

Analysis of long crystal channelling efficiency from hadron beam test data

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3° WORKSHOP ON ELECTROMAGNETIC DIPOLE MOMENTS OF UNSTABLE PARTICLES - 11/12/2023

1. SHORT CRYSTAL

Crystal properties:

Length	4 mm
Material	Silicon
Bend radius	80.0 m
X dimension	2 mm
Y dimension	35 mm

2. LONG CRYSTAL

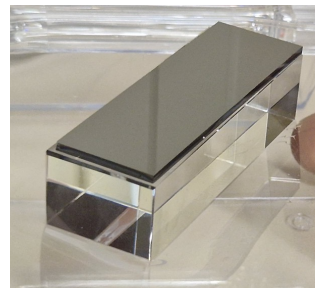
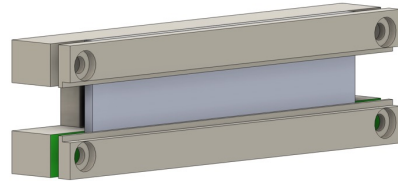
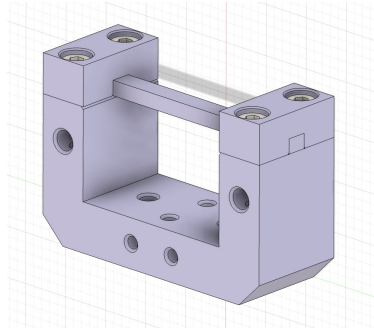
Crystal properties:

Length	70 mm
Material	Silicon
Bend radius	10.0 m
X dimension	2 mm
Y dimension	8 mm

3. ANODIC-BONDED

Crystal properties:

Length	70.5 mm
Material	Silicon
Bend radius	5.3 m
X dimension	2 mm
Y dimension	22.5 mm



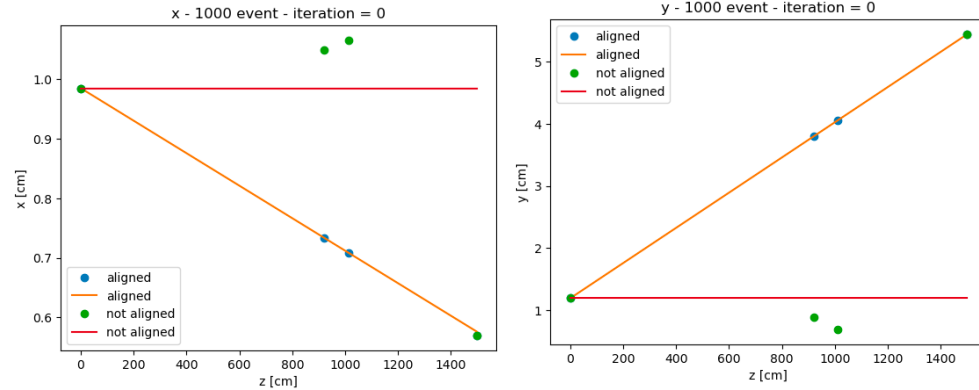
ANALYSIS PROCEDURE SHORT CRYSTAL

1. Alignment
2. Crystal position estimation
3. Channelling window estimation
4. Channelling efficiency estimation:
 - a. Double gaussian estimation
 - b. Right-sided single gaussian estimation

ANALYSIS PROCEDURE LONG +ANODIC CRYSTAL

1. Alignment
2. Crystal position estimation
3. Channelling window estimation
4. Channelling efficiency estimation:
 - a. Wide crystal face
 - b. Wide crystal face + angular scan
 - c. Small (x,y) bins +angular scan
 - d. Wide crystal face after correction map

Alignment algorithm developed by A. Merli



First station is fixed and z coordinate is fixed for all the planes

Alignment performed for all the crystals since the apparatus changed in between the data taking.

Rotations must be taken into account especially for the short crystal.

ALIGNMENT ALGORITHM

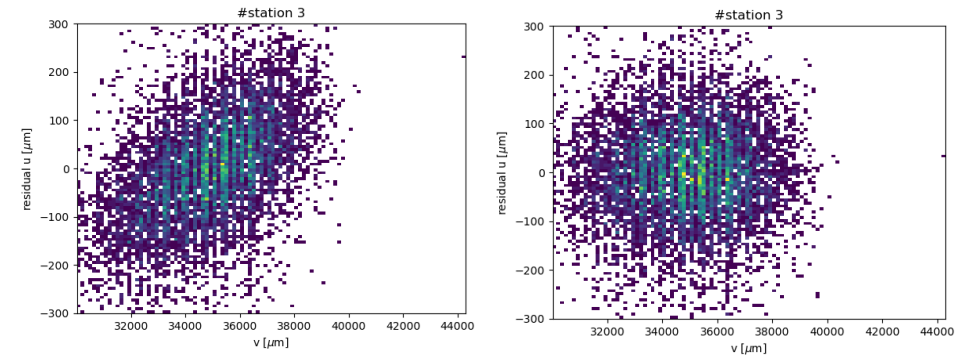
Unbiased fit of the alignment parameters

(2 translation + 1 rotation) x 3 planes

GLOBAL PARAMETER

event track parameters (intercept + slope)

LOCAL PARAMETER



Residual of the last station before and after including the rotation around z

1. SHORT CRYSTAL

Selected window

x [1, 1.2] cm

y [1.25, 1.75] cm

$\theta_L = 13.3 \mu\text{rad}$

2. LONG CRYSTAL

Selected window

x [0.88, 1.4] cm

y [1.9, 2.19] cm

$\theta_L = 12.9 \mu\text{rad}$

3. ANODIC-BONDED

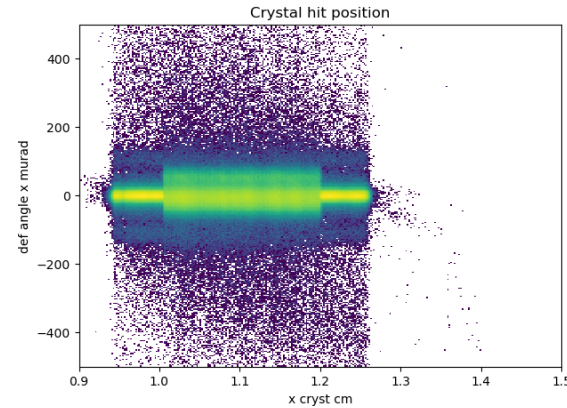
Selected window

x [0.86, 0.95] cm

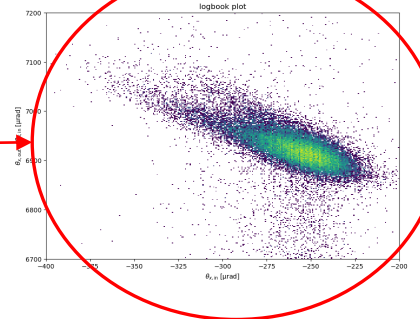
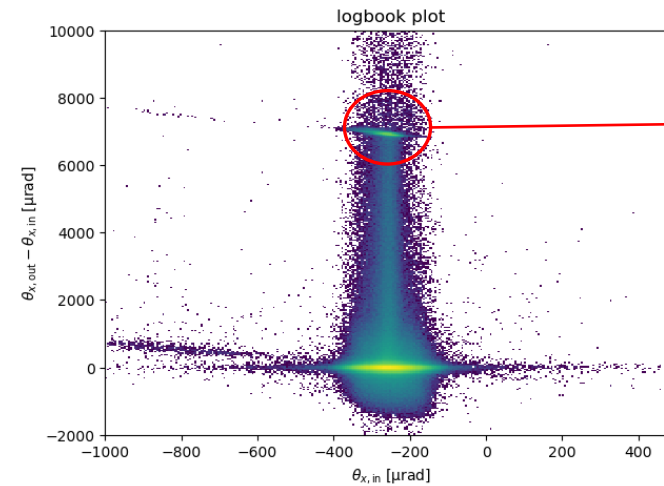
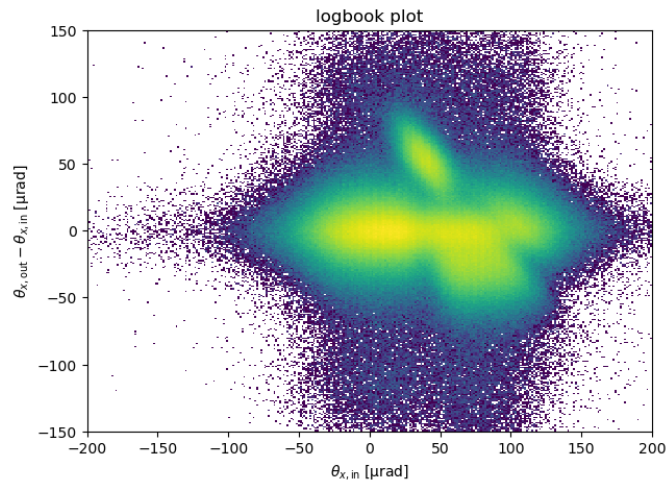
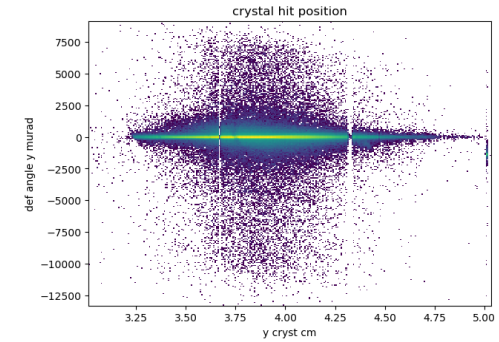
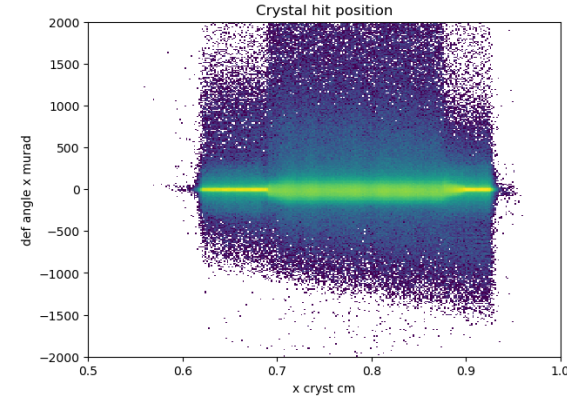
y [1.8, 2.3] cm

