Recycling cavity telescope design

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Daniel Brown, Julia Casanueva, Anna Green, Antonio Perreca

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Contents

Abstract

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1 Introduction

The aim of this document is to collect together the thoughts and calculations for designing the recycling cavity telescopes for ET. There are various requirements, often fighting against each other, which makes choosing a design challenging.

From a geometric view, the recycling cavities can either be operated in a stable or marginally stable configuration. Stable here means the roundtrip ABCD matrix of the cavity defines the cavity eigenmode [1]. This requires the higher order mode (HOM) separation frequency is large enough that no HOMs are resonant within the linewidth of the TEM00 mode. Marginally stable cavities, in theory, also have a well defined eigenmode but the mode separation is becoming ever closer to the linewidth of the TEM00. However, in this state the cavity eigenmode becomes very sensitive to optical aberrations. This places much stricter requirements on surface and optical path depth errors in the core interferometer optics. Additionally, due to the larger spatial extent of HOMs relative to the TEM00, the surface and OPD quality beyond the typical spot size, w, of the beam must also be controlled. Aberration errors for $r > w$ can modify the expected accumulated Gouy phases for HOMs. This can unexpectedly bring HOMs into resonance distorting the cavity eigenmode and generating a new pathway for loss and noise couplings. Such issues have been seen in Advanced LIGO already with 7th order modes becoming resonant in the arm cavities [3], and in Advanced Virgo with 8th order modes [4], exacerbating the effect of point absorbers. We expect with marginally stable cavities effects like these will become challenging to work with. Leading us to prefer a more geometrically stable design.

The primary aim of the recycling telescopes are to ensure the correct amount of roundtrip Gouy phase is accumulated in the power and signal recycling cavities to ensure we are geometrically stable. These telescopes must also magnify the input beam to match the arm cavities, and demagnify the arm cavity modes to use in the output path. This latter requirement however can be somewhat offset by additional telescopes external to the recycling cavities, albeit increasing the overall cost and complexity of the interferometer. However, many other technical requirements must also be satisfied which this document aims to collate, some of these are unique to each of the HF and LF detectors.

1.1 Starting point

As presented in previous section there are plenty of concerns that need to be addressed in order to converge towards a final design. Some studies have been already done, and together with the present experience in gravitational wave interferometers, we can already fix some important parameters to take as starting point for the design:

- Length of the Signal Recycling Cavity: too long a length of the signal recycling cavity can spoil the sensitivity at high frequencies, when its response time can't be neglected with respect to the arms one. It can be recovered by increasing the power, but it has other disadvantages. A tentative upper limit has been computed [12], 300m already reduces significantly the sensitivity above 2kHz while 100m is still a good compromise.
- Beam size at the Beam Splitter: in the case of ET HF, thermal lensing is a serious concern, in particular at the waist, where the power density is higher. This has been studied and a lower limit of 26mm of radius at the BS has been set [14].
- Gouy phase (RT) of the SRC: based on present detectors experience, the alignment of the Signal Recycling Mirror is a critical issue, in particular in the presence of HOMs due to optical aberrations (ex. point absorbers, mismatch). At LIGO, a new modulation frequency was added in order to build a robust error signal, but it required a minimal Gouy phase of 20deg [17].

The rest of important parameters are:

Parameter	ET-HF	ET-LF
ITM beam radius [cm]	12	9
ITM RoC [m]	5070	5580
ITM T [ppm]	7000	7000
PR T	0.046	0.054
SR T	0.1	0.2
HR L [ppm]	37.5	37.5

1.2 ET LF and HF requirements and concerns

In this section we have tried to list the different concerns that need to be studied in order to validate an optical design. This is an open list, that is/can be updated with new questions. Some requirements or limits are identical for both detectors, briefly these are:

- Noise at the beamsplitter, thermorefractive if beam is too small
- Beam should be collimated through BS, but how collimated is ok?
- The SRC and PRC are both geometrically stable
- No HOM resonance overlaps between the recycling and arm cavities
- Any expected thermal/substrate/surface changes does not accidentally bring HOMs intro resonance
- The cavities have the correct length for the RF control sidebands

- Where to pick-off balanced homodyne local oscillator, ideally where the spot size is small

