Three-nucleon force manifestations in the shell structure of atomic nuclei

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- Tokuro Fukui (Kyushu University- Fukuoka)
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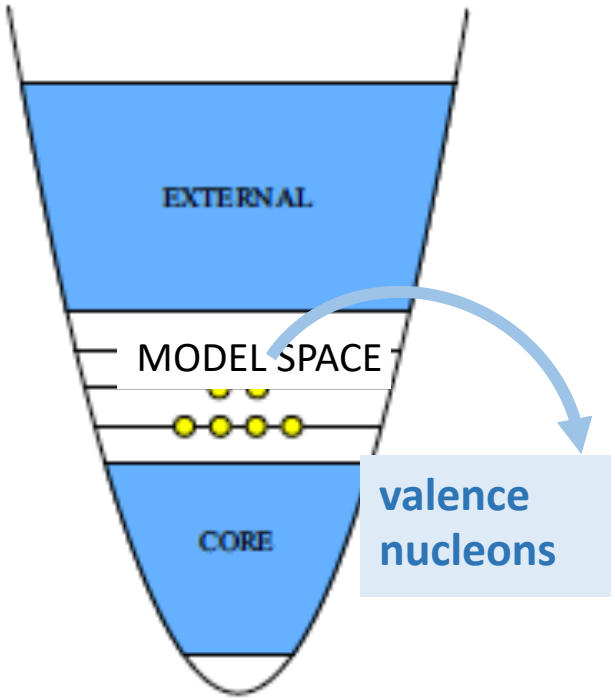
- Z. H. Cheng (Peking University)
- Furong Xu (Peking University)

L.Coraggio et al., *The role of three-nucleon potentials within the shell model: past and present*, Progress in Particle and Nuclear Physics 134, (2024) 104079

Shell model and realistic *NN* interactions



$$H_{\text{eff}} = \sum_a \epsilon_a N_a - \frac{1}{4} \sum_{abcdJ} \langle ab; J \mid V_{\text{eff}} \mid cd; J \rangle (-1)^J [a_a^\dagger a_b^\dagger]^J \cdot [\tilde{a}_c \tilde{a}_d]^J,$$

defined in the model space for only valence nucleons

SM effective interactions derived from realistic *NN* potentials

$$V_{NN} \Longrightarrow \text{many} - \text{body theory} \Longrightarrow \text{H}_{\text{eff}}$$

show some deficiencies in reproducing nuclear spectroscopy, especially when moving far from closed shells

phenomenological corrections to $\mathbf{H_{eff}}$

Monopole modified SM effective interactions

$H_{eff}'s$ from realistic *NN* potential need to be modified in their monopole components

Monopole component

$$H_{\rm mon} = \sum_{a\tau} \epsilon_{a\tau} N_{a\tau} + \frac{1}{2} \sum_{ab\tau\tau'} \frac{\bar{V}^{\tau\tau'}_{ab} N_{a\tau}(N_{b\tau'} - \delta_{ab}\delta_{\tau\tau'})}{\sum_J \hat{J}^2}$$

Centroid

$$\bar{V}^{\tau\tau'}_{ab} = \frac{\sum_J \hat{J} \langle a\tau b\tau'; J \mid V_{\rm eff} \mid a\tau b\tau'; J \rangle}{\sum_J \hat{J}}$$

$$\mathrm{ESPE}(a\tau) = \epsilon_{a\tau} + \sum_{b\tau'} \bar{V}^{\tau\tau'}_{ab} n^{\tau}_{b}$$

Responsible for the evolution of the SP energies

$n^{\tau}_{b} \equiv$ g.s occupation numbers

- Only centroids should be fitted to obtain results of a quality comparable with that provided by phenomenological interactions
- Deficiencies in centroids are related to the bad saturation and shell formation properties of the NN interaction and can be traced back to the lack of 3N forces

Initiated by Eduardo Pasquini in his Ph.D. thesis (1976) and pursued by the Strasbourg-Madrid group

3N forces and many-body systems

The explicit inclusion of 3NFs has historically been neglected in SM treatment due to

1. little knowledge of a mechanism providing many-body forces consistently with the nature of the NN interaction

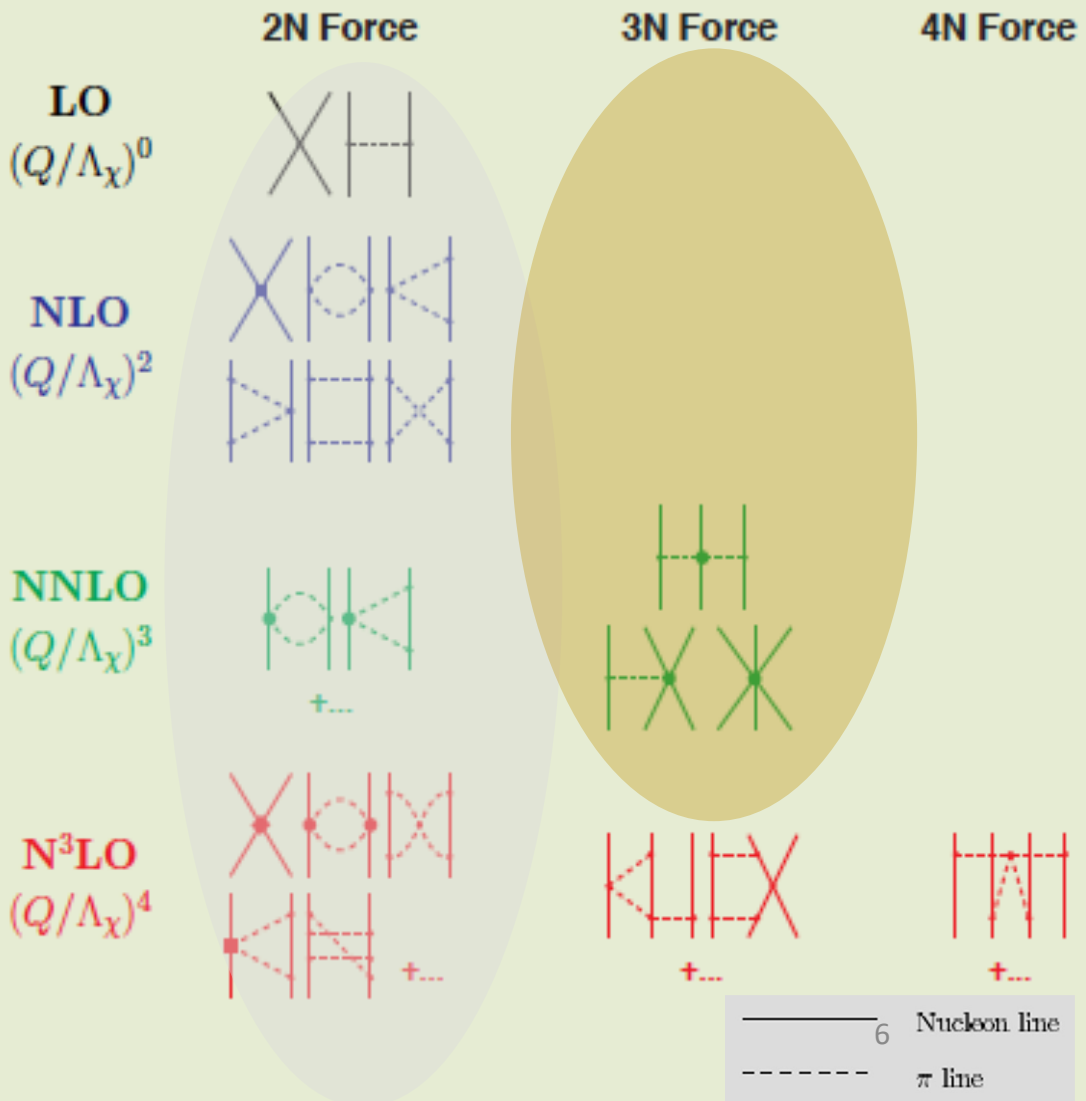
2. difficulty in handling a 3N term in many-body systems whose solution requires formalisms that are computationally extremely demanding

