Quantum Information in Analogue Black Holes

Analogue Hawking Radiation in BECs

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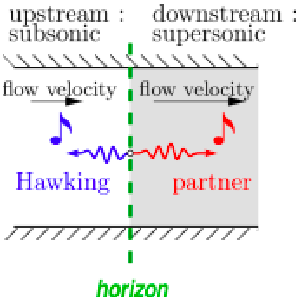
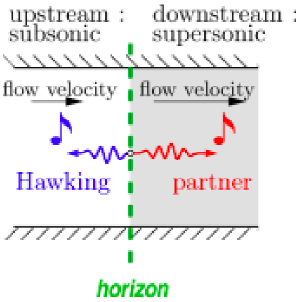
Analogue Gravity in BECs



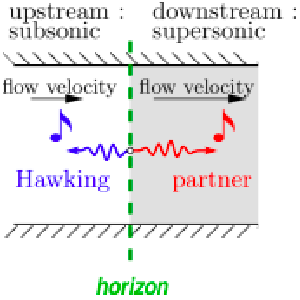
Analogue Gravity in BECs

⇒ GR and QFT : Hawking radiation



Analogue Gravity in BECs

- $\Rightarrow$ GR and QFT : Hawking radiation
- $\Rightarrow$ Acoustic black holes (ABH)



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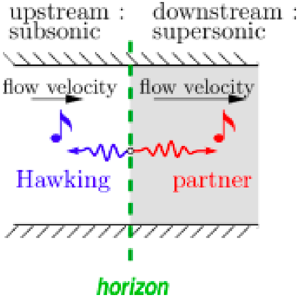

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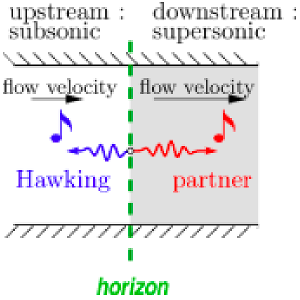

violation of Bell's inequalities in ABH

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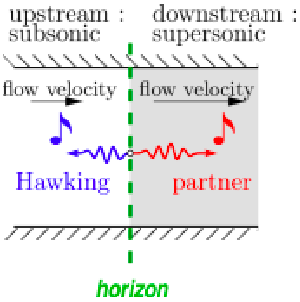

violation of Bell's inequalities in ABH

- ⇒ matter waves

Outline

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- ⇒ Acoustic black holes (ABH)
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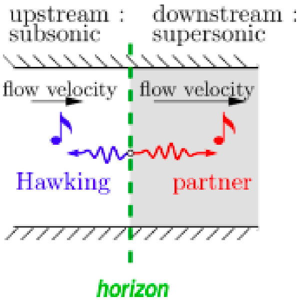
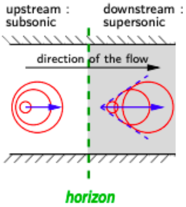


violation of Bell's inequalities in ABH

- ⇒ matter waves
- ⇒ continuous variables

Acoustic Black Holes in BECS and Analogue Hawking Radiation



$V < c$: subsonic region

$V > c$: supersonic region

$V = c$: acoustic horizon

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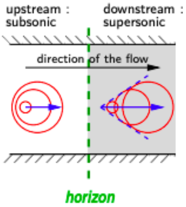

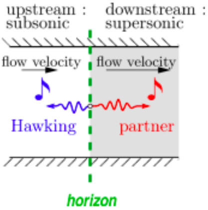

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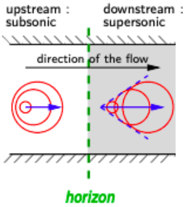

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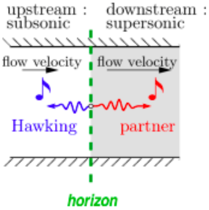
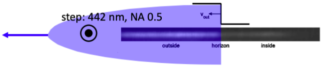


Emission from the horizon of pairs of correlated and entangled phonons

Experimental Acoustic Black Holes in BECs

Steinhauer [2016, 2019]



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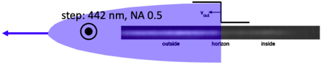

