Analogue preheating in a 1D condensate

PROJET FINANCÉ PAR L'ANR - PROJECT FUNDED BY THE ANR

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COPHY workshop

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Context and motivations

2 main predictions of *Quantum Field Theory in curved spacetime*:

- Black holes emit thermal radiation   S. Hawking, *Nature* **248**, 30 (1974)
- Expansion of universe produces particles   L. Parker, *Phys. Rev. Lett.* **21**, 562 (1968)

Associated with an inequivalence of "ingoing" and "outgoing" vacuum states:

$$\hat{a}_{\text{in}} \left| 0_{\text{in}} \right\rangle = 0$$

$$\hat{a}_{\text{out}} = \alpha\, \hat{a}_{\text{in}}^{(1)} + \beta\, \hat{a}_{\text{in}}^{(2)\dagger}$$

$$\Longrightarrow \quad \left\langle 0_{\text{in}} \right| \hat{a}^{\dagger}_{\text{out}} \hat{a}_{\text{out}} \left| 0_{\text{in}} \right\rangle = \left| \beta \right|^2$$

- Amplification of vacuum fluctuations
- Generation of squeezed states
- **Spontaneous creation** of quanta/particles, occurring in **entangled pairs**

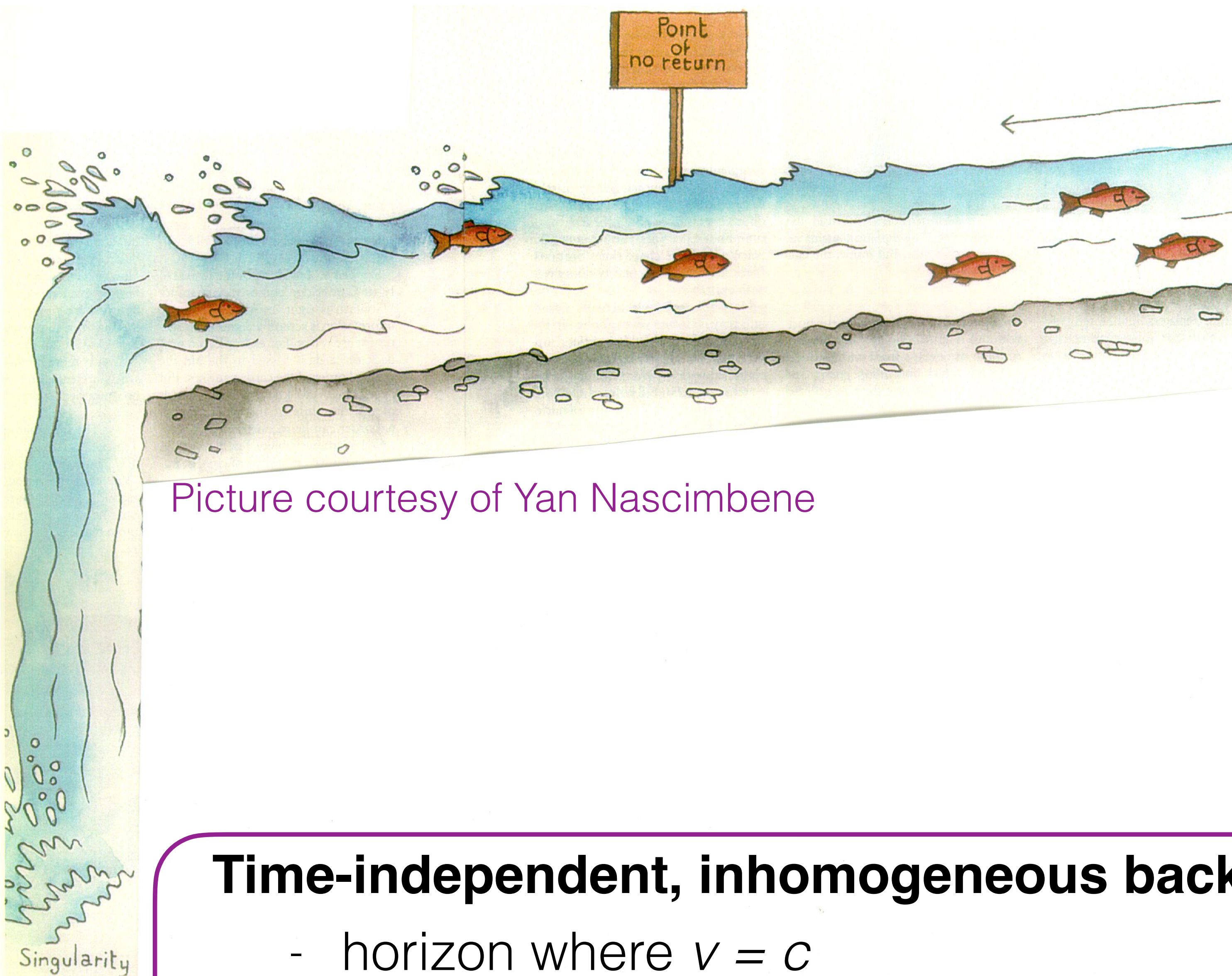
Analogue Gravity

Unruh (1981): on large scales, waves in moving media propagate as if in curved spacetime

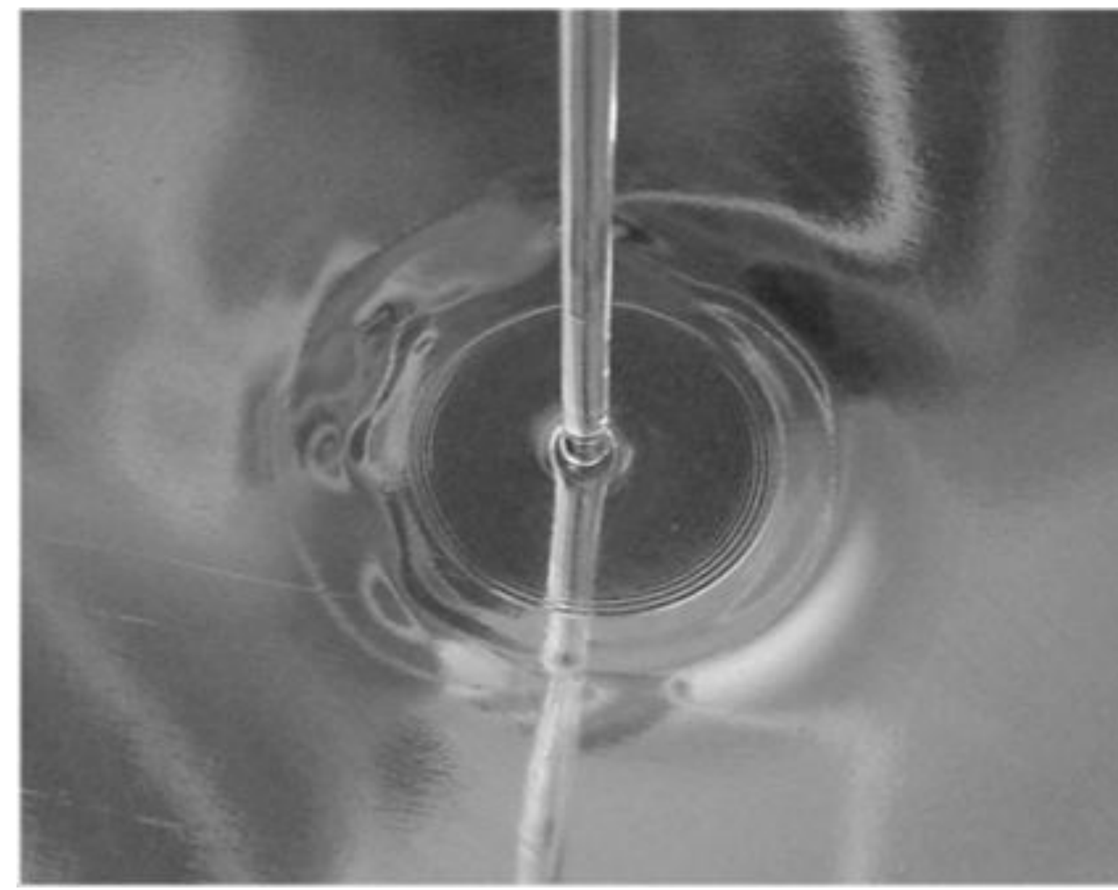
W. G. Unruh, *Phys. Rev. Lett.* **46**, 1351 (1981)

Ingredients:

- fixed background $$g_{\mu\nu}dx^\mu dx^\nu = \Omega^2\left[c^2dt^2 - \left(d\mathbf{x} - \mathbf{v}\,dt\right)^2\right]$$
- (linear) perturbations $$g^{\mu\nu}\nabla_\mu\nabla_\nu\phi = 0$$



