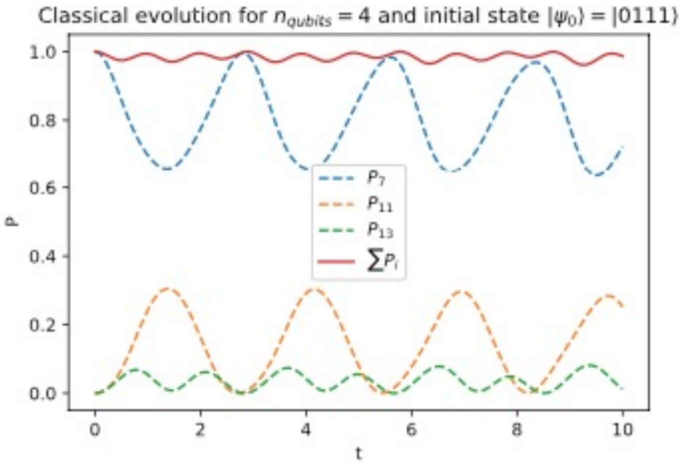


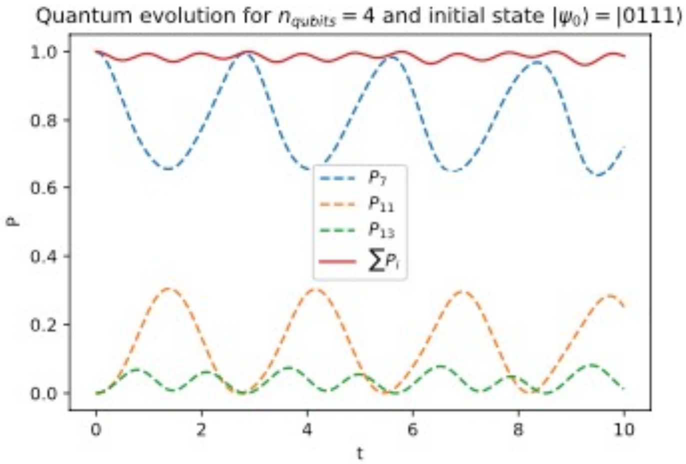
Illustration with our cooper pair problem

Results for 4 qubits


Classical evolution for $n_{qubits} = 4$ and initial state $|\psi_0\rangle = |0011\rangle$

