Fingerprint of the tensor interaction in N=20 isotones

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INTRANS workshop
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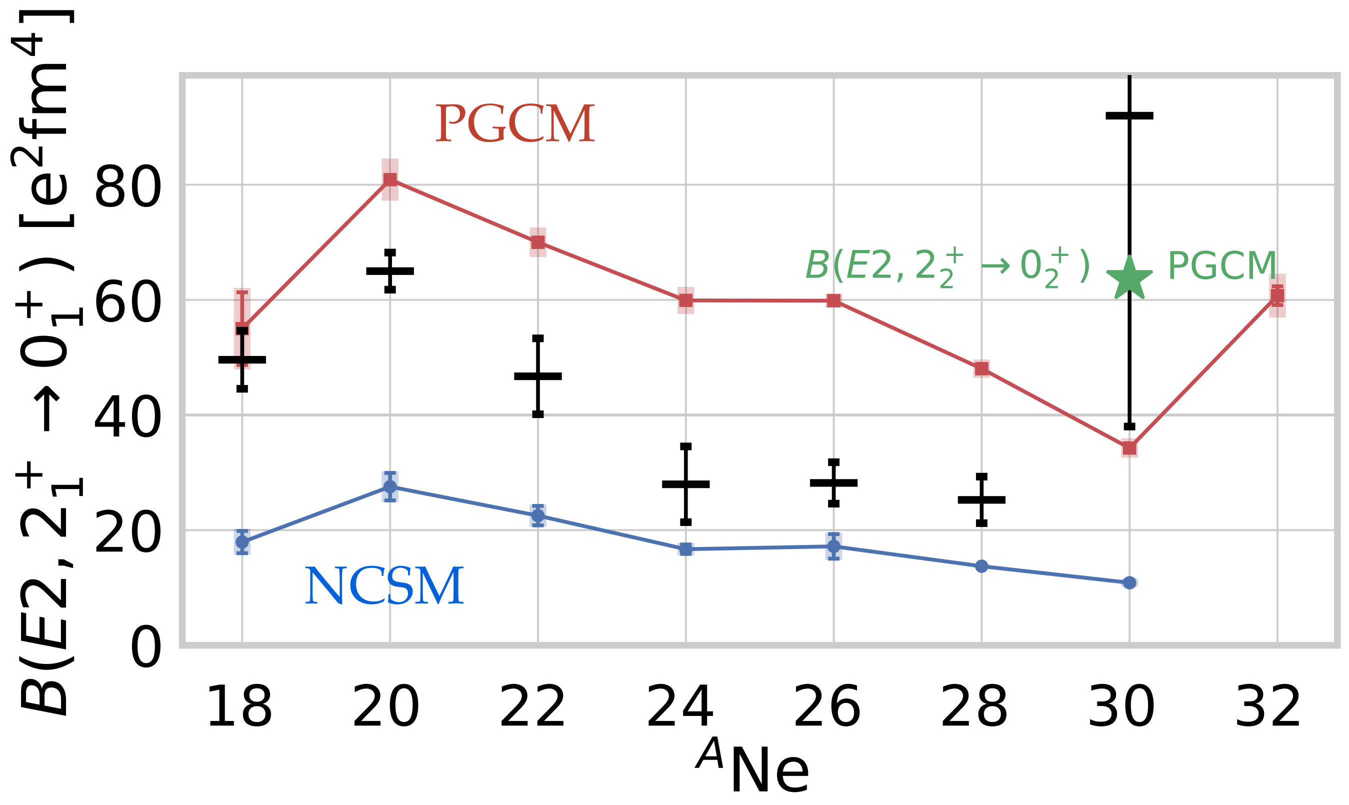
Advertisement: recent developments

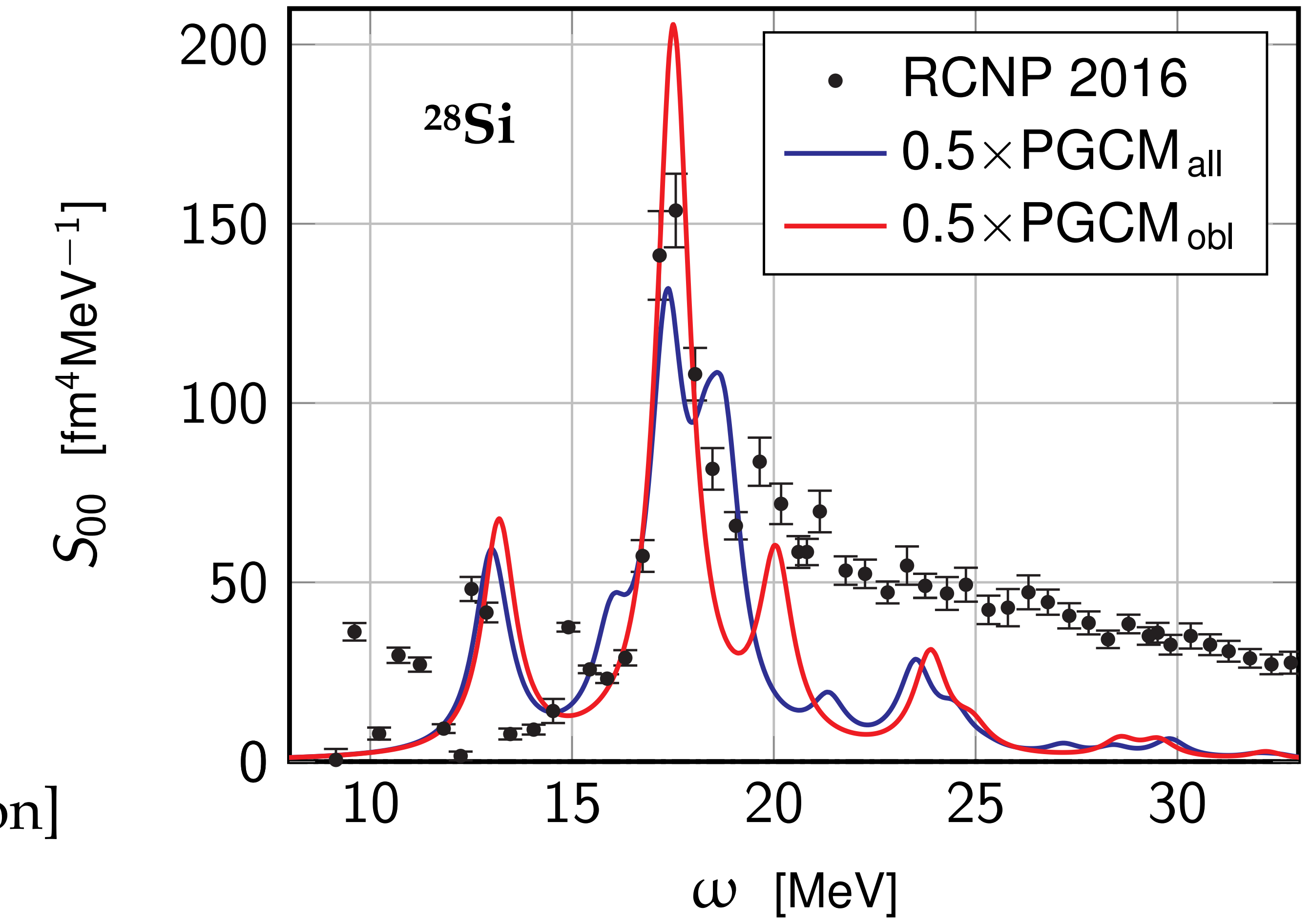
⦿ **Ab initio calculations**: significant progress in mid-mass nuclei & heavy closed-shell

- ○ How to extend to extend these approaches to heavy (doubly) open-shell nuclei?
  - ➟ Valence-space (shell model): diagonalisation → **factorial** scaling
  - ➟ Expansion combined with symmetry breaking / restoration → **polynomial** scaling