Picture courtesy of Yan Nascimbene

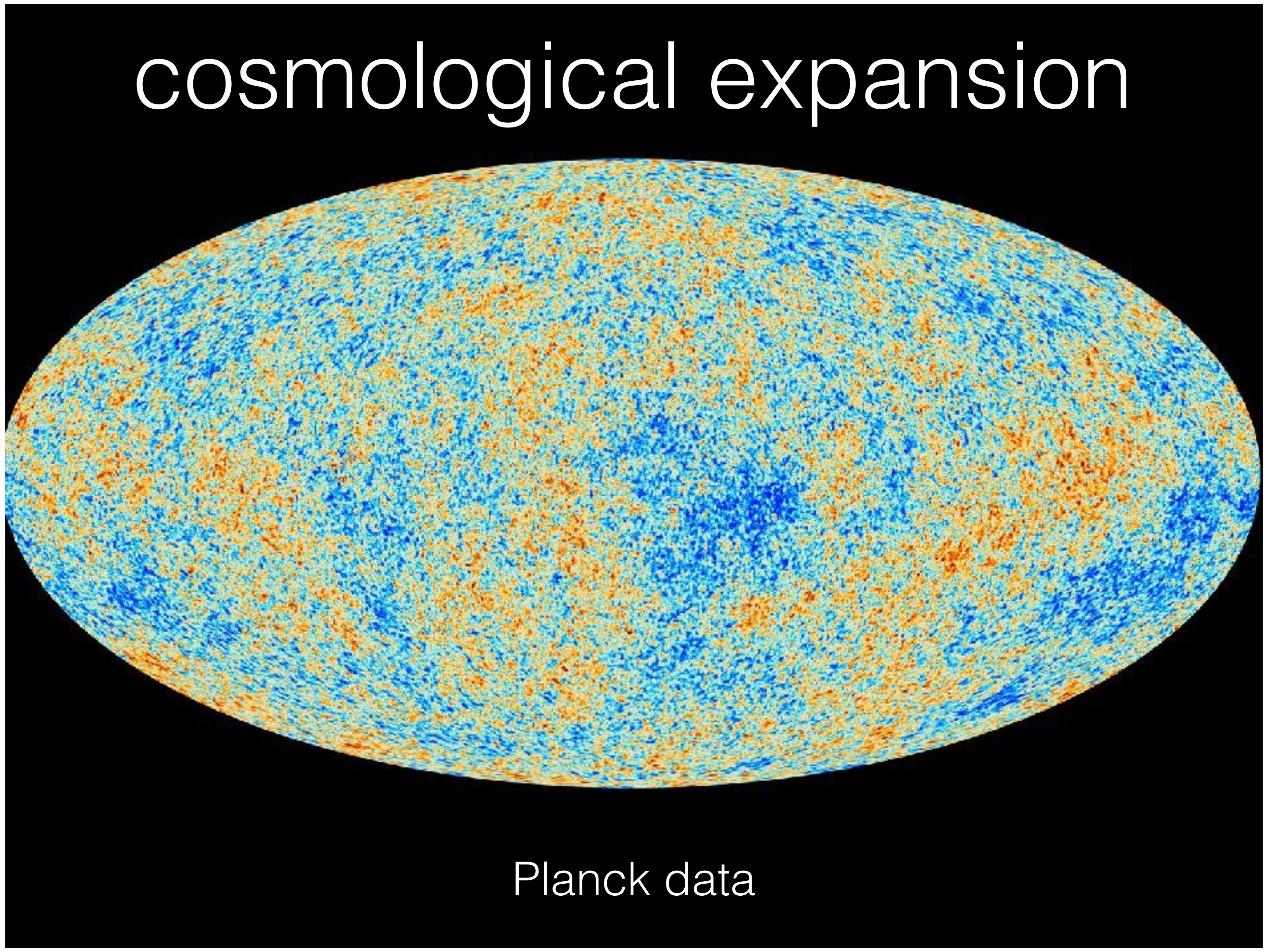


Picture courtesy of Piotr Pieranski

Time-independent, inhomogeneous backgrounds

- horizon where $v = c$
- analogue Hawking effect

Time-varying background

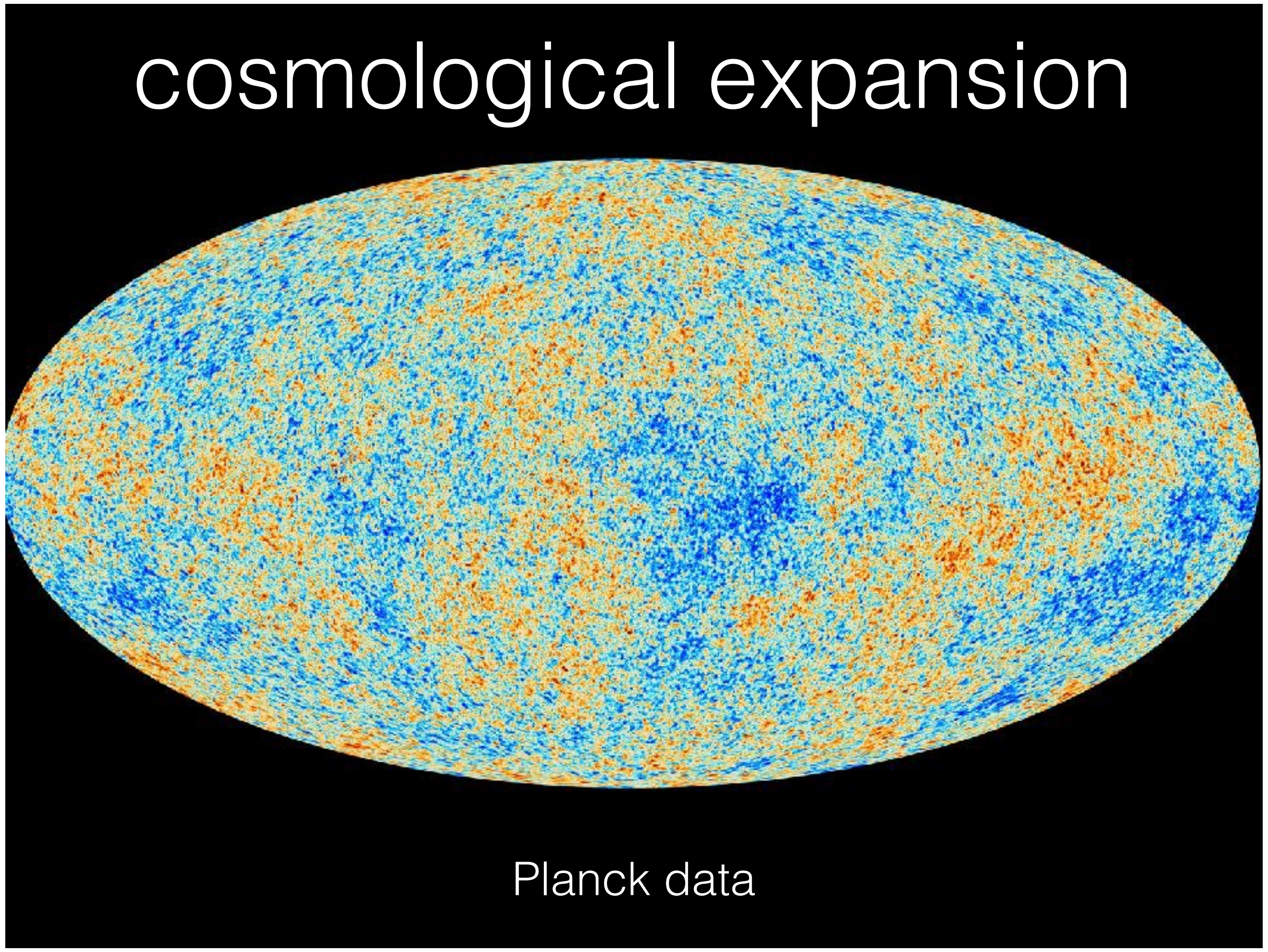


Two main types

“Cosmological expansion”

- Monotonic evolution
- Limited in time
- Broad excitation spectrum

Time-varying background



<u>Two main types</u>

“Cosmological expansion”

- Monotonic evolution
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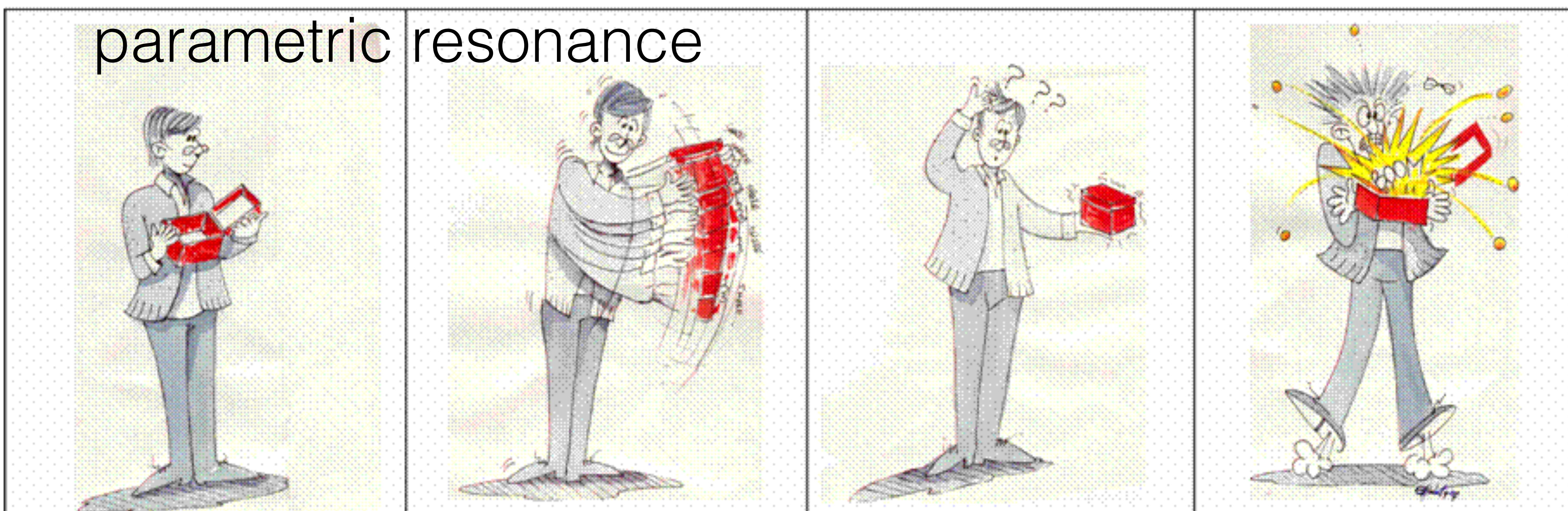
- Broad excitation spectrum

Parametric resonance (“Preheating”)

- Sinusoidal evolution
- Long-lasting
- Narrow excitation spectrum



from INFN Padova

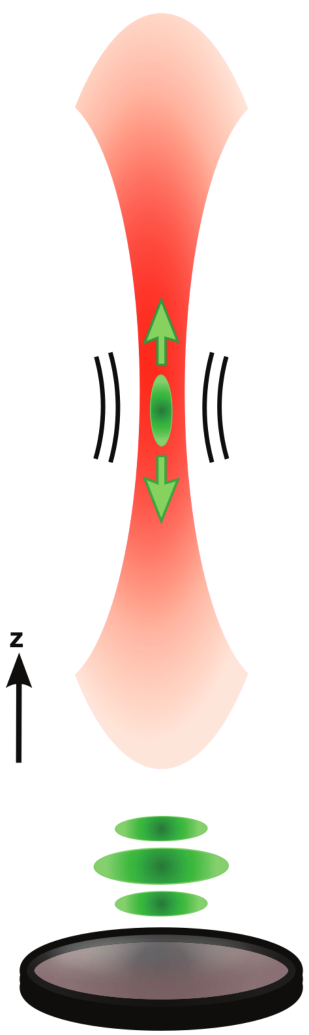
Outline

- BEC analogy - LCF experiment
- Mean field behaviour (effective spacetime)
- Linear perturbations (particle creation)
- Interactions (dissipation and decoherence)

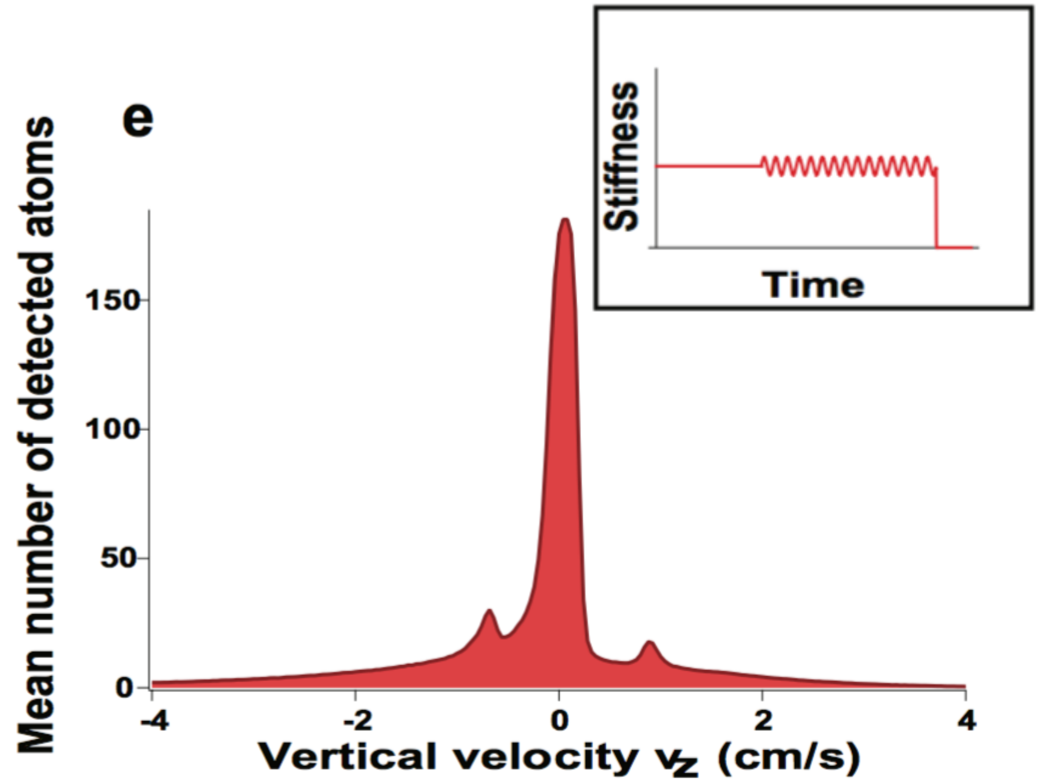
LCF experiment:

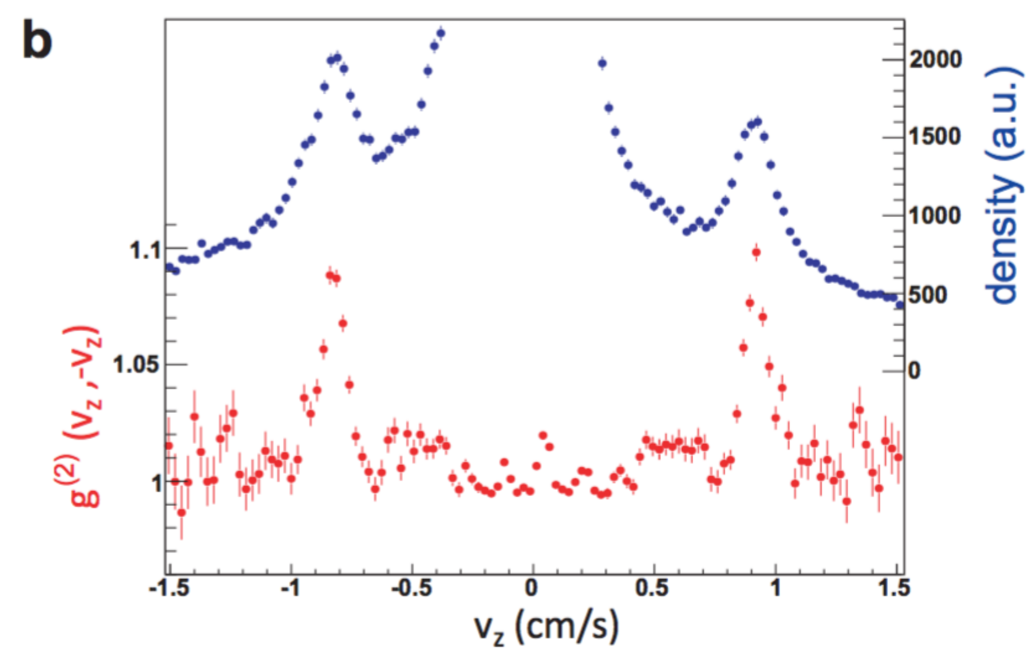
Time modulation of a quasi-1D Bose gas

J.-C. Jaskula *et al.*, *Phys. Rev. Lett.* **109**, 220401



$$g^{(2)}(k,-k) = \frac{\langle \hat{n}_k \hat{n}_{-k} \rangle}{\langle \hat{n}_k \rangle \langle \hat{n}_{-k} \rangle}$$

