Optical white-black hole horizons and entanglement

Dimitrios Kranas

Laboratoire de Physique de l'Ecole Normale Supérieure (LPENS)

Analogue Gravitation and Cosmology, LKB à Paris, 8 November 2023

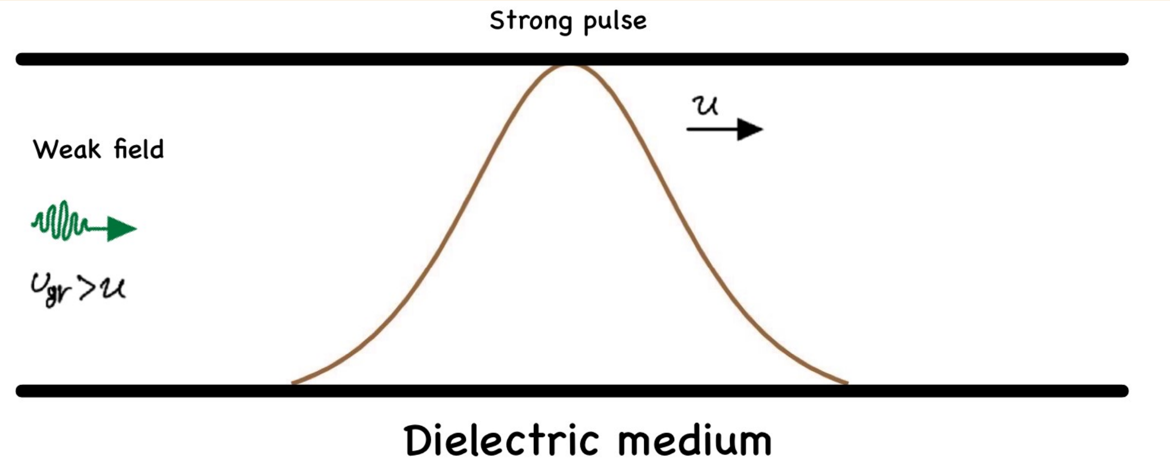
In collaboration with I. Agullo and A. J. Brady

- I. Agullo, A. J. Brady, and D. Kranas. **"Quantum Aspects of Stimulated Hawking Radiation in an Optical Analog White-Black Hole Pair"**. In: Phys. Rev. Lett. (2022).
- I. Agullo, A. J. Brady, and D. Kranas. **"Event horizons are tunable factories of quantum entanglement"**. In: J. Mod. Phys. D (2022).
- A. J. Brady, Ivan Agullo, and D. Kranas. **"Symplectic circuits, entanglement, and stimulated Hawking radiation in analog gravity"**. In: Phys. Rev. D 106 (2022).
- I. Agullo, A. J. Brady, and D. Kranas. **"Robustness of entanglement in Hawking radiation for optical systems immersed in thermal baths"**. In: Phys. Rev. D (2023).

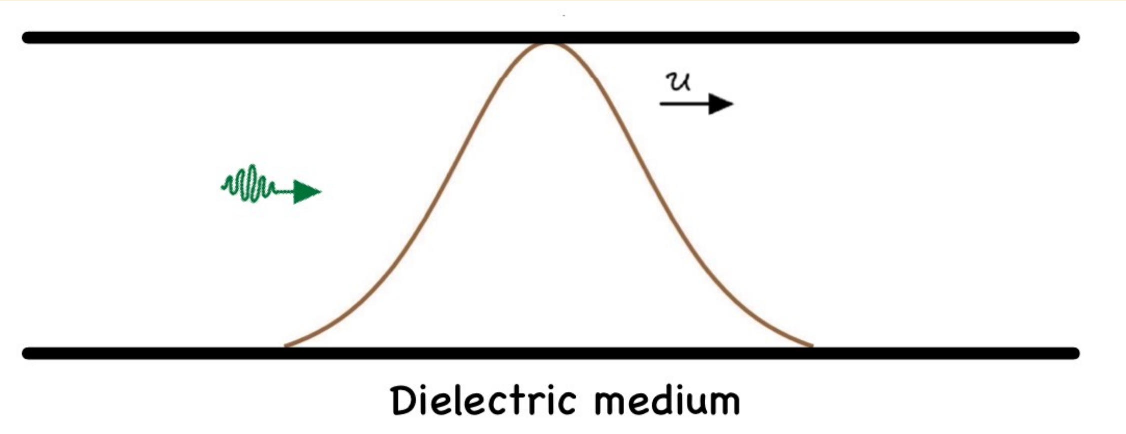
Apply tools from quantum information theory to quantify the amount of entanglement produced by causal horizons via the Hawking process and study its dependence on the input quantum state.

