Spontaneous generation of correlated photon pairs in hot atomic vapors

Interaction quenches and their consequences

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Introduction

Fluid light ?

- **Photons** as elementary constituants
- **Interactions** between photons → **collective** behavior
- Photons → **quantum** effects

Fluid of light à la Van Gogh, using Stable Diffusion

Numerous experimental platforms:

- Photorefractive crystals → Nice / Porto
- Exciton polaritons → Paris / Lille / Singapour
- Dye-filled cavities → Bonn

Striking properties:

- **Superfluidity**[1]
- Quantized vortices
- Bose-Einstein condensation

Fluid of light à la Van Gogh, using Stable Diffusion

[1] (Alberto Amo et al. **"Superfluidity of polaritons in semiconductor microcavities"**. In: *Nat. Phys.* 5 [2009])

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[2] (A. Amo et al. **"Polariton superfluids reveal quantum hydrodynamic solitons"**. In: *Science* 332 [2011])

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- Superfluidity
- Quantized vortices
- **Bose-Einstein condensation** [3], [4]

Fluid of light à la Van Gogh, using Stable Diffusion

[3] (Jan Klaers et al. **"Bose–Einstein condensation of photons in an optical microcavity".** In: *Nature* 468 [2010])

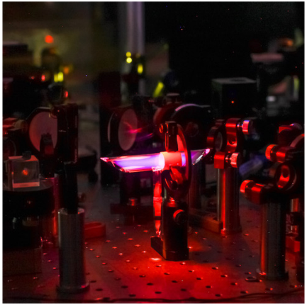
[4] (J. Kasprzak et al. **"Bose–Einstein condensation of exciton polaritons".** In: *Nature* 443 [2006])

Atomic vapors:

- **Alcali** → Hydrogen-like atoms
- **Rubidium** → well-known atomic structure
- **Optically** accesible transitions → laser

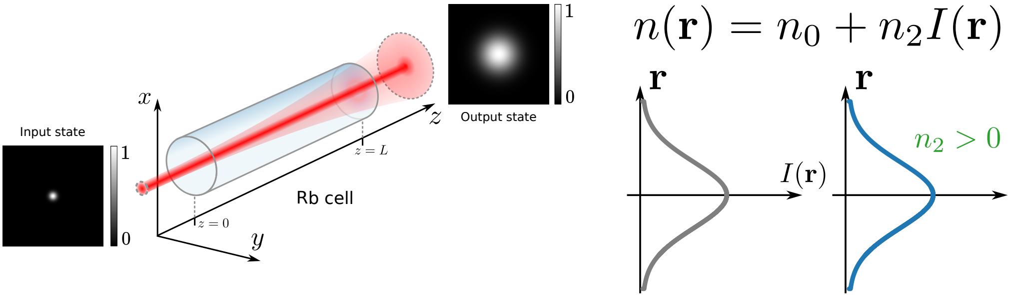
Hot Vapors :

- Glass cell → liquid / vapor equilibrium
- Temperature → atomic density control
- "Unrestricted" optical access



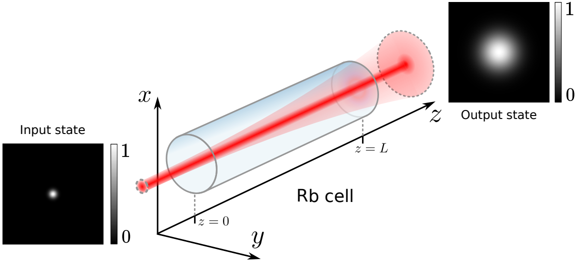
Resonantly pumped Rubidium cell

Quantum fluids of light

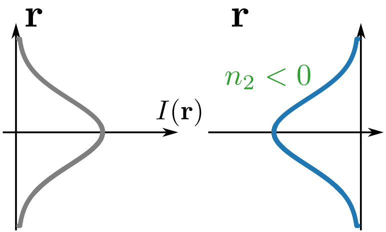


- Propagation of a gaussian beam in a **non-linear** medium
- Non-linear index of refraction n_2 $\leftrightarrow$ **non-trivial** propagation
- *Old* problem $\leftrightarrow$ **new vision**