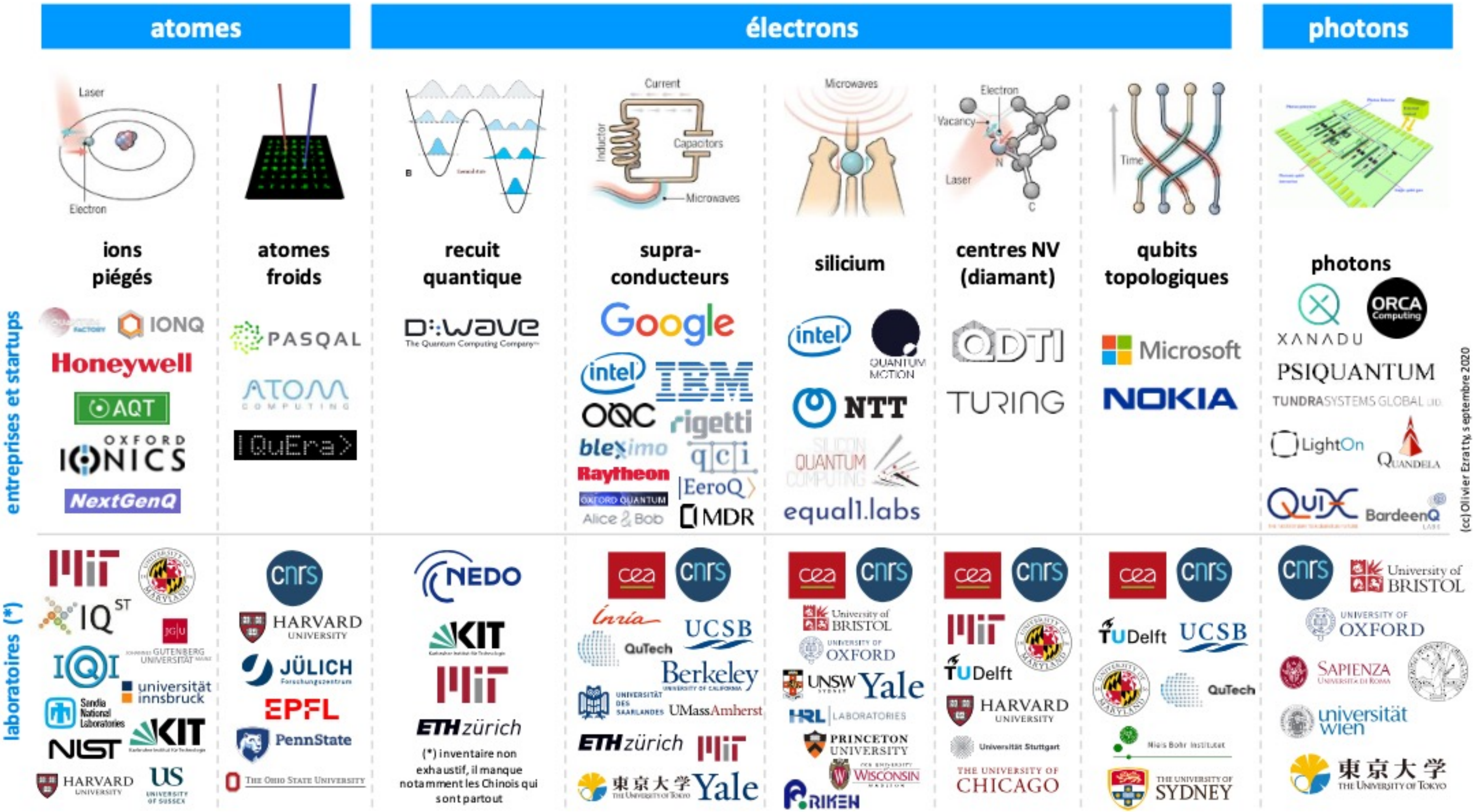

Quantum evolution for $n_{qubits} = 4$ and initial state $|\psi_0\rangle = |0011\rangle$


Classical evolution for $n_{qubits} = 4$ and initial state $|\psi_0\rangle = |0111\rangle$


Quantum evolution for $n_{qubits} = 4$ and initial state $|\psi_0\rangle = |0111\rangle$