Optical analogue white hole-black hole pairs

Optical WH-BH: Schematic representation

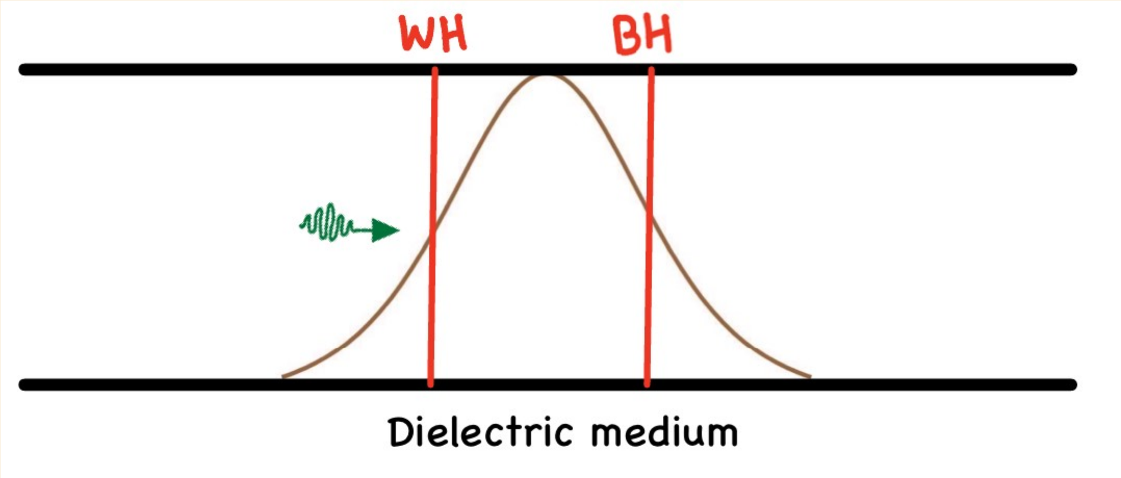


Optical WH-BH: Schematic representation



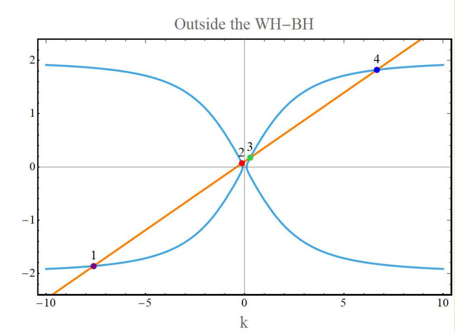
$$n_{eff}(\omega, x, t) = n(\omega) + \delta n(x, t), \quad \delta n(x, t) = \alpha E_s^2(x, t)$$

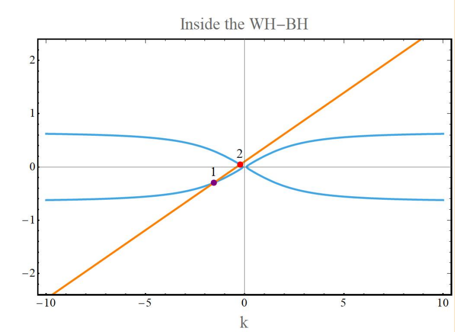
Optical WH-BH: Schematic representation



Dispersion relation in the comoving frame

$$\gamma(\omega + uk) = \pm\left[\Omega_o - \alpha E_o^2\,\mathrm{sech}^2\left(\frac{\chi}{\Delta}\right)\right]\sqrt{1 + \frac{g^2}{\omega^2 - k^2 - g^2}}$$