Paraxial fluids of light



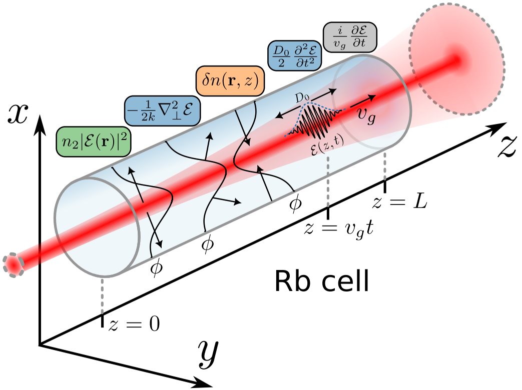
$$n(\mathbf{r}) = n_0 + n_2 I(\mathbf{r})$$



- Propagation of a gaussian beam in a **non-linear** medium
- Non-linear index of refraction n_2 ↔ **non-trivial** propagation
- *Old* problem ↔ **new vision**

To describe the electric field envelope $\mathcal{E}$ in a non-linear medium, we use the **non-linear Schrödinger equation** (NLSE) :

$$i\frac{\partial \mathcal{E}}{\partial z} = \underbrace{-\frac{i}{v_g}\frac{\partial \mathcal{E}}{\partial t}}_{\text{Drift}} + \underbrace{-\frac{1}{2k}\nabla_\perp^2\mathcal{E} + \frac{D_0}{2}\frac{\partial^2\mathcal{E}}{\partial t^2}}_{\text{Kinetic}} + \underbrace{k\frac{\delta n}{n}}_{\text{Potential}} + \underbrace{n_2\epsilon_0 c|\mathcal{E}|^2\mathcal{E}}_{\text{Interaction}} - \underbrace{i\frac{\alpha}{2}}_{\text{Losses}} \quad (1)$$



- $k(\omega) \rightarrow$ wavenumber
- $v_g \rightarrow$ group velocity
- $D_0 \rightarrow$ group velocity dispersion
- $\delta n \rightarrow$ index variation
- $\alpha \rightarrow$ absorption coefficient
- $n_2 \rightarrow$ non-linear index

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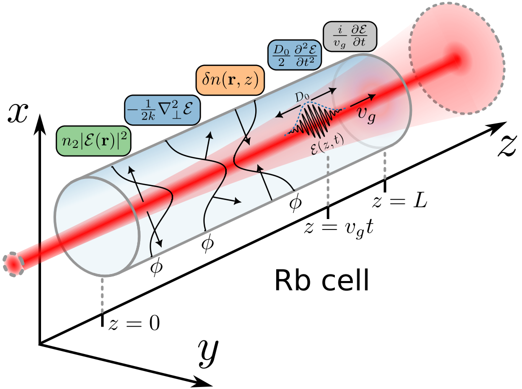

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For **continuous** regimes, we can ignore the **temporal** dynamics:

$$i\frac{\partial \mathcal{E}}{\partial z} = \underbrace{\boxed{-\frac{1}{2k}\nabla_\perp^2\mathcal{E} + \frac{D_0}{2}\frac{\partial^2\mathcal{E}}{\partial t^2}}}_{\textit{Kinetic}} + \underbrace{\boxed{k\frac{\delta n}{n}}}_{\textit{Potential}} + \underbrace{\boxed{n_2\epsilon_0 c|\mathcal{E}|^2\mathcal{E}}}_{\textit{Interaction}} - i\frac{\alpha}{2} \tag{3}$$

3 control "buttons" :

- **Kinetic** : dispersion, diffraction → initial state shaping
- **Potential** : linear index → optical pumping
- **Interaction** : non-linear index → temperature et laser detuning

Numerical solver : spectral method[5] , [6]

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[5] Quentin Glorieux et al. **"Hot atomic vapors for nonlinear and quantum optics"**. en. In: *New Journal of Physics* 25 (2023)