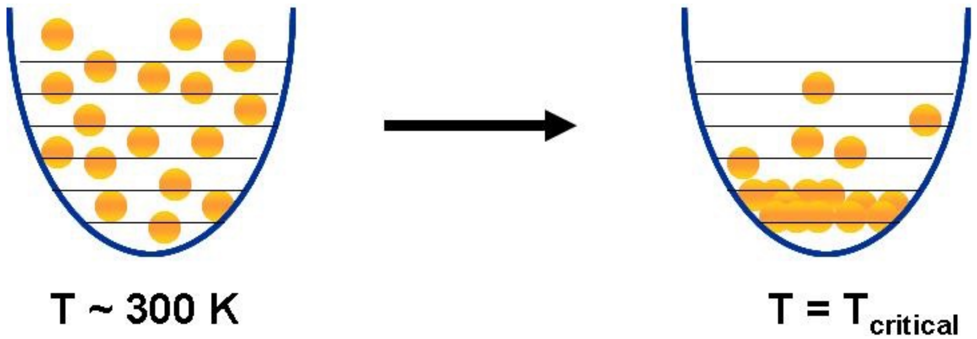




Correlation such that created pairs **not** entangled (g2 < 2)

The BEC/phonon analogy

L. J. Garay *et al.*, *PRL* **85**, 4643 (2000)



Condensed phase: macroscopic fraction of atoms in same state (usually ground state)

Atomic field operator $\hat{\Phi} = \sqrt{\hat{\rho}}\, e^{i\hat{\theta}}$

Atom density $\hat{\rho}$

Flow velocity $\hat{\mathbf{v}} = \frac{\hbar}{m}\nabla\hat{\theta}$

Advantages:

- theoretically simple
- experimentally well-controlled
- intrinsically quantum

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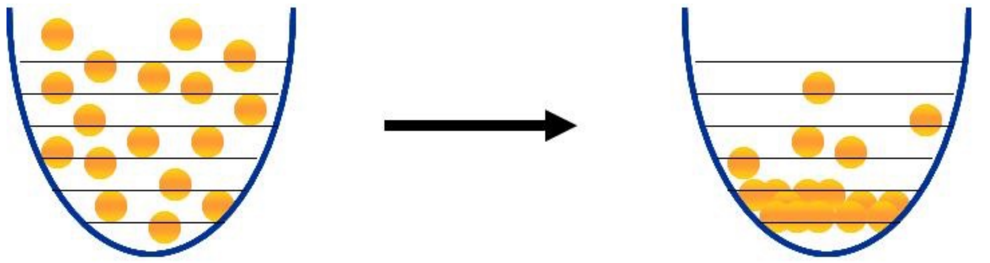

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Expansion of Hamiltonian

$$\hat{H} = H_0 + \hat{H}_2 + \hat{H}_3 + \ldots$$

Mean field: Large population, treated classically

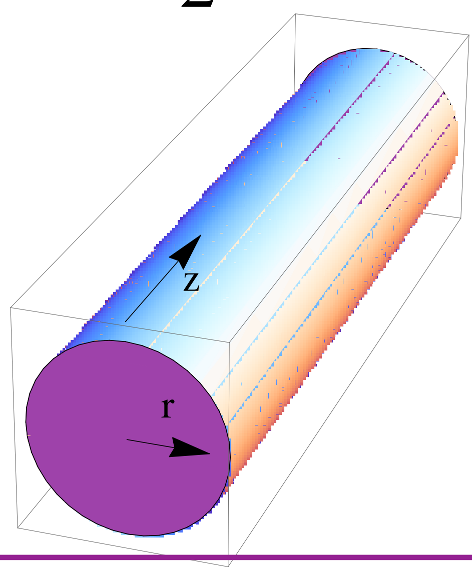
Quadratic part: Defines quasi-particles (phonons)

Cubic part: Describes lowest order quasi-particle interactions

Mean field dynamics in 1D

Harmonic radial potential

$$V = \frac{1}{2} m\omega_\perp^2 r^2$$



Mean field Hamiltonian: $$H_0 = \frac{\hbar^2}{2m} \left| \nabla \Phi_0 \right|^2 + \frac{g_{1D}}{2} \left| \Phi_0 \right|^4$$

where g_{1D} depends on the transverse width σ : $$g_{1D} \propto \frac{1}{\sigma^2}$$

⟶ Nonlinear Schrödinger equation:

$$i\,\partial_t \Phi_0 = -\frac{1}{2m} \partial_z^2 \Phi_0 + g_{1D} \left| \Phi_0 \right|^2 \Phi_0$$

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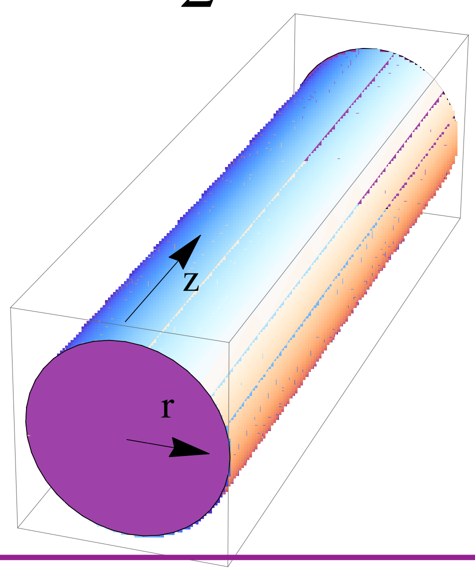

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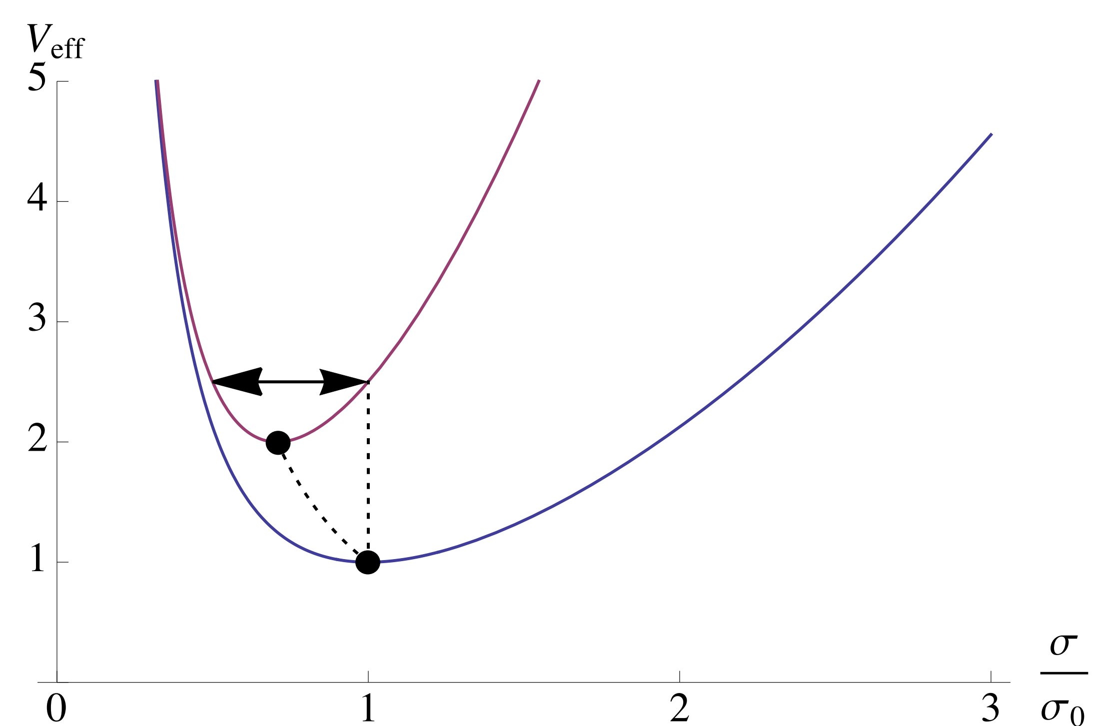
Quenched (or modulated) 1D Bose gas

Transverse width σ of cloud behaves like particle in the effective potential

$$V_{\text{eff}}(\sigma) = \tfrac{1}{2}\frac{\omega_\perp^2}{\omega_{\perp 0}^2}\left(\frac{\sigma}{\sigma_0}\right)^2 + \tfrac{1}{2}\left(\frac{\sigma}{\sigma_0}\right)^{-2}$$

(Kagan *et al.*, *PRA* **54**, R1753 (1996))

Engenders effective metric,
oscillating in time,
as seen by phonons

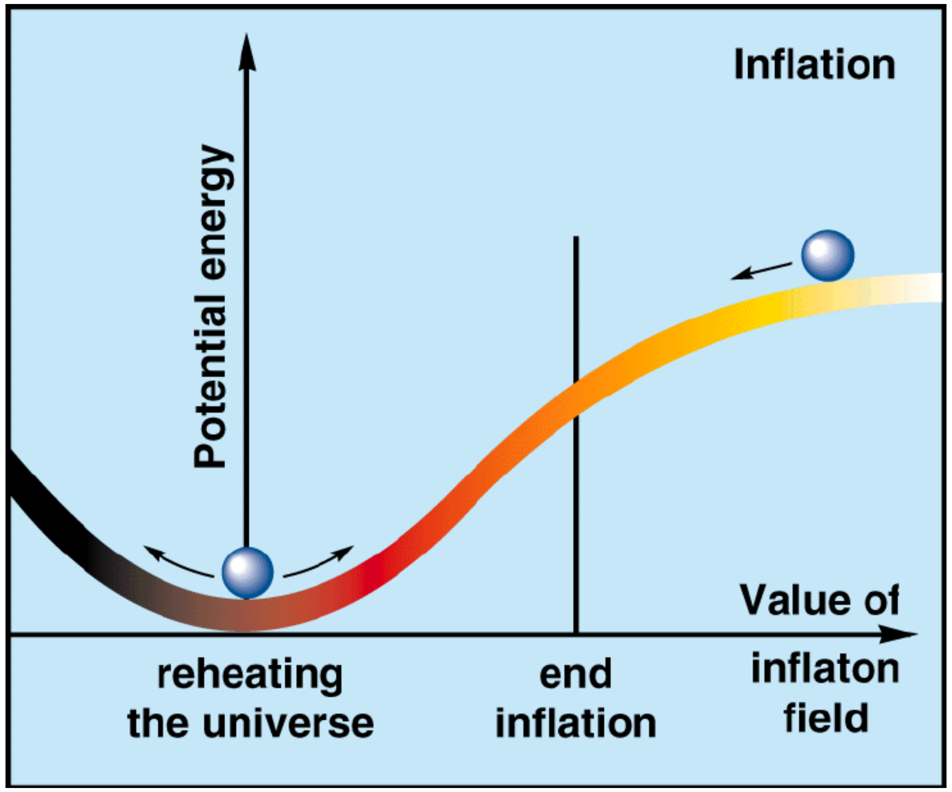


Inflaton field at end of inflationary era

Inflaton oscillates around minimum of its potential

(Kofman *et al.*, *PRL* **73**, 3195 (1994))

Engenders effective metric,
oscillating in time,
as seen by matter fields



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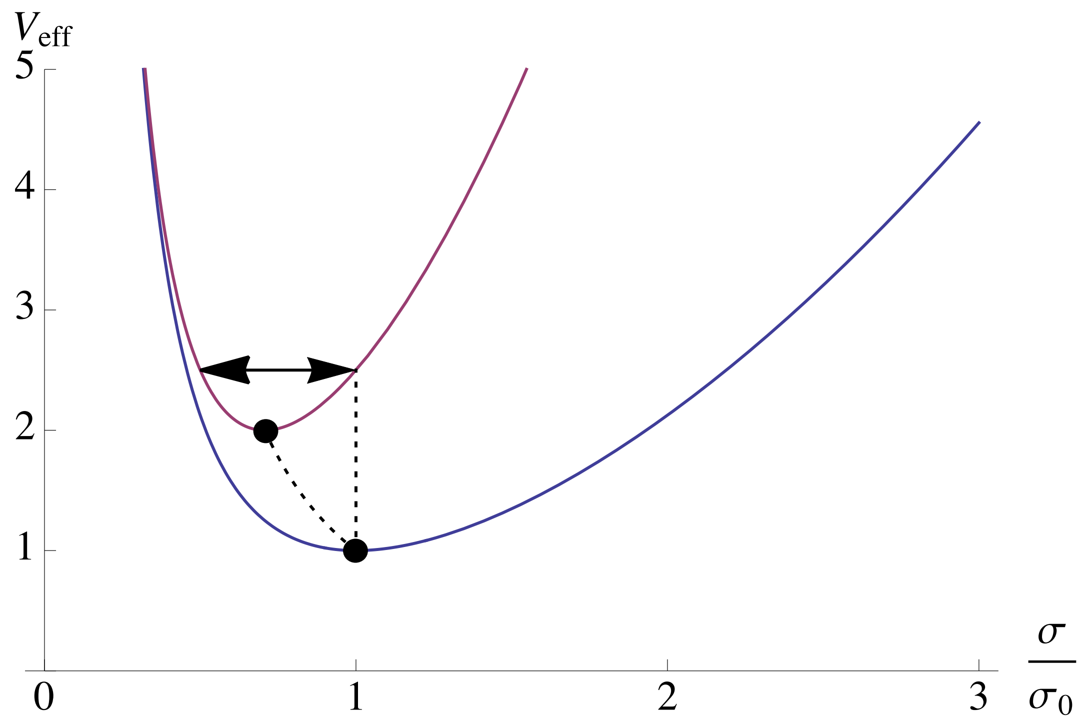

