Towards analogue black hole merger

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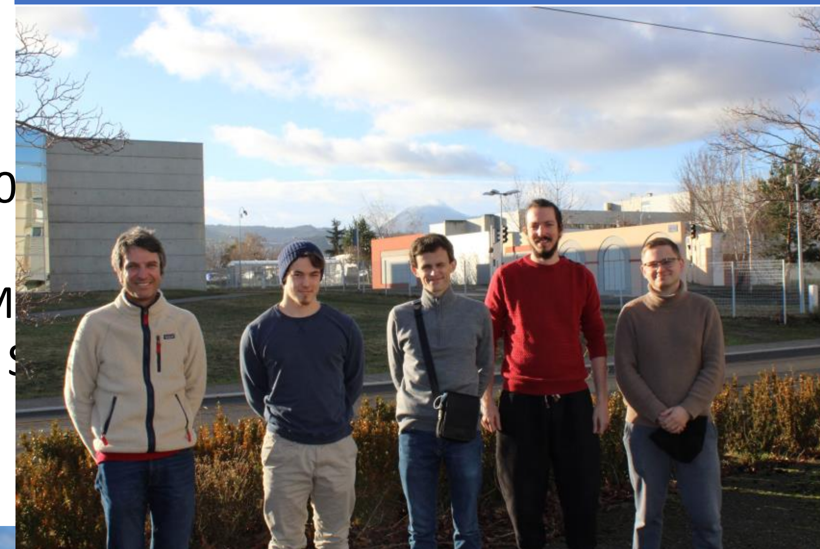
Clermont-Ferrand,

Quantum optoelectronics and nanophotonics

- Inhabitants: 300 0
- Students: 40 000
- Industry: HQ of M
- Science: 101-150 S



Plan

- Introduction
  - Exciton-polaritons
- Analogue black holes in polariton condensates
  - Kerr black hole and Penrose process
  - Black hole merger

Analogue physics

… is not the opposite of digital physics…

- Analogue gravity (Hawking, Penrose…)
- Early Universe (Kibble-Zurek, Inflation…)
- High-energy physics (Higgs field, Klein tunneling…)
- Quantum simulations (Heisenberg, Bose-Hubbard…)
- Topological photonics (Quantum Hall effects…)

Introduction

Exciton-polaritons

Strong coupling in microcavities: exciton-polaritons

High reflectivity R>0.9999

High fidelity resonator, Q=100 000

QW or bulk active layer with excitons

Quantized photons

$$E_C(k) = E_C(0) + \frac{\hbar^2 k^2}{2m_C}$$

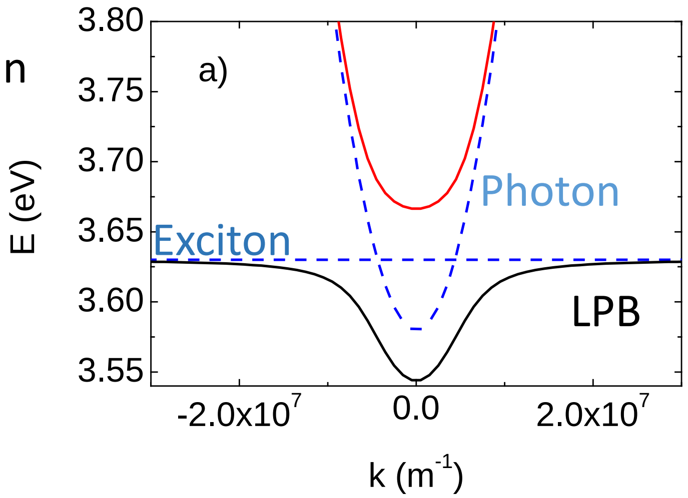
Quantized excitons

$$E_X(k) = E_X(0) + \frac{\hbar^2 k^2}{2m_X}$$

Strong coupling Hamiltonian

$$\begin{pmatrix} E_X(\vec{k}) & \hbar\Omega_R / 2 \\ \hbar\Omega_R / 2 & E_C(\vec{k}) \end{pmatrix}$$

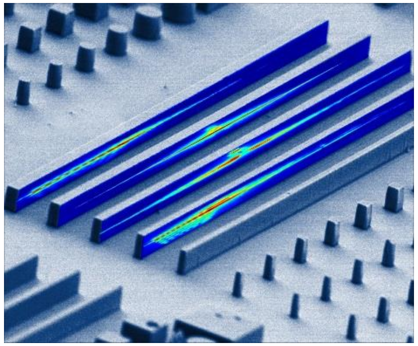
Rabi splitting

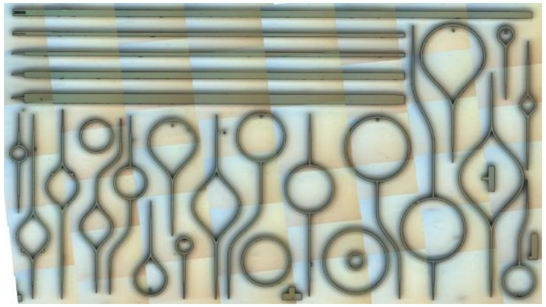


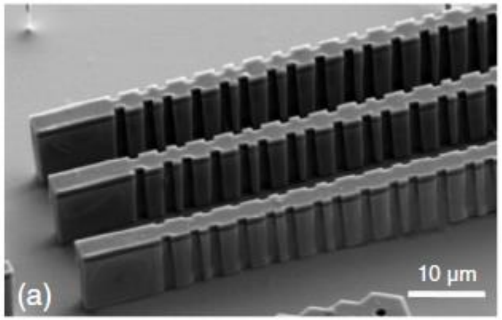
2D particles

The properties of exciton-polaritons

- Finite lifetime
- A non-parabolic dispersion
- A small effective mass
  - Slow light (v=1/300 c)
  - Light matter (10^{-5} of the free electron mass)
- Interactions
  - with the environment (phonons and electrons)
  - polariton-polariton interaction
- A bosonic character
  - Bose-Einstein condensates
  - Macrooccupied states
- A unique spin structure (2 spin projections)
  - Effective fields
  - Spin-anisotropic interactions
- Possibility to etch structures out of planar cavities



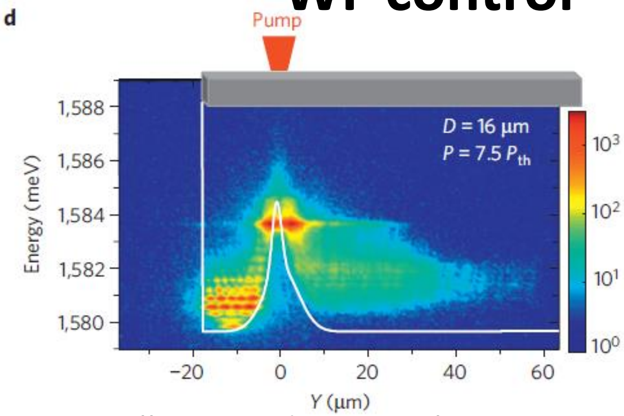




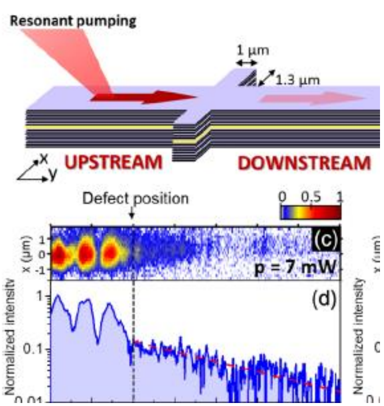
Structures obtained at LPN (CNRS, Paris)

Wavefunction engineering and measurement

WF control



Optically created potential in a wire, Nat. Phys. 6, 860 (2010).



Patterned wire cavity for 1D BH, PRL 114, 036402 (2015).

Optical measurements

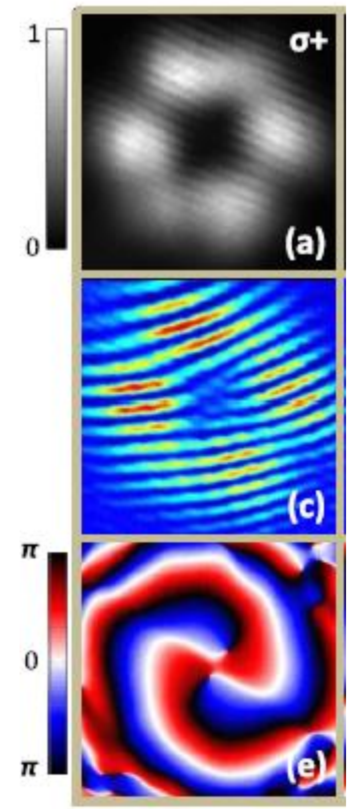
Intensity

$$n = |\psi|^2$$

Interference

Extracted phase

$$\varphi = \arg \psi$$



PRX 5, 011034 (2015).

Analogue black holes

A metric for a condensate

Gross-Pitaevskii equation

$$i\hbar\frac{\partial\psi}{\partial t}=-\frac{\hbar^2}{2m}\Delta\psi+\alpha|\psi|^2\psi$$

Madelung transform:

$$\psi=|\psi|e^{i\varphi}$$

Wave equation for weak excitations of a scalar Bose condensate:

$$\partial_\mu\left(\sqrt{-g}\,g^{\mu\nu}\partial_\nu\varphi\right)=0$$

Metric tensor for BEC excitations:

$$g_{\mu\nu}=\begin{pmatrix}-\left(c^2-\mathbf{v}^2\right) & -\mathbf{v}^T\\ -\mathbf{v} & 1\end{pmatrix}$$

Schwarzschild's metric tensor (for comparison):

$$g_{\mu\nu}=\begin{pmatrix}\left(1-\frac{2GM}{rc^2}\right) & 0 & 0 & 0\\ 0 & -\left(1-\frac{2GM}{rc^2}\right)^{-1} & 0 & 0\\ 0 & 0 & -r^2 & 0\\ 0 & 0 & 0 & -r^2\sin^2\theta\end{pmatrix}$$

Both tensors can exhibit **event horizons** (zero terms on the main diagonal)

Class. Q. Grav. 15, 1767 (1998).

