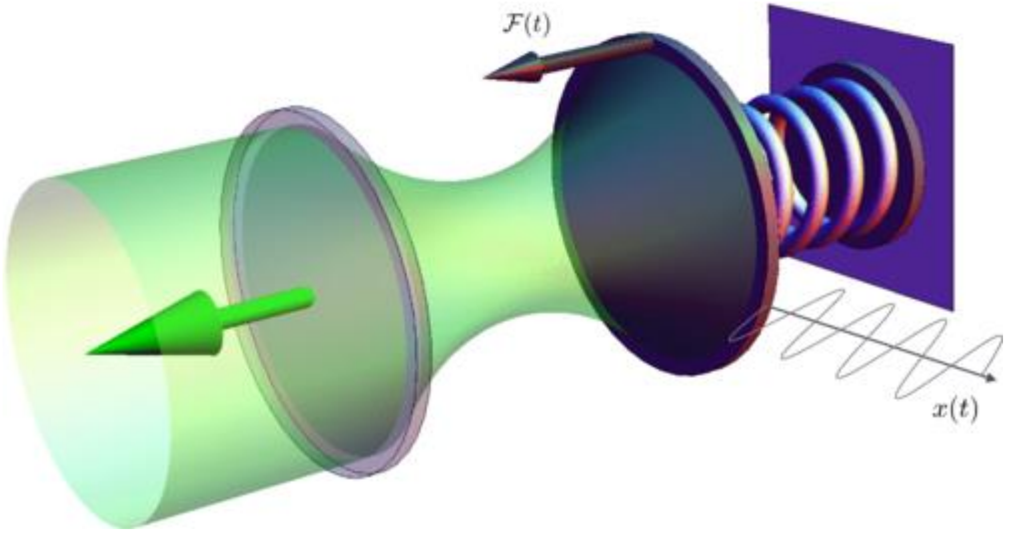
Non-separability of phonon pairs in a time modulated Bose-Einstein Condensate

Acoustic Analog of the Dynamical Casimir Effect

Victor Gondret, Charlie Leprince, Quentin Marolleau, Clothilde Lamirault, Denis Boiron & Chris Westbrook

Presentation at the GDR COPHY Transverse Task Force on Analog Gravity
Slides available at https://indico.ijclab.in2p3.fr/event/9888/

The Dynamical Casimir Effect (DCE)



Macrì, V. *et al*. PRX (2018).

Dynamical Casimir effect : fast oscillation of the mirror creates photons out of vacuum.

$$N_{photons} \sim \omega\tau \left(\frac{v}{c}\right)^2 F$$

Lambrecht *et al* PRL (1996)

Experimental plateforms to observe DCE

Move the mirror

Change the refraction index

Change the sound speed (in BEC)

Outline of the talk

① Detection of phonons in a Bose-Einstein Condensate

② Experimental procedure and phonon creation

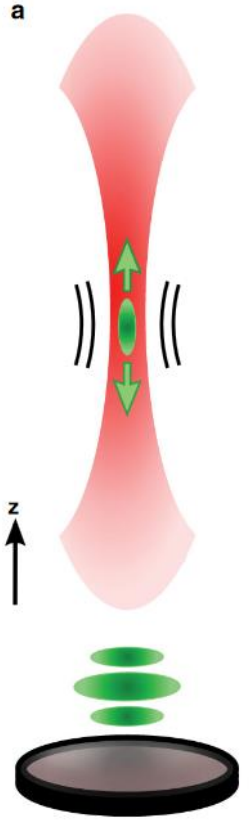
③ Correlation analysis of the phonon

$x(t) = x_0(1 + \epsilon \sin \omega t)$

$n(t) = n_0(1 + \epsilon \sin \omega t)$

Creation of pairs of phonon with frequency

$\omega_1 + \omega_2 = \omega$



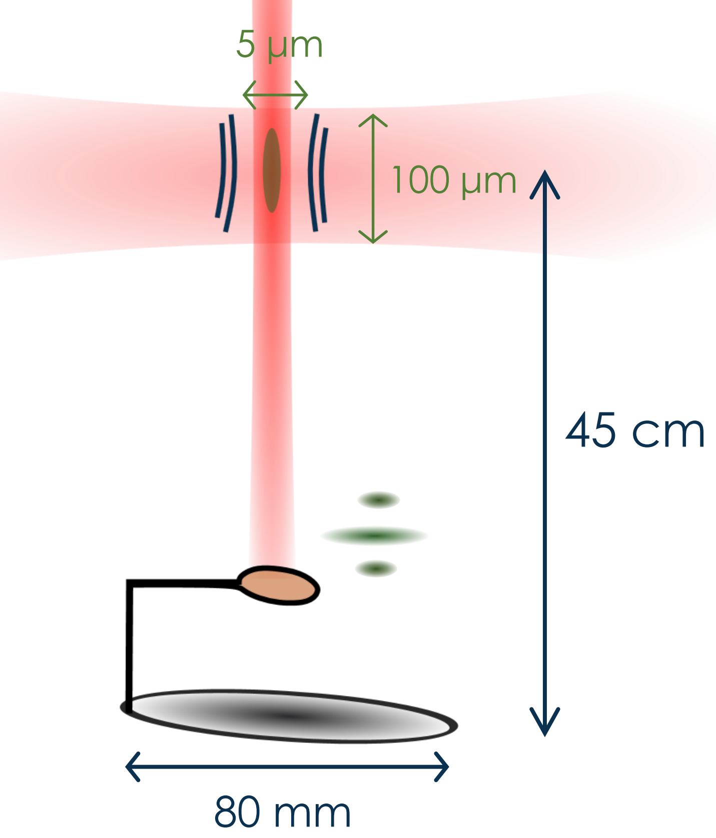
$c_s(t) = c_{s,0}(1 + \epsilon \sin \omega t)$

Wilson *et al.* Nature (2011).
Lähteenmäki *et al.* PNAS (2013)
Vezzoli, S. *et al.* Com Phys (2019).

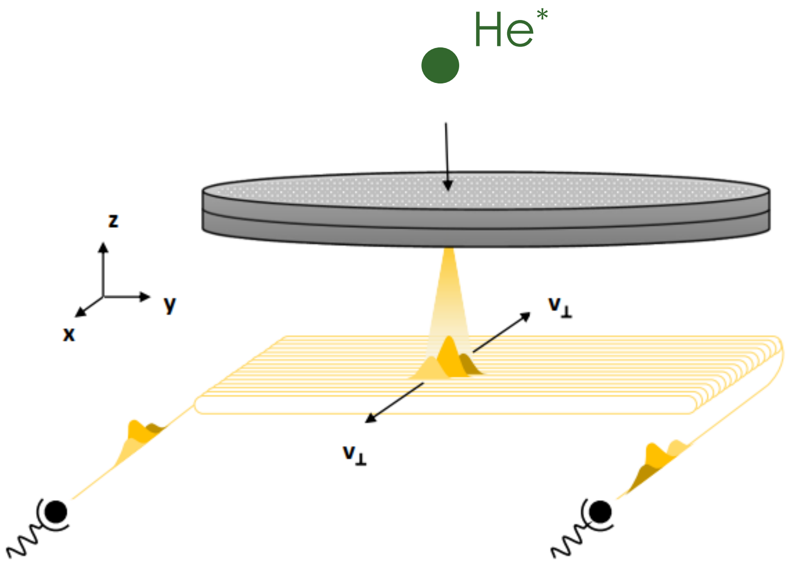
Jaskula *et al.* PRL (2012).

Detecting single atoms

Cigar shape BEC (10s)
- $\omega_{\parallel} = 10$ Hz
- $\omega_{\perp} = 1{,}6$ kHz



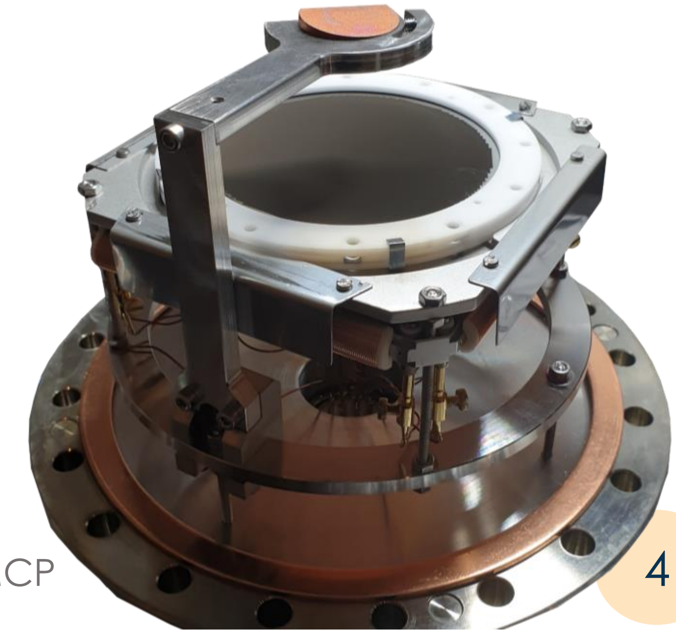
Metastable (20 eV , 2h) helium atom



A metastable helium atom hitting the detector triggers an electronic fountains :

→ Measure : X, Y, T.

80 µm *© Hamamatsu Photonics*



Picture of the MCP

Detecting single atoms

Cigar shape BEC (10s)

- $\omega_{\parallel} = 10$ Hz
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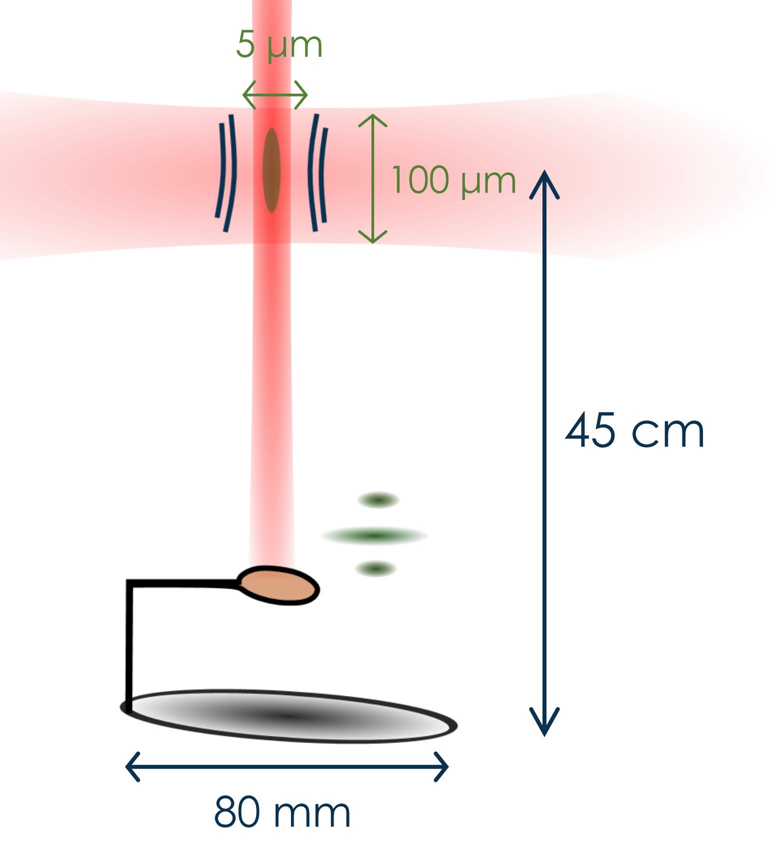
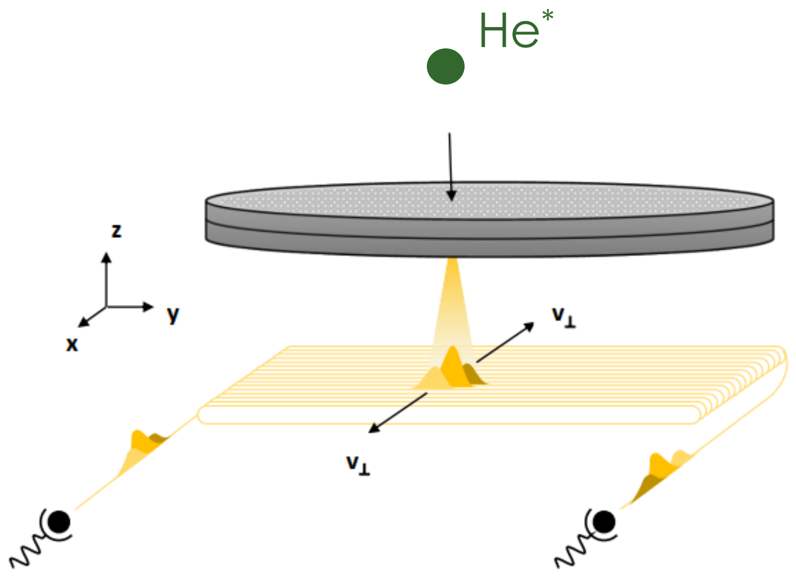
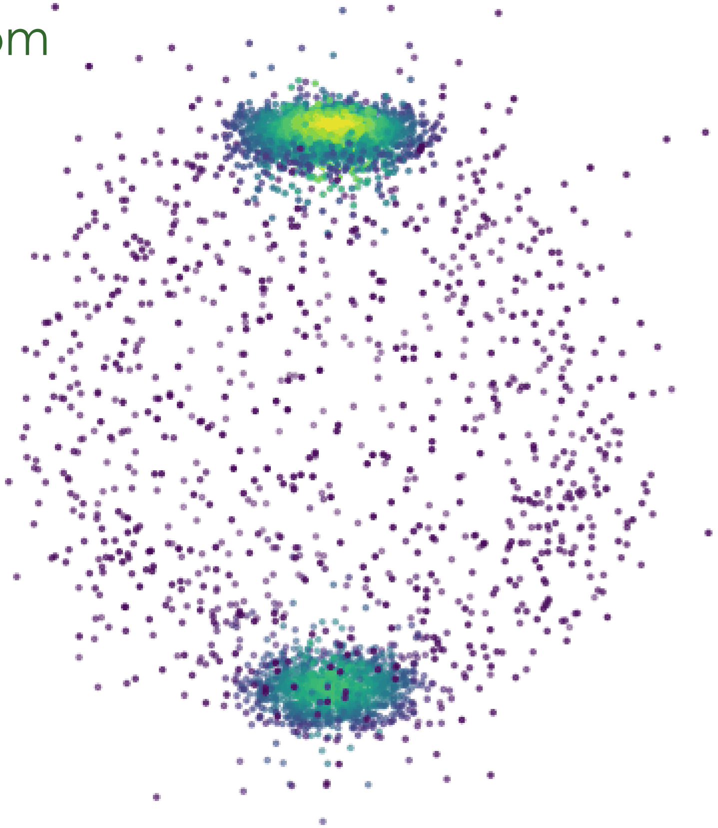


Metastable (20 eV , 2h) helium atom





3D reconstruction of the momentum of individual atoms (η = 10, 25, 50 % ?)

