

Three body forces revealed in strongly deformed nuclei

Frédéric Nowacki



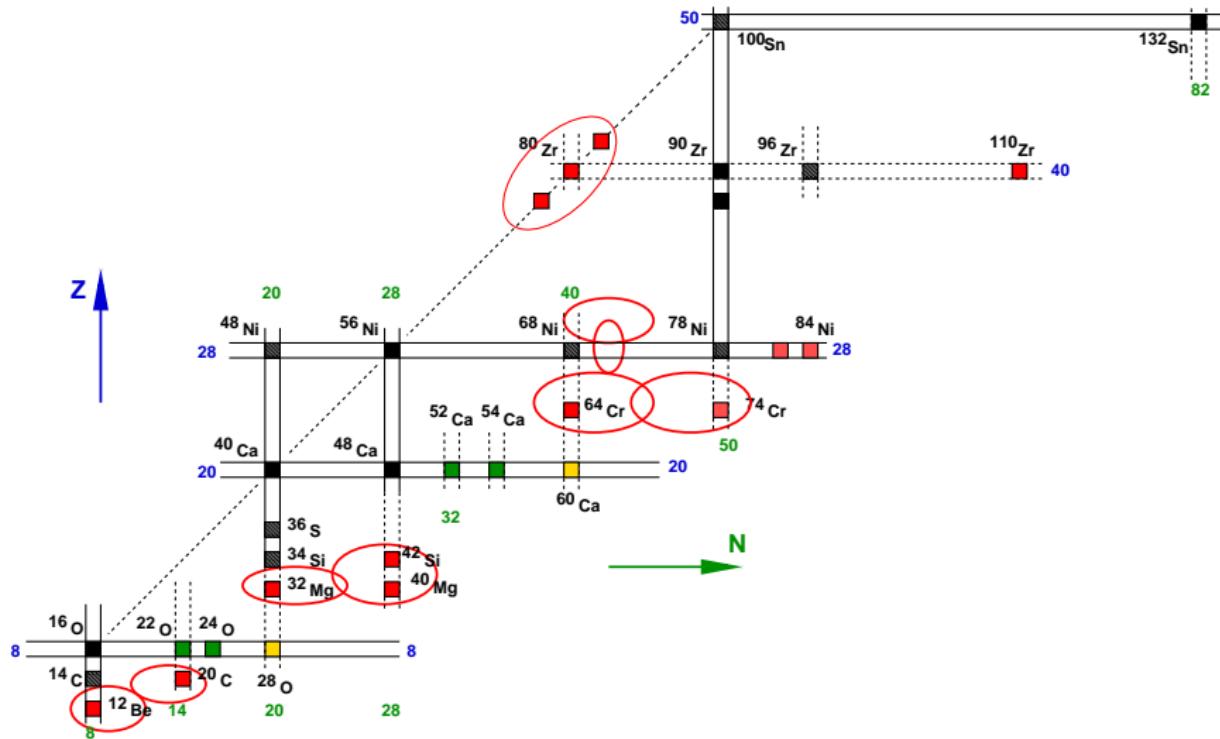
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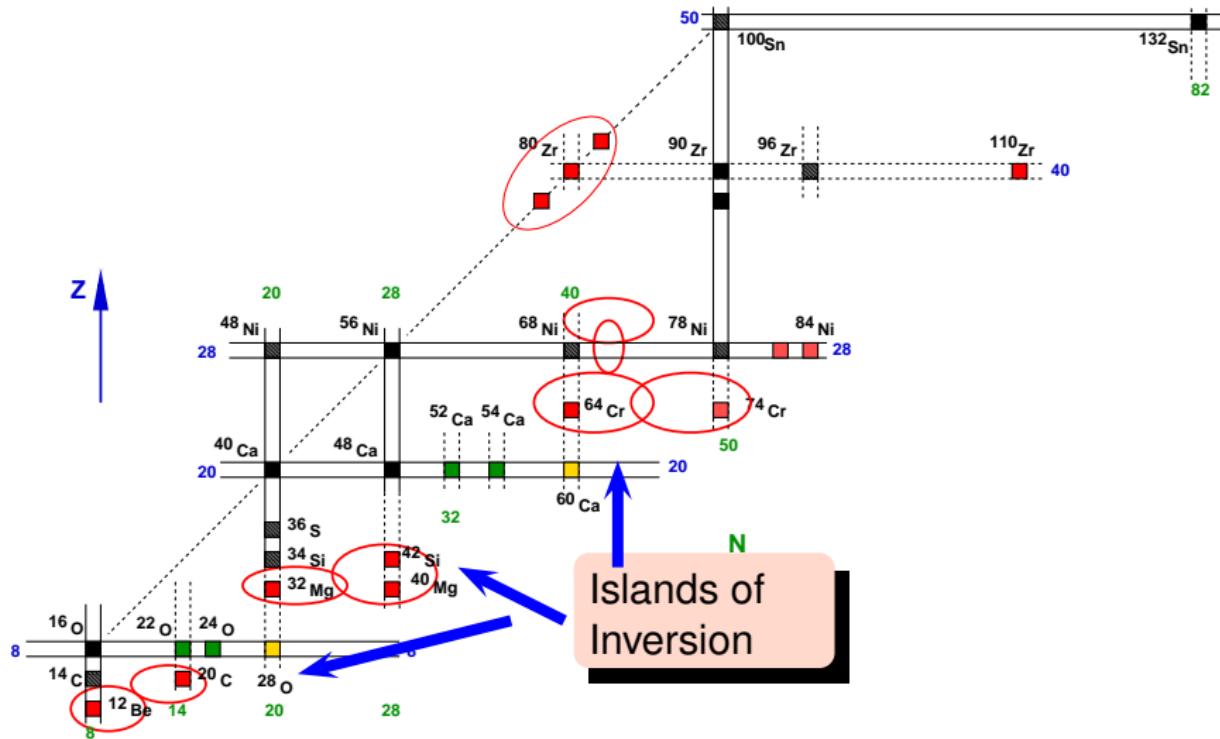
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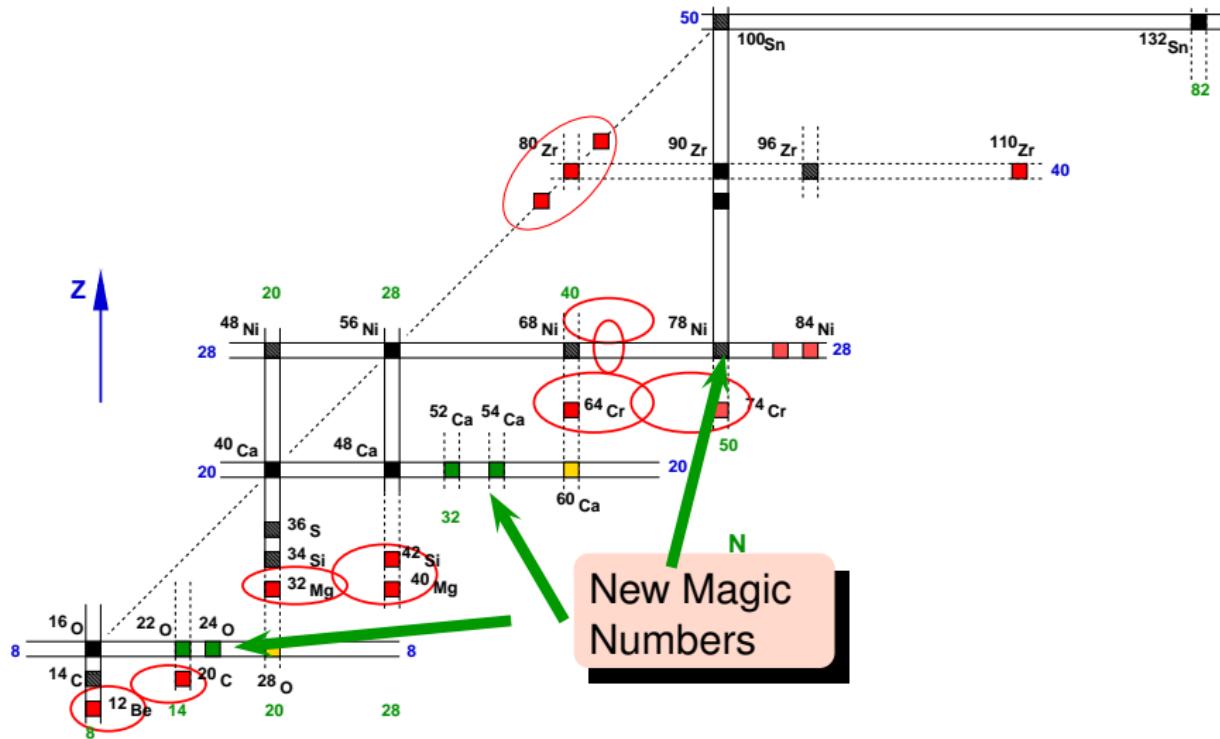
Landscape of medium mass nuclei



