Cosmology and inflation: where do we stand?

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Outline

- A brief description of the standard model of cosmology (ΛCDM)
- Open issues of the ΛCDM model
- Cosmic inflation
- Quantum cosmological perturbations
- Observational status of inflation in brief
- Discussion & Conclusions

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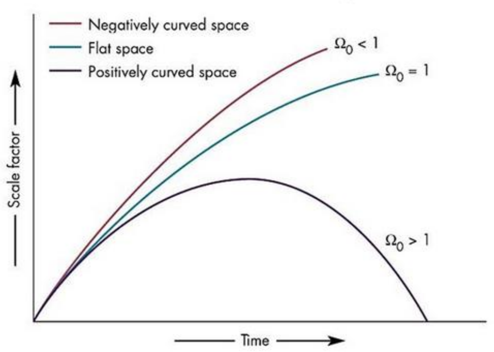
Scale factor



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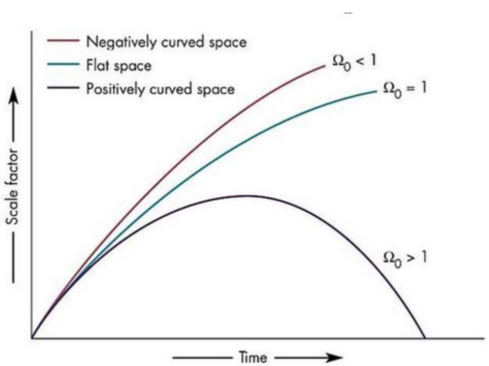
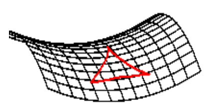
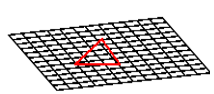


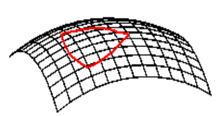


$\mathcal{K} = -1$



$\mathcal{K} = 0$

$\mathcal{K} = 1$



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Geometry CC Gavity +relat. Matter

Hubble parameter: $H = \frac{\dot{a}}{a}$

$$\left(\frac{\dot{a}}{a}\right)^2 + \frac{\mathcal{K}}{a^2} = \frac{\rho}{3M_{\mathrm{Pl}}^2} + \frac{\Lambda}{3}$$

$$\frac{\ddot{a}}{a} = -\frac{1}{6M_{\mathrm{Pl}}^2}(\rho + 3p) + \frac{\Lambda}{3}$$

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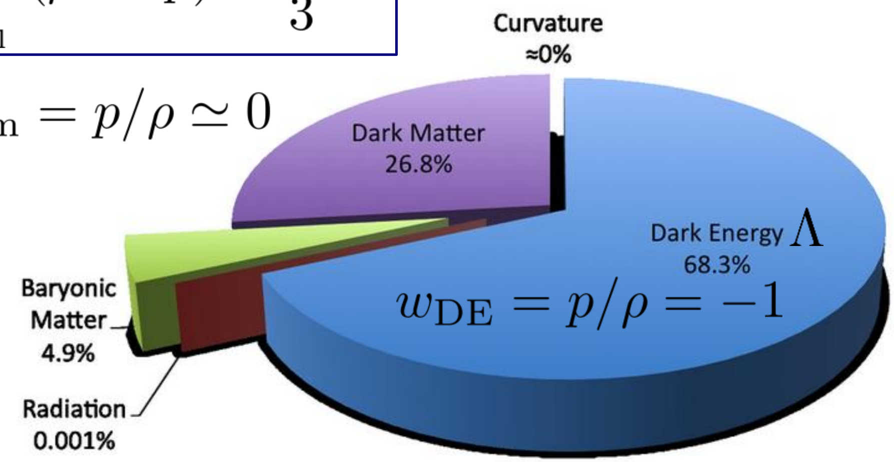

$$w_{\rm rad} = p/\rho = 1/3$$

w=p/ρ: equation of state

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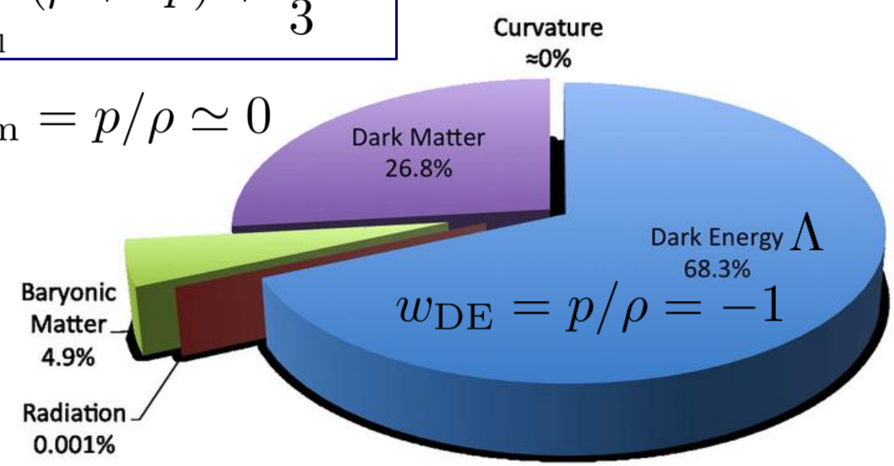
This allows us to trace back the history of the Universe



$w_{\rm DE} = p/\rho = -1$

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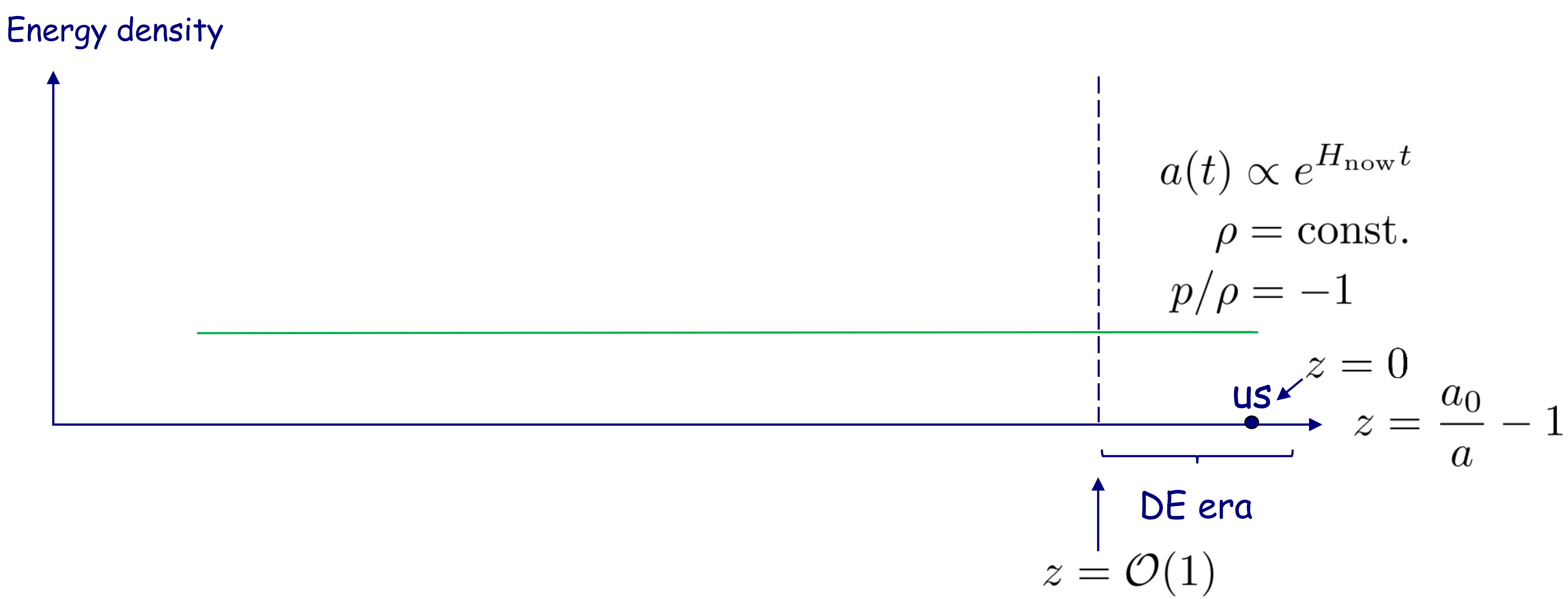

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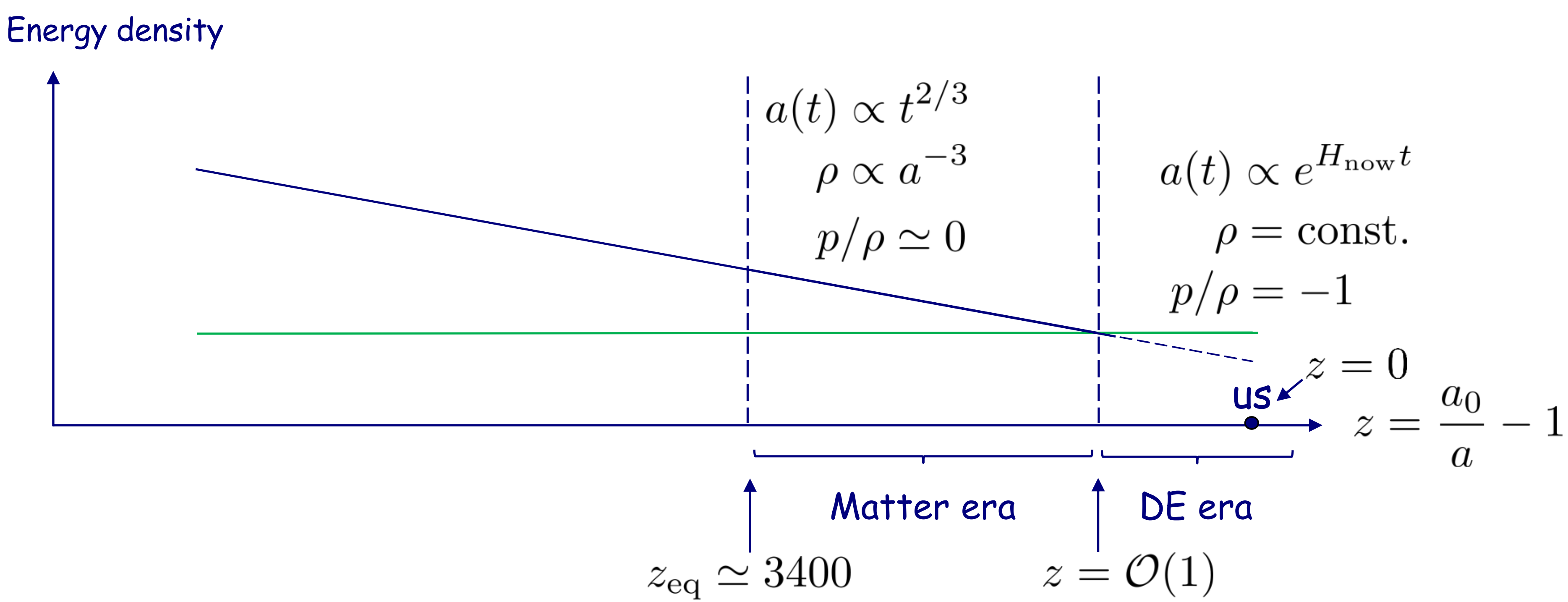

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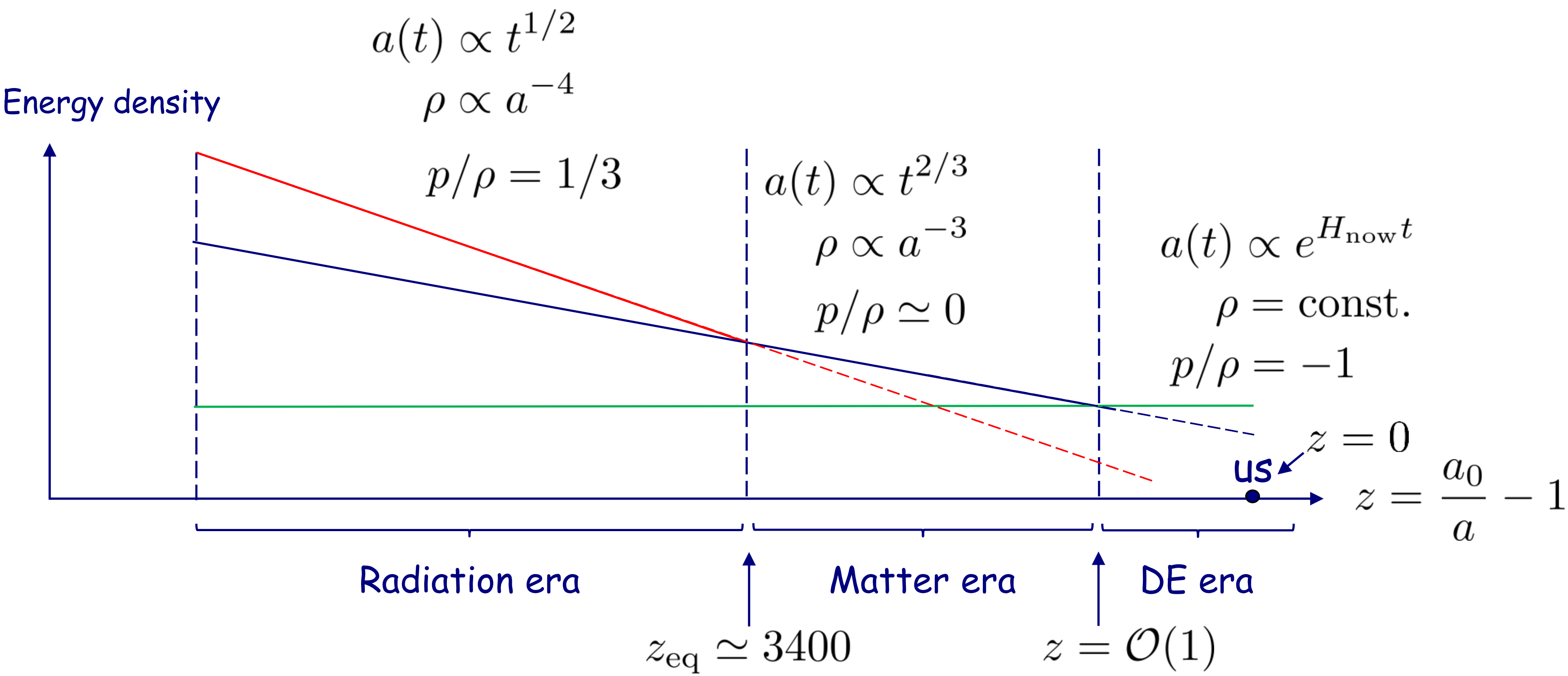