Linear perturbations: phonons

Homogeneity implies that each (k,-k) sector evolve independently:

$$\hat{H} = H_0 + \sum_k \hat{H}^{(2)}_{k,-k} + \dots \qquad \hat{H}^{(2)}_{k,-k} = \frac{\hbar^2 k^2}{m} \delta\hat{\theta}_k \delta\hat{\theta}_{-k} + \left[\frac{\hbar^2 k^2}{4m} + g_{1D}(t)\rho_0\right] \delta\hat{\rho}_k \delta\hat{\rho}_{-k}$$

Diagonalization defines quasi-particles (phonons):

$$\hat{\varphi}_k = \frac{1}{\sqrt{2}} \left(C_k^{-1}(t)\, \delta\hat{\rho}_k + i\, C_k(t)\, \delta\hat{\theta}_k \right) \longrightarrow \hat{H}^{(2)}_{k,-k} = \hbar\omega_k(t) \left[\hat{\varphi}_k^\dagger \hat{\varphi}_k + \hat{\varphi}_{-k}^\dagger \hat{\varphi}_{-k} \right]$$

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In response to time-variation $\omega_k = \omega_k(t)$

$$i\partial_t \begin{bmatrix} \hat{\varphi}_k \\ \hat{\varphi}^\dagger_{-k} \end{bmatrix} = \begin{bmatrix} \omega_k & i\dot{\omega}_k / 2\omega_k \\ i\dot{\omega}_k / 2\omega_k & -\omega_k \end{bmatrix} \begin{bmatrix} \hat{\varphi}_k \\ \hat{\varphi}^\dagger_{-k} \end{bmatrix} \longrightarrow$$

$$\hat{\varphi}_k(t) = \alpha(t, t_0)\hat{\varphi}_k(t_0) + \beta(t, t_0)\hat{\varphi}^\dagger_{-k}(t_0)$$

Bogoliubov transformation

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Bogoliubov transformation

Homogeneous, isotropic, Gaussian 2-mode state fully characterised by:

$n_k = \left\langle \hat{\varphi}_k^\dagger \hat{\varphi}_k \right\rangle = \left\langle \hat{\varphi}^\dagger_{-k}\hat{\varphi}_{-k} \right\rangle$ occupation number

$c_k = \left\langle \hat{\varphi}_k \hat{\varphi}_{-k} \right\rangle$ correlation amplitude

Entanglement (nonseparability) occurs when

$$\Delta_k = n_k - \left|c_k\right| < 0$$

Parametric resonance

Frequency modulation

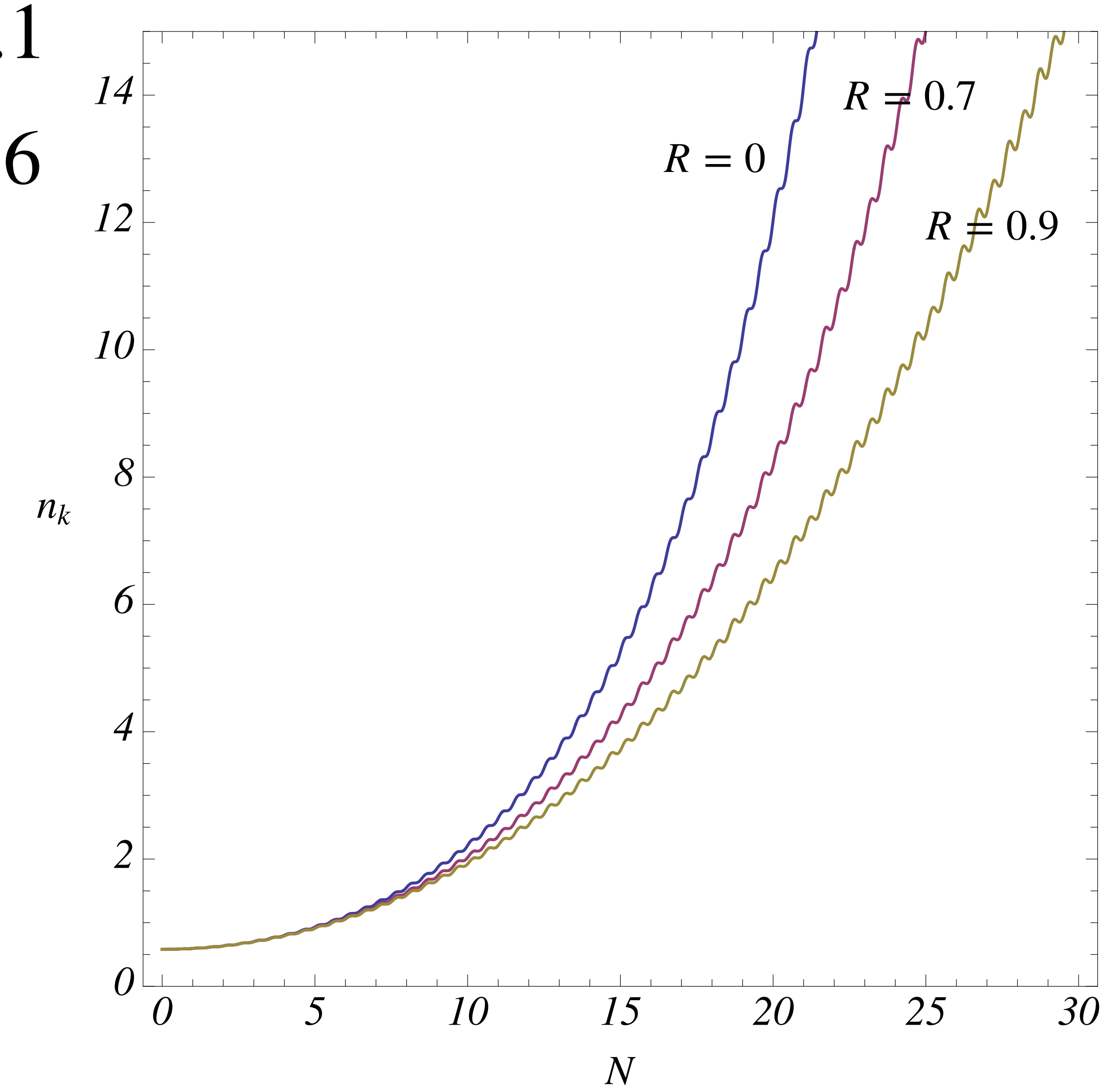
$$\frac{\omega_k^2(t)}{\omega_0^2} = 1 + A \sin\left(\omega_p t\right) \implies n_k \sim e^{Gt} \quad \text{for} \quad -1 < R < 1$$

where $\frac{G}{\omega_0} = \frac{A}{4}\sqrt{1 - R^2}$ and $\frac{A}{4}R = \frac{\omega_0 - \omega_p/2}{\omega_p/2}$

(X. Busch *et al.*, *Phys. Rev. A* **89**, 063606 (2014))

$A = 0.1$

$n_{\text{in}} = 0.6$



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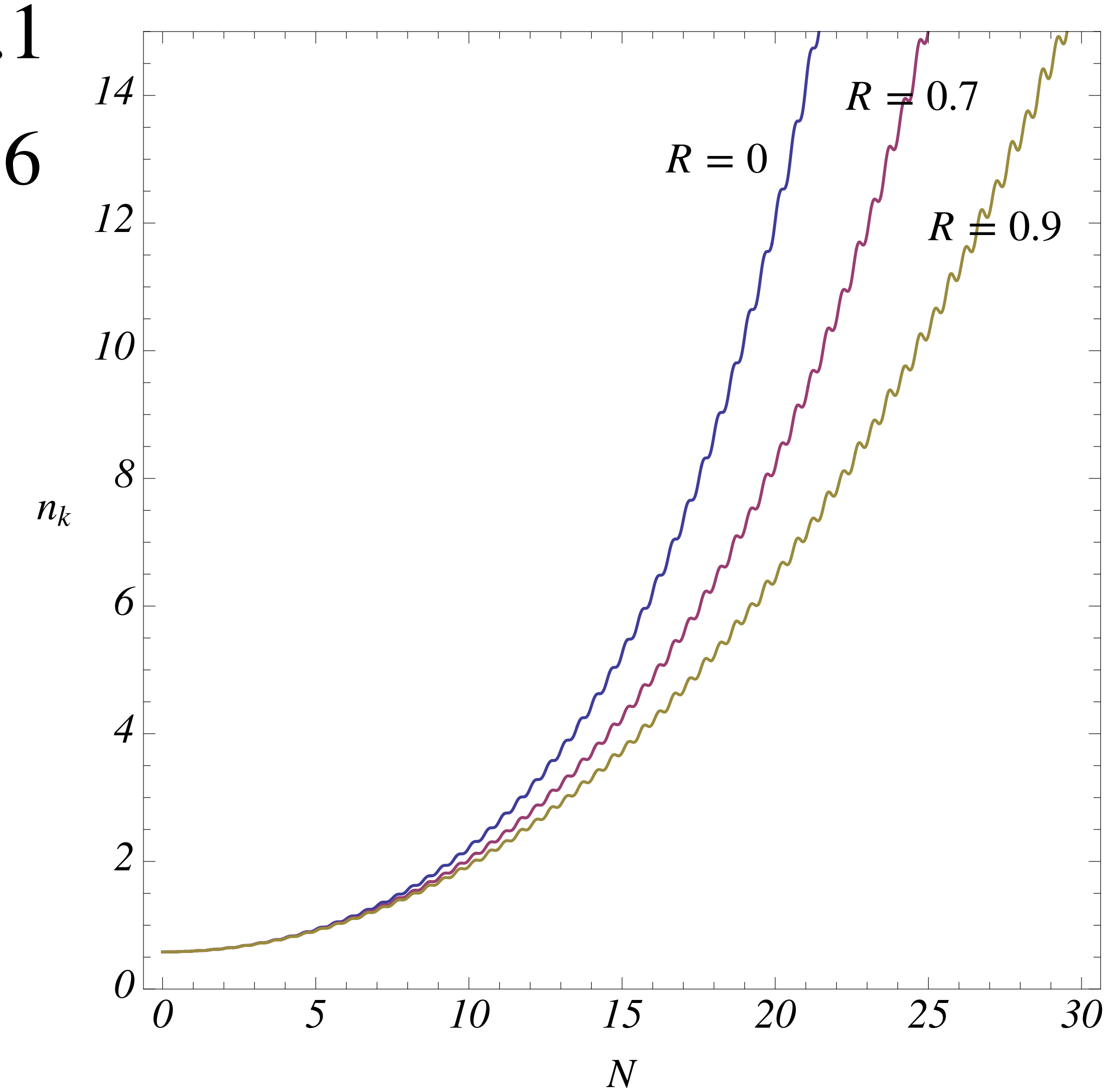