[6] https://github.com/Quantum-Optics-LKB/NLSE

Equivalence with the **Gross-Pitaevskii** equation [7] (GPE):

$$\text{(GPE)} \quad i\hbar\frac{\partial}{\partial t}\psi = \underbrace{\boxed{-\frac{\hbar^2}{2m}\nabla^2\psi}}_{\textit{Kinetic}} + \underbrace{\boxed{V\psi}}_{\textit{Potential}} + \underbrace{\boxed{g|\psi|^2\psi}}_{\textit{Interaction}}$$

$$\text{(NLSE)} \quad i\frac{\partial}{\partial z}\mathcal{E} = \underbrace{\boxed{-\frac{1}{2k}\nabla_\perp^2\mathcal{E}}}_{\textit{Kinetic}} - \underbrace{\boxed{k\frac{\delta n}{n}\mathcal{E}}}_{\textit{Potential}} + \underbrace{\boxed{n_2\epsilon_0 c|\mathcal{E}|^2\mathcal{E}}}_{\textit{Interaction}} - i\frac{\alpha}{2}\mathcal{E}$$

NLSE :

- Particles $\leftrightarrow$ photons
- Spatial evolution $\leftrightarrow$ z
- Interactions $\leftrightarrow$ non-linear medium
- Initial state $\leftrightarrow$ plane wave (gaussian)

GPE :

- Particles $\leftrightarrow$ atoms
- Temporal evolution $\leftrightarrow$ t
- Interactions $\leftrightarrow$ atom-atom diffusion
- Initial state $\leftrightarrow$ Bose-Einstein condensate

[7] Lev P. Pitaevskij et al. **Bose-Einstein condensation and superfluidity.** Oxford, 2016. 553 pp.

Using the Madelung[8] transform, one can rewrite the enveloppe $\mathcal{E}$ evolution equation as a fluid evolution equation with **density** $\rho \propto |\mathcal{E}|^2$ and **speed** $\mathbf{v} \propto \nabla_\perp \varphi$:

$$\mathcal{E}(\mathbf{r}_\perp, z) = \sqrt{\rho(\mathbf{r}_\perp, z)}e^{i\varphi(\mathbf{r}_\perp, z)}.$$

$$\frac{\partial \rho}{\partial z} = -\frac{1}{c}\nabla_\perp(\rho \mathbf{v}) \boxed{-\alpha\rho} \tag{4}$$

$$\frac{\partial \mathbf{v}}{\partial z} = -\boxed{\frac{1}{2c}\nabla_\perp \mathbf{v}^2} - \frac{c}{k_0}\nabla_\perp\left(\boxed{g\rho} - \underbrace{\frac{1}{2k_0}\frac{\nabla_\perp^2\sqrt{\rho}}{\sqrt{\rho}}}_{Quantum\ \ pressure} + \boxed{V}\right) \tag{5}$$

where $g = n_2\epsilon_0 c$

- Equation (4) : mass conservation
- Equation (5) : convective term

[8] E. Madelung. **"Eine anschauliche Deutung der Gleichung von Schrödinger".** In: *Naturwissenschaften* 14 (1926)

"Quantum" fluids ? One can convert eq.3 into a fully fledged Hamiltonian[9]:

$$\hat{H}(z) = \mathcal{N} \int \mathrm{d}\mathbf{r} \left[\frac{1}{2k_0} \nabla_\perp \hat{\mathcal{E}}^\dagger \cdot \nabla_\perp \hat{\mathcal{E}} + V(\mathbf{r}_\perp, z) \hat{\mathcal{E}}^\dagger \hat{\mathcal{E}} + \frac{g(\mathbf{r}_\perp, z)}{2} \hat{\mathcal{E}}^\dagger \hat{\mathcal{E}}^\dagger \hat{\mathcal{E}} \hat{\mathcal{E}} + i \frac{\alpha}{2} \hat{\mathcal{E}}^\dagger \hat{\mathcal{E}} \right]. \quad (6)$$