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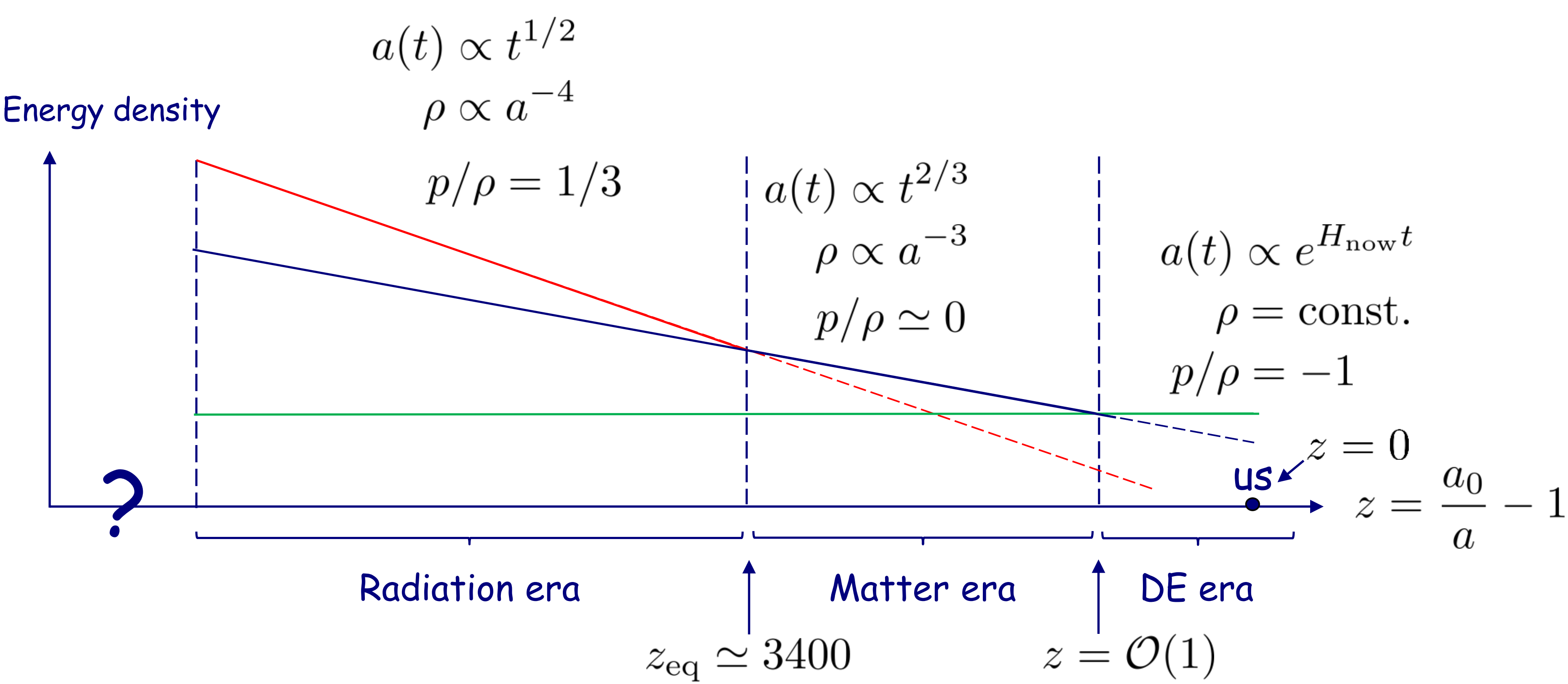
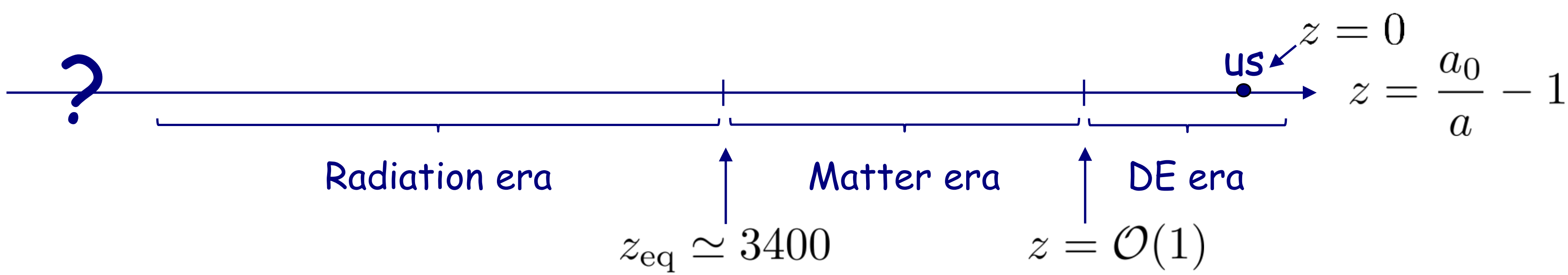
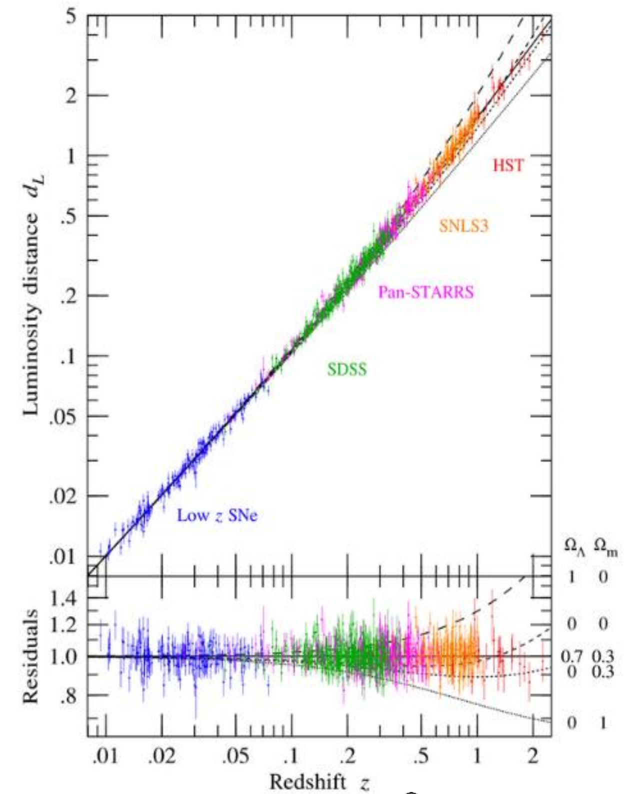


Three observational pillars



us — Energy

$$H_{\text{now}} \simeq 2 \times 10^{-42}\,\text{GeV}$$



$z = 0$

?

us

$$z = \frac{a_0}{a} - 1$$

Radiation era — Matter era — DE era

$$z_{\text{eq}} \simeq 3400$$

$$z = \mathcal{O}(1)$$

Three observational pillars

$T_{\rm rec} \simeq 0.26\,{\rm eV}$

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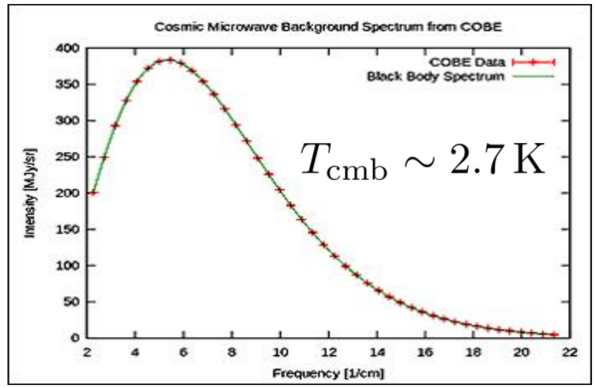
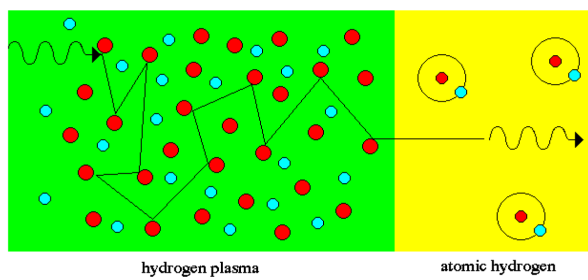
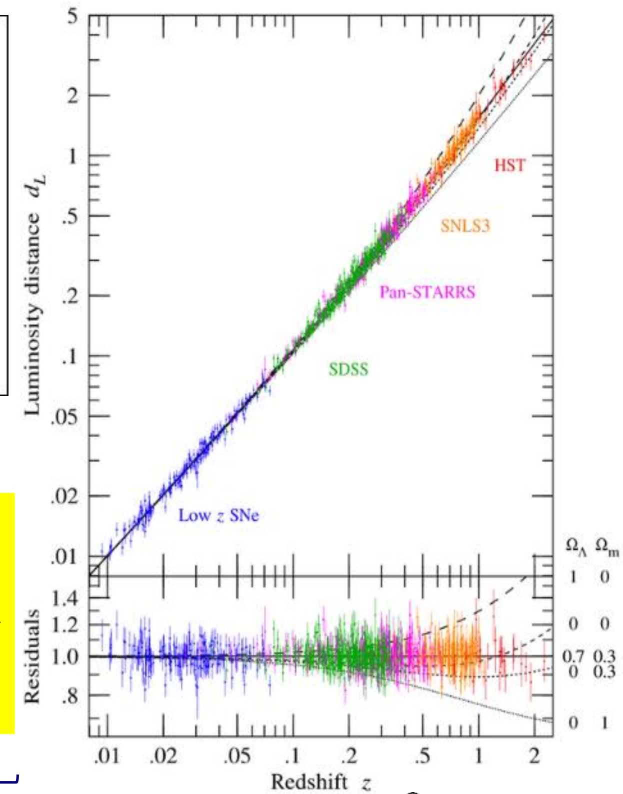


Last scattering surface (lss): prior, the Universe was opach





$z_{\rm lss} \simeq 1100$

$z = 0$ — us

$$z = \frac{a_0}{a} - 1$$

?

Radiation era — Matter era — DE era

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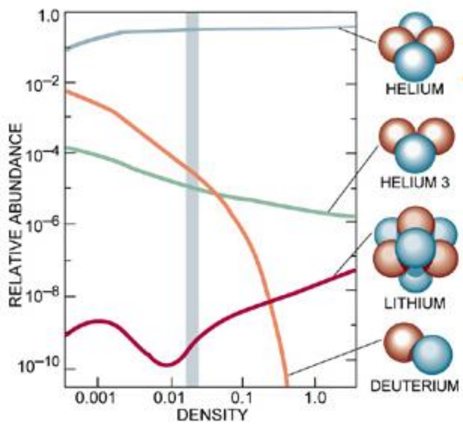
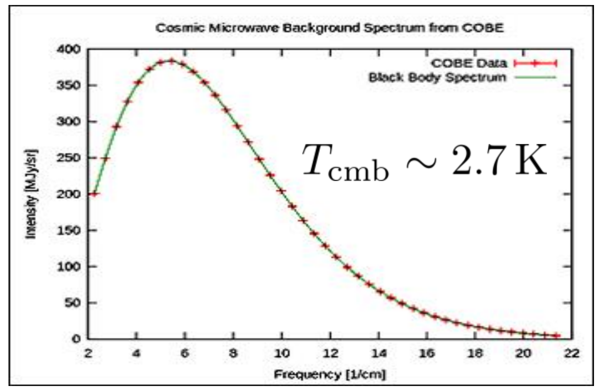
Three observational pillars

$T_{\rm bbn} \simeq 0.1\,{\rm MeV}$ $\qquad$ $T_{\rm rec} \simeq 0.26\,{\rm eV}$ $\qquad$ us $\qquad$ Energy

$H_{\rm bbn} \simeq 10^{-27}\,{\rm GeV}$ $\qquad$ $H_{\rm now} \simeq 2 \times 10^{-42}\,{\rm GeV}$





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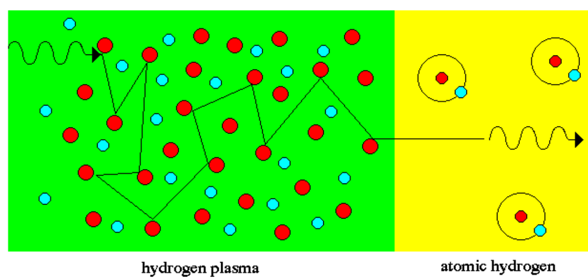

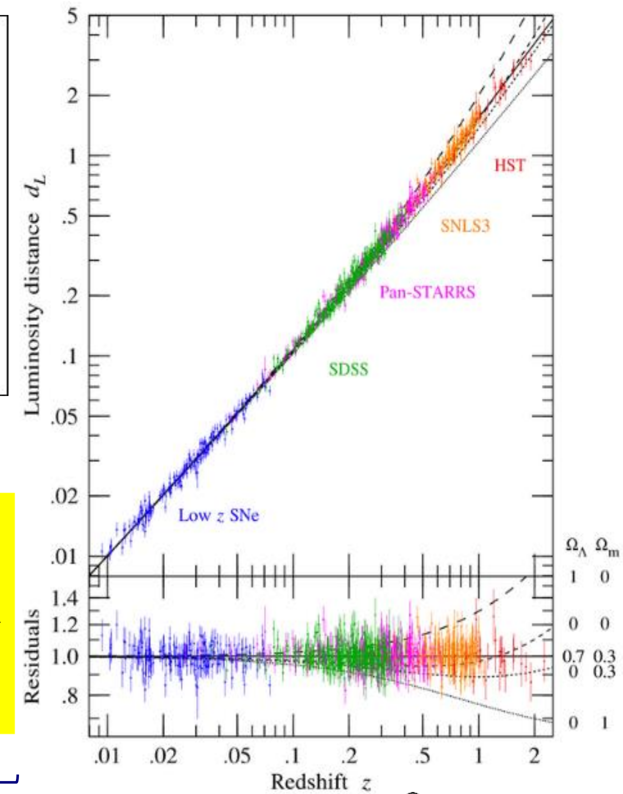

$z_{\rm bbn} \simeq 10^8$ $\qquad$ $z_{\rm lss} \simeq 1100$ $\qquad$ $z = 0$

? $\qquad$ us $\qquad$ $z = \frac{a_0}{a} - 1$

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Standard model of cosmology

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ρ_Λ^0, $\rho_{\rm cdm}^0$, $\rho_{\rm b}^0$, ρ_γ^0, ρ_ν^0	Composition of the Universe
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- Fits a wide range of data

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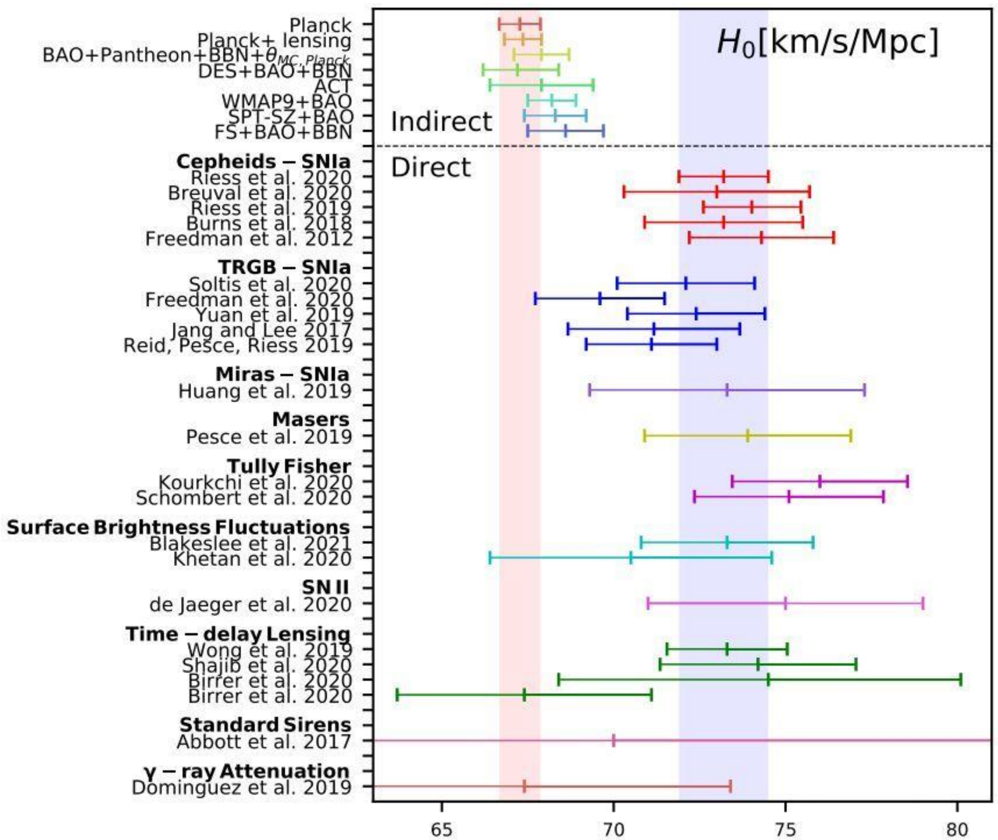