$\theta_L = 12.5 \mu\text{rad}$

SHORT CRYSTAL



LONG CRYSTAL



1. SHORT CRYSTAL

Selected window

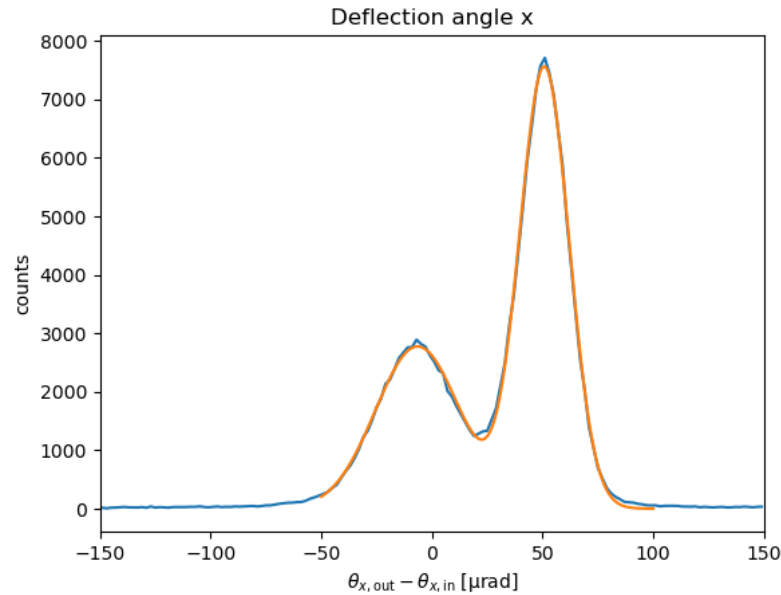
x [1, 1.2] cm

y [1.25, 1.75] cm

$\theta_L = 13.3 \mu\text{rad}$

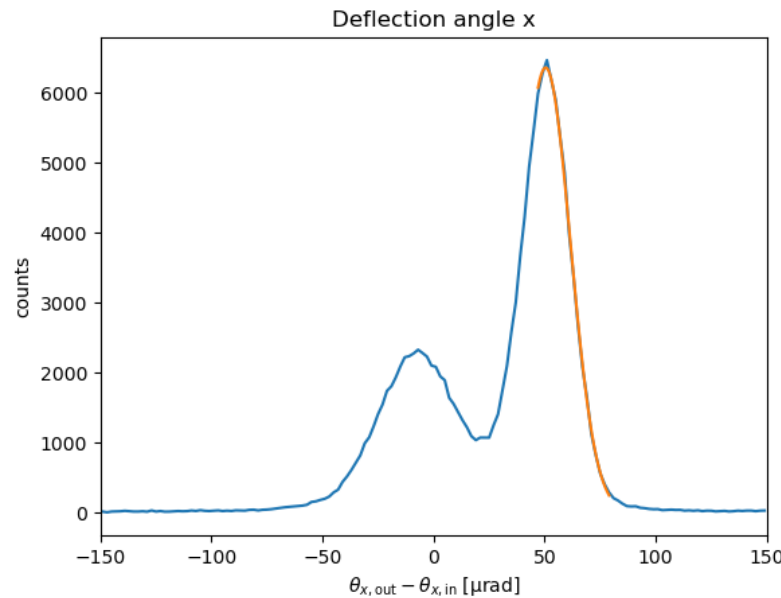
θ_{in} angular cut $42 \pm 0.5 * \theta_L \mu\text{rad}$

$$\varepsilon_{ch} = \frac{\text{number of channeled particles from gaussian fit}}{\text{n. particles with } \theta_{in} < 0.5 * \theta_L}$$



METHOD 1
Double gaussian fit

$$\varepsilon_{ch} = 61.7 \pm 0.3 \%$$



METHOD 2
FIT RANGE: $\theta_{out} - \theta_{in} \in [45, 80] \mu\text{rad}$

$$\varepsilon_{ch} = 61.0 \pm 0.2 \%$$

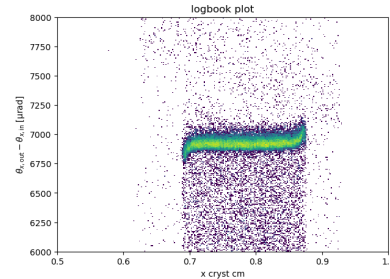
2. LONG CRYSTAL

Selected window

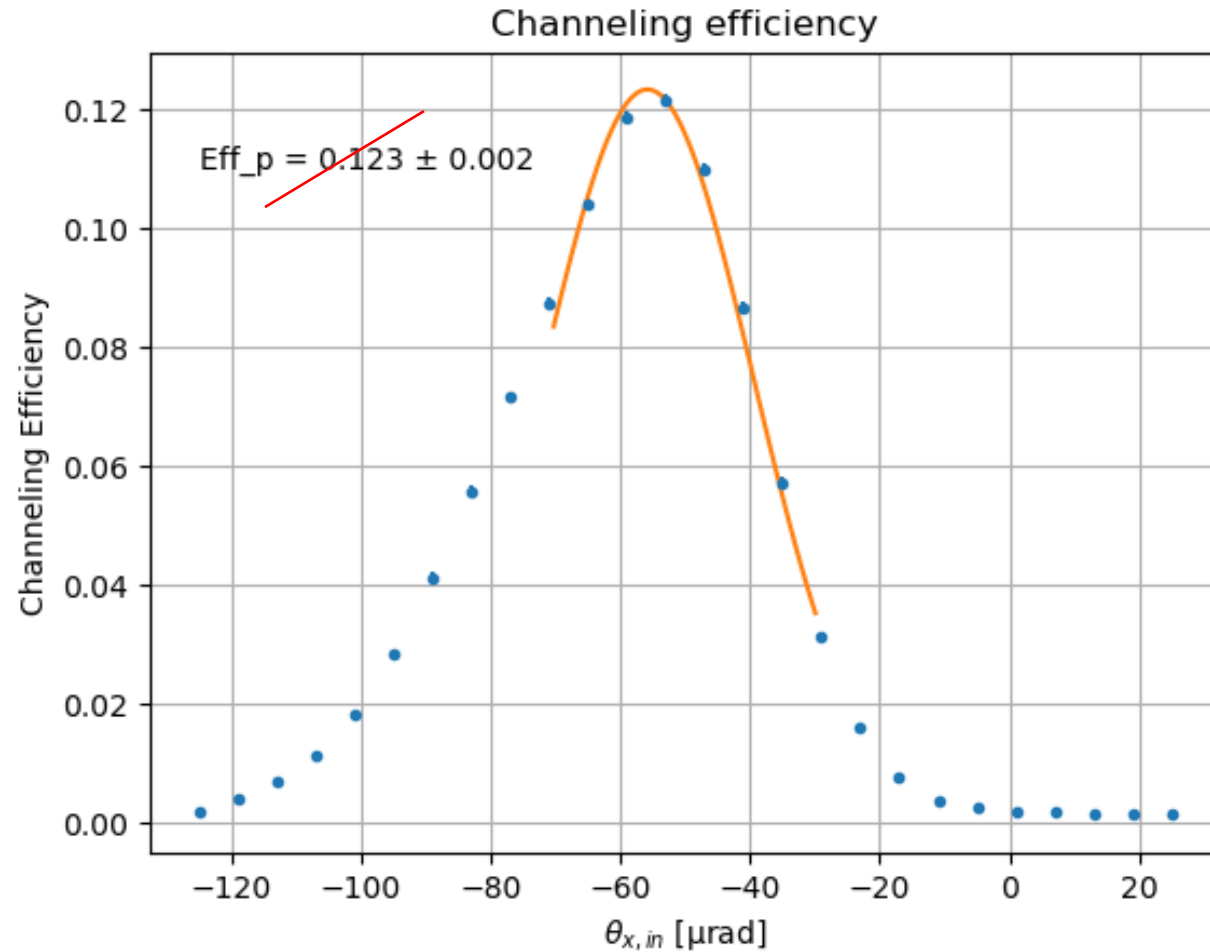
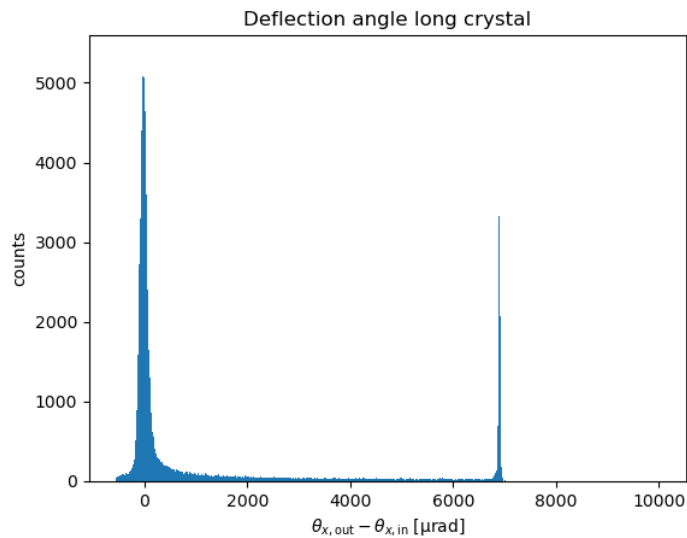
x [0.88, 1.4] cm