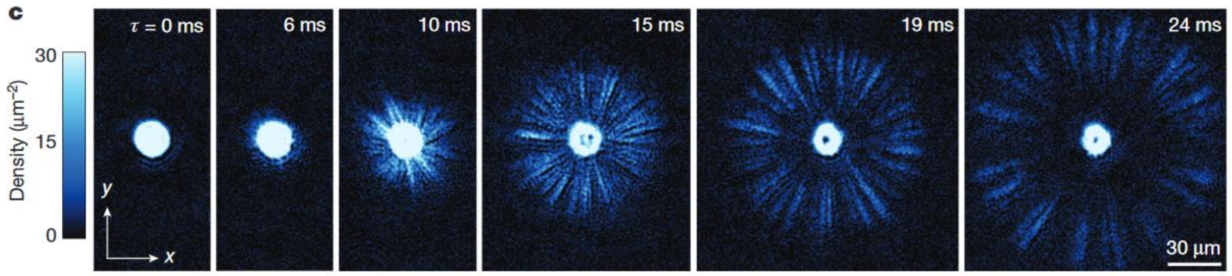
Saturation

How to create collective excitations ?

How to change the sound speed ? $c_s = \sqrt{g_1 n_1 / m} = \left(\frac{2\, a_s N}{L\, \sigma^2} \right)^{1/2}$

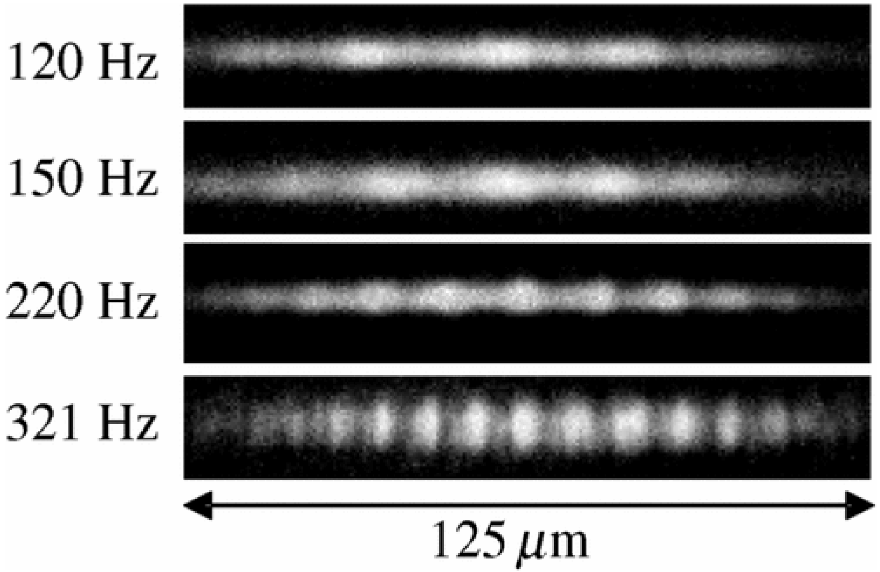
- a_s atomic scattering length
- $L\,,\sigma$ length and width of the BEC

Change the scattering length
Chicago, Heidelberg



Bose fireworks

Change the BEC width
Trento, Palaiseau



In situ imaging of Faraday waves

Clark *et al*. Nature (2017)

Engels *et al*. PRL (2007)

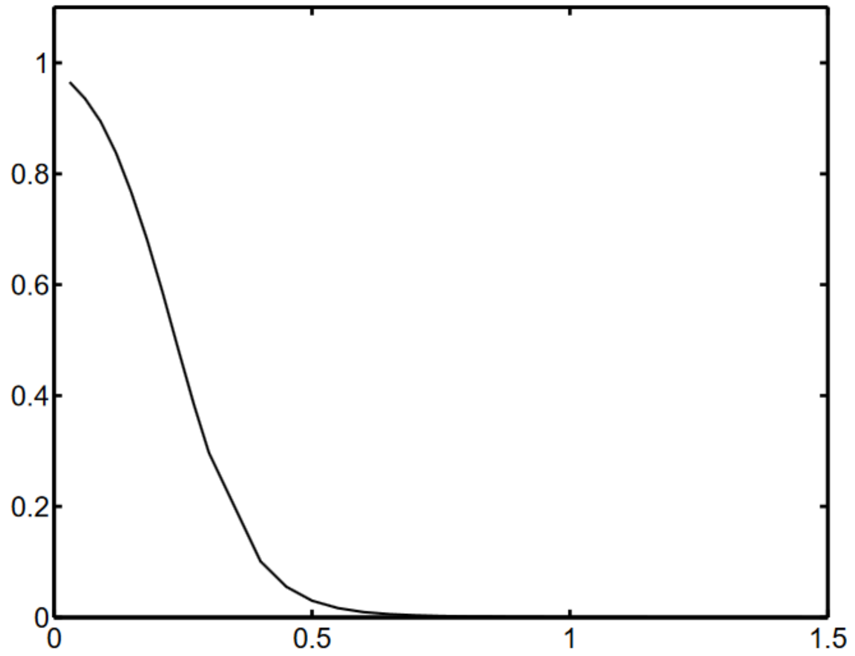
How to measure collective excitations ?

How to measure a_k ?

How collective excitation evolves when one release the BEC ?

Particles $$\begin{pmatrix} \hat{\phi}_k \\ \hat{\phi}^\dagger_{-k} \end{pmatrix} = \begin{pmatrix} u_k & v_k \\ v_k & u_k \end{pmatrix} \begin{pmatrix} \hat{a}_k \\ \hat{a}^\dagger_{-k} \end{pmatrix}$$ Quasi-particles

Ratio $\left.\frac{v_k}{u_k}\right|_{t\to\infty}$



Momentum k [a_ρ]

For high enough momentum, the collective excitation is mapped to a single atom.

(For us : k x a_ρ ~ 0.8)

Tozzo & Dalfovo, PRA (2004)

Outline of the talk

Outline of the talk

① Parametric creation of phonon in a cigar shape Bose-Einstein condensate

② Creation of quasi-particles in a Bose-Einstein condensate of He^*

③ Correlation analysis of the phonon

Summary of ①

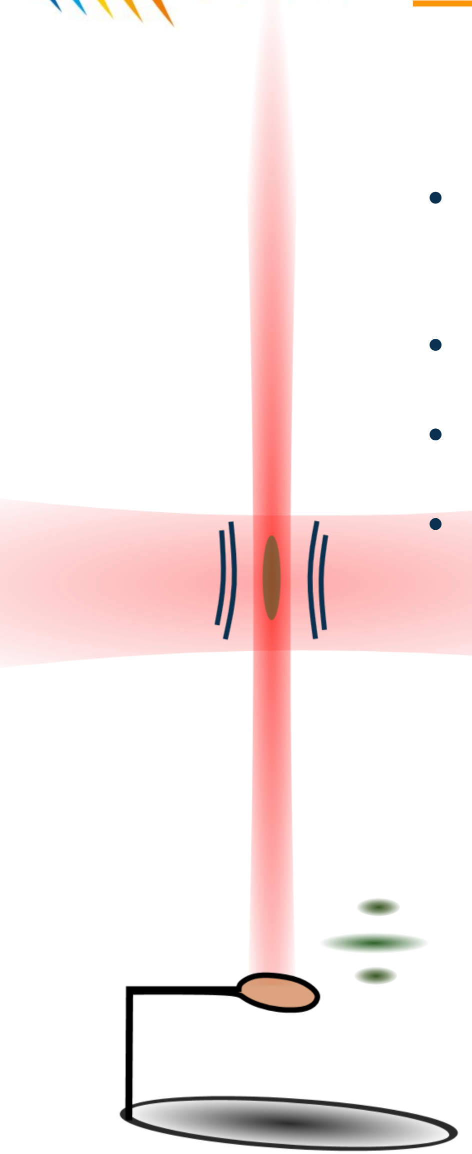
- Detecting the momentum of individual atoms
- Create entangled pairs of phonons when varies density
- Phonons adiabatically transform to atoms when release the trap

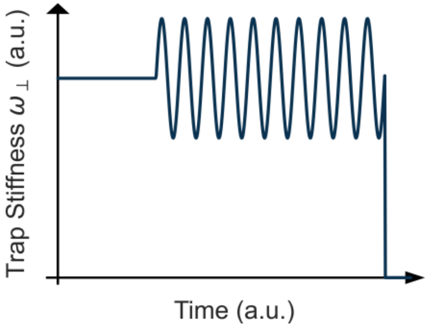
Let's do that !

