EFFECTIVE-FIELD THEORIES OF ANALOGUE GRAVITY

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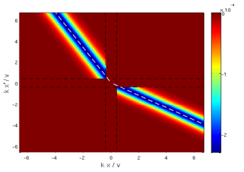
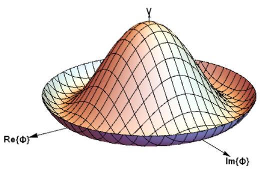
Introduction

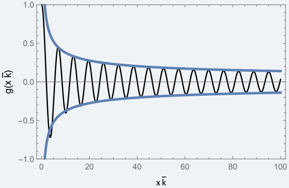
Motivations



Methodology



Results and Applications



Analogue Gravity

What is it?

Analogue Gravity is an approach to simulate the fields' propagation on top of a curved spacetime

The most well-known of these analogies is:

light waves in a curved spacetime $\Longleftrightarrow$ sound waves in a flowing fluid

The metric is obtained:

Einstein equation

Background

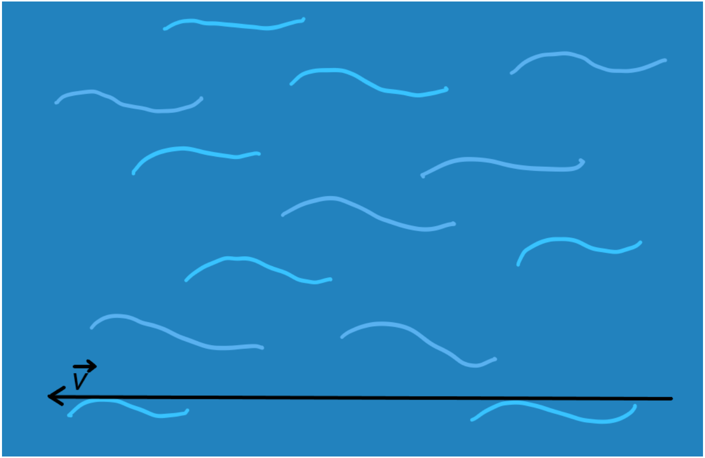
Acoustic Black Hole

Flowing fluid

- c_s: speed of sound
- v: fluid's velocity

W. G. Unruh

Found Phys 44, 532–545 (2014)



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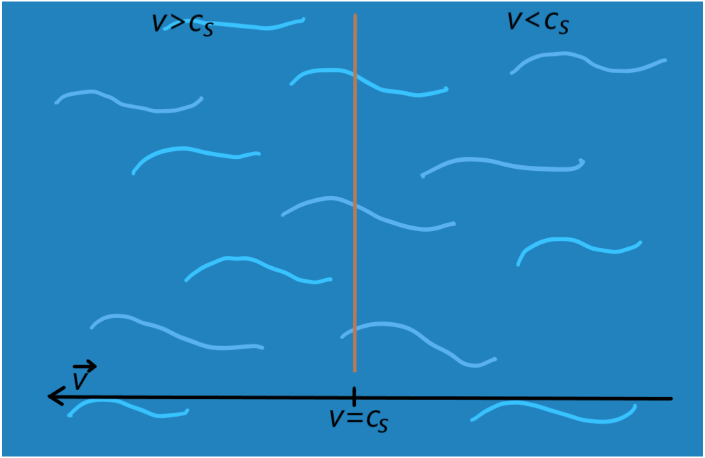

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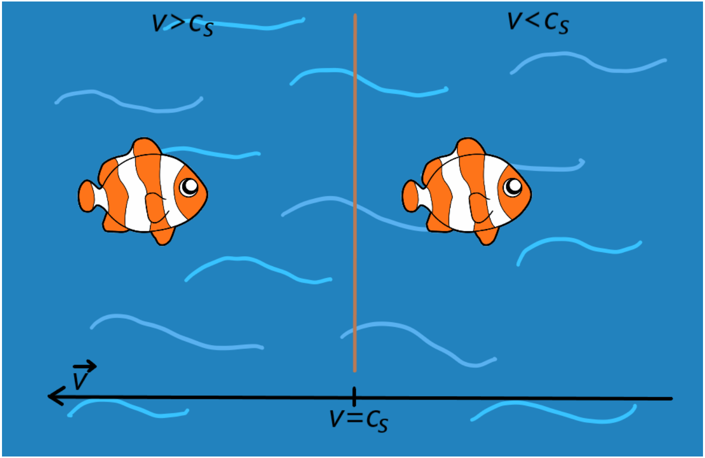