Topological defects

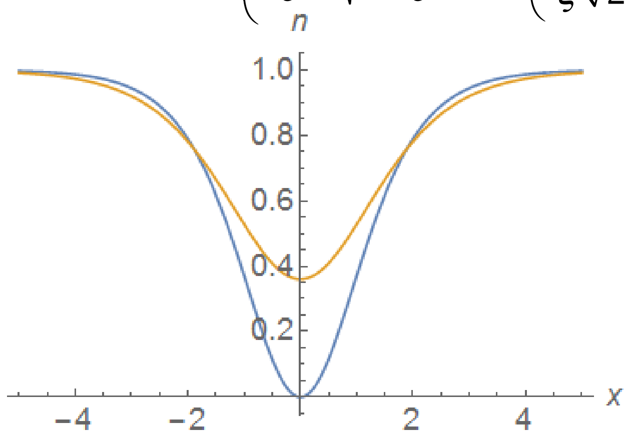
Healing length $\xi = \frac{\hbar}{\sqrt{2\alpha n m}}$

1D and 2D topological defects can be treated as particles!

1D: Soliton (quasi!)

- Dark soliton: local density minimum + phase jump
- Grey soliton *0<v<c*

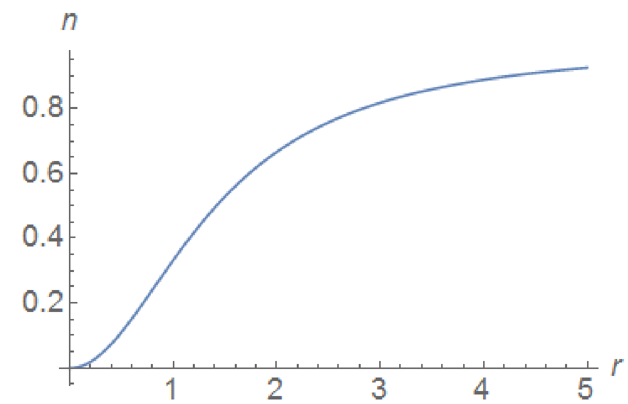
$$\psi(x-vt) = \sqrt{n}\left(i\frac{v}{c} + \sqrt{1-\frac{v^2}{c^2}}\tanh\left(\frac{x-vt}{\xi\sqrt{2}}\sqrt{1-\frac{v^2}{c^2}} \right) \right)$$



2D: vortex

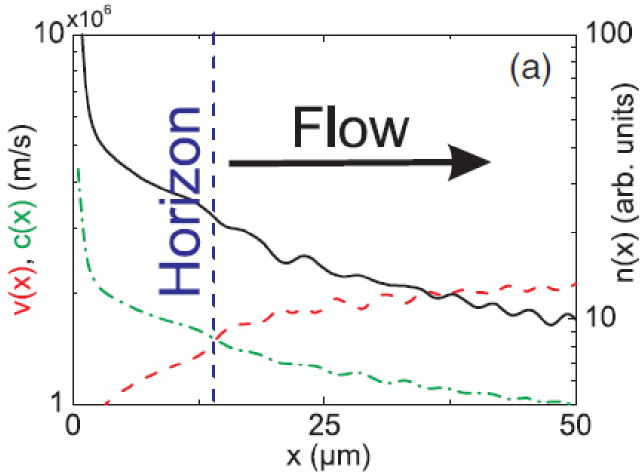
- Zero-density point + phase circulation
- Single-winding vortices

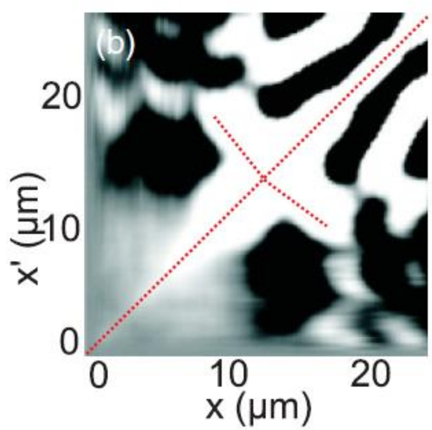
$$\psi \approx \frac{r/\xi}{\sqrt{2+(r/\xi)^2}} e^{i\phi}$$



Acoustic black hole for polaritons

Our proposal: PRB 84, 233405 (2011)

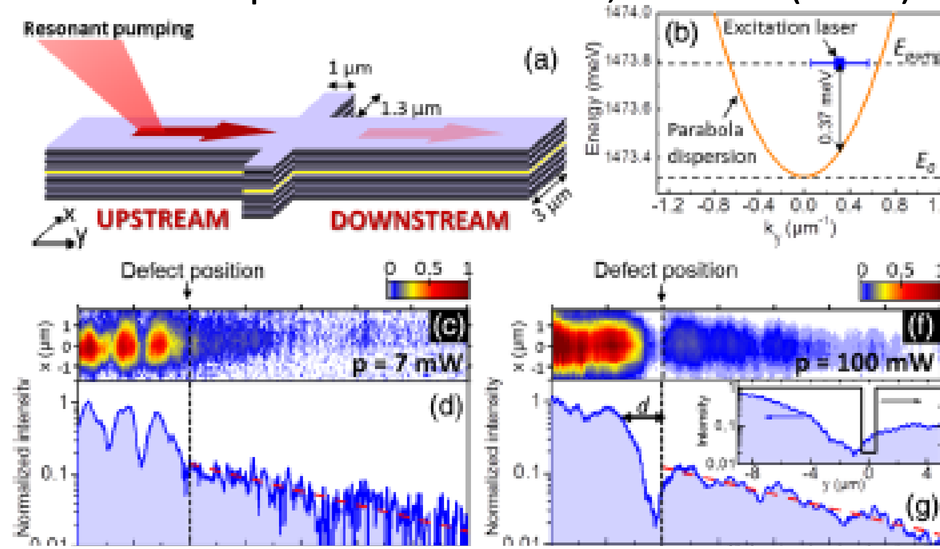




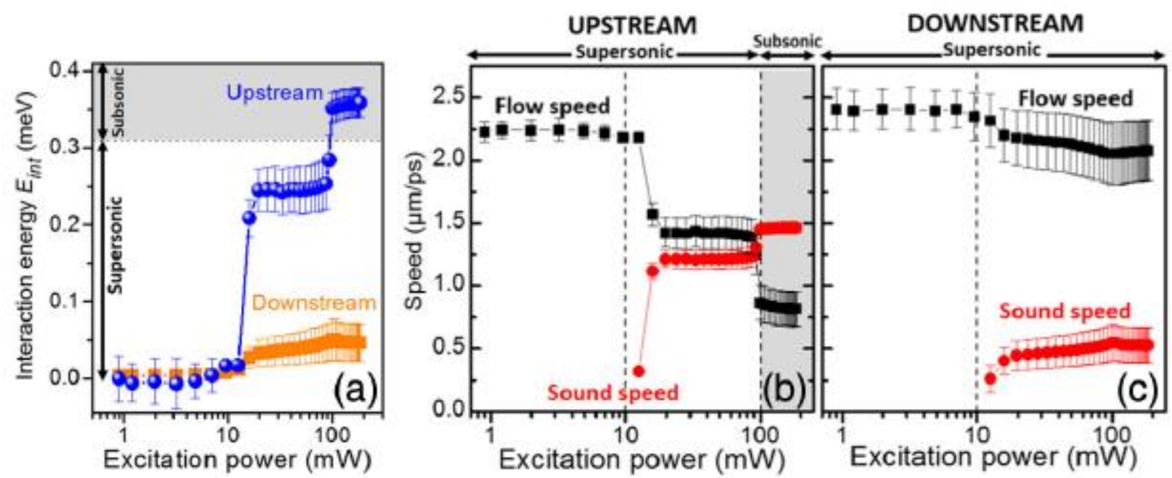
Correlated Hawking phonons

Suggested in R. Balbinot et al, PRA 78, 021603R (2008)

Experiment: PRL 114, 036402 (2015)



Experimental observation of the subsonic-supersonic transition in polariton condensates



Kerr Black Hole

Kerr black hole features

- Rotating black hole
- Maximal angular momentum $a/M=1$
- Strong frame dragging
- Two particular surfaces
  - Horizon (light cannot get out)
  - Static limit (light cannot go against the rotation)
- Penrose process (extraction of the rotation energy)
- Ergosphere

PRD 70, 124006 (2004).

Reproducing the Kerr metric in 2D

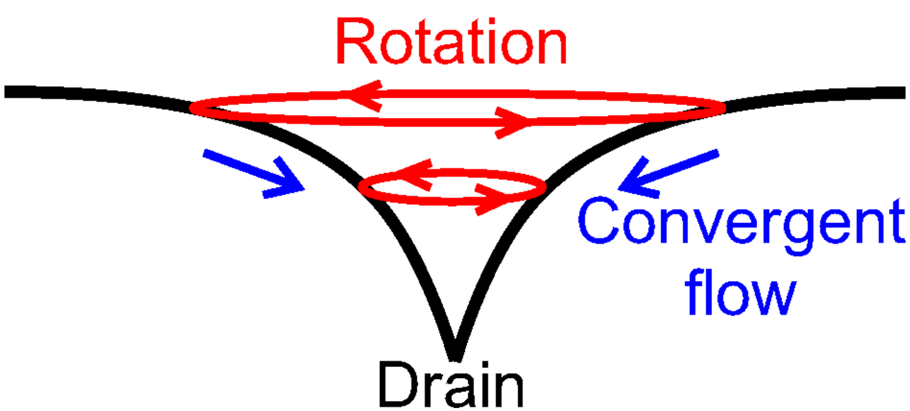
Equatorial plane only!