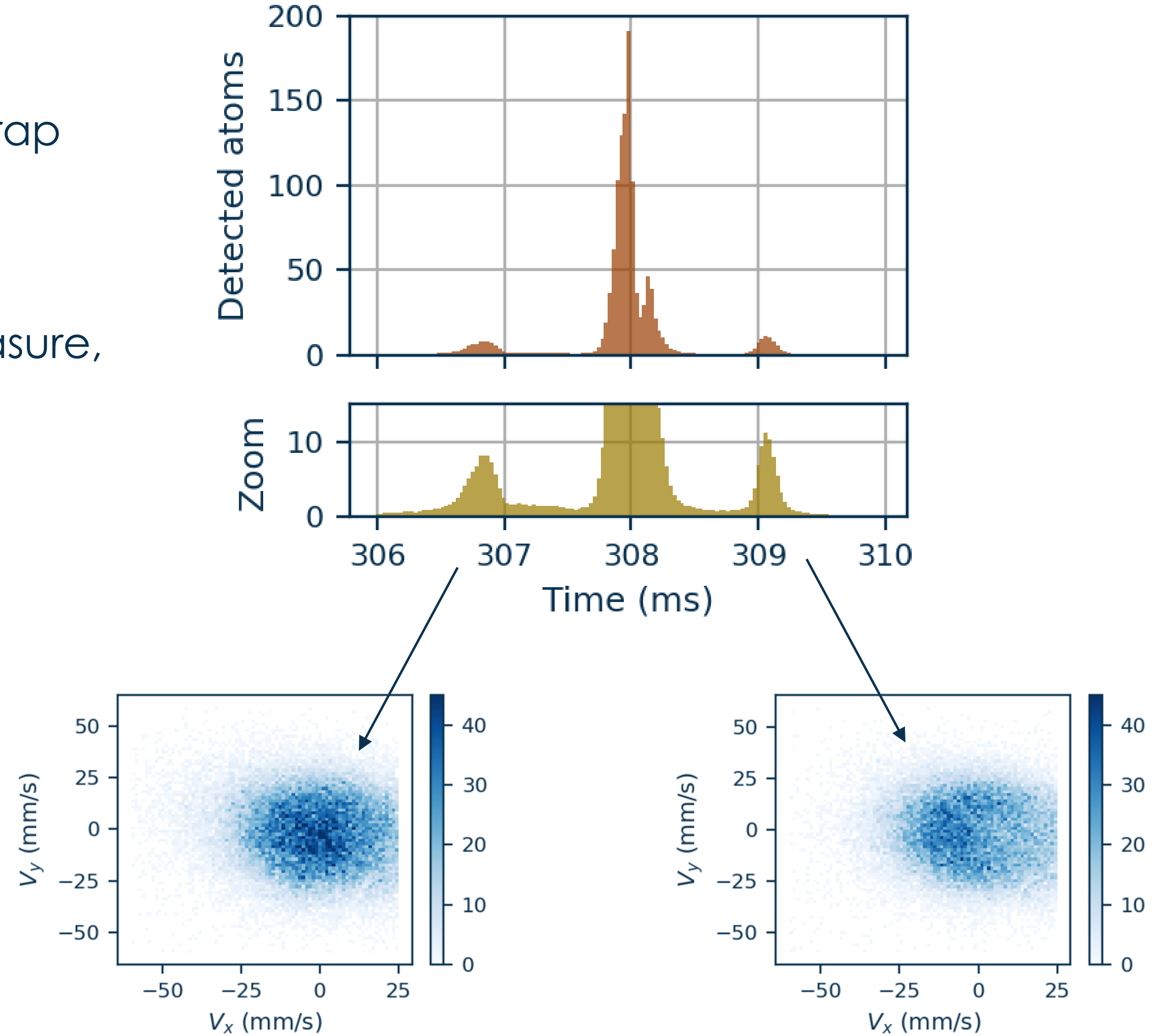
Experimental procedure

Recipe

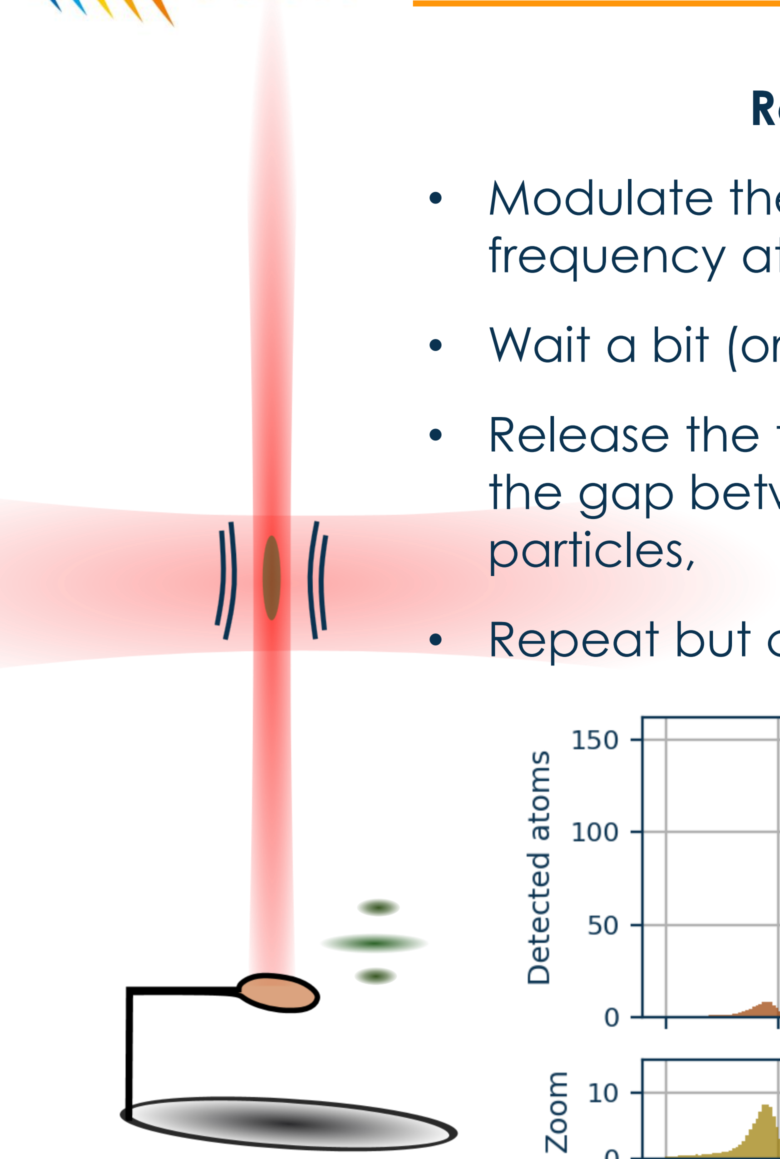
- Modulate the transverse trap frequency at ω,
- Wait a bit (or not),
- Release the trap and measure,
- Repeat





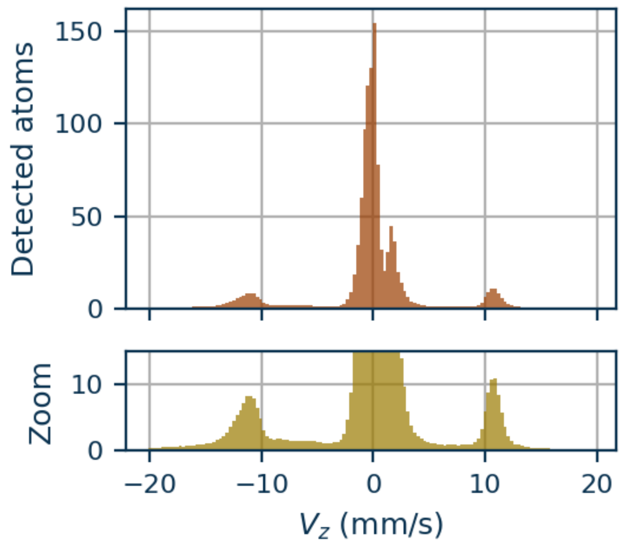


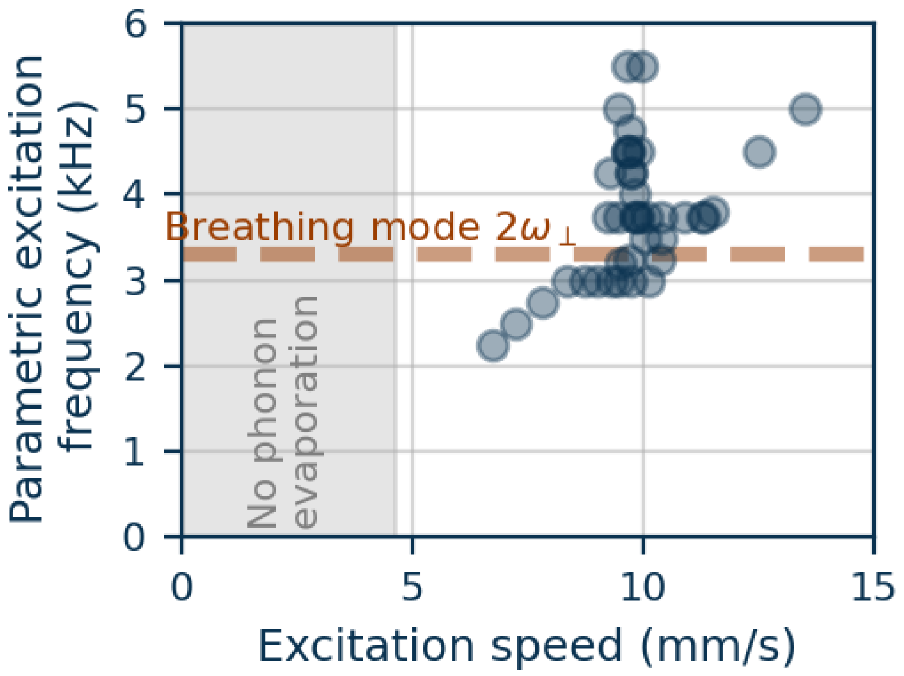
Measuring the dispersion relation



Recipe

- Modulate the transverse trap frequency at ω,
- Wait a bit (or not),
- Release the trap and measure the gap between the quasi-particles,
- Repeat but change ω

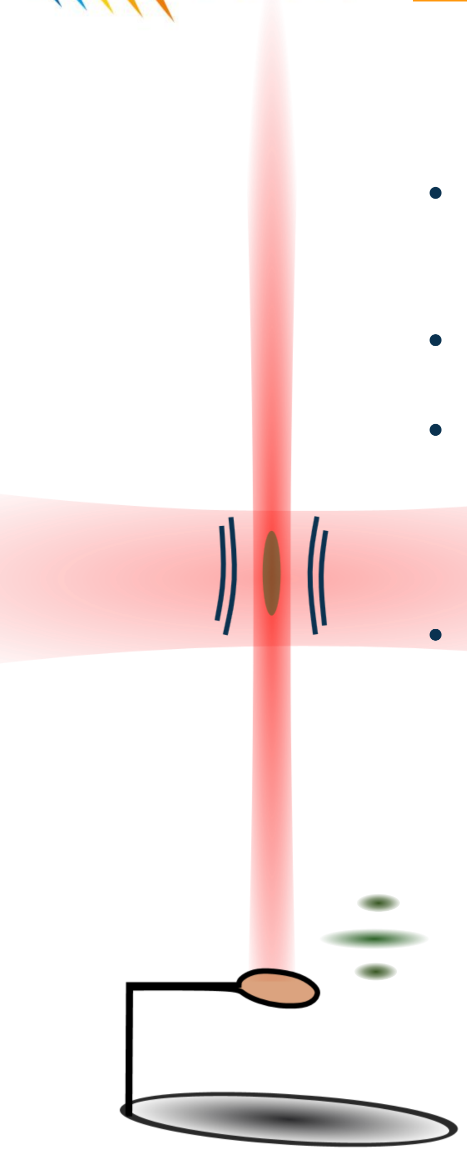




Parametric excitation near the breathing mode excites the BEC at resonance.

Tozzo & Dalfovo, PRA (2004)

Measuring the dispersion relation



Recipe

- Modulate the transverse trap frequency at ω,
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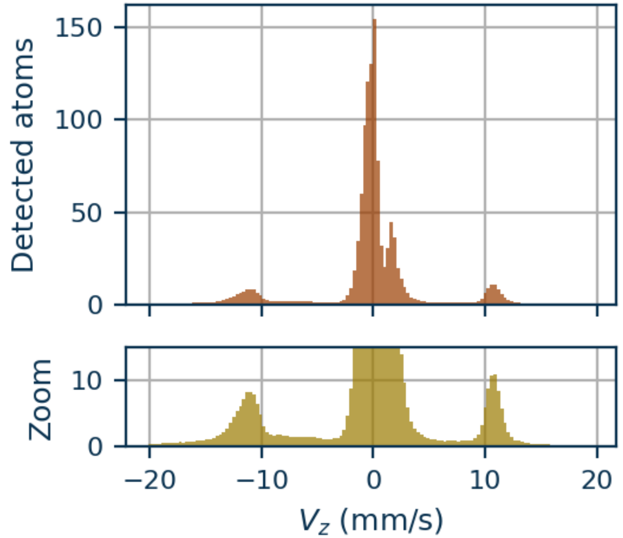
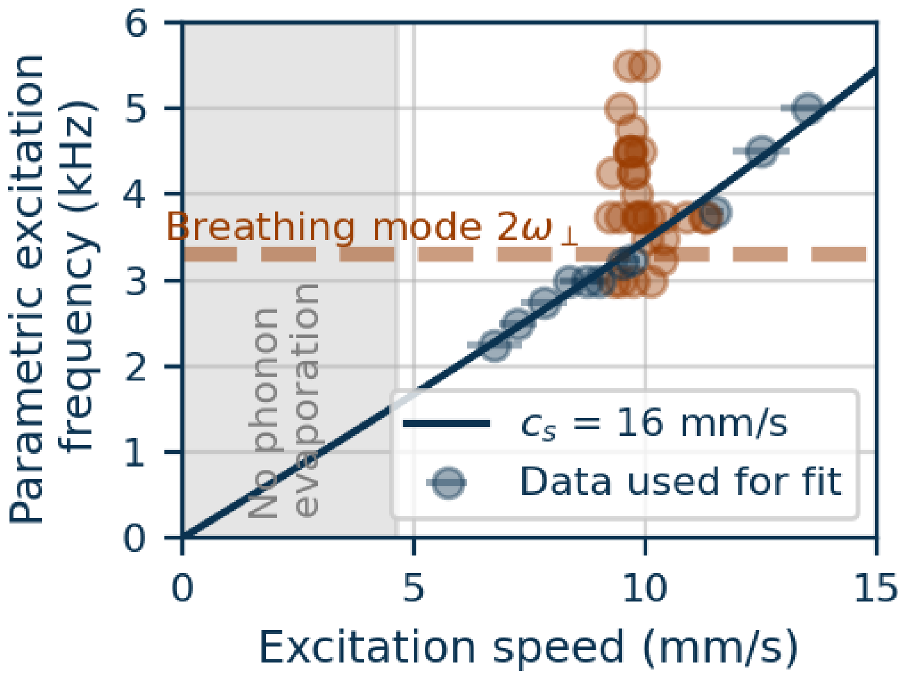
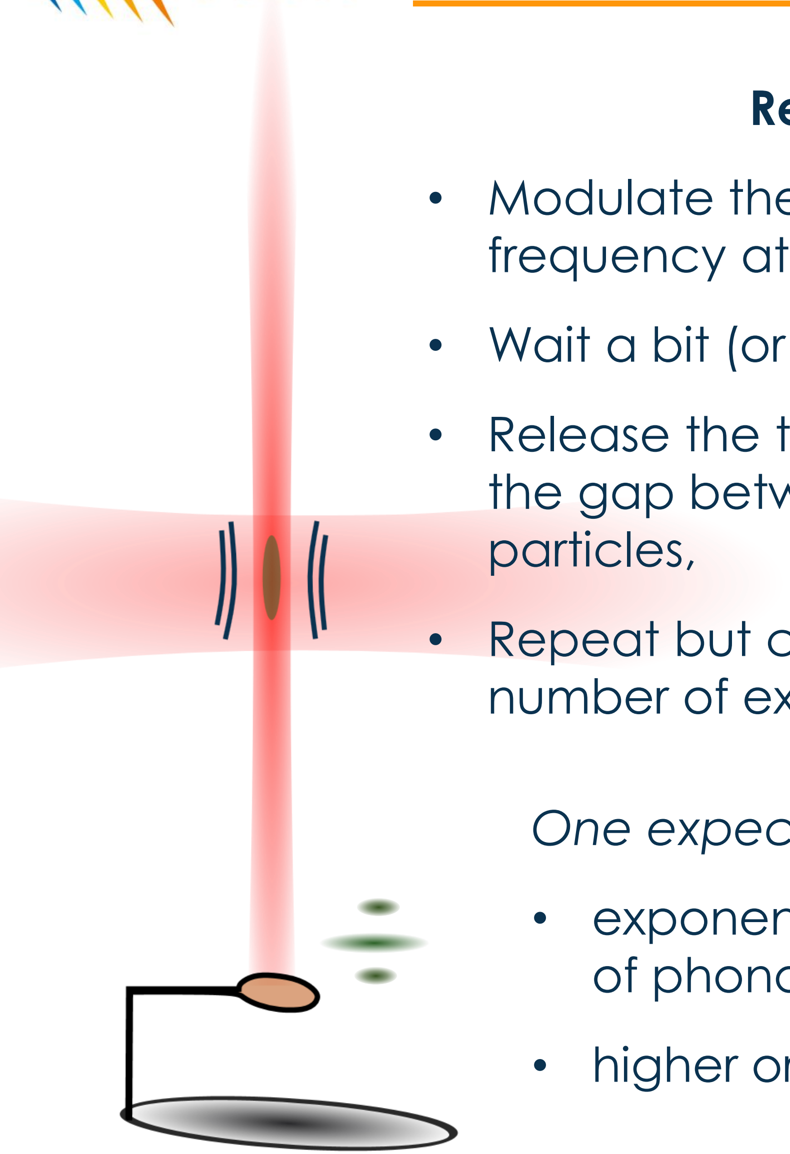
- Repeat but change ω