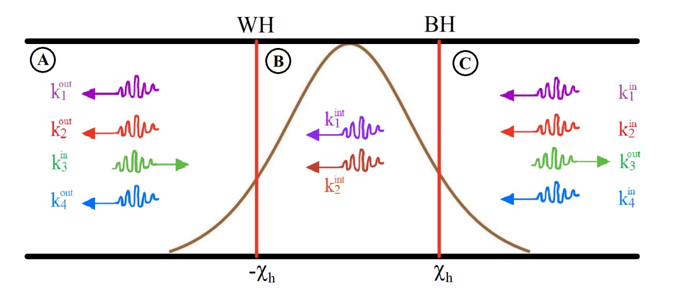


- There are two **short** wavelength modes k_1, k_4 and two **long** wavelength modes k_2, k_3.
- In the comoving frame k_1, k_2, k_4 move in the **left** direction while k_3 moves in the **right** direction.

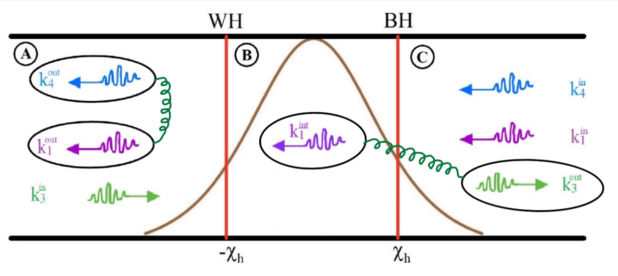
Modes and emergent causal structure

- In the comoving frame (χ, τ) w.r.t to the strong pulse, the system admits static solutions $\rightarrow$ Conservation of frequency.

- In the comoving frame (χ, τ) w.r.t to the strong pulse, the system admits static solutions $\rightarrow$ Conservation of frequency.



Modes and emergent causal structure



Each horizon behaves as a two-mode squeezer producing entangled Hawking pairs!!

$$\hat{a}_A^{out} = \cosh r\,\hat{a}_A^{in} + e^{i\varphi}\sinh r\,\hat{a}_B^{\dagger\,in},$$
$$\hat{a}_B^{out} = e^{i\varphi}\sinh r\,\hat{a}_A^{\dagger\,in} + \cosh r\,\hat{a}_B^{in}$$

Elements of quantum information theory of Gaussian states

Reference: A. Serafini, *Quantum Continuous Variables: A Primer of Theoretical Methods* (2017).

Elements of quantum information theory of Gaussian states

- Consider a system of N quantum bosonic degrees of freedom (harmonic oscillators): $\hat{\boldsymbol{R}} = (\hat{x}_1, \hat{p}_1, \ldots, \hat{x}_N, \hat{p}_N)$.

  $\rightarrow$ Commutation relations: $[\hat{x}, \hat{p}] = i\hbar \rightarrow [\hat{R}^i, \hat{R}^j] = i\hbar\Omega^{ij}, \qquad \Omega^{ij} = \bigoplus \begin{pmatrix} 0 & 1 \\ -1 & 0 \end{pmatrix}.$

Elements of quantum information theory of Gaussian states

- Consider a system of N quantum bosonic degrees of freedom (harmonic oscillators): $\hat{\boldsymbol{R}} = (\hat{x}_1, \hat{p}_1, ..., \hat{x}_N, \hat{p}_N)$.
  $\rightarrow$ Commutation relations: $[\hat{x}, \hat{p}] = i\hbar \rightarrow [\hat{R}^i, \hat{R}^j] = i\hbar\Omega^{ij}, \qquad \Omega^{ij} = \bigoplus \begin{pmatrix} 0 & 1 \\ -1 & 0 \end{pmatrix}.$

- **Gaussian state** $\hat{\rho}$: Completely characterized by the **first** and **second** moments.
  $\rightarrow$ $\mu^i \equiv \mathrm{Tr}\left[\hat{\rho}\hat{R}^i\right], \quad \sigma^{ij} \equiv \mathrm{Tr}\left[\hat{\rho}\{(\hat{R}^i - \mu^i), (\hat{R}^j - \mu^j)\}\right].$

Elements of quantum information theory of Gaussian states

- Consider a system of N quantum bosonic degrees of freedom (harmonic oscillators): $\hat{\boldsymbol{R}} = (\hat{x}_1, \hat{p}_1, ..., \hat{x}_N, \hat{p}_N)$.