Kerr

$$g_{\mu\nu}^{Kerr} = \begin{pmatrix} -\left(1-\frac{2M}{r}\right) & 0 & -\frac{4aM}{r} \\ 0 & \frac{r^2}{r^2-2Mr+a^2} & 0 \\ -\frac{4aM}{r} & 0 & \left(r^2+a^2+\frac{2a^2M}{r}\right) \end{pmatrix}$$

Condensate

$$g_{\mu\nu} = \frac{mn}{c}\begin{pmatrix} -\left(c^2-v^2\right) & 0 & -2rv_\phi \\ 0 & \left(1-\frac{v_r^2}{c^2}\right)^{-1} & 0 \\ -2rv_\phi & 0 & r^2 \end{pmatrix}$$



- Quantized rotation
- Need to determine c, v, v_r

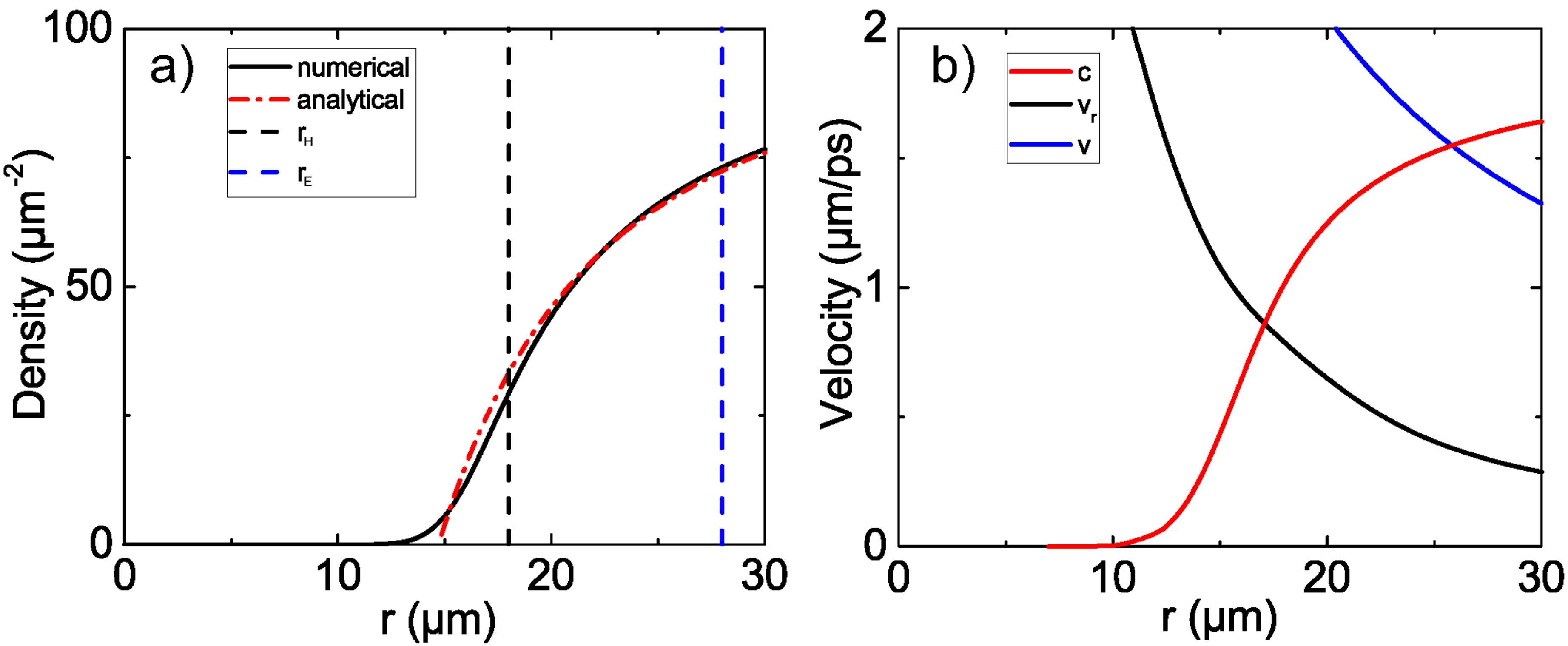
PRB 99, 214511 (2019)

Condensate wavefunction

$$\nabla \times \mathbf{v} \sim \nu\delta(\mathbf{r})$$
$$\nabla \cdot \mathbf{v} \sim -\zeta\delta(\mathbf{r})$$

$$\psi(r,\phi) = \sqrt{n_\infty}\left(1-\xi^2\frac{\nu^2+\zeta^2}{r^2}\right)\exp\left(i\left(\zeta\ln\frac{r}{\xi}+\nu\phi\right)\right)$$

Asymptotic series expansion at large r



PRB 99, 214511 (2019)

Analogue Kerr BH parameters

- Comparing the metric element g_{rr} we obtain

$$M_{cond} \sim r_H \sim \zeta\xi$$

Mass is controlled by the drain

- Maximal number of vortices in an analogue BH

$$\nu_{\max} \sim \frac{r_H}{\xi}, a_{\max} \sim r_H$$

- Maximal angular momentum $\frac{a_{\max}}{M} \sim 1$

Vortices inside the BH are distributed along the horizon

PRB 99, 214511 (2019)

From light to matter

- Density waves – null geodesics (light)
- Topological defects – stable particles
- We can test time-like geodesics (particle trajectories)

Analogue physics with vortices – Maxwell (1860), Feynman (1948), Dirac (1951), Popov (1973)…

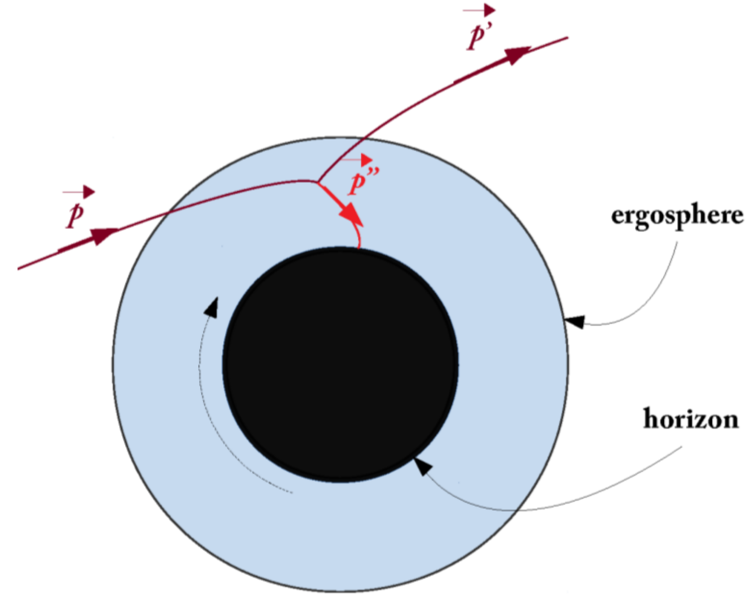
V.N. Popov, Sov. Phys. JETP 37, 341 (1973) - independent prediction of the BKT transition

2+1 Relativistic electrodynamics in which phonons play the role of photons, and quantum vortices the role of charged particles (flat space).

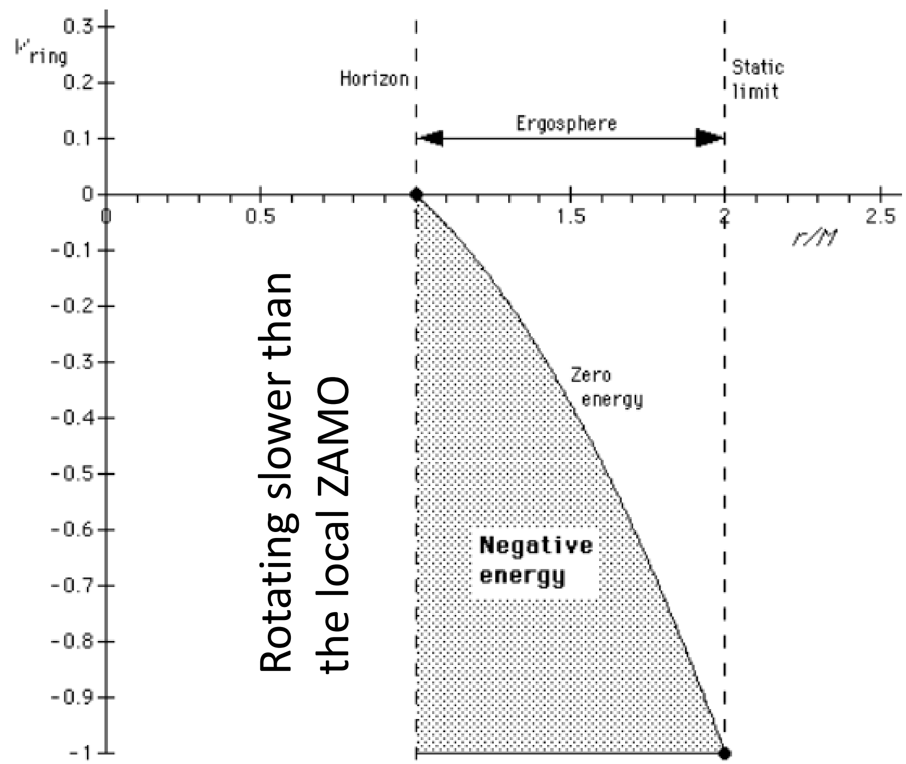
$$S_0 = -m_V c \sum_i \int ds_i - iq \int A j d^3 x - \frac{1}{2c} \int (\nabla \times A)^2 d^3 x \qquad E_V = m_V c^2$$

Penrose process

- A particle p falls into the ergosphere
- It splits into two (p’ and p’’)
- p’’ falls into the BH
  - p’’ had negative energy!
- p’ escapes