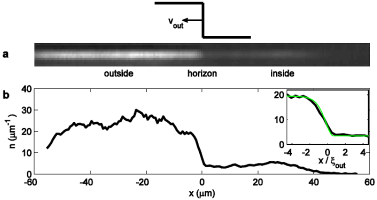

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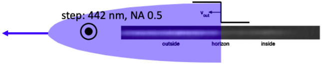
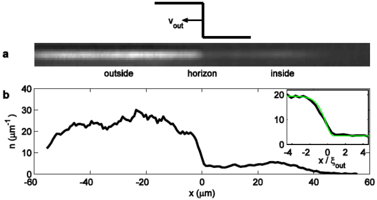
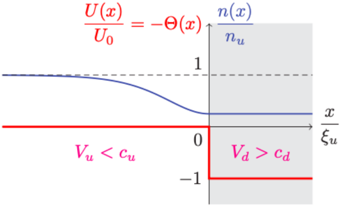






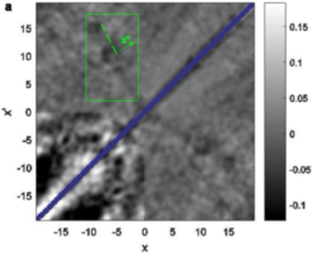
Waterfall configuration

The two-point density correlation function

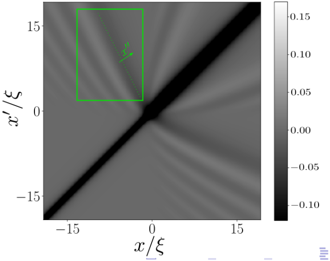
$$G^{(2)}(x,x') = \langle : \hat{n}(x,t)\hat{n}(x',t) : \rangle - \langle \hat{n}(x,t)\rangle\langle \hat{n}(x',t)\rangle$$

Analogue HR in BEC

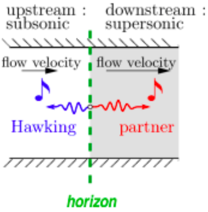
Experiments
Steinhauer [2019]



Simulations
Isoard [2020]



Why studying Bell's inequalities in ABH ?



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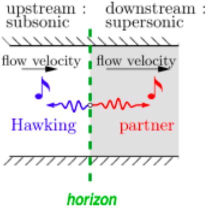

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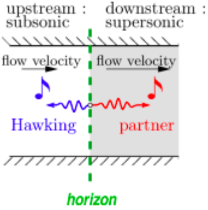

⇒ first time with matter waves (continuous variables) in ABH

⇒ propose realistic experimental observables

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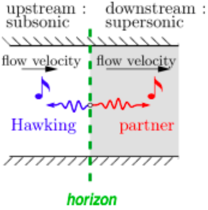

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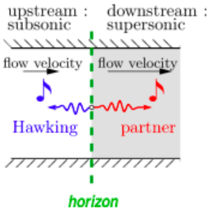

- $\Rightarrow$ first time with matter waves (continuous variables) in ABH
- $\Rightarrow$ propose realistic experimental observables
- $\Rightarrow$ test the quantum non-locality of our ABH
- $\Rightarrow$ theoretical tools to do it

Hidden variables and local realism [CHSH 1969]

$$A = \pm 1 \quad A' = \pm 1 \quad B = \pm 1 \quad B' = \pm 1$$

BELL'S INEQUALITIES
EPR 1935 ⇒ Bell 1964

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$$|\langle \hat{C} \rangle| = 2\sqrt{2}$$

Linearizing the Gross-Pitaevskii equation : the Bogoliubov approximation

$$i\partial_t\hat{\Phi}(x,t) = -\frac{1}{2}\partial_x^2\hat{\Phi}(x,t) + [U(x) + g\hat{n} - \mu]\hat{\Phi}(x,t)$$

with $\hat{n} = \hat{\Phi}^\dagger\hat{\Phi}$

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Fluctuations around the condensate stationary profile

$$\hat{\Phi}(x,t) = \langle \hat{\Phi}(x) \rangle + \delta \hat{\Psi}(x,t)$$

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$$L\begin{pmatrix}u_\ell\\ v_\ell\end{pmatrix} = \varepsilon(q_\ell)\begin{pmatrix}u_\ell\\ v_\ell\end{pmatrix}$$