- The beamsplitter suspension is limited in weight, smaller aperture BS would be preferable
- Minimise angle of incidence on any telescope mirrors to reduce astigmatism loss
- Gouy phase separation between mirrors for good alignment control
- Accurately sense the required degrees of freedom in whatever telescope we make
- Adding too many mirrors introduce more degrees of freedom that need to be controlled, damped, or sensed
- Physically, all these telescopes must exist within one cavern at the ET site, these need all need to fit together

- Scattered light risk is reduced
- Ensure ghost beams can be separated from the main beams

In particular for the HF detector:

- Do we have enough gouy phase between mirrors for any adaptive wavefront control actuators

In particular for the LF detector:

- Long Cryo-baffles limits the distance between ITM and next optic given some angle of incidence

2 Beamsplitter requirements

Perhaps the most limiting requirement is that of the BS [5]. In 2G detectors the spot size at the BS is about the same size at the ITM—as there are no strong focusing elements between the ITMs and the beamsplitter. As the 3G detector spot sizes are so much larger, the non-normal incidence angle θ at the BS mean it must be $1/\cos(\theta)$ larger in diameter than the ITMs to ensure clipping losses are not problematic. This has already been seen in LIGO and as part of the A+ upgrade a larger BS is being installed.

The obvious solution here is to then to place the telescopes between the BS and the ITMs (referred to as ”telescopes in the arms”) to reduce the spot size at the BS. This is what has been suggested in [15] and [5]. The design in [15] aimed for small spots at the BS, 6mm. For the lower power ET-LF this would be workable, however for the higher power HF detector this could be problematic in the current configuration.

Questions that need answering:

- How heavy can the BS be?
- HF: Angle of incidence, which is best if thermal lensing is present from a small spot? [4.1?]
- HF: What is an acceptable BS thermal lensing amount? [Wavefront sensing]
- HF: Can we do BS thermal actuation? [Wavefront sensing]
- Estimate the thermorefractive noise at the BS as a function of the BS, when it becomes limiting [6]
- A collimated beam is preferred through the BS, limit on divergence?

3 Beamsplitter thermal deformations

In the case of ET HF, thermal lensing is a particular concern. This has been studied and a lower limit of 26mm of radius at the BS has been suggested [14].

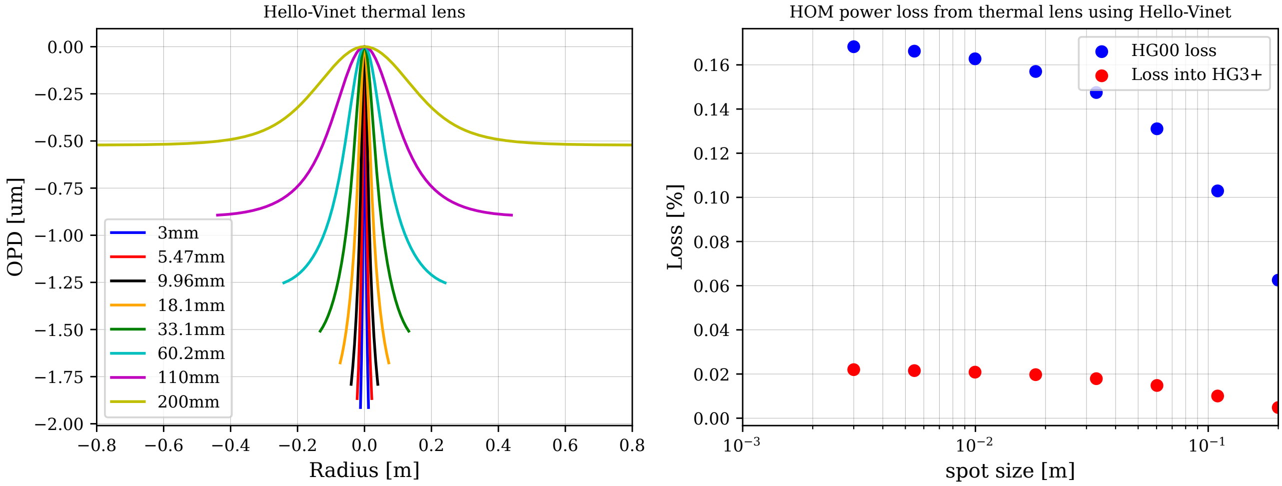

Figure 1: Power loss into higher order modes for different sized thermal lenses, on-axis. Shown is how much is lost out of the HG00, compared to how much is loss out of the HG00+HG20+HG02. Assumes 0.1W of coating heating into a fused silica 10cm substrate from Hello-Vinet.