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Three-Body Forces and the Limit of Oxygen Isotopes

Takaharu Otsuka,^{1,2,3} Toshio Suzuki,⁴ Jason D. Holt,⁵ Achim Schwenk,⁵ and Yoshinori Akaishi⁶¹*Department of Physics, University of Tokyo, Hongo, Tokyo 113-0033, Japan*²*Center for Nuclear Study, University of Tokyo, Hongo, Tokyo 113-0033, Japan*³*National Superconducting Cyclotron Laboratory, Michigan State University, East Lansing, Michigan, 48824, USA*⁴*Department of Physics, College of Humanities and Sciences, Nihon University, Sakurajosui 3, Tokyo 156-8550, Japan*⁵*TRIUMF, 4004 Wesbrook Mall, Vancouver, BC, V6T 2A3, Canada*⁶*RIKEN Nishina Center, Hirosawa, Wako-shi, Saitama 351-0198, Japan*

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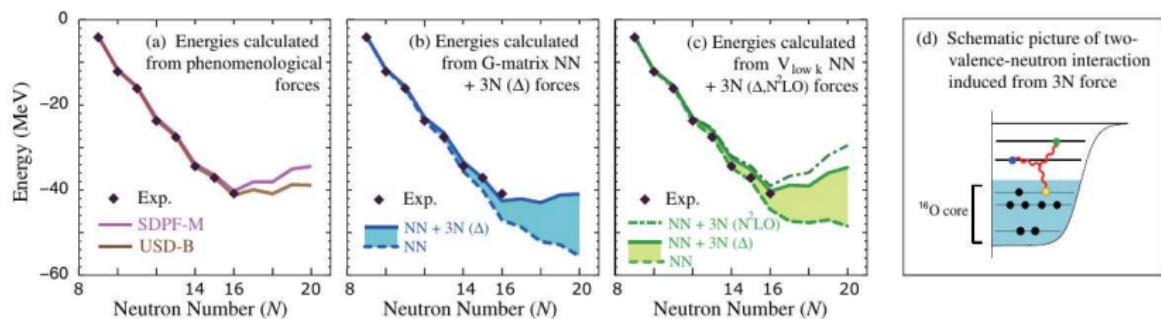


FIG. 4 (color online). Ground-state energies of oxygen isotopes measured from ^{16}O , including experimental values of the bound 16–24 O. Energies obtained from (a) phenomenological forces SDPF-M [13] and USD-B [14], (b) a G matrix and including FM 3N forces due to Δ excitations, and (c) from low-momentum interactions $V_{\text{low } k}$ and including chiral EFT 3N interactions at $N^2\text{LO}$ as well as only due to Δ excitations [25]. The changes due to 3N forces based on Δ excitations are highlighted by the shaded areas. (d) Schematic illustration of a two-valence-neutron interaction generated by 3N forces with a nucleon in the ^{16}O core.

Evolution of Shell Structure in Neutron-Rich Calcium Isotopes

G. Hagen,^{1,2} M. Hjorth-Jensen,^{3,4} G. R. Jansen,³ R. Machleidt,⁵ and T. Papenbrock^{1,2}

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²*Department of Physics and Astronomy, University of Tennessee, Knoxville, Tennessee 37996, USA*

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⁴*National Superconducting Cyclotron Laboratory and Department of Physics and Astronomy,*

Michigan State University, East Lansing, Michigan 48824, USA

⁵*Department of Physics, University of Idaho, Moscow, Idaho 83844, USA*

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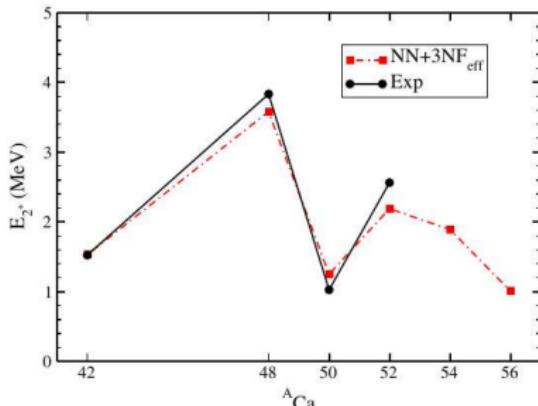


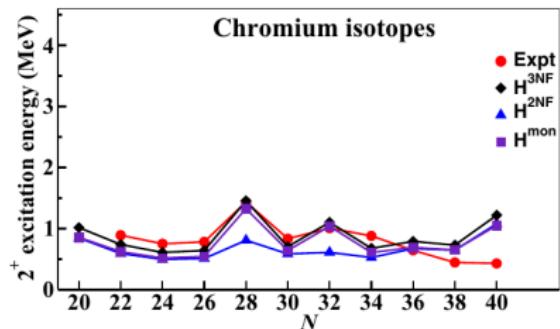
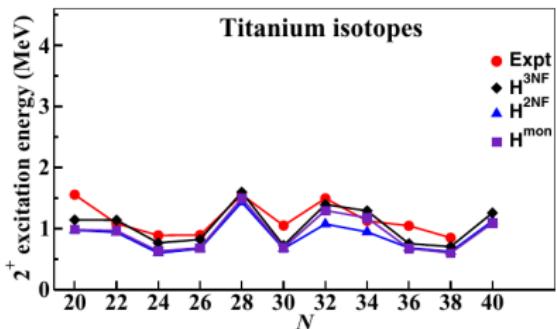
FIG. 2 (color online). (Excitation energies of $J^\pi = 2^+$ states in the isotopes $^{42,48,50,52,54,56}\text{Ca}$ (experiment: black circles, theory: red squares))

3N forces in light nuclei

PHYSICAL REVIEW C 100, 034324 (2019)

Contribution of chiral three-body forces to the monopole component of the effective shell-model Hamiltonian