Doppler shifted Bogoliubov dispersion relation

$$(\varepsilon(q_\ell) - Vq_\ell)^2 = \omega_B^2(q_\ell) \text{ with } \omega_B(q_\ell) = q_\ell\sqrt{1 + \frac{q_\ell^2}{4}}$$

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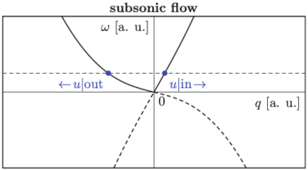

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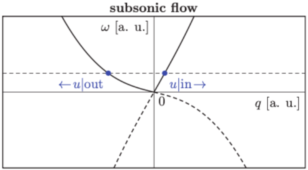

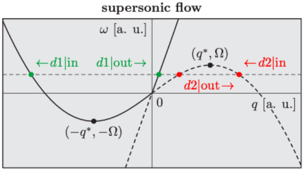

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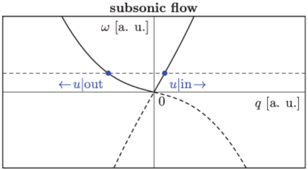

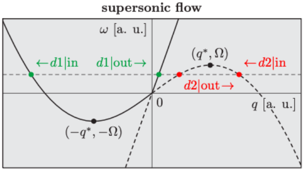

Scattering (ingoing and outgoing) modes

$$\hat{\mathbf{c}} = S\,\hat{\mathbf{b}}$$

outgoing : $\hat{\mathbf{c}} = (\hat{c}_0, \hat{c}_1, \hat{c}_2^\dagger)^T$

ingoing : $\hat{\mathbf{b}} = (\hat{b}_0, \hat{b}_1, \hat{b}_2^\dagger)^T$

Wigner Function of Gaussian States

$$\hat{q}_i = \frac{1}{\sqrt{2}}(\hat{c}_i + \hat{c}_i^\dagger) \quad \hat{p}_i = \frac{1}{i\sqrt{2}}(\hat{c}_i - \hat{c}_i^\dagger)$$

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$$\hat{\mathbf{R}} = (\hat{R}_0, \hat{R}_1, ..., \hat{R}_5)^T = \sqrt{2}(\hat{q}_0, \hat{p}_0, \hat{q}_1, \hat{p}_1, \hat{q}_2, \hat{p}_2)^T$$

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Covariance Matrix

$$\sigma_{kl} = \frac{1}{2}\langle \hat{R}_k \hat{R}_l + \hat{R}_l \hat{R}_k \rangle - \langle \hat{R}_k \rangle \langle \hat{R}_l \rangle$$

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Wigner function of the density matrix $\hat{\rho}$

$$W_{\hat{\rho}}(q,p) = \frac{1}{\pi^3 \sqrt{\det \sigma}} \exp\left\{ -\frac{1}{2} R^T \sigma^{-1} R \right\}$$

Components of the covariance matrix

Violation of Bell's inequalities in an Acoustic Black Hole

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$\langle \hat{c}_i^\dagger \hat{c}_i \rangle$

$|\langle \hat{c}_0 \hat{c}_1^\dagger \rangle|$

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three-mode pure state

$$|0\rangle_b \propto e^{\left(X_{02}\hat{c}_0^\dagger + X_{12}\hat{c}_1^\dagger\right)\hat{c}_2^\dagger} |0\rangle_c$$

squeezed vacuum

Violation of Bell's inequalities in an Acoustic Black Hole

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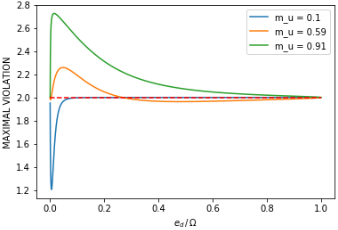
squeezed vacuum

two-mode mixed state

$$\rho_{(ij)} = \mathrm{Tr}_{(k)}\{|0\rangle_b\langle 0|\}$$

Two-mode violation of Bell's inequalities in an Acoustic Black Hole

Hawking-Partner at $T = 0$



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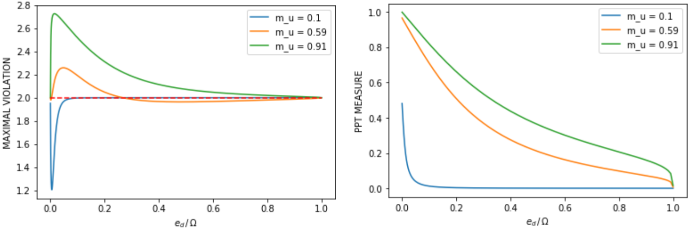