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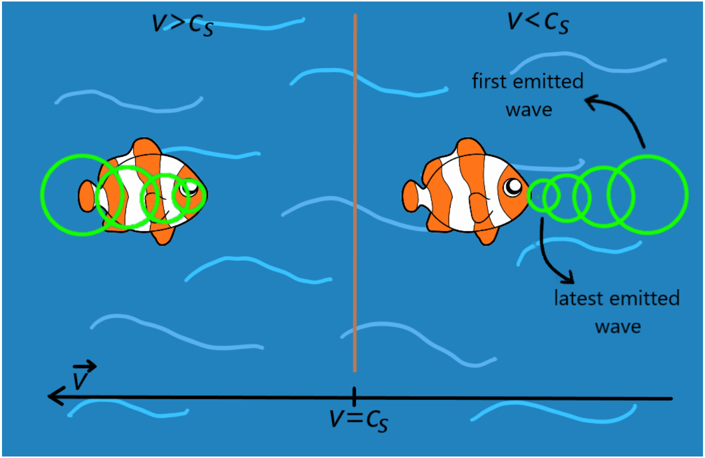
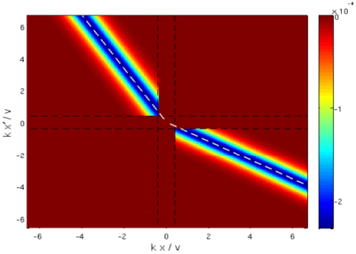


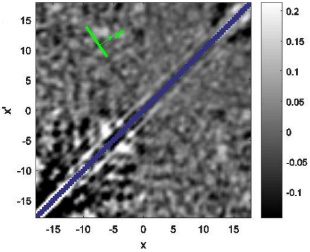
Density-Density Correlation Function

Phonon emission $\Longleftrightarrow$ Long-range correlations between density fluctuations

- R. Balbinot *et al.* (2008)
  Phys.Rev.A78,021603(R)



- J. Steinhauer
  Nature Phys 12, 959–965 (2016)



Surface gravity

$$G_2^{1D}(x,x') \propto \cosh^{-2}\left[\frac{k}{2}\left(\frac{x}{c_r - v} + \frac{x'}{v - c_l}\right)\right]$$

On the right

On the left

Our Research Questions

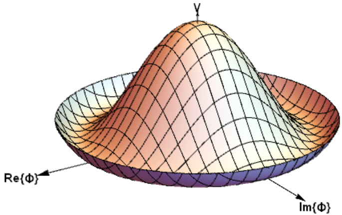
1. How to set up a most general effective-field theory for an analogue model?

2. How to design versatile methods to keep together general relativity and quantum field theory features?

Effective-Field Theory for an Analogue Model

Analog model with a relativistic BEC = Effective Low Energy Phonon Lagrangian

$\mu > m$



Broken Symmetries:

- $U(1)$, spontaneous
- Lorentz, explicit

Madelung: $\Phi = \frac{\rho}{\sqrt{2}} e^{i\theta}$

The superfluid's velocity is $\mathbf{v} = \frac{\hbar}{\mu}\boldsymbol{\nabla}\theta$

$$\mathcal{L} = \frac{1}{2}\partial_\nu\rho\partial^\nu\rho + \frac{1}{2}\rho^2\partial_\nu\theta\partial^\nu\theta - \underset{\substack{\downarrow\\ \text{Chemical}\\ \text{potential}}}{\mu}\rho^2\partial_t\theta - \frac{1}{2}({\underset{\substack{\downarrow\\ \text{Mass}\\ \text{of } \Phi}}{m}}^2 - \mu^2)\rho^2 - \frac{\overset{\substack{\text{Coupling}\\ \text{constant}\\ \curvearrowright}}{\lambda}}{4}\rho^4.$$

The Most General BEC Effective Lagrangian

$$\mathcal{L}(\rho, \partial_\mu \rho, \partial_\mu \theta) \qquad \begin{cases} \rho = \rho_0 + \tilde{\rho} + .. \\ \theta = \theta_0 + \tilde{\theta} + .. \end{cases}$$

- $\tilde{\rho}$: density fluctuation (Higgs)
- $\tilde{\theta}$: phonon field (Goldstone)

Assumptions:

- Background superflow velocity $\vec{v} = \vec{\nabla}\theta_0/\mu$ ($\hbar = 1$).
- The system is inviscid
- Local density approximation