One recognizes a **Bose gas** Hamiltonian and the **Gross-Pitaevskii** equation (GPE):

$$i \frac{\partial \hat{\mathcal{E}}}{\partial z} = -\frac{1}{2k_0} \nabla_\perp^2 \hat{\mathcal{E}} + V(\mathbf{r}_\perp, z) \hat{\mathcal{E}} + g(\mathbf{r}_\perp, z) \hat{\mathcal{E}}^\dagger \hat{\mathcal{E}} \hat{\mathcal{E}} - i \frac{\alpha}{2} \hat{\mathcal{E}}. \quad (7)$$

With creation and annihilation operators $\hat{a}^\dagger$ and $\hat{a}$ such that:

$$\hat{\mathcal{E}} = \int \mathrm{d}\mathbf{k}_\perp \hat{a}_{\mathbf{k}_\perp} e^{i\mathbf{k}_\perp \cdot \mathbf{r}_\perp}$$

Plane wave expansion

[9] Pierre-Élie Larré et al. **"Propagation of a quantum fluid of light in a cavityless nonlinear optical medium"**. In: *Phys. Rev. A* 92 (2015)

LKB

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Possibility to model arbitrary space-time geometries [11]

[10] Pierre-Élie Larré et al. **"Propagation of a quantum fluid of light in a cavityless nonlinear optical medium".** In: *Phys. Rev. A* 92 (2015)

[11] M. J. Jacquet et al. **"The next generation of analogue gravity experiments".** In: *Transactions of the Roy. Soc. A* 378.2177 (2020), p. 20190239

We develop up to the first order in field fluctuations in order to diagonalize the hamiltonian[12]:

$$\hat{\mathcal{E}} = \underbrace{\mathcal{E}_0}_{\text{Mean field}} + \underbrace{\delta\hat{\mathcal{E}}}_{\text{Fluctuations}}$$

$$g(\mathbf{r}_\perp, z)\hat{\mathcal{E}}^\dagger\hat{\mathcal{E}}\hat{\mathcal{E}} = g(\mathbf{r}_\perp, z)|\mathcal{E}_0|^2\delta\hat{\mathcal{E}}$$

Introducing the new creation and annihilation operators $\hat{b}^\dagger$ and $\hat{b}$ such that:

$$\begin{pmatrix} \hat{a}_{\mathbf{k}_\perp} \\ \hat{a}^\dagger_{-\mathbf{k}_\perp} \end{pmatrix} = \begin{pmatrix} u_{\mathbf{k}_\perp} & v_{\mathbf{k}_\perp} \\ v_{\mathbf{k}_\perp} & u_{\mathbf{k}_\perp} \end{pmatrix} \begin{pmatrix} \hat{b}_{\mathbf{k}_\perp} \\ \hat{b}^\dagger_{-\mathbf{k}_\perp} \end{pmatrix}$$

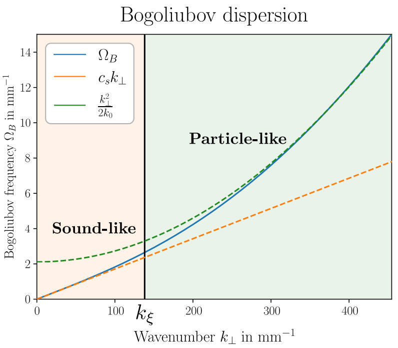
$$\delta\hat{\mathcal{E}} = \int \mathrm{d}\mathbf{k}_\perp u_{\mathbf{k}_\perp} \hat{b}_{\mathbf{k}_\perp} e^{i\mathbf{k}_\perp\cdot\mathbf{r}_\perp} + v_{\mathbf{k}_\perp} \hat{b}^\dagger_{-\mathbf{k}_\perp} e^{-i\mathbf{k}_\perp\cdot\mathbf{r}_\perp} \quad (10)$$

[12] N. N. Bogoljubov. **"On a New Method in the Theory of Superconductivity".** In: *Journal of Experimental and Theoretical Physics* 34 (1958)

One can obtain the new eigenmodes of the propagation with a dispersion relation for the fluctuations called **Bogoliubov dispersion** [13, 14]:

$$\Omega_{\mathcal{B}}(\mathbf{k}_\perp) = \sqrt{\frac{\mathbf{k}_\perp^2}{2k_0}\left(\frac{\mathbf{k}_\perp^2}{2k_0} + 2g|\mathcal{E}_0|^2\right)} - i\frac{\alpha}{2}. \quad (11)$$