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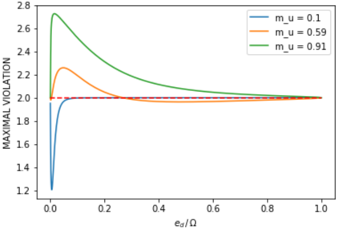

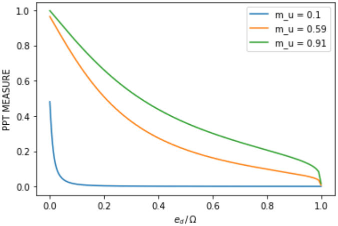

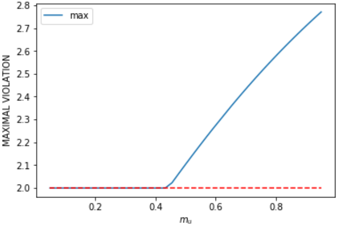

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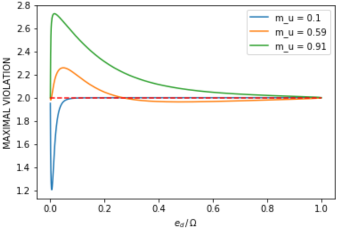
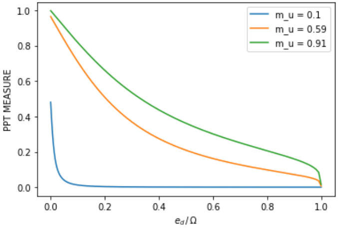
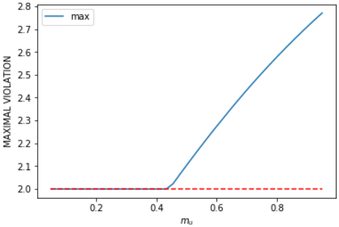
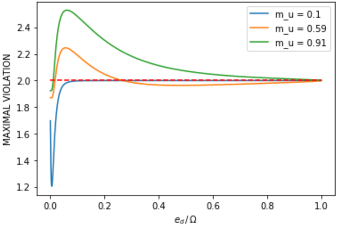