y [1.9, 2.19] cm

$\theta_L = 12.9 \mu\text{rad}$

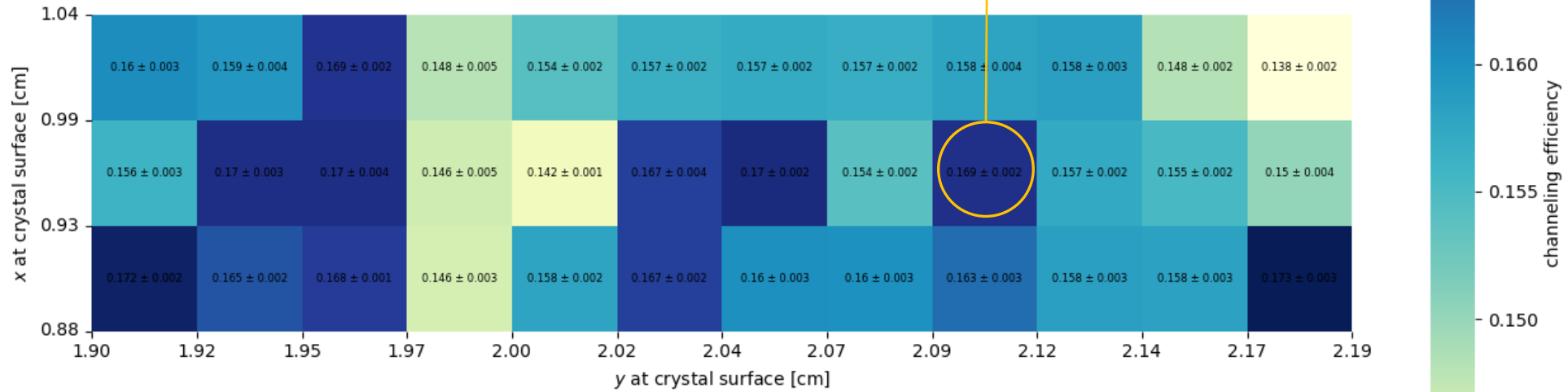
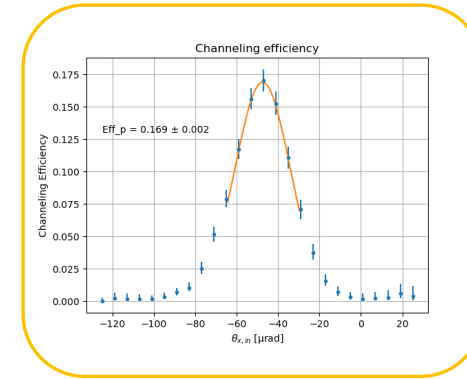
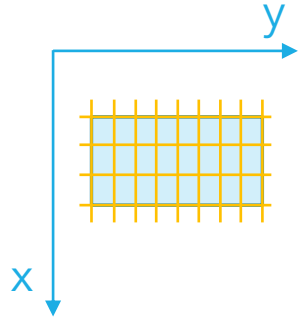