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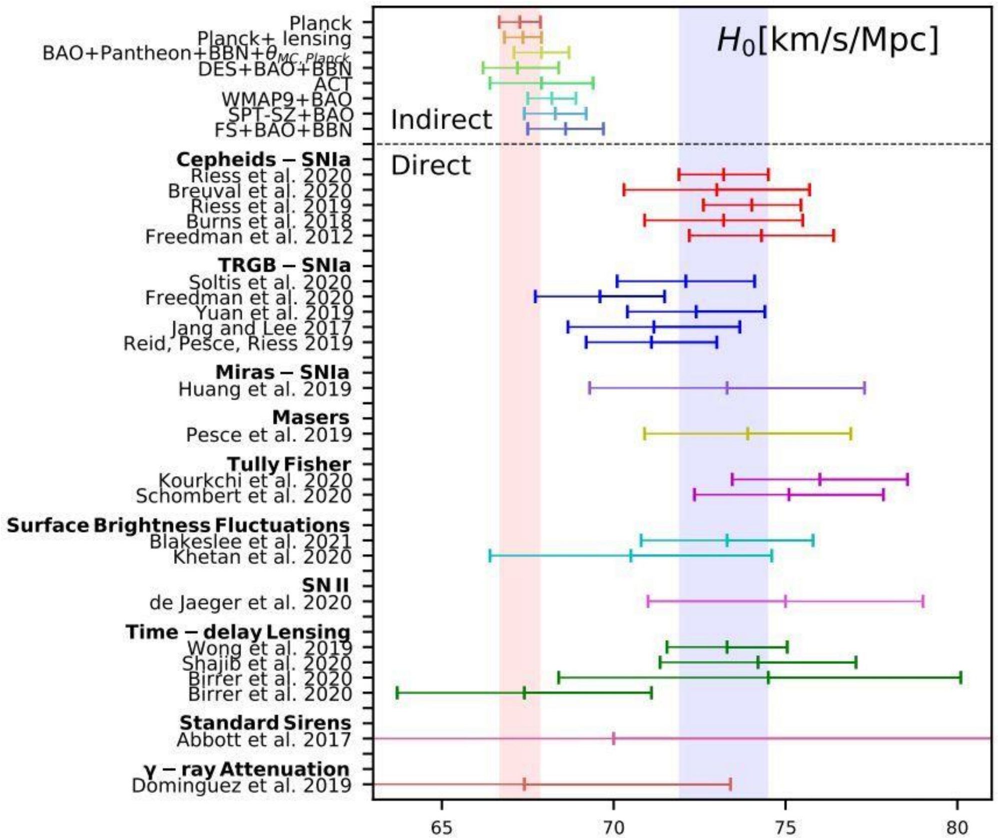

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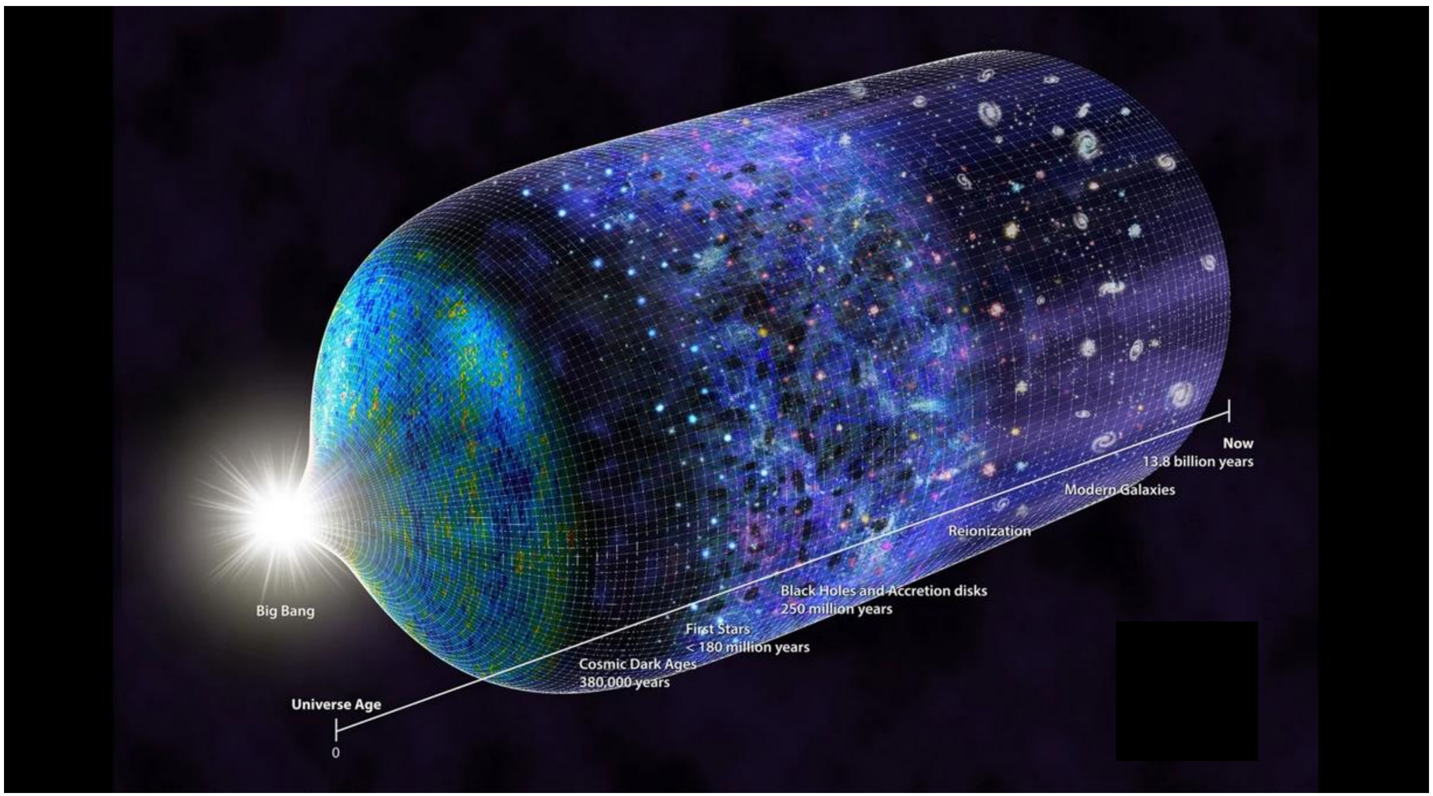

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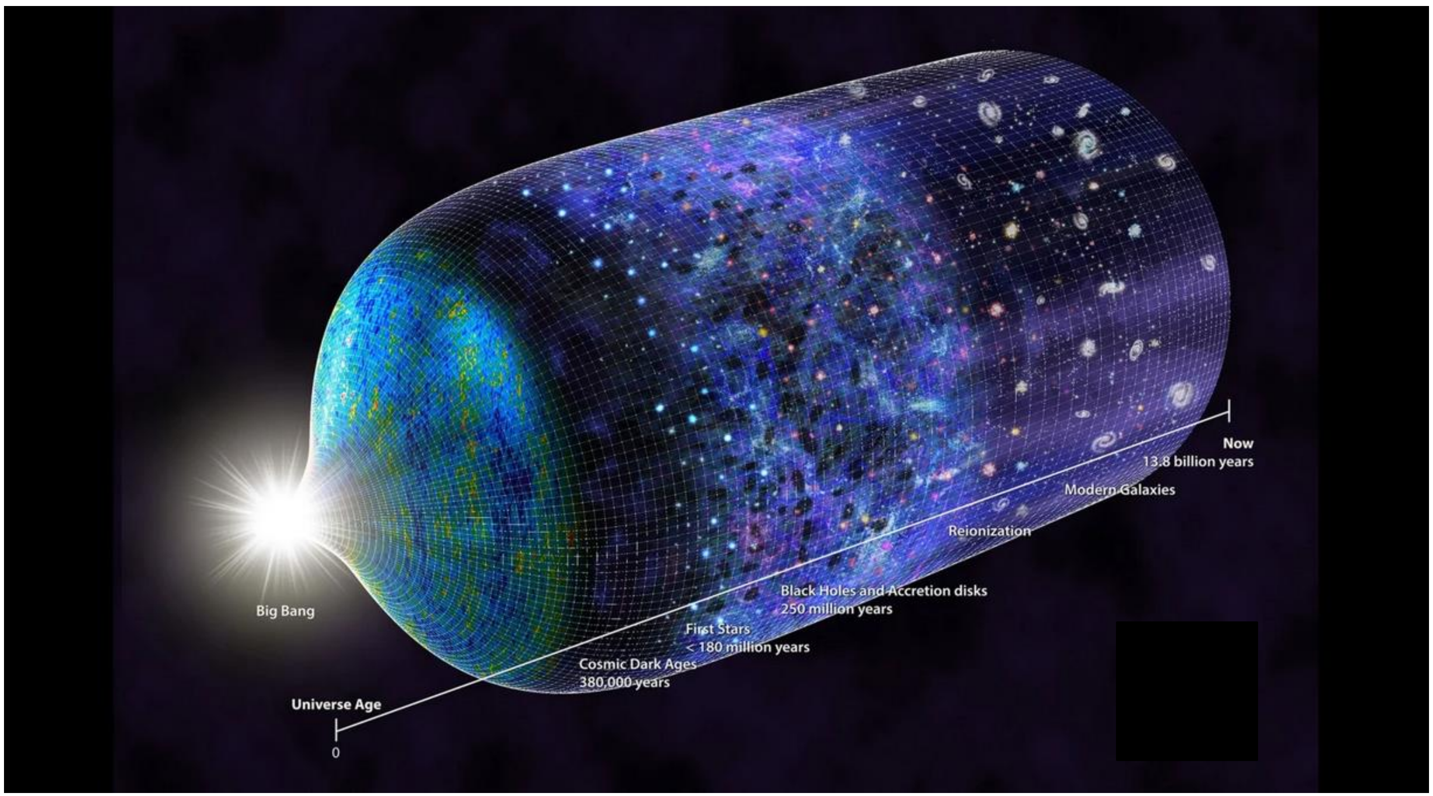

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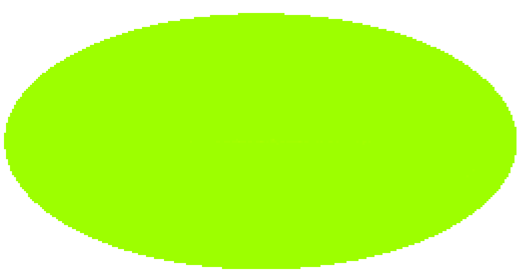

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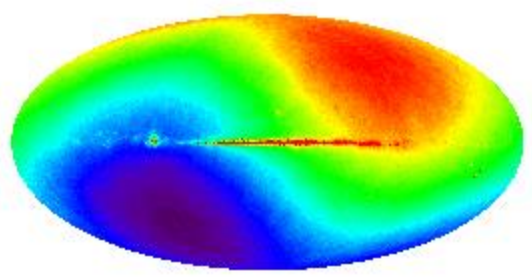

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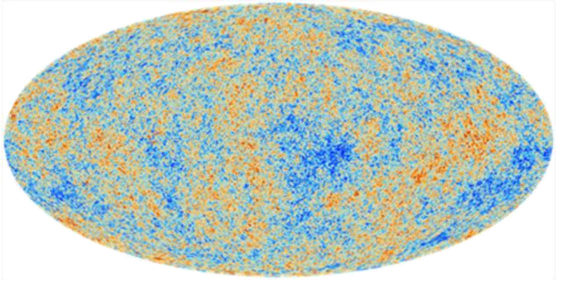

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precision

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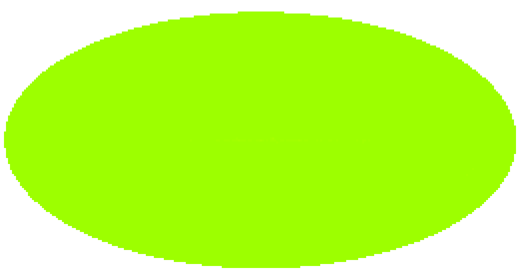

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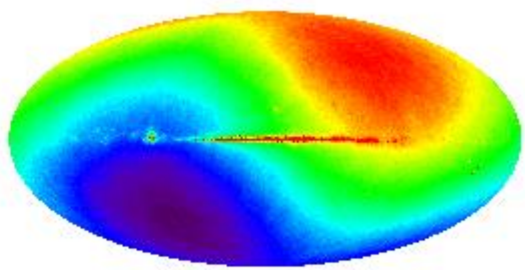

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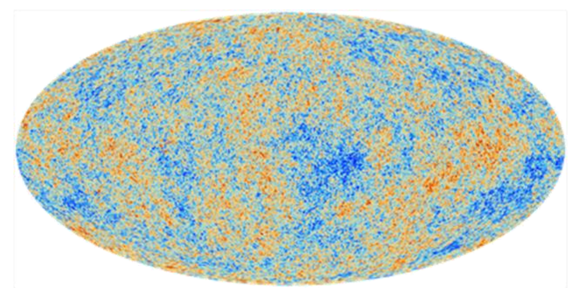