- Parametric excitation near the breathing mode excites the BEC at resonance.
- Excitations on the phonon branch

Measuring the production rate of phonon

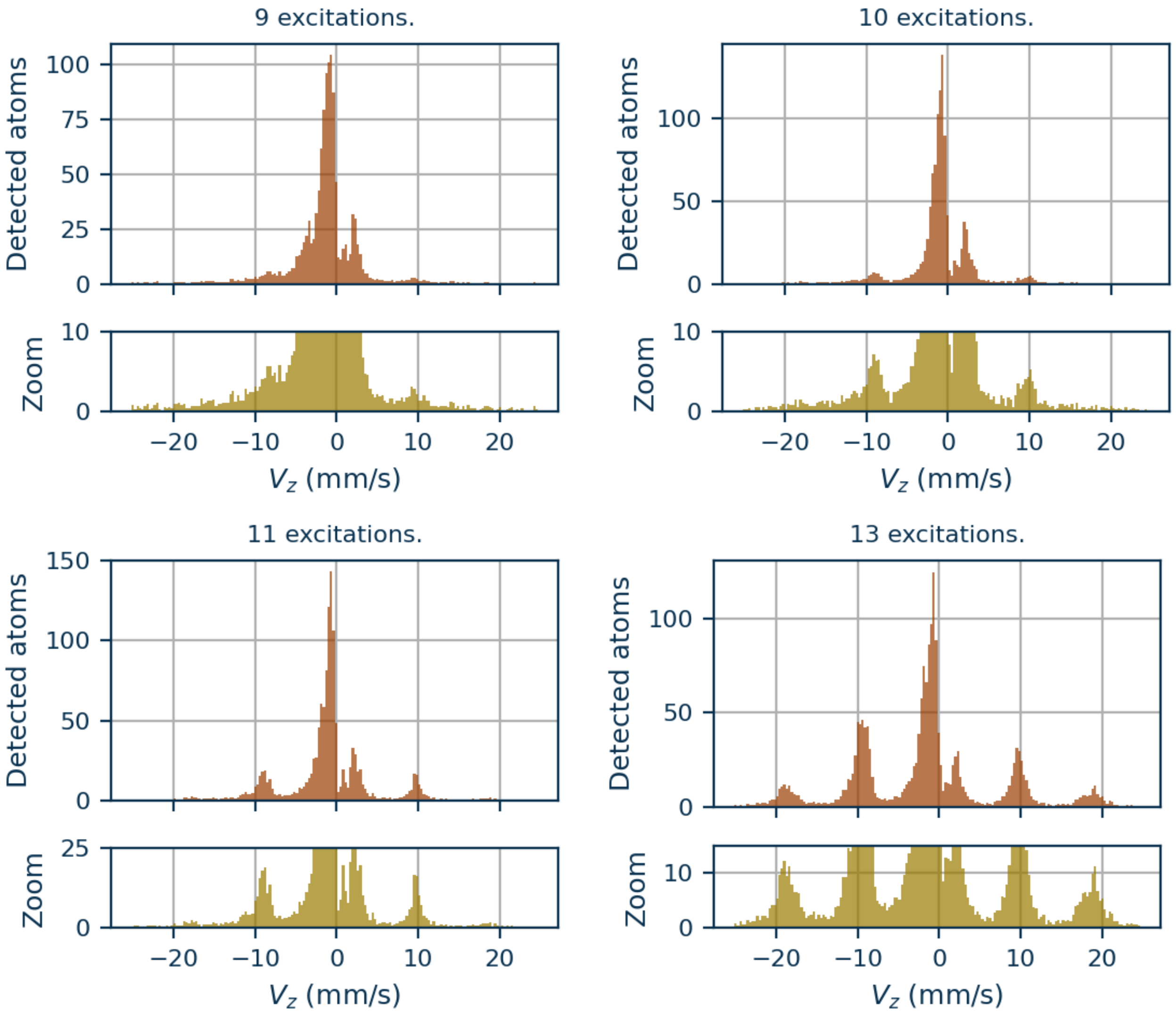


Recipe

- Modulate the transverse trap frequency at ω,
- Wait a bit (or not),
- Release the trap and measur the gap between the quasi-particles,
- Repeat but change the number of excitations

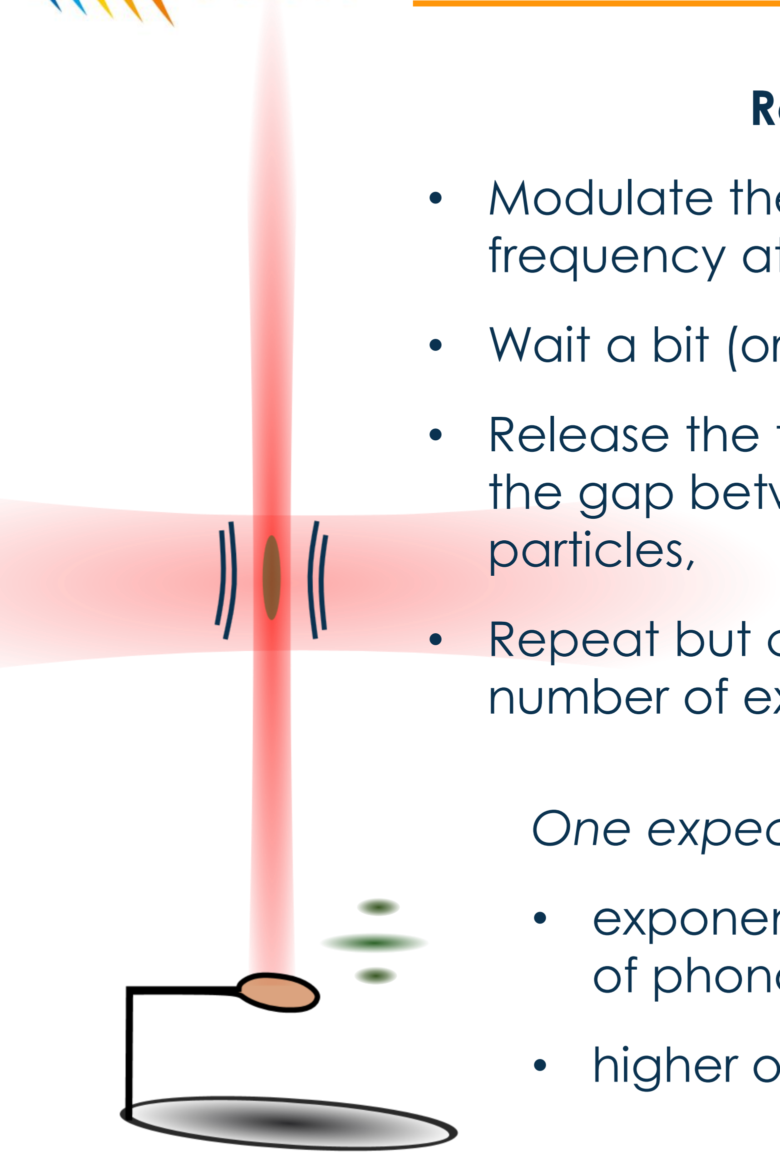
One expects[(1)] :

- exponential creation of phonons,
- higher orders phonons



(1) Robertson *et al*. PRD (2017)

Measuring the production rate of phonon



Recipe

- Modulate the transverse trap frequency at ω,
- Wait a bit (or not),
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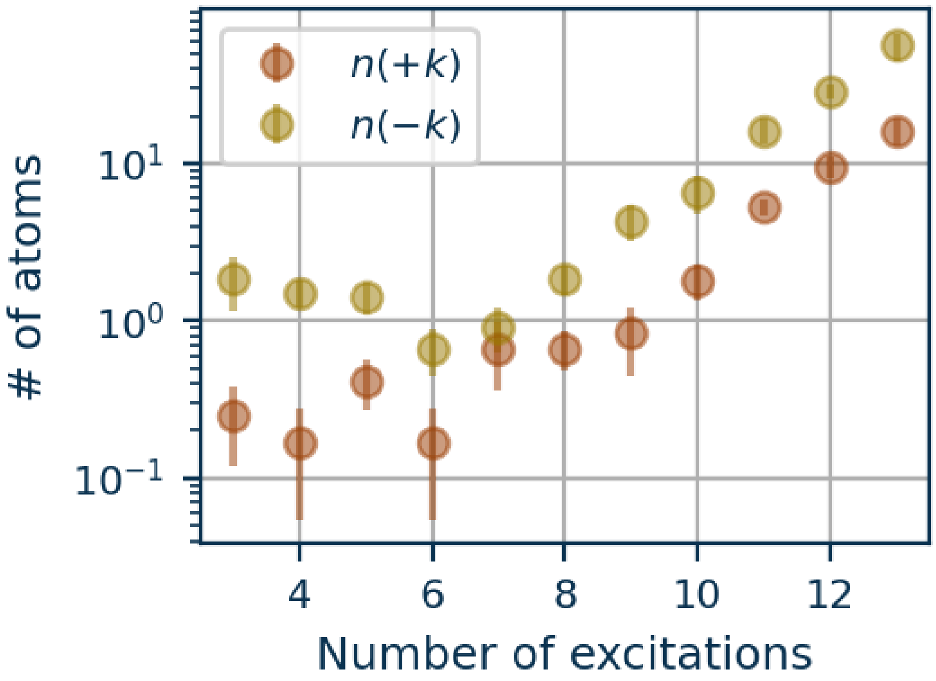


Add to the to do list of the task force

- ➢ Check the decay of the production rate[2]

(1) Robertson *et al*. PRD (2017)
(2) Micheli & Robertson, PRB (2022)

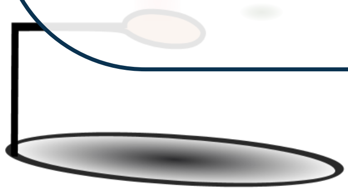
Measuring the production rate of phonon

Recipe

- Modulate the transverse trap frequency at ...
- Wait a bit (or not),
- Release the trap and measure the ... between the quasi-particles,
- Repeat ... number of excitations

One expects(1) :

- exponential creation of phonons,
- higher orders phonons



Outline of the talk

① Detection of phonons in a Bose-Einstein Condensate

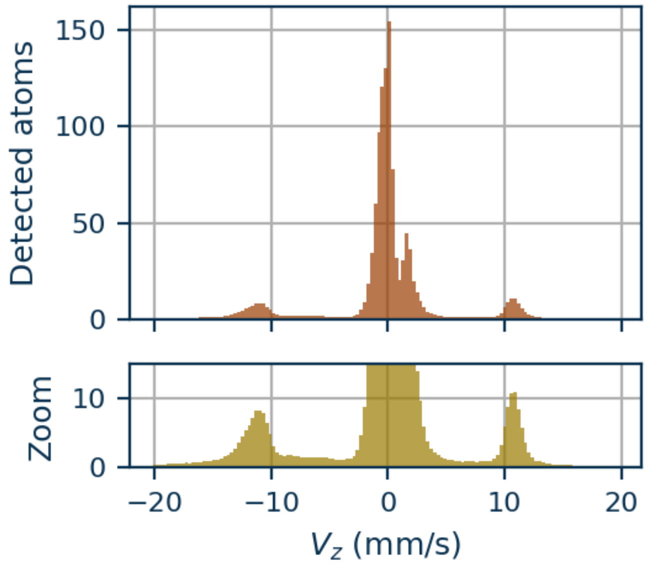
② Experimental procedure and phonon creation

③ Correlation analysis of the phonon

Summary of ②

- Creation and detection of quasi-particles in a BEC
- Population grows exponentially with excitation parameters
- Quasi-particles are phonon, in a BEC with c_s = 16 mm/s

Two mode squeezed state



Experimental parameters *(2012)*
12 oscillations, 4.5 ms *(25ms)*
ω = 2.8 kHz *(2.2 kHz)*
A = 7% *(10%)*
c_s= 16 mm/s *(13 mm/s)*
T = 90 nK *(200 nK)*

We expect to create :

$$|TMS\rangle = \frac{1}{\cosh r}\sum_n \left(e^{-i\phi}\tanh r\right)^n |n\rangle_k \otimes |n\rangle_{-k}$$