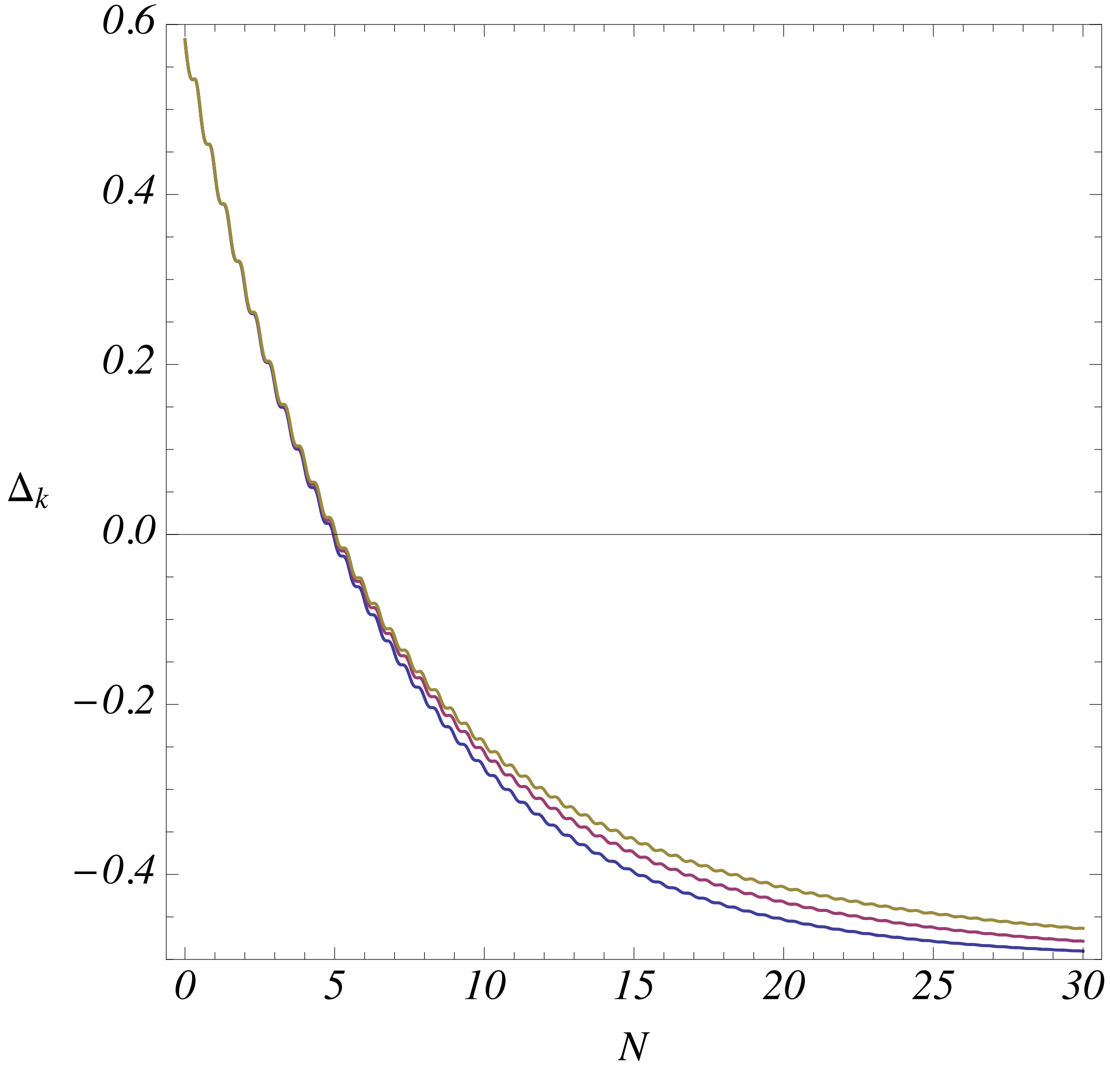

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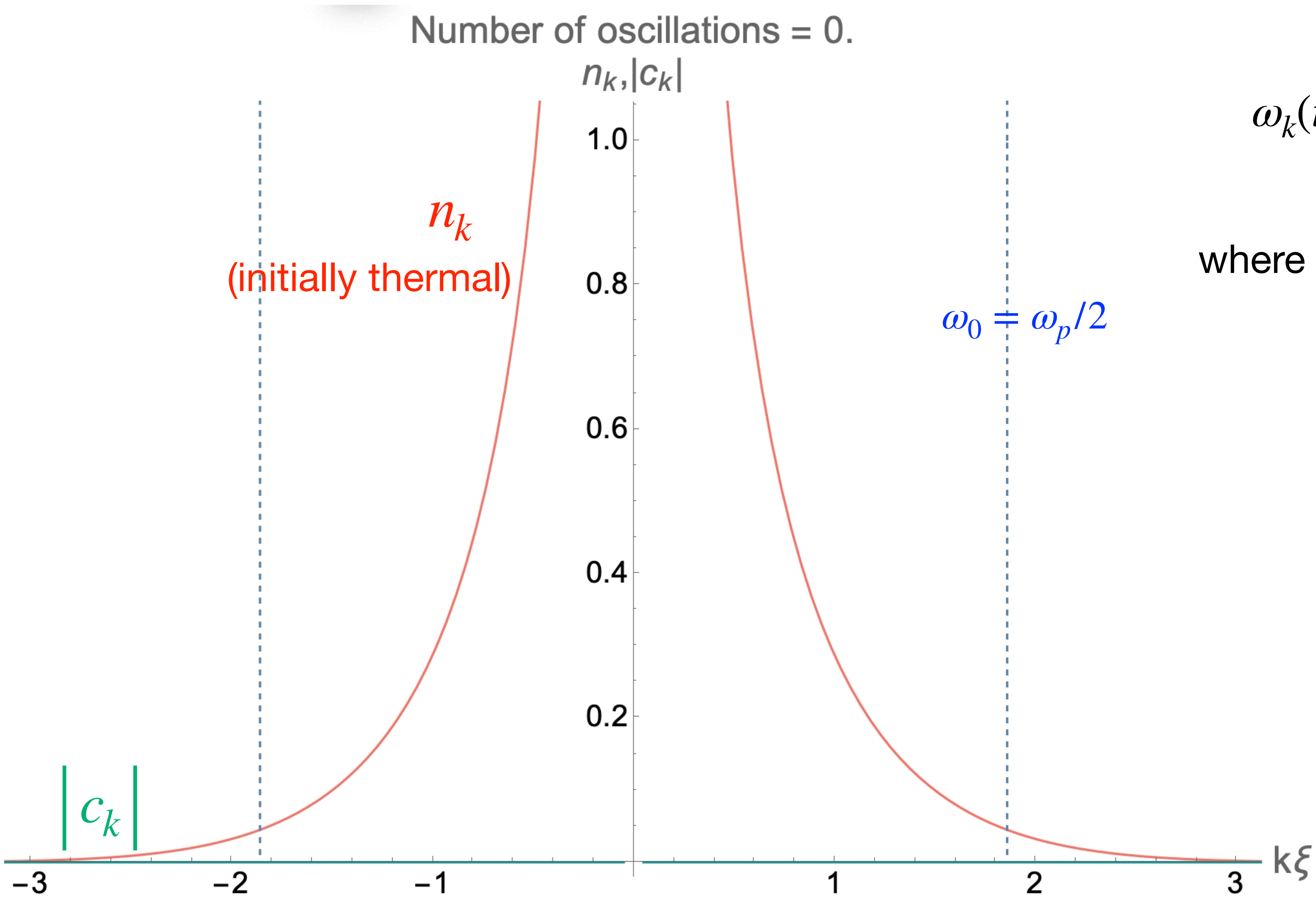

(Video courtesy of Amaury Micheli)

$$\omega_k(t) = \sqrt{\frac{g_{1D}(t)\rho_0}{m}k^2 + \frac{\hbar^2 k^4}{4m^2}}$$

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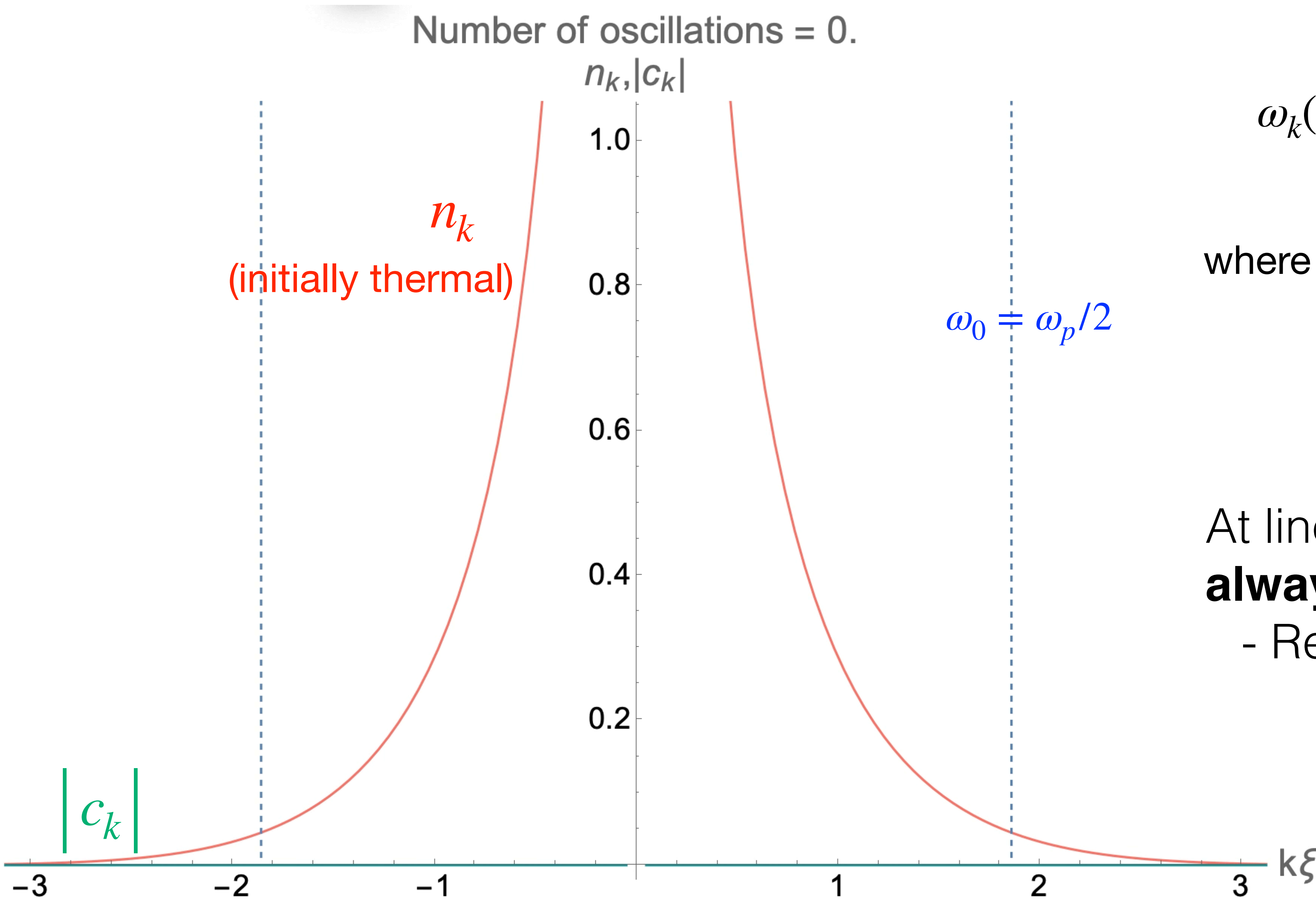

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At linear level, resonant modes **always** reach an entangled state

- Reasonable?

Loss of phonons from resonant mode

Phenomenological approach to damping

Introducing a **linear** damping rate, Γ

(X. Busch *et al.*, *Phys. Rev. A* **89**, 063606 (2014))

At each time step, $n_k \to n_k \, e^{-\Gamma \Delta t}$

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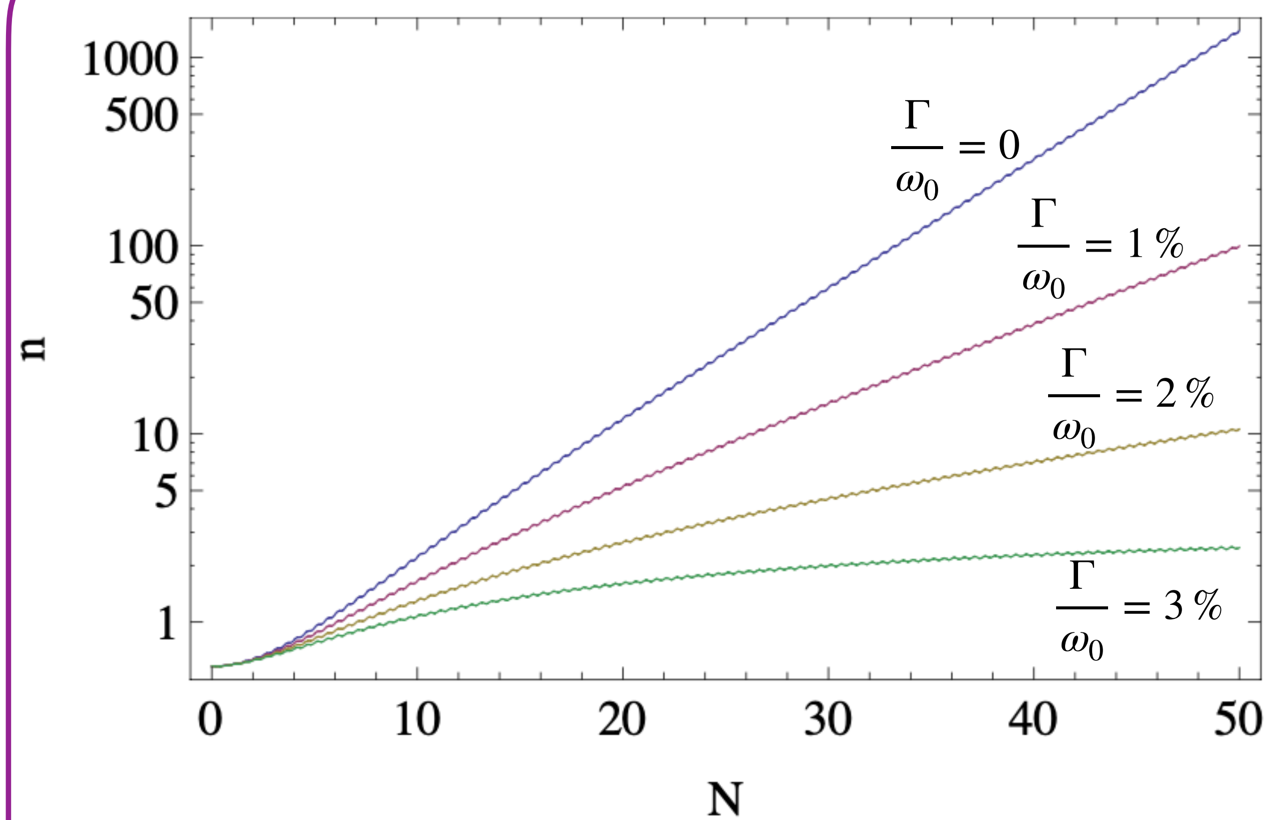