Figure 1 shows how much power is scattered into higher order modes from a thermal lens for different spot sizes. Small beams on the whole appear to have a higher loss. Although much of this loss is into 2nd order modes, which should be corrected for with adaptive optics. Higher order losses seem to be fairly constant however and do not drop until the beam is much larger.

This is calculated assuming a beamsplitter radius $4w$, where w is the spot size at the beamsplitter. Numerically the produced thermal lenses are much stronger, however the beam interacting with it is much smaller.

This is of course not the full picture, the non-normal incidence of the optical field through the beamsplitter means the thermal lens generated will be astigmatic slightly.

The SRC beam path through the beamsplitter can also sample the MICH-X path thermal lens in the beamsplitter substrate. This can have substantial higher order mode content as the thermal lens is offset from the SRC path.

4 Telescope design

One of the key points of the design is the telescope inside the recycling cavities. Its role is to provide the required magnification for the beam to match the arm cavities and to provide enough Gouy phase for the recycling cavities to be stable. Work on this topic has already been done [15], that proposes a folded telescope between the beam splitter and the input test mass with two curved mirrors. This telescope allows a cavity length of 182 m, a Gouy phase of more than 20 degrees (52° for the LF and 36° for the HF). However, in this configuration the beam radius at the beamsplitter is only 6 mm, which makes it very challenging for the HF interferometer.

Questions that need answering:

- If we add a telescope between the PR and the BS mirror, can we find a combination that allows to keep a good Gouy phase, a good SRC length but with a beam radius at the beam splitter closer to 3-4 cms?
- Is this design good also for the LF configuration? Can their parameters be relaxed, while still meeting the requirements?

For a given telescope design:

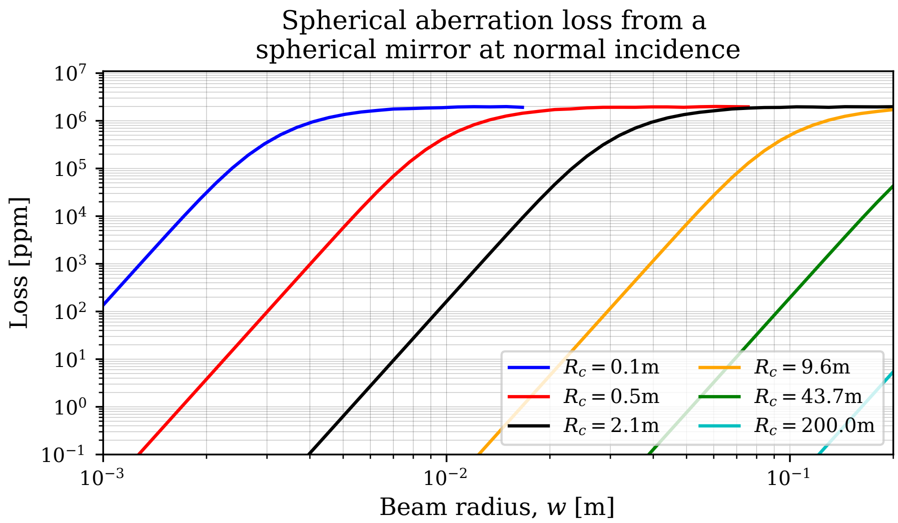

Figure 2: Numerical calculation of loss a TEM00 at normal incidence from a spherical mirror experiences due to spherical aberrations. This is primarily into 4th and higher order Hermite-Gaussian modes.

- A telescope off-axis will introduce aberrations, can we optimize the design (focal lenses vs AoI) to minimize these losses (targeting 0)? See 4.1.
- Which would be the impact of a residual movement (longitudinal or angular - also translation) on the telescope mirrors in terms of aberrations? Important for understanding the requirements for the suspensions

4.1 Off-axis spherical mirror aberrations

To minimise substrate absorption and deformations, along with back-scattering due to imperfect AR coatings, reflective focusing of beams is preferable. This means we must use off-axis mirrors in the telescopes. Spherical mirrors when used off axis introduce a variety of higher order wavefront aberrations in the reflected field. This scatters light away from an ideal TEM00 mode into higher orders which will then not match the arm cavity eigenmodes and act as a source of loss. Free-form optical surfaces may offer a solution to minimising aberrations, however here we first see what can be achieved with spherical mirrors. If we cannot reach our goals with spherical mirrors then free-form optics may be necessary, from which the best-case spherical mirror design would likely be a useful starting point.