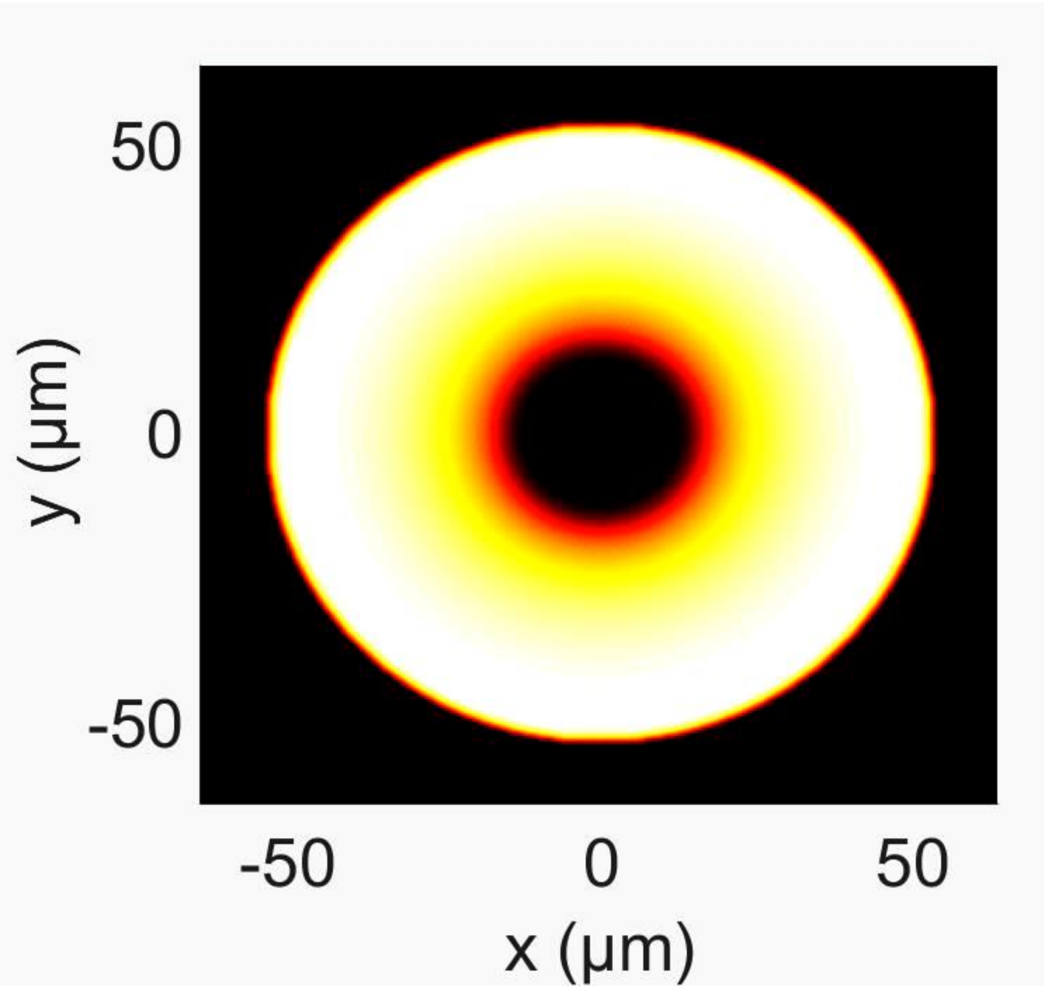
- Negative energy does not mean an antiparticle
- One needs more than mc^2 to get the particle away from BH

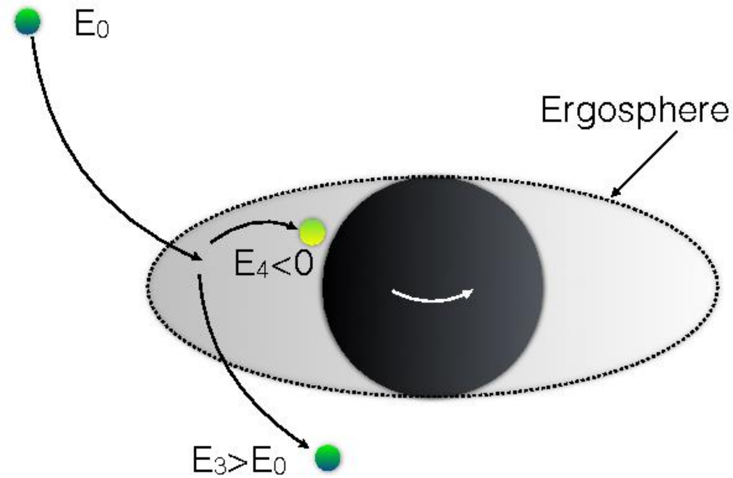


E.F. Taylor, Exploring Black Holes

PRB 99, 214511 (2019)

Penrose process

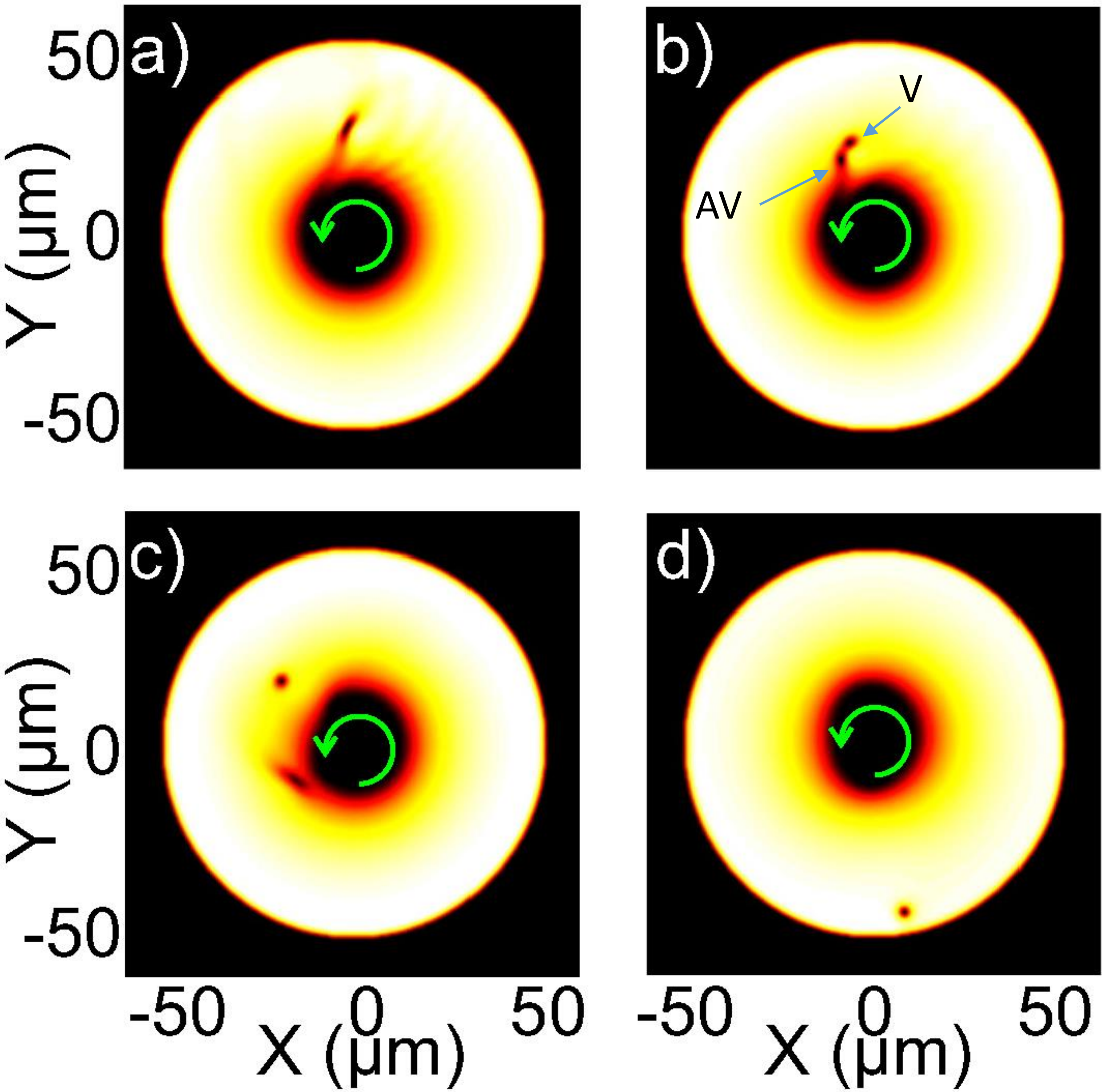




- A vortex-antivortex pair is formed from a density dip
- The anti-vortex falls into BH
- The vortex escapes to infinity
- BH rotation decreases (energy loss)

PRB 99, 214511 (2019)

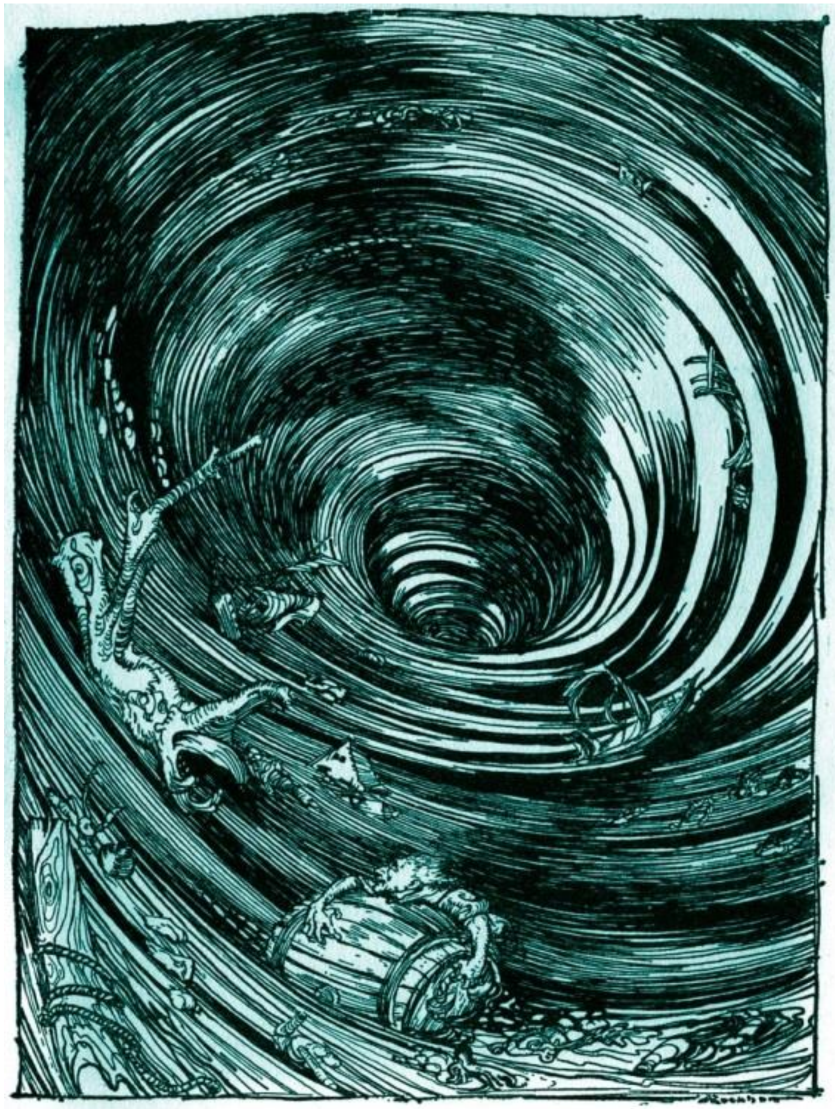
Penrose process: snapshots



- V/AV interaction slows the AV
- AV rotates *slower* than the condensate
- $E_{AV}<0$
- V gains energy from AV and escapes