Y. Z. Ma,¹ L. Coraggio,² L. De Angelis,² T. Fukui,² A. Gargano,² N. Itaco,^{2,3} and F. R. Xu¹



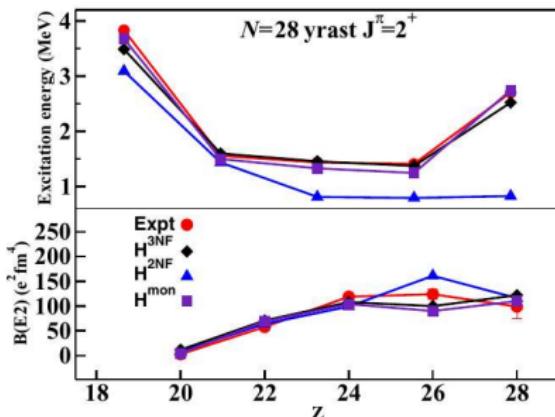
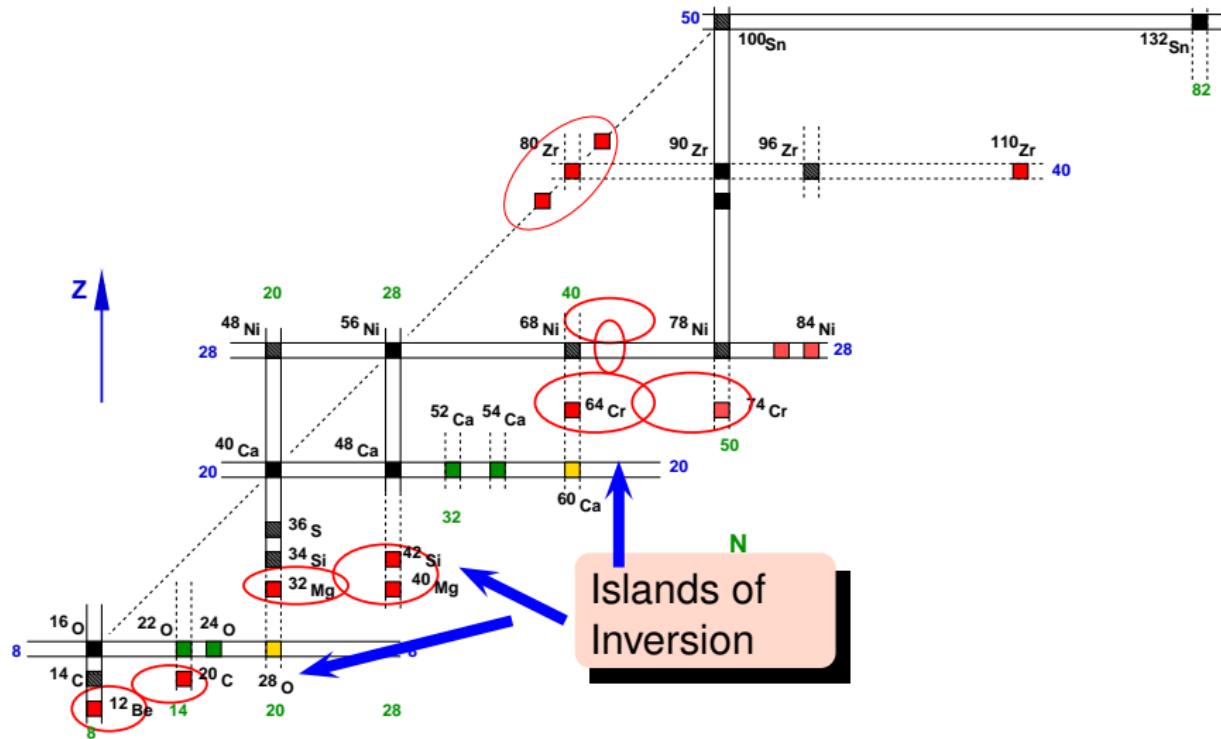
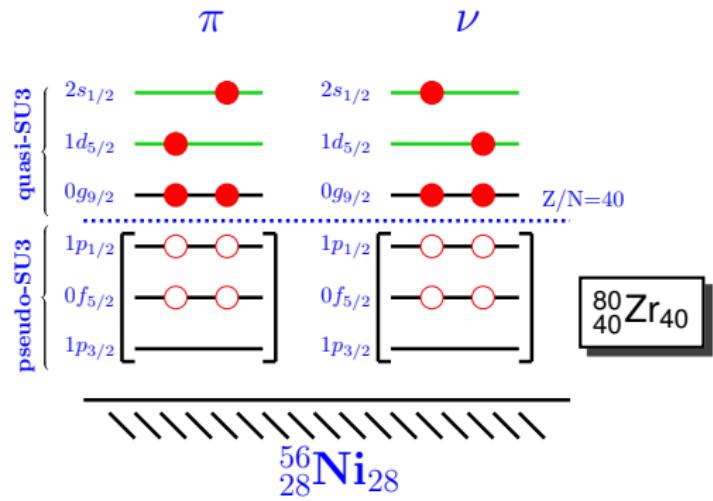
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FIG. 22. Experimental and calculated excitation energies of the yrast $J^\pi = 2^+$ states and $B(E2; 2_1^+ \rightarrow 0_1^+)$ transition rates for the $N = 28$ isotones. See text for details.

Landscape of medium mass nuclei



H.O. vs Spin-Orbit shell closure at N=Z



- p shell: ^{16}O
spherical/doubly magic
- sd shell: ^{40}Ca
spherical/doubly magic
- pf shell: ^{80}Zr
deformed nucleus

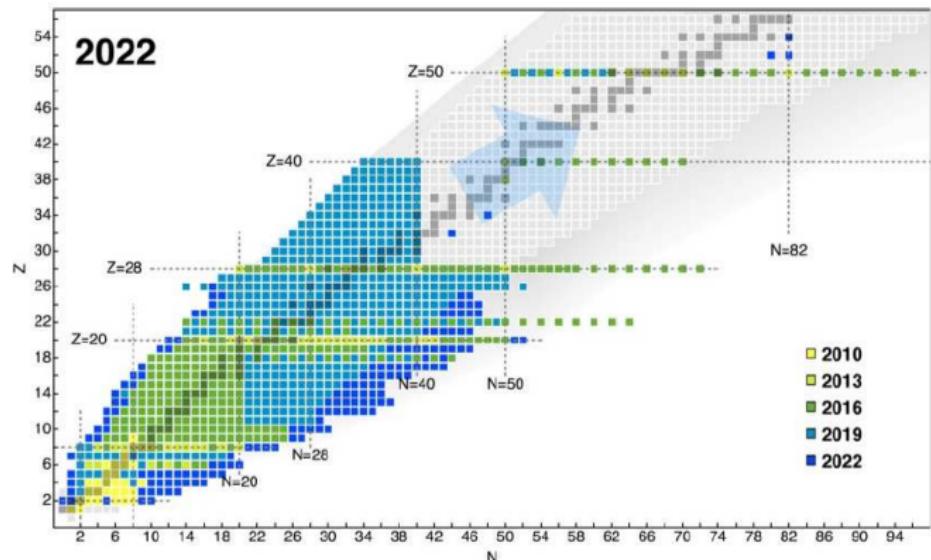
- Low-lying states in H.O. N=Z=8: CS , 4p4h, 8p8h
- Low-lying states in H.O. N=Z=20: CS , 4p4h, 8p8h
- Low-lying states in H.O. N=Z=40: 4p4h ? 8p8h ? 12p12h ?

Ab-initio predictions ?



Ab Initio Progress: How Heavy Can We Go?

Tremendous progress in ab initio reach, largely due to polynomially scaling methods!



Island of Inversion at the N=Z line

Strongly deformed states at $N = Z$:

- Configuration mixing in ^{72}Kr
- Most deformed cases for ^{76}Sr , ^{80}Zr
- Shape transition between ^{84}Mo and ^{86}Mo

NSCL/GRETINA Experiment

R.D.O. Llewellyn *et al.*, Phys. Rev. Lett. **124**, 152501

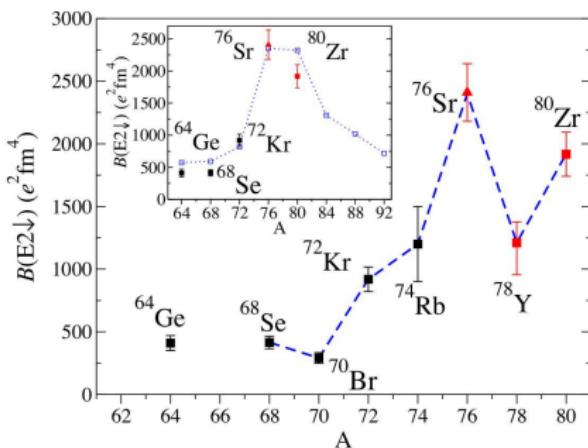
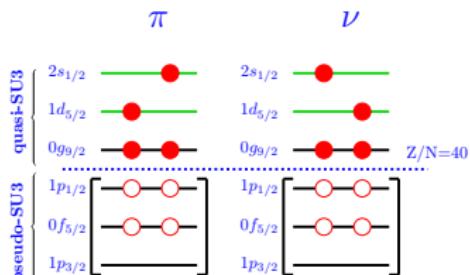


FIG. 3. Schematics of the $B(E2\downarrow)$ values for the $N = Z$ nuclei



- ZBM3 valence space:
extension of JUN45
to pseudo-SU3 + Quasi-SU3
- New effective interactions:
 - Realistic TBME + Monopole “3N” constraints
 - ab-initio N3LO (2N) interaction
- SM + DNO-SM for most deformed cases