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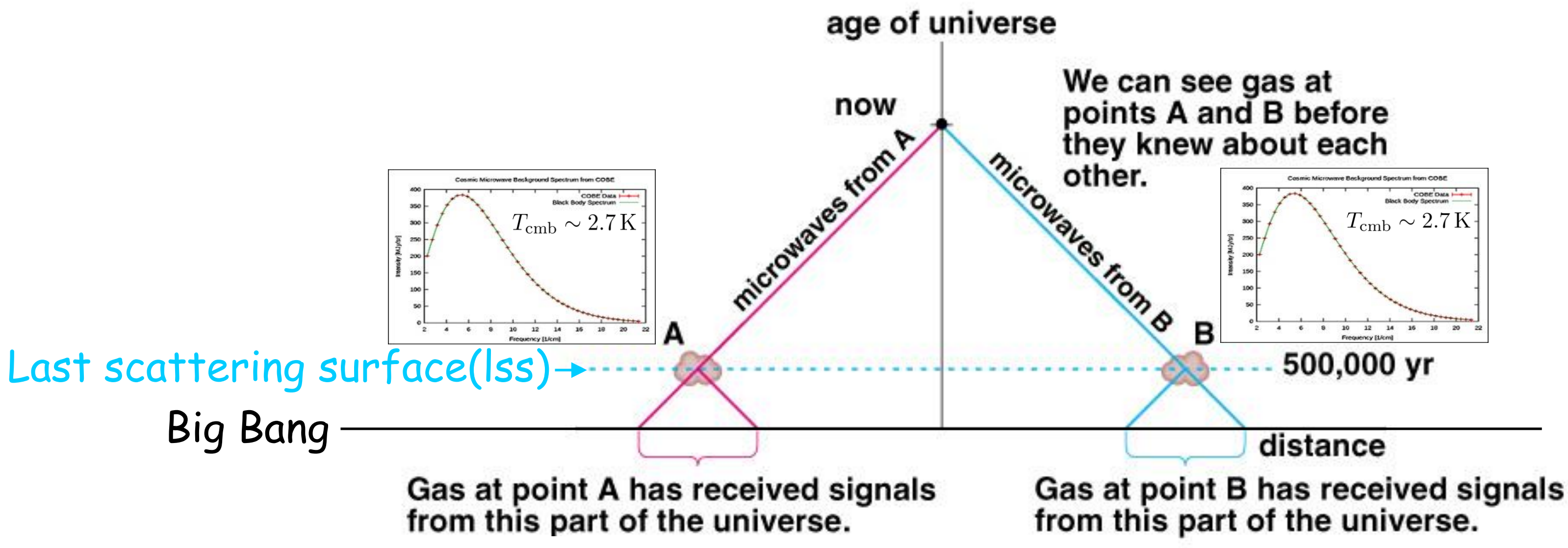

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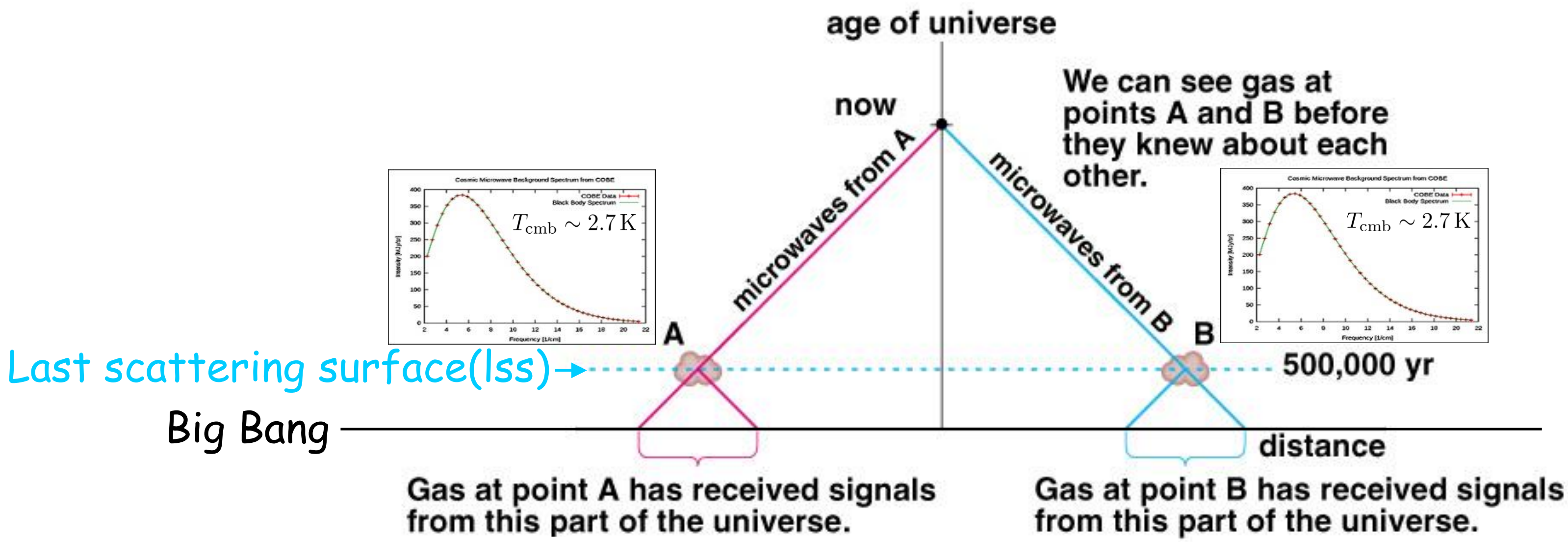

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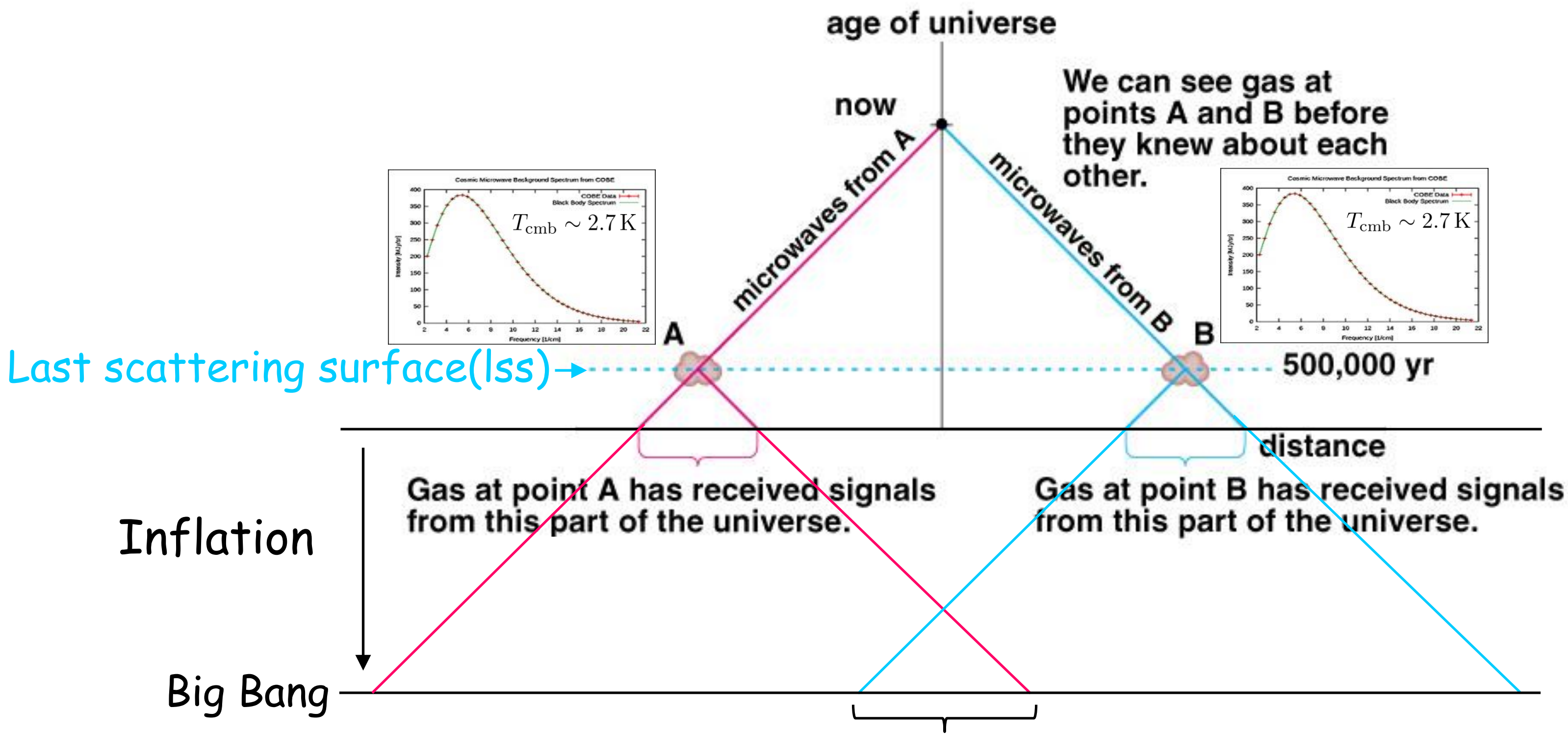

With inflation A and B are in causal contact

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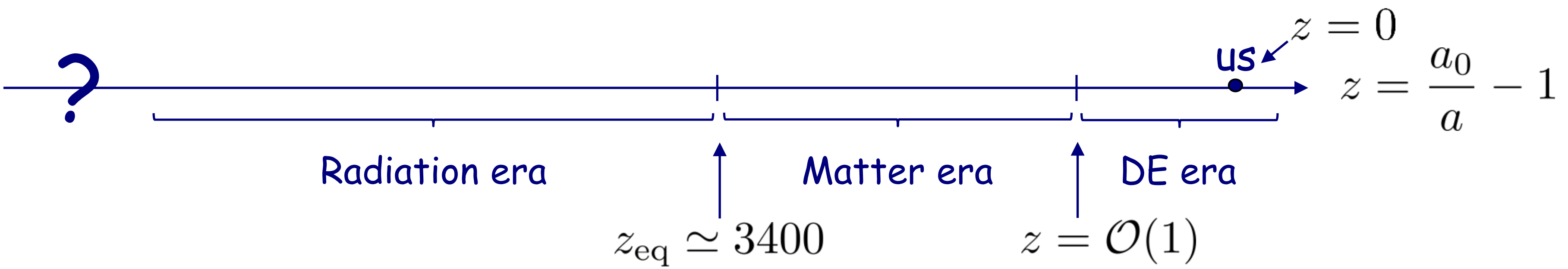

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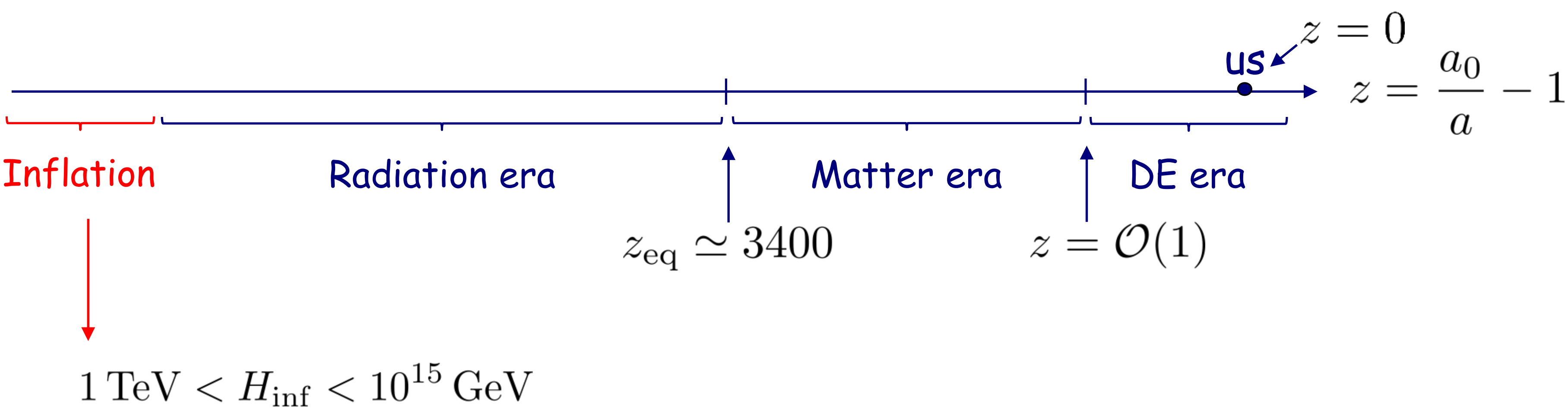

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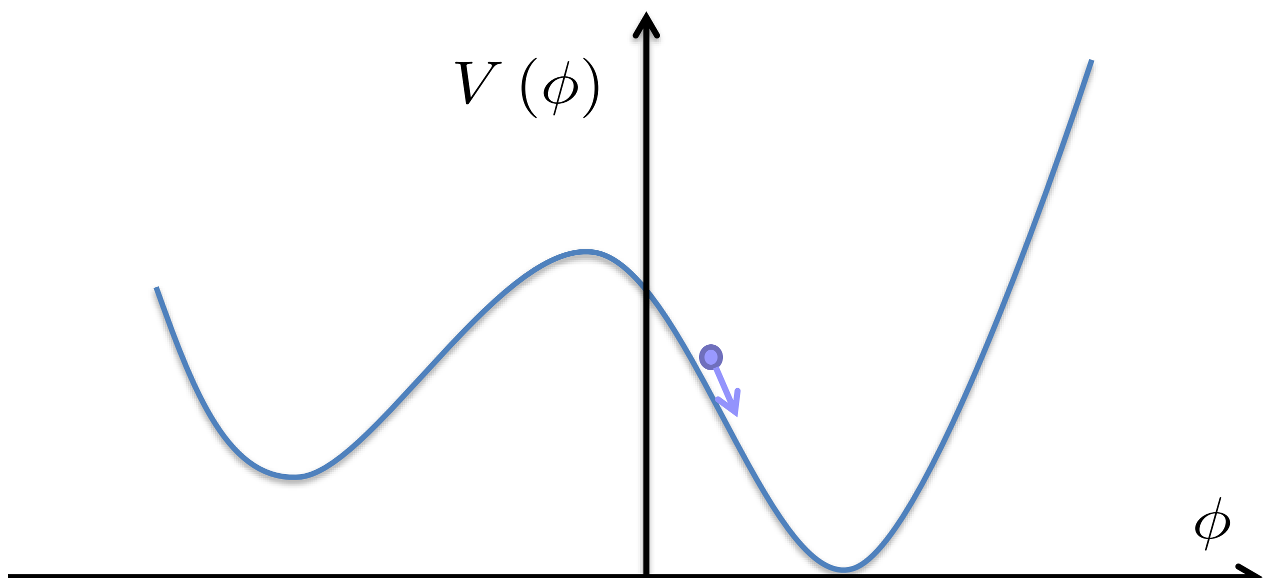

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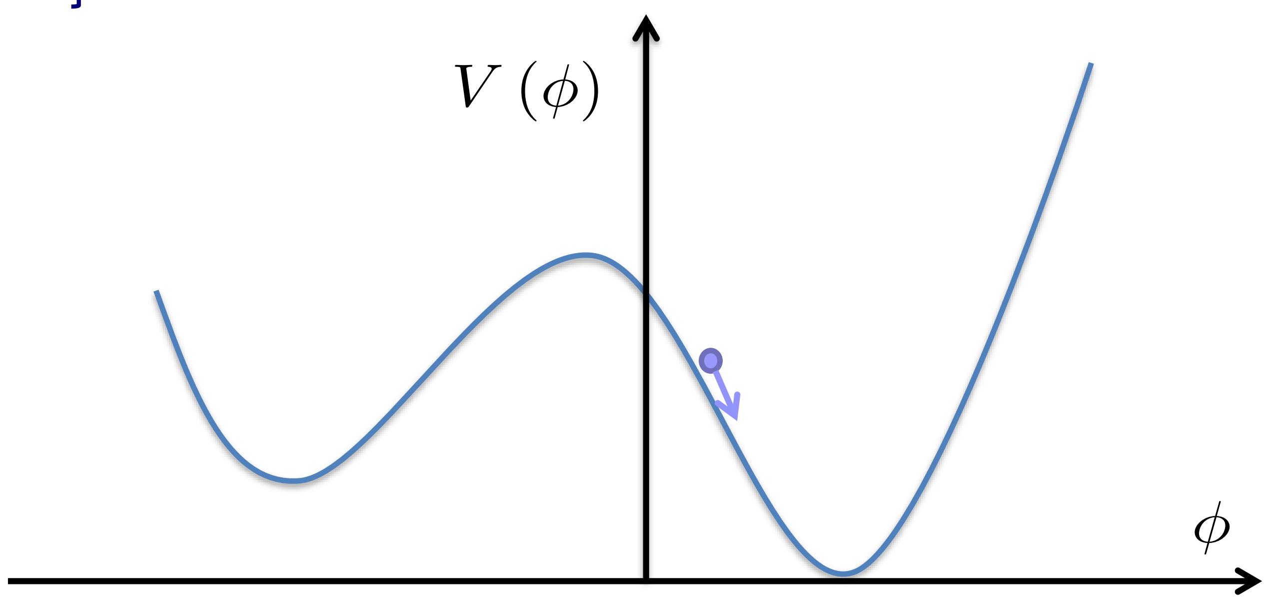
$$\left.\begin{aligned} \rho &= \frac{\dot{\phi}^2}{2} + V(\phi) \\ p &= \frac{\dot{\phi}^2}{2} - V(\phi) \end{aligned}\right] V(\phi) \gg \frac{\dot{\phi}^2}{2} \Rightarrow p \simeq -\rho < 0$$