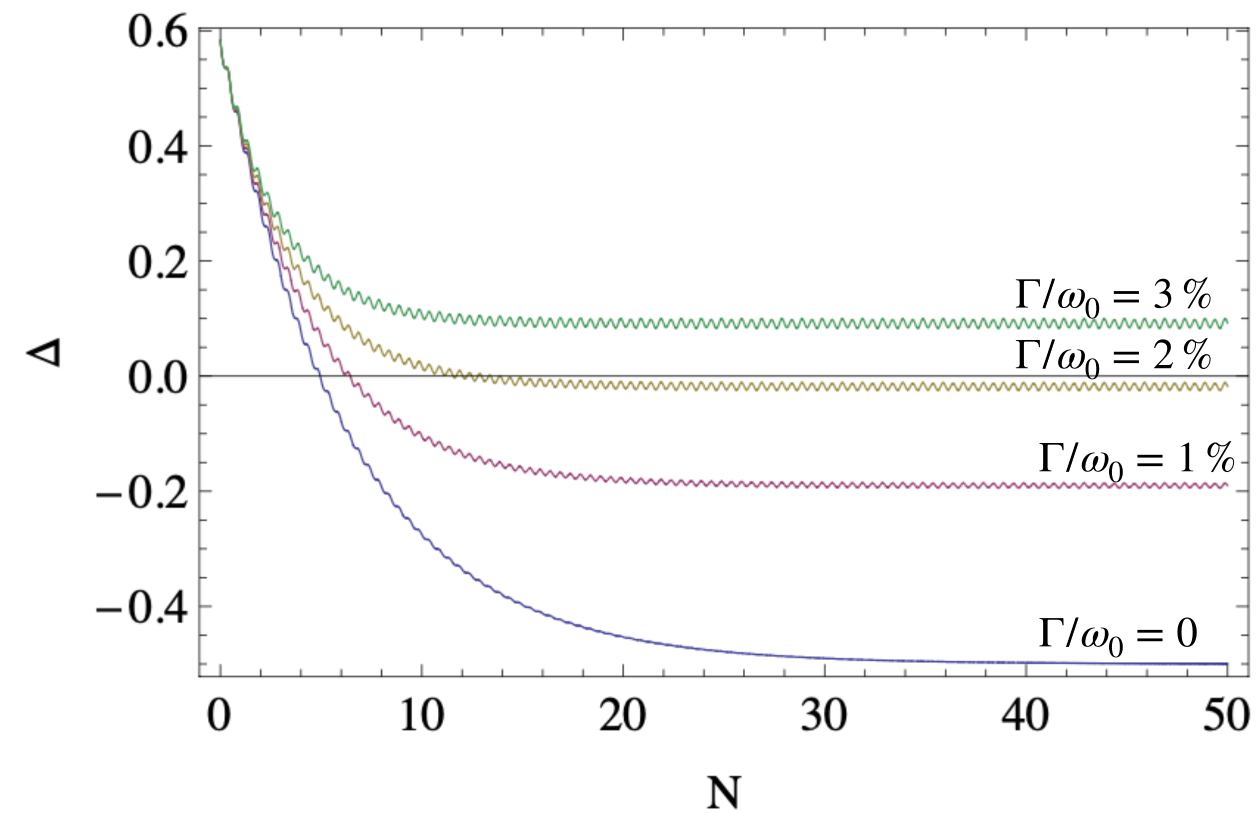

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$G/\omega_0 = 2.5\,\%$

Observations:
- Growth eventually saturates (for $\Gamma > G$)
- For large enough Γ, **entanglement never achieved**

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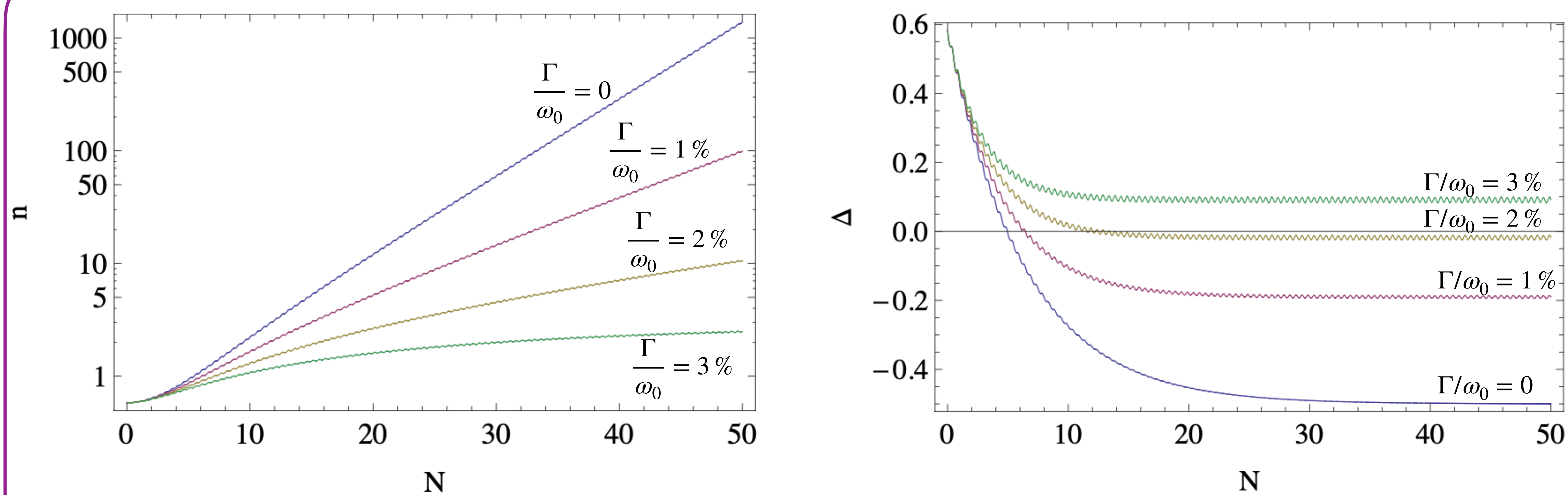

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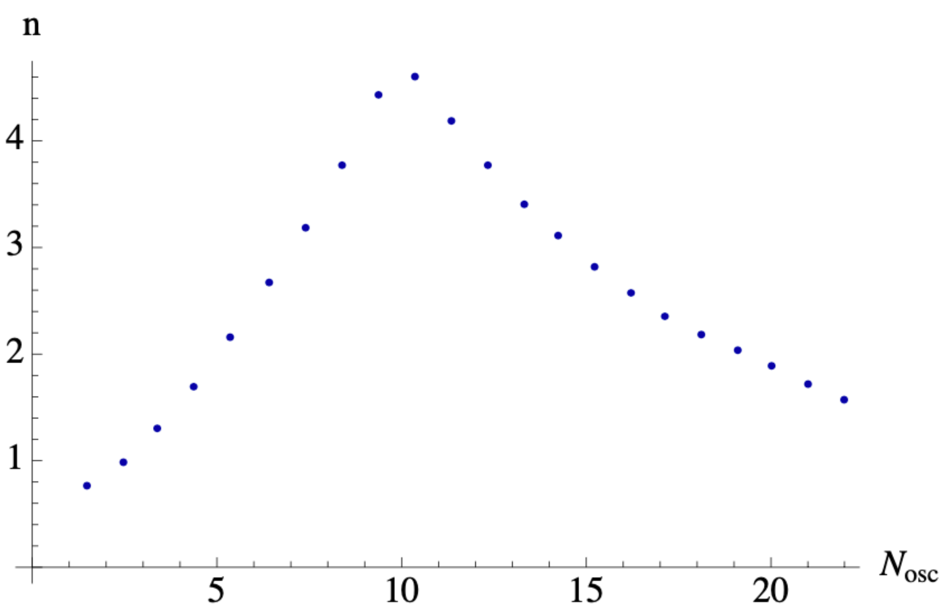
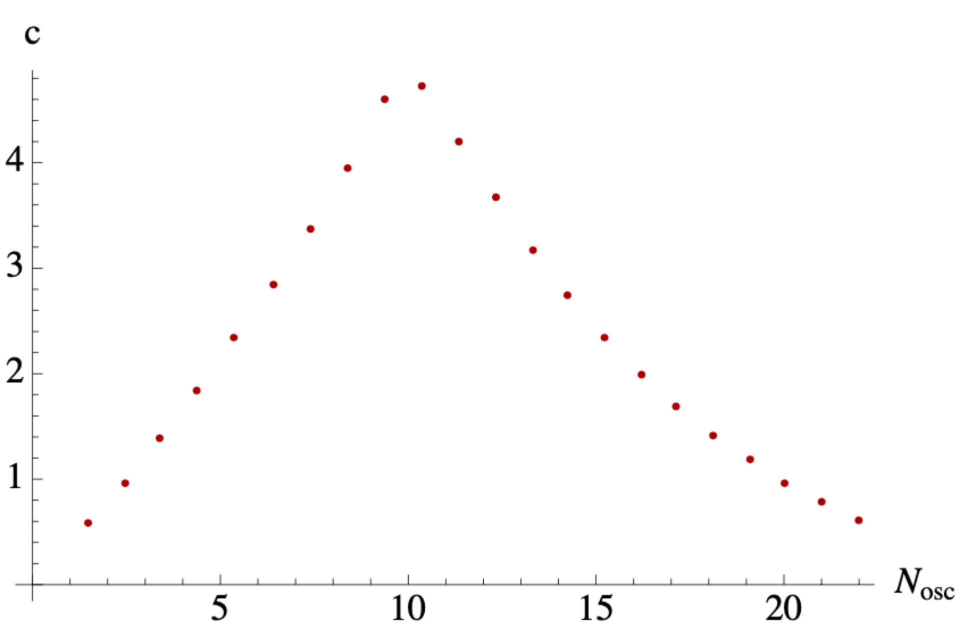
What is the microscopic mechanism responsible for the damping?

Numerical study of dissipative rate

Using classical-field approximation on the **fully nonlinear** equation of motion

$$i\,\partial_t \Psi = -\frac{1}{2m}\partial_x^2 \Psi + g(t)\left|\Psi\right|^2 \Psi$$





Soon after end of modulation,
we observe **exponential decay** of *n* and *c*
(as assumed in phenomenological approach)

Must be due to coupling with “environment”, *i.e.*, resonant modes with other (non-resonant) modes

Can we explain the observed diss. rate?

Interaction with thermal phonons

$$\hat{H} = H_0 + \hat{H}_2 + \hat{H}_3 + \ldots$$

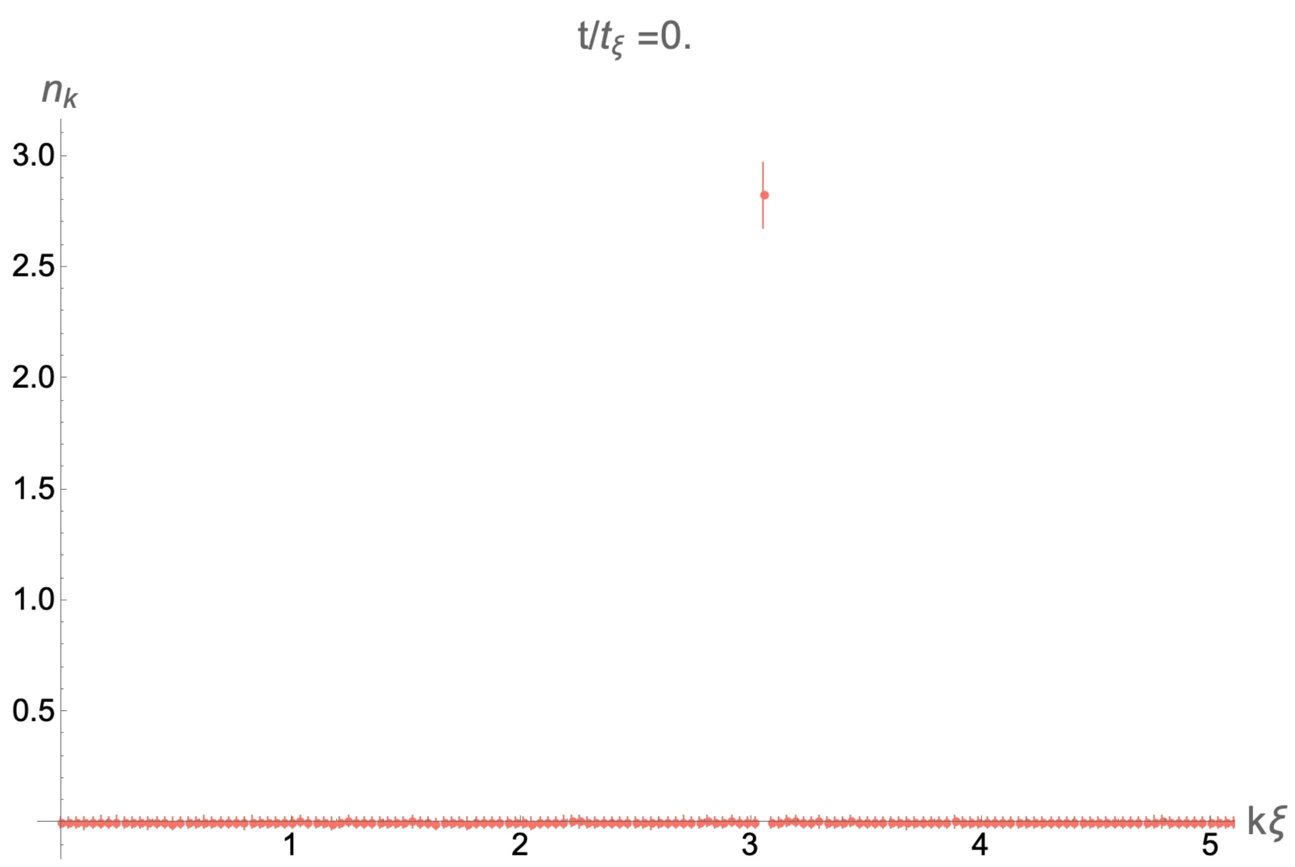
$$\hat{H}_3 = \sum_{k,q} H_3(k,q) \left[\hat{\varphi}^\dagger_{k+q} \hat{\varphi}_q \hat{\varphi}_k + \hat{\varphi}^\dagger_k \hat{\varphi}^\dagger_q \hat{\varphi}_{k+q} \right]$$