  $\rightarrow$ Commutation relations: $[\hat{x}, \hat{p}] = i\hbar \rightarrow [\hat{R}^i, \hat{R}^j] = i\hbar\Omega^{ij}, \qquad \Omega^{ij} = \bigoplus \begin{pmatrix} 0 & 1 \\ -1 & 0 \end{pmatrix}.$

- **Gaussian state** $\hat{\rho}$: Completely characterized by the **first** and **second** moments.

  $\rightarrow$ $\mu^i \equiv \mathrm{Tr}\left[\hat{\rho}\hat{R}^i\right], \quad \sigma^{ij} \equiv \mathrm{Tr}\left[\hat{\rho}\{(\hat{R}^i - \mu^i), (\hat{R}^j - \mu^j)\}\right].$

- **Evolution**: $(\boldsymbol{\mu}^{\text{in}}, \boldsymbol{\sigma}^{\text{in}}) \longrightarrow (\boldsymbol{\mu}^{\text{out}}, \boldsymbol{\sigma}^{\text{out}}), \quad \boldsymbol{\mu}^{\text{out}} = \boldsymbol{S}\boldsymbol{\mu}^{\text{in}}, \quad \boldsymbol{\sigma}^{\text{out}} = \boldsymbol{S}\boldsymbol{\sigma}^{\text{in}}\boldsymbol{S}^{\mathsf{T}}, \quad \boldsymbol{S}\cdot\boldsymbol{\Omega}\cdot\boldsymbol{S}^{\mathsf{T}} = \boldsymbol{\Omega}.$

Elements of quantum information theory of Gaussian states

- Consider a system of N quantum bosonic degrees of freedom (harmonic oscillators): $\hat{\boldsymbol{R}} = (\hat{x}_1, \hat{p}_1, ..., \hat{x}_N, \hat{p}_N)$.
  - $\rightarrow$ Commutation relations: $[\hat{x}, \hat{p}] = i\hbar \rightarrow [\hat{R}^i, \hat{R}^j] = i\hbar\Omega^{ij}$, $\quad \Omega^{ij} = \bigoplus \begin{pmatrix} 0 & 1 \\ -1 & 0 \end{pmatrix}$.
- **Gaussian state** $\hat{\rho}$: Completely characterized by the **first** and **second** moments.
  - $\rightarrow$ $\mu^i \equiv \mathrm{Tr}\left[\hat{\rho}\hat{R}^i\right], \quad \sigma^{ij} \equiv \mathrm{Tr}\left[\hat{\rho}\{(\hat{R}^i - \mu^i), (\hat{R}^j - \mu^j)\}\right]$.
- **Evolution**: $(\boldsymbol{\mu}^{\text{in}}, \boldsymbol{\sigma}^{\text{in}}) \longrightarrow (\boldsymbol{\mu}^{\text{out}}, \boldsymbol{\sigma}^{\text{out}}), \quad \boldsymbol{\mu}^{\text{out}} = \boldsymbol{S}\boldsymbol{\mu}^{\text{in}}, \quad \boldsymbol{\sigma}^{\text{out}} = \boldsymbol{S}\boldsymbol{\sigma}^{\text{in}}\boldsymbol{S}^{\mathsf{T}}, \quad \boldsymbol{S} \cdot \boldsymbol{\Omega} \cdot \boldsymbol{S}^{\mathsf{T}} = \boldsymbol{\Omega}$.
- **Entanglement**: Quantified by *Logarithmic Negativity LN*.
  - $\rightarrow$ Based on the Positivity of Partial Transposition (PPT) criterion.
  - $\rightarrow$ Can be used to quantify the entanglement of mixed states.
  - $\rightarrow$ Can be computed from $\boldsymbol{\sigma}$.
  - $\rightarrow$ For Gaussian states where either subsystem is made of a single degree of freedom, LN is a **faithful** entanglement quantifier.
  - $\rightarrow$ Measures entanglement in units of Bell states.

Experimental status

- Observing the spontaneous Hawking effect, even in highly controllable analog gravity platforms, has proven to be an extremely challenging task.