Potentials derived in ChPT

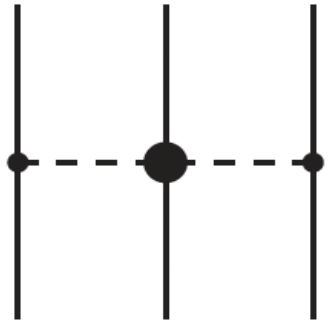
- Nucleons interact via pion exchanges and short-range contact interactions. The long-range forces are ruled by the symmetries of QCD, while short-range forces - which are not resolved - are absorbed into contacts terms proportional to low-energy constants (LEC)

- Chiral potentials are organized in a systematic low-momentum expansion, where two- and many-body forces are generated on an equal footing

- Most interaction vertices that appear in the 3N force also occur in the *NN* force → consistency requires that the parameters LECs carried by these vertices have the same values for *NN* and 3N terms



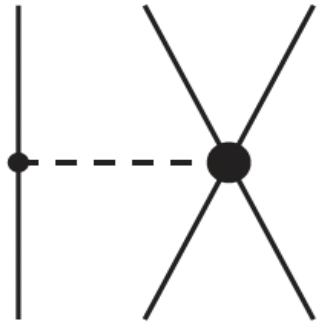
3N chiral potential at N^2LO



two-pion exchange

No new parameters with respect to those appearing

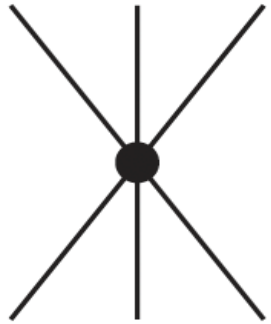
in the expression of the N^3LO 2NF



one-pion exchange +

NN-contact interaction

New LEC parameter: c_D



pure 3N-contact interaction

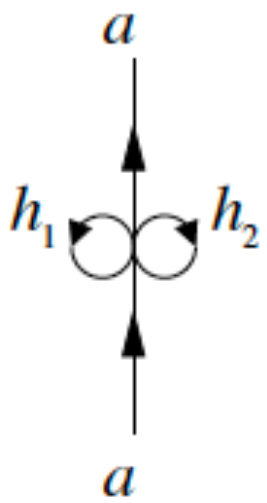
New LEC parameter: c_E

c_D and c_E may be fixed by an optimal over-all fit of the properties of light nuclei, as for instance in P. Navrátil et al, PRL 99 (2007) 042501

Normal-ordered decomposition of the 3N component

Starting from a reference state and using the Wick theorem, the three-body component of the nuclear Hamiltonian can be re-arranged into a sum of zero-, one-, two-, and three-body terms → only normal-ordered one- and two-body parts of 3N forces are included

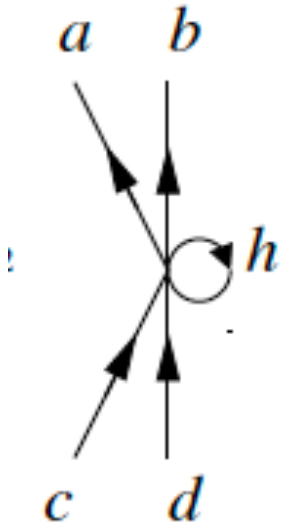
normal-ordered one-body part of the 3N force



interactions among one-valence and two-core nucleons

$$\epsilon_a^{(3\mathrm{NF})} = \sum_{\substack{h_1,h_2 \\ J_{12}J}} \frac{\hat{J}^2}{2\hat{j}_a{}^2} {}_A\langle (h_1,h_2),a;J_{12}J\,|V_{3N}|\,(h_1,h_2),a;J_{12}J\rangle_A$$

normal-ordered two-body part of the 3N force



interactions among two-valence and one-core nucleons

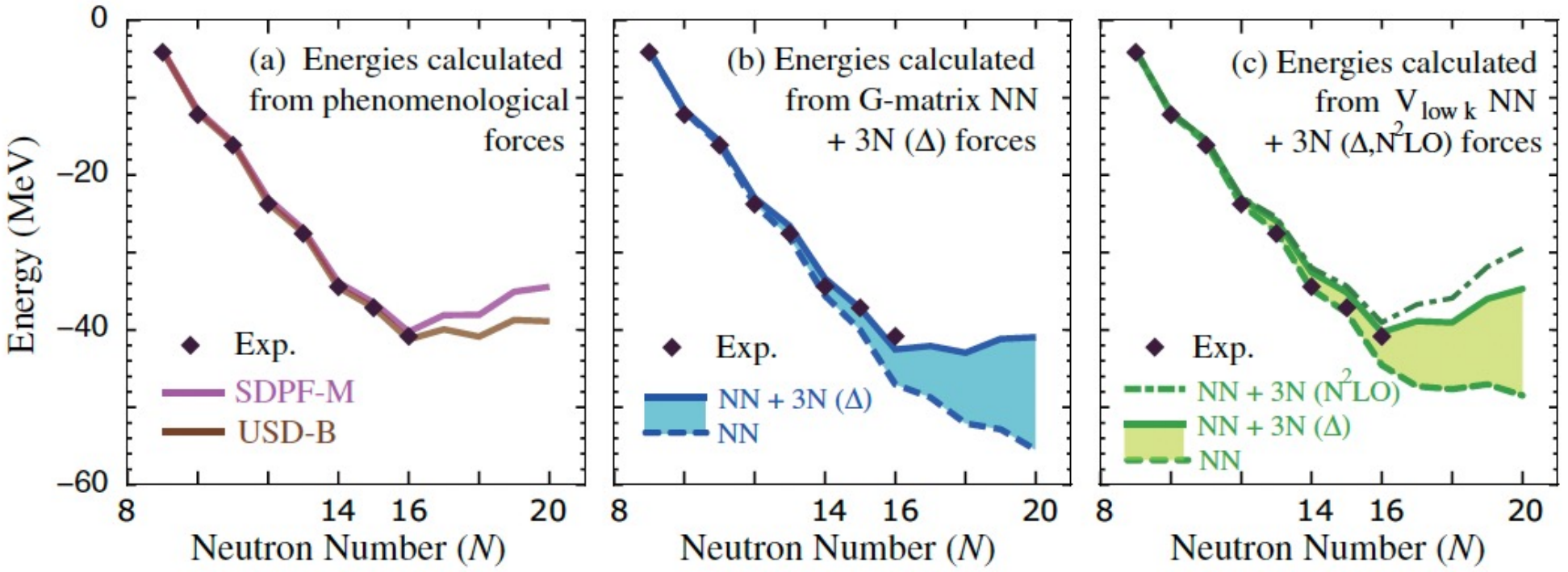
$$V_{ab,cd,J}^{(3\mathrm{NF})} = \sum_{h,J'} \frac{\hat{J}'^2}{\hat{J}^2} {}_A\langle (a,b),h;JJ'\,|V_{3N}|\,(c,d),h;JJ'\rangle_A$$