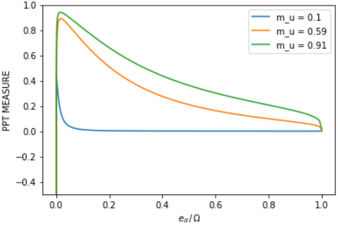


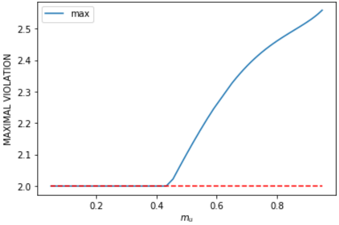


Two-mode violation of Bell's inequalities in an Acoustic Black Hole

Hawking-Partner at $k_B T / mc_u^2 = 0.1$

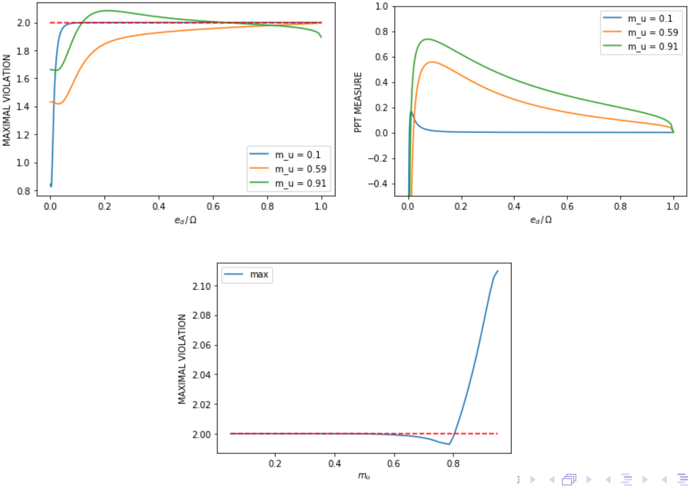






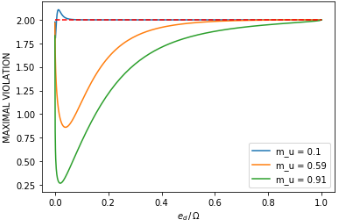
Two-mode violation of Bell's inequalities in an Acoustic Black Hole

Hawking-Partner at $k_B T / mc_u^2 = 0.2$



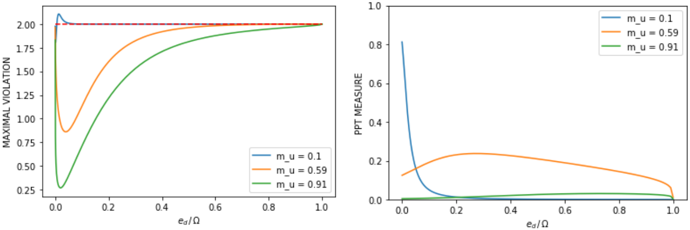
Two-mode violation of Bell's inequalities in an Acoustic Black Hole

Companion-Partner at $T = 0$



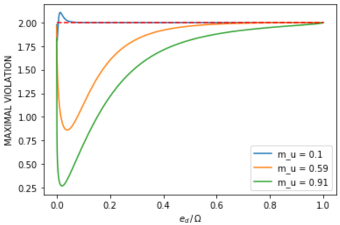
Two-mode violation of Bell's inequalities in an Acoustic Black Hole

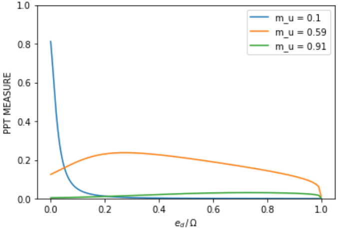
Companion-Partner at $T = 0$

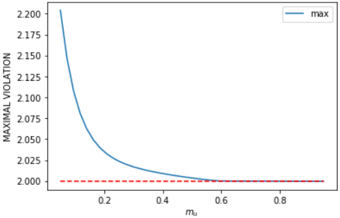


Two-mode violation of Bell's inequalities in an Acoustic Black Hole

Companion-Partner at $T = 0$

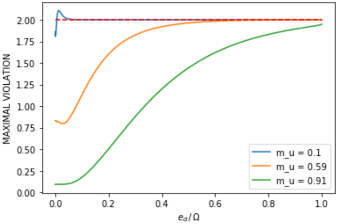


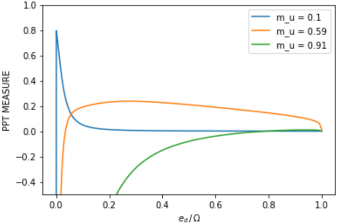


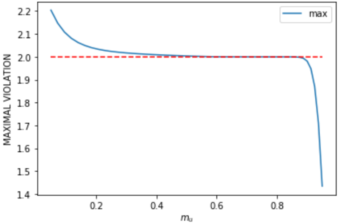


Two-mode violation of Bell's inequalities in an Acoustic Black Hole

Companion-Partner at $k_B T / m c_u^2 = 0.1$







Three-mode violation of Bell's inequalities in an Acoustic Black Hole

Maximization of a three-measurement correlation

$$C_{\text{corr}}(A, B, D, A', B', D') > 2 \iff \text{violation}$$

Three-mode violation of Bell's inequalities in an Acoustic Black Hole

Maximization of a three-measurement correlation

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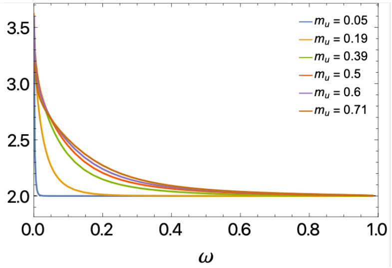
Maximized violation at T = 0