Discrete Non-Orthogonal Shell Model

Generator Coordinate Method: $|\Psi_{\text{eff}}\rangle = \sum_i f_i |\Phi_i\rangle$

1) Deformed Hartree-Fock (HF) Slater determinants

2) Restoration of rotational symmetry

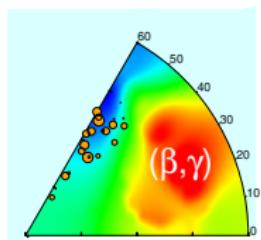
3) Mixing of shapes:

$$|\Psi_{\text{eff}}\rangle = \text{shape}_1 + \text{shape}_2 + \text{shape}_3 + \dots$$

Intrinsic/Laboratory Description

- **Deformation structure of nuclear states:** $\{J_\alpha^\pi\}$, $q = (\beta, \gamma)$

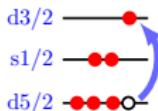
$$M_\alpha^{(J)}(q, K) = \sum_{q', K'} [\hat{N}^{1/2}]_{K' K}^{(J)}(q', q) f_\alpha^{(J)}(q', K')$$



- ◊ Probability of a configuration (β, γ) :

$$P_\alpha^{(J)}(q) = \sum_K |M_\alpha^{(J)}(q, K)|^2$$

- **particle-hole interpretation:**



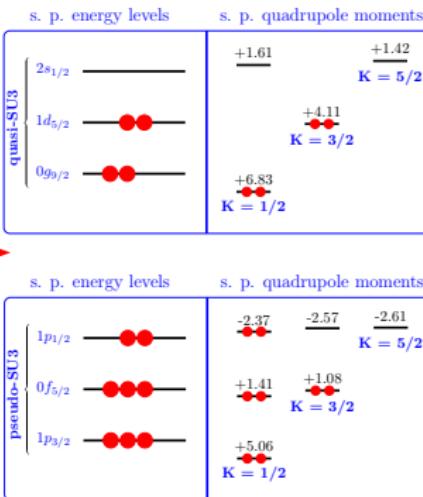
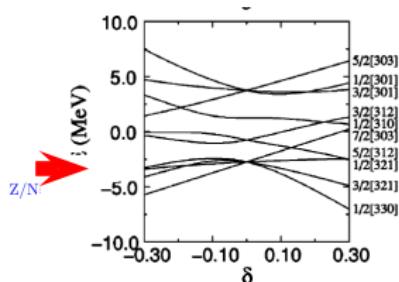
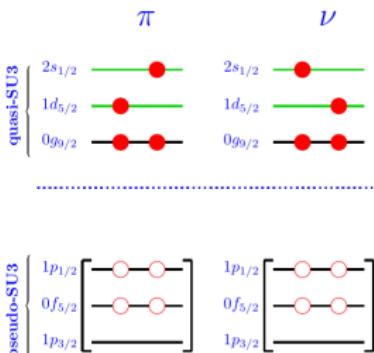
M-scheme

- **K-quantum numbers:**

$$P_\alpha^{(J)}(K) = \sum_q |M_\alpha^{(J)}(q, K)|^2$$

Nilsson-SU3 estimates

single particle energy levels



Wavy lines below the nucleus indicate the ground state.

$^{56}_{28}\text{Ni}_{28}$

nucleus	NpNh*	ZRP	PHF	B(E2)(e ² .fm ⁴)	Exp.	DNO-SM
^{76}Sr	4p-4h	924	806			
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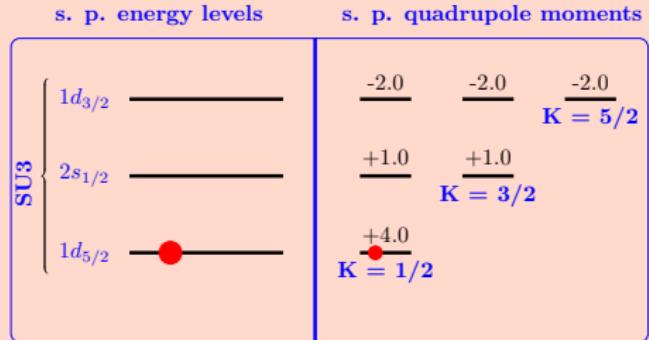
single-particle energy levels

s. p. energy levels	s. p. quadrupole moments
$2s_{1/2}$	$+1.61$ $+1.42$ $K = 5/2$

PHYSICAL REVIEW C 92, 024320 (2015)