ANGULAR SCAN: $5.9 \mu\text{rad}$ step $\pm 0.5 * \theta_L$

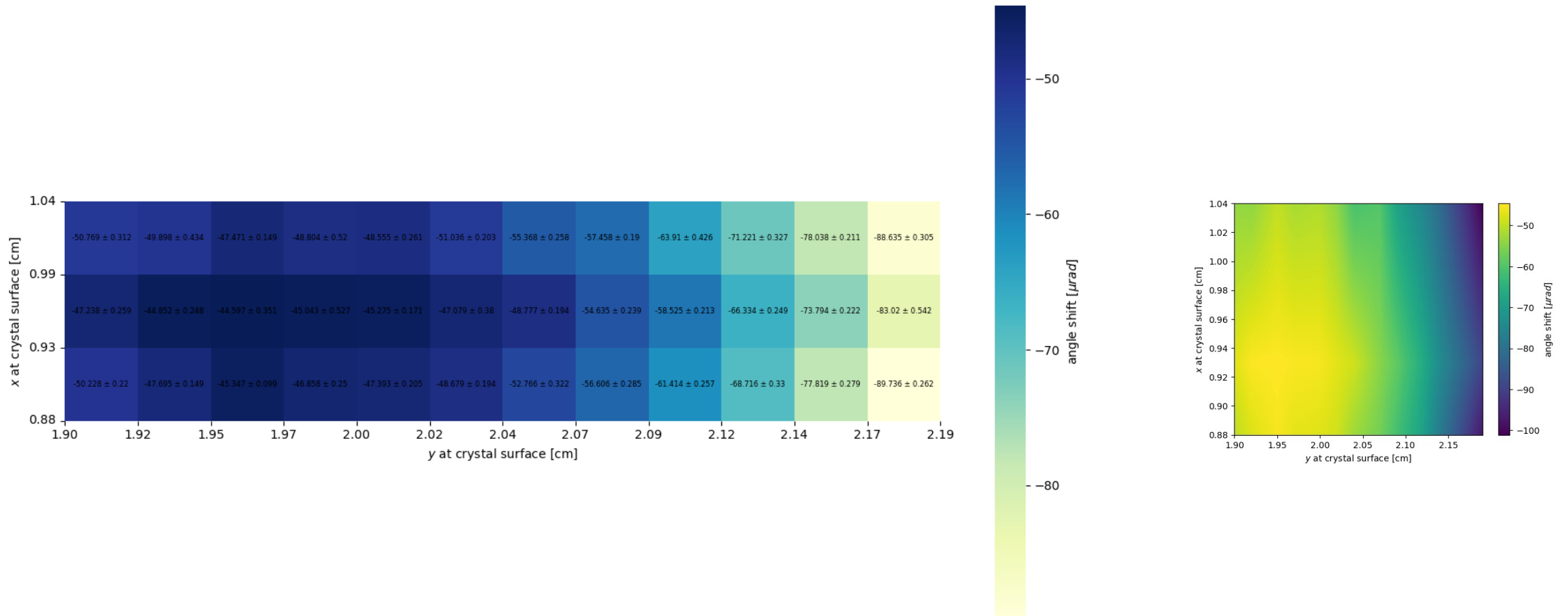


Asymmetry due to torsion effect

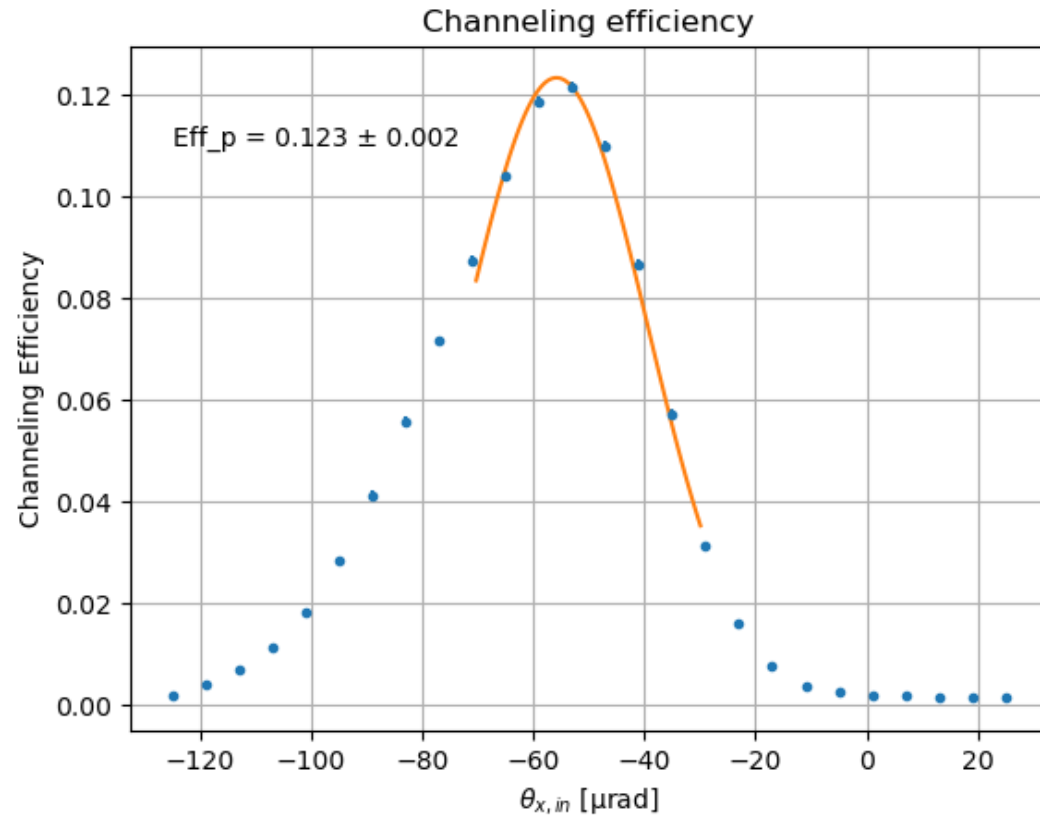
LONG CRYSTAL : efficiency map



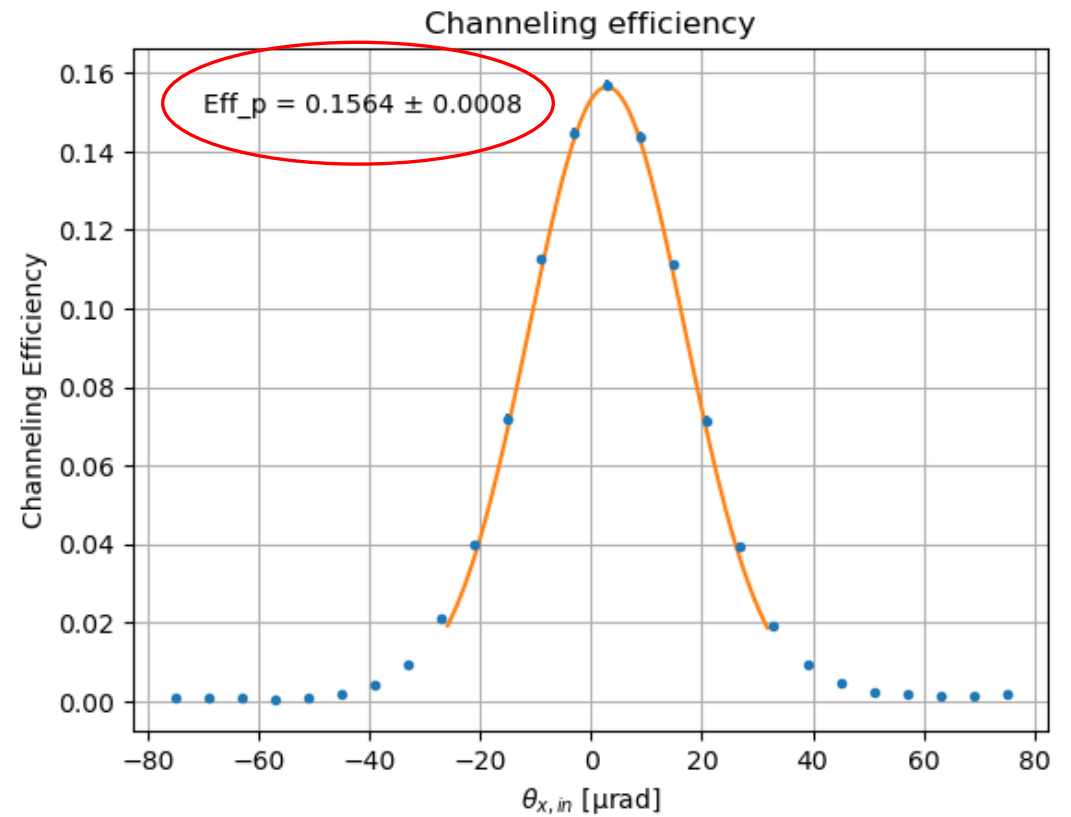
LONG CRYSTAL : torsion map



CHANNELLING EFFICIENCY



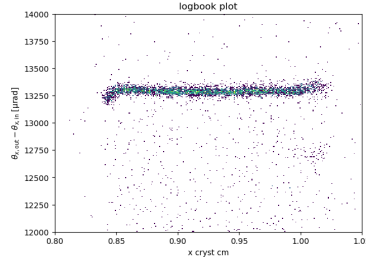
**BEFORE TORSION
CORRECTION**



**AFTER TORSION
CORRECTION**

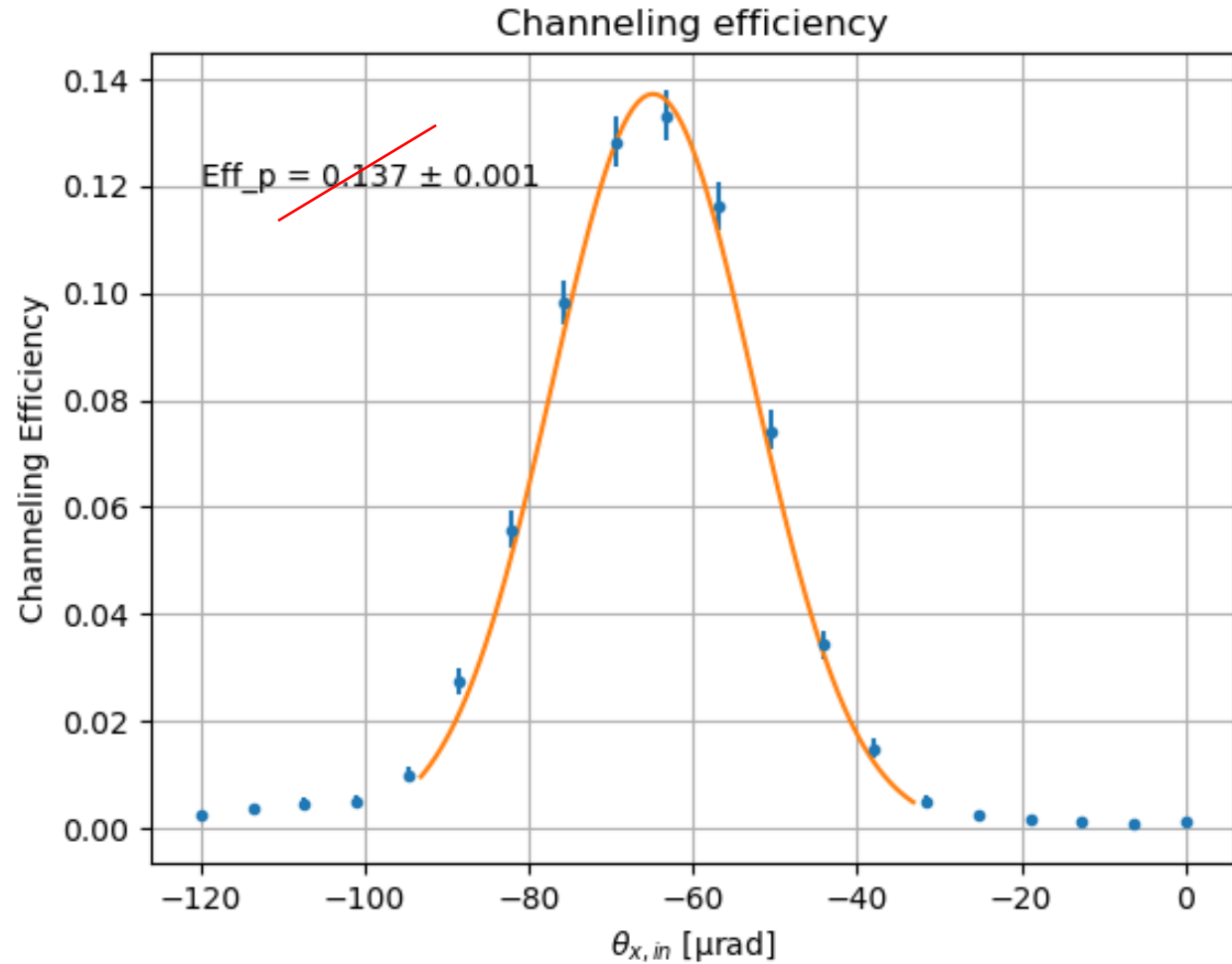
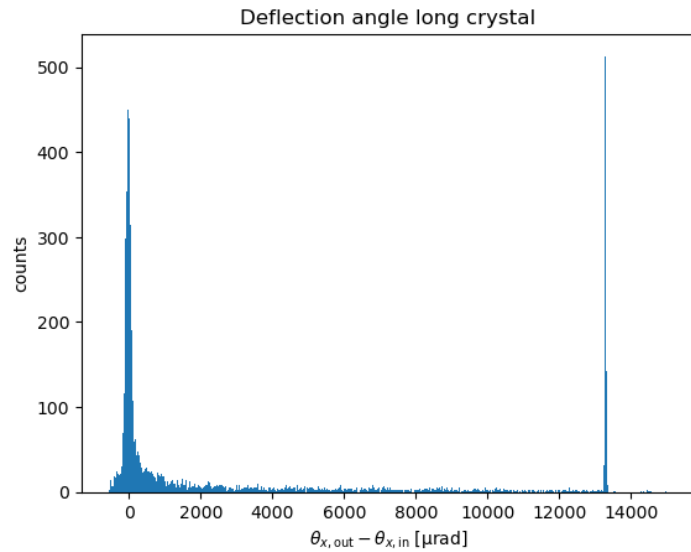
3. ANODIC BONDING

Selected window
x [0.86, 0.95] cm
y [1.8, 2.3] cm



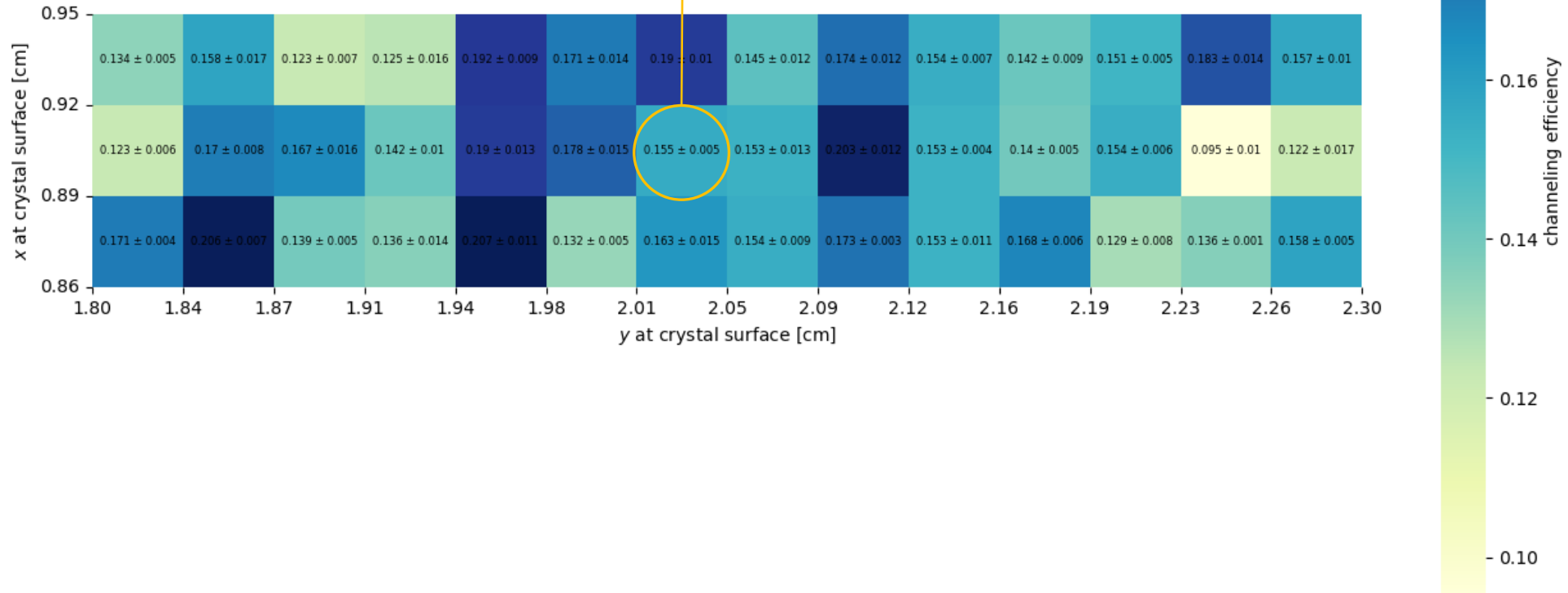
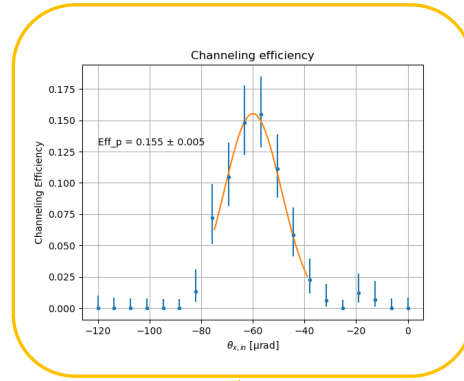
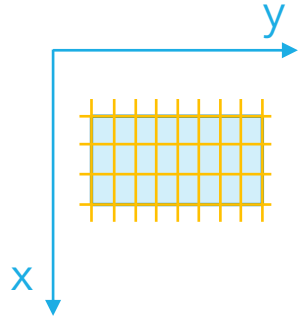
$\theta_L = 12.5 \mu\text{rad}$