The Penrose effect - Getting out of the Maelstrom

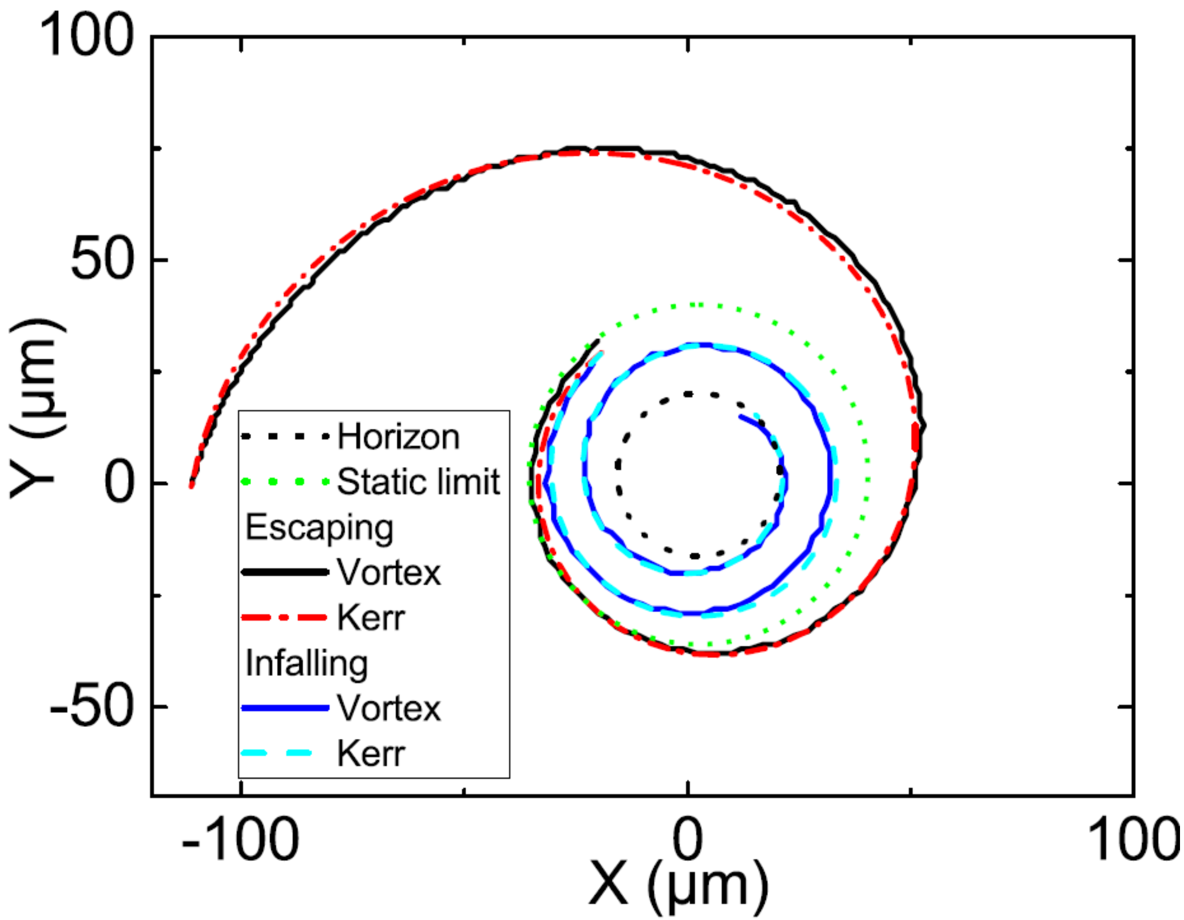
A Descent into the Maelstrom (E. A. Poe)



20 000 leagues under the seas (J. Verne)

PRB 99, 214511 (2019)

Vortex trajectories and time-like geodesics of the Kerr metric



Hamilton equations for time-like Kerr geodesics

$$\dot{r} = \frac{\Delta}{\Sigma} p_r$$

$$\dot{p}_r = -\left(\frac{\Delta}{2\Sigma}\right)' p_r^2 + \left(\frac{R}{2\Delta\Sigma}\right)'$$

$$\dot{\phi} = -\frac{1}{2\Delta\Sigma}\frac{\partial}{\partial L} R$$

where

$$\Sigma = r^2$$

$$\Delta = r^2 - 2Mr + a^2$$

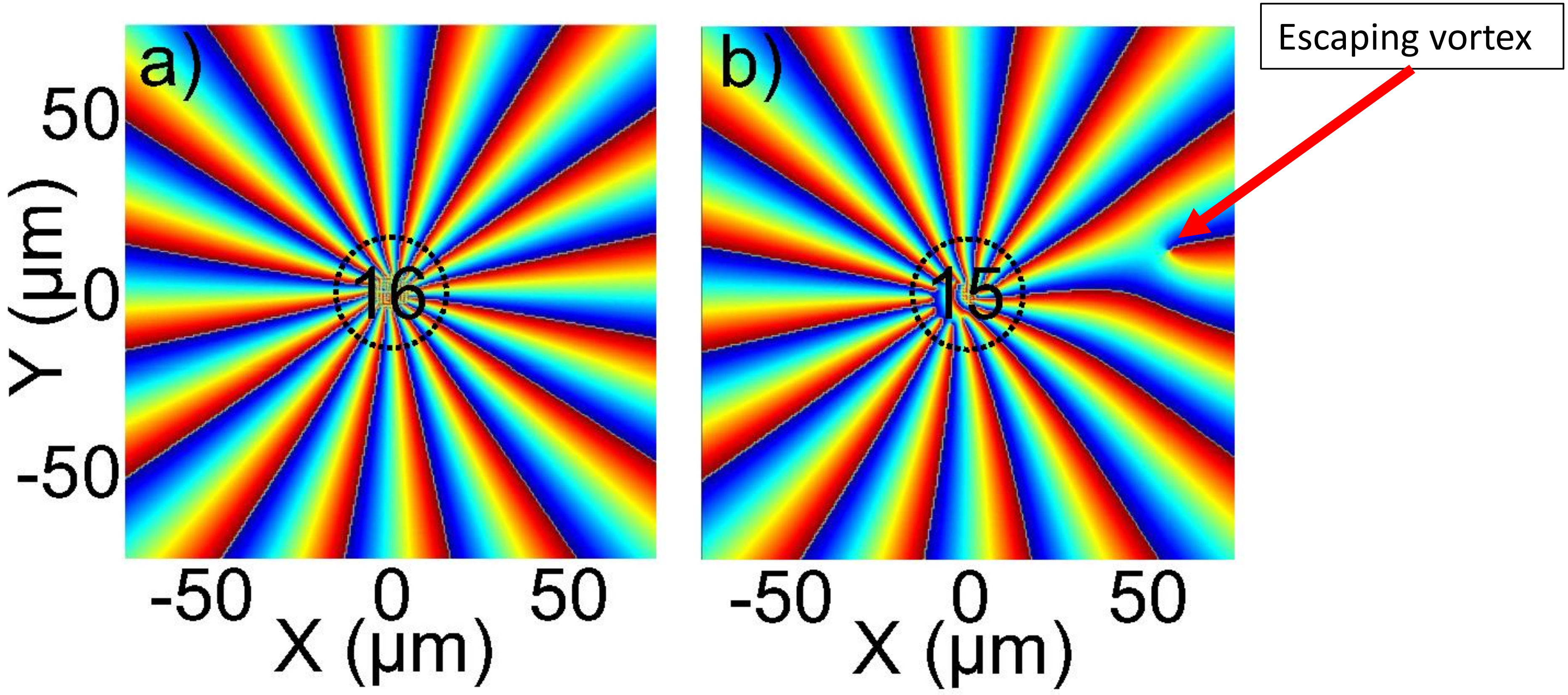
$$R = P^2 - \Delta\left(r^2 + \left(L - aE\right)^2\right)$$

$$P = E\left(r^2 + a^2\right) - aL$$

Both metrics dominated by the divergent term $g_{rr} \sim (r - r_H)^{-1}$

PRB 99, 214511 (2019)

Phase of the condensate



Outlook

- Dynamical metric
  - The angular momentum is not fixed externally
- Natural presence of quantum fluctuations
  - Towards quantum gravity
  - Comparable scales of quantum and gravitational effects
- Control of quantum fluctuations
  - Interactions
  - Particle mass

$$\frac{n^{(1)}(0)-n^{(1)}(\infty)}{n^{(1)}(0)} \sim \frac{\alpha m}{\hbar^2}$$

- Thermal fluctuations negligible

$$n^{(1)}(s) \sim \left(\frac{s_T}{s}\right)^{\nu},\ \nu = \frac{k_B T m}{2\pi\hbar^2 n_s}$$

Towards analogue black hole merger

1st BH merger detection: 2016

PRL 116, 061102 (2016)