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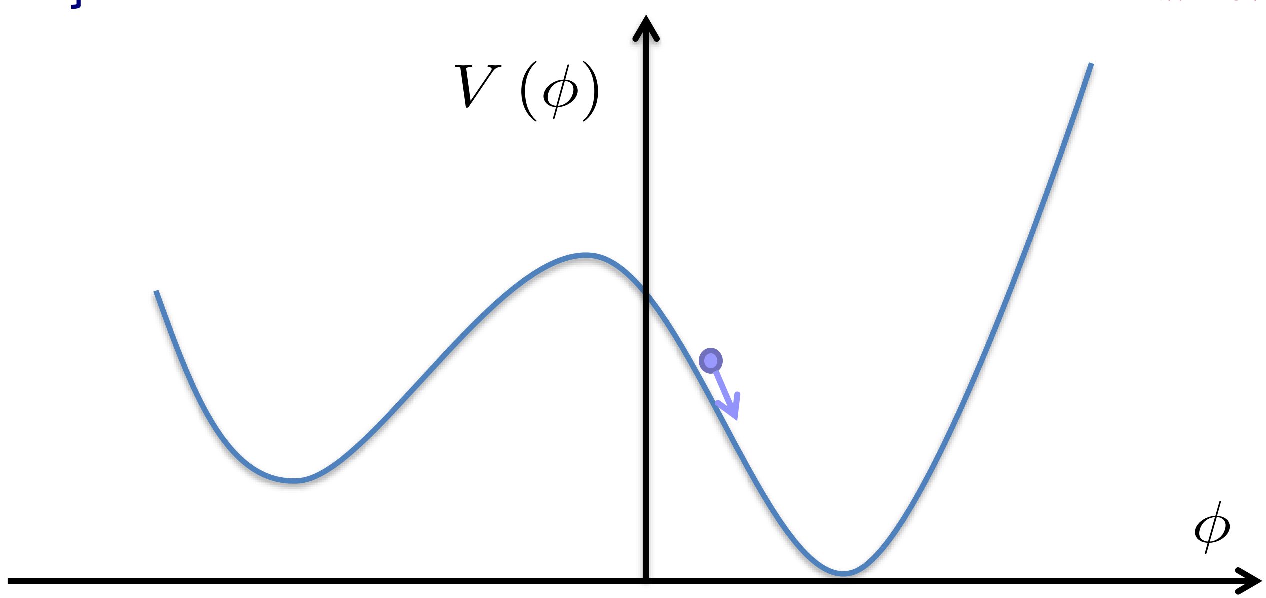

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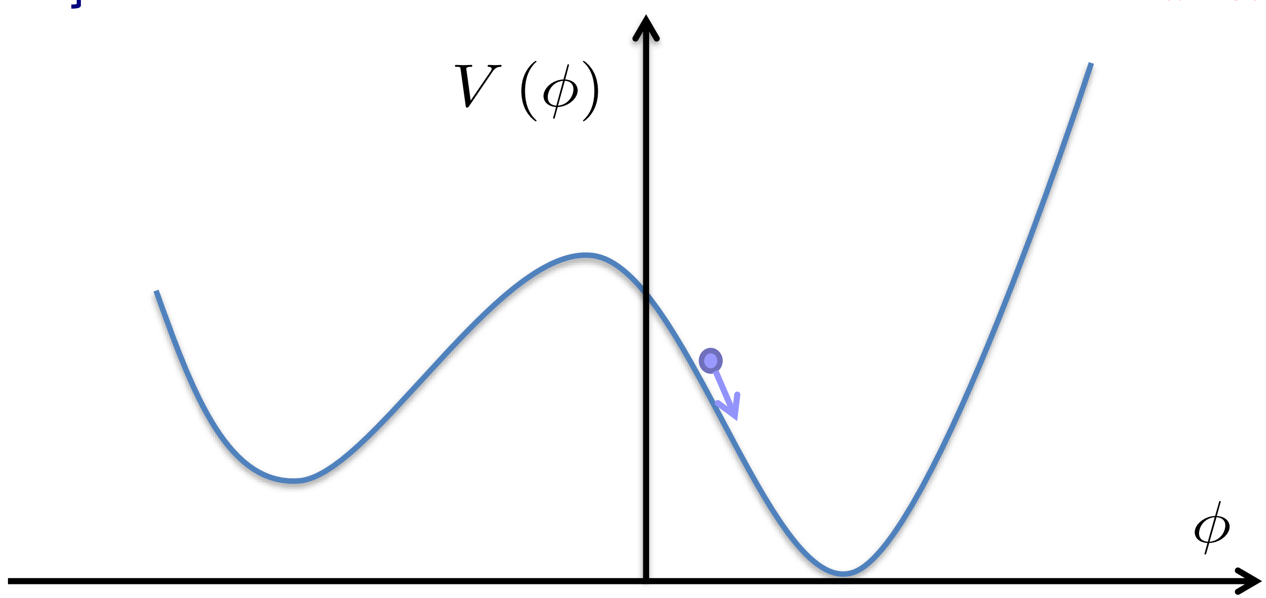
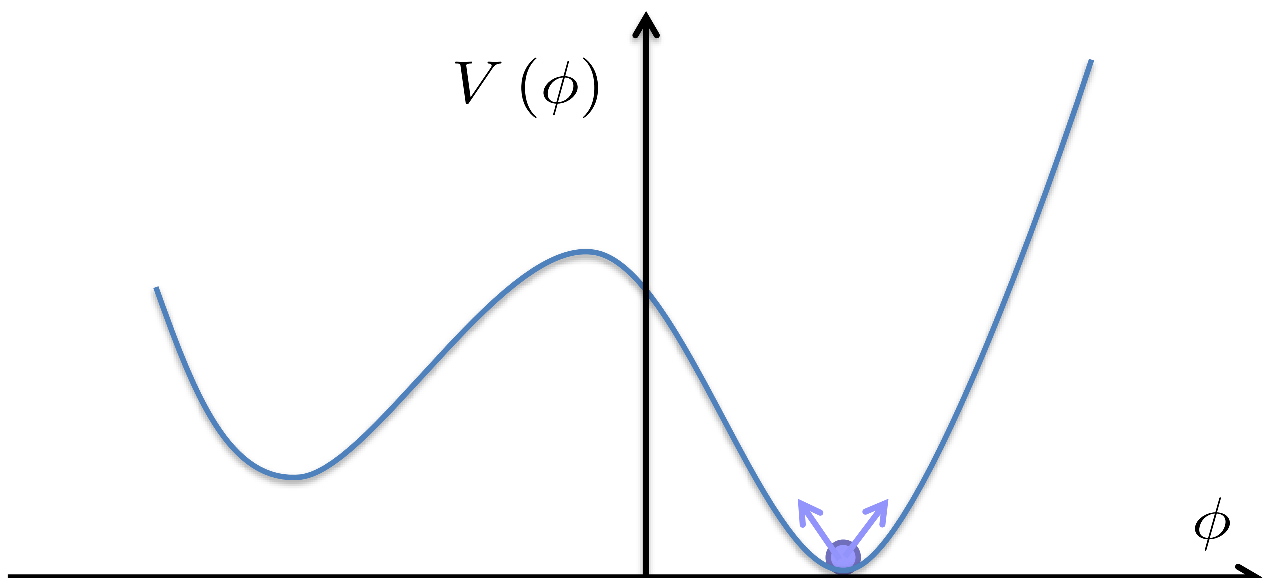


If the scalar field moves slowly (the potential is flat), then pressure is negative which, in the context of GR, means accelerated expansion and, hence, inflation takes place.

Stopping inflation: reheating epoch

Inflation (usually) stops when the field reaches a part of the potential which is no longer flat enough to support inflation; this happens in the vicinity of the minimum of the potential



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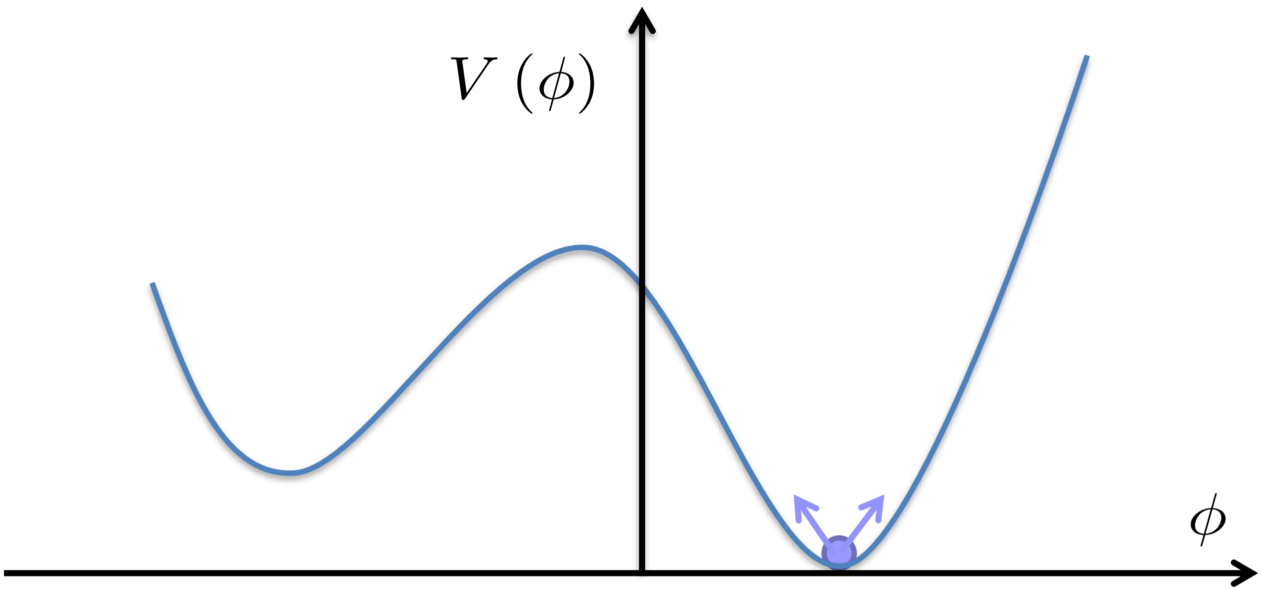

Then, the field oscillates, decays and the decay products thermalize …then the radiation dominated era starts … Reheating phase

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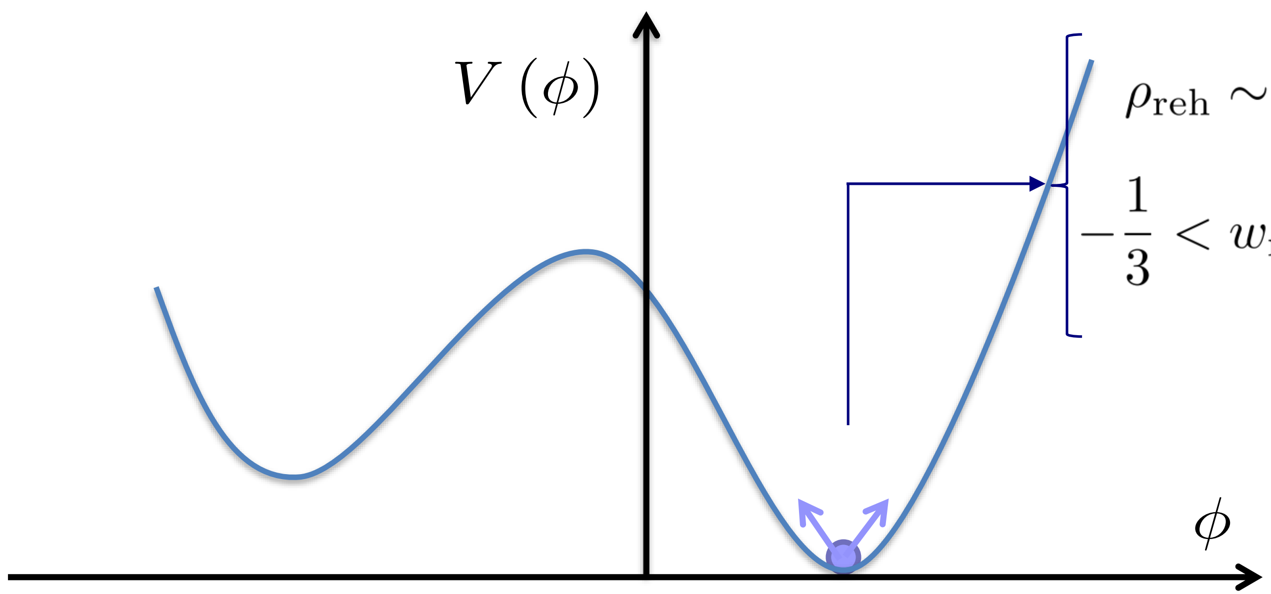

$$\rho_{\rm reh} \sim T^4_{\rm reh} < \rho_{\rm inf}$$

$$-\frac{1}{3} < w_{\rm reh} = \frac{p_{\rm reh}}{\rho_{\rm reh}} < 1$$

Then, the field oscillates, decays and the decay products thermalize …then the radiation dominated era starts … Reheating phase

Questions & open issues

- Who is the inflaton field? Can it be the Higgs field? Relation with high energy physics (SUSY, string theory, etc …)
- Is it a single field scenario or a more complicated model (several scalar fields, non minimal kinetic term, etc …)
- How did inflation start?
- Reheating, preheating etc …
- Are we sure it is inflation? Alternatives to inflation …