First study explicitly including the 3NF in the derivation of the effective SM Hamiltonian in the *sd* space

With an *NN* force-only, g.s. energies do not stop to decrease putting the dripline at N = 20

3N contributions correct the g.s behavior bringing a significant raise from N = 16 to 18 and then provide the correct location of the drip line ↔ increase the $d_{5/2}$ - $d_{3/2}$ SO splitting

Ground-state energies of oxygen isotopes measured from ^{16}O



T. Otsuka et al, PRL 105, 032501 (2010)

Outline of calculations

NN **and 3N forces within the ChPT framework at N^3LO and at N^2LO, respectively**

[D. R. Entem, R. Machleidt, PRC 66 (2002) 014002; and c_D c_E from P. Navrátil et al, PRL 99 (2007)042501]

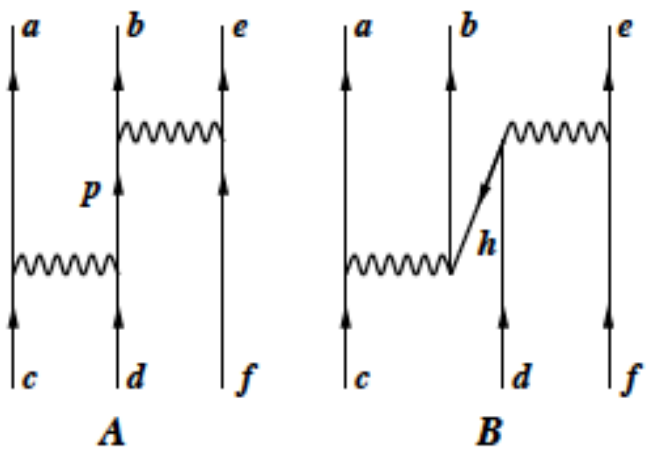
H_{eff} within the framework of the MBPT ($\hat{Q}$-box folded diagram expansion)

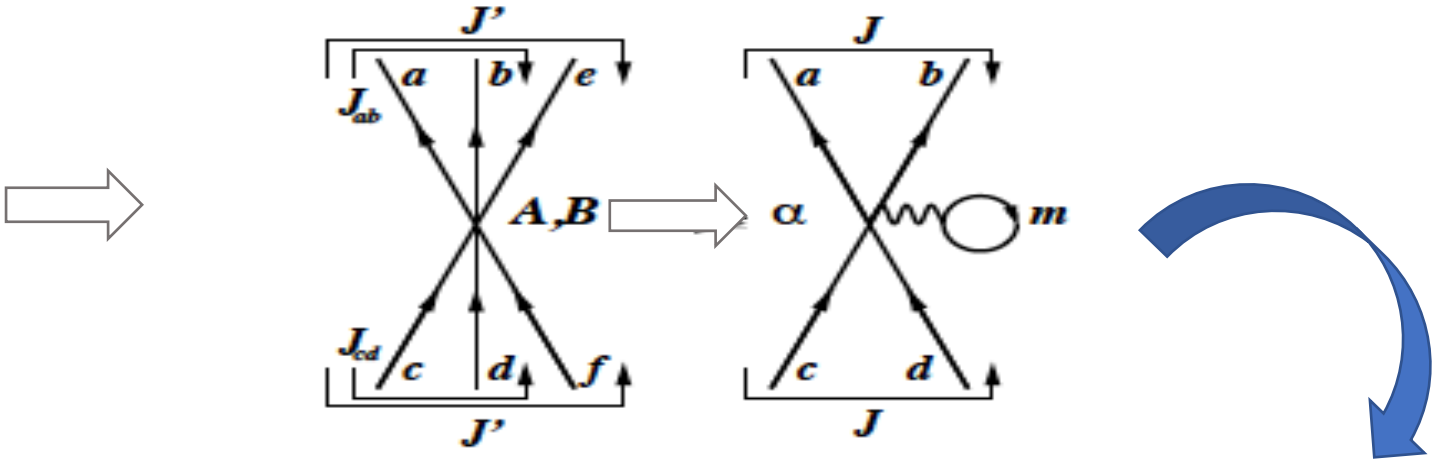
by including in the $\hat{Q}$-box one- and two-body Goldstone diagrams up to third order in the *NN* potential and at first order in the 3N one

[T. Fukui et al, PRC 98 (2018) 044305; Y. Z. Ma et al, PRC 100 (2019) 034324]

Induced 3N forces

For systems with more than 2-valence nucleons H_{eff} needs to account for the progressive filling of the model space orbitals → the $\hat{Q}$-box should include many-body diagrams arising from the interaction - via the 2NF - of the valence nucleons with virtual excitations outside the model space





Second order 3-body diagrams

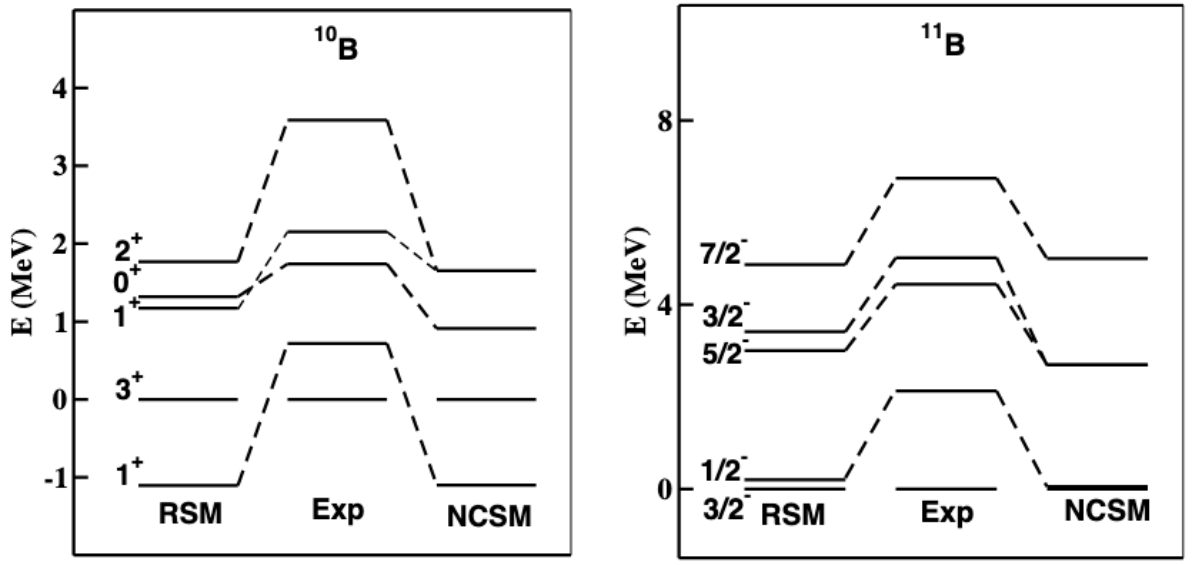
Density-dependent two-body contributions

$$\langle (j_a j_b)_J | V^{\alpha} | (j_c j_d)_J \rangle = \sum_{m\, J'} \rho_m \frac{\hat{J'}^2}{\hat{J}^2} \left\langle [(j_a j_b)_J, j_m]_{J'} \middle| V^{A,B} \middle| [(j_c j_d)_J, j_m]_{J'} \right\rangle$$

m runs in the valence space, $\rho_m \equiv$ unperturbed occupation of the orbital m according to the number of valence nucleons