General Lagrangian expansion

We expand the action around the stationary point (ρ_0, θ_0)

$$
\begin{aligned}
\mathcal{L} = & \overbrace{\mathcal{L}(\rho_0, \partial_\mu \rho_0, \partial_\mu \theta_0)}^{\text{background pressure}} + \\
& + \overbrace{\tilde{\rho} \left(\frac{\delta \mathcal{L}}{\delta \rho} - \partial_\mu \frac{\delta \mathcal{L}}{\delta \partial_\mu \rho} \right) \Bigg|_{\rho_0, \theta_0} + \partial_\mu \tilde{\theta} \frac{\delta \mathcal{L}}{\delta \partial_\mu \theta} \Bigg|_{\rho_0, \theta_0}}^{Eulero-Lagrange\ equation} + \\
& + \mathcal{L}_2 + \ldots
\end{aligned}
$$

We consider $\mathcal{L}_{\text{eff}} = \mathcal{L}_2$ in terms of $\tilde{\rho}$, $\tilde{\theta}$ and their products
From the equation of motion

Leading order in p^μ

$\tilde{\rho} \simeq \frac{1}{\tilde{m}^2} V^\mu \partial_\mu \tilde{\theta}$

Next-to-leading order in p^μ

$\tilde{\rho} \simeq \frac{1}{\tilde{m}^2} \left(1 - \frac{\Box}{\tilde{m}^2}\right) V^\mu \partial_\mu \tilde{\theta}$

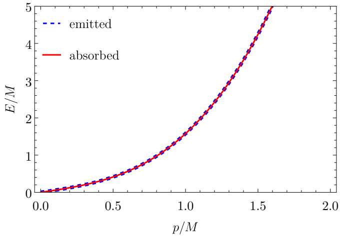
Phonon Dispersion Law

$$E_{\pm,\mathrm{lab}} \simeq \mathbf{v}\cdot\mathbf{p} \pm \left(c_s|\mathbf{p}| + \frac{\mathcal{A}}{M^2}|\mathbf{p}|^3 \right)$$

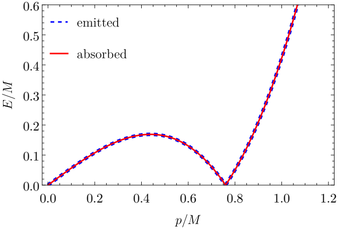
Cut-off

Far from the horizon $v = 0$

Monotonic ($\mathcal{A} > 0$)



Non-Monotonic ($\mathcal{A} < 0$)



I consider $|\mathcal{A}| = 1$ and $c_s = c/\sqrt{3}$

Density-Density Correlation Function

In a homogeneous 1+1D BEC

Field
propagator

$$\begin{cases} \mathcal{L} = \frac{1}{2}\psi^t G^{-1}\psi \\ \psi^t = (\tilde{\rho}, \tilde{\theta}) \end{cases} \Longrightarrow \begin{cases} Z[J] \propto e^{i \int d^2x \int d^2y \frac{1}{2} J^t(x) G(x,y) J(y)} \\ J^t = (J_1, J_2) \end{cases}$$

External current
associated to $\tilde{\rho}$

External current
associated to $\tilde{\theta}$

$$\Longrightarrow \langle \tilde{\rho}(x)\tilde{\rho}(y) \rangle = - \frac{1}{Z[0]} \frac{\delta^2 Z[J]}{\delta J_1(x) \delta J_1(y)}\bigg|_{J=0} = -iG_{11}(x,y)$$

Results: Effective Lagrangian at NLO

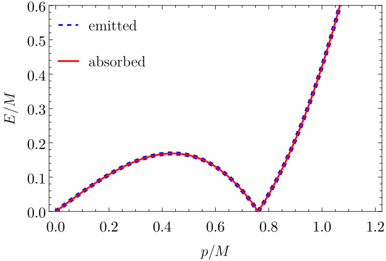
$$\mathcal{L} = \underbrace{\frac{1}{2}\overbrace{\rho_0^2\left(\eta^{\mu\nu} + \left(\frac{1}{c_s^2} - 1\right) v^\mu v^\nu\right)}^{\sqrt{-g}g^{\mu\nu}}\partial_\mu\tilde{\theta}\partial_\nu\tilde{\theta}}_{A} - \underbrace{\frac{\rho_0^2}{8}\left(\frac{\gamma}{\mu}\left(\frac{1}{c_s^2} - 1\right)\right)^2 v^\mu v^\nu \partial_\mu\tilde{\theta}\Box\partial_\nu\tilde{\theta}}_{B}\,.$$