Nilsson-SU3 self-consistency in heavy $N = Z$ nuclei

A. P. Zuker,¹ A. Poves,^{2,3} F. Nowacki,¹ and S. M. Lenzi⁴



180 190 200 210

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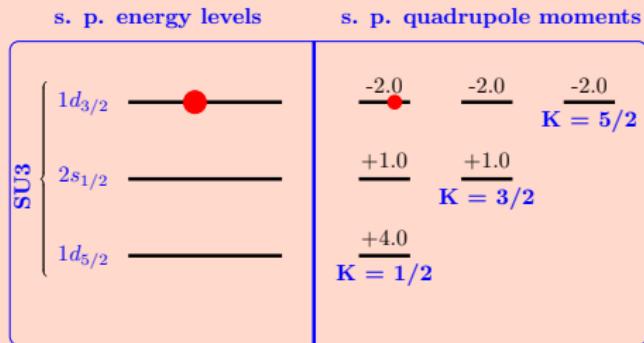
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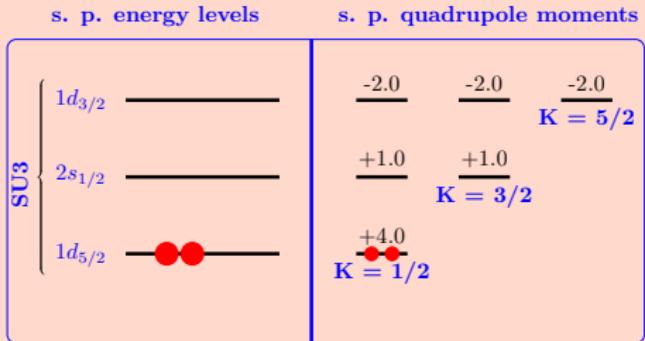
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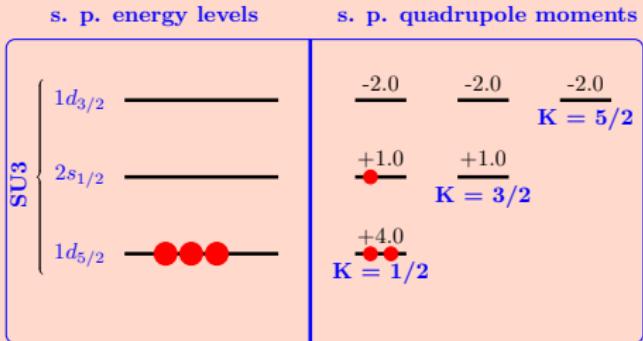
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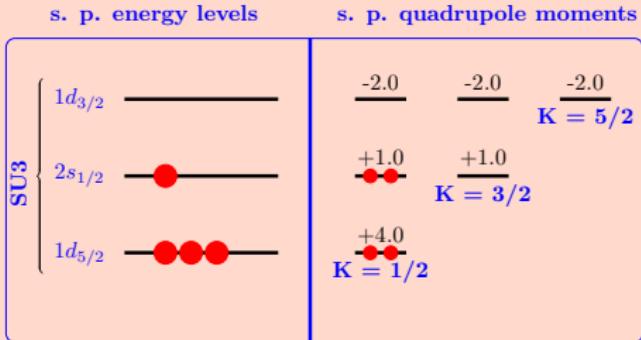
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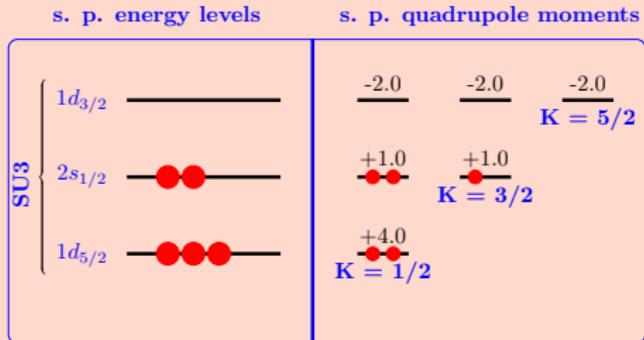
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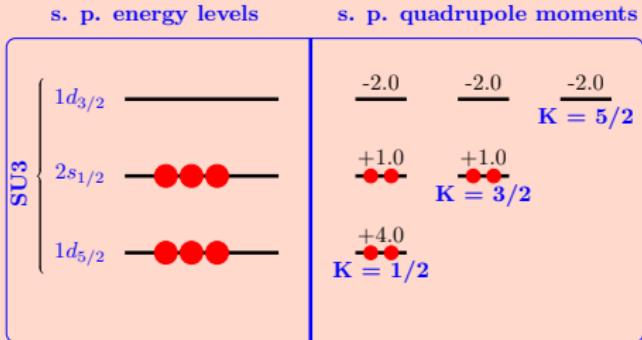
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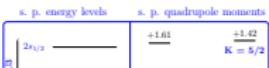
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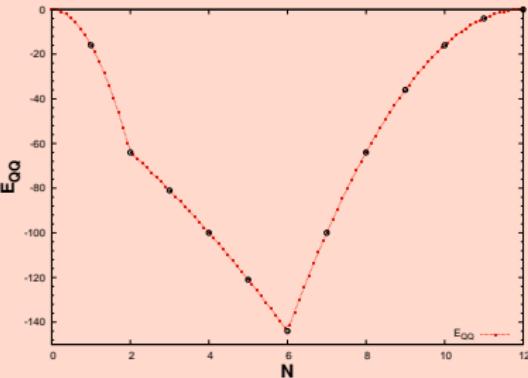
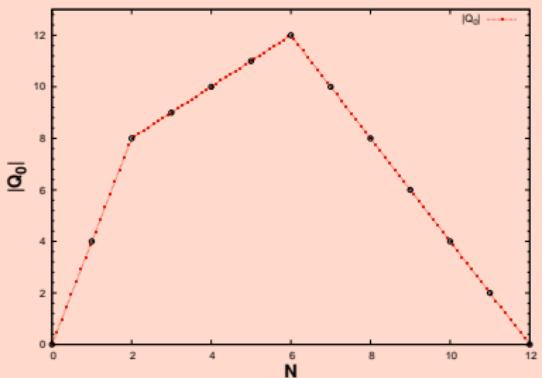
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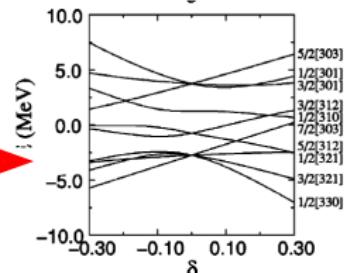
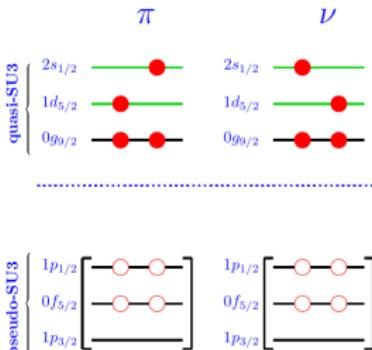
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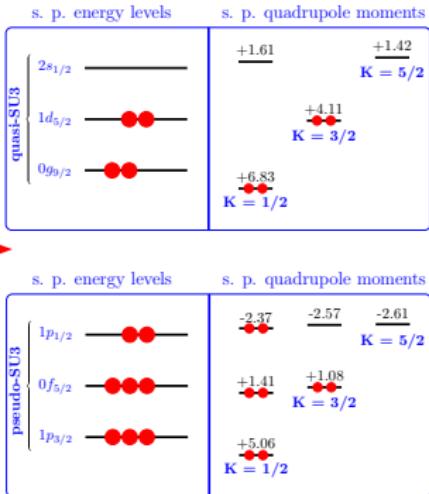


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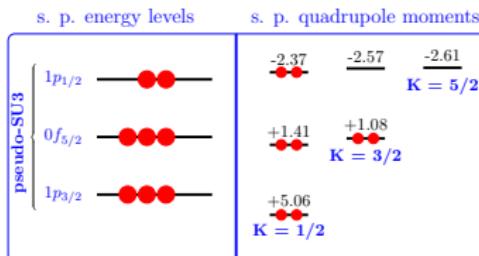
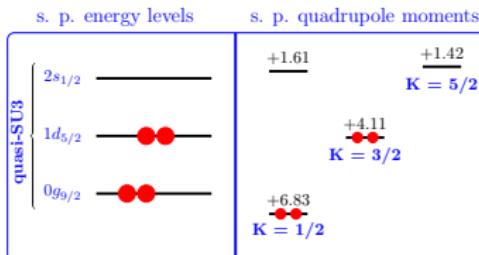


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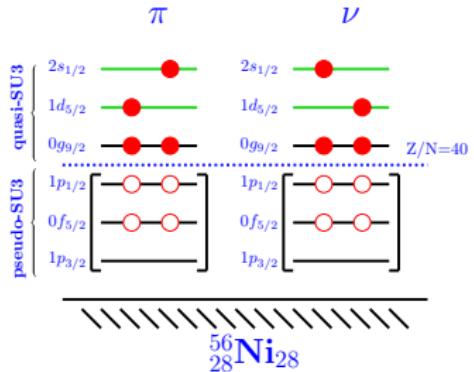
Island of Inversion at the N=Z line

Strongly deformed states at $N = Z$

- Configuration mixing in ^{72}Kr
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- NSCL/GRETINA Experiment**



!



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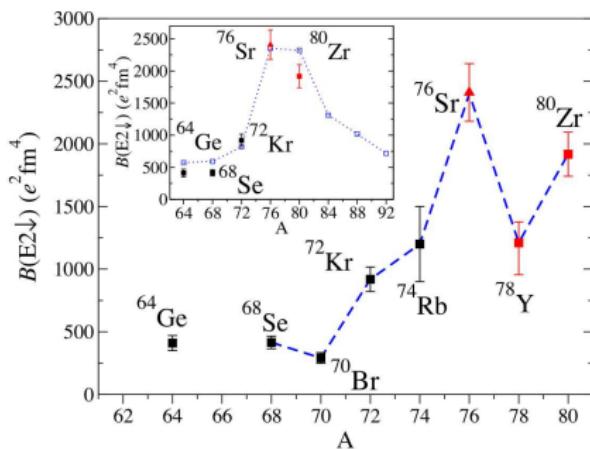
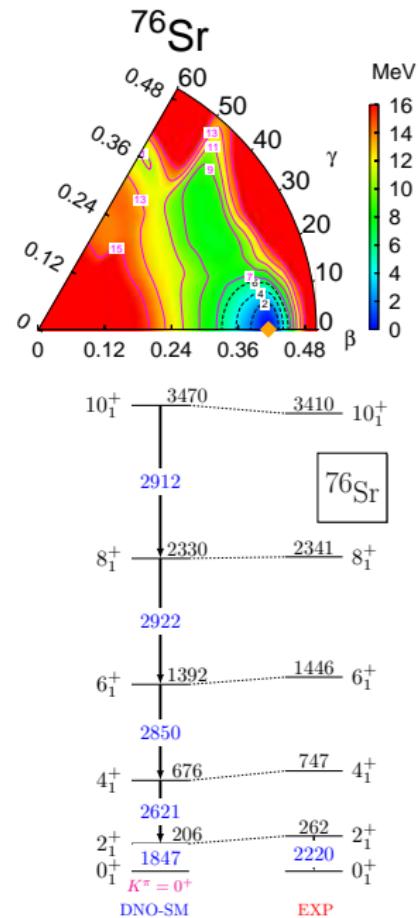