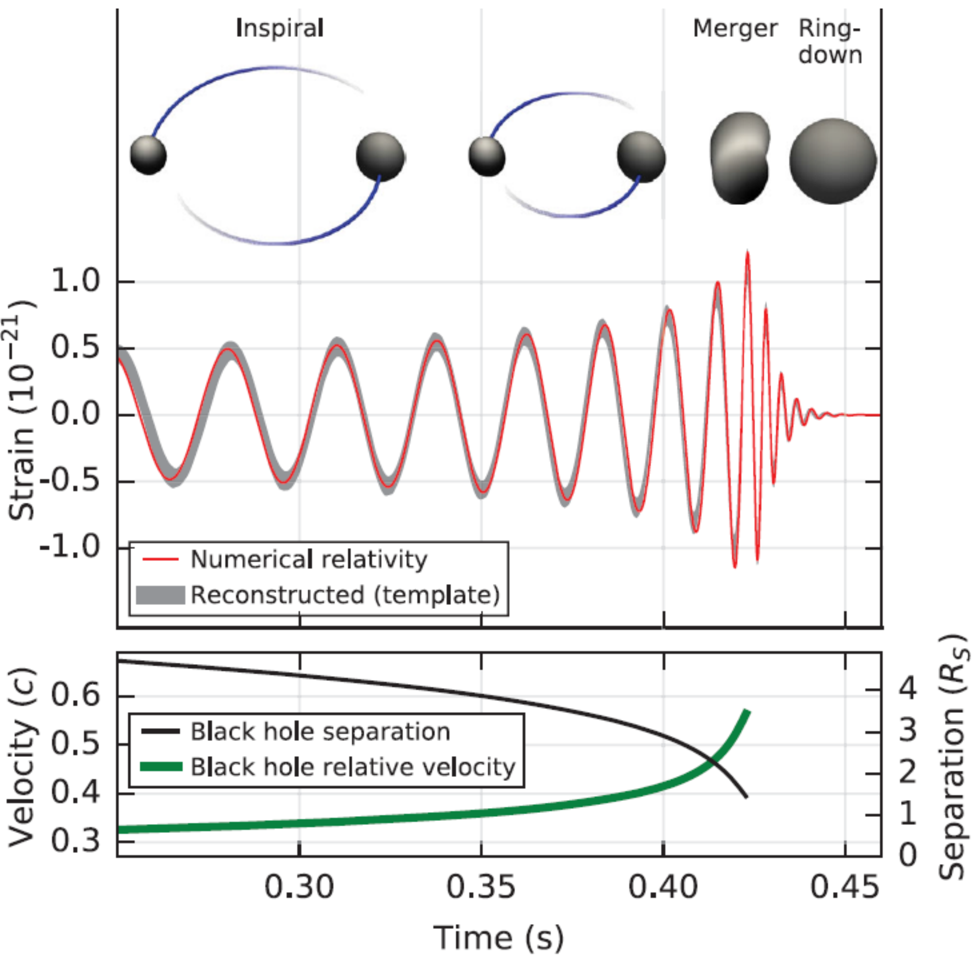


FIG. 2. *Top:* Estimated gravitational-wave strain amplitude from GW150914 projected onto H1. This shows the full

Energy losses in GR:

$$\frac{dE}{dt} = -\frac{32}{5r^5}\left(M_1 M_2\right)^2 \left(M_1 + M_2\right)$$

A. Einstein, 1918

Radial velocity:

$$\frac{dr}{dt} = -\frac{64}{5r^3} M_1 M_2 \left(M_1 + M_2\right)$$

Solution: $r \sim \sqrt[4]{t_0 - t}$

Gross-Pitaevskii equation for polaritons

- Many modifications of the GPE available
- Exemple: damped GPE by Pitaevskii (1959) – decay of the second sound

$$i\hbar \frac{\partial \psi}{\partial t} = (1 - i\Lambda)\left(-\frac{\hbar^2}{2m}\Delta\psi + \alpha|\psi|^2\psi - \mu\psi \right)$$

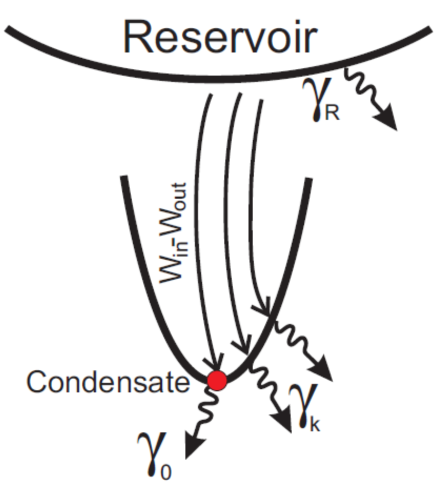
L.P. Pitaevskii, Sov. Phys. JETP 35, 282 (1959).

- Modified GPE for polaritons with k^2-decay

$$i\hbar \frac{\partial \psi}{\partial t} = -(1 - i\Lambda)\frac{\hbar^2}{2m}\Delta\psi + \alpha\left(|\psi|^2 + n_R\right)\psi$$

- Phenomenological description of experiments:
  - Phys. Rev. Lett. 109, 216404 (2012)
  - Phys. Rev. B 88, 035313 (2013)
- k^2 losses as a key feature in experiment:
  - Nature 608, 687 (2022)
- Theoretical justification:
  - PRA 89, 033626 (2014)

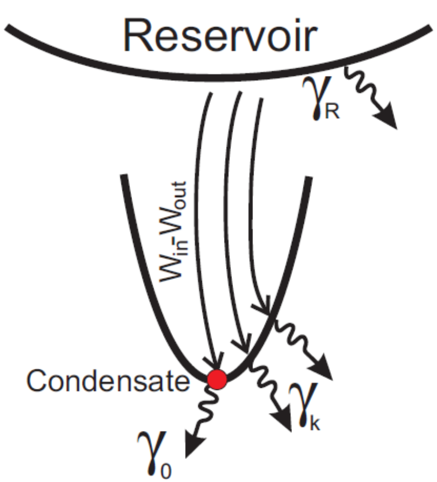
Convergent flow expected for a vortex!



k^2 losses: origins

- Boltzmann factor
- Zero net gain/decay for k=0
- Decay Γ~E for k>0 because

$$\frac{W_{out}}{W_{in}} \sim e^{-\frac{E_R-E}{k_BT}} \sim e^{\frac{E}{k_BT}}$$



- Exciton-photon fractions

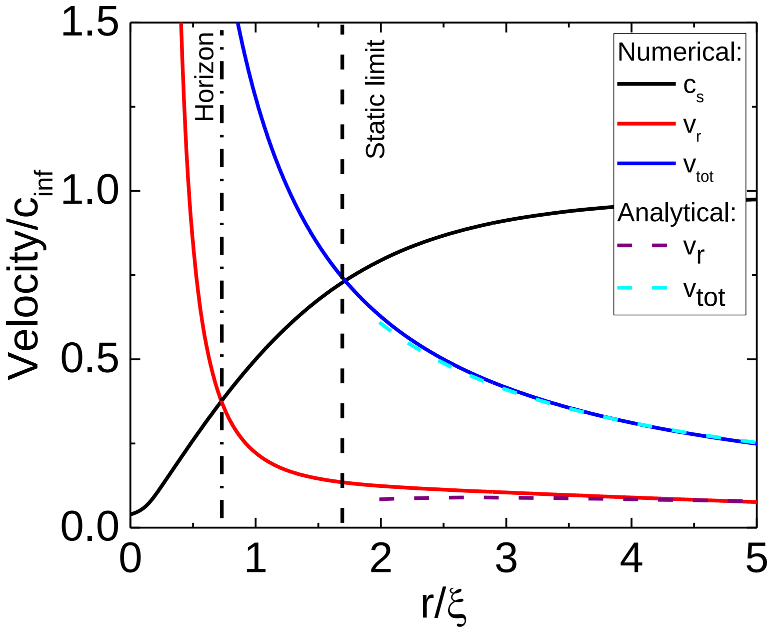
$$\Gamma_k = |x_k|^2 \Gamma_x + \left(1 - |x_k|^2\right)\Gamma_c$$

- Close to zero detuning:

$$\Gamma_k \approx \frac{\Gamma_x + \Gamma_c}{2} + \frac{\hbar k^2}{2m_c\Omega_R}\frac{\Gamma_x - \Gamma_c}{2}$$

Single vortex BH

Numerical solution (1 vortex):



Large distances: $\psi = \sqrt{n}\left(1 - \xi^2 / r^2\right) e^{i\theta}$

$$\Delta\psi = \left(\frac{1}{r}\frac{\partial}{\partial r} r \frac{\partial}{\partial r}\psi + \frac{1}{r^2}\frac{\partial^2 \psi}{\partial \theta^2}\right) \sim \left(\frac{-\xi^2}{r^4} - \frac{1}{r^2}\right)\psi \sim -\frac{1}{r^2}\psi$$

Losses behave as $1/r^2$

We use the particle conservation law (continuity equation):

Losses inside a circle (radius R):

$$\Gamma_{tot}(R) = \int_{\xi}^{R} \frac{1}{r^2} 2\pi r\, dr \sim \ln(R/\xi)$$

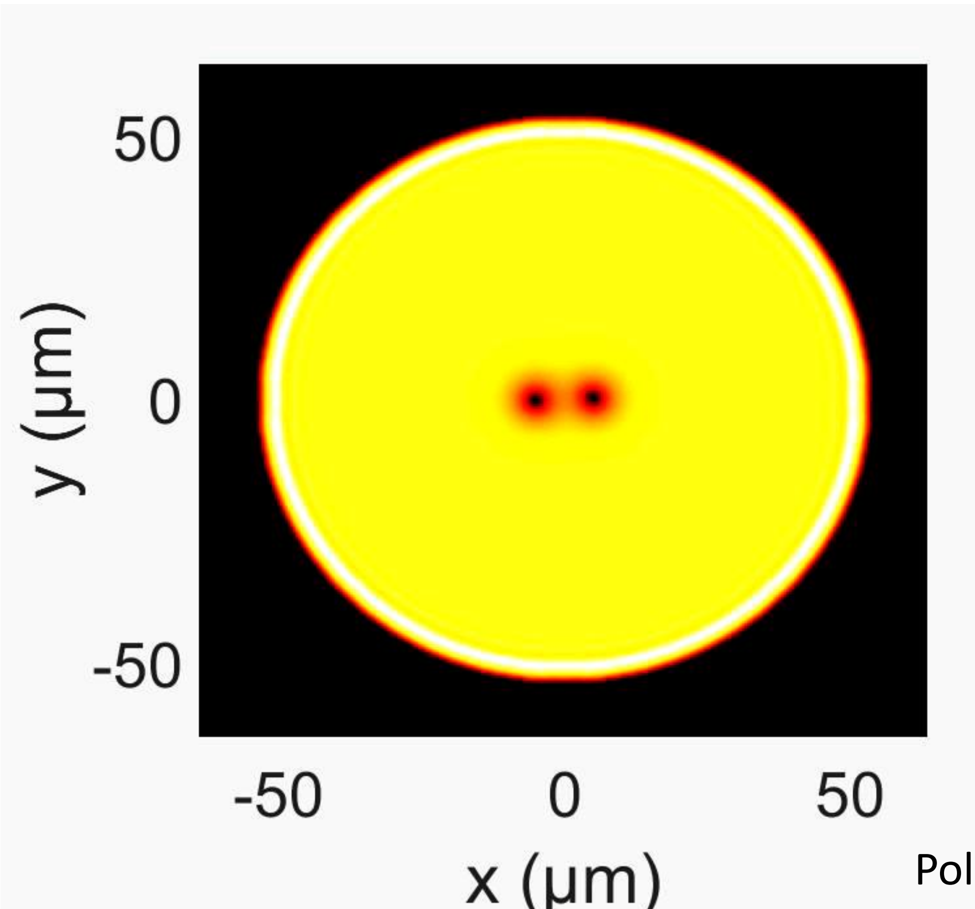
Compensated by inward flow:

$$\Gamma_{tot} = v_r(R) \times 2\pi R$$

Therefore: $$v_r(R) \sim \frac{\ln(R/\xi)}{R}$$

Black Hole merger with GPE

$$i\hbar\frac{\partial\psi}{\partial t}=-(1-i\Lambda)\frac{\hbar^2}{2m}\Delta\psi+(U-i\Gamma)\psi+\alpha|\psi|^2\psi+i\gamma e^{-n_{tot}/n_0}\psi$$