- **phononic** branch at low $\mathbf{k}_\perp$
- **particle** branch at high $\mathbf{k}_\perp$
- **Speed of sound** $c_s = c\sqrt{g|\mathcal{E}_0|^2}$
- **Healing length** $\xi = \frac{2\pi}{k_0\sqrt{g|\mathcal{E}_0|^2}}$



[13] N. N. Bogoljubov. **"On a New Method in the Theory of Superconductivity".** In: *Journal of Experimental and Theoretical Physics* 34 (1958)

[14] Q. Fontaine et al. **"Observation of the Bogoliubov dispersion relation in a fluid of light".** In: *Phys. Rev. Lett.* 121 (2018)

Quenches et out-of-equilibrium dynamics

Our non-linear medium is **finite**, when the beam enters or exits the cell, it undergoes a **violent index step** or interaction **quench**.

- Outside of the cell, the dispersion relation is the vaccuum one $\kappa(\mathbf{k}_\perp) = \frac{\mathbf{k}_\perp^2}{2k_0}$
- Inside the cell, the dispersion relation is the Bogoliubov dispersion $\Omega_B(\mathbf{k}_\perp)$

$\rightarrow$ The eigenstates of the propagation are **different**.

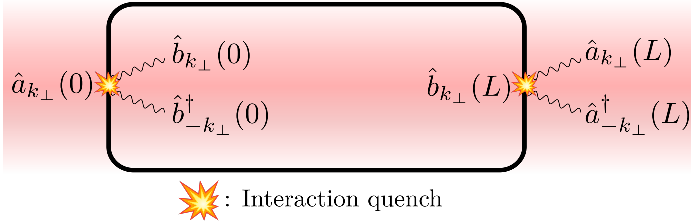
But a plane wave is solution of both equations ?

$$i\frac{\partial}{\partial z}\mathcal{E} = \boxed{-\frac{1}{2k}\nabla_\perp^2\mathcal{E}} + \boxed{k\frac{\delta n}{n}\mathcal{E}} + \boxed{g|\mathcal{E}|^2\mathcal{E}} - i\frac{\alpha}{2}\mathcal{E}$$

$$i\frac{\partial}{\partial z}\mathcal{E} = \boxed{-\frac{1}{2k}\nabla_\perp^2\mathcal{E}} + \boxed{k\frac{\delta n}{n}\mathcal{E}} - i\frac{\alpha}{2}\mathcal{E}$$

Quantum fluctuations create **correlated** excitations at $+\mathbf{k}_\perp$ and $-\mathbf{k}_\perp$ in the fluid **from the vacuum**:

$$\left\langle \hat{b}^\dagger_{\mathbf{k}_\perp} \hat{b}_{\mathbf{k}_\perp} \right\rangle = u^2_{\mathbf{k}_\perp} \left\langle \hat{a}^\dagger_{\mathbf{k}_\perp} \hat{a}_{\mathbf{k}_\perp} \right\rangle + v^2_{\mathbf{k}_\perp} \underbrace{\left\langle \hat{a}_{-\mathbf{k}_\perp} \hat{a}^\dagger_{-\mathbf{k}_\perp} \right\rangle}_{\hat{a}^\dagger_{-\mathbf{k}_\perp} \hat{a}_{-\mathbf{k}_\perp} + \left[\hat{a}^\dagger_{-\mathbf{k}_\perp}, \hat{a}_{-\mathbf{k}_\perp}\right]} + 2u_{\mathbf{k}_\perp} v_{\mathbf{k}_\perp} \mathrm{Re}\left(\langle \hat{a}_{-\mathbf{k}_\perp} \hat{a}_{\mathbf{k}_\perp} \rangle\right) \tag{12}$$