ANGULAR SCAN: $6 \mu\text{rad}$ step $\pm 0.5 * \theta_L$

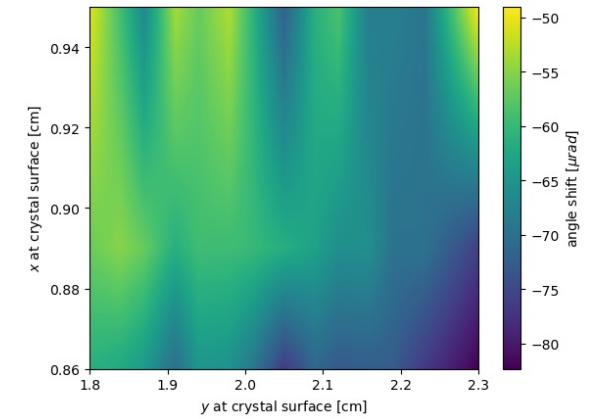
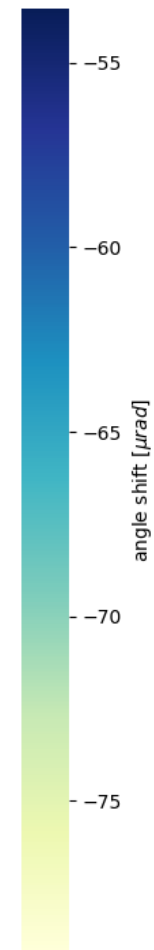
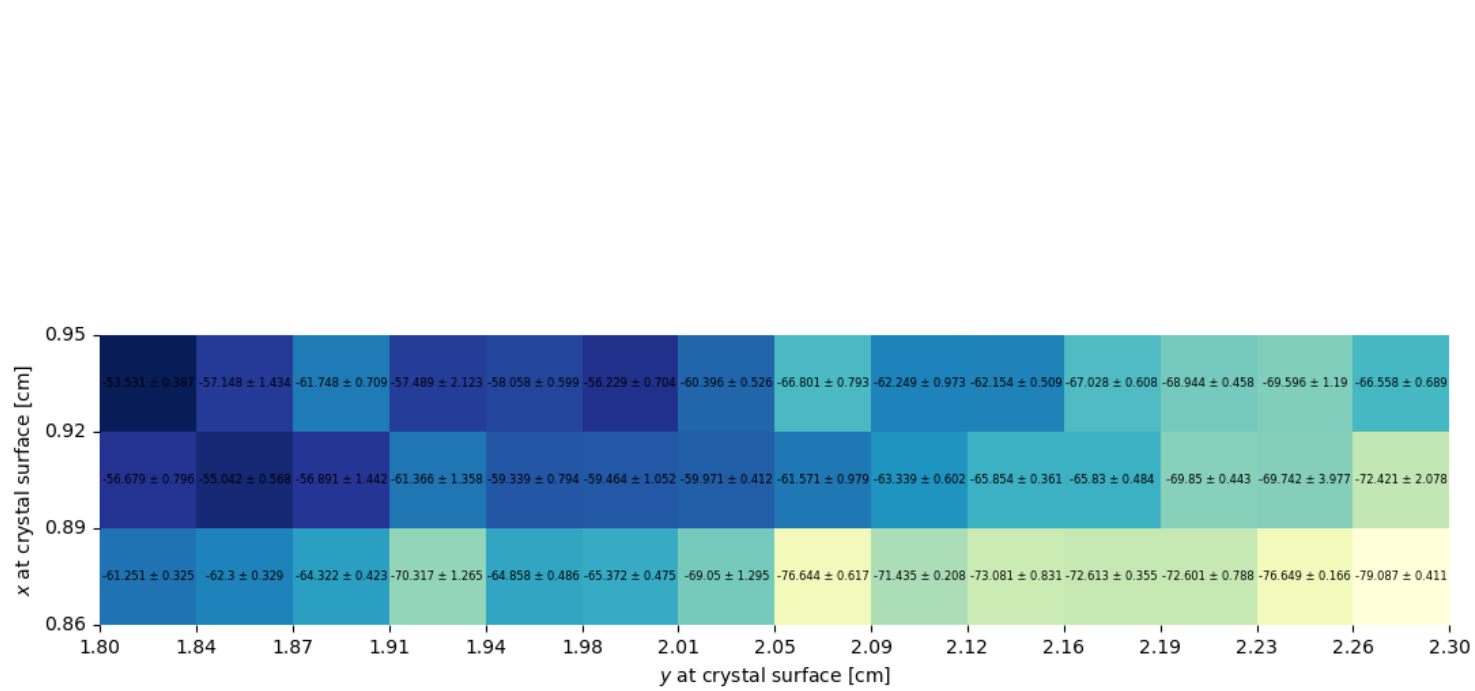