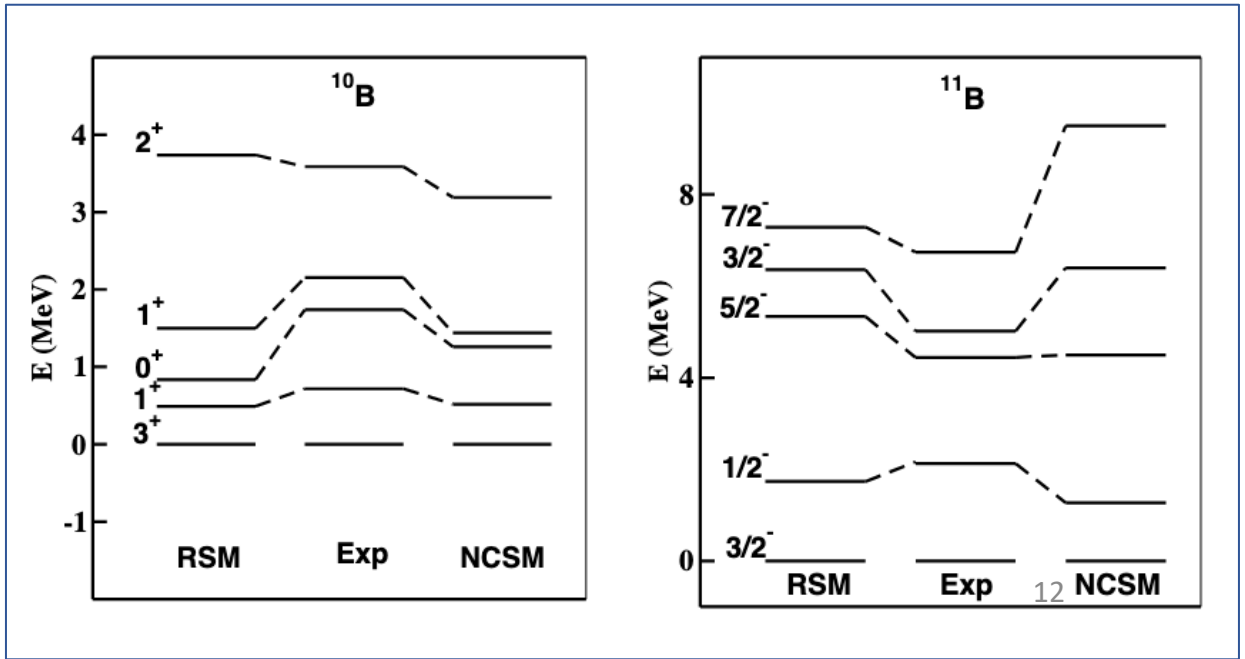
p-shell nuclei: B isotopes

Benchmark calculations: comparison of SM and ab initio NCSM results



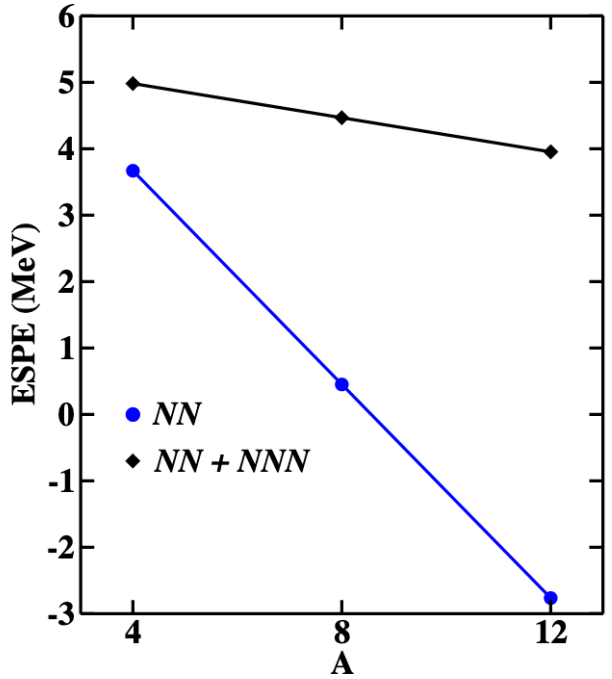
NN force

L. Coraggio, et al, Ann. Phys. (NY) 327 (2012) 2125

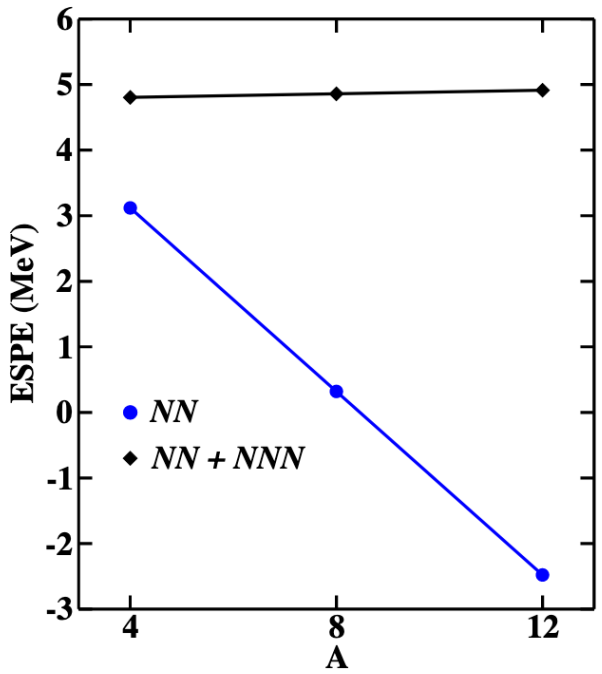


NN+3N forces

T. Fukui et al, PRC 98 (2018) 044305



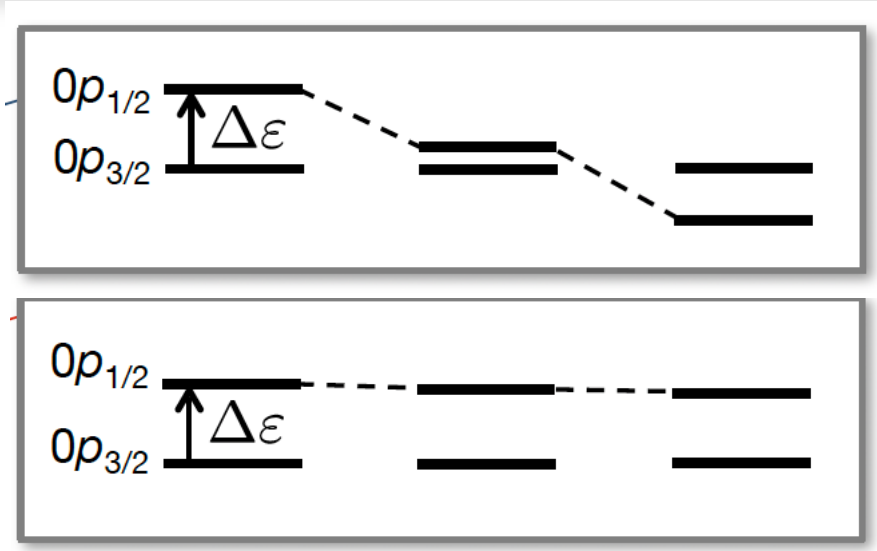
proton $0p_{1/2}$ ESPE relative to $0p_{3/2}$



neutron $0p_{1/2}$ ESPE relative to $0p_{3/2}$

Relative ESPE rapidly drops down when considering only the N^{3}LO *NN* potential, even becoming negative around A=8, while relative ESPE is almost constant (4 ~ 5 MeV) when the 3N potential is taken into account → which reflects on the spectra of ^{10}B and ^{11}B