Lowest-order phonon/phonon interactions

At linear order in resonant mode

- Interaction with thermal bath
- **Broadening**

(A. Micheli and S. Robertson, *Phys. Rev. B* **106**, 214528 (2022))



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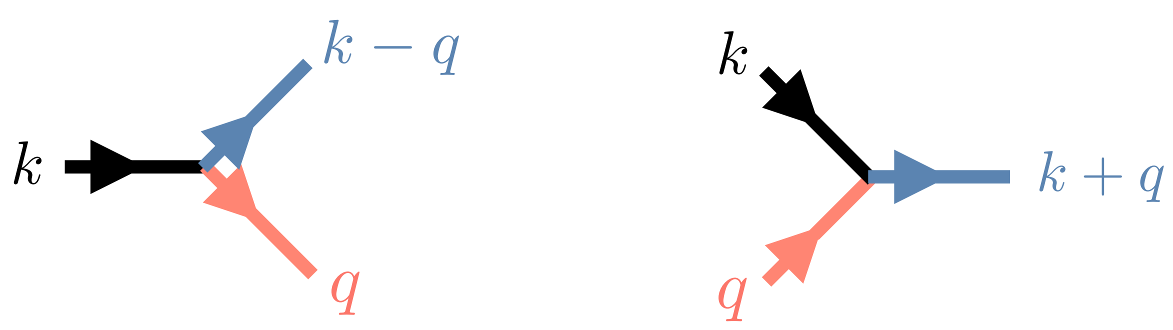


Fermi Golden Rule: Elastic channel? $\omega_k + \omega_q = \omega_{k+q}$

Only at $q = 0$

$$\Gamma_k \propto \lim_{q \to 0} \left| H_3(k,q) \right|^2 n_q^{\text{th}}$$

$$\left| H_3(k,q) \right|^2 \propto q \qquad n_q^{\text{th}} \propto \frac{1}{q}$$

Yields finite prediction for damping rate: $\Gamma_k \propto \dfrac{k_B T}{\rho_0 \xi}$

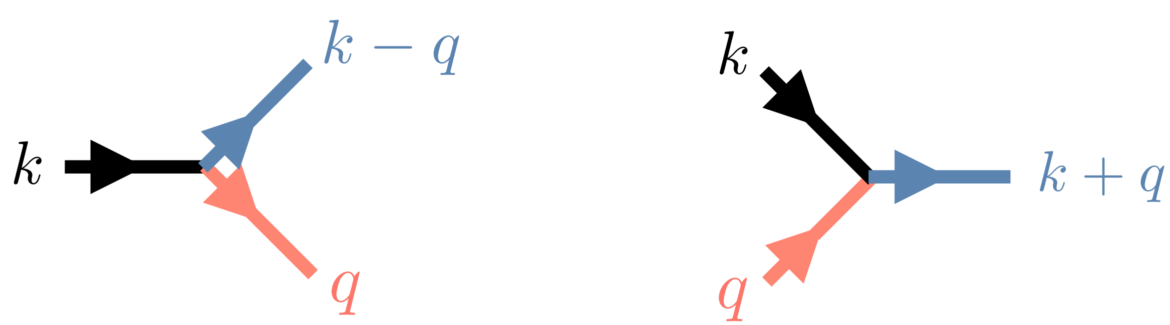
Interaction with thermal phonons

$$\hat{H} = H_0 + \hat{H}_2 + \hat{H}_3 + \dots$$

$$\hat{H}_3 = \sum_{k,q} H_3(k,q) \left[\hat{\varphi}^\dagger_{k+q}\hat{\varphi}_q\hat{\varphi}_k + \hat{\varphi}^\dagger_k\hat{\varphi}^\dagger_q\hat{\varphi}_{k+q}\right]$$

Lowest-order phonon/phonon interactions
At linear order in resonant mode
- Interaction with thermal bath
- **Broadening**

(A. Micheli and S. Robertson, *Phys. Rev. B* **106**, 214528 (2022))



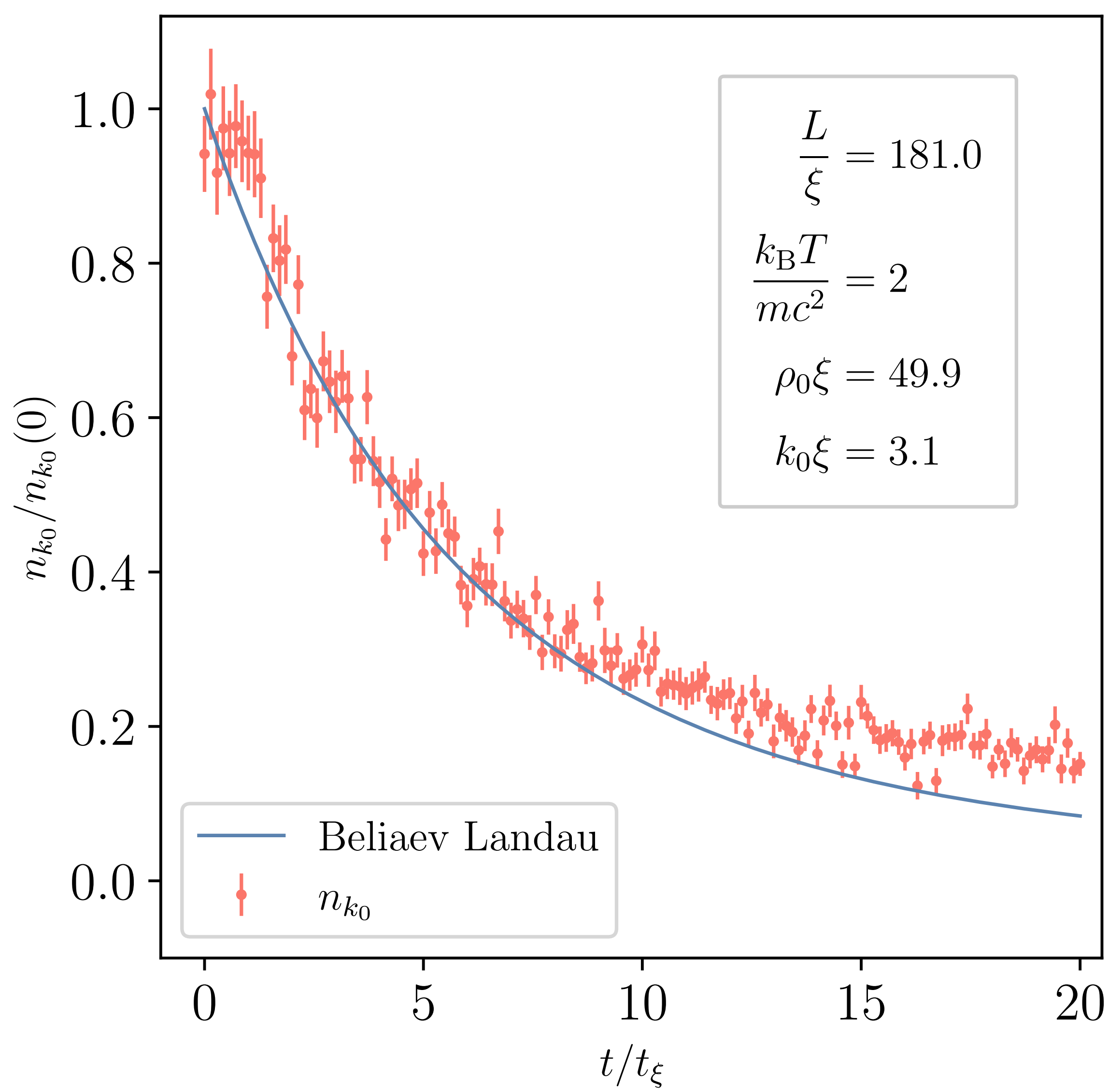
Fermi Golden Rule: Elastic channel? $\omega_k + \omega_q = \omega_{k+q}$

Only at $q = 0$

$$\Gamma_k \propto \lim_{q\to 0} \left|H_3(k,q)\right|^2 n^{\text{th}}_q$$

$\left|H_3(k,q)\right|^2 \propto q$ $\qquad n^{\text{th}}_q \propto \frac{1}{q}$

Yields finite prediction for damping rate: $\Gamma_k \propto \frac{k_B T}{\rho_0 \xi}$



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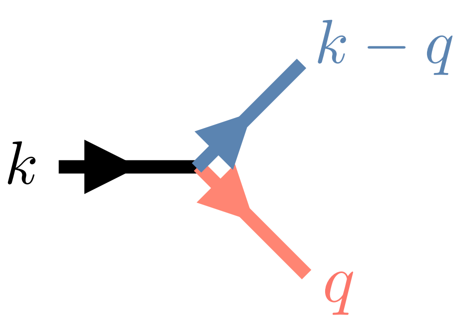

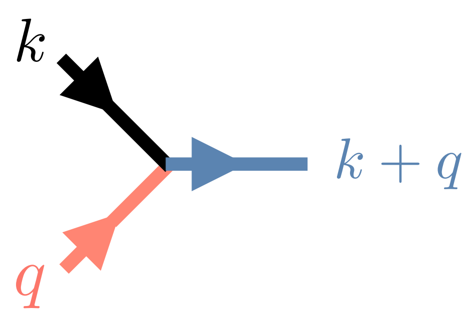

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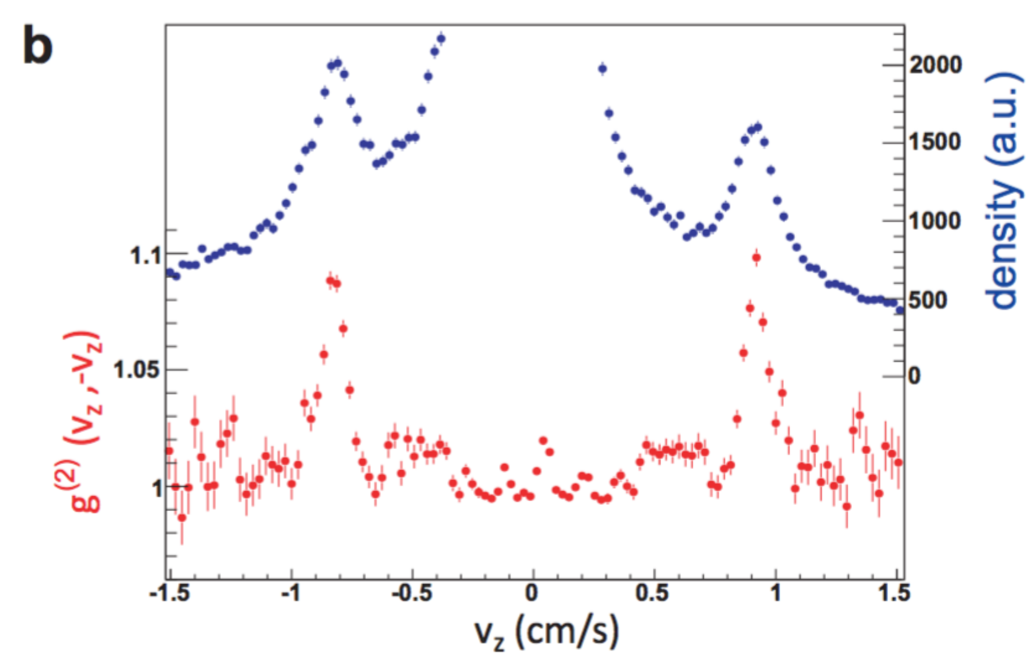
J.-C. Jaskula *et al.*, *Phys. Rev. Lett.* **109**, 220401



Phenomenological approach:

- threshold $\Gamma/\omega > 4.2\,\%$ for absence of entanglement

Prediction: $\Gamma/\omega \sim 5\,\%$

Summary

Preheating can be simulated in the response of phonons to a modulated 1D BEC

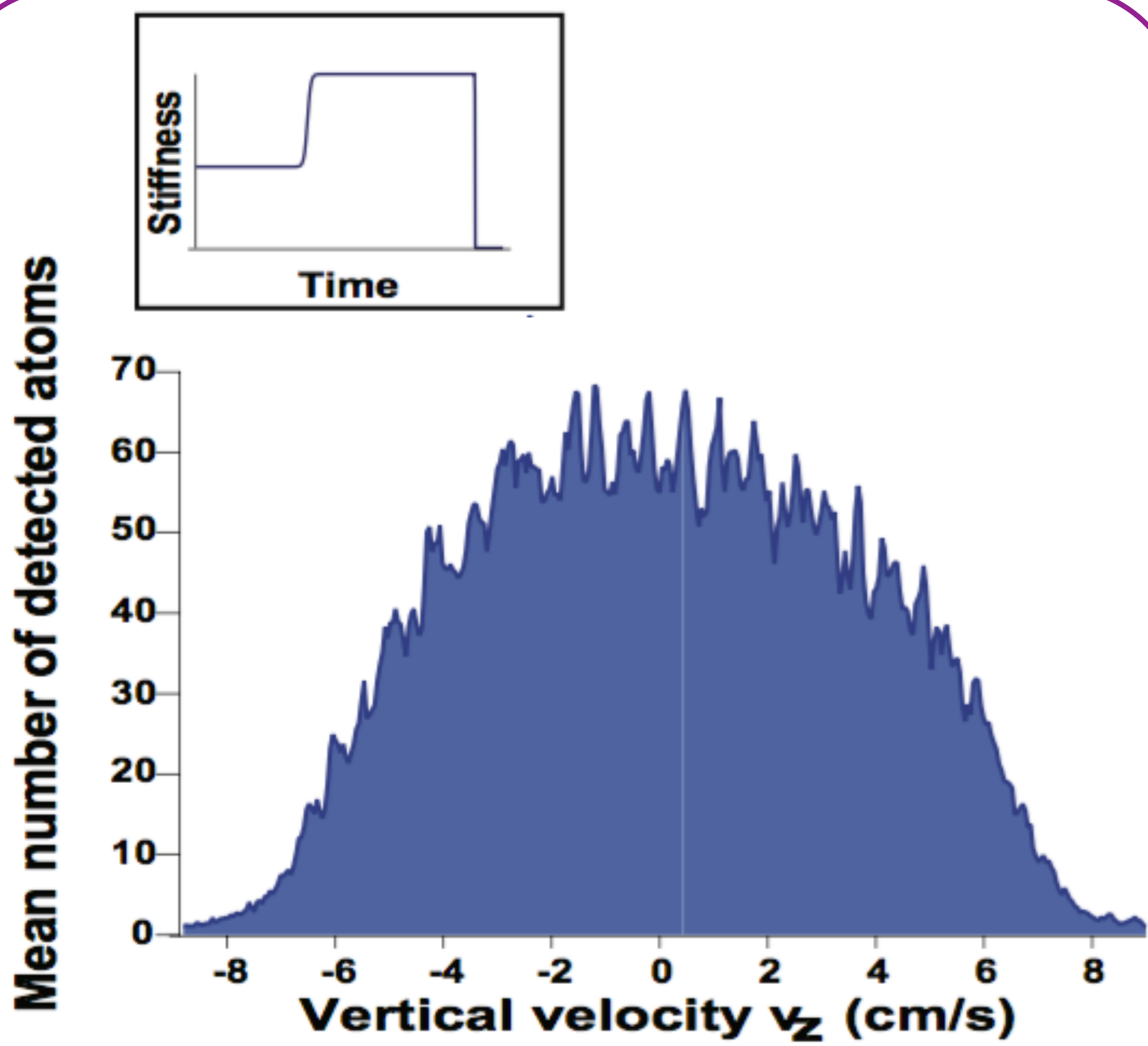
Observed: Growth of phonon number showing pair creation

To observe: Entanglement of created pairs

Dissipation occurs through **interaction with thermal bath** of phonons

- sufficient to explain absence of entanglement in previous experiment
- help to optimise design of future experiments

Have we already reached a strongly nonlinear regime?



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- No peak at resonance
- No peak at k=0 (condensate)

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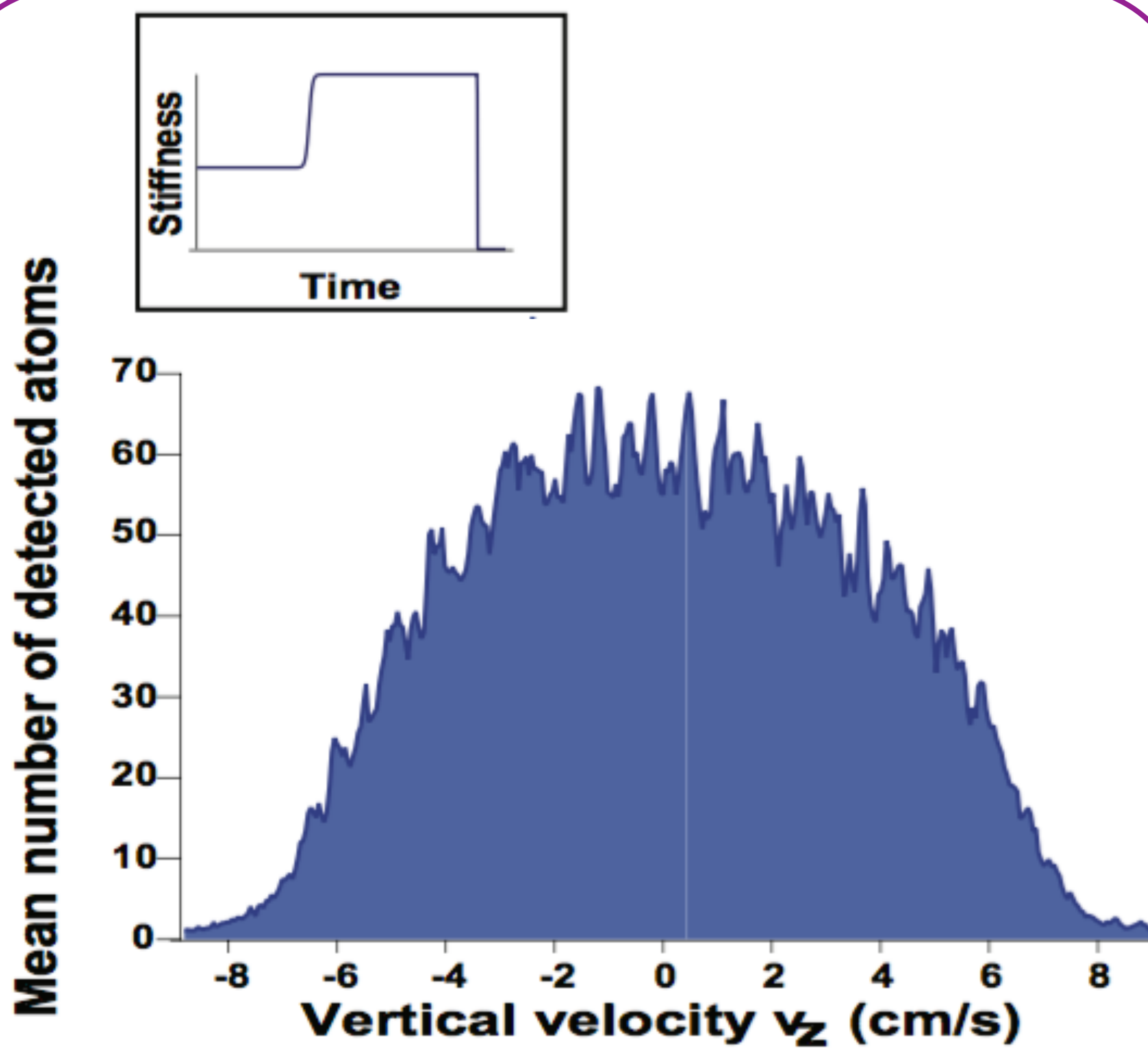

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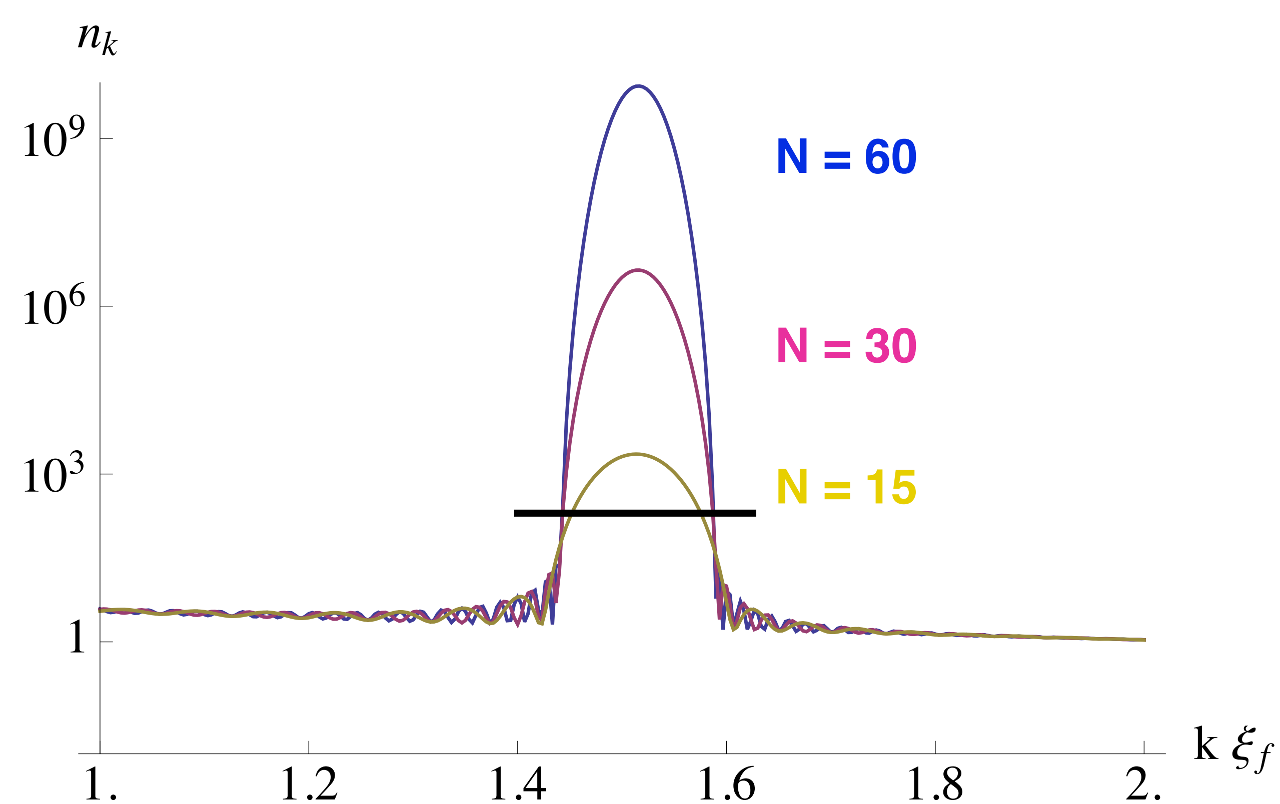
Prediction of linear theory:
$n_k \gg$ no. of atoms (!!)

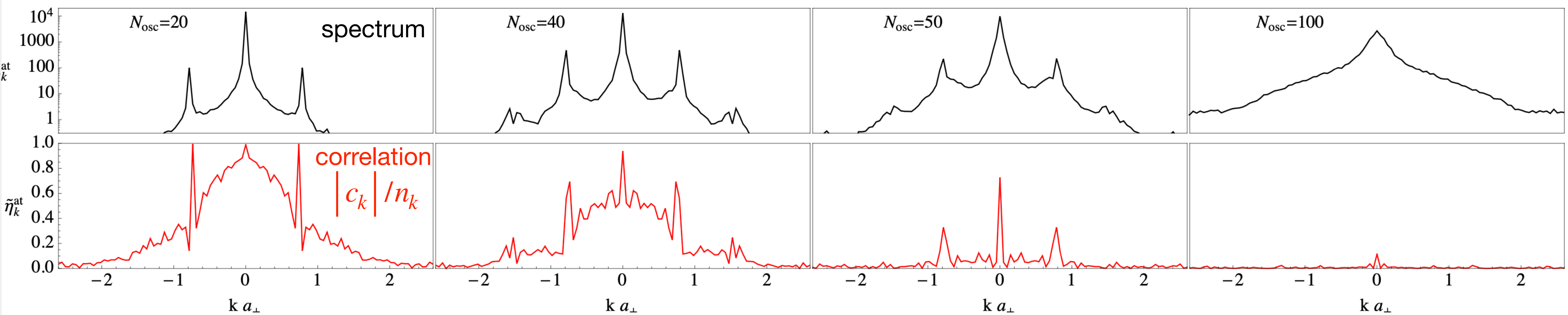


$k\xi = v_z / c$ $c \sim 1$ cm/s

Nonlinear effects: Long-time behaviour

SR, F. Michel and R. Parentani, arXiv:1802.00739 (2018)

time



entangled　　not entangled　　not even correlated

Redistribution of energy: spectrum becomes broad and featureless

Decoherence: *(k,-k)* correlations (even classical) completely lost