Quantum fluctuations create **correlated** excitations at $+\mathbf{k}_\perp$ and $-\mathbf{k}_\perp$ in the fluid **from the vaccuum**:

$$\begin{aligned}
&\left\langle \hat{a}^\dagger_{\mathbf{k}_\perp} \hat{a}_{\mathbf{k}_\perp} \right\rangle = 0 \\
&\langle \hat{a}_{-\mathbf{k}_\perp} \hat{a}_{\mathbf{k}_\perp} \rangle = 0 \\
&\left\langle \hat{b}^\dagger_{\mathbf{k}_\perp} \hat{b}_{\mathbf{k}_\perp} \right\rangle = u^2_{\mathbf{k}_\perp} \left\langle \hat{a}^\dagger_{\mathbf{k}_\perp} \hat{a}_{\mathbf{k}_\perp} \right\rangle + v^2_{\mathbf{k}_\perp} \left(\left\langle \hat{a}^\dagger_{-\mathbf{k}_\perp} \hat{a}_{-\mathbf{k}_\perp} \right\rangle + \mathbf{1} \right) + \\
&\qquad 2 u_{\mathbf{k}_\perp} v_{\mathbf{k}_\perp} \mathrm{Re} \left(\langle \hat{a}_{-\mathbf{k}_\perp} \hat{a}_{\mathbf{k}_\perp} \rangle \right)
\end{aligned} \tag{13}$$

Finally:

$$\begin{aligned}
\left\langle \hat{b}^\dagger_{\mathbf{k}_\perp} \hat{b}_{\mathbf{k}_\perp} \right\rangle &= v^2_{\mathbf{k}_\perp} \neq \mathbf{0} \\
\left\langle \hat{b}_{-\mathbf{k}_\perp} \hat{b}_{\mathbf{k}_\perp} \right\rangle &= u^2_{\mathbf{k}_\perp} \neq \mathbf{0}
\end{aligned} \tag{14}$$

How can we detect this emission of correlated excitations ?

We use the **structure factor** $S(\mathbf{k}_\perp)$[15]:

- Spectrum of **density fluctuations**
- Fourier transform of the autocorrelation function $g^{(2)}(\mathbf{r}_\perp)$

Considering a density $\hat{\rho}_{\mathbf{k}_\perp}$

$$S(\mathbf{k}_\perp) = \langle \hat{\rho}_{\mathbf{k}_\perp} \hat{\rho}_{-\mathbf{k}_\perp} \rangle \,. \tag{15}$$

Which simplifies for a plane wave (or a condensate):

$$S(\mathbf{k}_\perp) = \frac{1}{N_0} \left\langle |\delta \hat{\rho}_{\mathbf{k}_\perp}|^2 \right\rangle . \tag{16}$$

where $\delta\hat{\rho}_{\mathbf{k}_\perp} = \hat{\rho}_{\mathbf{k}_\perp} - \langle \hat{\rho}_{\mathbf{k}_\perp} \rangle$ are the density fluctuations and N_0 the noise of a coherent state of the same number of particles.

[15] V. I. Yukalov. **"Structure factor of Bose-condensed systems".** en. In: *Journal of Physical Studies* 11 (2007)

One can get an analytical expression for $S(\mathbf{k}_\perp)$ [16, 17]:

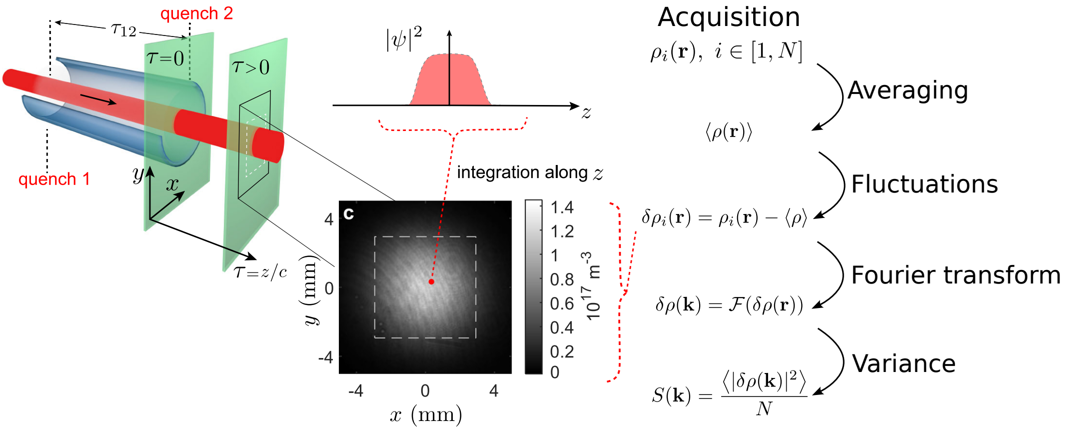
$$\begin{aligned} S(\mathbf{k}_\perp) &= 1 + 2N(\mathbf{k}_\perp) + 2\mathrm{Re}\left[C(\mathbf{k}_\perp)\right] \\ N(\mathbf{k}_\perp) &= \left\langle \hat{a}^\dagger_{\mathbf{k}_\perp} \hat{a}_{\mathbf{k}_\perp} \right\rangle \\ C(\mathbf{k}_\perp) &= \langle \hat{a}_{-\mathbf{k}_\perp} \hat{a}_{\mathbf{k}_\perp} \rangle \end{aligned} \tag{17}$$

- The structure factor allows to give information on the **populations** and **correlations** of the system fluctuations.
- As we can solve the **kinetic equation**, we can compute these correlators.

[16] Chen-Lung Hung et al. **"Extracting density–density correlations from in situ images of atomic quantum gases"**. en. In: *New J. Phys.* 13 (2011)

[17] Chen-Lung Hung et al. **"From Cosmology to Cold Atoms: Observation of Sakharov Oscillations in a Quenched Atomic Superfluid"**. In: *Science* 341 (2013)

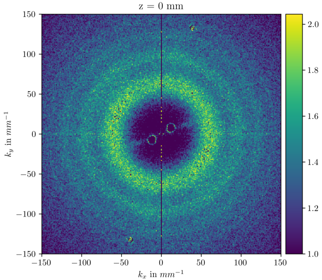
In order to measure $S(\mathbf{k}_\perp)$, we simply measure the spectrum of the density fluctuations[18].

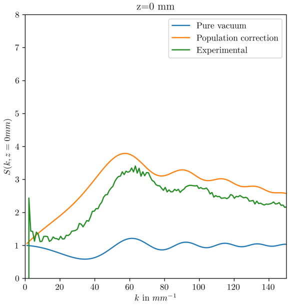


[18] Jeff Steinhauer et al. **"Analogue cosmological particle creation in an ultracold quantum fluid of light"**. In: *Nat. Comm.* 13 (2022)

Modulation of the structure factor with the wavenumber $\mathbf{k}_{\perp}$:

- Two central peaks: corrected parasitic fringes
- Cylindrical symmetry: concentric rings
- Azimuthal average in order to denoise the signal
- Free evolution after the cell reveals the correlation signal

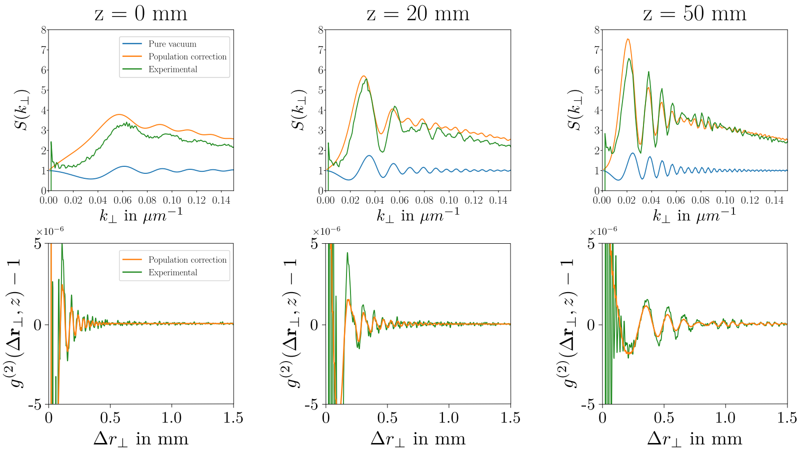




Experimental results

Conversion of the structure factor into a spatial $g^{(2)}$:

$$g^{(2)}(\Delta\mathbf{r}_\perp, z) = 1 + \int \mathrm{d}\mathbf{k}_\perp e^{i\mathbf{k}_\perp \cdot \Delta\mathbf{r}_\perp} \left[S(\mathbf{k}_\perp, z) - 1\right]$$