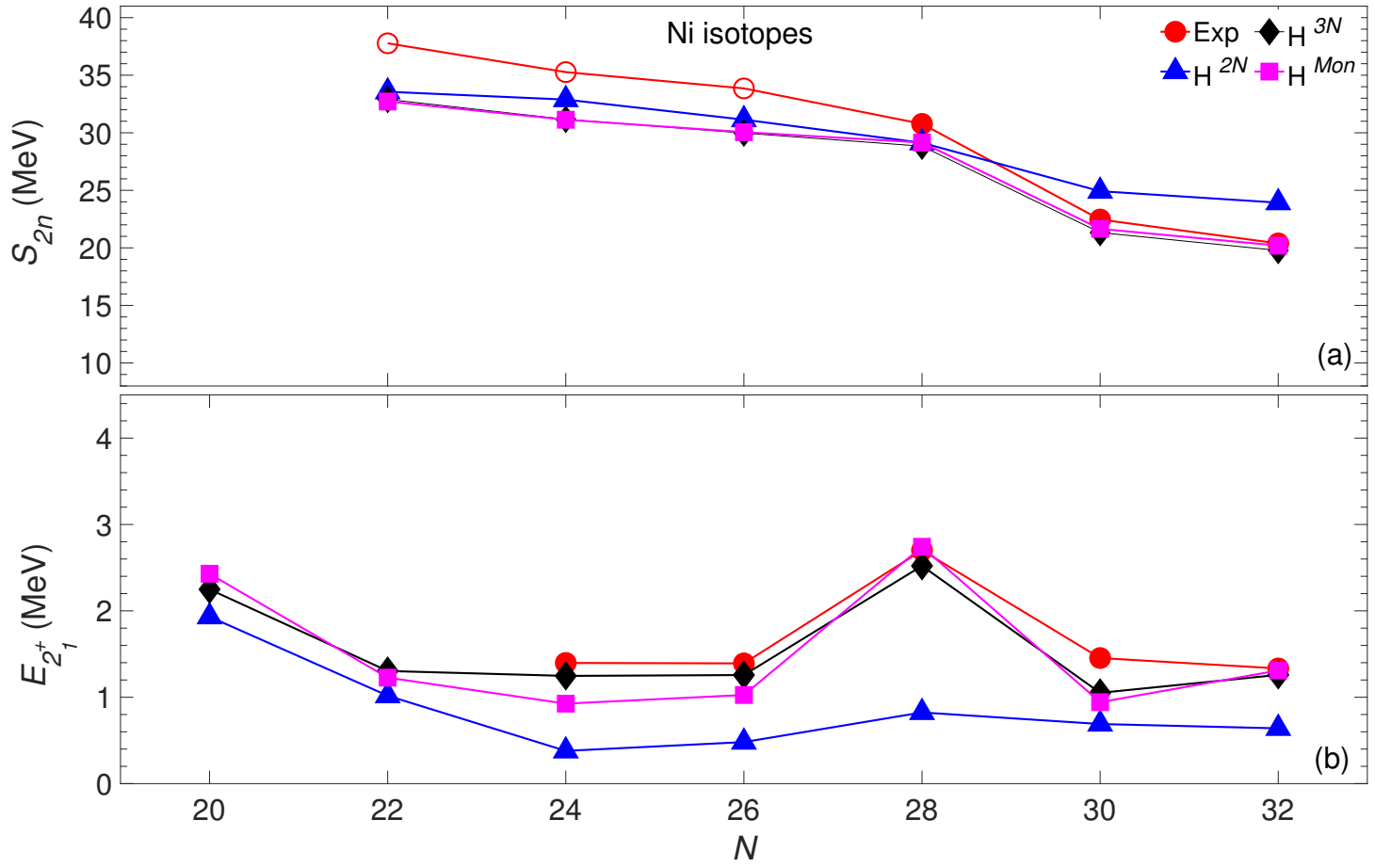
3NF is essential to make explicit the spin-orbit splitting of the 0p orbitals and ensure better closure properties



T. Fukui et al, PRC 98 (2018) 044305

Two-neutron separation energies and 2_1^+ excitation energies for nickel isotopes from N = 20 to 32

fp valence space

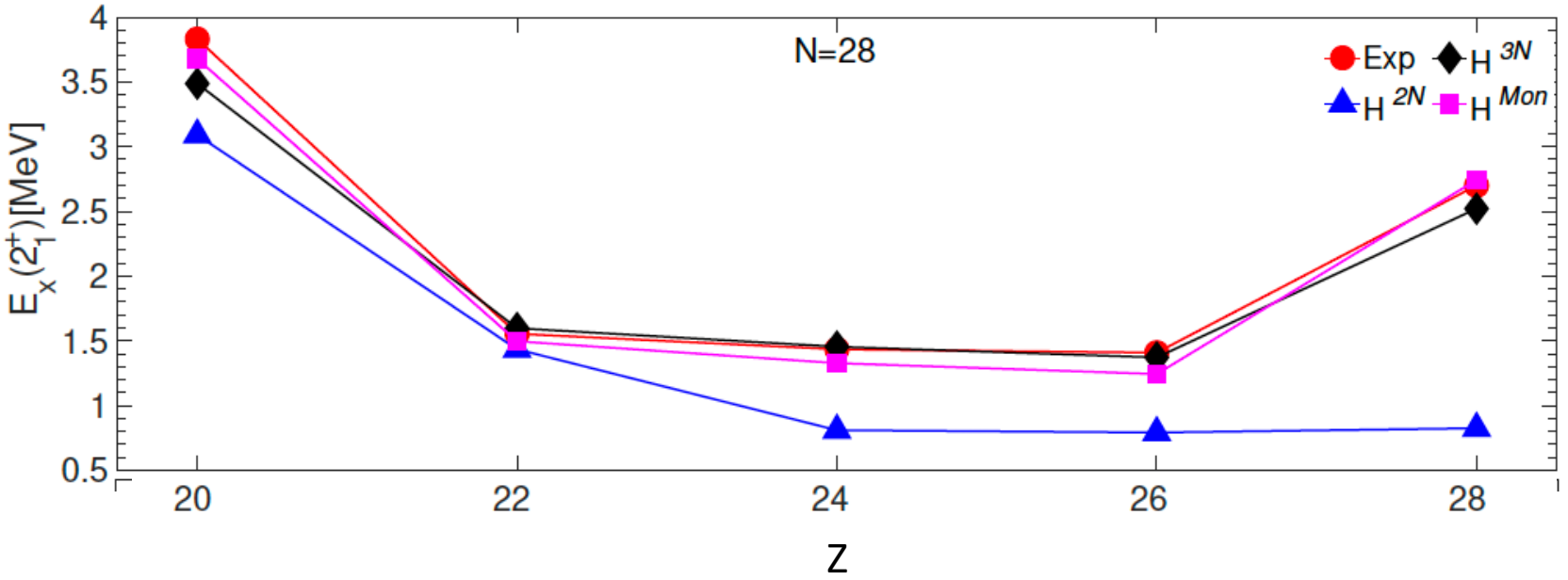


H^{2N}	only induced 3N forces
H^{3N}	genuine + induced 3N forces
H^{Mon}	monopole component of H^{3N} + multipole component of H^{2N}