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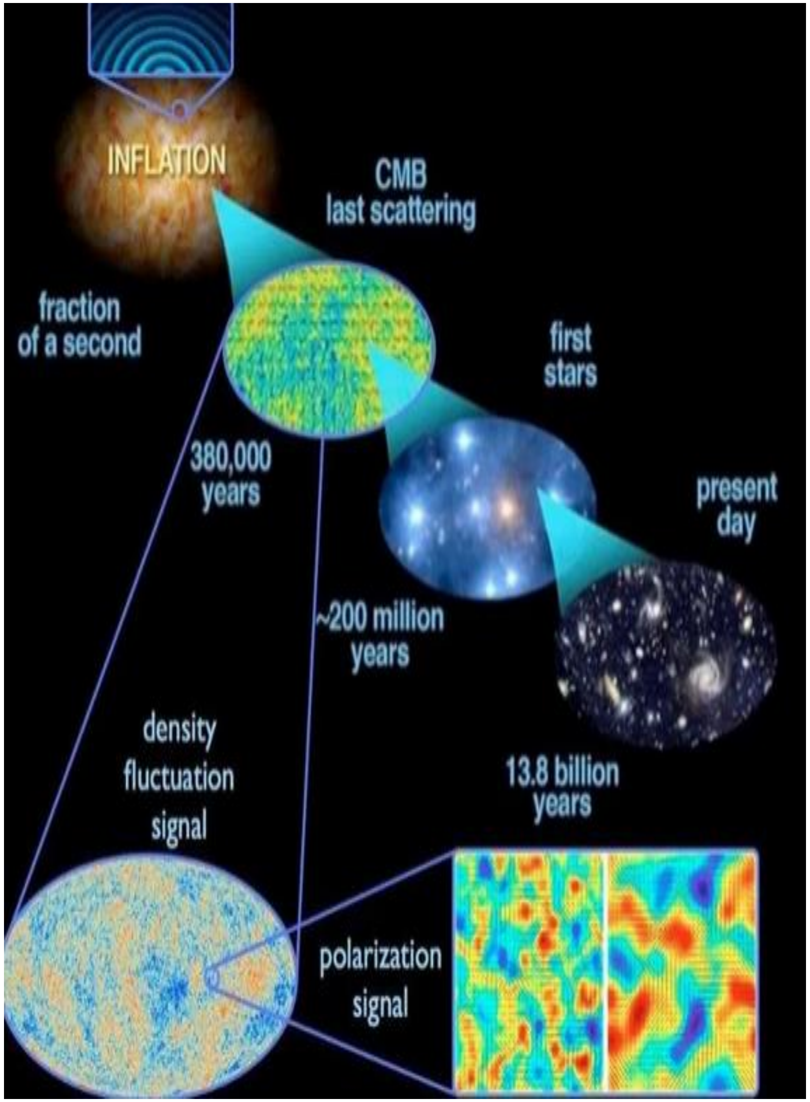
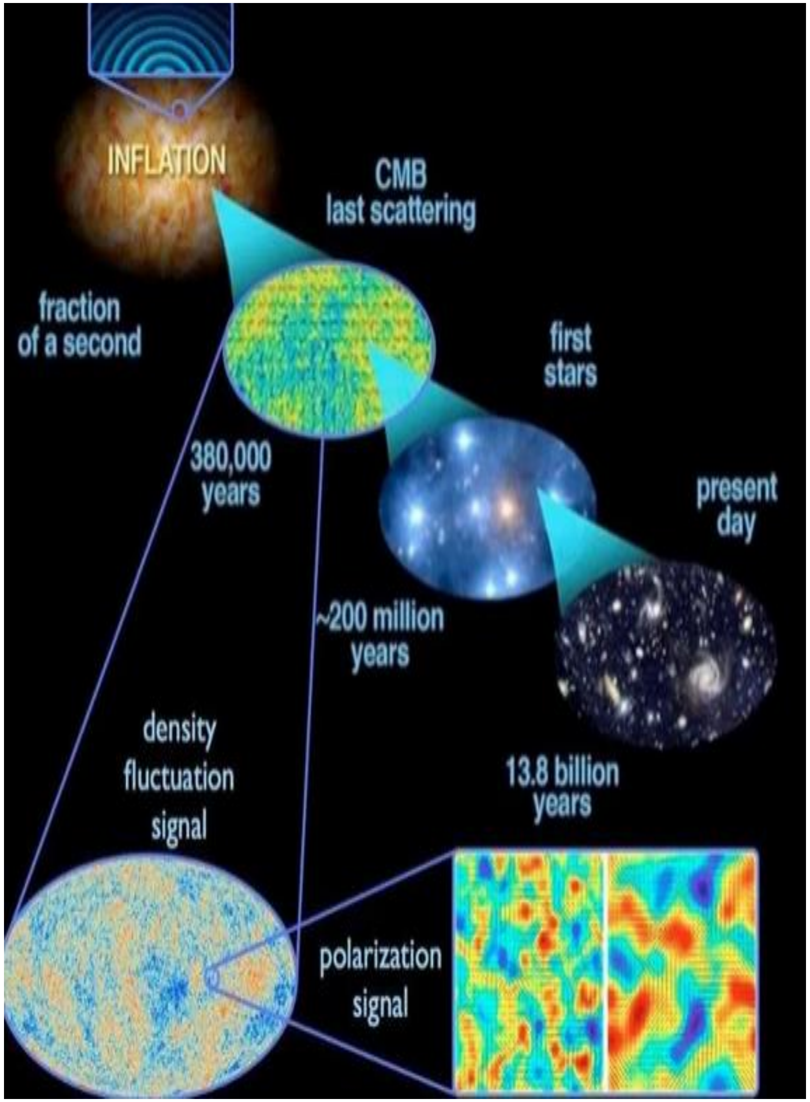


- Inflation not only solves the puzzles of the standard model but also gives an explanation of the origin of the structures (galaxies, CMB anisotropies etc …) observed in the Universe

Quantum perturbations



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- Main idea: quantum fluctuations of the gravitational and scalar (inflaton) fields during inflation amplified by gravitational instability and stretched by cosmic expansion

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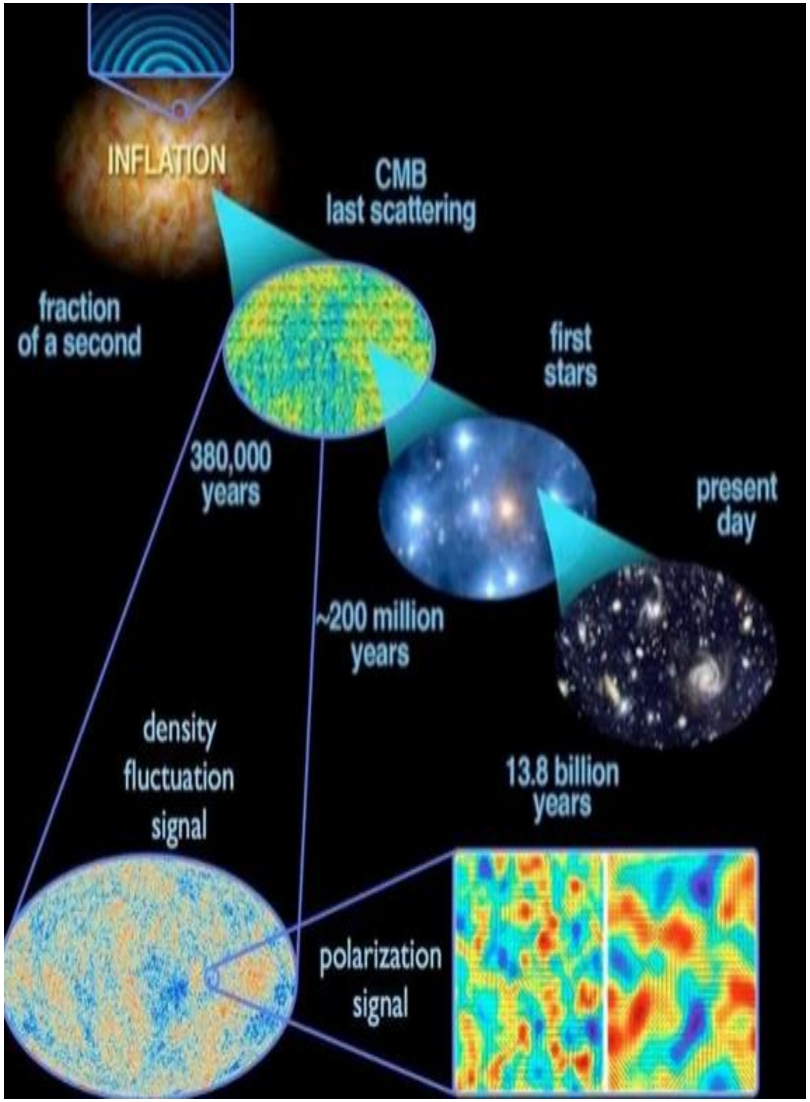

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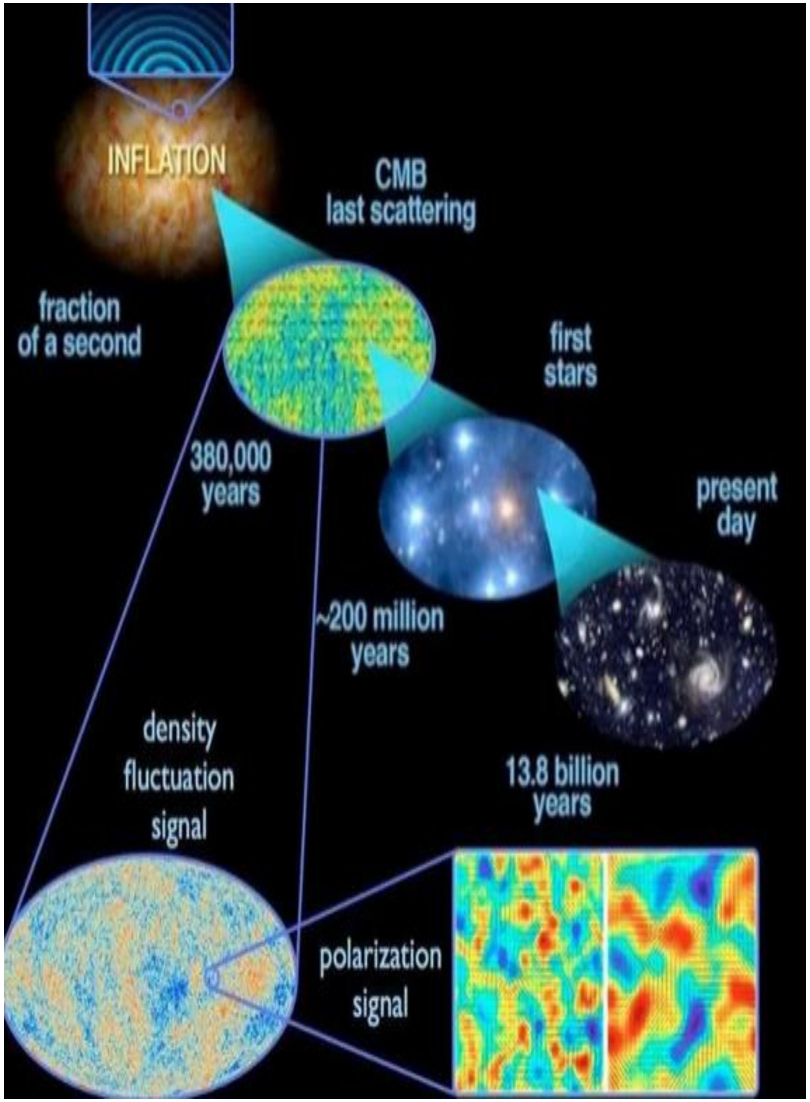
All structures in the Universe are of quantum-mechanical origin!

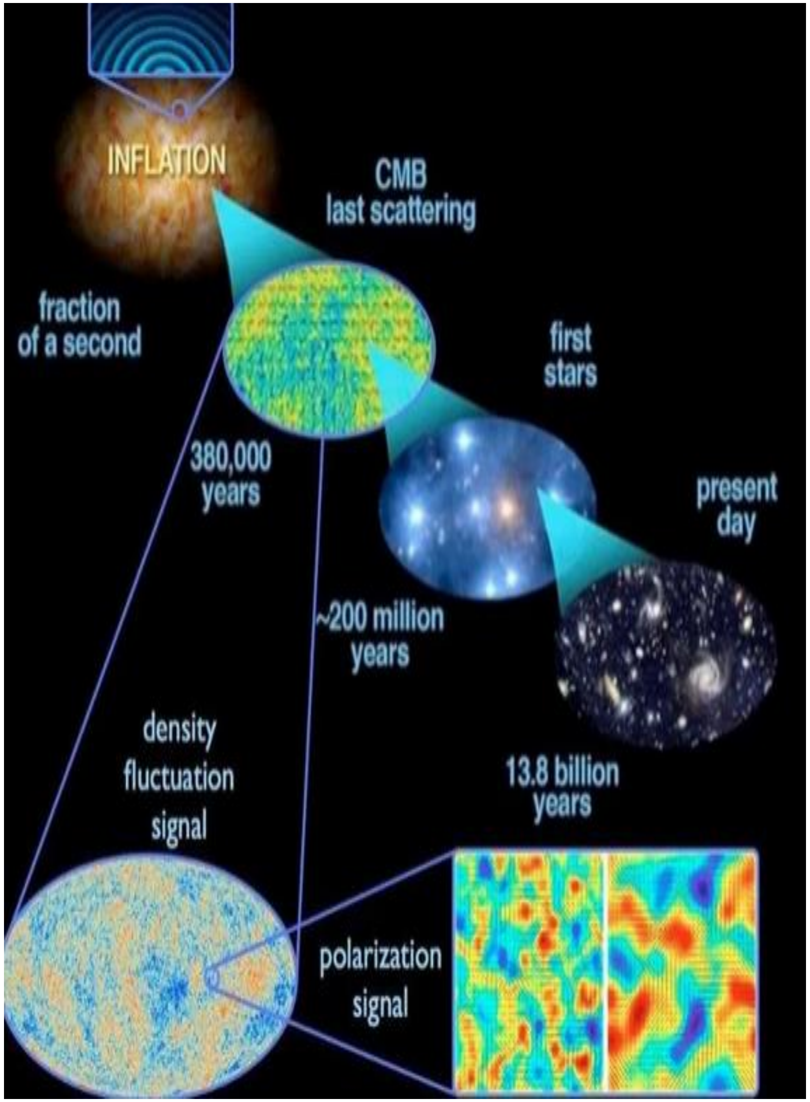


- In the early Universe, the perturbations are small so a perturbative approach is possible

$$g_{\mu\nu} = g_{\mu\nu}(t) + \delta\hat{g}_{\mu\nu}(t, \boldsymbol{x}) + \cdots$$

$$\phi = \phi(t) + \delta\hat{\phi}(t, \boldsymbol{x}) + \cdots$$

Quantum perturbations



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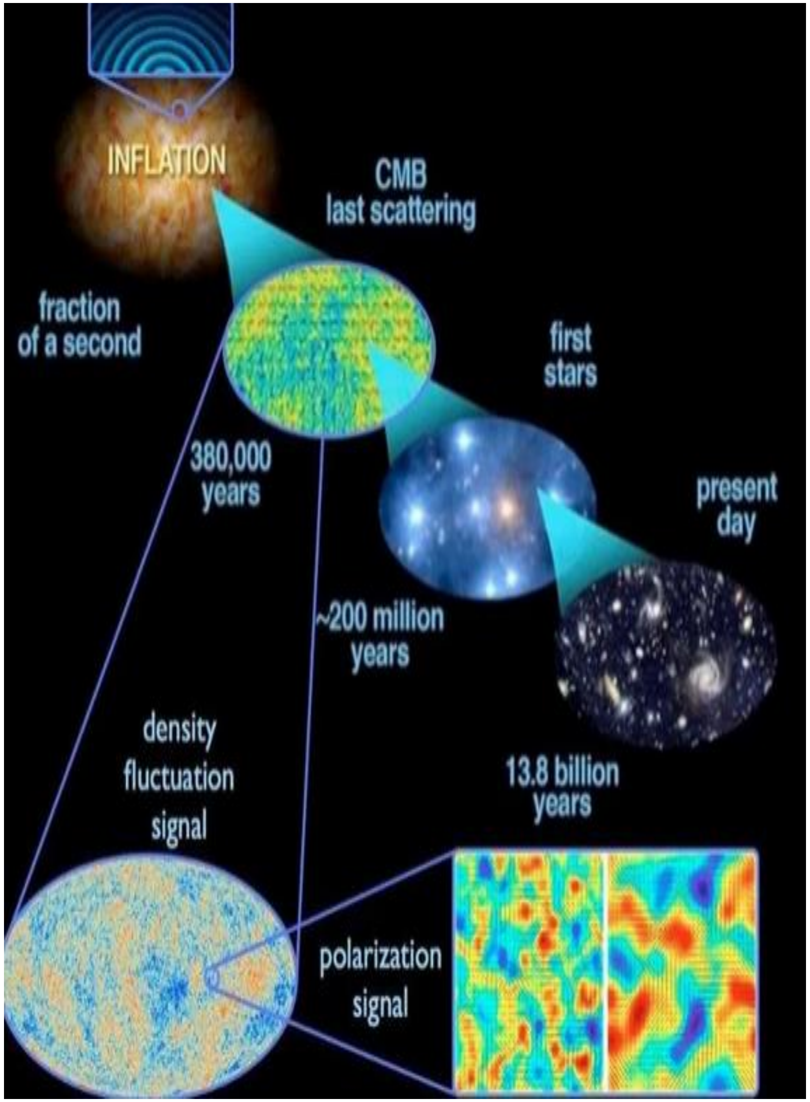
- Two types of perturbations are produced