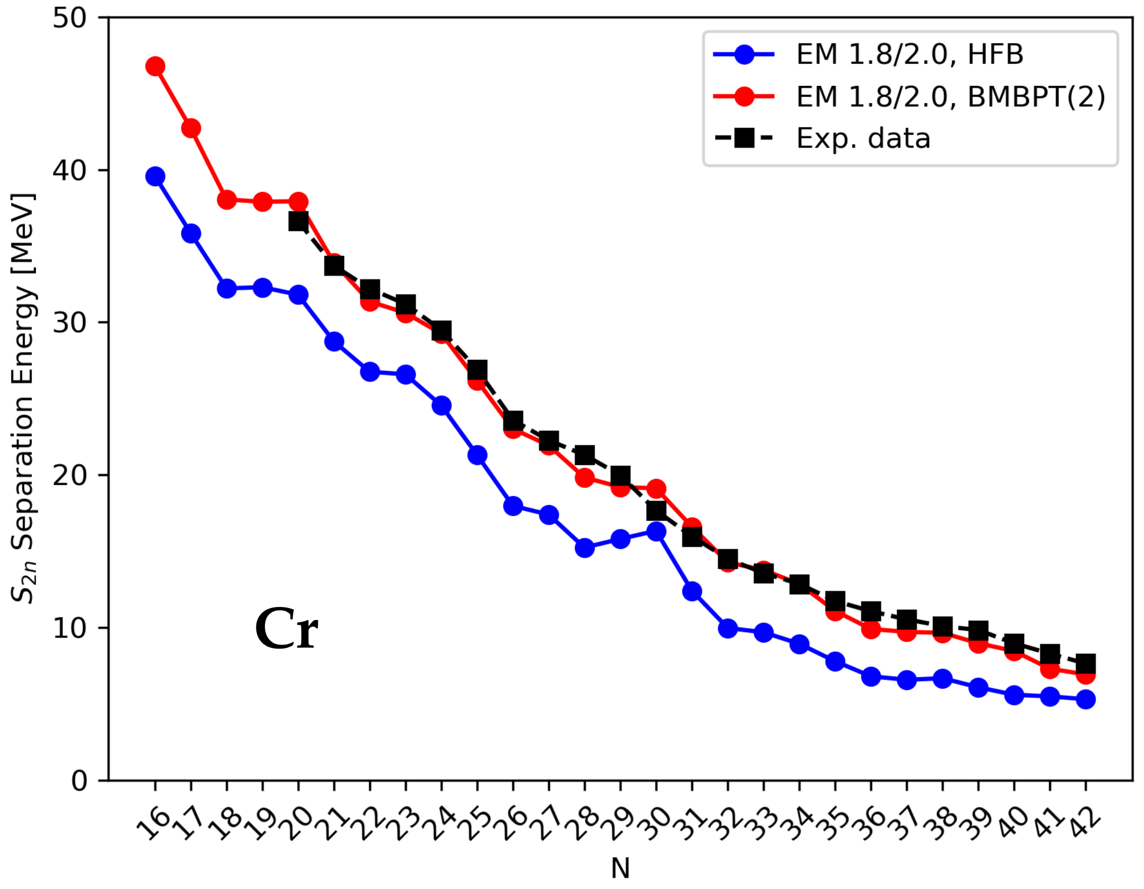
[Frosini *et al.* 2022]

PGCM



[Porro *et al.* in preparation]

MBPT



[Scalesi *et al.* in preparation]

SCGF

→ This talk

Outline

Motivation

⦿ One-neutron transfer reaction $^{36}S(p,d)^{35}S$ (iThemba LABS)

○ Goal: assess variation of **spin-orbit splitting in *N*=20 isotones** (cf. with known value in ^{40}Ca)

➟ Highlight the effect of **tensor** interaction

Objectives

⦿ Ab initio (self-consistent Green's function) calculations along *N*=20 isotonic chain

1) How do they perform, what can we learn?

2) Can we characterise the **scheme dependence** of non-observables ESPEs?

Articles

○ S. Jongile *et al.*, submitted (2023)

○ V. Somà & T. Duguet, submitted (2024)

Self-consistent Green's functions

- Many-body Schrödinger eq. → Dyson eq. $g_{\alpha\beta}(\omega) = g_{0\,\alpha\beta}(\omega) + \sum_{\gamma\delta} g_{0\,\alpha\gamma}(\omega)\,\Sigma^{\star}_{\gamma\delta}(\omega)\,g_{\delta\beta}(\omega)$
- One-body propagator displays **spectral representation**

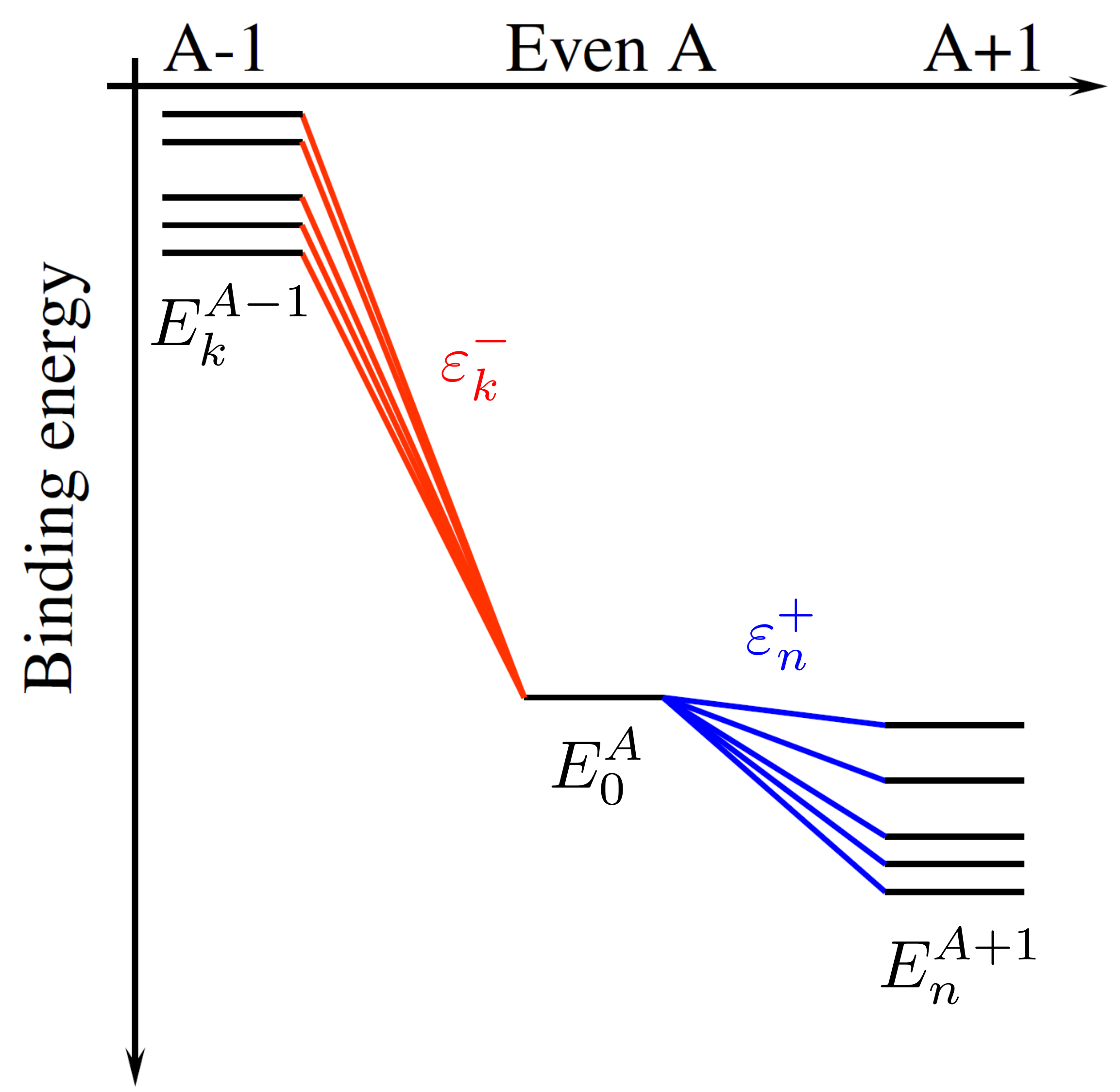
$$g_{\alpha\beta}(\omega) = \sum_n \frac{(\mathcal{X}^n_\alpha)^* \mathcal{X}^n_\beta}{\omega - \varepsilon^+_n + i\eta} + \sum_k \frac{\mathcal{Y}^k_\alpha (\mathcal{Y}^k_\beta)^*}{\omega - \varepsilon^-_k - i\eta}$$