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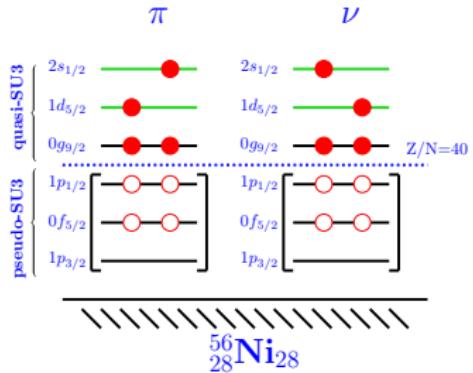
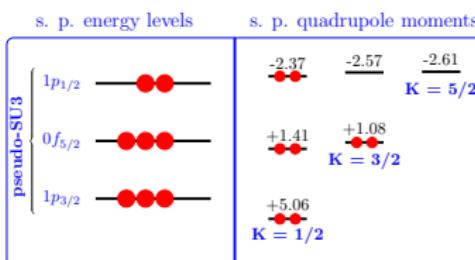
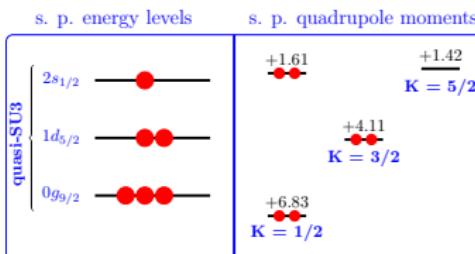


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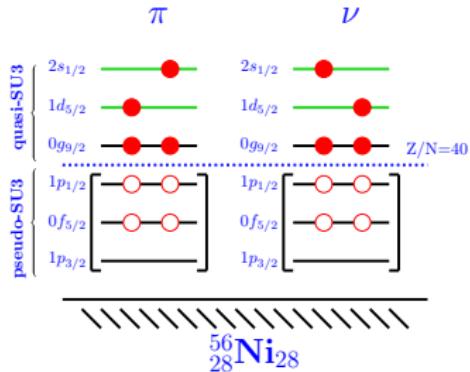
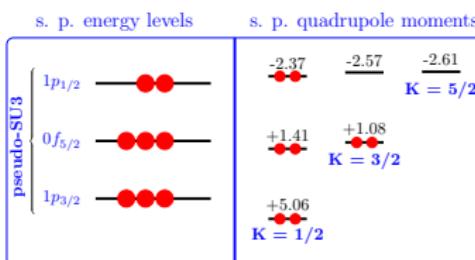
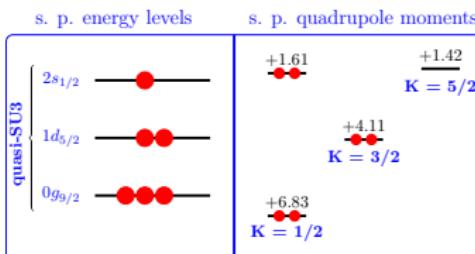
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	8p-8h	1891	1732				
^{86}Mo	0p-0h	542	196				
	2p-2h	1030	871				
	4p-4h	1416	1179				
	6p-6h	1858	1655				

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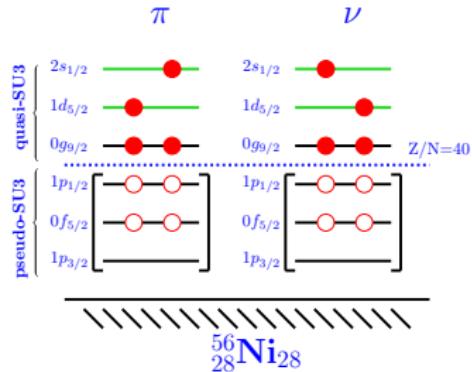
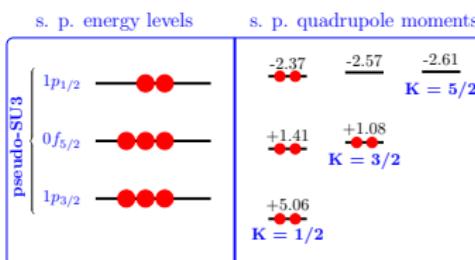
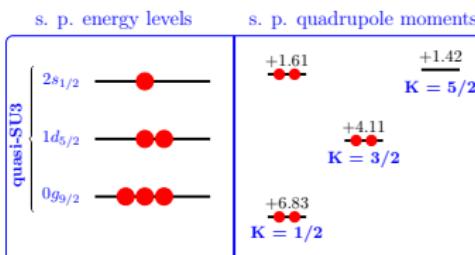
nucleus	Np-Nh*	ZRP	PHF	B(E2)(e ² .fm ⁴)	Exp.	DNO-SM*	SM
^{84}Mo	4p-4h	1104	1193				
	8p-8h	1891	1732				
^{86}Mo	0p-0h	542	196				
	2p-2h	1030	871				
	4p-4h	1416	1179				
	6p-6h	1858	1655				
				1740^{+580}_{-430}	1765		-
						707(71)	1184 731

Island of Inversion at the N=Z line

Strongly deformed states at $N = Z$

- Configuration mixing in ^{72}Kr
- Most deformed cases for ^{76}Sr , ^{80}Zr
- Shape transition between ^{84}Mo and ^{86}Mo

NSCL/GRETINA Experiment

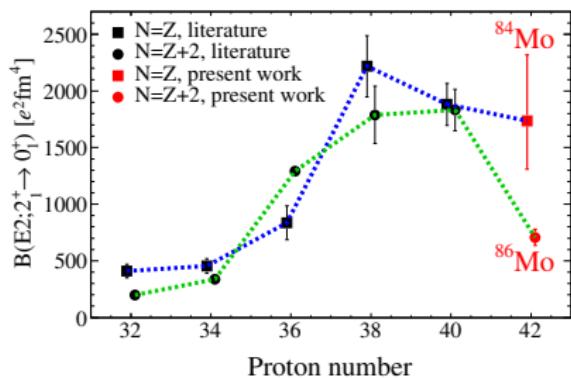


nucleus	Np-Nh*	ZRP	PHF	B(E2)(e ² .fm ⁴)	Exp.	DNO-SM*	SM
^{84}Mo	4p-4h	1104	1193	1740^{+580}_{-430}	1765	1765	$-$
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	2p-2h	1030	871				
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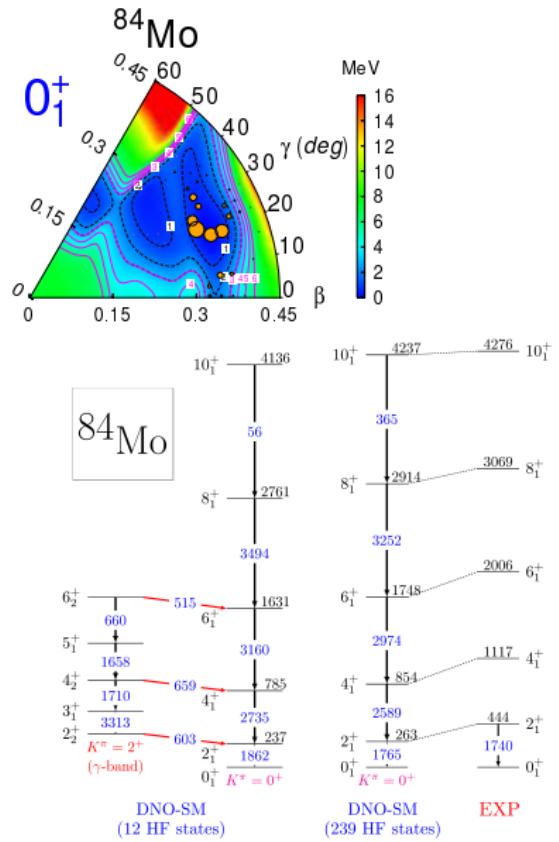
Island of Inversion at the N=Z line