Experimental status

- Observing the spontaneous Hawking effect, even in highly controllable analog gravity platforms, has proven to be an extremely challenging task.
- Only one group has claimed observation of the spontaneous Hawking process in the BEC [J. Steinhauer (2016)] .

Experimental status

- Observing the spontaneous Hawking effect, even in highly controllable analog gravity platforms, has proven to be an extremely challenging task.
- Only one group has claimed observation of the spontaneous Hawking process in the BEC [J. Steinhauer (2016)] .
- Further confirmation needed!!

A promising avenue

Let's stimulate particle creation by illuminating the horizons!!! [S. Weinfurtner et al (2011)], [Drori et al (2019)]

A promising avenue

Let's stimulate particle creation by illuminating the horizons!!! [S. Weinfurtner et al (2011)], [Drori et al (2019)]

- Coherent state: $\boldsymbol{\mu} \neq 0$, $\boldsymbol{\sigma} = \boldsymbol{I}$.

A promising avenue

Let's stimulate particle creation by illuminating the horizons!!! [S. Weinfurtner et al (2011)], [Drori et al (2019)]

- Coherent state: $\boldsymbol{\mu} \neq 0$, $\boldsymbol{\sigma} = \boldsymbol{I}$.
  — Increase intensity of the Hawking radiation? ✓
  — Enhances the generated entanglement? ✗

A promising avenue

Let's stimulate particle creation by illuminating the horizons!!! [S. Weinfurtner et al (2011)], [Drori et al (2019)]

- Coherent state: $\boldsymbol{\mu} \neq 0$, $\boldsymbol{\sigma} = \boldsymbol{I}$.
  - Increase intensity of the Hawking radiation? ✓
  - Enhances the generated entanglement? ✗
- Single-mode squeezed state: $\boldsymbol{\mu} = \mathbf{0}$, $\boldsymbol{\sigma} \neq \boldsymbol{I}$.

A promising avenue

Let's stimulate particle creation by illuminating the horizons!!! [S. Weinfurtner et al (2011)], [Drori et al (2019)]

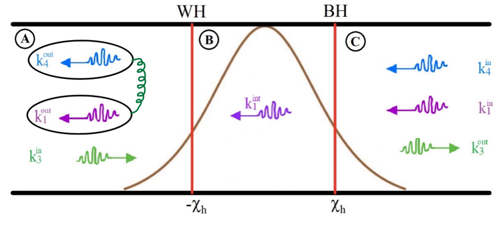
- Coherent state: $\boldsymbol{\mu} \neq 0$, $\boldsymbol{\sigma} = \boldsymbol{I}$.
  - Increase intensity of the Hawking radiation? ✓
  - Enhances the generated entanglement? ✗
- Single-mode squeezed state: $\boldsymbol{\mu} = \mathbf{0}$, $\boldsymbol{\sigma} \neq \boldsymbol{I}$.
  - Increase intensity of the Hawking radiation? ✓
  - Enhances the generated entanglement? ✓

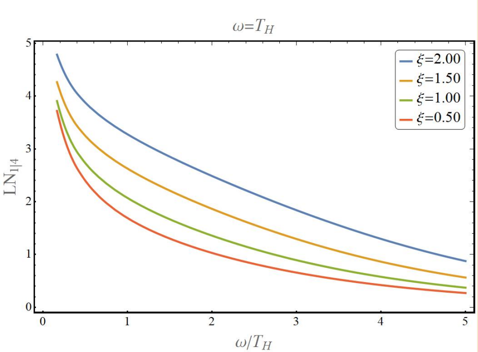
Single-mode squeezing and entanglement

- Squeeze k_3^{in} mode. $\boldsymbol{\mu}^{\text{in}} = \mathbf{0}, \quad \boldsymbol{\sigma}^{\text{in}} = \boldsymbol{I}_2 \oplus \boldsymbol{I}_2 \oplus \begin{pmatrix} e^{-\xi} & 0 \\ 0 & e^{\xi} \end{pmatrix} \oplus \boldsymbol{I}_2$