Transition amplitudes

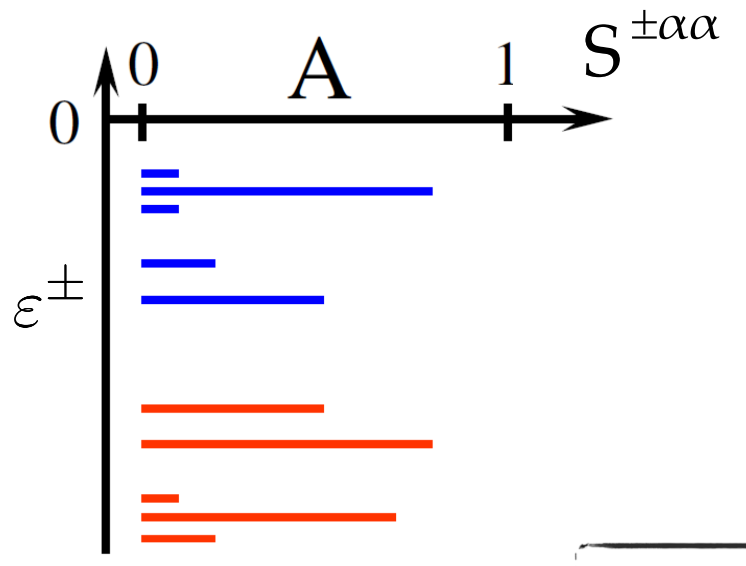
$$\mathcal{X}^n_\alpha = \langle \Psi^{\text{A+1}}_n | a^\dagger_\alpha | \Psi^{\text{A}}_0 \rangle$$
$$\mathcal{Y}^k_\alpha = \langle \Psi^{\text{A-1}}_k | a_\alpha | \Psi^{\text{A}}_0 \rangle$$

Separation energies

$$\varepsilon^+_n = E^{A+1}_n - E^A_0$$
$$\varepsilon^-_k = E^A_0 - E^{A-1}_k$$



Numerator + denominator



Spectroscopic matrices

$$\mathbf{S}^+_n \equiv \mathcal{X}_n \mathcal{X}^\dagger_n$$
$$\mathbf{S}^-_k \equiv \mathcal{Y}^*_k \mathcal{Y}^T_k$$

Spectral function

$$\mathbf{S}(\omega) \equiv \sum_{n \in \mathcal{H}_{A+1}} \mathbf{S}^+_n\, \delta(\omega - \varepsilon^+_n) + \sum_{k \in \mathcal{H}_{A-1}} \mathbf{S}^-_k\, \delta(\omega - \varepsilon^-_k)$$

Effective single-particle energies (I)

⊙ **Many-body observables often difficult to interpret**

○ E.g. separation energies can not, in general, be used to define a single-nucleon shell structure

➟ Resort to simpler/reduced quantities, e.g. effective single-particle energies (ESPEs)

⊙ Well-defined procedure to **compute ESPEs from a correlated wave function** [Baranger 1970]

○ Moments of the spectral function

$$\mathbf{M}^{(p)} \equiv \int_{-\infty}^{+\infty} \omega^p \, \mathbf{S}(\omega) \, d\omega$$

○ First moment define centroid Hamiltonian

$$\mathbf{M}^{(1)} = \sum_{n \in \mathcal{H}_{A+1}} \mathbf{S}_n^+ \varepsilon_n^+ + \sum_{k \in \mathcal{H}_{A-1}} \mathbf{S}_k^- \varepsilon_k^- \equiv \mathbf{h}^{\text{cent}}$$

○ Eigenvalues of $\mathbf{h}^{\text{cent}}$ represent ESPEs

$$\mathbf{h}^{\text{cent}} \, |\psi_\beta^{\text{cent}}\rangle = e_\beta^{\text{cent}} \, |\psi_\beta^{\text{cent}}\rangle$$

○ In Baranger basis, ESPEs are energy centroids

$$e_\beta^{\text{cent}} \equiv \sum_{n \in \mathcal{H}_{A+1}} S_n^{+\beta\beta} \varepsilon_n^+ + \sum_{k \in \mathcal{H}_{A-1}} S_k^{-\beta\beta} \varepsilon_k^-$$

➟ Baranger procedure is **independent of the underlying theoretical approach**

➟ However, ESPEs values **depend on the scheme and scale** of the theoretical approach

Effective single-particle energies (II)

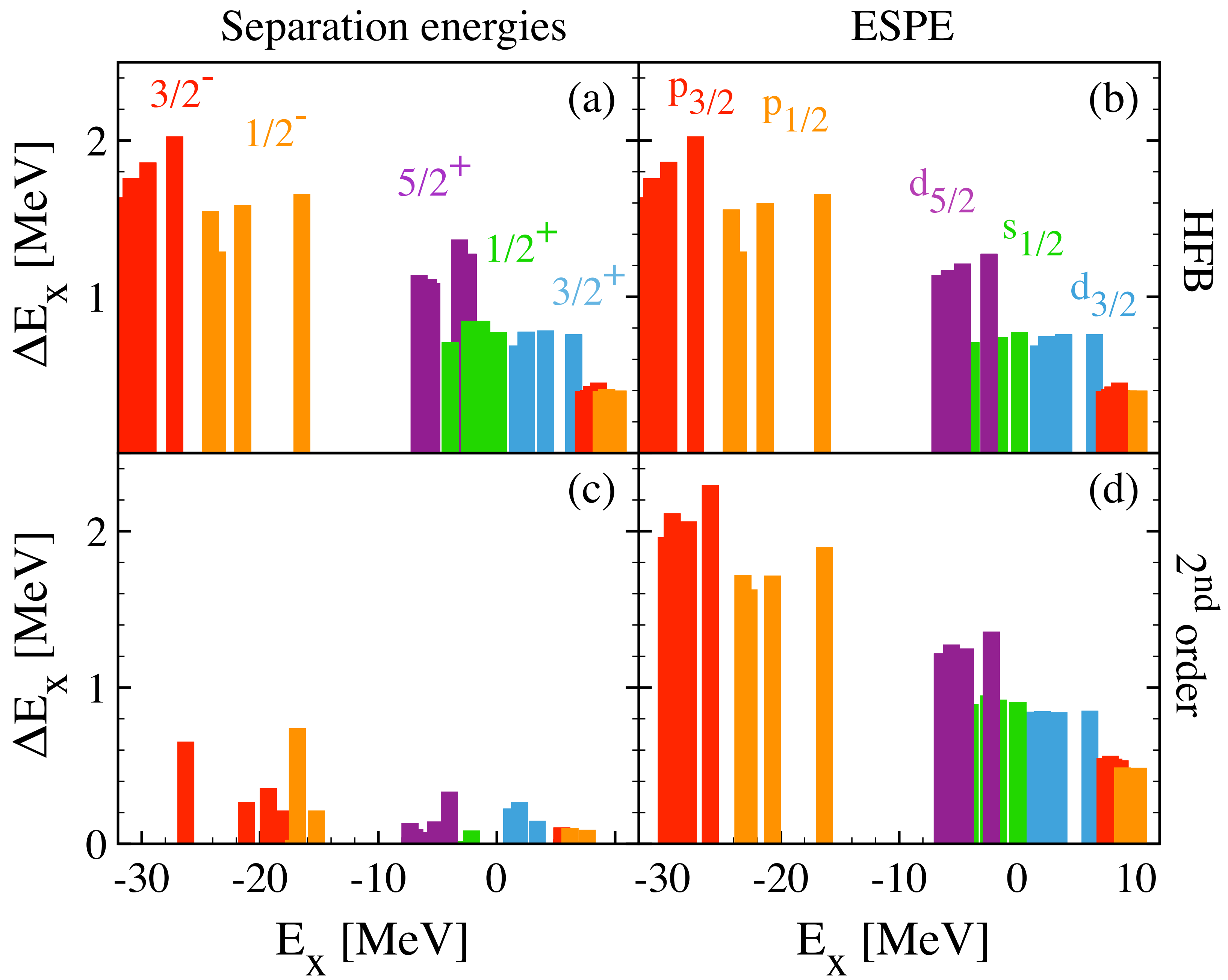
⦿ **At fixed scheme, scale dependence relates to changes in the input interaction**

○ E.g. via a unitary transformation of the Hamiltonian

➟ Proven to affect ESPEs

➟ While observables ~ unchanged

[Duguet *et al.* 2015]



⦿ **At fixed scale, scheme dependence relates to degrees of freedom, model assumptions, …**

○ E.g. valence-space vs full-space approaches

○ Also concerns **"experimental" ESPEs** (entering e.g. via DWBA calculations)

○ Ultimately relates to (non-observable) "correlations" in the nuclear wave functions

➟ **No "true/correct" theoretical scheme**, all equally valid!