Second order correlation function

$$g^{(2)}(q,k) \stackrel{\text{def}}{=} \frac{\langle : \hat{n}_q \hat{n}_k : \rangle}{\langle \hat{n}_q \rangle \langle \hat{n}_k \rangle}$$

$$g^{(2)}(k,k) = 2$$

1. Bosonic bunching

$$\langle n \rangle = \sinh r$$

$$P(n) = \frac{\tanh^n r}{\cosh r}$$

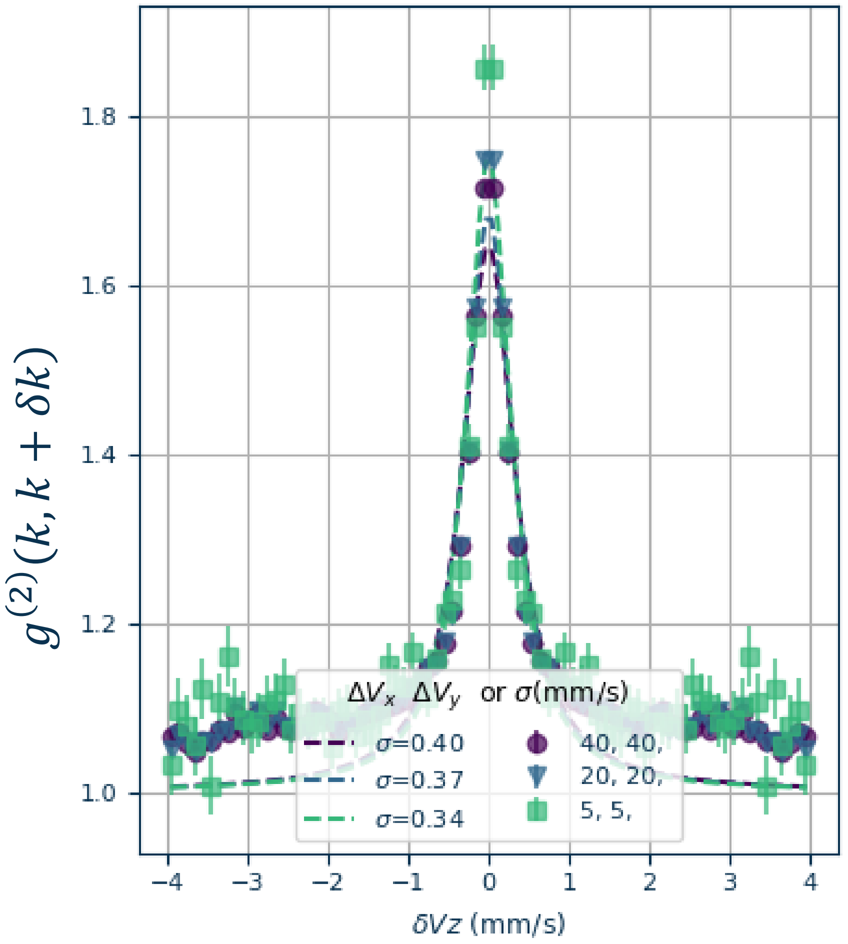
2. Counting statistics

$$g^{(2)}(k,-k) = 2 + \frac{1}{\langle n \rangle}$$

3. Entanglement

Tracing over one of the modes leaves the remaining mode in a thermal state

Probing (local) correlations



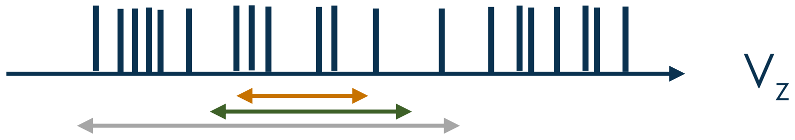
Local correlations (normal)

$g^{(2)}(k,k) = 2$ in the limit where the integration volume goes to 0.

(In the limit where pixels size goes to zero)

How to measure the number of particles in mode ? *(for counting statistics)*

How to define a mode size ? ($2\pi/L$)

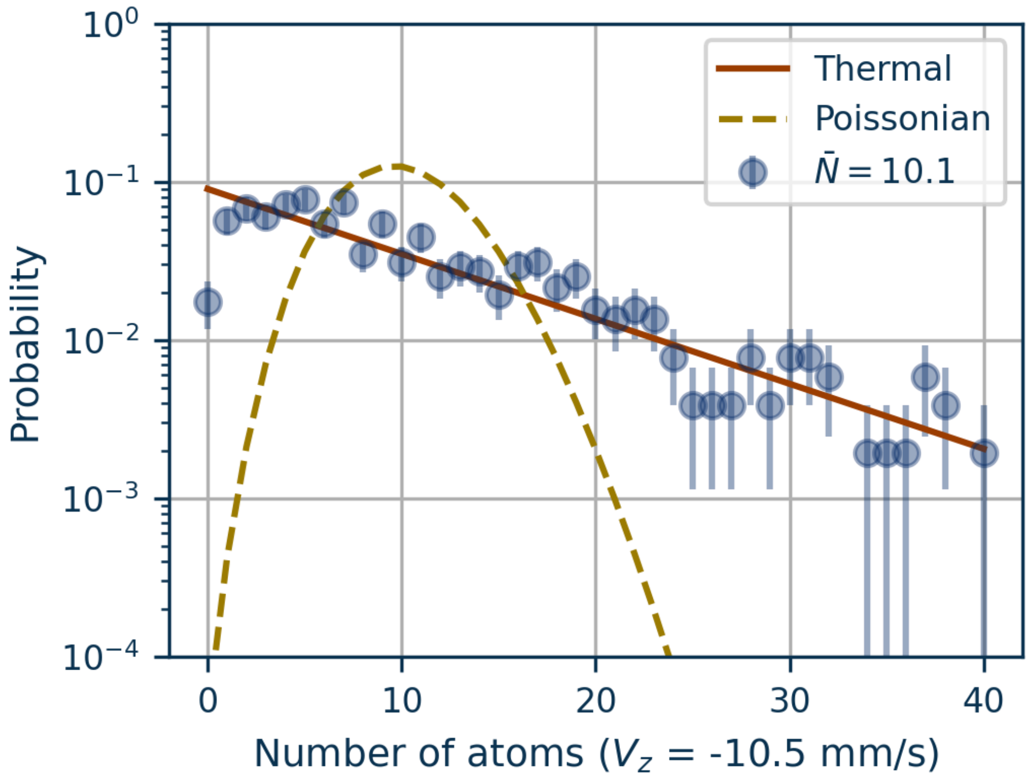


Use the HBT effect !
Bosonic bunching

The width of the auto-correlation gives the mode size.

Counting statistics

For a thermal state, the mean number of particles determines the distribution statistics.



$$\langle n \rangle = \sinh r$$

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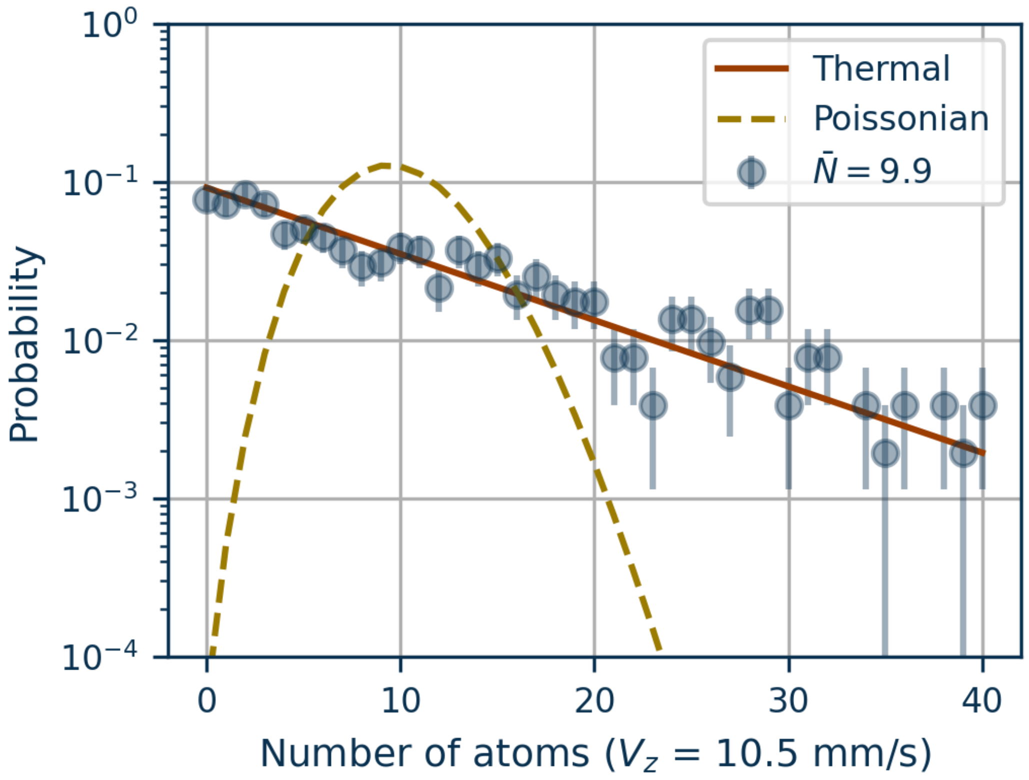
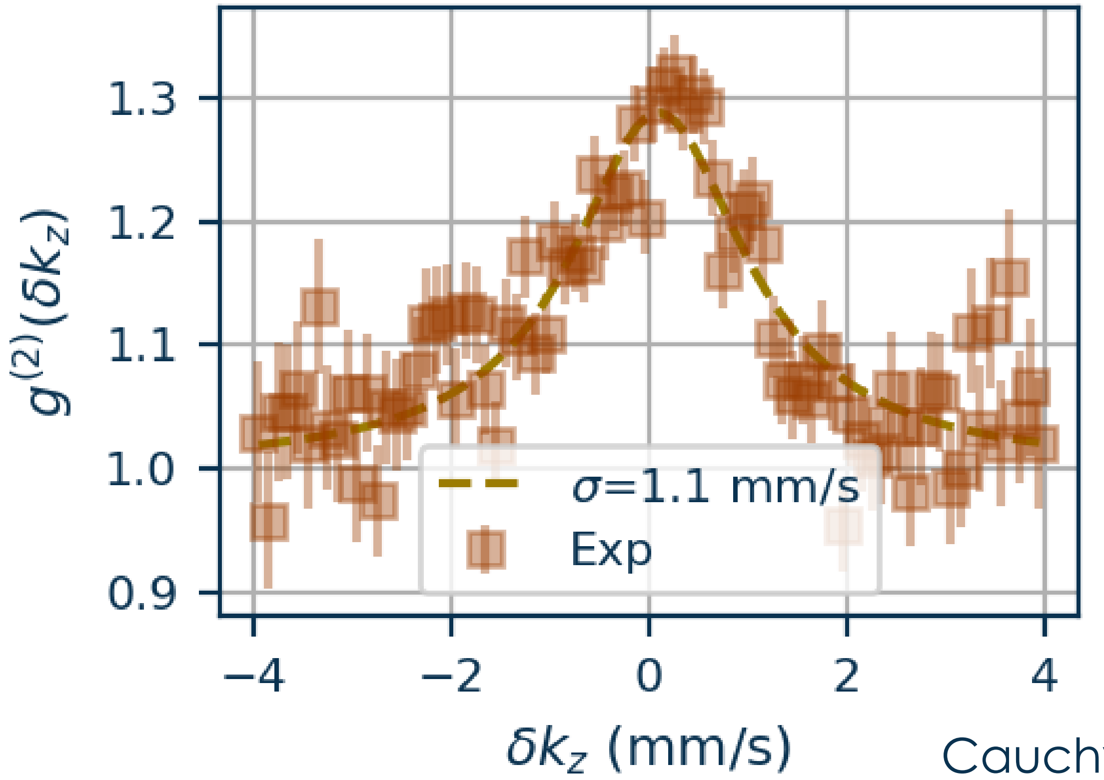


Probing (cross) correlations

$g^{(2)}(-k, k+\delta k)$



- $g^{(2)}(k, -k) < 2$
- Pic value of 1.3 (< 1.8 of local correlations)

WHY ???? ☹☹☹

Shot-to-shot fluctuations of the BEC initial speed kills cross-correlations

Use other entanglement witnesses

Cauchy-Schwarz inequality

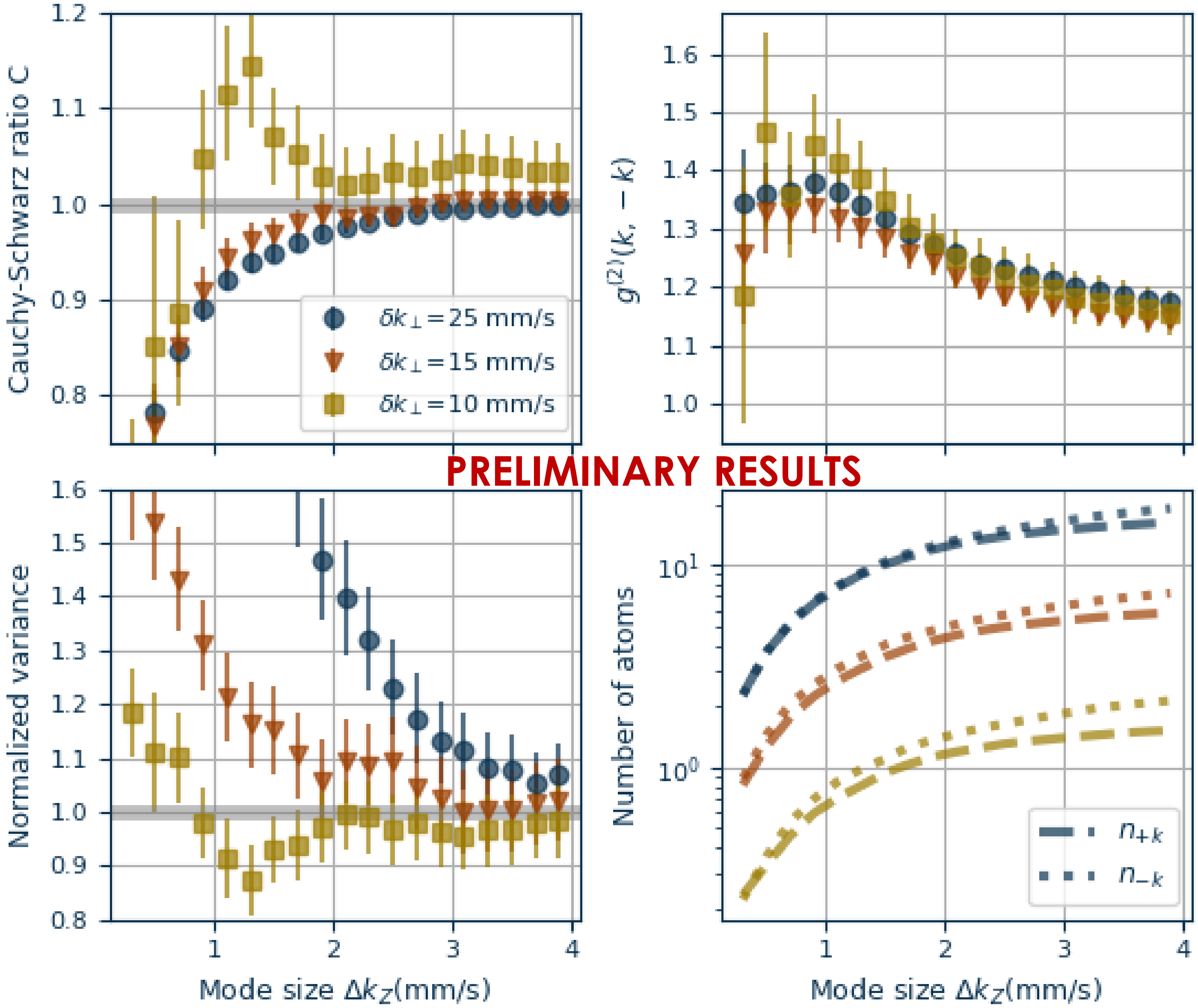
$$C = \frac{G^{(2)}(k,-k)}{\sqrt{G^{(2)}(-k,-k) \times G^{(2)}(k,k)}} < 1$$