Single-mode squeezing and entanglement

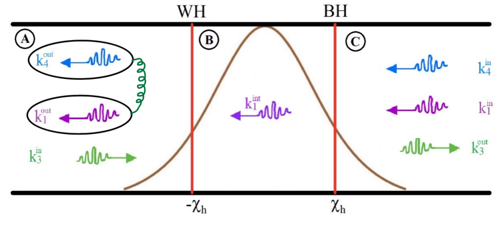
- Squeeze k_3^{in} mode. $\boldsymbol{\mu}^{\text{in}} = \mathbf{0}, \quad \boldsymbol{\sigma}^{\text{in}} = \boldsymbol{I}_2 \oplus \boldsymbol{I}_2 \oplus \begin{pmatrix} e^{-\xi} & 0 \\ 0 & e^{\xi} \end{pmatrix} \oplus \boldsymbol{I}_2$

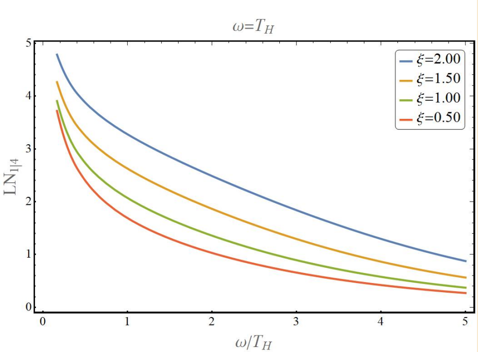




Single-mode squeezing and entanglement

- Squeeze k_3^{in} mode. $\boldsymbol{\mu}^{\text{in}} = \mathbf{0}, \quad \boldsymbol{\sigma}^{\text{in}} = \boldsymbol{I}_2 \oplus \boldsymbol{I}_2 \oplus \begin{pmatrix} e^{-\xi} & 0 \\ 0 & e^{\xi} \end{pmatrix} \oplus \boldsymbol{I}_2$





- **Single-mode squeezed inputs amplify the entanglement generated by the horizons.**

Thermal noise in the lab frame

Thermal noise in the lab frame

- Consider thermal fluctuations being isotropic in the lab frame.

Thermal noise in the lab frame

- Consider thermal fluctuations being isotropic in the lab frame.
- $\bar{n}_{i,\mathrm{lab}}^{\mathrm{env}} = \left(e^{|\omega_{i,\mathrm{lab}}|/T_{\mathrm{lab}}^{\mathrm{env}}} - 1\right)^{-1}, \quad i = 1, 2, 3, 4.$

Thermal noise in the lab frame

- Consider thermal fluctuations being isotropic in the lab frame.
- $\bar{n}_{i,\mathrm{lab}}^{\mathrm{env}} = \left(e^{|\omega_{i,\mathrm{lab}}|/T_{\mathrm{lab}}^{\mathrm{env}}} - 1\right)^{-1}, \quad i = 1, 2, 3, 4.$
- $\omega_{i,\mathrm{lab}} = \gamma(\omega + uk_i)$.

Thermal noise in the lab frame

- Consider thermal fluctuations being isotropic in the lab frame.
- $\bar{n}_{i,\text{lab}}^{\text{env}} = \left(e^{|\omega_{i,\text{lab}}|/T_{\text{lab}}^{\text{env}}} - 1\right)^{-1}, \quad i = 1, 2, 3, 4.$
- $\omega_{i,\text{lab}} = \gamma(\omega + uk_i).$
- But, evolution is performed in the comoving frame $\rightarrow$ Lorentz transform $\bar{n}_{i,\text{lab}}$.