➟ ESPEs from different schemes must be compared **with care**

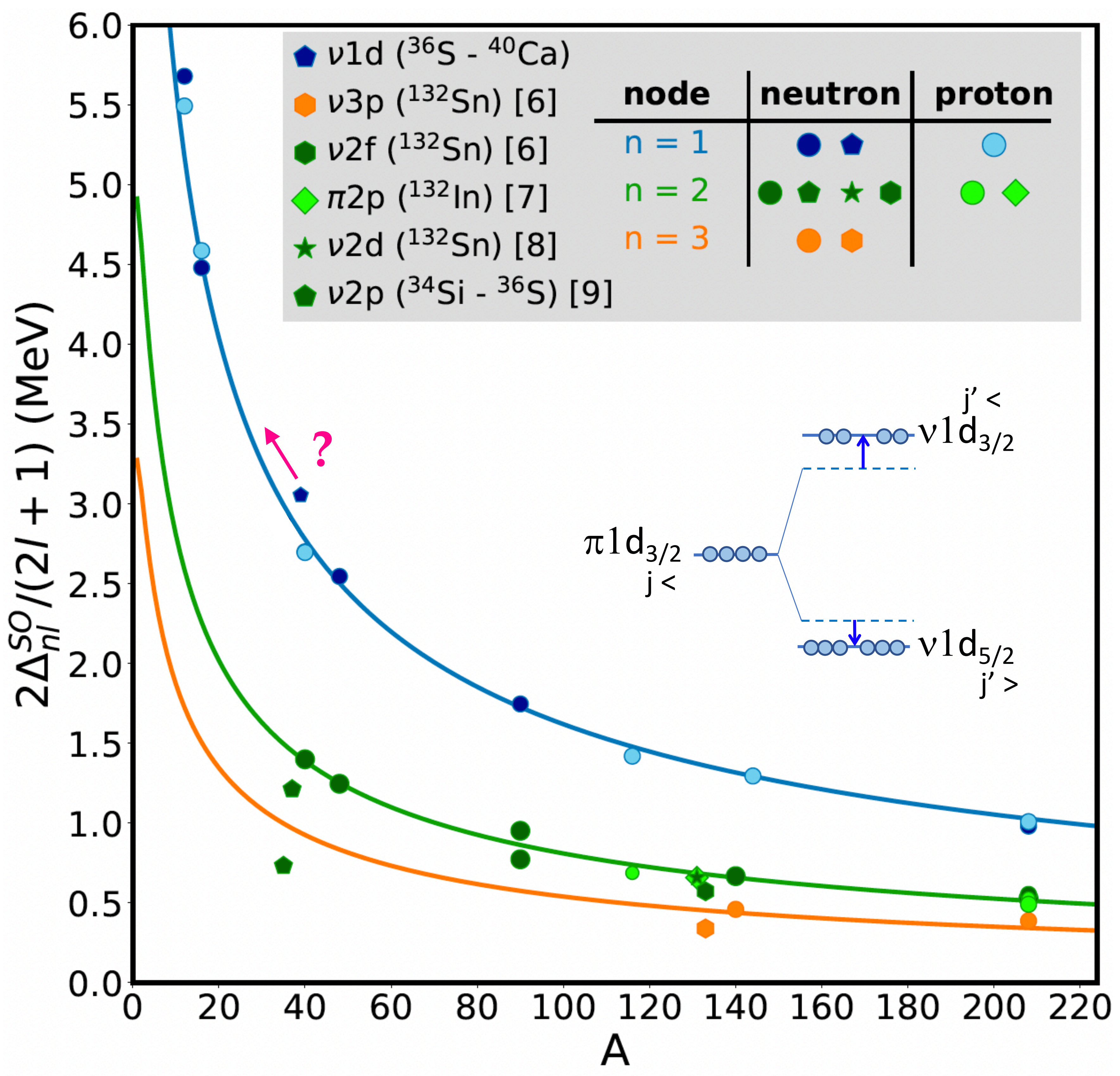
Physics case

- **Central**, **spin-orbit** and **tensor** operators at play in nuclear interactions
  - ➟ How do they impact nuclear shell structure (and its evolution with N & Z)?
- E.g., evolution of energy splitting between spin-orbit partners well understood

$$\Delta^{SO}_{n\ell} \equiv e^{cent}_{n\ell_{j_<}} - e^{cent}_{n\ell_{j_>}} \quad \text{with} \quad \begin{array}{l} j_> = \ell + s \\ j_< = \ell - s \end{array}$$

[Mairle *et al.* 1993]

  - ➟ Smooth evolution with *A* and *n*
- **Tensor force** expected to perturb this picture
  - **Repulsive** between $j_<$ and $j'_<$
  - **Attractive** between $j_<$ and $j'_>$



[S. Jongile *et al.* submitted]

- Focus on $1d_{5/2}$-$1d_{3/2}$ splitting in N=20
  - When going from Z=20 to Z=16
    - ➟ Mairle evolution predicts **increase** of Δ^{SO}
    - ➟ Tensor force induces a **decrease** of Δ^{SO}

What happens?