Y..Z. Ma et al, PRC 100. 034324 (2019)

2_1^+ excitation energies for *N*=28 isotones from *Z* = 20 to 28

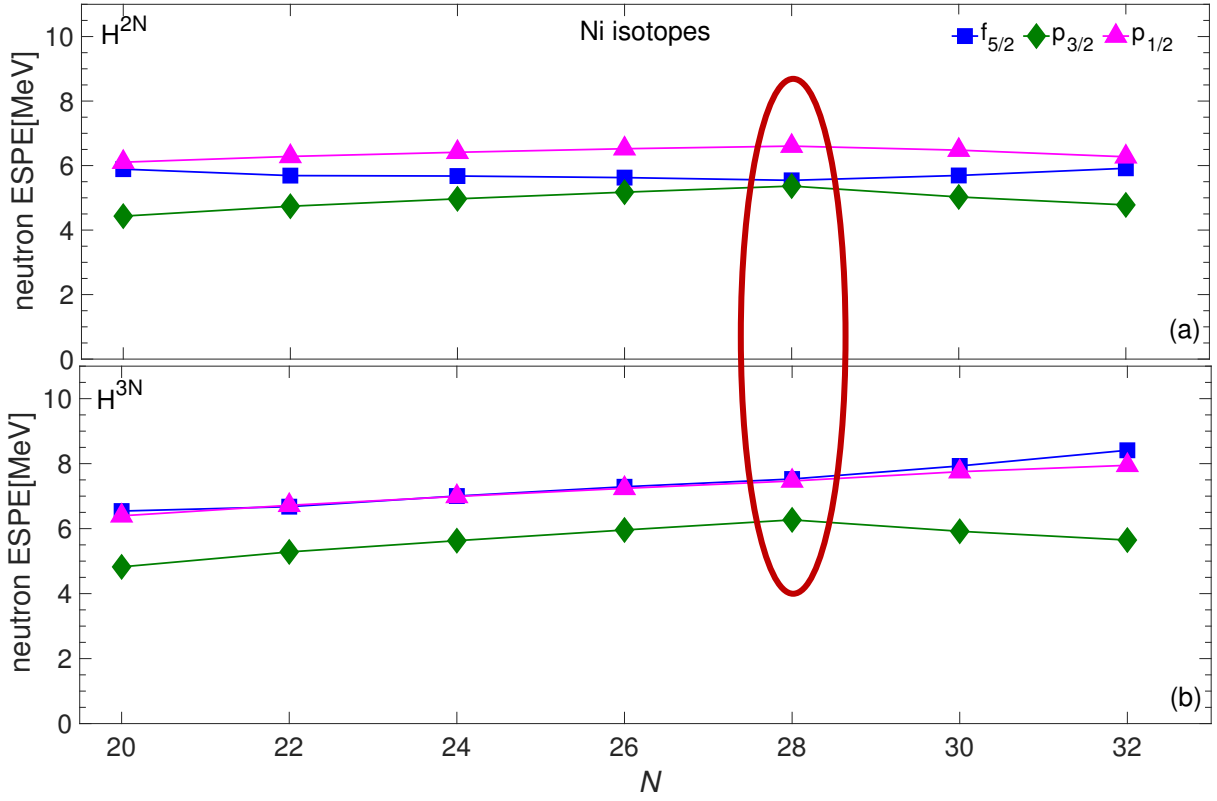
fp valence space

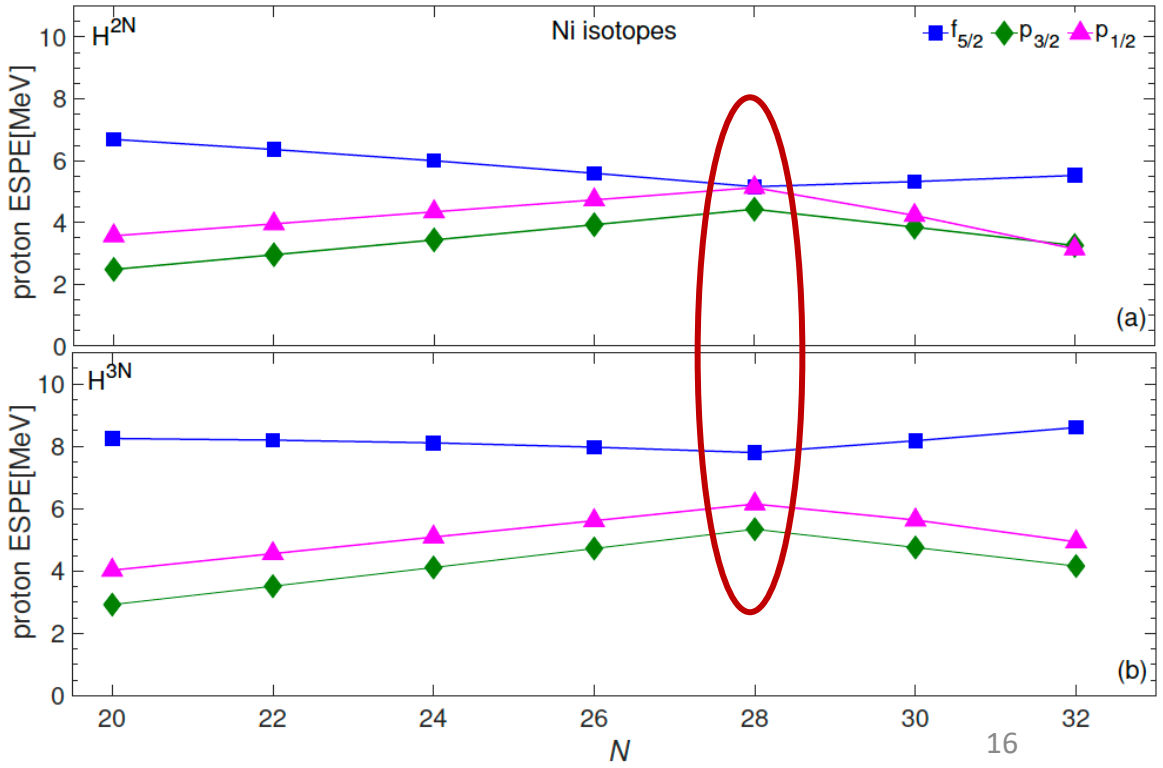


H^{2N}	only induced 3N forces
H^{3N}	genuine + induced 3N forces
H^{Mon}	monopole component of H^{3N} + multipole component of H^{2N}

Y..Z. Ma et al, PRC 100. 034324 (2019)