The issue with spherical mirrors is that they suffer from a variety of higher order aberrations when used at non-normal incidences. Eq.29 in [10] provides the optical path depth, $s(x, y, R_c, \theta)$, on reflection from a spherical mirror at an angle of incidence θ and curvature R_c. The reflected field will have an additional the phase front given by $iks(x, y)$. The overlap integral between this and a TEM00 beam reflected from an ideal parabolic mirror:

$$s'(x, y, R_c, \theta) = \frac{x^2}{R_c} + \frac{y^2}{R_c} \tag{1}$$

can be used to compute the loss. The difference, $s - s'$, is the key quantity here to observe and will contain all aberration terms greater than the defocus. Whilst some effort can be put into trying to solve these integrals analytically, for now they will be solved numerically.

For $\theta = 0$, only the spherical aberration term will be present. This sets a best case loss which is dependant on R_c and the spot size at the mirror, w. The results of these calculations are shown in figure 2. Here we can see that losses can increase dramatically for larger beams on strongly focusing mirrors, as expected.

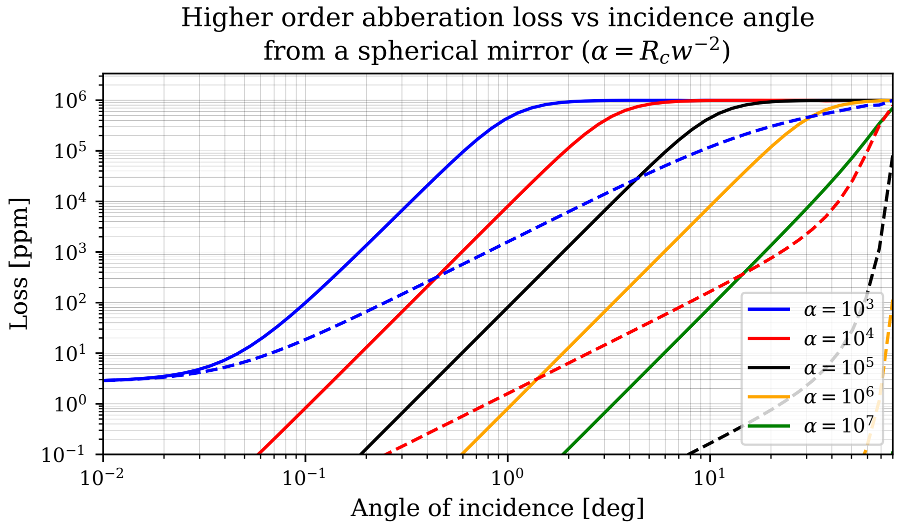

Figure 3: Dashed line shows aberration loss with astigmatism removed. Coma increases linearly with θ whereas astigmatism is θ^2 [9], therefore the dashed line is likely showing coma scattering [TODO: could double check this by looking at the actual residual distortion]. $R_c = 10^4$ was used for this plot, and chosen to depict the low and high θ contributions.

Let us consider the Rowlinson design [15] briefly, at both ZM1 and ZM2 we would have much less than < 1 ppm of loss.

The issue arises with astigmatism at non-normal angles, as shown in figure 3. The astigmatic loss can be parameterised by a ratio of the curvature and spot size, $\alpha \equiv \frac{R_c}{w^2}$. For ZM1 and ZM2, α is approximately $10^{5.8}$ and $10^{4.7}$ respectively. Rowlinson does not specify the exact angles to use, but from figure 3 we can see that we are in the region of a few 100ppm of loss (ZM2 would be 400ppm).

An interesting option to consider is that astigmatism can be cancelled out in a telescope. A simple argument is given in [11]. This requires carefully choosing the distances and curvatures of ZM1 and ZM2 and will limit the possible design choices but should be explored.

- Free form optics: How well do we realistically expect we could do with current polishing capabilities? How well centred would the beam need to be on such an optic?
- HOM scattering: Whilst a single loss value is one possible metric we can use, the scattering into higher order modes is more complicated than just a pure loss. Here we will apply these higher order aberrations to simulated recycling cavity to observe the global effect.