The experiment

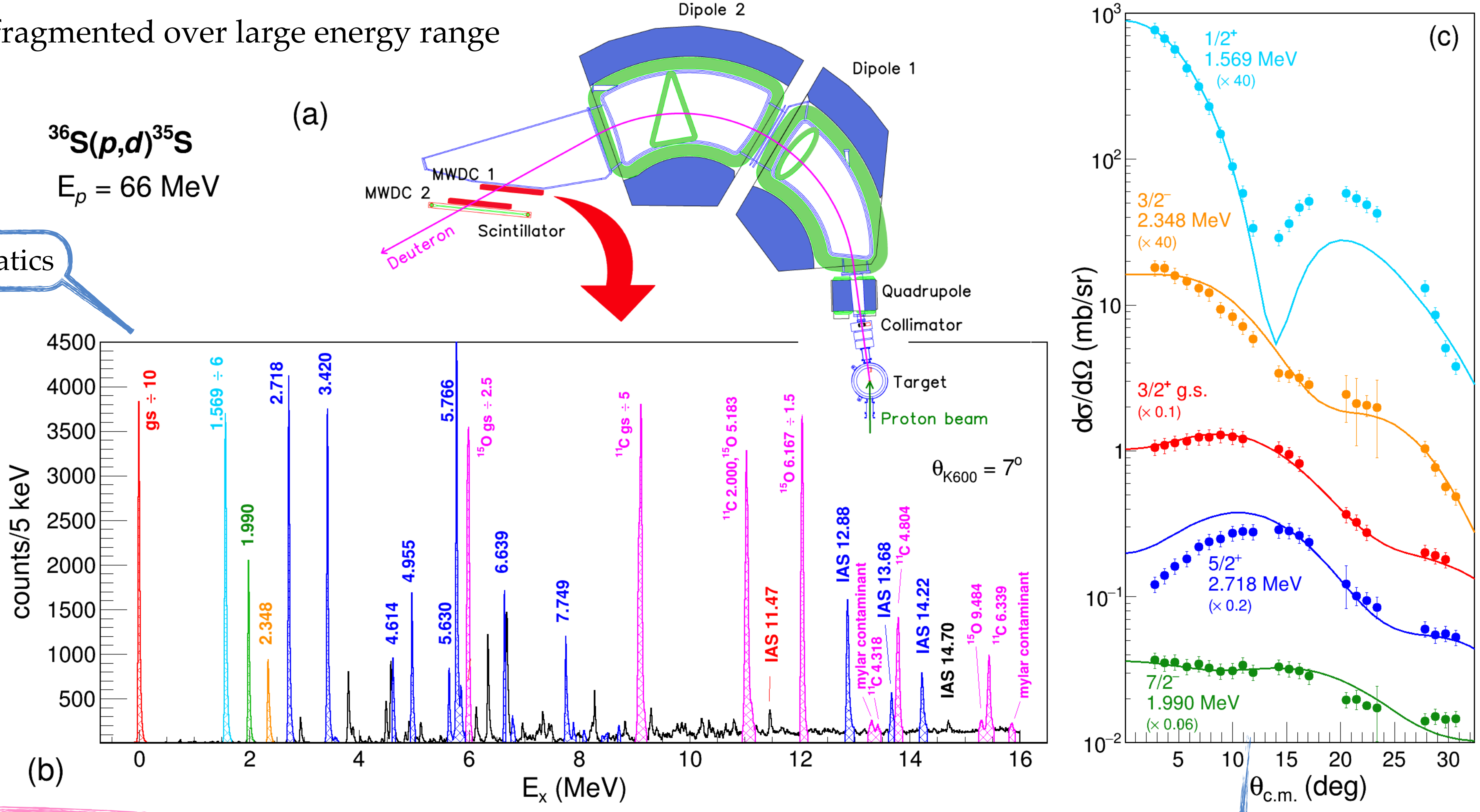
⊙ **One-neutron transfer reaction $^{36}S(p,d)^{35}S$ at E_p=66 MeV @ iThemba LABS**

[S. Jongile *et al.* submitted]

○ Challenge: strength typically fragmented over large energy range

Excitation spectrum from d kinematics

S. Jongile
R. Neveling
O. Sorlin
M. Wiedeking
et al.



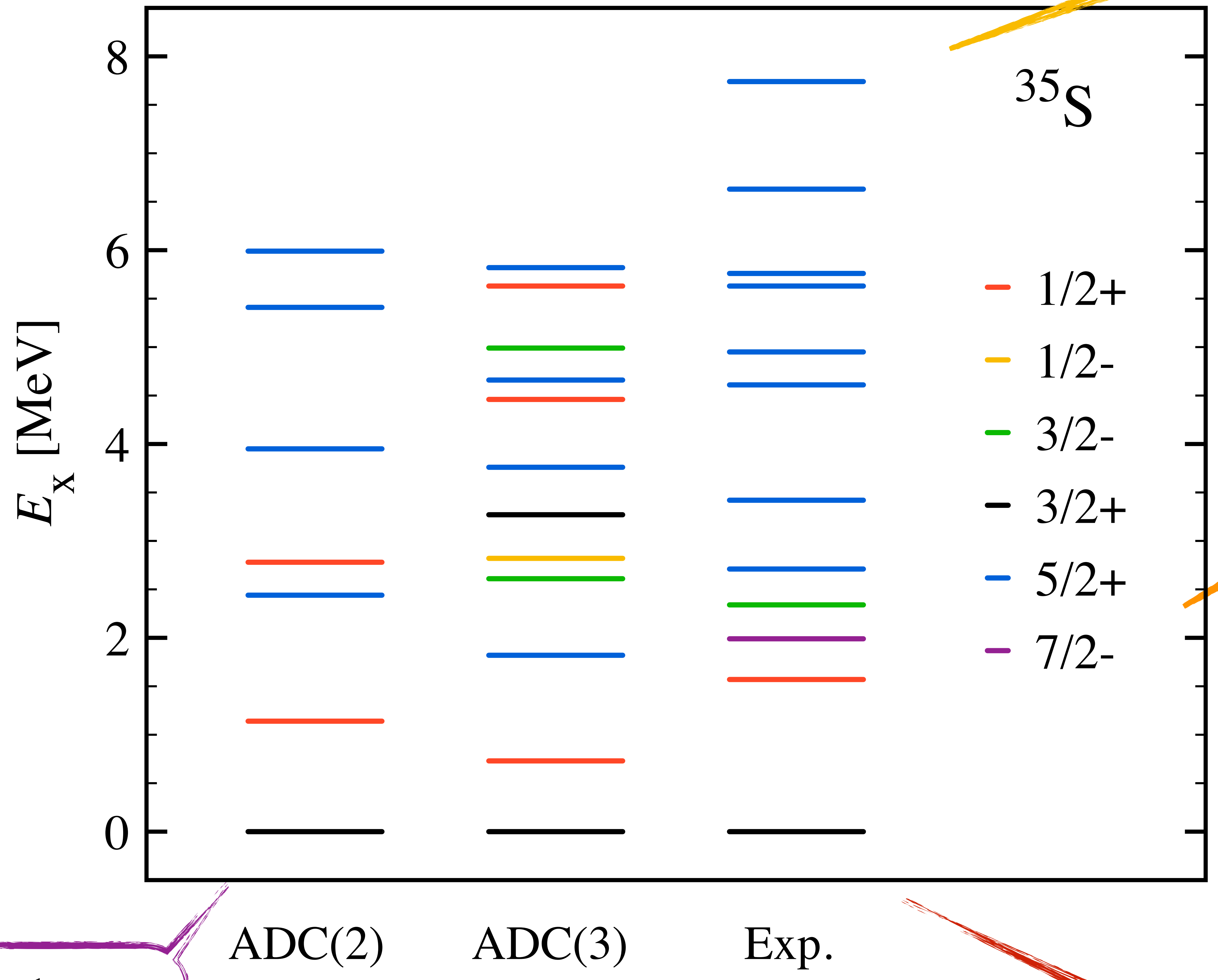
→ **Identification of 98 states** in ^{35}S up to E_x=16 MeV

L & J assignment from d angular distributions

Excitation spectra

- **Excitation energies** can be cleanly compared between experiment & theory
- Theoretical set-up: SCGF in ADC(2) & ADC(3), $NNLO_{sat}$ (2N+3N) interaction

Strong fragmentation of $5/2^+$



Some J^π absent in ADC(2,3) → Collective character?

Compression of the spectrum when going ADC(2) → ADC(3)