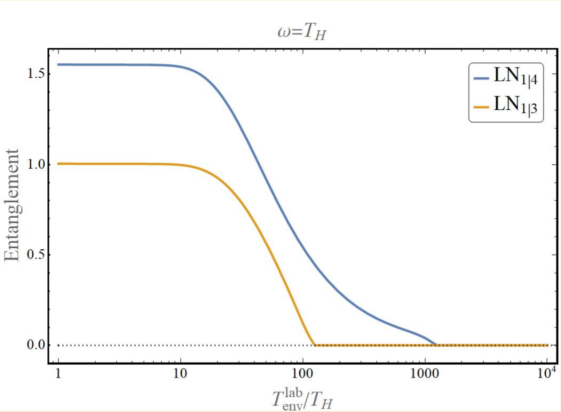
Thermal noise in the lab frame

- Consider thermal fluctuations being isotropic in the lab frame.
- $\bar{n}^{\text{env}}_{i,\text{lab}} = \left(e^{|\omega_{i,\text{lab}}|/T^{\text{env}}_{\text{lab}}} - 1\right)^{-1}, \quad i = 1, 2, 3, 4.$
- $\omega_{i,\text{lab}} = \gamma(\omega + u k_i)$.
- But, evolution is performed in the comoving frame $\rightarrow$ Lorentz transform $\bar{n}_{i,\text{lab}}$.
- $\bar{n}_{i,\text{co}}\, d^3x_{\text{co}}\, d^3k_{\text{co}} = \bar{n}_{i,\text{lab}}\, d^3x_{\text{lab}}\, d^3k_{\text{lab}} \rightarrow \bar{n}_{i,\text{co}} = \bar{n}_{i,\text{lab}}$.

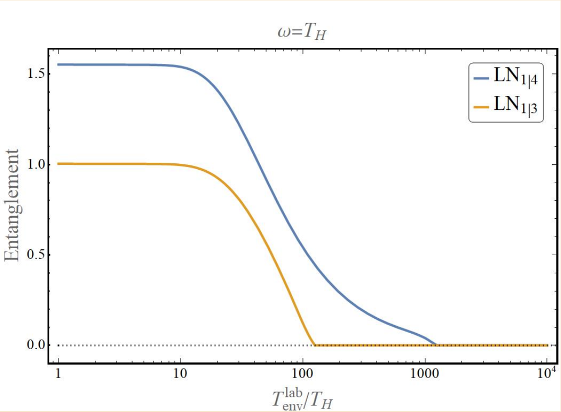
Thermal noise in the lab frame

- Consider thermal fluctuations being isotropic in the lab frame.
- $\bar{n}_{i,\mathrm{lab}}^{\mathrm{env}} = \left(e^{|\omega_{i,\mathrm{lab}}|/T_{\mathrm{lab}}^{\mathrm{env}}} - 1\right)^{-1}, \quad i = 1, 2, 3, 4.$
- $\omega_{i,\mathrm{lab}} = \gamma(\omega + uk_i)$.
- But, evolution is performed in the comoving frame $\rightarrow$ Lorentz transform $\bar{n}_{i,\mathrm{lab}}$.
- $\bar{n}_{i,\mathrm{co}}\, d^3x_{\mathrm{co}}\, d^3k_{\mathrm{co}} = \bar{n}_{i,\mathrm{lab}}\, d^3x_{\mathrm{lab}}\, d^3k_{\mathrm{lab}} \rightarrow \bar{n}_{i,\mathrm{co}} = \bar{n}_{i,\mathrm{lab}}$.
- **Input state**: $\boldsymbol{\mu}^{\mathrm{in}} = \mathbf{0}, \quad \boldsymbol{\sigma}^{\mathrm{in}} = \bigoplus_{i=1}^{4}(2\bar{n}_{i,\mathrm{co}}^{\mathrm{env}} + 1)\boldsymbol{I}_2$.

Thermal noise in the lab frame

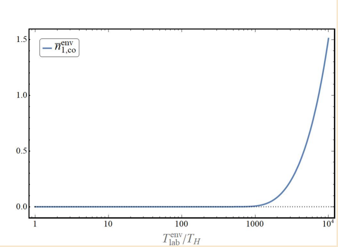


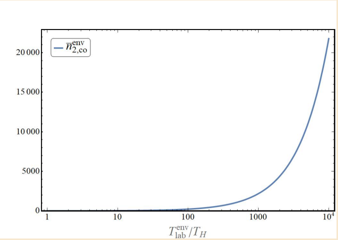
Thermal noise in the lab frame

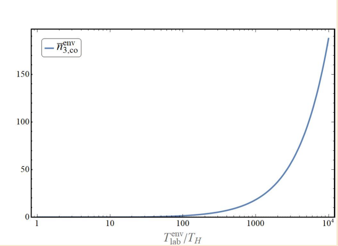


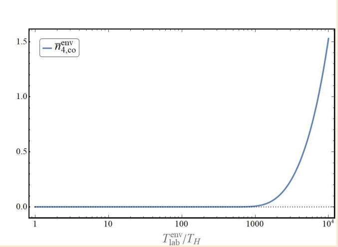
Message: Entanglement is shielded against thermal fluctuations in the lab frame.