  - Gravitational waves

  - Scalar perturbations



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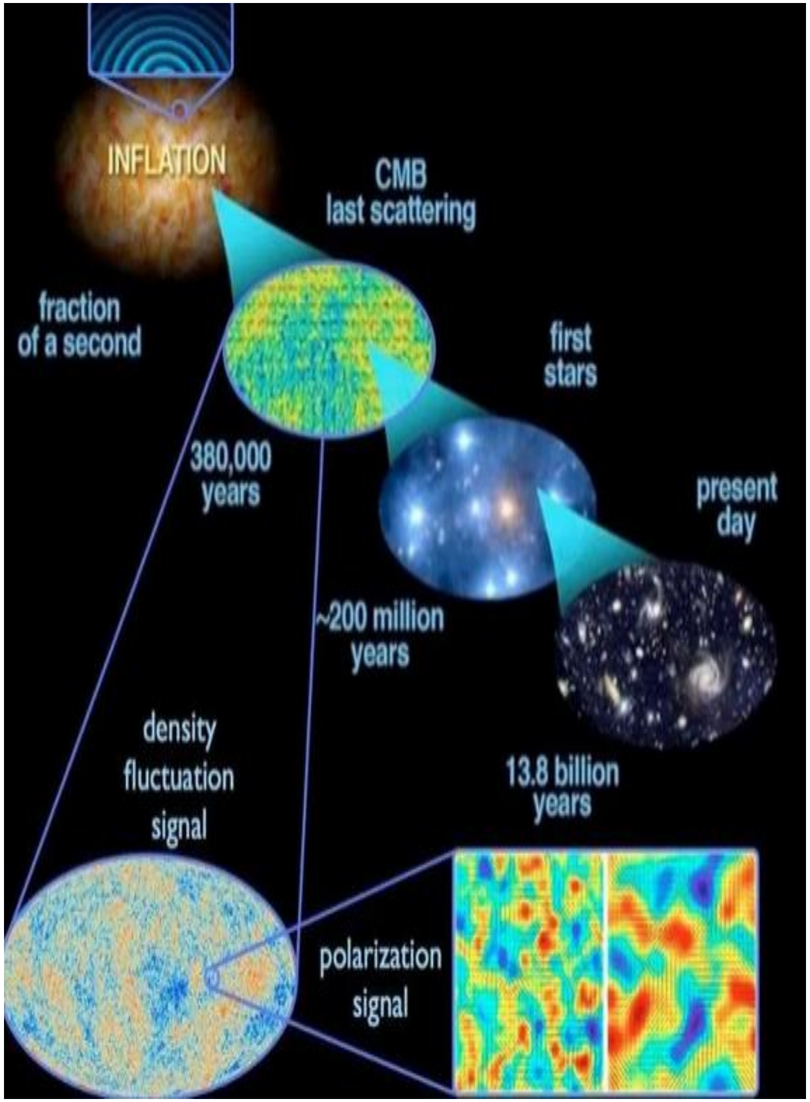
- Scalar perturbations are characterized by a one quantity: <u>curvature perturbations</u> $\hat{\zeta}(\eta, \boldsymbol{x})$

Quantum perturbations



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- Two types of perturbations are produced

  - Gravitational waves

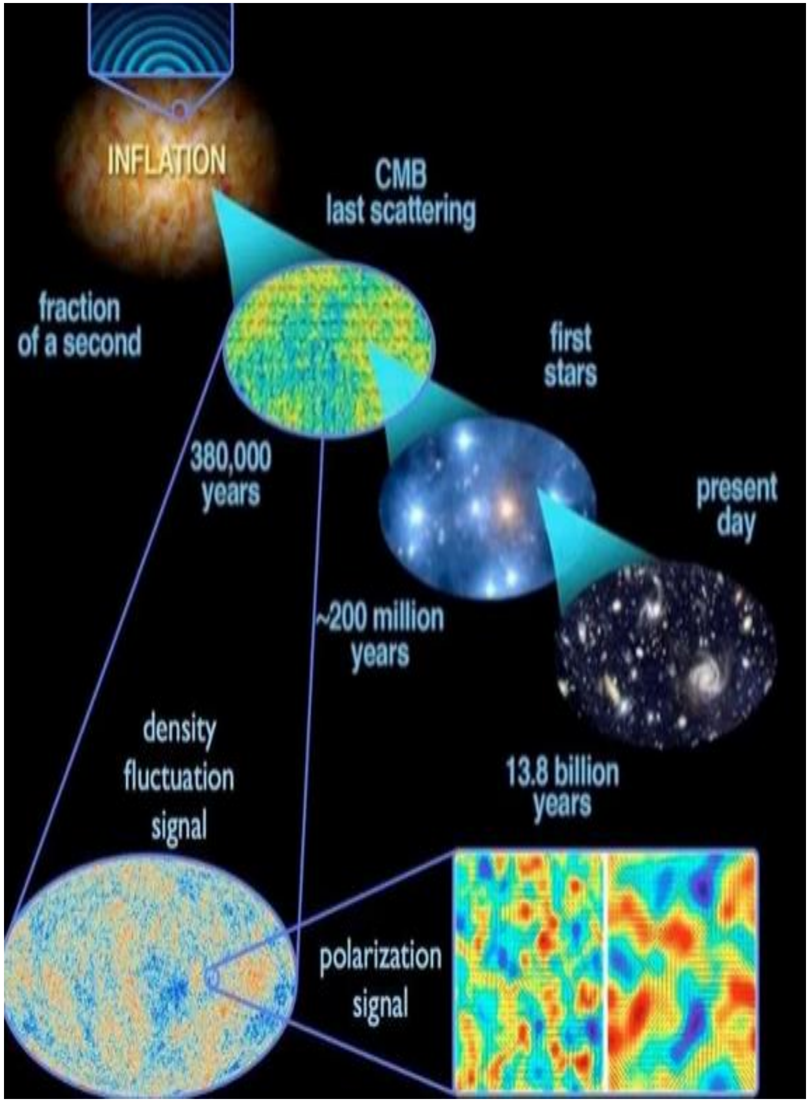
  - Scalar perturbations

- Scalar perturbations are characterized by a one quantity: curvature perturbations $\hat{\zeta}(\eta, \boldsymbol{x})$

- In Fourier space, this is a collection of oscillators, each mode k being described by a "position" and a momentum

$$(\hat{q}_{\boldsymbol{k}}, \hat{\pi}_{\boldsymbol{k}})$$

Quantum perturbations

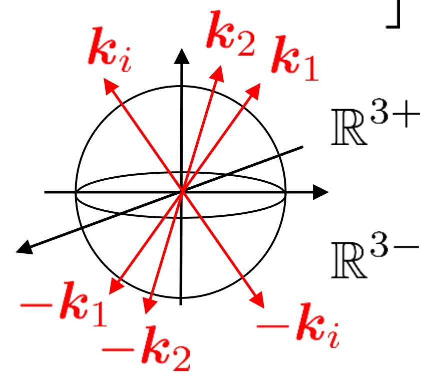


Hamiltonian of the cosmological perturbations:

$$S = \frac{M_{\mathrm{Pl}}^2}{2}\int \mathrm{d}^4x\,\sqrt{-g}\,R - \int \mathrm{d}^4x\sqrt{-g}\left[\frac{1}{2}g^{\mu\nu}\partial_\mu\phi\partial_\nu\phi + V(\phi)\right]$$

↓

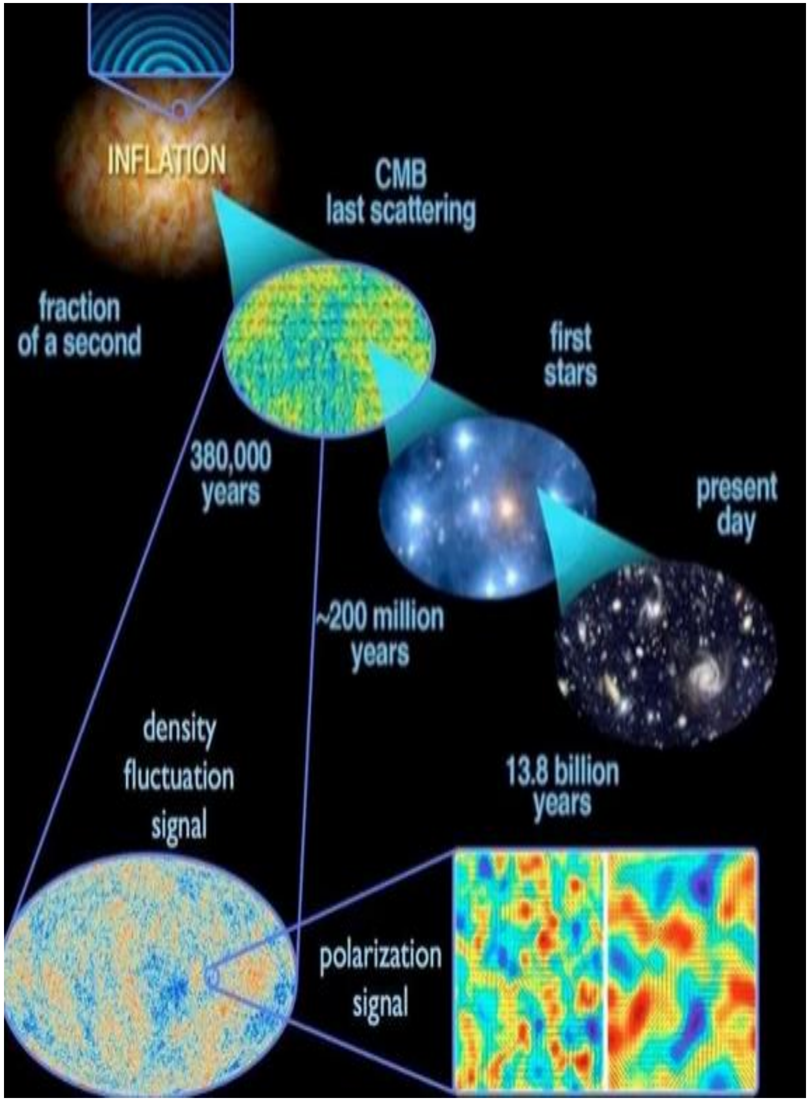
$$\hat{\mathcal{H}} = \int_{\mathbb{R}^{3+}} \mathrm{d}^3\boldsymbol{k}\,\hat{\mathcal{H}}_{\pm\boldsymbol{k}}$$



with

$$\hat{\mathcal{H}}_{\pm\boldsymbol{k}} = \frac{1}{2}\hat{\pi}_{\boldsymbol{k}}^2 + \frac{1}{2}k^2\hat{q}_{\boldsymbol{k}}^2 + \frac{1}{2}\hat{\pi}_{-\boldsymbol{k}}^2 + \frac{1}{2}k^2\hat{q}_{-\boldsymbol{k}}^2 + \frac{z'}{z}\left(\hat{q}_{\boldsymbol{k}}\hat{\pi}_{-\boldsymbol{k}} + \hat{q}_{-\boldsymbol{k}}\hat{\pi}_{\boldsymbol{k}}\right)$$

$$z(\eta) = a(\eta)\sqrt{2\epsilon_1}M_{\mathrm{Pl}}\,,\; \epsilon_1 = \frac{M_{\mathrm{Pl}}^2}{2}\left(\frac{V_\phi}{V}\right)^2 \ll 1$$



- The (pure) state of the system is a Gaussian two-mode squeezed state

$$\Psi[\zeta(\eta, \boldsymbol{x})] = \prod_{\boldsymbol{k}\in\mathbb{R}^{3+}} \psi\left(\eta, q_{\boldsymbol{k}}, q_{-\boldsymbol{k}}\right)$$

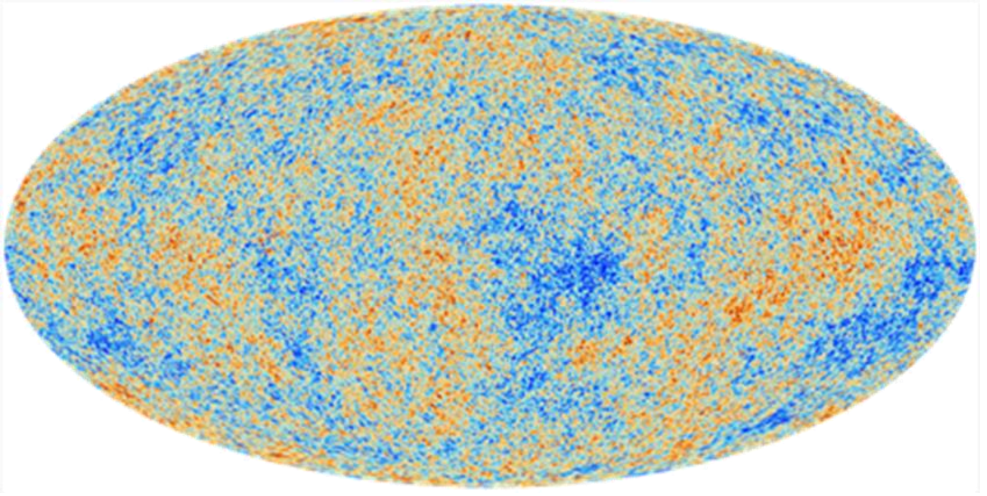
with

$$\psi = \frac{e^{A(r_{\boldsymbol{k}},\varphi_{\boldsymbol{k}})k(q_{\boldsymbol{k}}^2+q_{-\boldsymbol{k}}^2)-B(r_{\boldsymbol{k}},\varphi_{\boldsymbol{k}})kq_{\boldsymbol{k}}q_{-\boldsymbol{k}}}}{\sqrt{\pi}\cosh(r_{\boldsymbol{k}})\sqrt{1-e^{-4i\varphi_{\boldsymbol{k}}}\tanh^2(r_{\boldsymbol{k}})}}$$