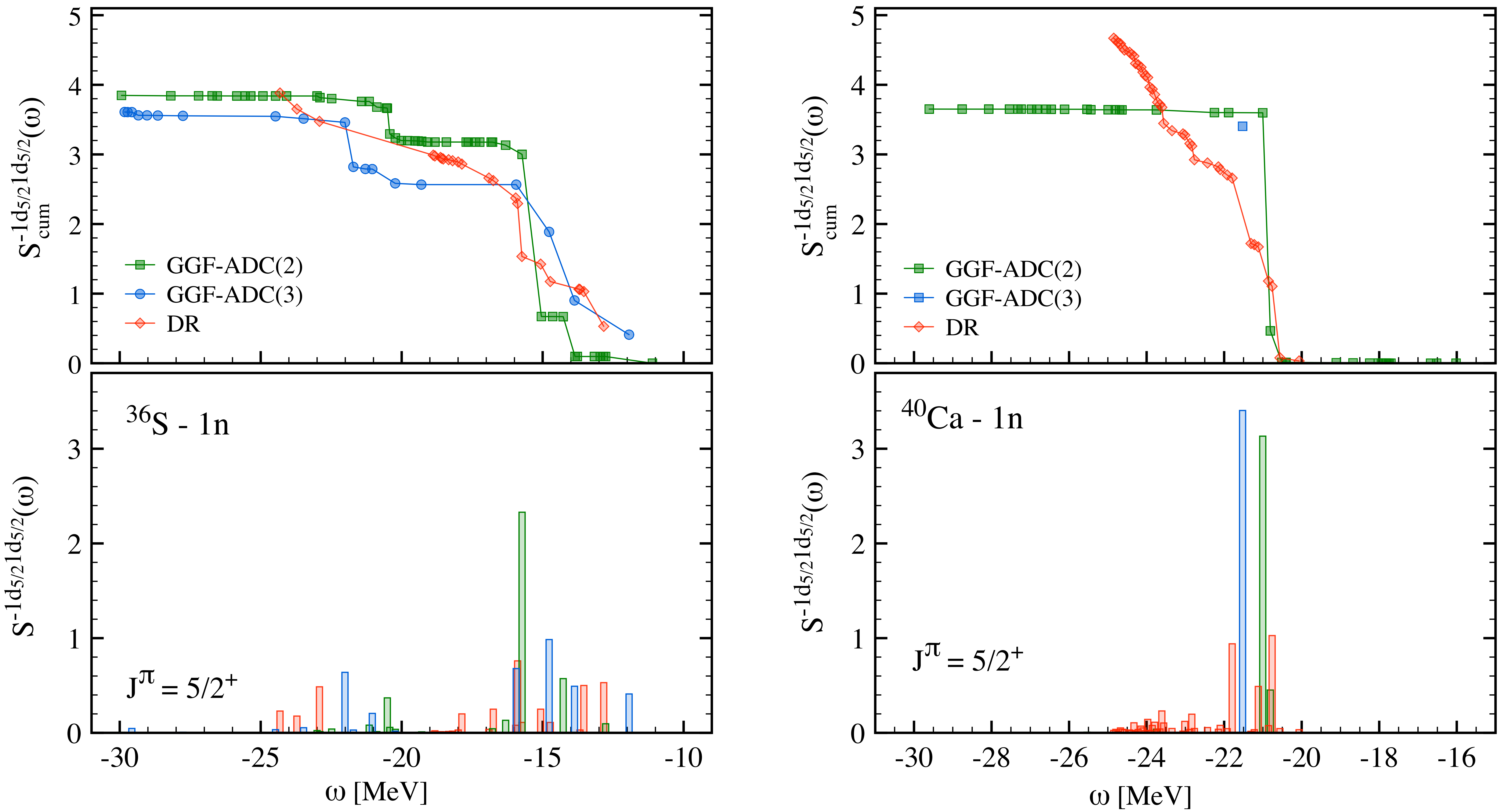
First few states too low in ab initio calculations

Spectral function

- **DWBA calculations** to obtain spectroscopic amplitudes from measured cross sections
  - ➟ "Experimental" spectral function is in fact **theoretical-scheme-dependent**
  - ➟ Consistent calculation performed for ^{40}Ca(p,d)^{39}Ca reaction (+ 1n addition channels)

[S. Jongile *et al.* submitted]

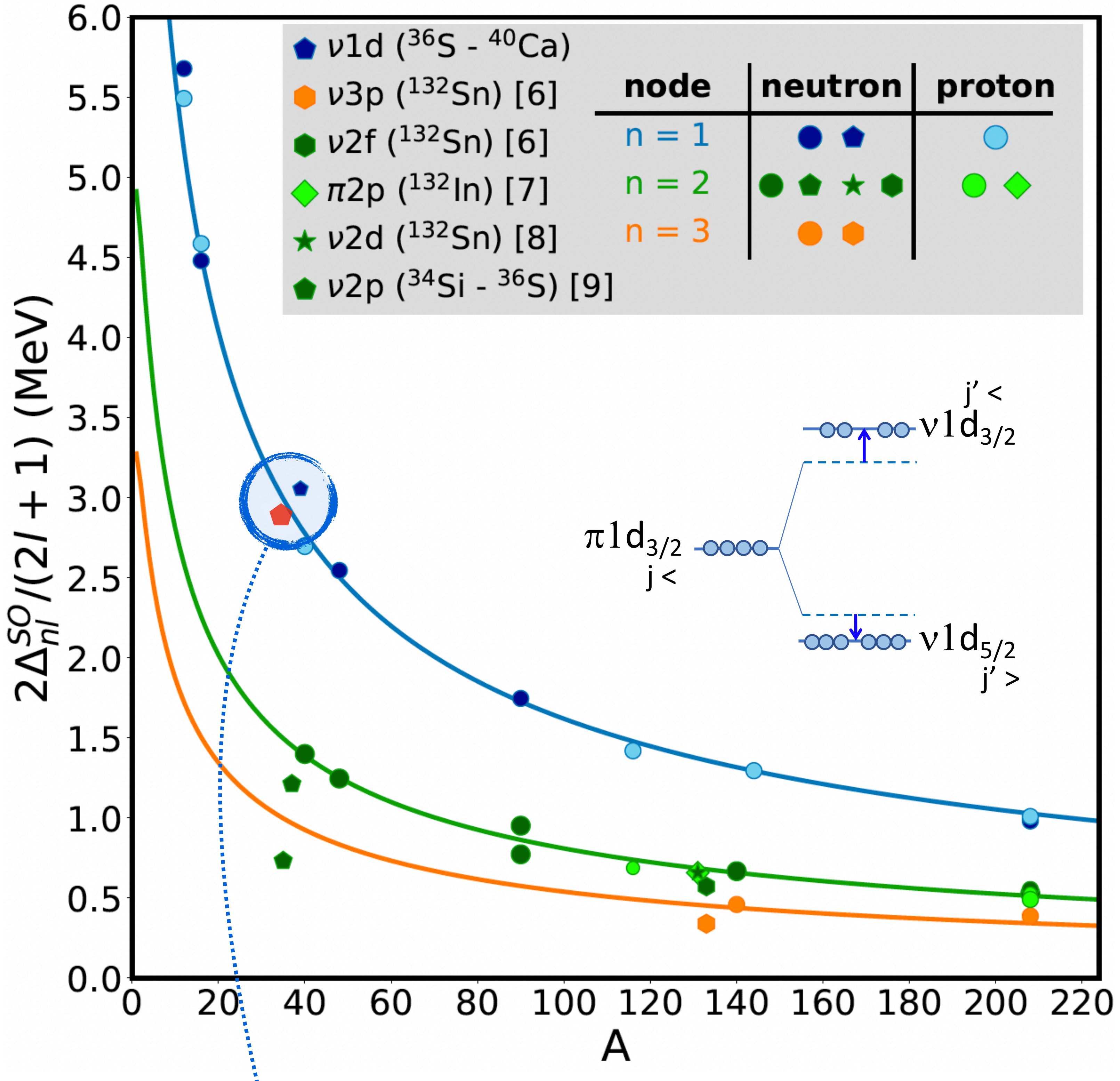
Direct-reaction (DR)
vs
ab initio calculations