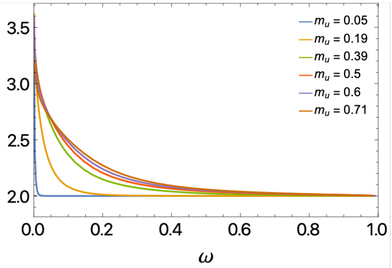


Three-mode violation of Bell's inequalities in an Acoustic Black Hole

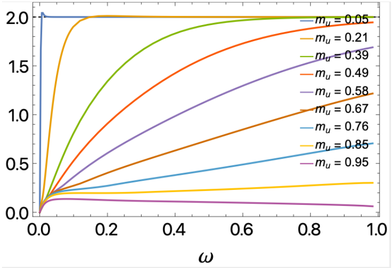
Maximization of a three-measurement correlation

$$C_{\text{corr}}(A, B, D, A', B', D') > 2 \iff \text{violation}$$

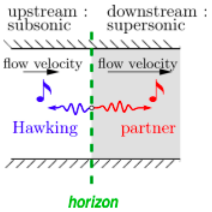
Maximized violation at T = 0



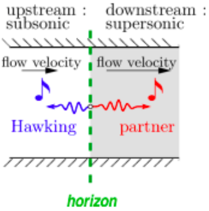
Maximized violation at $k_B T / m c_u^2 = 0.3$



To sum up

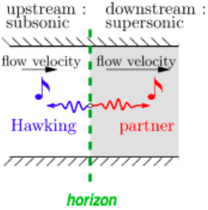

upstream : subsonic
downstream : supersonic
flow velocity
flow velocity
Hawking
partner
horizon

To sum up



$\Rightarrow$ violation of Bell's inequalities in a two-mode ABH

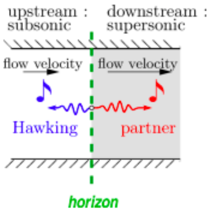
To sum up



⇒ violation of Bell's inequalities in a two-mode ABH

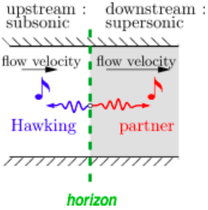
⇒ violation resilient to temperature

To sum up



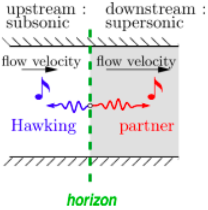
- ⇒ violation of Bell's inequalities in a two-mode ABH
- ⇒ violation resilient to temperature
- ⇒ hope for experiments with matter waves in ABH config.

To sum up



- ⇒ violation of Bell's inequalities in a two-mode ABH
- ⇒ violation resilient to temperature
- ⇒ hope for experiments with matter waves in ABH config.
- ⇒ generalization of Bell's violation to three-modes

To sum up



⇒ violation of Bell's inequalities in a two-mode ABH

⇒ violation resilient to temperature

⇒ hope for experiments with matter waves in ABH config.

⇒ generalization of Bell's violation to three-modes

Beyond analogy with GR : testing QM predictions

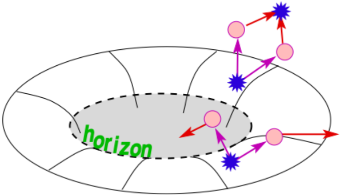
People working on the project

Supervisors

Nicolas PAVLOFF, LPTMS (Paris-Orsay)
Andreas BUCHLEITNER, QOS (Freiburg)

Collaborator

Mathieu ISOARD, LKB (Paris)