5 Error signals for Longitudinal and Angular control

In present gravitational wave detectors, the longitudinal and angular control signals are built using the PDH technique. The frequency of the sidebands is chosen strategically to provide the right information when beating with the carrier. For the longitudinal error signals these conditions are quite straightforward, always supposing that all the sidebands are resonant in the different mode cleaner cavities:

- Sideband 1: to control the arm cavities. This sidebands need to be resonant inside the PRC, antiresonant inside the PRC and antiresonant in the arm cavities.
- Sideband 2: to control the PRC. In Virgo this sideband is anti-resonant on the interferometer.
- Sideband 3: to control the SRC. This needs to be resonant in both PRC and SRC but antiresonant on the arm cavities.

In this choice, the Schnupp asymmetry plays an important role. First, it determines the losses of the sidebands inside the PRC, which changes their finesse. Also, it determines how much back-coupling into the PRC will be. Finally, the ratio between the PR mirror reflectivity and the Schnupp determines which frequencies are resonant in the SRC and which are not.

- Which frequencies will be resonant in both recycling cavities as a function of the Schnupp asymmetry? Is there any combination of modulation frequencies and Schnupp that can be used as a starting point? [16]
- For a given length of the recycling cavities and for a given PR reflectivity, how much transmission towards the SRC and back-scattering towards the PRC there will be as a function of the Schnupp asymmetry?
- Do we want LIGO-like, slightly asymmetric recycling cavities, or Virgo-like symmetric recycling cavities?
- For a given Gouy phase of the SRC, and a Schnupp asymmetry, can we find modulation frequencies that could build a good SR alignment signal? [17]
- Detuned vs tuned SRC? As a starting point can we study both and establish residual movement requirements for reaching the target sensitivity?

6 Signal extraction

- Where do we need to extract beams in order to produce error signals? Do the beam geometry/AOI allow to do this and where would we need optical benches?
- Balanced Homodyne Detection, where it is better to take the pick-off?

7 Higher Order Modes separation

One of the critical points in gravitational wave detectors is the scattering into Higher Order Modes due to defects and how do they couple to the interferometer and its error signals. This behaviour will depend strongly on the round trip Gouy phase inside the recycling cavities.

- What is the HOMs separation on the PRC and the SRC? How do they couple to the recycling cavities? How do this change as a function of the RT Gouy phase and the Finesse of the recycling cavities?
- How do the sidebands HOMs couple into the interferometer and how do they affect the longitudinal error signals? Do we need to worry about mode hopping? [8]
- How the RT Gouy phase of the SRC affects the squeezing, does it get reduced?

8 ISB Workshop working space

During the October 2022 ISB workshop several groups looked into various parts of the recycling telescope design.

8.1 Problems to be tackle in smaller groups

- brainstorming on recycling cavity design (where/how to focus)
- simple code to calculate the round trip Gouy phase in the recycling cavities, ABCD matrix/Finesse
- minimal beam size on the arm cavity mirrors (already done, see below)
- maximum astigmatism in the RC, could be defined as coupling loss for the arm cavity
- estimate of the beamsplitter transmissive thermorefractive noise

8.2 Minimal beam size

The minimal beam size in a symmetric linear cavity is given by [7]:

$$w^2 = \frac{L\lambda}{\pi}\sqrt{\frac{1}{1-g^2}} = \frac{\lambda}{\pi}\sqrt{\frac{RL}{2-L/R}}$$

For a symmetric cavity the minimal values ($g = 0$) are:

L [km]	λ [nm]	Beam radius [cm]	Mirror diameter [mm]
10	1064	5.8	350
15	1064	7.1	430
20	1064	8.2	500
10	1550	7.0	420
15	1550	8.6	520
20	1550	9.9	600
10	2000	8.0	480
15	2000	9.8	600
20	2000	11.3	680

The mirror diameter is assumed to be around 6 times the beam radius.

8.3 Recycling cavity brainstorming

Think about the compensation of astigmatism with two curved mirrors with opposite RoC [11].