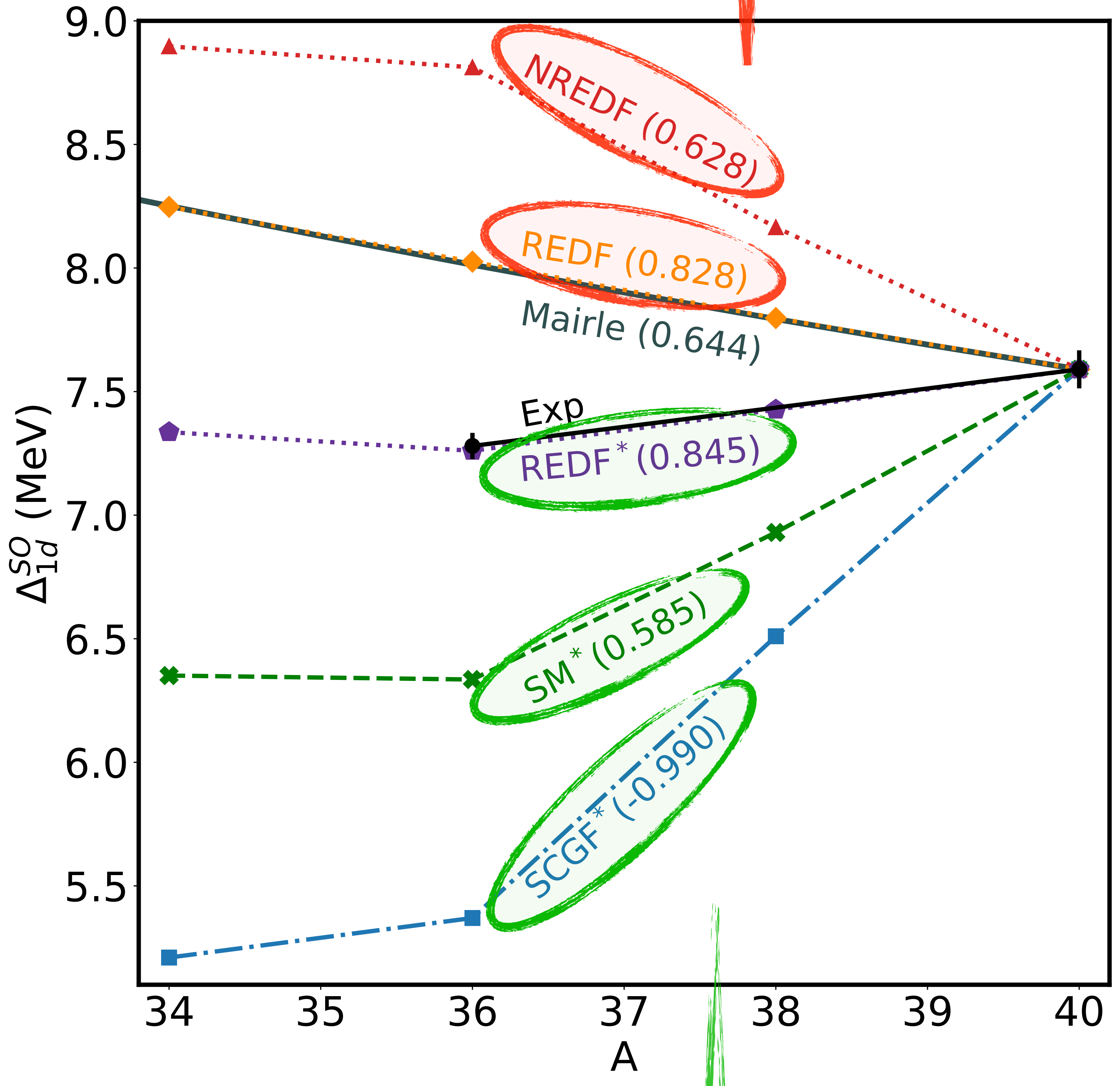
Spin-orbit splitting

- Resulting ESPEs determine spin-orbit splitting in the two nuclei

[S. Jongile *et al.* submitted]



Calculations **without** tensor operator → $\Delta^{SO}(^{40}Ca) < \Delta^{SO}(^{36}S)$



Calculations **with** tensor operator → $\Delta^{SO}(^{40}Ca) > \Delta^{SO}(^{36}S)$

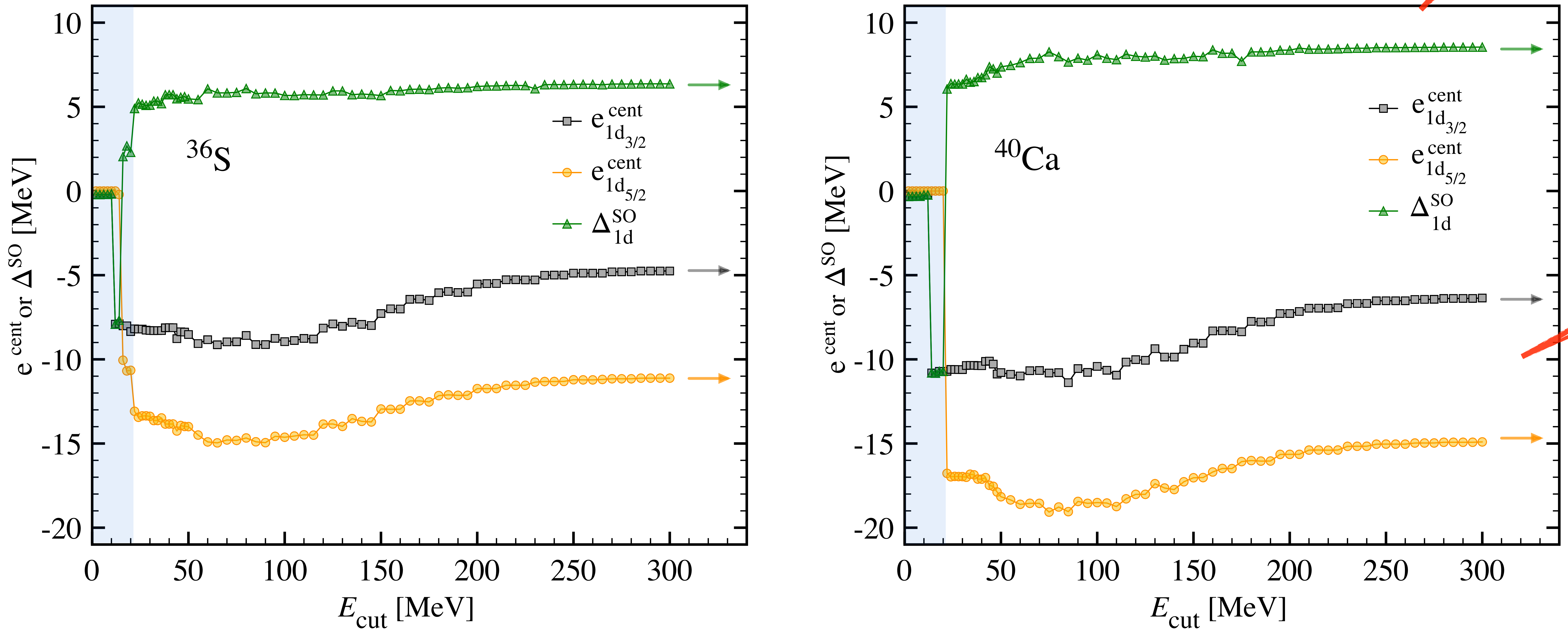
$\Delta^{SO}(^{40}Ca) - \Delta^{SO}(^{36}S)$ deviates from the trend → **signature of tensor interaction**

Stability of ESPEs (I)

- **Direct-reaction** approach restricted to ω accessible via one-nucleon removal / addition experiments
- **Shell model** restricted by construction to energy range of the valence space
- Within the ab initio approach, one can examine the impact of limiting the energy range

$$\mathbf{M}^{(1)} = \sum_{n \in \mathcal{H}_{A+1}} \mathbf{S}_n^+ \varepsilon_n^+ + \sum_{k \in \mathcal{H}_{A-1}} \mathbf{S}_k^- \varepsilon_k^- \equiv \mathbf{h}^{\text{cent}} \qquad \text{imposing} \qquad |\varepsilon_p^\pm| \leq E_{\text{cut}}$$

Δ^{SO} more stable but also varies



ESPE non-trivially modified

[V. Somà & T. Duguet submitted]

Stability of ESPEs (II)

⦿ Further hypothesis of DR and SM approaches → use of a **single harmonic oscillator shell**

○ Ab initio: approximation of omitting off-diagonal elements in spectroscopic matrices