Normalized variance

$$V = \frac{Var(n_k - n_{-k})}{n_k + n_{-k}} > 1$$

Kheruntsyan *et al.* PRL (2012)

Sub-shot noise variance



Cauchy-Schwarz inequality

$$C = \frac{G^{(2)}(k,-k)}{\sqrt{G^{(2)}(-k,-k) \times G^{(2)}(k,k)}} < 1$$

Normalized variance

$$V = \frac{Var(n_k - n_{-k})}{n_k + n_{-k}} > 1$$

We observe for the « *right* » pinhole :

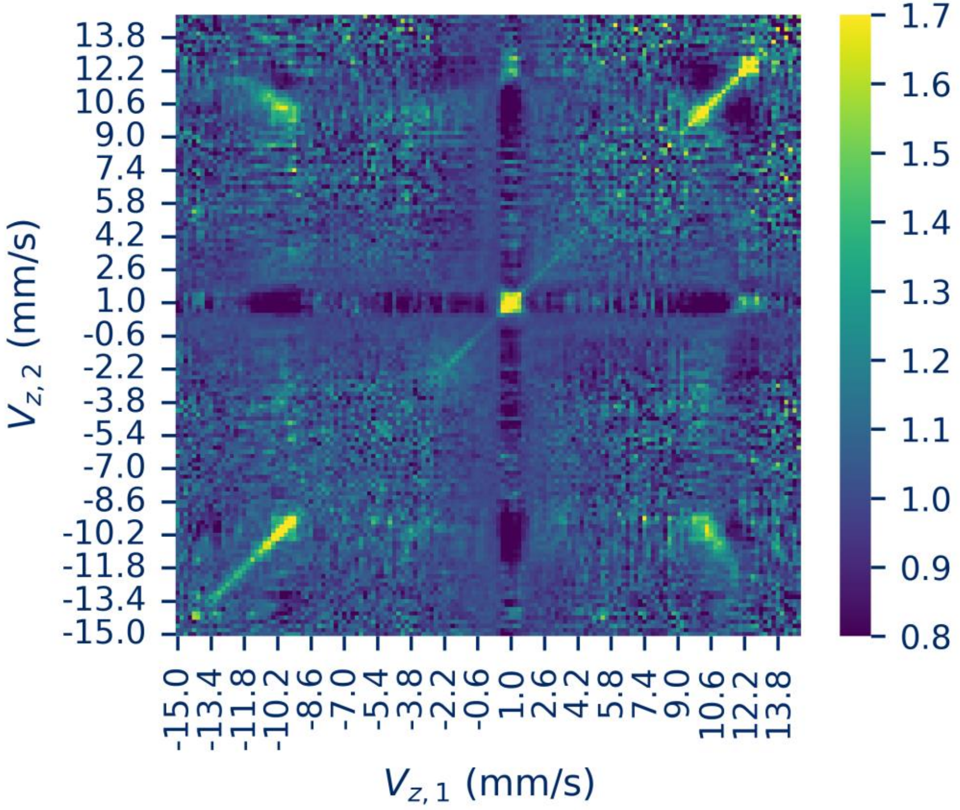
- Violation of the Cauchy-Schwarz inequality,
- Sub-shot noise variance

Conclusion

What we saw

- Creation and detection of (evaporated) phonons in a Bose-Einstein Condensate,
- Bogoliubov dispersion relation and exponential creation of phonons,
- Counting statistics compatible with a two-mode squeezed state,
- Preliminary results on non-separability.

2D correlation map of $g^{(2)}(V_{z,1}, V_{z,2})$

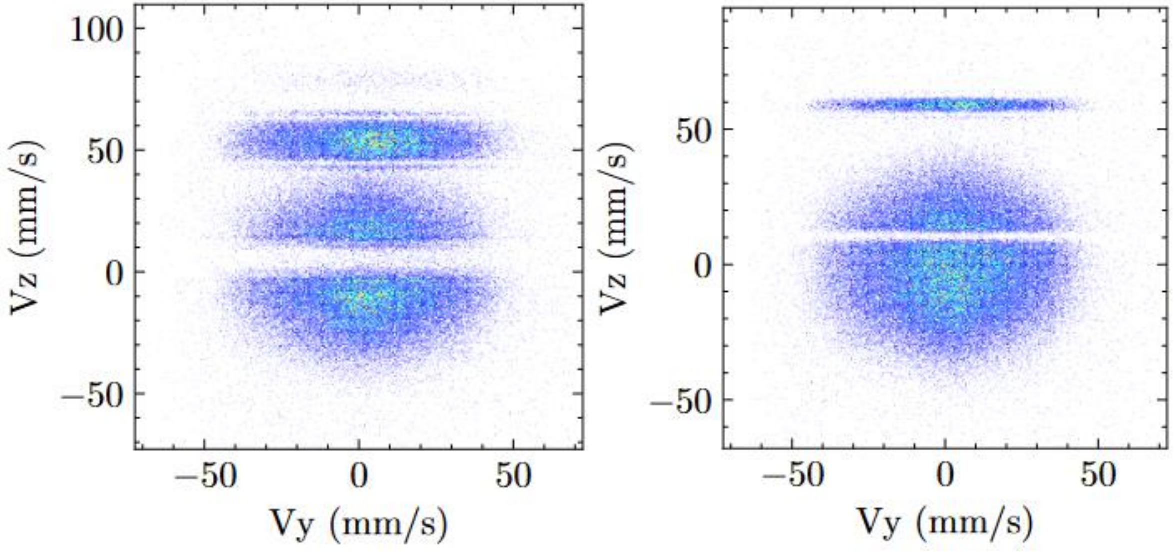


Outlooks

What's next ?

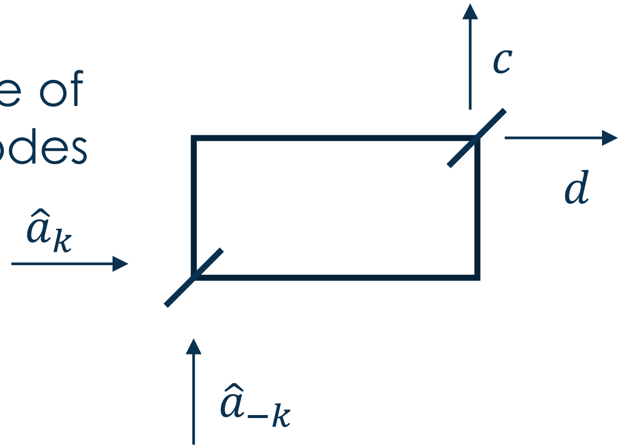
- Check robustness and replicability of preliminary results,
- Dependance of entanglement with number of excitation, amplitude…
- Add a lattice to change effective mass
- Study the exponential (decayed ?) creation process
- Probe the coherence with Bragg beams (atomic interferometer)

…….



Broadband and narrowband Bragg mirrors along z.

Interference of the two modes



Outlooks

What's next ?

- ✔ Universe origin : done.
- ? What about the future universe ?

This project :

5 T $CO_{2,\ eq}$/researcher/year

Paris 2012 agreement
→ 2T $CO_{2,\ eq}$/year

- Check robustness and replicability of preliminary results;
- Dependance of entanglement with number of excitation, amplitude…
- Add a lattice to change effective mass
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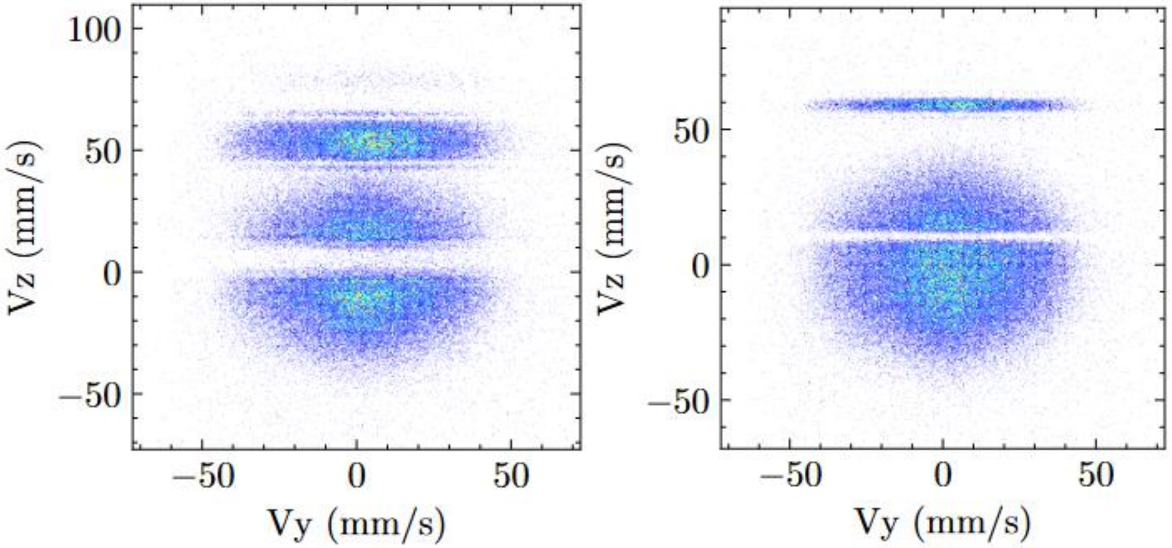

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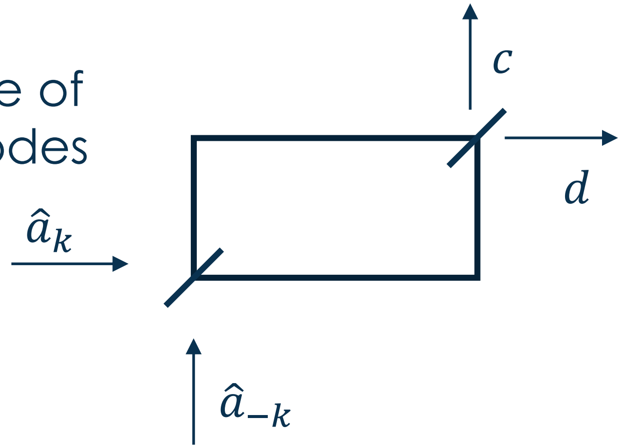
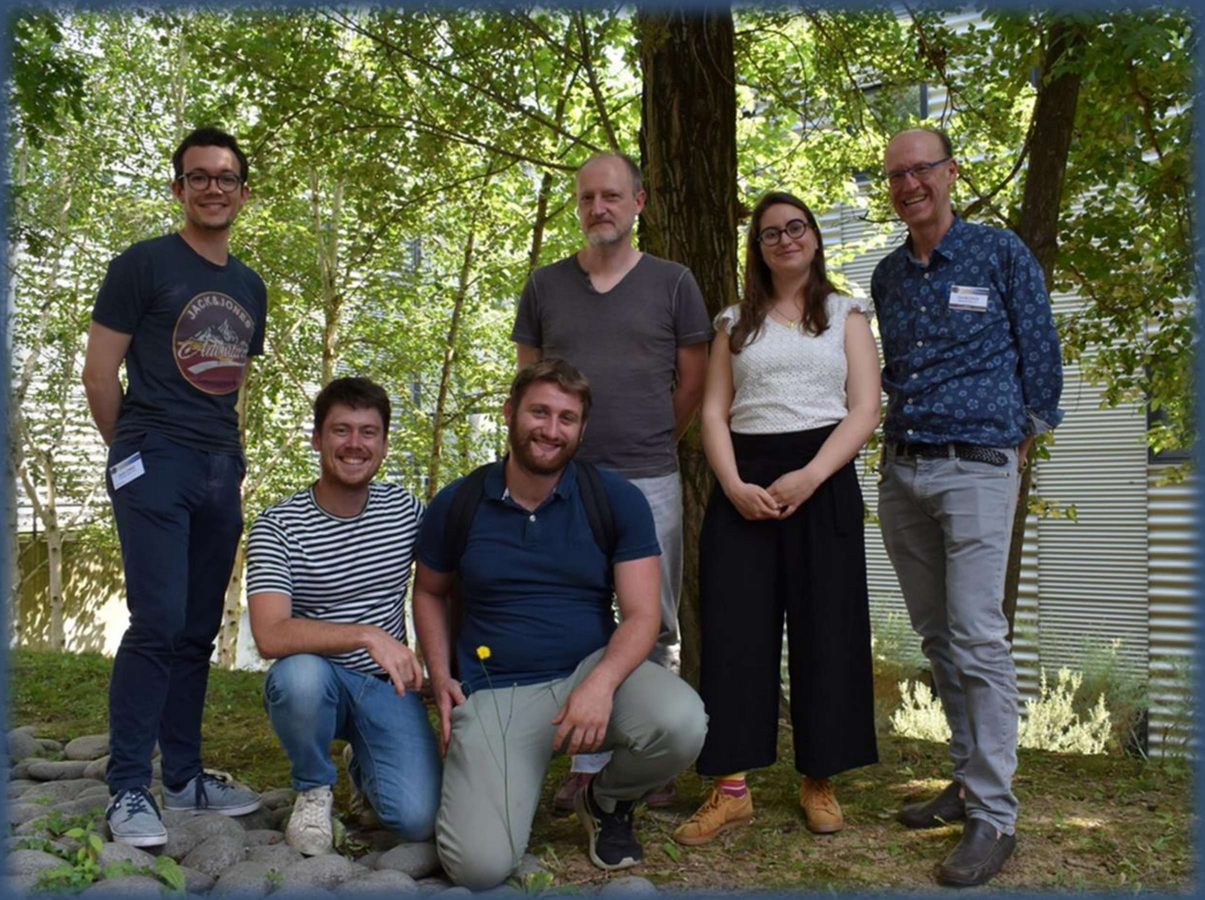


Thank you for your time !