Pseudo-Spins [GKMR 2004]

continuous variables

$$|\mathcal{E}_i\rangle = \frac{1}{\sqrt{2}}(|q_i\rangle + |-q_i\rangle) \quad |\mathcal{O}_i\rangle = \frac{1}{\sqrt{2}}(|q_i\rangle - |-q_i\rangle)$$

$$\hat{\mathbb{I}}_{E_i} = \int_0^\infty dq_i\, |\mathcal{E}_i\rangle \langle \mathcal{E}_i| \qquad \hat{\mathbb{I}}_{O_i} = \int_0^\infty dq_i\, |\mathcal{O}_i\rangle \langle \mathcal{O}_i|$$

$$\hat{S}_z^{(i)} = \hat{\mathbb{I}}_{E_i} - \hat{\mathbb{I}}_{O_i}$$

$$\hat{S}_z^{(i)} |\psi_e\rangle = + |\psi_e\rangle$$

$$\hat{S}_z^{(i)} |\psi_o\rangle = - |\psi_o\rangle$$

$$[\hat{S}_r^{(i)}, \hat{S}_s^{(i)}] = 2i\epsilon_{rst}\hat{S}_t^{(i)}$$

$$\{\hat{S}_r^{(i)}, \hat{S}_s^{(i)}\} = 2\delta_{rs}\hat{\mathbb{I}}$$

GKMR pseudo-spins and their Wigner functions

$$|\mathcal{E}\rangle = \frac{1}{\sqrt{2}}(|q\rangle + |-q\rangle) \quad |\mathcal{O}\rangle = \frac{1}{\sqrt{2}}(|q\rangle - |-q\rangle)$$

$$\hat{S}_z = \int_0^\infty dq\,|\mathcal{E}\rangle\langle\mathcal{E}| - \int_0^\infty dq\,|\mathcal{O}\rangle$$

$$\hat{S}_x = \int_0^\infty dq\,|\mathcal{E}\rangle\langle\mathcal{O}| + \int_0^\infty dq\,|\mathcal{O}\rangle\langle\mathcal{E}|$$

$$\hat{S}_y = i\int_0^\infty dq\,|\mathcal{O}\rangle\langle\mathcal{E}| - i\int_0^\infty dq\,|\mathcal{E}\rangle\langle\mathcal{O}|$$

$$\begin{aligned} W_{\hat{S}_z}(q_i, p_i) &= \pi\,\delta(q_i)\delta(p_i) \\ W_{\hat{S}_y}(q_i, p_i) &= i\,\delta(q_i)\int_{-\infty}^{+\infty} dq_i'\,\mathrm{sgn}(q_i')e^{-2ip_iq_i'} \\ W_{\hat{S}_x}(q_i, p_i) &= \mathrm{sgn}(q_i) \end{aligned}$$

Averaging Observables

$$\langle \hat{A} \rangle = \mathrm{Tr}\{\hat{\rho}\hat{A}\} = \int_{-\infty}^{+\infty} dq \int_{-\infty}^{+\infty} dp \, W_{\hat{\rho}}(q,p) \, W_{\hat{A}}(q,p)$$

Wigner Function of Gaussian States

Averaging Observables

$$\langle \hat{A} \rangle = \mathrm{Tr}\{\hat{\rho}\hat{A}\} = \int_{-\infty}^{+\infty} dq \int_{-\infty}^{+\infty} dp \, W_{\hat{\rho}}(q,p) \, W_{\hat{A}}(q,p)$$

$$W_{\hat{A}}(q,p) = \int_{-\infty}^{+\infty} dz \, e^{ipz} \, \langle q - \frac{1}{2}z | \, \hat{A} \, | q + \frac{1}{2}z \rangle$$

Wigner Function of Gaussian States

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Averaging the pseudo-spin correlations

Wigner Function of Gaussian States

Averaging Observables

$$\langle \hat{A} \rangle = \mathrm{Tr}\{\hat{\rho}\hat{A}\} = \int_{-\infty}^{+\infty} dq \int_{-\infty}^{+\infty} dp \, W_{\hat{\rho}}(q,p) \, W_{\hat{A}}(q,p)$$

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Averaging the pseudo-spin correlations

three-mode pure state

$$\langle \hat{S}_r^{(i)} \otimes \hat{S}_s^{(j)} \otimes \hat{S}_t^{(k)} \rangle$$