Thermal noise in the lab frame



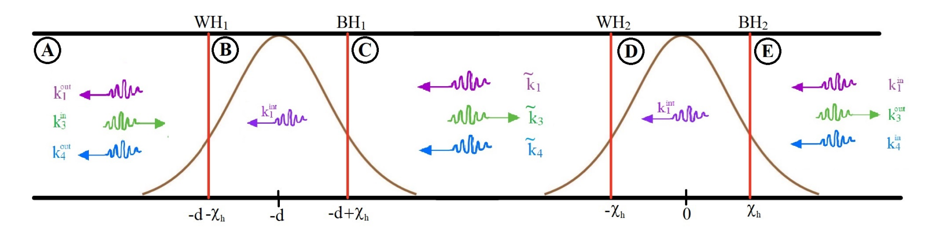






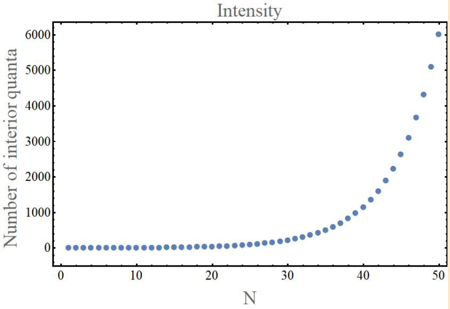
Hawking laser effect

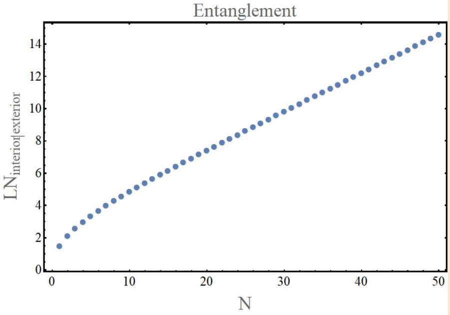
The setup



- Consider the configuration of two strong electric pulses each reproducing an analog white-black hole.
- The BH_1 horizon and the WH_2 horizons exchange Hawking quanta stimulating each other.
- We numerically solve the scattering problem and compute intensities and entanglement.

Intensity and entanglement in the optical laser setup





- The intensity of the trapped mode increases exponentially in time (manifestaton of the lasing effect). [U. Leonhardt and T. G. Philbin (2008), S. Finazzi and R. Parentani (2010), A. Coutant and R. Parentani (2010), D. Bermudez and U. Leonhardt (2018), H. Katayama (2021)] .
- In addition, we find that the entanglement shared between the interior and the exterior modes increases linearly in time.
- Laser configuration behaves as an entanglement factory!!!

Take-home messages

- We have leveraged sharp tools from quantum information theory to quantify the amount of entanglement generated in the Hawking effect.

Take-home messages

- We have leveraged sharp tools from quantum information theory to quantify the amount of entanglement generated in the Hawking effect.
- **Stimulated Hawking effect:** Entanglement is tunable. We can use this to our advantage to increase not only the classical but also the quantum aspects of the process.

Take-home messages

- We have leveraged sharp tools from quantum information theory to quantify the amount of entanglement generated in the Hawking effect.

- **Stimulated Hawking effect:** Entanglement is tunable. We can use this to our advantage to increase not only the classical but also the quantum aspects of the process.

- Entanglement generated by the optical Hawking process is **robust** against background thermal fluctuations being isotropic in the **lab frame** thanks to the combination of **dispersion** and the **Lorentz boost**.

Take-home messages

- We have leveraged sharp tools from quantum information theory to quantify the amount of entanglement generated in the Hawking effect.
- **Stimulated Hawking effect:** Entanglement is tunable. We can use this to our advantage to increase not only the classical but also the quantum aspects of the process.
- Entanglement generated by the optical Hawking process is **robust** against background thermal fluctuations being isotropic in the **lab frame** thanks to the combination of **dispersion** and the **Lorentz boost**.
- The optical laser setup is a promising alternative platform that not only boosts Hawking particle production but also increases the entanglement between interior and exterior degrees of freedom.