Symmetric already before torsion correction

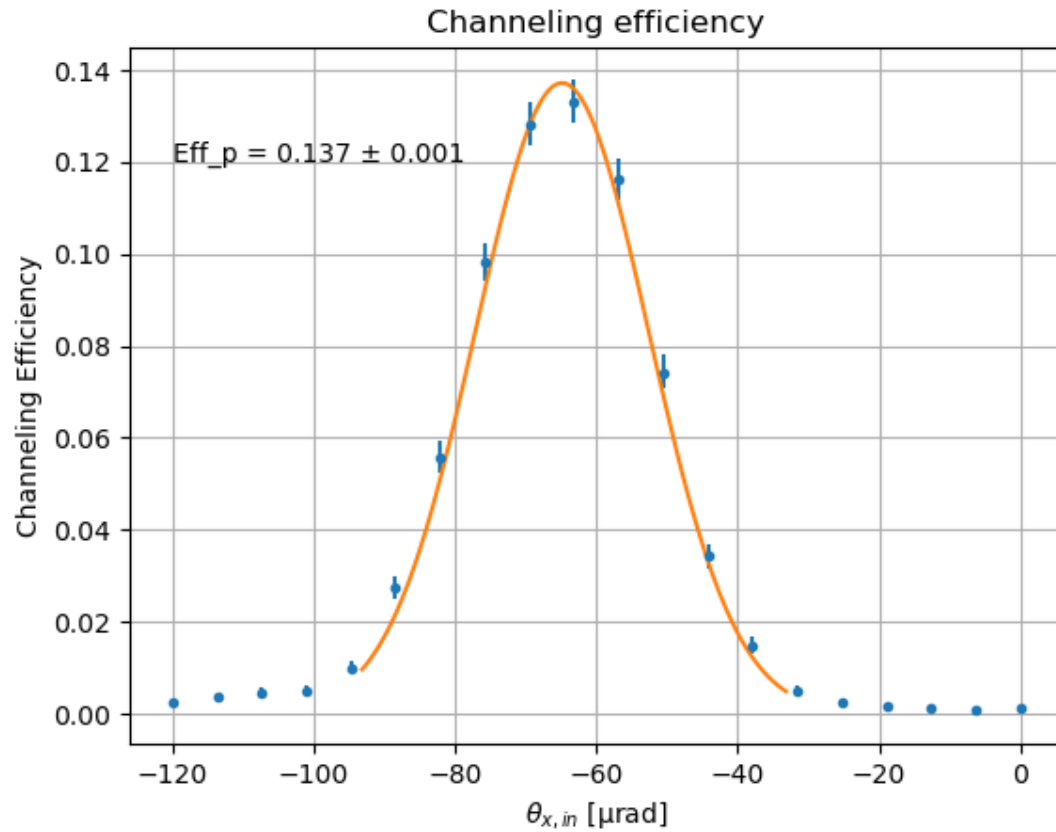
ANODIC BONDING: efficiency map



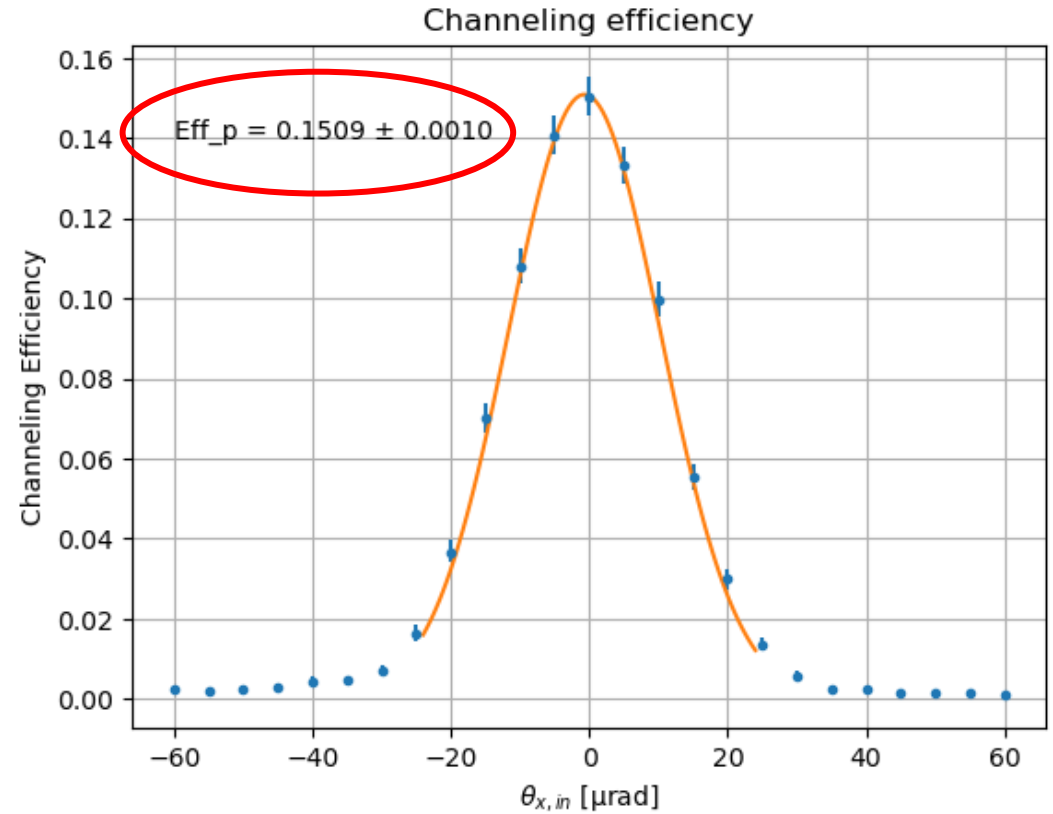
ANODIC BONDING: torsion map



CHANNELLING EFFICIENCY



**BEFORE TORSION
CORRECTION**



**AFTER TORSION
CORRECTION**

1. SHORT CRYSTAL

$$\epsilon_{ch1} = 0.617 \pm 0.003$$

$$\epsilon_{ch2} = 0.610 \pm 0.002$$

2. LONG CRYSTAL

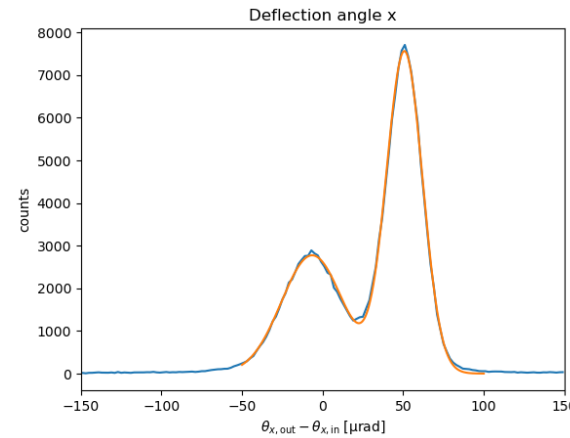
$$\epsilon_{ch_pre} = 0.123 \pm 0.002$$

$$\epsilon_{ch_post} = 0.1568 \pm 0.0007$$

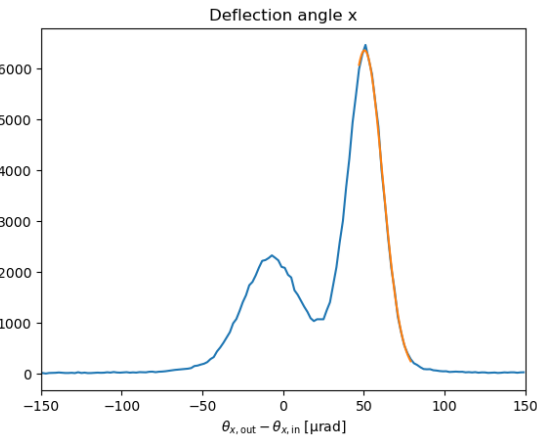
3. ANODIC-BONDED

$$\epsilon_{ch_pre} = 0.137 \pm 0.001$$

$$\epsilon_{ch_post} = 0.1509 \pm 0.0010$$

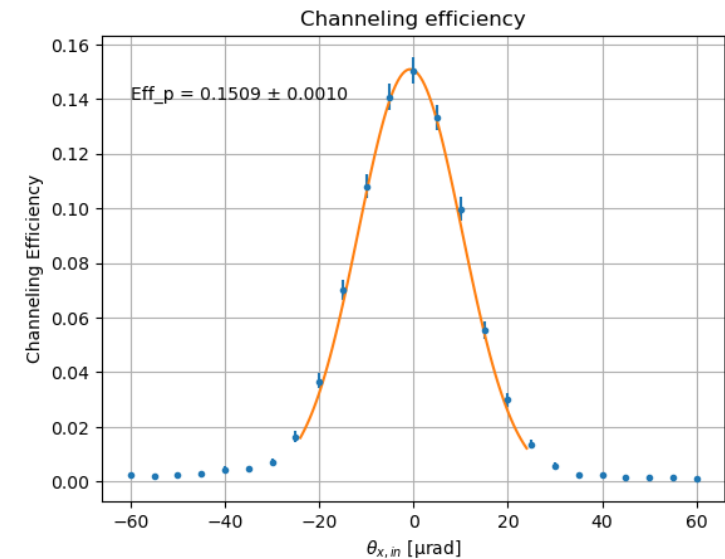
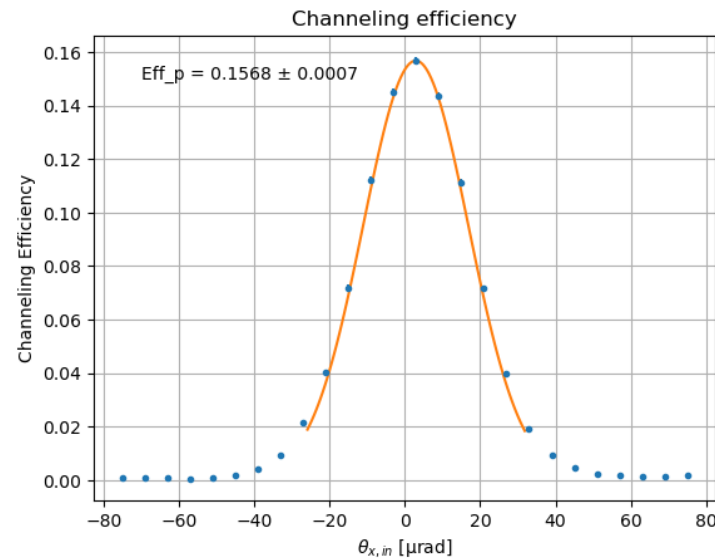


LONG CRYSTAL



SHORT CRYSTAL

ANODIC-BONDED

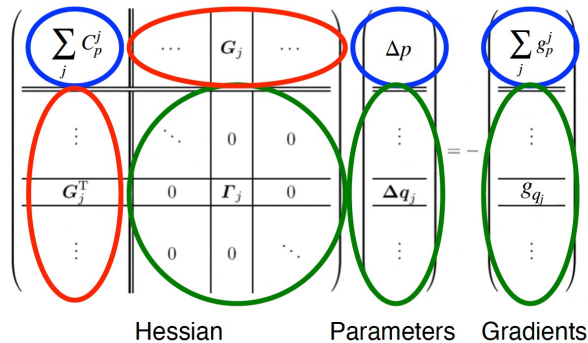


Alignment algorithm developed by A. Merli

- Optimisation problem: determine alignment constants that minimise $\chi^2 = \sum_i \frac{(x_i^{pred} - x_i)^2}{\sigma_i^2}$
- \vec{q} local parameters: parameters that describe the single event (tracks slope x/y, track coordinate x/y at 0)
- \vec{p} global parameters: parameters that describe the whole dataset (translation and rotation of sensors)
- Approximate the χ^2 around $\vec{x} = (\vec{p}, \vec{q})$

$$\chi^2(\vec{x} + \Delta\vec{x}) \approx \chi^2(x) + \underbrace{\left(\frac{\partial\chi^2}{\partial\Delta\vec{x}} \right)^T}_{g} \Delta\vec{x} + \frac{1}{2} (\Delta\vec{x})^T \underbrace{\left(\frac{\partial^2\chi^2}{\partial\Delta\vec{x}\partial\Delta\vec{x}} \right)}_H \Delta\vec{x}$$

- Solve $\frac{\partial\chi^2}{\partial\Delta\vec{x}}(\Delta\vec{x}^{best}) = 0$ with the Newton method $\rightarrow \Delta\vec{x}^{best} = -H^{-1}g$
- n_{q_j} local parameters for each track j : 4 x nTracks
- n_p global parameters: (2 translations + 1 rotation) x 3 planes
- The inverse of hessian (dimension = $(n_p + n_{q_j} \times nTracks)^2$) is computational very expensive
- Solution: Schur complement (https://en.wikipedia.org/wiki/Schur_complement) thanks to the particular structure of the hessian

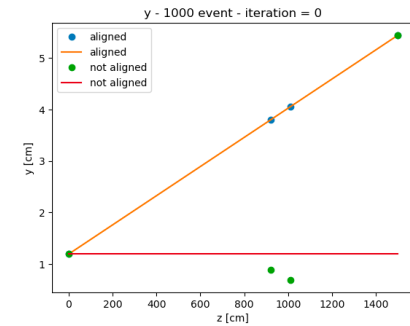
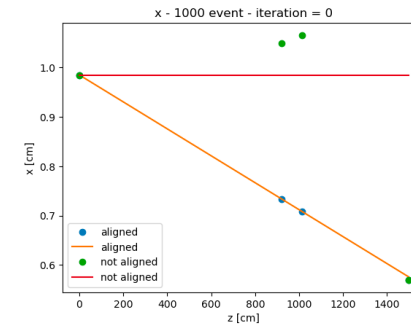


Related to global parameters
 Related to local parameters
 Related to global-local parameters

First station is fixed and z coordinate is fixed for all the planes

Alignment performed for all the crystals since the apparatus changed in between the data taking.

Rotations must be taken into account especially for the short crystal.



Schur complement

- The solution only use inverse of nTracks small matrices Γ_j (each with dimension $n_{q_j} \times n_{q_j}$) and $S = \sum_j C_p^j - \sum_j G_j \Gamma_j^{-1} G_j^T$ matrix (dimension $n_p \times n_p$) \rightarrow very fast!

$$H^{-1} = \begin{pmatrix} S^{-1} & -S^{-1}G_j\Gamma_j^{-1} \\ -\Gamma_j G_j^T S^{-1} & \Gamma_j^{-1} + \Gamma_j^{-1} G_j^T S^{-1} G_j \Gamma_j^{-1} \end{pmatrix}$$