Some lecture

- J.-C. Jaskula *et al.*, Phys. Rev. Lett. **109**, 220401 (2012).
- S. Robertson, F. Michel, and R. Parentani, Phys. Rev. D **95**, 065020 (2017).
- A. Micheli and S. Robertson, Phys. Rev. B **106**, 214528 (2022).



Fundings

anr

QUANTUM FLAGSHIP

PALM Laboratoire d'Excellence Physique : Atomes Lumière Matière

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QUANTERA

INSTITUT d'OPTIQUE GRADUATE SCHOOL ParisTech

université PARIS-SACLAY

Theoretical description

Fourier + Bogoliubov transformation gives the evolution of annihilation operator for collective excitations a_k

$$i\partial_t a_k = \Omega_k a_k + \frac{-i\partial_t \Omega_k}{2\Omega_k} a_{-k}^\dagger$$

where $\Omega_k^2 = \frac{g_1 n_1 k^2}{m} + \left(\frac{k^2}{2m}\right)^2$

- recover Bogoliubov dispersion relation with sound speed
  $$c_s = \sqrt{g_1 n_1 / m}$$
- modes k and –k evolve independently when c_s= cst
- k/–k mixing when the sound speed varies !

Describe the system as

$$\hat{\Psi} = \Phi_0(r,t)(1 + \hat{\phi}(z,t))$$

Φ_0 : gaussian ansatz for the transverse profile with width σ

$\hat{\phi}$: small perturbation of the field, depending only on z and time.

The perturbation obeys the Bogoliubov - de Gennes equation

$$i\hbar\partial_t \hat{\phi} = \frac{-1}{2m}\partial_{zz}\hat{\phi} + g_1 n_1(\hat{\phi} + \hat{\phi}^\dagger)$$

where $g_1 n_1$ depends on transverse profile Φ_0.

Busch *et al.* PRA (2013)

How to create collective excitations ?

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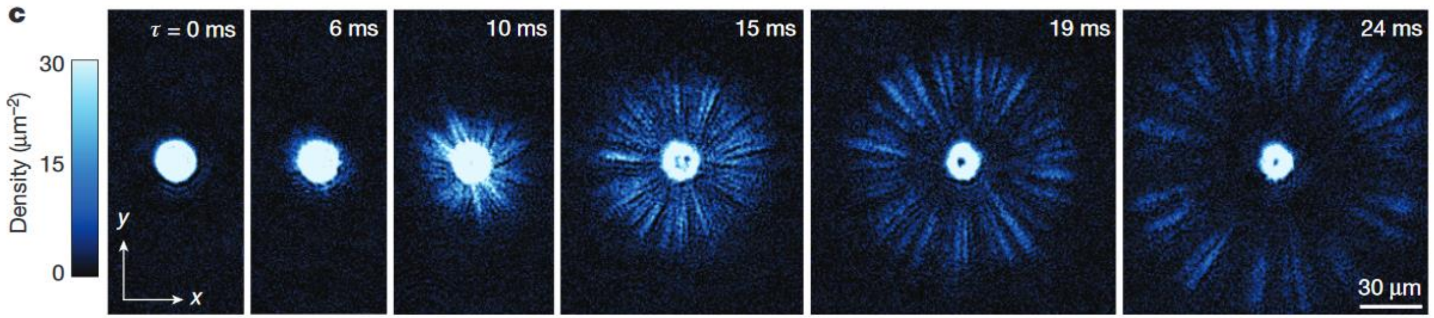
How to change c_s ?

$$c_s = \sqrt{g_1 n_1 / m} = \left(\frac{2\, a_s N}{L\, \sigma^2}\right)^{1/2}$$

- a_s atomic scattering length
- L , σ length and width of the BEC



Bose fireworks

Clark *et al*. Nature (2017)

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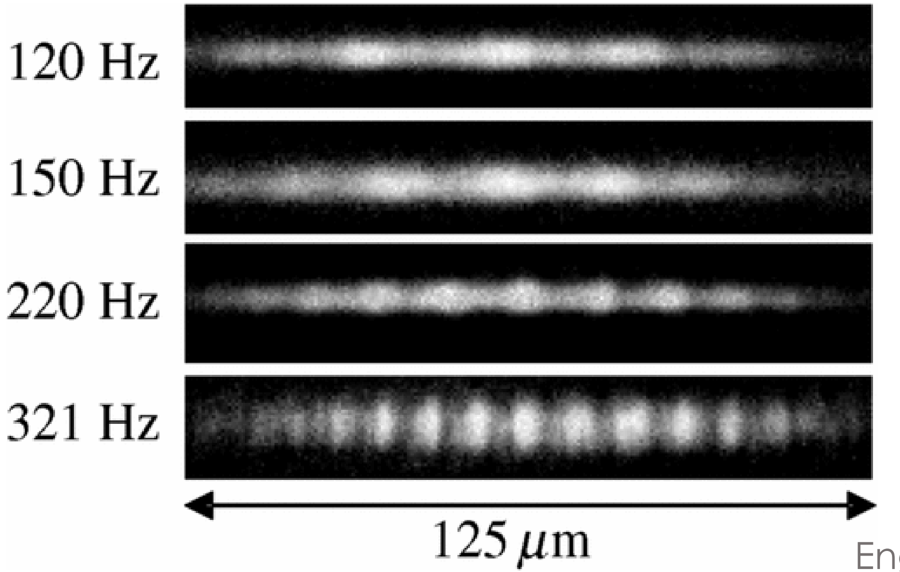


In situ imaging of Faraday waves

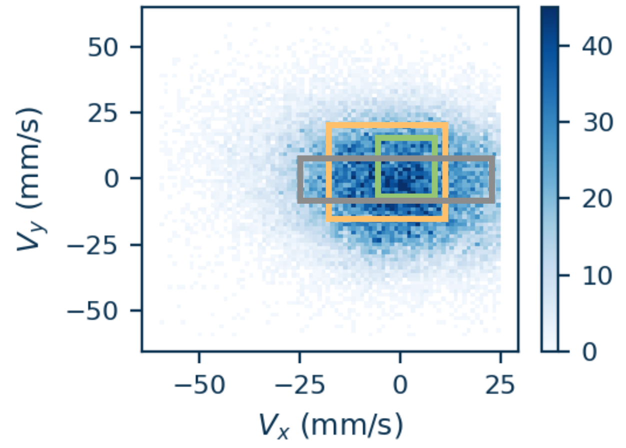
Engels *et al.* PRL (2007)

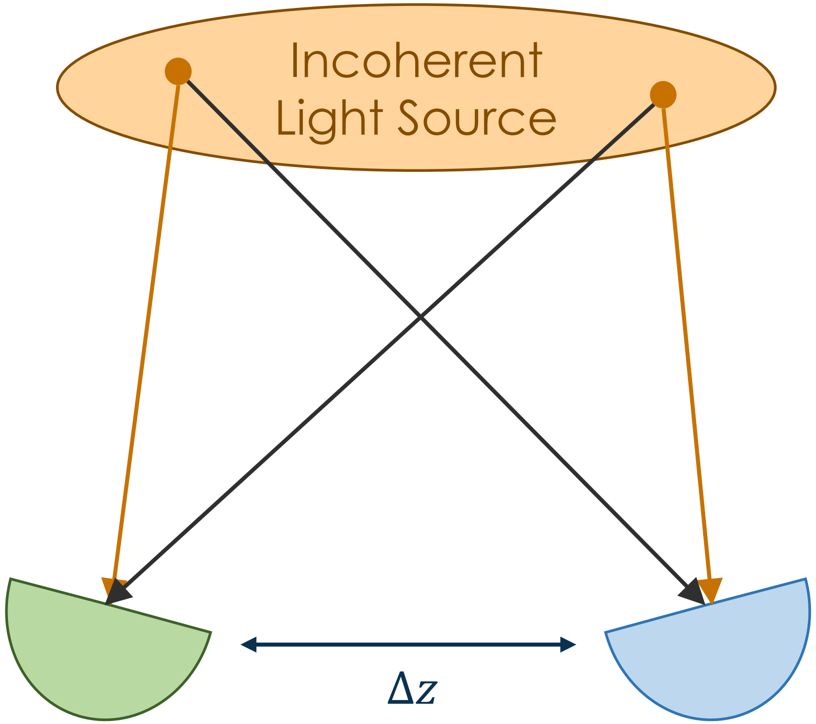
Probing (local) correlations

How to measure

$$g^{(2)}(k,q) = \langle : \hat{n}_q \hat{n}_k : \rangle / \langle \hat{n}_k \rangle \langle \hat{n}_q \rangle$$

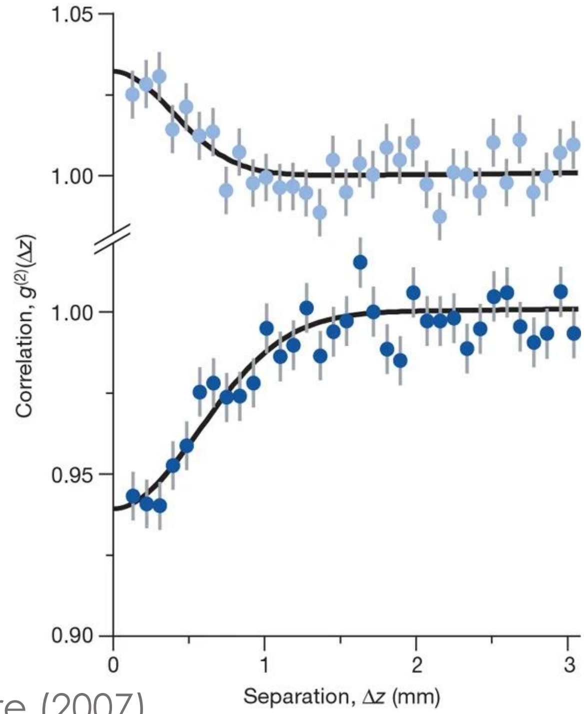
How to define a mode size ? ($2\pi/L$)