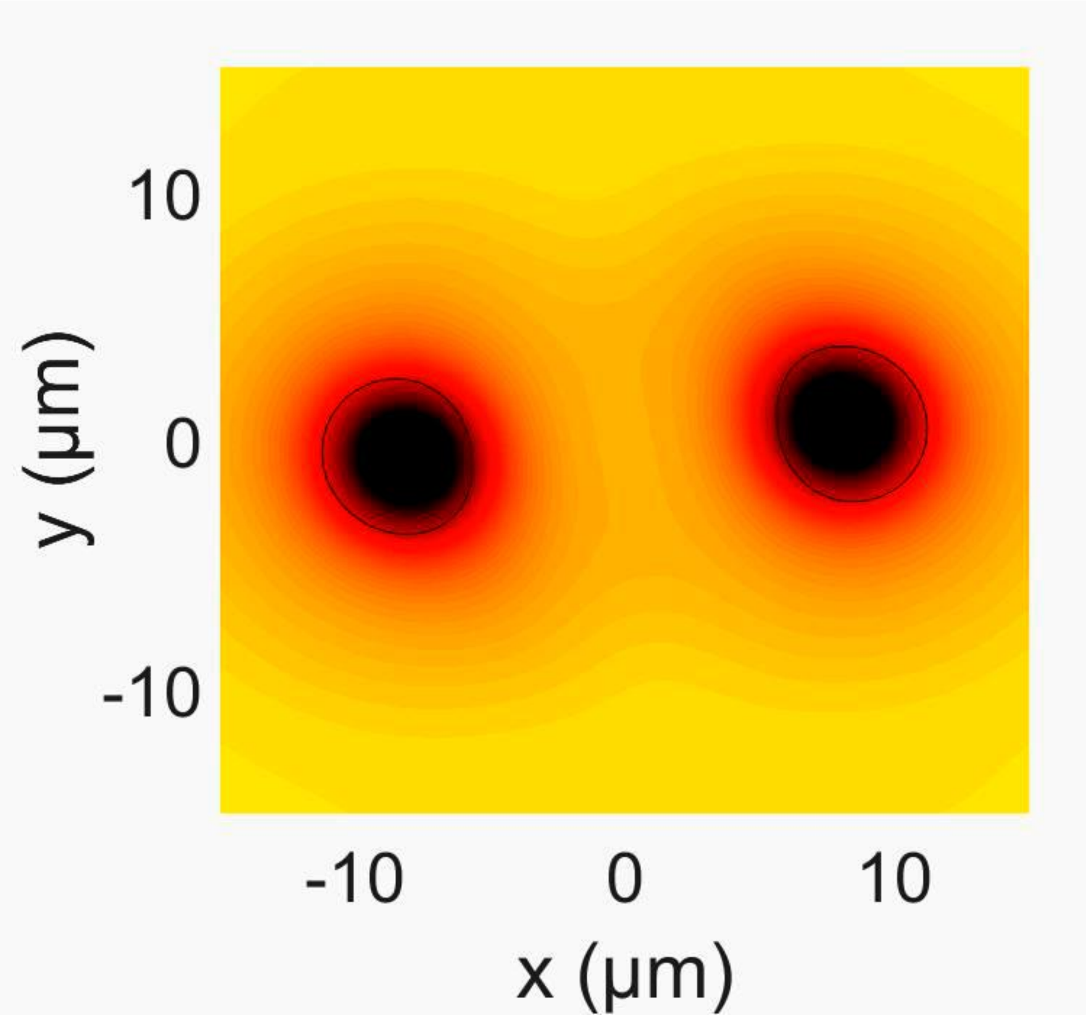
- 1st results: 18/07/2018 on YouTube https://www.youtube.com/watch?v=dF863h5zDBA
- Wave vector-dependent losses Λk^2
- Same-sign vortices
- Attractive interaction
- Emergent gravity
- Dynamical, fully self-consistent metric
- Horizon merging?
- Graviational waves?

Polariton condensates with co-rotating vortices: Sci. Adv. 9, eadd1299 (2023).

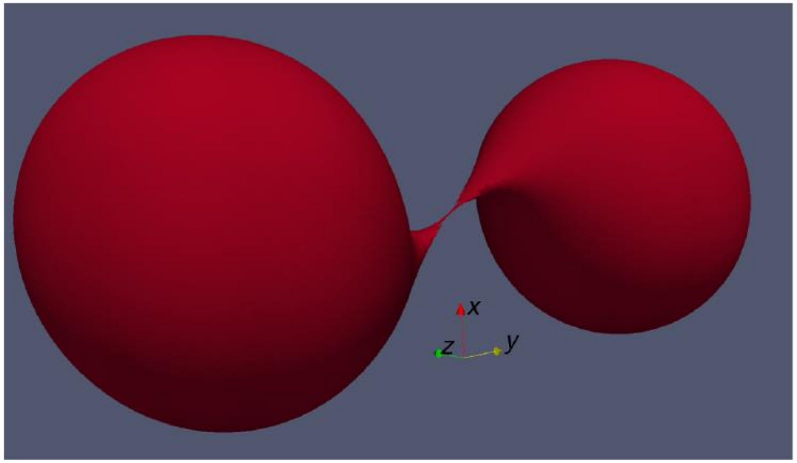
Merger: static limit / horizon

Analogue BH merger:

- Static limit merges
- The horizons do not merge



GR results:



Calculated shape of the horizon at merger

PRD 85, 024031 (2012).

Zero angular momentum case

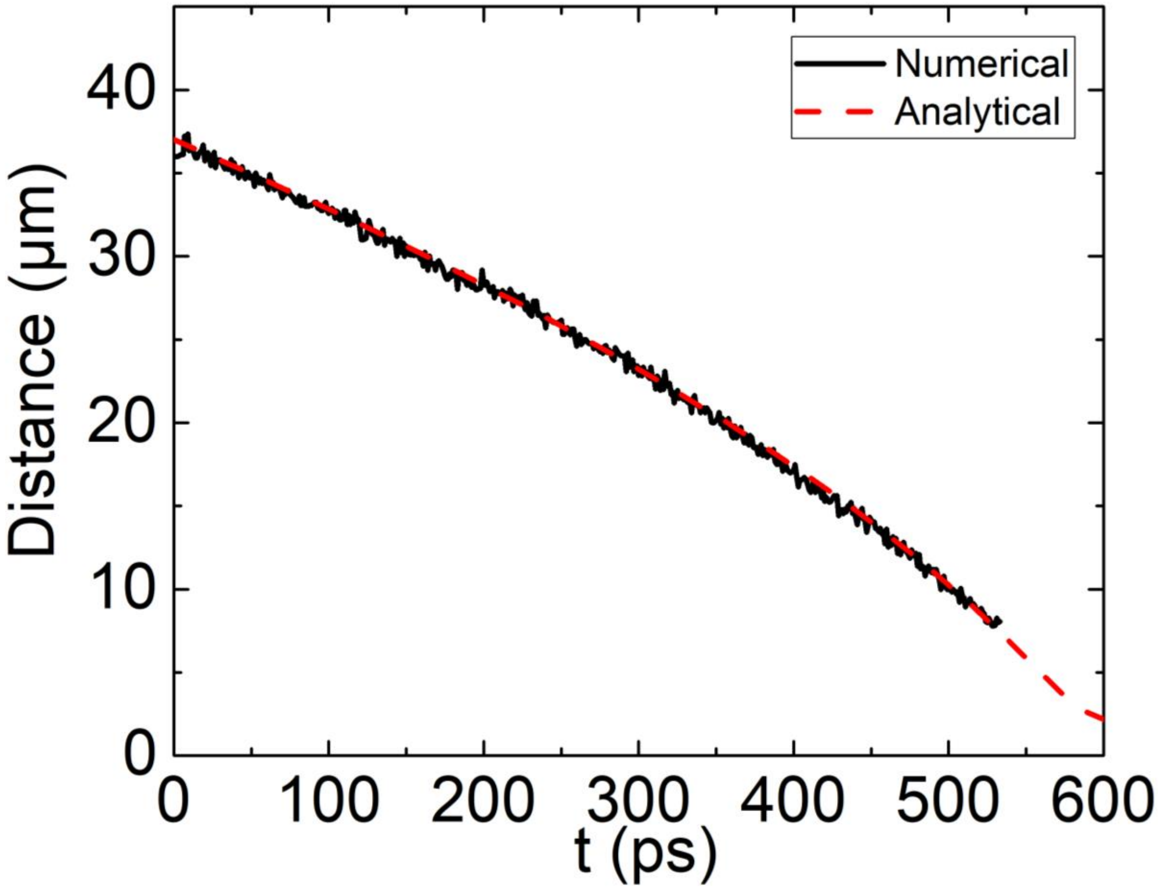
$$v_r(R) \sim \frac{\ln(R/\xi)}{R}$$

Equation of motion for ZAM case:

$$\frac{dr}{dt} = -a\frac{\ln(r/\xi)}{r} \Leftrightarrow \frac{-rdr}{a\ln(r/\xi)} = dt \Leftrightarrow t = -\xi^2 a^{-1}\,\mathrm{Ei}\left(2\ln(r/\xi)\right)$$