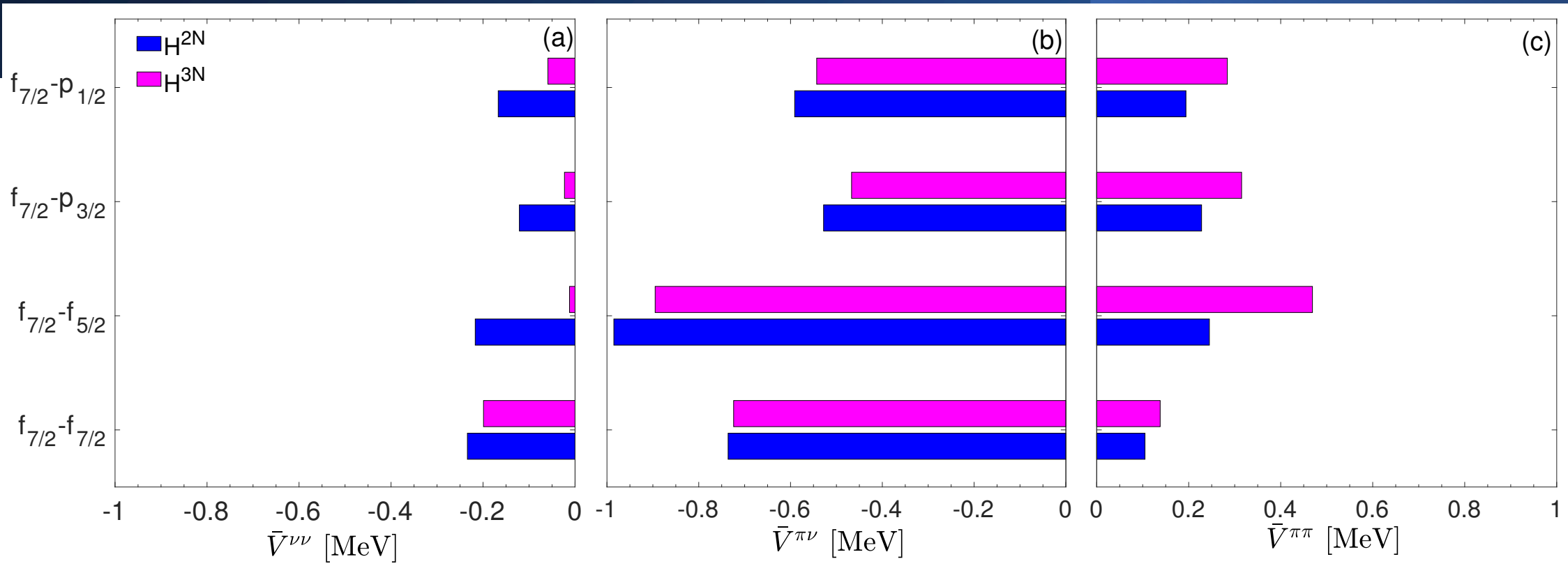
Neutron and proton ESPE for nickel isotopes with and without 3N force





H^{2N}	only induced 3N forces
H^{3N}	genuine + induced 3N forces

Monopole matrix elements $\bar{V}^{\tau\tau'}_{f_{7/2b}}$ of the effective interactions with and without 3N force



- 3NF provides a repulsive contribution to all matrix elements, which makes the $\nu\nu$ and $\pi\nu$ matrix elements less attractive and $\pi\pi$ ones more repulsive.
- The size of the contributions depends on the involved orbitals, ranging from few 10 to ~ 200 keV → substantial changes in the spacings between the ESPE's in correspondence of a sizable occupation of a specific orbital
- Changes produced by the 3N force are larger for non diagonal with respect to diagonal matrix elements, which leads to a larger gap between the $f_{7/2}$ and the remaining orbitals

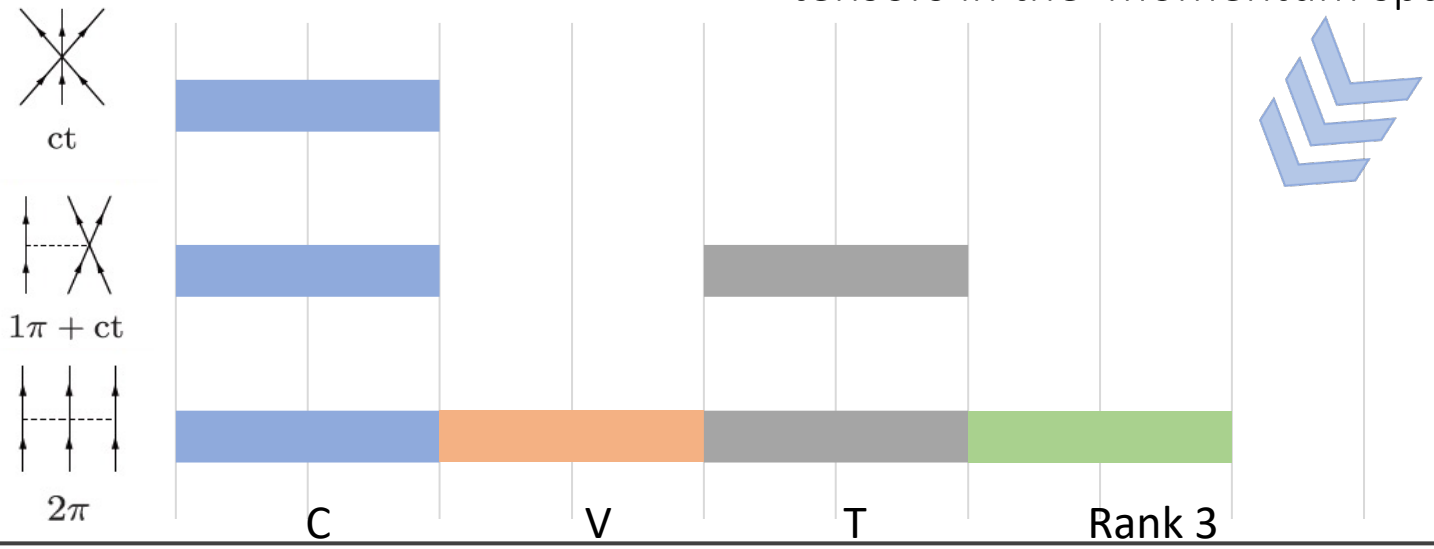
What is the specific mechanism behind the increase in the SO splitting produced by the chiral 3NF? Are there specific components of the 3NF leading to such an increasing?