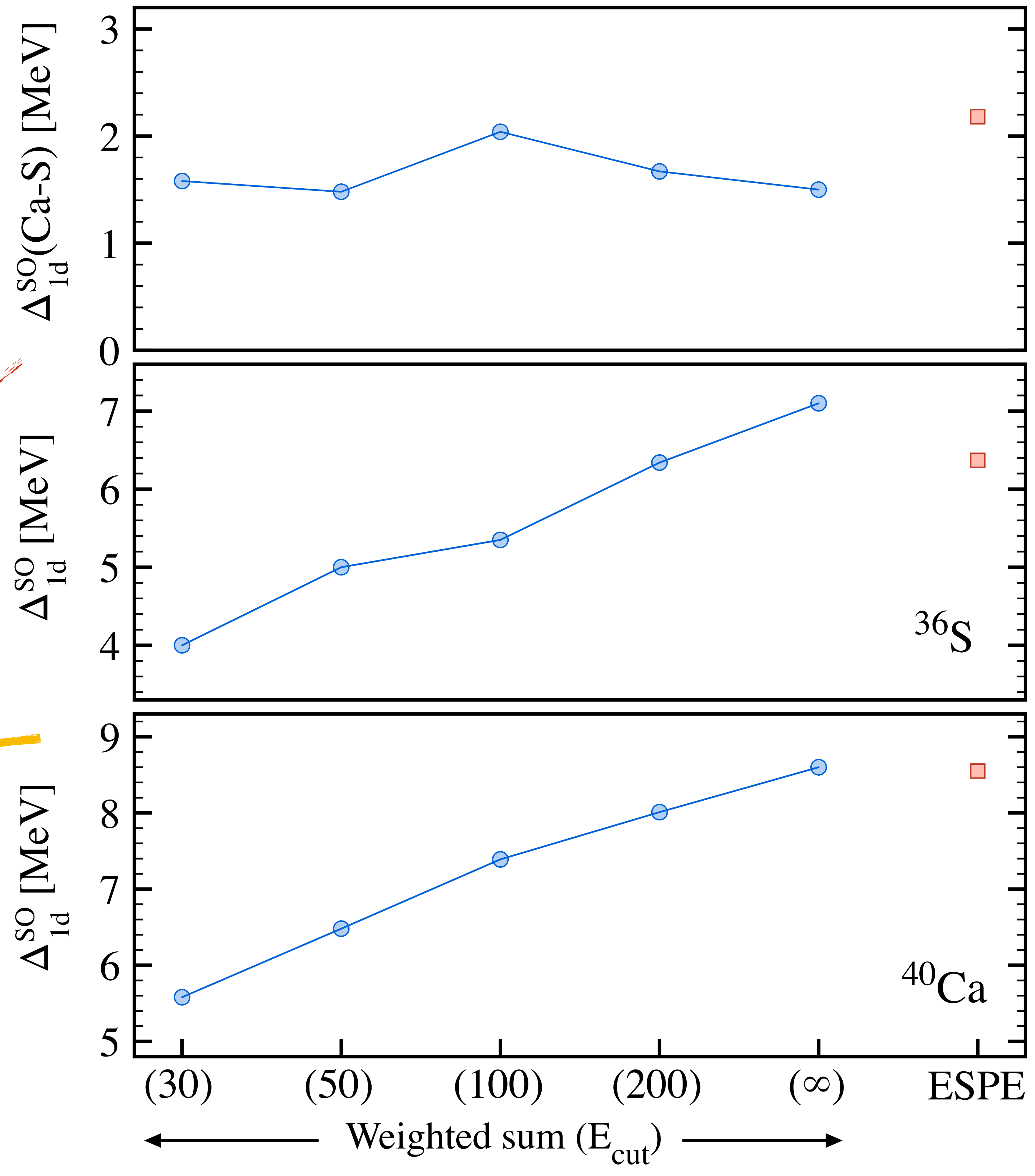
[V. Somà & T. Duguet submitted]

$$e_\mu^{\text{cent}} \approx \sum_{n\in\mathcal{H}_{A+1}} S_n^{+\mu\mu}\varepsilon_n^+ + \sum_{k\in\mathcal{H}_{A-1}} S_k^{-\mu\mu}\varepsilon_k^-$$

Would be exact in direct-reaction or shell-model schemes

Net effect of diagonal approximation: reduction of Δ^{SO} variation

Similar trend, but different end result → $5/2^+$ more fragmented in ^{36}S

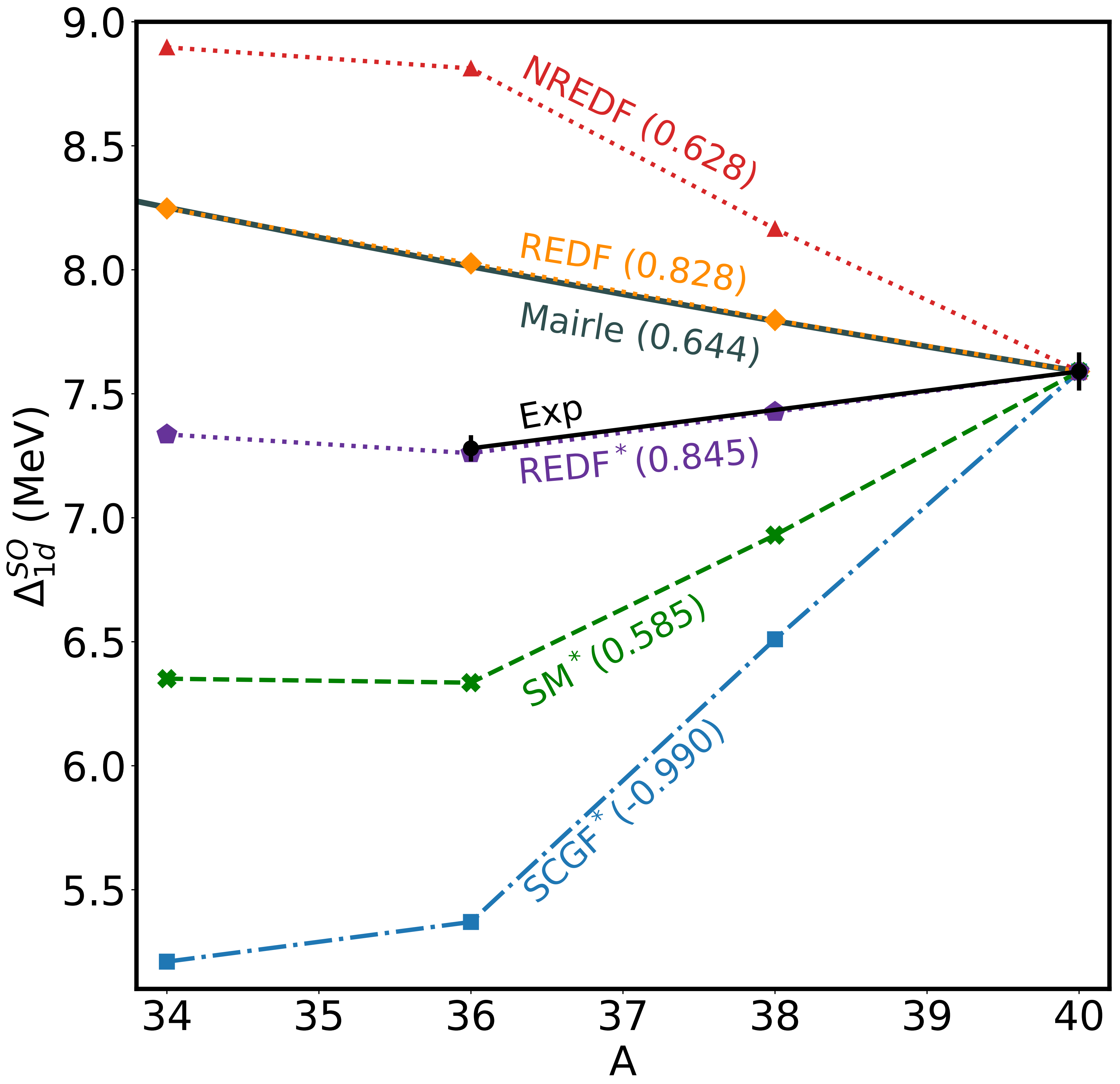


Conclusions qualitatively unchanged

Conclusions

⦿ Notion of shell structure (& its evolution) based on effective single-particle energies

○ **Unambiguous procedure**, but result does depend on **scheme** and **scale** of the theory



⦿ Present work illustrates **dependence on the theoretical scheme**

○ Ab initio (full-space) SCGF calculations

○ Application to evolution of neutron ℓ=2 spin-orbit energy splitting in N=20 nuclei

→ Reduction interpreted as **fingerprint of tensor force**

→ Qualitative understanding of scheme dependence

→ Approximations make ab initio closer to DR & SM

(To be taken with a grain of salt!)