FIRST ESTIMATION OF THE EFFICIENCY

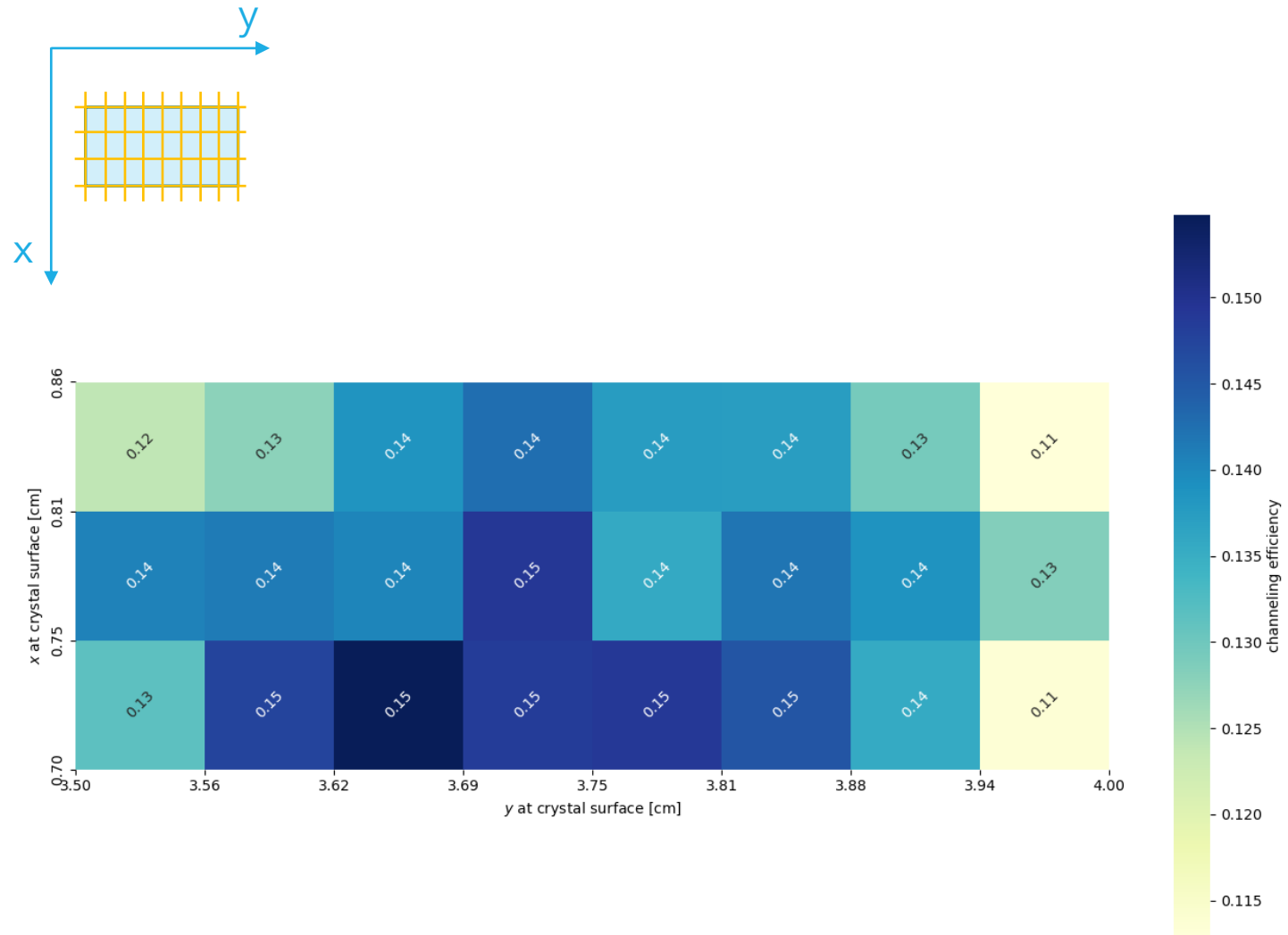
APPLIED CUTS:

- x position on the crystal [0.7, 0.86] cm
- y position on the crystal [3.5, 4.0] cm

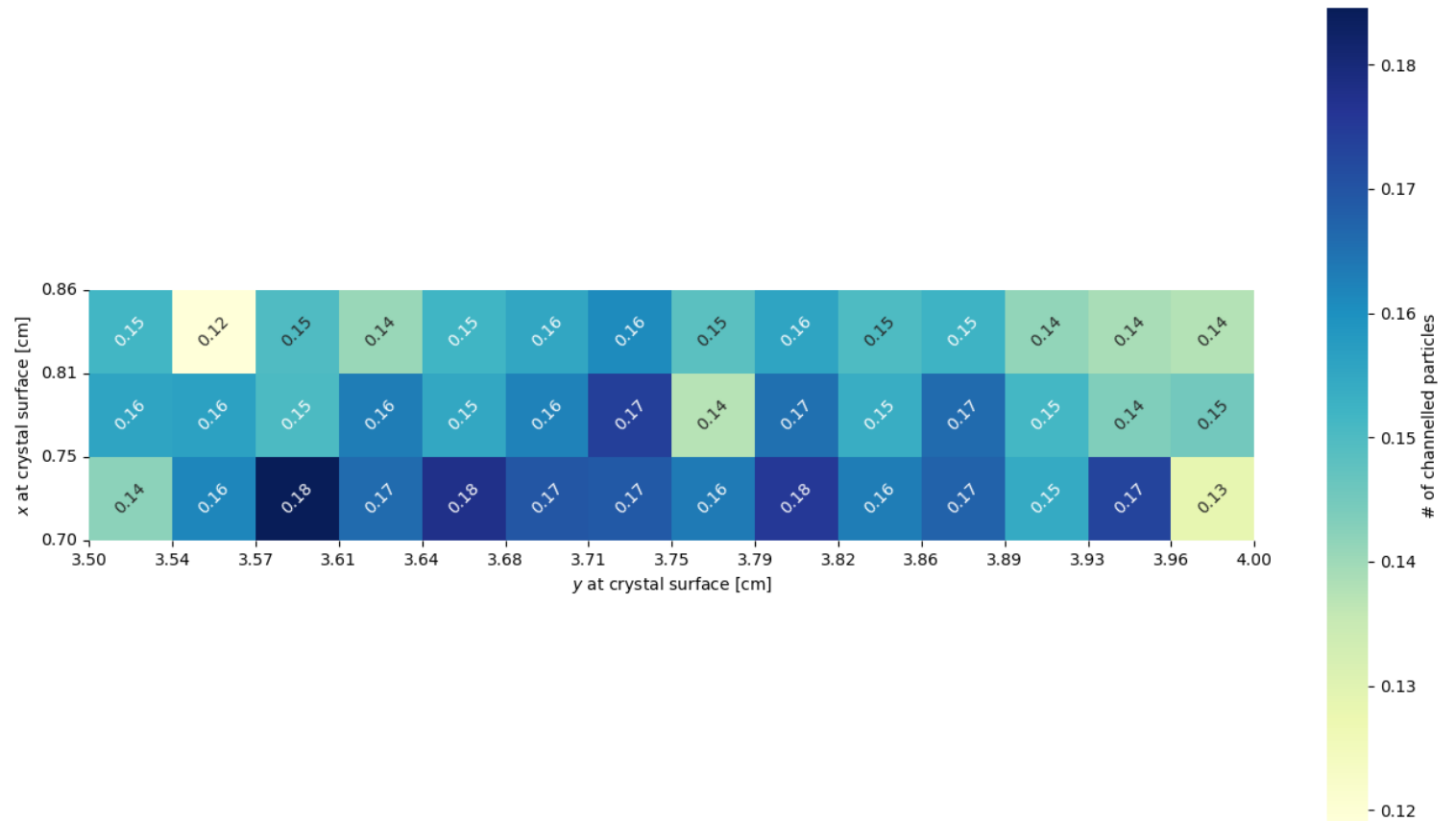
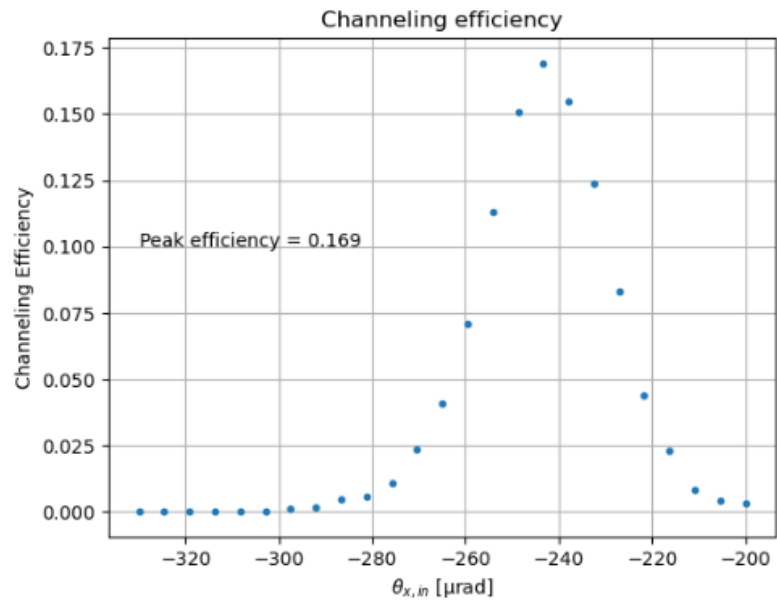
Scan in the hit position at the crystal with 8 bins in the y direction and 3 in the x direction.

For each step the efficiency was estimated as before within a $\pm \theta_L$ window.

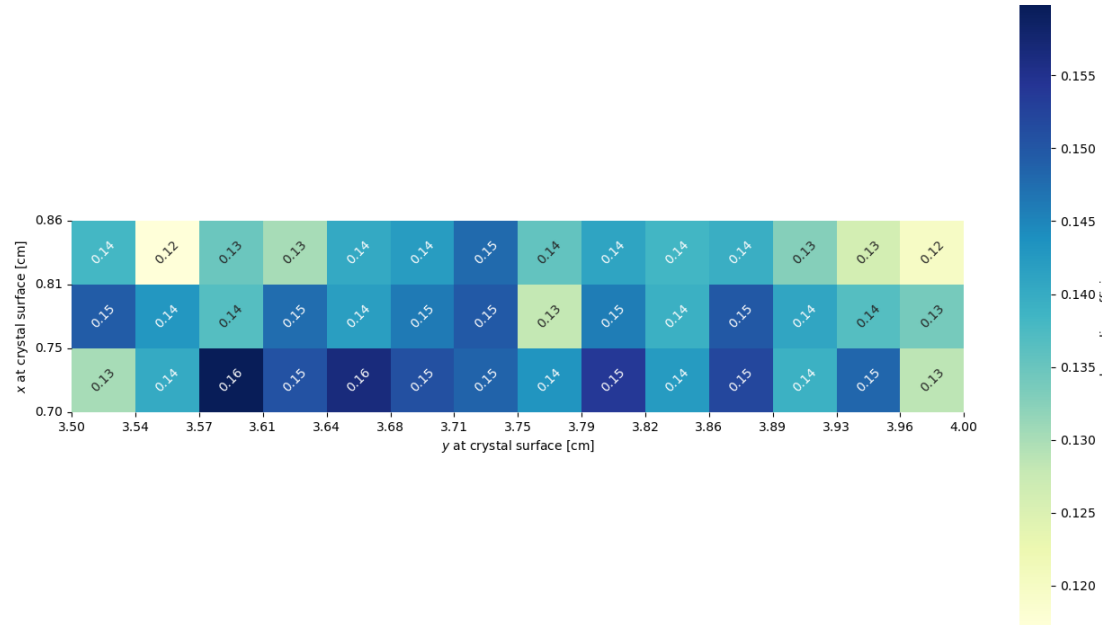
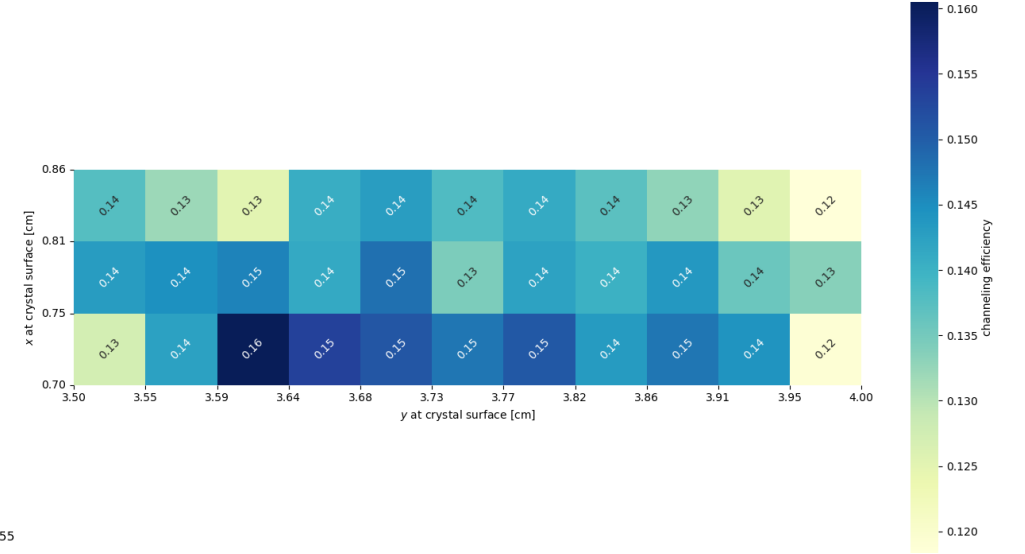
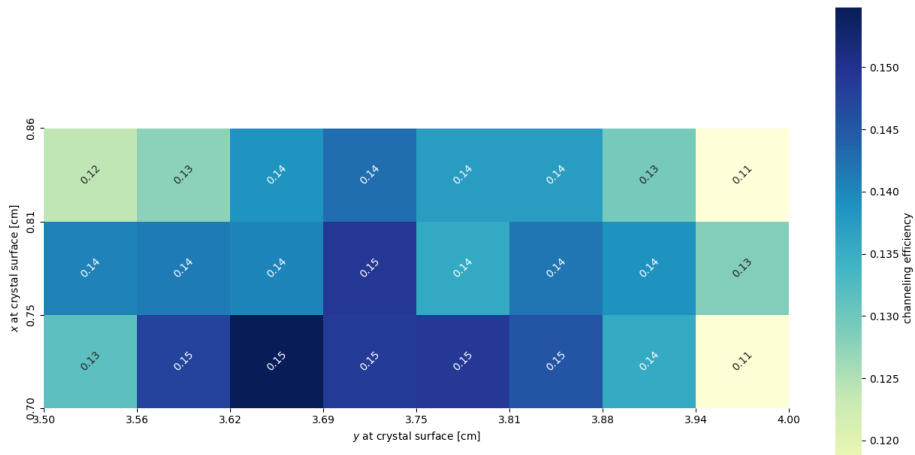
As expected the peak values is higher than before.