Tensor decomposition of the chiral 3NF at N^{2}LO

3NF can be schematically written as

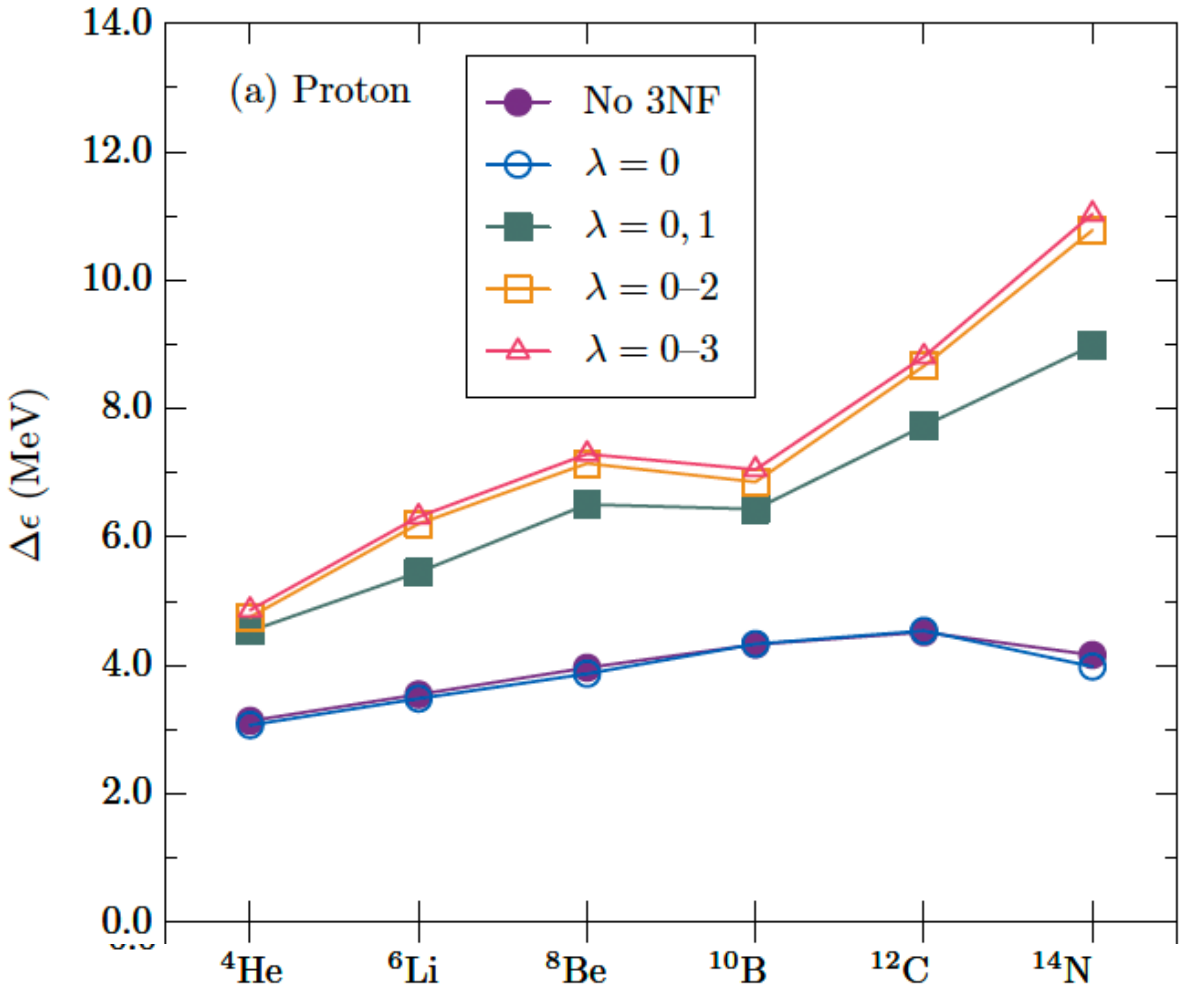
$$v_{3N}^{\alpha} \Longrightarrow T^{\alpha} \sum_{\lambda} S_{\lambda}^{\alpha} \cdot Q_{\lambda}^{\alpha} \qquad \alpha \in \{ct, 1\pi + ct, 2\pi\}$$

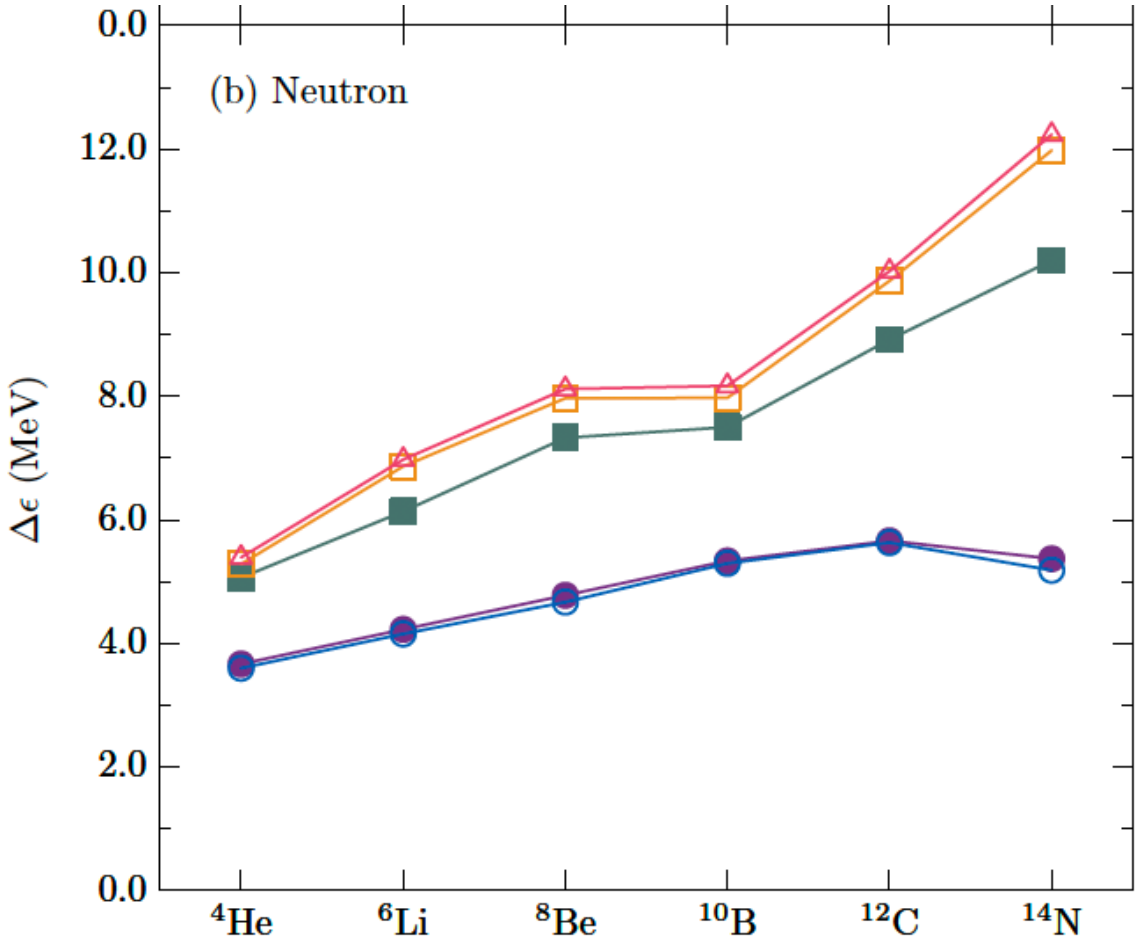
by coupling the spin tensor operators S_{λ}^{α} with the corresponding rank tensors in the momentum space Q_{λ}^{α}. T^{α} is the isospin part.



Formalism developed by Tokuro Fukui

Very preliminary results





$0p_{1/2}$ -$0p_{3/2}$ ESPE

Summary

- Genuine 3NFs provide a repulsive contribution, which is essential in determining the location of the neutron dripline and the evolution of the shell structure

- 3NFs affect essentially the monopole component, then confirming the need to introduce monopole adjustments when using effective SM Hamiltonians derived from NN potentials

- Changes induced by the 3NF in the monopole component of the effective Hamiltonian are essential to explain the enlargement of the SO splitting

- A crucial role in the SO splitting is played by the vector (rank-1) component arising from the the 1π+ct exchange term of the chiral 3NF

Thanks for your attention