- A: LO term ($\sim p^2$). It is the kinetic term $\Longrightarrow$ analogue metric
- B: NLO term ($\sim p^4$)

Results: Non-Monotonic Dispersion Law

$$\mathcal{L}_2 = \frac{1}{2}\partial_\mu\tilde{\rho}\partial^\mu\tilde{\rho} - \frac{\tilde{m}^2}{2}\tilde{\rho}^2 + \frac{B^2}{2}\partial_\mu\tilde{\theta}\partial^\mu\tilde{\theta} + V^\mu\tilde{\rho}\partial_\mu\tilde{\theta} + (C_1\eta^{\mu\nu} + C_2 V^\mu V^\nu)\,\partial_\mu\tilde{\rho}\partial_\nu\tilde{\theta}.$$

Non-monotonic dispersion law $\iff |C_1| > |B| \implies \mathcal{A} < 0$



Non-monotonic dispersion law $\Longrightarrow$ Getting to a phase transition?

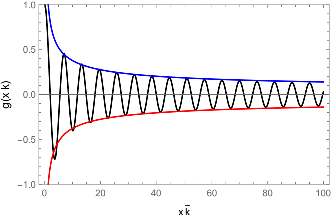
Results: Density-Density Correlation Function

New method for the density-density correlation function using field theory

$\mathcal{A} > 0$

$$\langle \tilde{\rho}(x)\tilde{\rho}(0)\rangle = \frac{c_s}{2\pi \tilde{m}^2 x^2}.$$

$\mathcal{A} < 0$



🟦 $\sim \frac{1}{\sqrt{x}}$ 🟥 $\sim -\frac{1}{\sqrt{x}}$

$$\langle \tilde{\rho}(x)\tilde{\rho}(0)\rangle \propto g(x\bar{k})$$

Results: Density-Density Correlation Function

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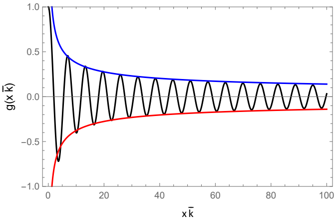


🟦 $\sim \frac{1}{\sqrt{x}}$ 🟥 $\sim -\frac{1}{\sqrt{x}}$

Oscillations $\implies$ breaking of translational symmetry

Conclusions

- Innovative approach to construct an analogue model with BEC
- Building of the phonon low energy effective Lagrangian at NLO in phonons' momentum
- Phonon dispersion law with p^3 corrections has:
  - monotonic behaviour
  - non-monotonic behaviour
    $\Longrightarrow$ translation-symmetry breaking
    $\Longrightarrow$ phase transition to supersolid state (?)
- Density-density correlation function for a homogeneous BEC in (1+1)D for the system with:
  - monotonic dispersion law: new method! (see instead Haldane PhysRevLett.47.1840)
  - non-monotonic dispersion law: new physics!

Applications

- Calculate the density-density correlation function with
  - a horizon
  - current density approximation
- Calculate the phonon-condensate correlation function $\Longrightarrow$ starting state of the system
- Study the phonons’ dynamic in non-homogeneous systems with a horizon
- Mean field approximation $\Longrightarrow$ Local field, beyond mean-field approximation

The End

Thank you for your attention!

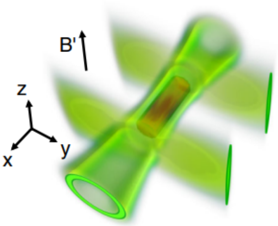
Speed of sound

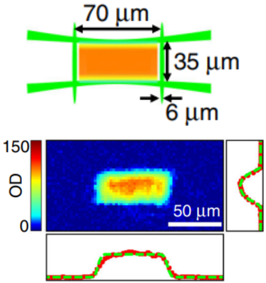
In the model $\lambda\Phi^4$

$$c_s^2 = \frac{\lambda\rho_0^2}{3\lambda\rho_0^2 + 2m^2}$$

- maximum when $\rho_0 \to \infty \implies c_s \to 1/\sqrt{3} \implies$ first sound
- minimum when $\rho_0 \to 0 \implies c_s \to 0 \implies$ normal phase

Optical-Box Trap





Steinhauer Experiment