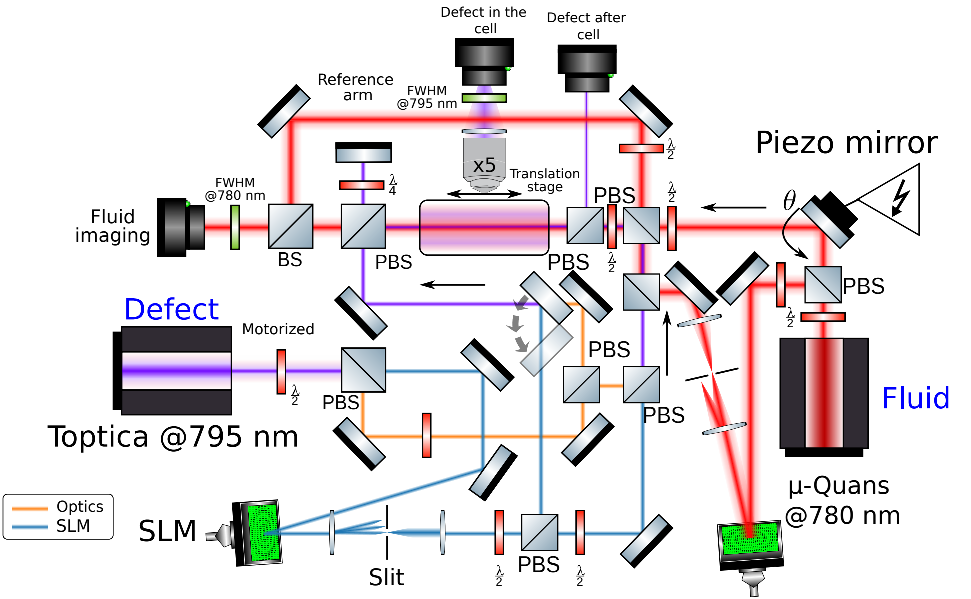


Observation of a **correlation signal** induced by the **quantum fluctuations** of the vacuum by measuring density fluctuations:

- **Elegant** measurement technique
- **Analytical** model
- **Temporal** dynamics ?

Fluids of light as credible testbeds for analogue experiments

Antisèches


Defect in the cell
Defect after cell
Reference arm
FWHM @795 nm
x5
Translation stage
Piezo mirror
θ
PBS
λ/2
λ/4
Fluid imaging
FWHM @780 nm
BS
PBS
Defect
Motorized
Toptica @795 nm
Fluid
μ-Quans @780 nm
Optics
SLM
SLM
Slit

$$i\frac{\partial}{\partial z}\begin{pmatrix}\tilde{u}_{\mathbf{k}_\perp}(z)\\ \tilde{v}_{\mathbf{k}_\perp}(z)\end{pmatrix} = \underbrace{\begin{pmatrix}\frac{i\alpha}{2}+\frac{\mathbf{k}_\perp^2}{2k_0}+k_0\Delta n(z) & k_0\Delta n(z)\\ -k_0\Delta n(z) & \frac{i\alpha}{2}-\frac{\mathbf{k}_\perp^2}{2k_0}-k_0\Delta n(z)\end{pmatrix}}_{\mathcal{A}_{\mathbf{k}_\perp}(z)-\frac{i\alpha}{2}\mathbb{I}}\begin{pmatrix}\tilde{u}_{\mathbf{k}_\perp}(z)\\ \tilde{v}_{\mathbf{k}_\perp}(z)\end{pmatrix} \tag{18}$$

$\Delta n(z) = \epsilon_0 c n_2 I e^{-\alpha z}$

Spectroscopie de Bragg