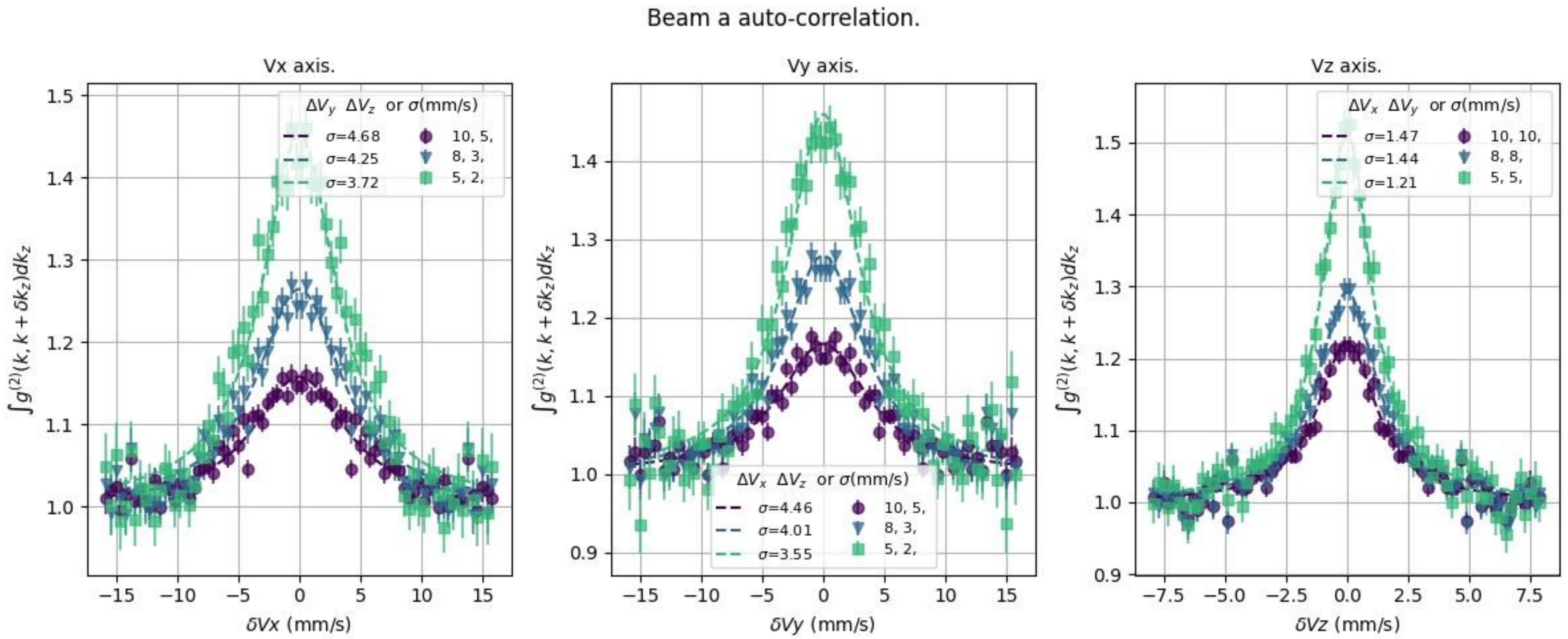
Use the HBT effect !
Bosonic bunching

- Constructive interferences for bosons,
- Destructive interference for fermions

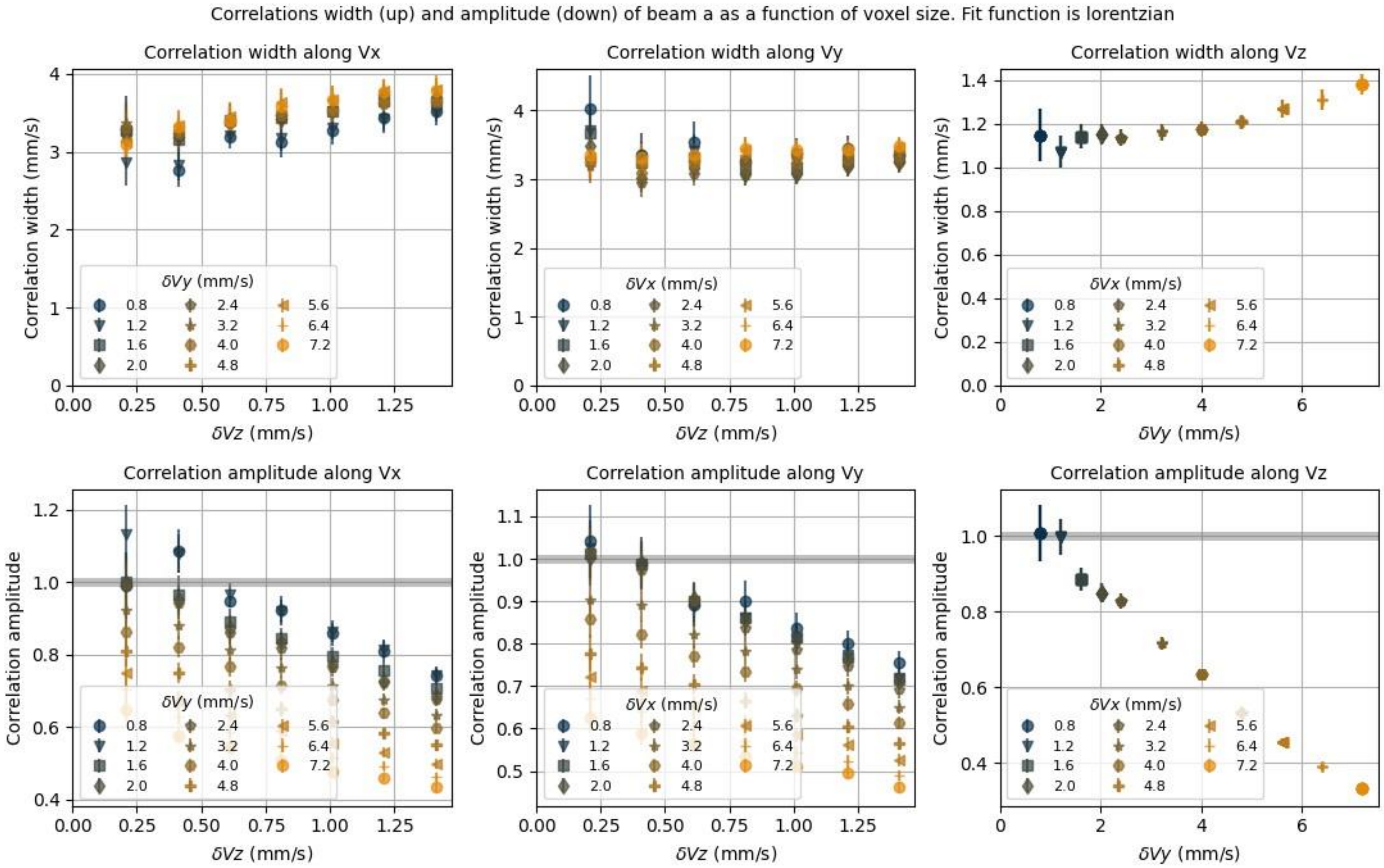


Jeltes *et al*, Nature (2007)

HBT : Correlation width (1)



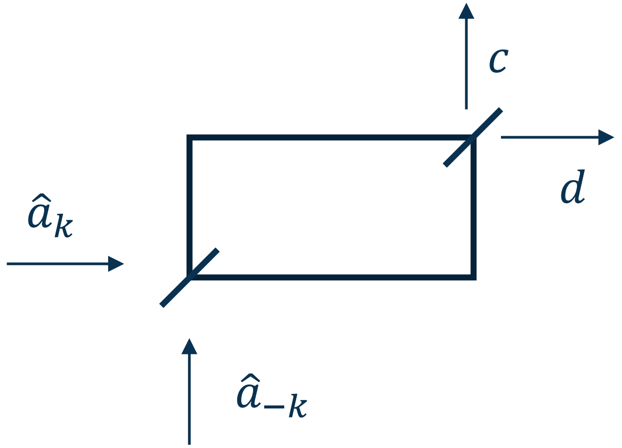
HBT : Correlation width (2)



Check the $g^{(2)} > 2$ criteria

$$\langle \hat{n}_k \hat{n}_{-k} \rangle = \langle \hat{b}_k^\dagger \hat{b}_{-k}^\dagger \hat{b}_k \hat{b}_{-k} \rangle = n_k n_{-k} + |\langle \hat{b}_k \hat{b}_{-k} \rangle|^2 + |\langle \hat{b}_k^\dagger \hat{b}_{-k} \rangle|^2$$

if the state is separable : $\underbrace{\leq n_k n_{-k}}$ $\underbrace{0 \; ????}$

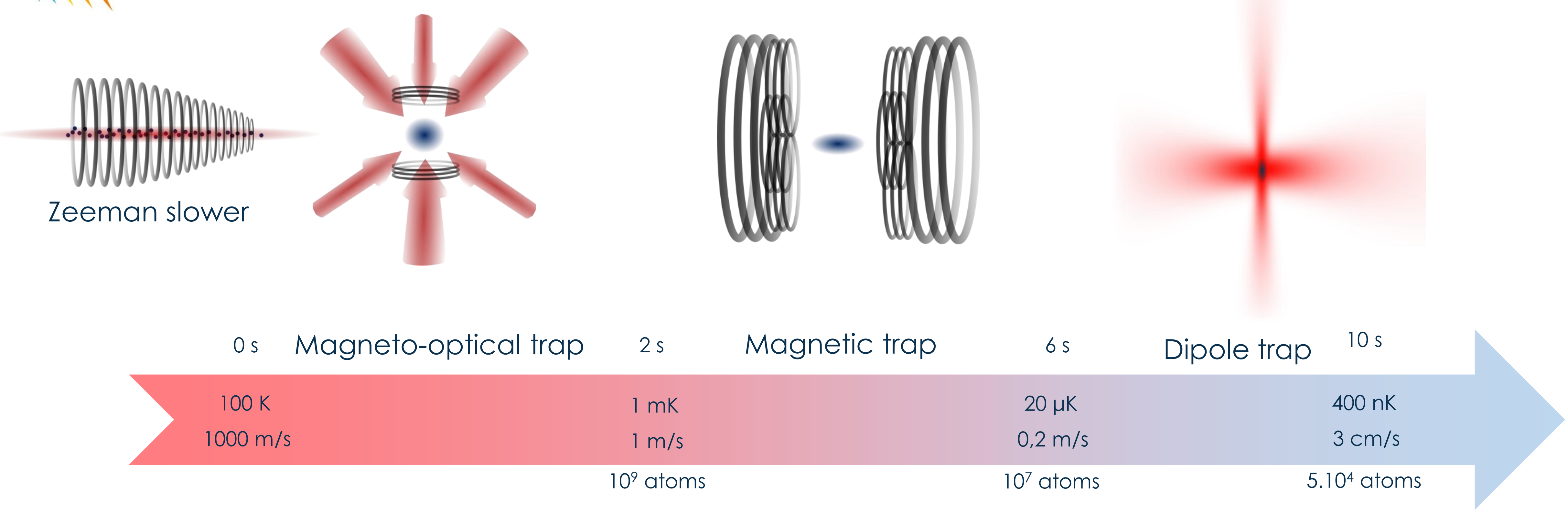


$$c = \cos(\phi)\, a + \sin(\phi)\, b$$

$$n_c = \langle c^\dagger c \rangle = \cos(\phi)^2\, a^\dagger a + \sin(\phi)^2\, b^\dagger b + \cos\phi \sin\phi\, (a^\dagger b + b^\dagger a)$$

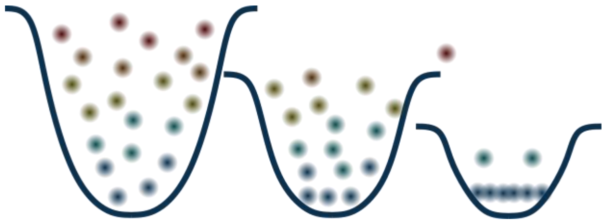
→ Check that this term is zero.

(Not that fast) Production of He* BEC



Plasma to excite He to metastable state : 20 eV, 2h.

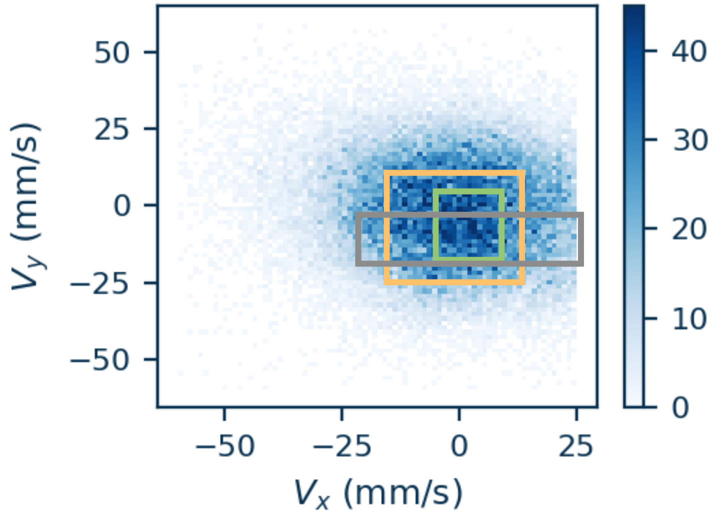
Evaporative cooling to obtain a BEC

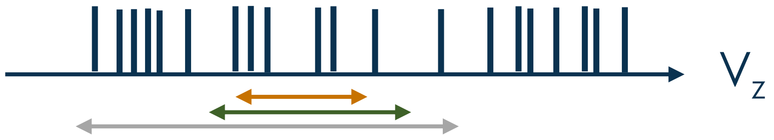


Probing (local) correlations

How to measure the number of particles in mode ? *(for counting statistics)*

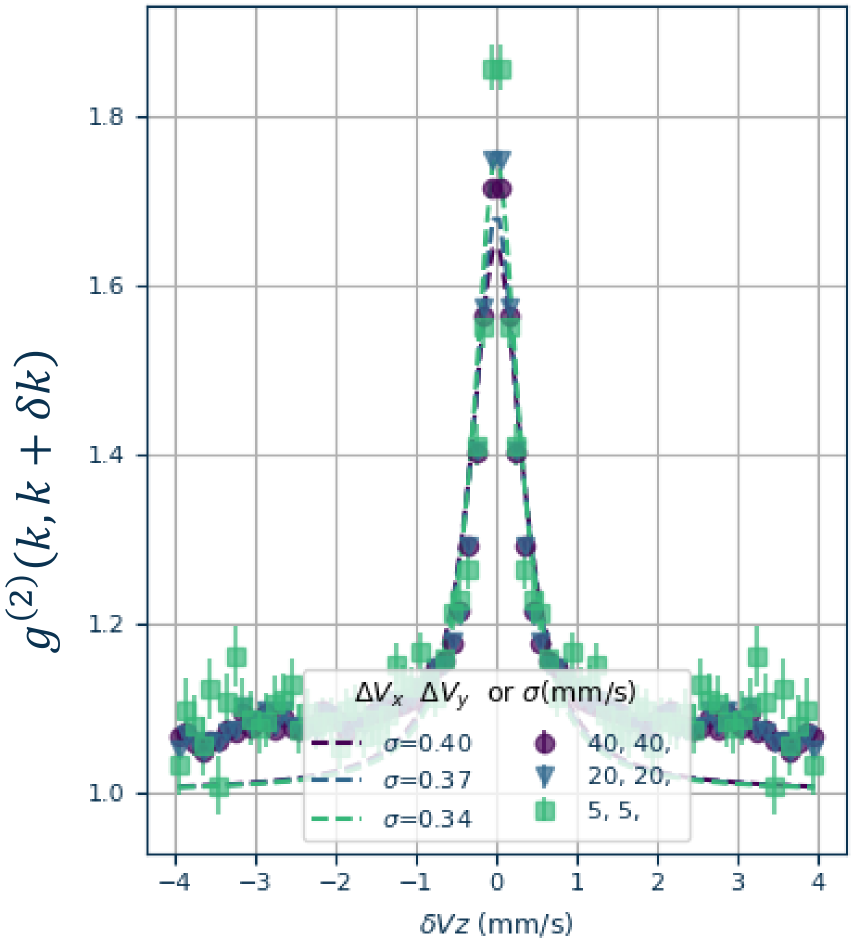
How to define a mode size ? $(2\pi/L)$





Use the HBT effect !

Bosonic bunching



Local correlations (normal)

$g^{(2)}(k,k) = 2$ in the limit where the integration volume goes to 0.

The width of the auto-correlation gives the mode size.