FIRST ESTIMATION OF THE EFFICIENCY



FIRST ESTIMATION OF THE EFFICIENCY



FIRST ESTIMATION OF THE EFFICIENCY

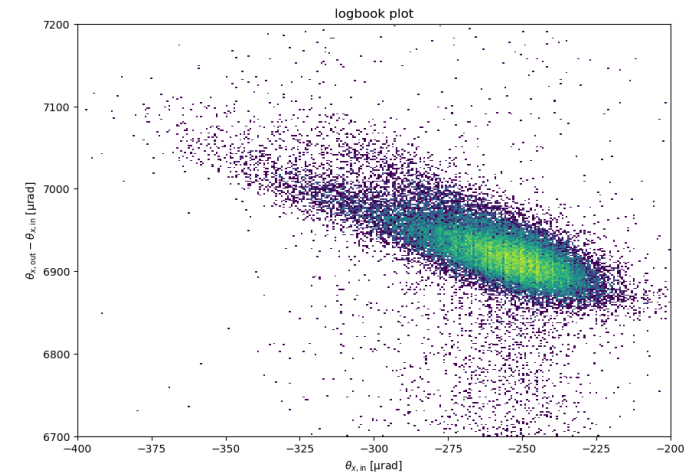
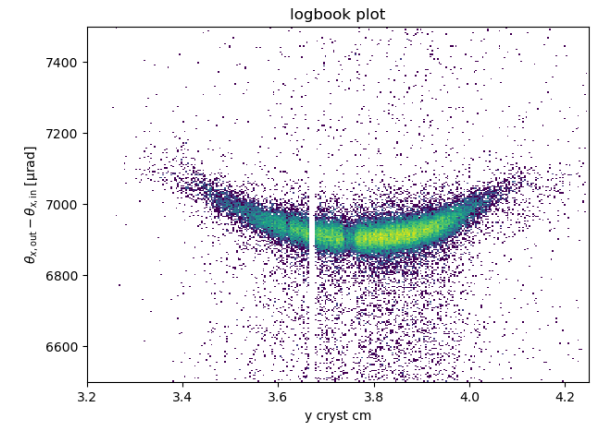
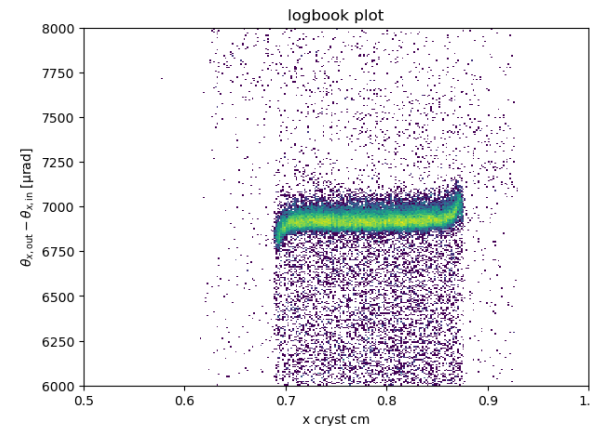
Crystal specifications:

- Dimensions: 2 mm x, 20 mm y, 70 mm z
- Deflection angle: 7 mrad

Lindhart Angle: $\theta_L(180\text{GeV}) = 12.9 \mu\text{rad}$

Selected run for alignment: 720454

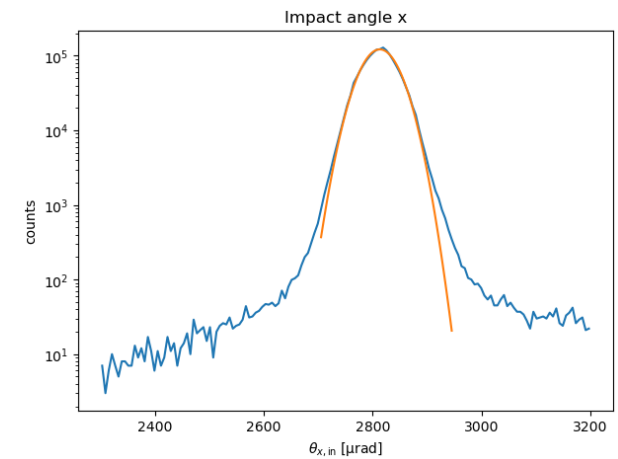
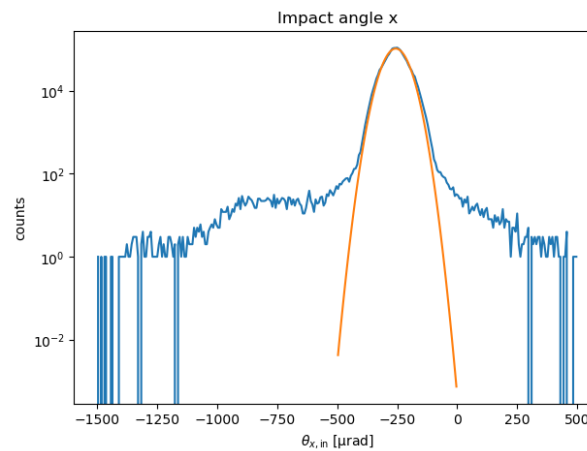
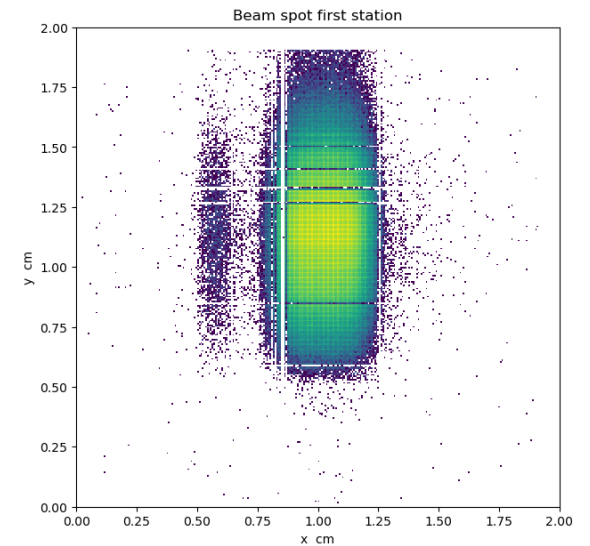
Selected run for channeling: 720462



FIRST ESTIMATION OF THE EFFICIENCY

From aligned data, run

```
momentum=180e3,  
angular_div_x=41.3e-6,  
angular_div_y=31.8e-6,  
section_x=5e-3,  
section_y=12e-3,  
mean_angle_x=-256e-6,  
mean_angle_y=2813e-6
```



FIRST ESTIMATION OF THE EFFICIENCY

Biryukov et al, Crystal Channeling and Its Application at High-Energy Accelerators, Springer 1997, page 16

Acceptance for a divergent beam: $\phi = 41 \mu\text{rad}$

$$A(pv/R) = \frac{2x_c}{d_p} \frac{\pi}{4} \frac{\theta_{c,0}}{\Phi} \left(1 - \frac{R_c}{R}\right)^2 = A_S \left(1 - \frac{R_c}{R}\right)^2, \quad (2.12)$$

Taking into account also the dechanneling effect

$$F(\Theta, pv/R) = A_S A_B \left(\frac{pv}{R}\right) \exp\left(-\frac{R\Theta}{L_D}\right). \quad (2.19)$$

$$F(\Theta, \rho) = A_S (1 - \rho)^2 \exp\left(-\frac{\Theta}{\Theta_D \rho (1 - \rho)^2}\right). \quad (2.20)$$

↓

$$\rho = \frac{R_c}{R}$$

Maximal deflection efficiency = 18.9%

$$\epsilon_{sim} = \epsilon_{max} * \sqrt{1 - \theta_{in}^2 / \theta_c^2}$$