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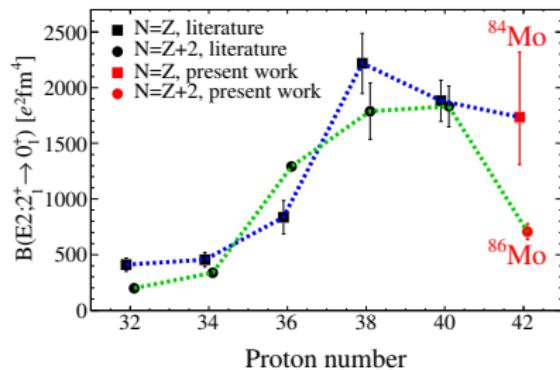
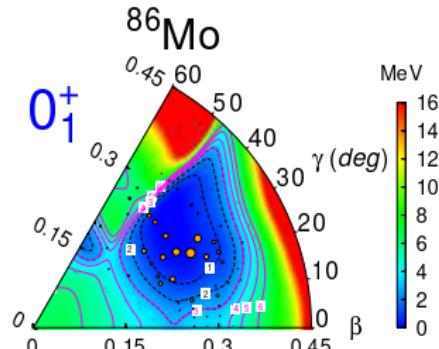
J. Ha, F. Recchia *et al.*, to be submitted



Island of Inversion at the N=Z line

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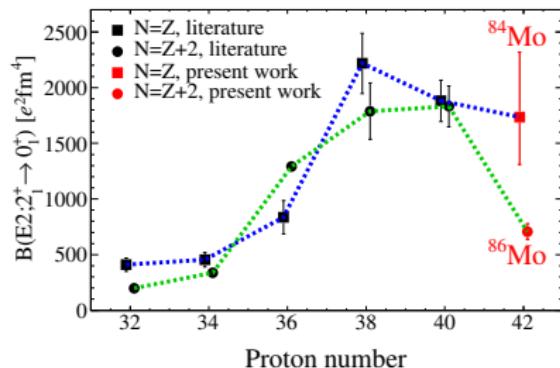
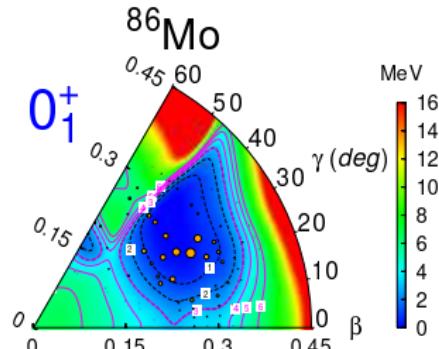


nucleus	Np-Nh*	$B(E2)(e^2 \cdot \text{fm}^4)$				DNO-SM*	SM
		ZRP	PHF	Exp.			
^{84}Mo	4p-4h	1104	1193	1740^{+580}_{-430}	1765	1740^{+580}_{-430}	-
	8p-8h	1891	1732				
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	2p-2h	1030	871				
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Island of Inversion at the N=Z line

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Shell closures and 2N forces only

PHYSICAL REVIEW C 74, 061302(R) (2006)

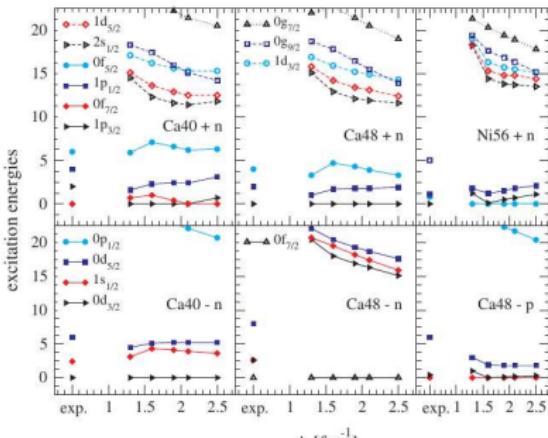
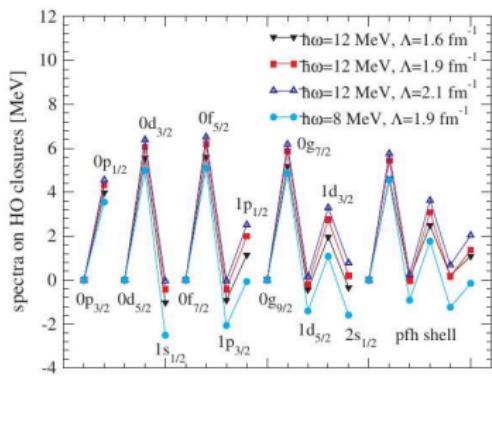
Shell-model phenomenology of low-momentum interactions

Achim Schwenk^{1,*} and Andrés P. Zuker^{2,†}

¹Nuclear Theory Center, Indiana University, 2401 Milo B. Sampson Lane, Bloomington, Indiana 47408, USA

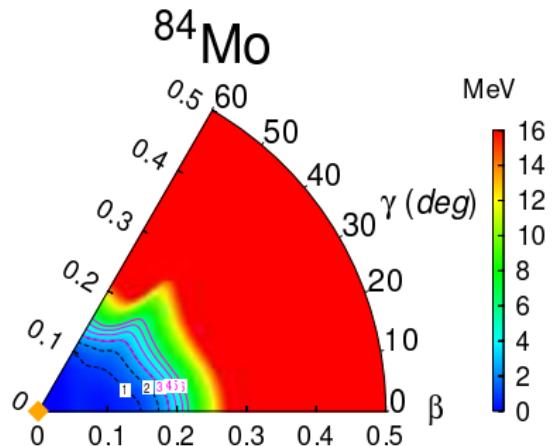
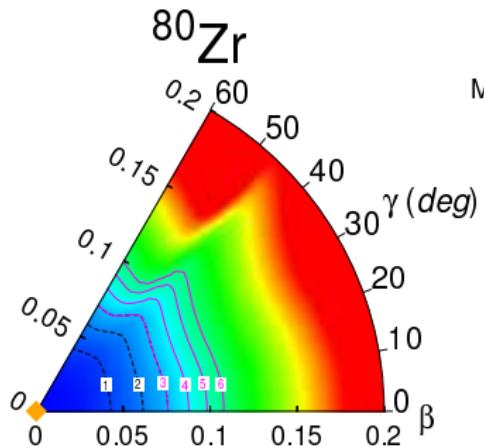
²Institut de Recherches Subatomiques, IN2P3-CNRS, Université Louis Pasteur, F-67037 Strasbourg, France

(Received 14 January 2005; revised manuscript received 20 September 2006; published 12 December 2006)



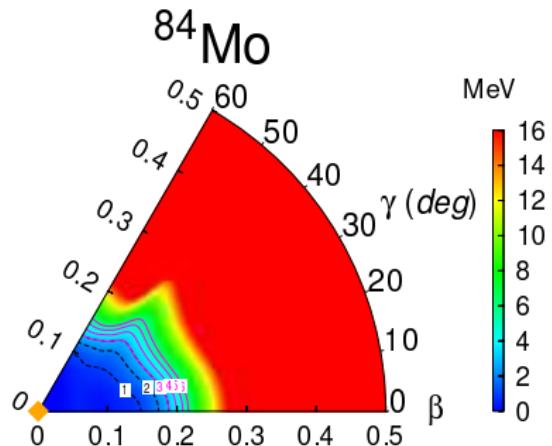
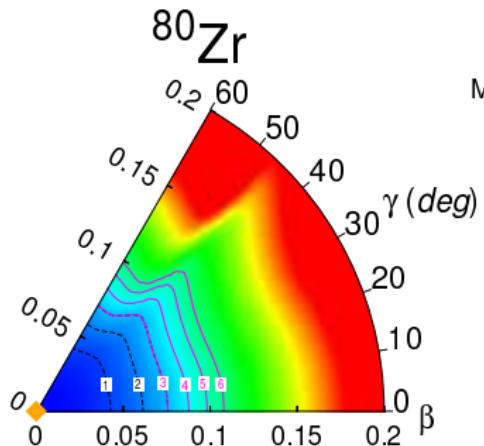
- no Spin-orbit shell closures in ^{12}C , ^{22}O , ^{48}Ca , ^{56}Ni
- too strong H. O. shell closures ^{16}O , ^{40}Ca , ... and ^{80}Zr !!!