It is an entangled state

$$\psi(\eta, q_{\boldsymbol{k}}, q_{-\boldsymbol{k}}) \neq \psi(\eta, q_{\boldsymbol{k}})\psi(\eta, q_{-\boldsymbol{k}})$$

The cosmological two-mode squeezed state is (very!) strongly squeezed



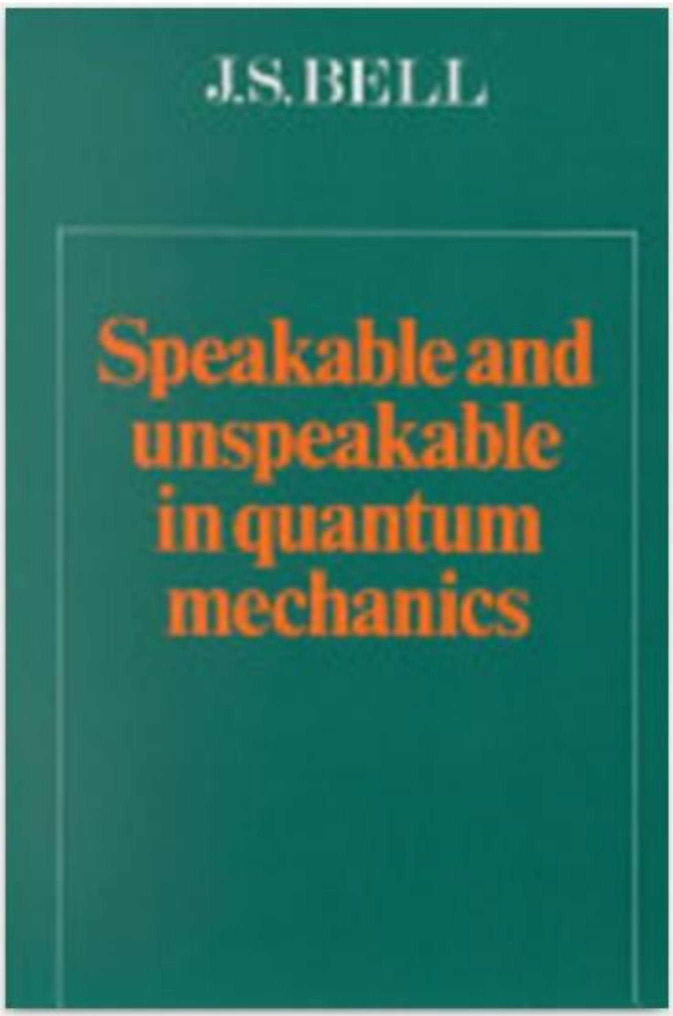
CMB anisotropy is the strongest
squeezed state ever produced in Nature

$$r_k = \mathcal{O}\left(10^2\right)$$

$$-10\log_{10}\left(e^{-2r_k}\right)\ \mathrm{dB}$$

- ~ 15 dB in the lab
- > 500 dB inflation

Are the perturbations quantum or classical?



- The Wigner function is positive since the state is Gaussian

- But the state is entangled and discord is large …

- The very same paradox was studied by John Bell at about the same time cosmic inflation was invented …

21

EPR correlations and EPW distributions

Dedicated to Professor E. P. Wigner

It is known that with Bohm's example of EPR correlations, involving particles with spin, there is an irreducible non-locality. The non-locality cannot be removed by the introduction of hypothetical variables unknown to ordinary quantum mechanics. How is it with the original EPR example involving two particles of zero spin? Here we will see that the Wigner phase space distribution[1] illuminates the problem.

"Cosmic inflation, quantum information and the pioneering role of John Bell in Cosmology", Universe 5 (2019), arXiv:1904.00083

Other important questions

- The quantum-to-classical transition

- Can we obtain a direct observational signature of the quantum origin of the perturbations?

- The role of decoherence. Attempts to write a master equation for cosmological perturbations, impact for the quantum to classical transition …

- The quantum measurement problem in Cosmology. The quantum state of perturbations is homogeneous and isotropic (e.g. it is invariant under the translation operator). How do we produce a state which is not homogeneous and isotropic?

- Can we use quantum perturbations to probe quantum mechanics itself?

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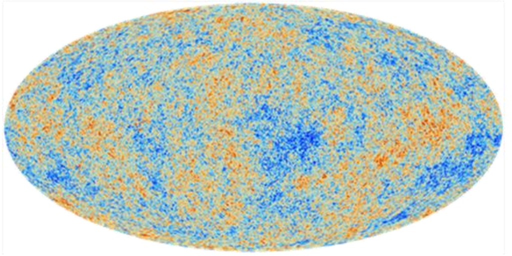
How to probe inflation?

Inflation can be observationally probed by measuring correlations functions



$$\left\langle \frac{\delta T}{T}(\boldsymbol{e}_1)\frac{\delta T}{T}(\boldsymbol{e}_2)\right\rangle \quad \text{related to} \quad \left\langle \Psi|\hat{\zeta}(\eta,\boldsymbol{x})\hat{\zeta}(\eta,\boldsymbol{x}+\boldsymbol{r})|\Psi\right\rangle$$

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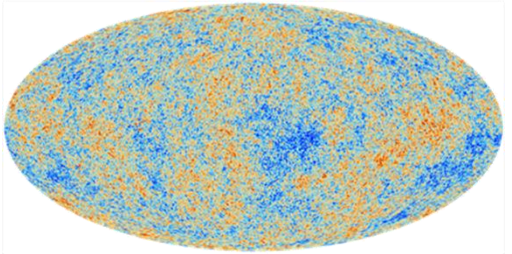

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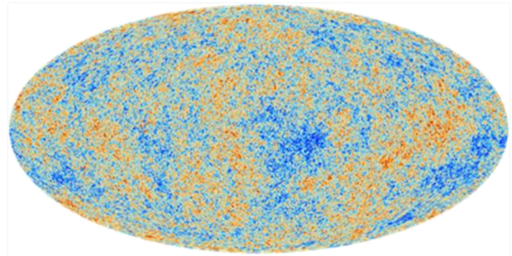
$$\left\langle \Psi|\hat{\zeta}\left(\eta,\boldsymbol{x}\right)\hat{\zeta}\left(\eta,\boldsymbol{x}+\boldsymbol{r}\right)|\Psi\right\rangle = \int \prod_{\boldsymbol{k}\in\mathbb{R}^{3+}} \mathrm{d}\zeta_{\boldsymbol{k}}^{\mathrm{R}}\mathrm{d}\zeta_{\boldsymbol{k}}^{\mathrm{I}}\Psi^{\star}[\zeta(\eta,\boldsymbol{x})]\zeta(\eta,\boldsymbol{x})\zeta(\eta,\boldsymbol{x}+\boldsymbol{r})\Psi[\zeta(\eta,\boldsymbol{x})]$$

$$= \int_0^{+\infty}\frac{\mathrm{d}k}{k}\frac{\sin(kr)}{kr}\mathcal{P}_{\zeta}(k)$$

Power spectrum

Inflation can be observationally probed by measuring correlations functions



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Power spectrum

$$\mathcal{P}_\zeta(k) = \frac{H^2}{8\pi^2\epsilon_1 M^2_{\mathrm{Pl}}}\left[1-2(C+1)\epsilon_1 - C\epsilon_2 - \underbrace{(2\epsilon_1+\epsilon_2)}\ln\frac{k}{k_*}\right]$$

$$= n_\mathrm{S} - 1 \ll 1$$

$$\epsilon_1 = -\frac{\dot{H}}{H^2} = \frac{M^2_{\mathrm{Pl}}}{2}\left(\frac{V_\phi}{V}\right)^2$$

$$\epsilon_2 = 2M^2_{\mathrm{Pl}}\left[\left(\frac{V_\phi}{V}\right)^2 - \frac{V_{\phi\phi}}{V}\right]$$

Prediction: the spectral index should be close to one but different from one

Planck Measurements

- Universe spatially flat: $\Omega_{\mathcal{K}} = -0.040^{+0.038}_{-0.041}$

- Adiabatic perturbations: $\alpha^{(2,2500)}_{\mathcal{R}\mathcal{R}} \in [0.985, 0.999]$

- Gaussian perturbations:

$$\left\{ \begin{aligned} f_{\rm NL}^{\rm loc} &= -0.9 \pm 5 \\ f_{\rm NL}^{\rm eq} &= -26 \pm 47 \\ f_{\rm NL}^{\rm ortho} &= -38 \pm 24 \end{aligned} \right.$$

- Almost scale invariant power spectrum: $n_{\rm S} = 0.9645 \pm 0.0049$

- Background of quantum gravitational waves: $r = \dfrac{T}{S} < 0.035$

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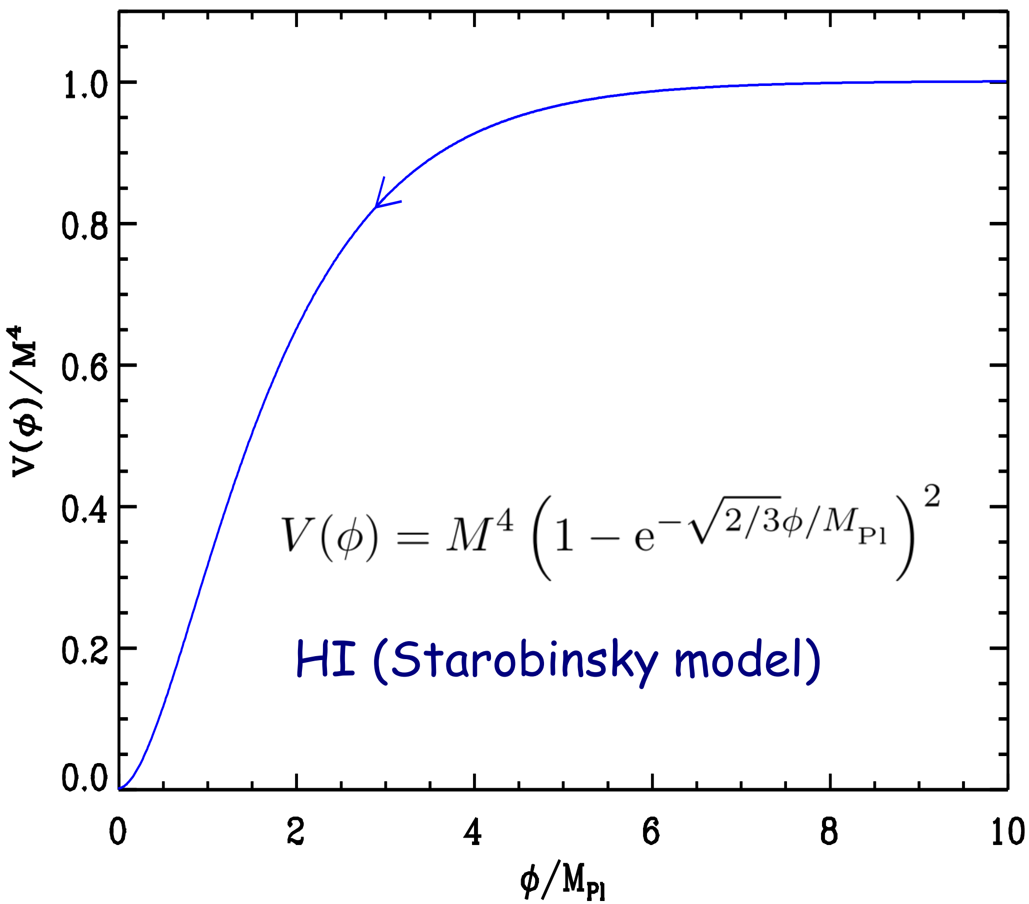
So far, all observations are consistent with single field scenarios

Plateau inflationary models are the winners!



J. Martin, C. Ringeval and V. Vennin, Phys. Dark Univ. 5-6 (2014) 75, arXiv:1303.3787

J. Martin, C. Ringeval, R. Trotta and V. Vennin, JCAP 1403 (2014) 039, arXiv1312.3529

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Recap

- Despite some issues, the ΛCDM is a very good model, explaining a wide range of data

- Inflation completes this model, makes it much more consistent and provides a convincing model for structure formation, in agreement with cosmological observations

- According to inflation, structures are nothing but quantum fluctuations amplified by gravitational instability and stretched to cosmological scales

- Inflation is the only situation in Physics where GR and QM are needed to understand the theory and derive predictions and where, at the same time, we have high accuracy data; can it be used to probe the interface between GR and QM?

Future (inflationary theory)

- Physical nature of inflation. Which energy scale? Is inflation related to the Higgs field? Do we deal with single field inflation or is the inflationary mechanism more complicated?

- How did inflation start?

- Reheating: how did inflation stop? What is the coupling of the inflaton with the rest of the world?

- Can we find a direct proof of the quantum origin of the perturbations?

- Can inflation be useful for explaining other aspects of cosmology, eg magnetogenesis, PBHs, …

Future (observations to further probe inflation)

- Primordial gravitational waves

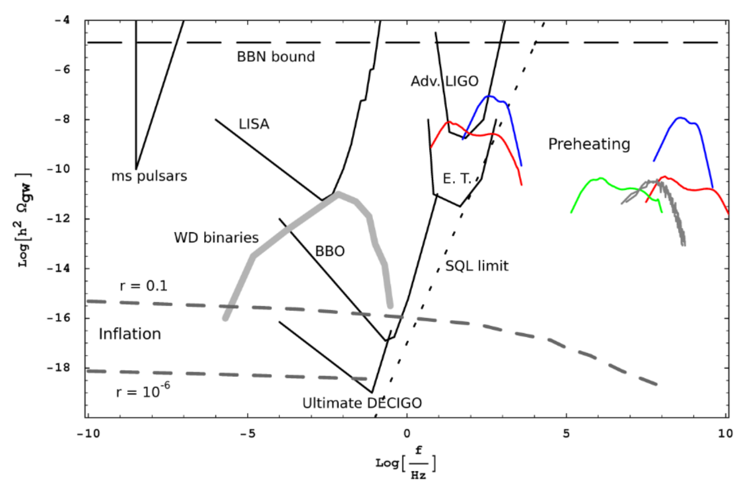
  - Produced during slow-roll inflation but not yet detected

  - No general prediction (no lower bound); model-dependent

  - Hope to detect the signal through CMB B-mode polarization (S4, LiteBIRD satellite)

  - Threshold $r = \frac{T}{S} \sim 10^{-3}$ to be compared with $r_{\text{staro}} \sim 2 - 4 \times 10^{-3}$

  - Important consequences:
    - Energy scale of inflation
    - First derivative of inflaton's potential
    - Field excursion
    - Model selection
    - More on reheating
    - Gravity must be quantized



  - GW also produced during reheating (different amplitude and frequency) direct detection?

Future (observations to further probe inflation)

- Primordial Non-Gaussianities

  - Produced during (slow-roll) inflation thanks to non-linear terms in perturbation theory but not yet detected

  - No general prediction; model-dependent

    - Single field: $f_{\rm NL} \sim 10^{-2}$

    - More complicated models: larger signal [$f_{\rm NL} \sim \mathcal{O}(1)$] and, moreover, the "structure" of the signal allows us to identify the underlying models

  - Important since gives access to the next-to-leading order of perturbation theory; window on new degrees of freedom in the early Universe

  - Future CMB and LSS surveys; hope to reach: $f_{\rm NL} \sim \mathcal{O}(1)$

Conclusions

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Thank you for your attention