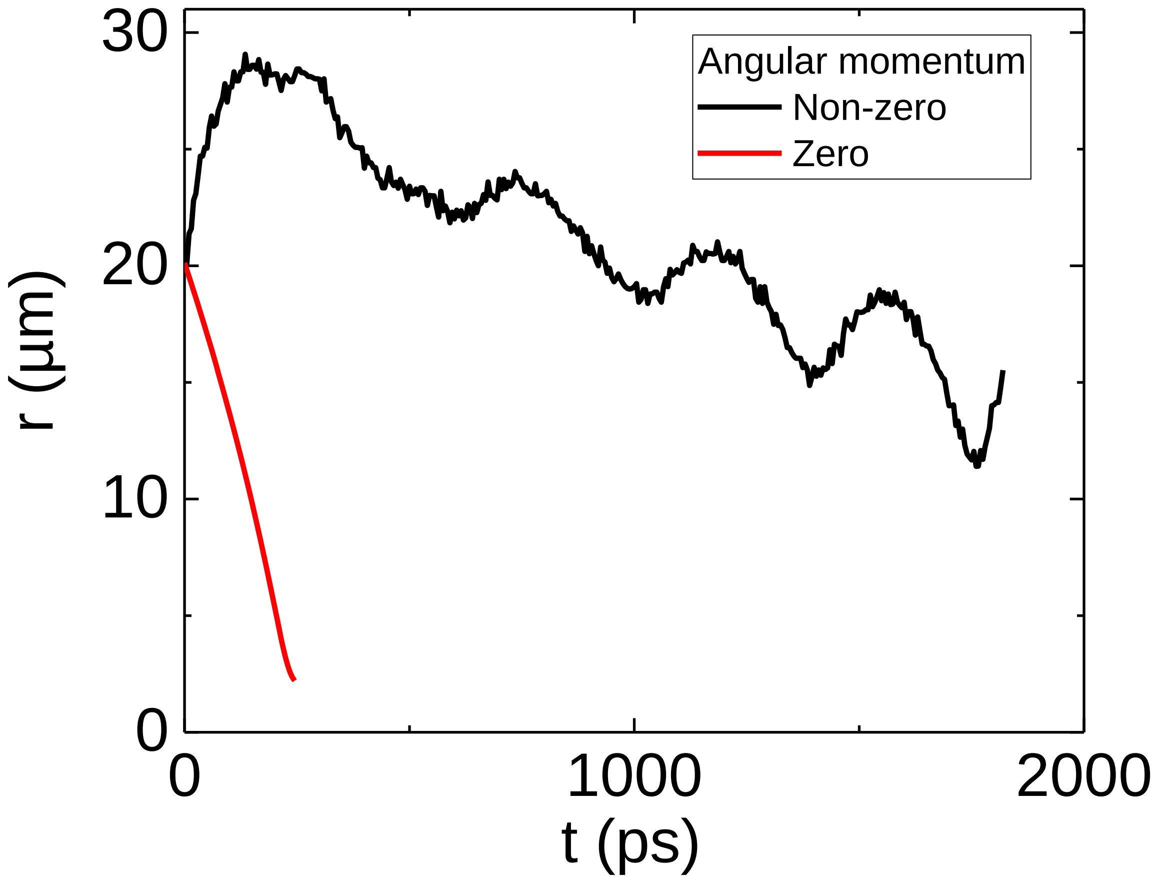
Special function:

$$Ei(z) = -\int_{-z}^{\infty}\frac{e^{-t}}{t}dt$$



- Fits very well
- No merger possible
- Describes two rotating black holes with zero relative rotation (in the center of mass system)
- All rotation is due to frame dragging
- No gravitational waves
- Inspiral due to attraction

Non-zero angular momentum case



- Elliptic trajectory
- Much slower distance decrease
- Energy loss by gravitational waves
- Kepler's law?
- Equation of motion?

Analogue Kepler's laws

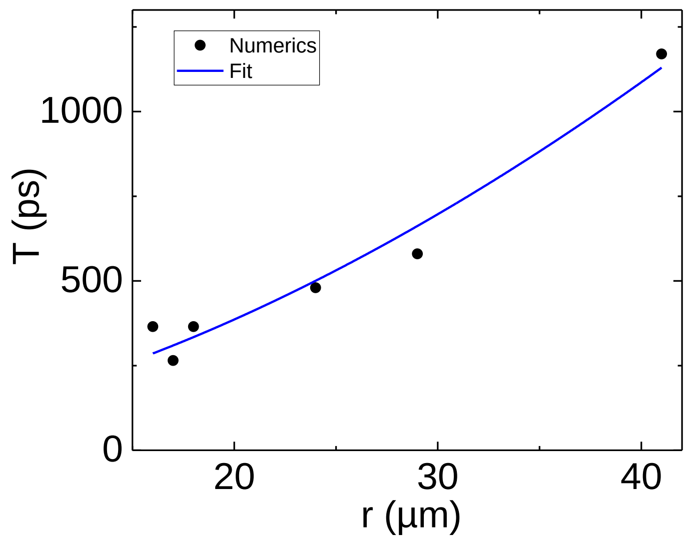
Since log varies slowly, we write $\alpha = \log \frac{r}{\xi}$ - approx. constant for 1 period

From the radial velocity we can extract acceleration and force:

$F(r) \sim \frac{\alpha^2}{r^3}$ which gives a potential energy $U \sim -\frac{\alpha^2}{r^2}$

From the force and centripetal acceleration we can get the rotation period:

$T \sim \frac{r^2}{\alpha} = \frac{r^2}{\log r/\xi}$ to be compared with $T \sim r^{3/2}$ (Kepler)



Rotation period as a function of orbit radius

Dependence weaker than r^2

Wave emission and energy loss

- No dipole emission thanks to momentum conservation

$$\left(\sum m r_i\right)'' = \left(\sum m v_i\right)' = 0$$

- Quadrupolar emission:

$$\frac{dE}{dt} \sim -\dddot{Q}^2 \qquad Q_{xx} = m r^2 \left(3\cos^2\theta - 1\right)$$

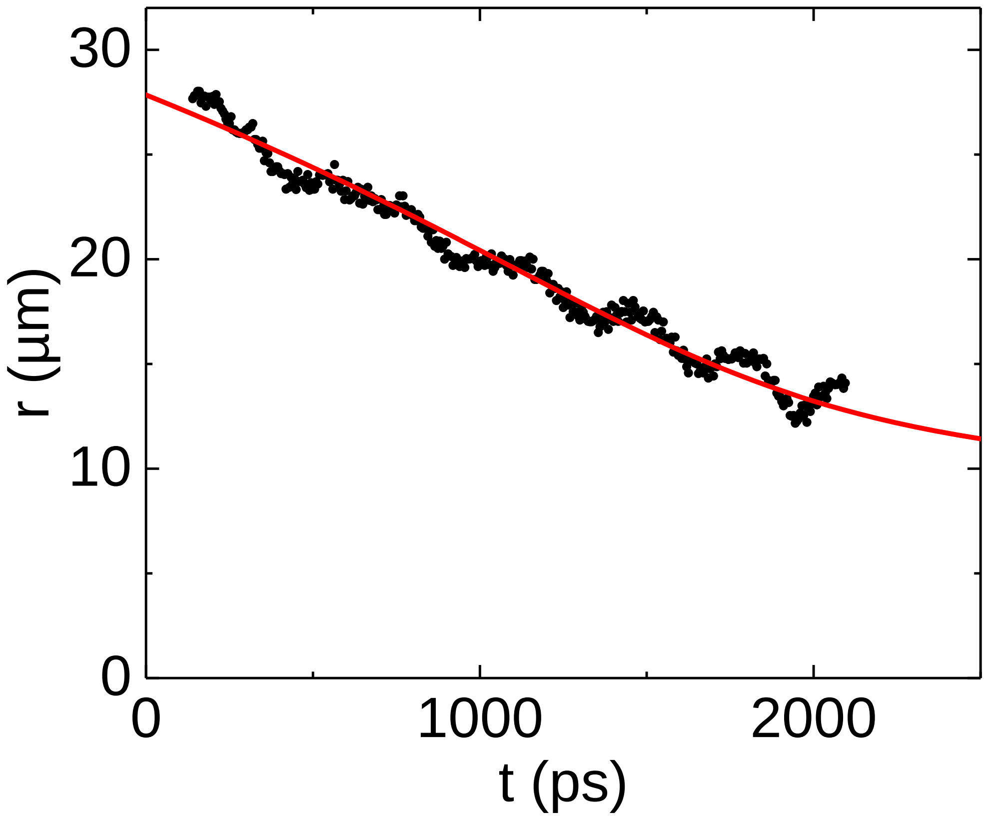
$$\dddot{Q}_{xx} = 24 m r^2 \omega^3 \cos\omega t \sin\omega t \sim r^2 \omega^3$$

$$\frac{dE}{dt} \sim -r^4 \omega^6 \sim -\frac{\alpha^6}{r^8}$$ thanks to the modified Kepler's law: $$T \sim \frac{r^2}{\alpha} = \frac{r^2}{\log r/\xi}$$

Using $U \sim -\frac{\alpha^2}{r^2}$ we get the equation of motion $$\frac{r^5}{\log^5 r/\xi}\, dr \sim dt$$

Intervortex distance (non-zero angular momentum)

$$t = t_0 - A\left(54\xi^6\,\mathrm{Ei}\left(6\log\frac{r}{\xi}\right) - r^6\left(\frac{1}{4\log^4\frac{r}{\xi}} + \frac{1}{2\log^3\frac{r}{\xi}} + \frac{3}{2\log^2\frac{r}{\xi}} + \frac{9}{\log\frac{r}{\xi}}\right)\right)$$



If the logarithmic variation is neglected (large distance):

$$r \sim \sqrt[6]{t_0 - t}$$

Compared with GR:

$$r \sim \sqrt[4]{t_0 - t}$$

Possible experimental obstacles

Non-resonant pumping (continuous)

- Repetition? (probe?)
- Vortex creation during the condensate formation? (long non-resonant pulses)

Quasi-resonant pumping (pulsed)

- Long rotation periods incompatible with high Λk^2 decay
- Radial dynamics due to initial density distribution

Minimal value of Λ=0.03 corresponds to 30 μeV extra broadening at 1 meV.

Conclusions

- Natural vortex attraction thanks to k^2 losses
- Fully dynamical analogue metric (emergent gravity)
- Analogue BH merger (incomplete)
- Solutions for zero and non-zero angular momentum
- Outlook: other power laws for decay (k^1, k^3, k^4…)