Wigner Function of Gaussian States

Averaging Observables

$$\langle \hat{A} \rangle = \mathrm{Tr}\{\hat{\rho}\hat{A}\} = \int_{-\infty}^{+\infty} dq \int_{-\infty}^{+\infty} dp\, W_{\hat{\rho}}(q,p)\, W_{\hat{A}}(q,p)$$

$$W_{\hat{A}}(q,p) = \int_{-\infty}^{+\infty} dz\, e^{ipz} \langle q - \frac{1}{2}z | \hat{A} | q + \frac{1}{2}z \rangle$$

Averaging the pseudo-spin correlations

three-mode pure state

$$\langle \hat{S}_r^{(i)} \otimes \hat{S}_s^{(j)} \otimes \hat{S}_t^{(k)} \rangle$$

two-mode mixed state

$$\langle \hat{S}_r^{(i)} \otimes \hat{S}_s^{(j)} \rangle$$

Hawking radiation in GR

Quantum Field Theory in Curved Space-Time

Massless Real Scalar Field : $\nabla_\mu \nabla^\mu \varphi_i = 0$

$$\nabla_\mu \nabla^\mu \varphi_i = \frac{1}{\sqrt{-g}} \partial_\mu \left(\sqrt{-g} g^{\mu\nu} \partial_\nu \varphi_i \right)$$

Hawking Radiation (1974) with Planck Spectrum at T_H

$$\hat{\phi} = \sum_i [\varphi_i \hat{a}_i + \varphi_i^* \hat{a}_i^\dagger] = \sum_i [\psi_i \hat{b}_i + \psi_i^* \hat{b}_i^\dagger]$$

$$\hat{b}_i = \sum_j [\alpha_{ij}^* \hat{a}_j - \beta_{ij}^* \hat{a}_j^\dagger]$$

$$\langle 0_{in} | \hat{a}_i^\dagger \hat{a}_i | 0_{in} \rangle = 0 \quad \langle 0_{in} | \hat{b}_i^\dagger \hat{b}_i | 0_{in} \rangle = \sum_j |\beta_{ij}|^2 = \frac{1}{e^{\hbar\omega_i / k_B T_H} - 1}$$

1D Acoustic BH in BECs : The Analogue Metric

Gross-Pitaevskii equation

$$i\hbar\partial_t\phi = -\frac{\hbar^2}{2m}\partial_x^2\phi + [U + g|\phi|^2]\phi$$

$$\phi = \sqrt{\rho_0 + \rho_1}e^{i(\theta_0+\theta_1)} \approx \sqrt{\rho_0}e^{i\theta_0}\left(1 + \frac{\rho_1}{2\rho_0} + i\theta_1\right)$$

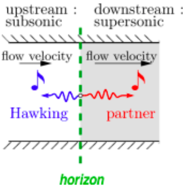
$$\frac{1}{\sqrt{-g}}\partial_\mu\left(\sqrt{-g}g^{\mu\nu}\partial_\nu\theta_1\right) = 0$$

$$g_{\mu\nu} = \frac{\rho_0}{mc}\begin{pmatrix} [c^2 - v^2] & -v \\ -v & -1 \end{pmatrix}$$

where $v = \frac{\hbar}{m}\partial_x\theta_0$ and $mc^2 = \rho_0 g$.

The Mapping

ANALOGUE HAWKING RADIATION



Gravitational Black Hole $\longleftrightarrow$ **Acoustic Black Hole**

Gravitational Black Hole	Acoustic Black Hole
light speed c	sound speed $c(x)$
radius $r_s = 2GM/c^2$	x_h such that $V(x_h) = c(x_h)$
surface gravity $\kappa = c^2/2r_s$	$\kappa_{\rm ac} \propto \partial_x (c^2 - V^2)\|_{x_h}$
temperature $T_H \propto \kappa$	$T_H\|_{\rm Acoustic} \propto \kappa_{\rm ac}$

Fitting $S_{u,d2}$ with a thermal occupation number

$$\Pi_0 = -\int_0^{\Omega} \frac{d\omega}{2\pi} \hbar\omega |S_{u,d2}(\omega)|^2$$

$$\Gamma \times n_{T_H}(\omega) = \Gamma \times \frac{1}{\exp\left(\frac{\hbar\omega}{k_B T_H}\right) - 1}$$

$$\Gamma = 1.143 \quad \frac{k_B T_H}{m c_u^2} = 0.067$$