8.4 Thermorefractive noise

- *Contributors: Stefan Danilishin, Mikhail Korobko, Severin Nadji, Teng Zhang*
- Mathematica code is available in the overleaf folder
- Main publication [2] and Eq.2 can be used directly to calculate the thermorefractive noise. Two correction need to be made: i) redefining the angle $i = \arcsin(\sin(\phi)/n)$, where $\phi = \pi/3$ is the angle of incidence of the beam on the BS. ii) Dividing the (amplitude) spectral density by the factor $\mathcal{F}/\pi$, where $\mathcal{F}$ is the finesse of the arm cavity. Eq.2 in [2] was derived for GEO600 where no arm cavities are used, and it uses the total length of the unfolded arm, which is double the arm length in ET. The modified equation we used here is:

$$S_h(\omega) = \frac{\pi^2}{L^2\mathcal{F}^2}\frac{4k_b\kappa T^2\beta^2\alpha'}{\pi(C\rho r_0^2\omega)^2}\frac{\eta+\eta^{-1}}{2\eta^2}\left[1+\frac{2k^2r_0^2\eta}{(\eta+\eta^{-1})(1+(2kl_{th})^4)}\right], \quad (2)$$

  where $a' = a/\cos i$, and $i = \arcsin(\sin(\phi)/n)$, where $\phi = \pi/3$ is the angle of incidence of the beam on the BS.
- The resulting Figure 4 (left panel) shows the thermorefractive noise in the units of strain for different values of beam size. We found that for beam size $r_0 \simeq 1.6mm$, thermorefractive noise becomes comparable to the current projection of ET-HF sensitivity at 40Hz, whereas the value of $r_0 \simeq 2.2$mm was found to be dangerous for the ET-LF projected sensitivity.
- Paper [13] does the calculation of the TR noise as well, although with different parameters (assuming no telescope between the ITM and BS).
- pyGWINC currently doesn't include the BS thermorefractive noise
- The equation for the noise in pyGWINC doesn't account for the cavity standing wave and the different angle of incidence (i.e. is only suitable for ITM).

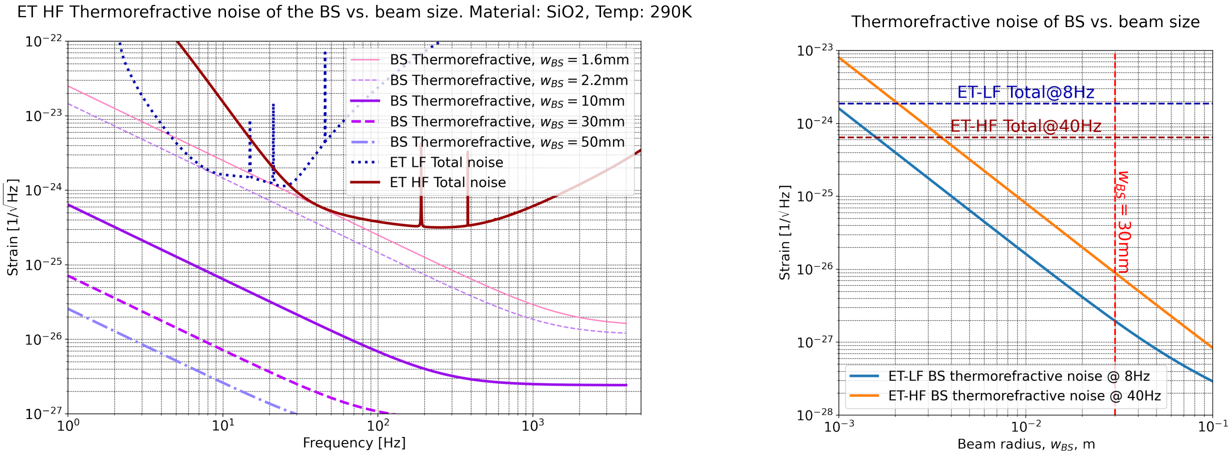

Figure 4: Thermorefractive noise for the parameters for different beam radius r_0 at the BS, compared to the ET-D design curve (red). BS thickness: $a = 43mm$. Other parameters match the pyGWINC material values.

9 Signal Recycling design: constraints

In this section we collect a list of constraints to be followed for the design of the SRC. Here we refer to the SRC, considering that the PRC and SRC will be symmetric.

- *SRC length*: The SRC cavity length should be between 80 and 120 meter;
- *PRC length*: No constraints but we think to have symmetric cavities (PRC=SRC);
- *Gouy phase*: separation between the TEM00 and the TEM01 of 20 deg (one way);
- *Beam radius on BS*: 20 mm $< w <$ 50 mm;
- *Beam radius on SRM (PRM)*: $w>$ 10 mm;
- *BS position*: The BS position is flexible.

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