N3LO NN calculations



nucleus	NpNh*	$B(E2)(e^2 \cdot \text{fm}^4)$				
		ZRP	PHF	Exp.	DNO-SM	N3LO
^{80}Zr	4p-4h	587	637			
	8p-8h	1713	1509	1910(180)	2325	0.03
	12p-12h	2663	2396			
^{84}Mo	4p-4h	1104	1193	1740^{+580}_{-430}	1740	174
	8p-8h	1891	1732			

N3LO NN calculations

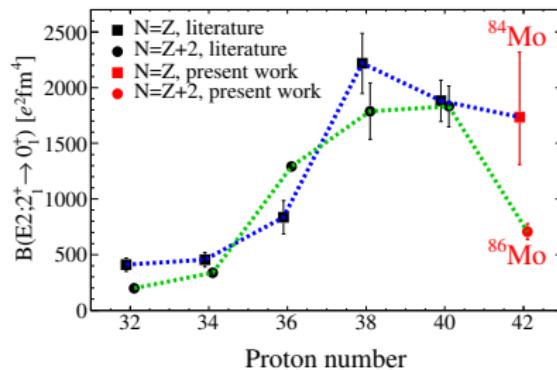
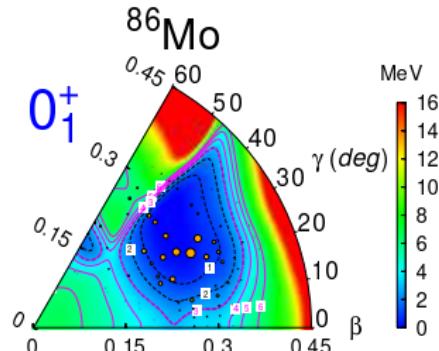


nucleus	NpNh*	ZRP	PHF	$B(E2)(e^2 \cdot \text{fm}^4)$		
				Exp.	DNO-SM	N3LO
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Island of Inversion at the N=Z line

Strongly deformed states at $N = Z$

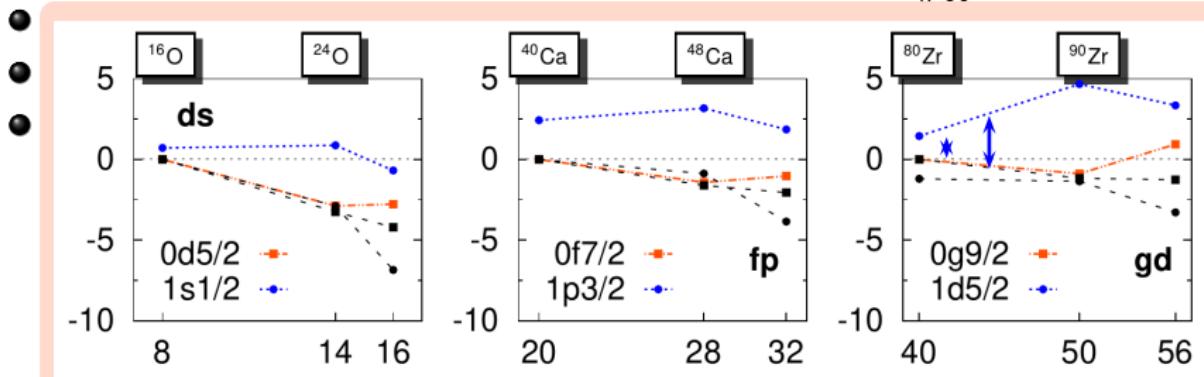
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Island of Inversion at the N=Z line

◇ Strongly deformed states at $N = Z$

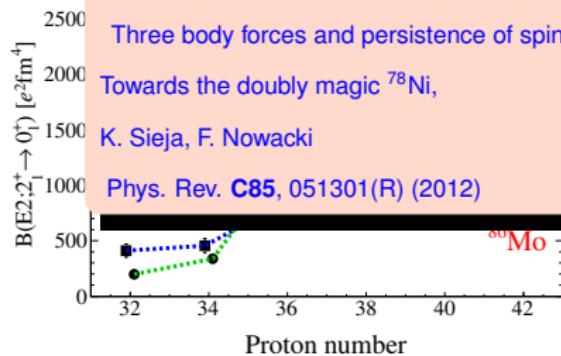


Three body forces and persistence of spin-orbit shell gaps in medium-mass nuclei:

Towards the doubly magic ^{78}Ni ,

K. Sieja, F. Nowacki

Phys. Rev. C85, 051301(R) (2012)



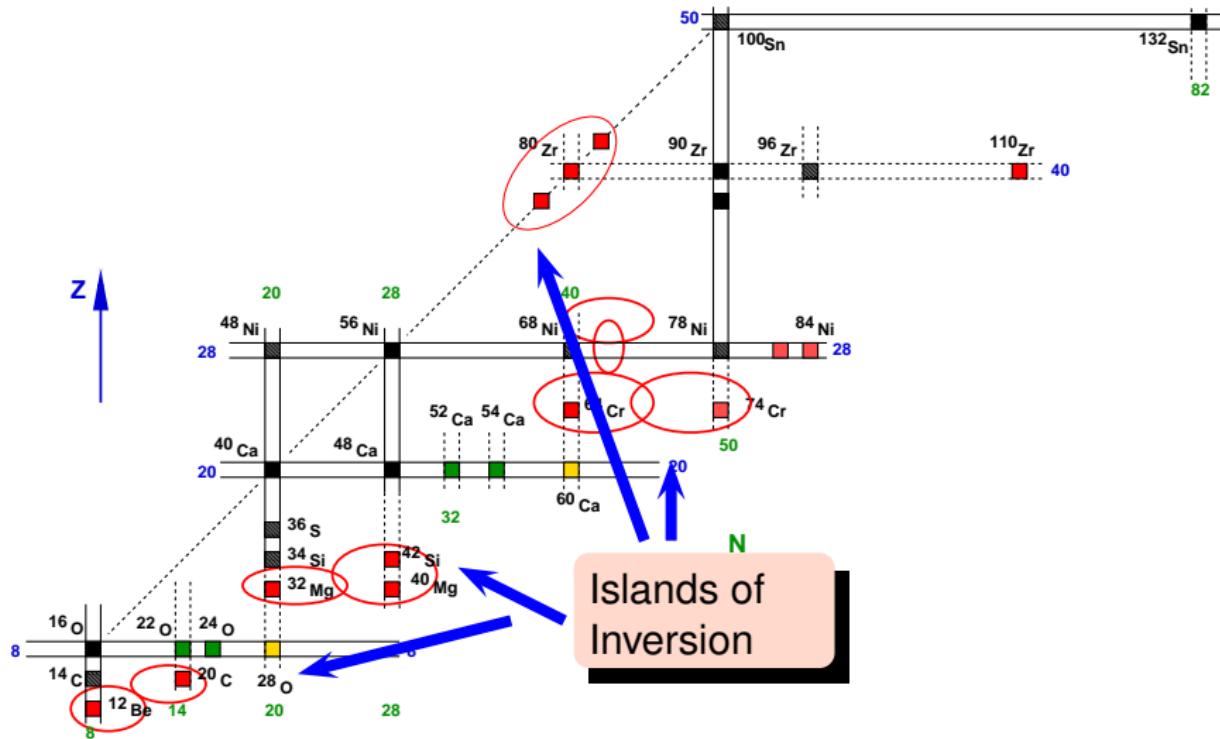
^{86}Mo	2p-2h	1030	871
	4p-4h	1416	1179
	6p-6h	1858	1655

707(71)

1184

731

Landscape of medium mass nuclei



Summary

- Monopole drift develops in all regions but the Interplay between correlations (pairing + quadrupole) and spherical mean-field (monopole field) determines the physics.
- New “island of inversion” or “island of deformation” present for neutron-rich systems show up also at N=Z line with very deformed rotors dominated by Many-particles-Many-holes configurations.
- Shape transition between ^{84}Mo and ^{86}Mo and first fingerprint of 3N forces in deformed systems
- Around $A \sim 80$, an “island of enhanced collectivity” show very deformed rotors dominated by Many-particles-Many-holes configurations.

Summary

Special thanks to:

- D. D. Dao, K. Sieja
- G. Martinez-Pinedo, A. Poves, S. Lenzi
- A. Gade, O. Sorlin, A. Obertelli