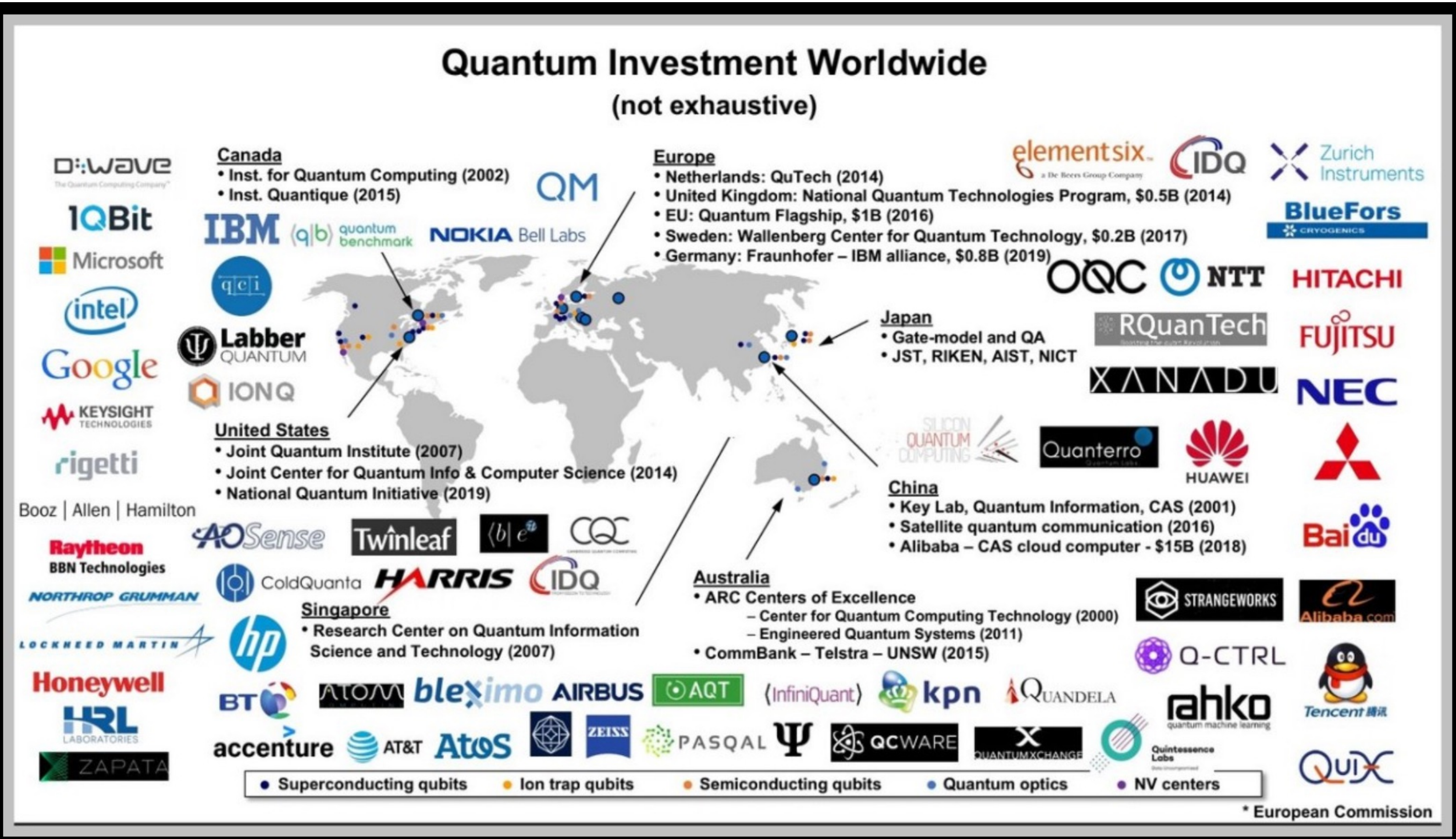
(From E. A. Ruiz Guzman, in PhD@IJCLab)

Building quantum computers: companies



From article O. Ezratty

Building quantum computers: companies



Quantum Investment Worldwide

(not exhaustive)

Canada
- Inst. for Quantum Computing (2002)
- Inst. Quantique (2015)

Europe
- Netherlands: QuTech (2014)
- United Kingdom: National Quantum Technologies Program, $0.5B (2014)
- EU: Quantum Flagship, $1B (2016)
- Sweden: Wallenberg Center for Quantum Technology, $0.2B (2017)
- Germany: Fraunhofer – IBM alliance, $0.8B (2019)

Japan
- Gate-model and QA
- JST, RIKEN, AIST, NICT

United States
- Joint Quantum Institute (2007)
- Joint Center for Quantum Info & Computer Science (2014)
- National Quantum Initiative (2019)

China
- Key Lab, Quantum Information, CAS (2001)
- Satellite quantum communication (2016)
- Alibaba – CAS cloud computer - $15B (2018)

Australia
- ARC Centers of Excellence
  - – Center for Quantum Computing Technology (2000)
  - – Engineered Quantum Systems (2011)
- CommBank – Telstra – UNSW (2015)

Singapore
- Research Center on Quantum Information Science and Technology (2007)

● Superconducting qubits ● Ion trap qubits ● Semiconducting qubits ● Quantum optics ● NV centers

* European Commission

William Oliver, MIT -